Univariate Time Series Prediction of Solar Power using a Hybrid Wavelet ARMA-NARX Prediction Method

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Abstract*—This paper proposes a new hybrid method for super short-term solar power prediction. Solar output power usually has a complex, nonstationary, and nonlinear characteristic due to intermittent and time varying behavior of solar radiance. In addition, solar power dynamics is fast and is inertia less. An accurate super short-time prediction is required to compensate for the fluctuations and reduce the impact of solar power penetration on the power system. The objective is to predict one step-ahead solar power generation based only on historical solar power time series data. The proposed method incorporates discrete wavelet transform (DWT), Auto-Regressive Moving Average (ARMA) models, and Recurrent Neural Networks (RNN), while the RNN architecture is based on Nonlinear Auto-Regressive models with eXogenous inputs (NARX). The wavelet transform is utilized to decompose the solar power time series into a set of richer-behaved forming series for prediction. ARMA model is employed as a linear predictor while NARX is used as a nonlinear pattern recognition tool to estimate and compensate the error of wavelet-ARMA prediction. The proposed method is applied to the data captured from UCLA solar PV panels and the results are compared with some of the common and most recent solar power prediction methods. The results validate the effectiveness of the proposed approach and show a considerable improvement in the prediction precision.

***Index Terms* –Univariate Time Series Prediction, Solar Power, Wavelet Transform, Auto-Regressive Moving Average (ARMA), Nonlinear Auto-Regressive Models with eXogenous Inputs (NARX), Neural Network (NN).**

I. Introduction

Extensive propagation of intermittent renewable energy sources to address the electricity power demand and the environmental concerns raises new challenges for power system operation due to the stochastic nature of these resources [1]. There is no doubt that renewable energy resources are essential to meet our current energy demand. Reliable prediction of renewable energy is essential for its efficient use as a part of our energy supply. Embedding renewable energy prediction techniques in grid operation procedure facilitates the massive integration of renewables. Real time control of renewables and compensating devices would be practical and more efficient if super short-time prediction is available. Reliable super short-term prediction methods may also improve the power quality and reliability of the power grid by enabling prompt compensation of negative consequences of renewable dynamics and fluctuations [2]. By evolving smart grids technology which expands hybrid renewables integration, the operation of distribution networks will also benefit from state-of-the-art renewables outputs prediction [3].

The necessity of renewables prediction from one side and its complexity from other side have motivated many researchers in this area to find a practical solution. Renewables power predication methods are mainly multivariate model based methods. These prediction methods are normally based on physical and atmospheric parameters which influence renewables generation such as temperature, irradiance, wind speed, relative humidity, etc., [4-5]. For solar power prediction which is the focus of this paper, multivariate model-based methods estimate the solar power, solar irradiance, cloudiness and clearness indices based on spatial, temporal and climatic conditions. Authors in [6] use measured air temperature, relative humidity and sunshine hours to predict daily global solar radiation. In [7], prediction inputs to generalized fuzzy model are the day of the year, sun shine hour, ambient temperature, relative humidity and atmospheric pressure. Assi et al. [8] use air temperature, wind speed, sun duration, and relative humidity as the inputs of an ANN to predict solar radiation. Aerosol index which indicates the particulate matter in the atmosphere is proposed besides the other meteorological data to get more accurate forecasting results [9]. Mathematically, different statistical models are adopted in these prediction methods. Among the linear models, Auto-Regressive (AR) and Auto-Regressive Moving Average (ARMA) are commonly used for wind and solar power prediction [10], [11]. However, linear time series models are addressed notably in the literature, behavior of renewable power generation series may not be entirely captured by the linear techniques due to time varying and inherent nonlinear characteristic of renewables generation [12]. Thus, nonlinear methods have been proposed by many scientists. Takagi Sugeno (TS) fuzzy model [13], wavelet-based methods [14], neural networks (NNs) [15], support vector machine [12], echo state network [16], k-nearest neighbors (kNN) [17-18], and adaptive neural fuzzy inference systems (ANFIS) [19], are some of the examples of the prediction methods. Although the superiority of nonlinear statistical models over linear models has been shown before, researchers are still not satisfied with the prediction precision and generalization capability of the existing advanced methods [20].

