SAND2015-9448PE

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Quantum Error Correcting Code Simulation

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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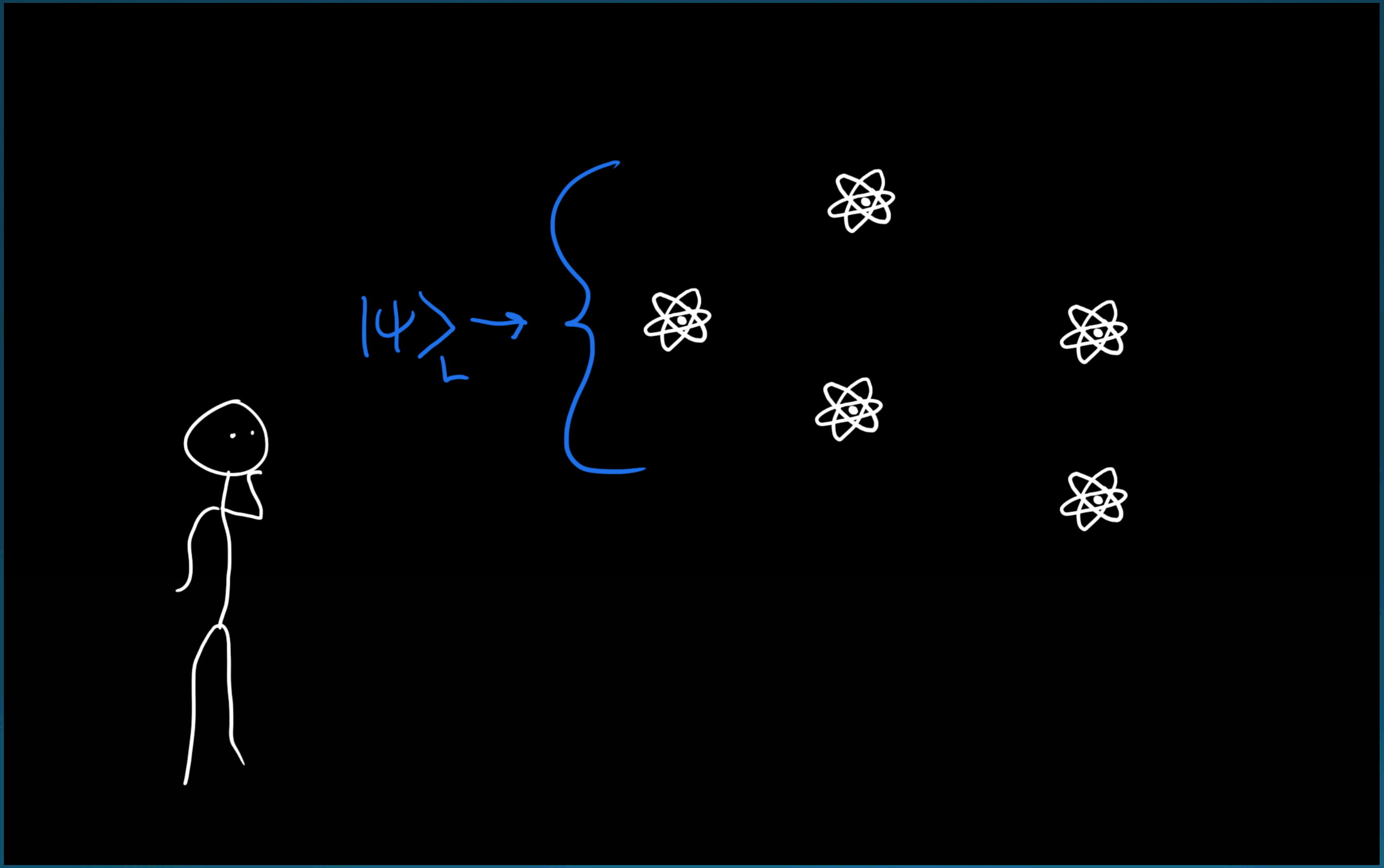
The Wish



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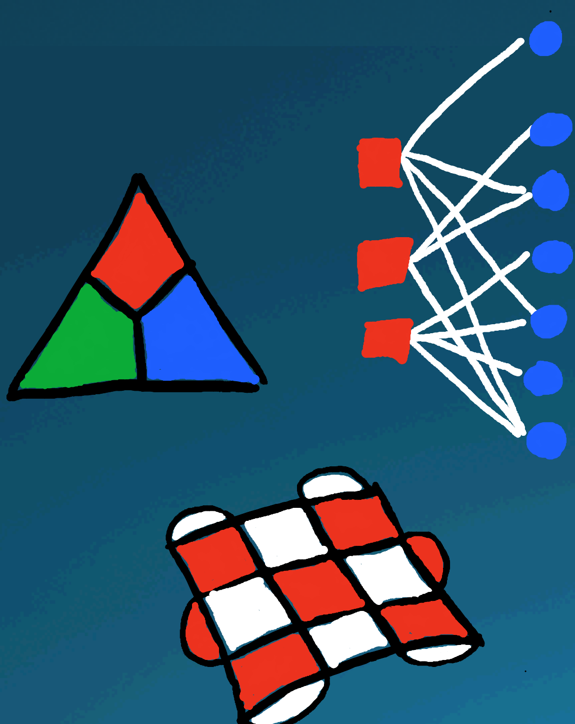
Motivation

- Simulate practical quantum error correcting codes (QECCs) against more realistic noise



PECOS

- Logic checker
- Distance checker
- Threshold calculation
  - Any QECCs with Clifford circuits and Pauli measurements
  - Flexible Error Model
  - Decoders

What type of QECCs?

- Spatially-local, parity measurements
- Embeddable on a plane
- Efficient determination of recovery operations (decoding)
- High error tolerance

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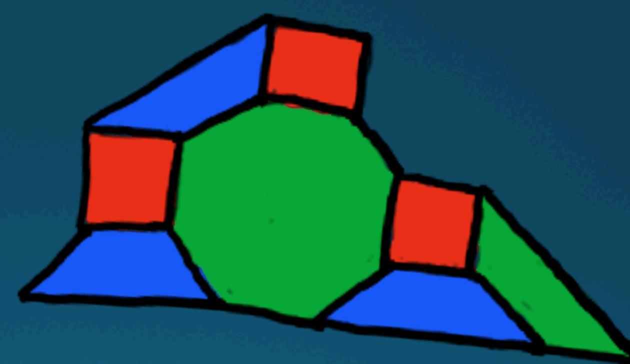