Intermittent characteristics of generated power by solar, outliers, and inconsistent mean and variance, turn the solar

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prediction problem into a complicated problem. To solve this problem, a variety of complex solutions with multivariate inputs are applied so far. However, for super short–time prediction horizons, the cost of acquiring and maintaining a weather station which is able to collect and store solar radiation, sun duration, temperature, wind speed, and relative humidity data is exorbitant. Satellite imagery and numerical weather prediction are also restricted [21]. Moreover, the effectiveness of employing individual meteorological variable has not been studied yet [22]. The focus of this paper is on a univariate model based method which is economic and realistic for super short-time prediction. In this method a prediction model is a function of the current or past values of the power time series but not of any other time series such as temperature, wind speed, and relative humidity.

In this paper, a new univariate hybrid strategy is presented. This method uses linear and nonlinear models including discrete wavelet transform (DWT), Auto-Regressive Moving Average (ARMA) models and Nonlinear Auto-Regressive models with eXogenous inputs (NARX) architecture of Recurrent Neural Networks (RNN) to predict solar power. The wavelet transform is used to decompose the historical power series into the better-behaved series for prediction. ARMA model is applied as a linear predictor and NARX is utilized as a nonlinear pattern recognition tool to estimate and compensate the error of wavelet-ARMA prediction. The proposed method is applied to the UCLA solar power dataset for nearly two hundred thousand observations, sampled at a rate of once per minute. This dataset is large enough to implement the proposed prediction algorithms.

The rest of the paper is organized as follows. In Section II, the principles of the proposed algorithm are described. Then, simulation results obtained from applying the proposed algorithm on real solar data are presented in Sections III. Finally, concluding remarks are in Sections IV.

II. METHODOLOGY

The majority of the prediction methods require a wide range of input data (multivariate). The drawback of these approaches is that the limited availability of input data causes loss in accuracy. In addition, inaccuracy in input data propagates to the output and those errors are accumulated. There is a need for prediction methods that are reasonably accurate and work with limited input data. In the prediction literature it is well-known that a single method cannot provide the best solution for different situation. Several models are incorporated in practice. Both experimental and theoretical analysis show that combining different methods can increase prediction accuracy [23]. In this paper a RNN is integrated with an ARMA model and the wavelet method is employed to decompose the input data to a series with better feature for prediction. The proposed hybrid method includes the following prediction stages:

Stage 1) Decomposition of historical power series into the constitutive series through the wavelet transforms.

Stage 2) Modeling each of the constitutive series by ARMA model for recognition of their linear pattern.

Stage 3) Recomposition of estimated series obtained from ARMA model for each constitutive series through the inverse wavelet transform.

Stage 4) Compensating the error of the wavelet-ARMA model prediction by recurrent neural networks through capturing the nonlinear patterns buried in residual part.

A. Wavelet Transform

When the time series like solar power series is non-stationary [24], the identification of an appropriate global model is very difficult if not impossible [25]. In order to cope with this challenge, wavelet transform technique has been introduced as an effective way for better analysis and prediction in the preprocessing step. The Wavelet transform (WT) is a time–frequency decomposition that provides useful basis of time series in both time and frequency. This approach allows the collected measurements to be smoothed until some hidden trend is identified. This capability is performed by diagnostic the main frequency component from the time series and mining abstract local information. Recently, WT has been frequently utilized for analysis and forecasting time series [26], [27].

In discrete wavelet series the so-called “approximated series” (A_i) and the “detail series” (D_i) are calculated as [28]:

$$A_{i+1}(k) = \sum_{l} H(l).A_i(2.k+l) \tag{1}$$

$$D_{i+1}(k) = \sum_{l} G(l).A_i(2.k+l) \tag{2}$$

where $i=0,1,2,\ldots..M$ are the wavelet decomposition levels. The functions G and H are the high-pass and low-pass filters' finite impulse responses, respectively. These two filters are related to each other and they are known as quadrature mirror filter. A_0, is equal to the original finite-length sequence $S(n)$. Figure 1 depicts the decomposition structure of wavelet transforms for level 3.

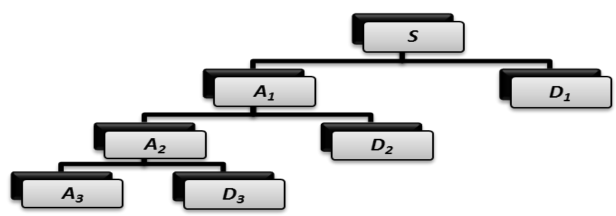

Figure 1. Wavelet decomposition tree at level 3

The inverse discrete wavelet transform can be obtained from equation (3).

$$A_i(k) = \sum_{l} H(l).A_{i+1}(2.k+l) + \sum_{l} G(l).D_{i+1}(2.k+l) \tag{3}$$

In this transformation the data series is broken down by the transformation into its wavelets that are scaled and shifted version of the mother wavelet which is the main function of transformation. Figure 2 depicts the signal and corresponding wavelet including low scale and high scale concept of the signal decomposition. Wavelet transform projects a signal into different scale components while multiplying each wavelet coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelets of the original signal.

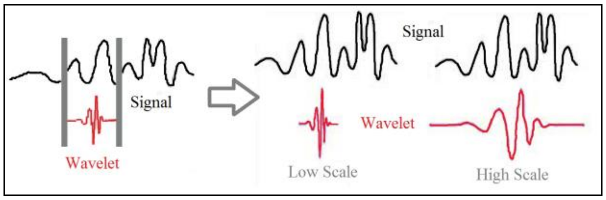

Figure 2. Signal and wavelet representation

B. ARMA

Autoregressive and Moving Average (ARMA) model is capable to extract useful statistical properties of the data, it is simple, cost effectiveness and it incorporates of the well-known Box–Jenkins methodology [24]. The ARMA model is a stochastic process coupling autoregressive component (AR) to a moving average component (MA). The AR component computes the summation by adding past samples, while the MA component integrates a random process representing noise in the data. This kind of model is called ARMA *(p, q)* and is expressed with *p* and *q* parameter as:

$$x(t) - \sum_{k=1}^{p} \varphi_t(k) x(t-k) = \varepsilon_t + \sum_{i=1}^{q} \theta_t(i) \varepsilon(t-i) \tag{4}$$

where, x_t is a time series, ε is an error term distributed as a Gaussian white noise, and φ_t and θ_t are the parameters of the autoregressive and moving average part respectively.

C. RNN-NARX

There are two major types of neural networks, feed-forward and recurrent. Unlike feed-forward neural networks, RNNs can use their internal memory to process arbitrary sequences of inputs. In this paper, in order to deal with nonlinearity of time series the proposed RNN architecture used here is one based on Nonlinear Autoregressive models with eXogenous input (NARX model). Since ARMA models are linear, here NARX is employed to model the nonlinear part of the time series which is not parametric.

As opposed to other recurrent network architecture, NARX networks have a limited feedback and it comes from only the output neuron rather than from hidden states. NARX models are formalized by [29]:

$$y(t) = f(u(t-n_u), ..., u(t-1), u(t), y(t-n_y), ..., y(t-1)) \tag{5}$$

where function *f* is a nonlinear function, *u(t)* and *y(t)* represent input and output of the network respectively at time *t*, n_u and n_y are order of input and output. Figure 3 shows the dynamic structure of NARX.

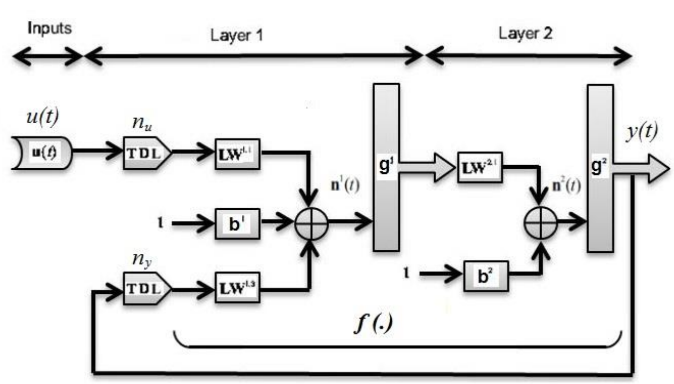

Figure 3. NARX dynamic architecture of two layers

D. Hybrid Algorithm

With the aim of improving the prediction accuracy this paper proposes the idea of considering the time series as the composition of a linear stationary and autocorrelation component (S_L), and a nonlinear structure (S_N).

$$S(t) = S_L(t) + S_N(t) \tag{6}$$

Thus the linear (S_L) and nonlinear (S_N) components have to be predicted according to the current and past data separately. The idea is that the decomposed time series by wavelet, first is modeled through ARMA. That is the wavelet-ARMA method models the linear component of the time-series. Then referring to (6), the residual of the wavelet-ARMA model (linear model) contains only the nonlinear part of time series. That means after estimation of predicted value of linear component from time series ($\hat{S}_L$), the error of prediction (e_t) is calculated from (7).

$$e(t) = S(t) - \hat{S}_L(t) \tag{7}$$

The next step is the residual analysis for mining nonlinear pattern in the error. Extracting any meaningful nonlinear pattern in the error compensates for the wavelet-ARMA limitation and leads to improvement of the prediction algorithm. In this work the NARX is proposed to capture the nonlinearity in the error. Therefore the NARX model of error at each time can be expressed by a nonlinear function of past errors as a feedback target and power times series changes as non-feedback inputs.

$$e(t) = f(S(t-n_u), ..., S(t-1), S(t), e(t-n_y), ..., e(t-1)) + \varepsilon(t) \tag{8}$$

where function *f(.)* is a nonlinear function defined by NARX and *ε(t)* is the random remaining error, *S(t)* is power time series and *e(t)* is wavelet-ARMA error. In fact the *f(.)* is modeling the prediction of nonlinear component of time series ($\hat{S}_N$). Therefore, the proposed method is benefiting from the privilege of wavelet-ARMA model for extracting linear feature of data, and it is taking advantage from NARX model for exploring nonlinear characteristic of the error. The flowchart of the proposed algorithm is shown in Figure 4.

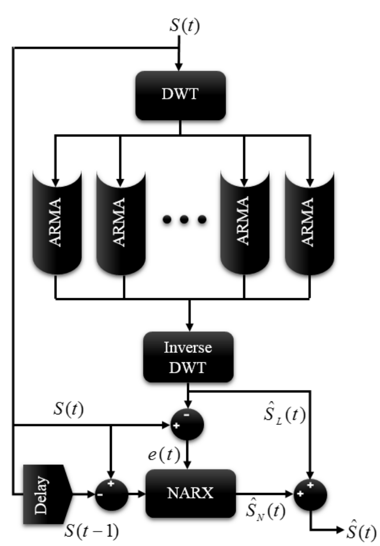

Figure 4. Proposed hybrid prediction algorithm

E. Prediction Accuracy

In order to evaluate the prediction accuracy of the proposed method and assess its prediction capability the mean absolute normalized error (MAE), mean squared normalized error (MSE), and maximum of normalized error (MAXE) are introduced as the performance indices. Normalized error is calculated from (9).

$$E_{norm} = \left(S(t) - \hat{S}(t)\right) / \bar{S}(t) \tag{9}$$

where $\bar{S}(t)$ is the mean value of $S(t)$.

III. Simulation Results

The solar power data captured every minute from the solar panels at UCLA Student Union is considered as the case study for the proposed prediction method. The algorithm is applied to two different days one is cloudy and one sunny. The proposed hybrid method is compared with some common prediction algorithms.

The implemented prediction method is developed based on flow chart in Figure 4. It uses a wavelet function of type Daubechies, order 20 and 3 level of decomposition. Figure 5 shows three level wavelet decomposition of solar power time series in a cloudy day. The ARMA model orders are p=8, and q=8. Finally NARX includes 45 hidden layers and one output layers. The following parameters are tuned by running several experiments to reach the acceptable accuracy. There exist different systematic approaches for designing the NARX such as particle swarm optimization but that is outside of scope of this work.

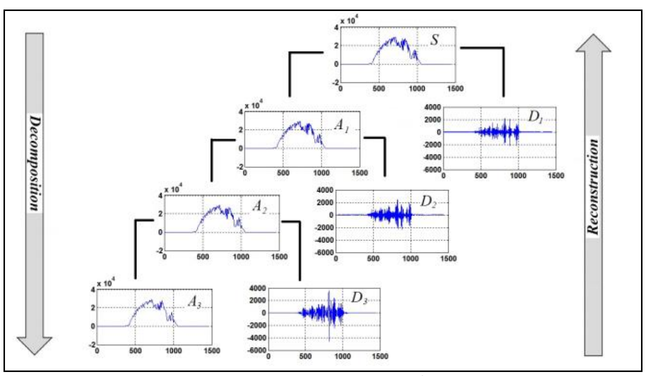

Figure 5. Three level wavelet decomposition of solar power time series

Figure 5 shows the decomposition of the signal achieved using wavelet transformation. The level three A-series (D-series) data has mostly low frequency (high frequency) content that can be used for accurate prediction.

Figure 6 depicts the solar power captured in a sunny and a cloudy day. It can be seen that power fluctuates is more in cloudy day compared to sunny day and the level of maximum power is less.

Figure 7 shows the actual solar power and predicted curves for a sunny day and Figure 8 plots the percentage of prediction error over the maximum capacity of solar. Figures 9, 10 also show the solar power prediction and prediction error percentage for a cloudy day. Figures 8 and 10 compare the error of wavelet-ARMA approach and wavelet-ARMA-NARX to show the improvement provided by employing the neural network for nonlinear pattern recognition of residuals.

The correlation analysis depicts that the correlation of the NARX final error at each point is less than 0.1 with previous data which reflects no correlation between data to improve the prediction.

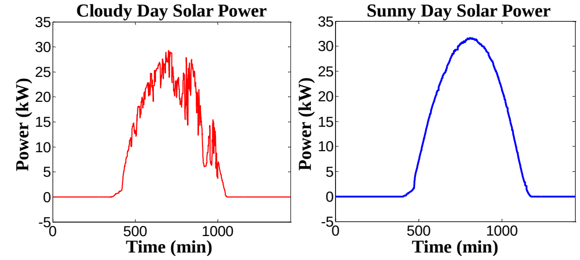

Figure 6. Solar power in a sunny and a cloudy day

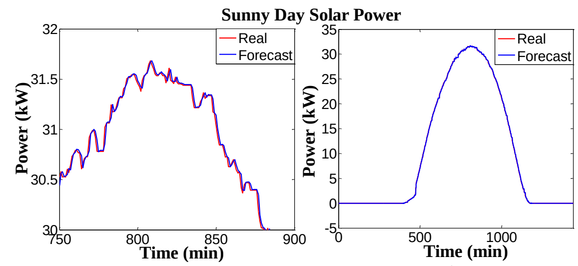

Figure 7. Actual solar power and predicted curves for a sunny day

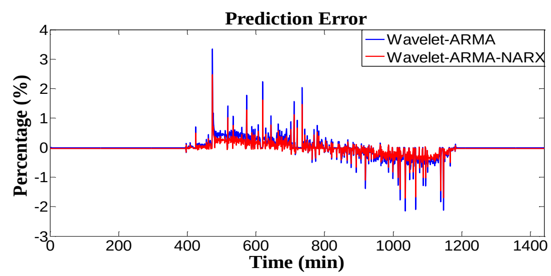

Figure 8. Prediction error for sunny day

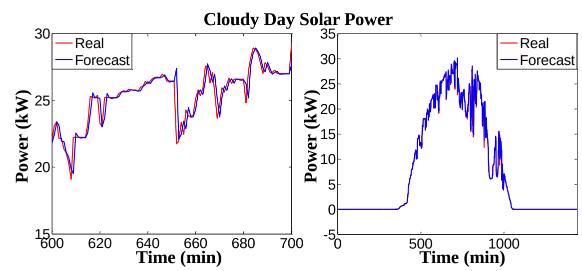

Figure 9. Actual solar power and predicted curves for a cloudy day

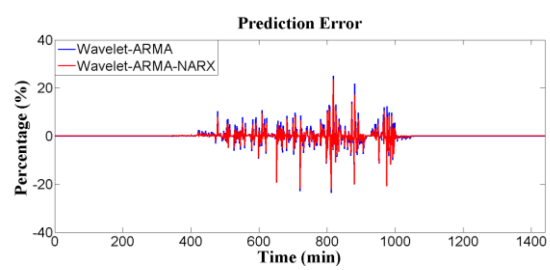

Figure 10. Prediction error for cloudy day

Tables I and II depict the comparison results between the performance of the proposed hybrid approach and some other approaches for a sunny and a cloudy day. The comparison is based on calculated error through MAE, MSE and MAXE criteria. The obtained results validate the effectiveness and better performance of wavelet-ARMA-NARX compared with ARMA, wavelet-ARMA, and NN.

TABLE I COMPARISON OF DIFFERENT PREDICTION METHOD FOR SUNNY DAY

	MAE	MSE	MAXE
ARMA	0.9303	0.0251	0.1087
Wavelet-ARMA	0.4703	0.0095	0.1061
NN	1.0329	0.0247	0.1321
Wavelet-ARMA-NRX	0.3668	0.0050	0.0783

TABLE II COMPARISON OF DIFFERENT PREDICTION METHOD FOR CLOUDY DAY

	MAE	MSE	MAXE
ARMA	4.7791	1.1514	0.9887
Wavelet-ARMA	3.7517	1.0289	0.9907
NN	4.8410	1.0687	1.0088
Wavelet-ARMA-NRX	3.3742	0.8971	0.9603

IV. Conclusions

In this paper, a new univariate hybrid method is developed to predict the solar output power and model the intermittent characteristic of solar power generation. This method incorporates wavelet, ARMA and RNN to predict one minute ahead solar power. The wavelet algorithm is used to decompose the time series to better behaved components for prediction, ARMA is employed for modeling the linear features of the data, and finally NARX has been included to compensate the error of wavelet-ARMA and capture the nonlinear pattern in the residual. One of the attractions of this method is that it requires only the historical output solar power time series. This method is simple, affordable, and practical for super short-time prediction. The numerical results validate the accuracy and effectiveness of this method.

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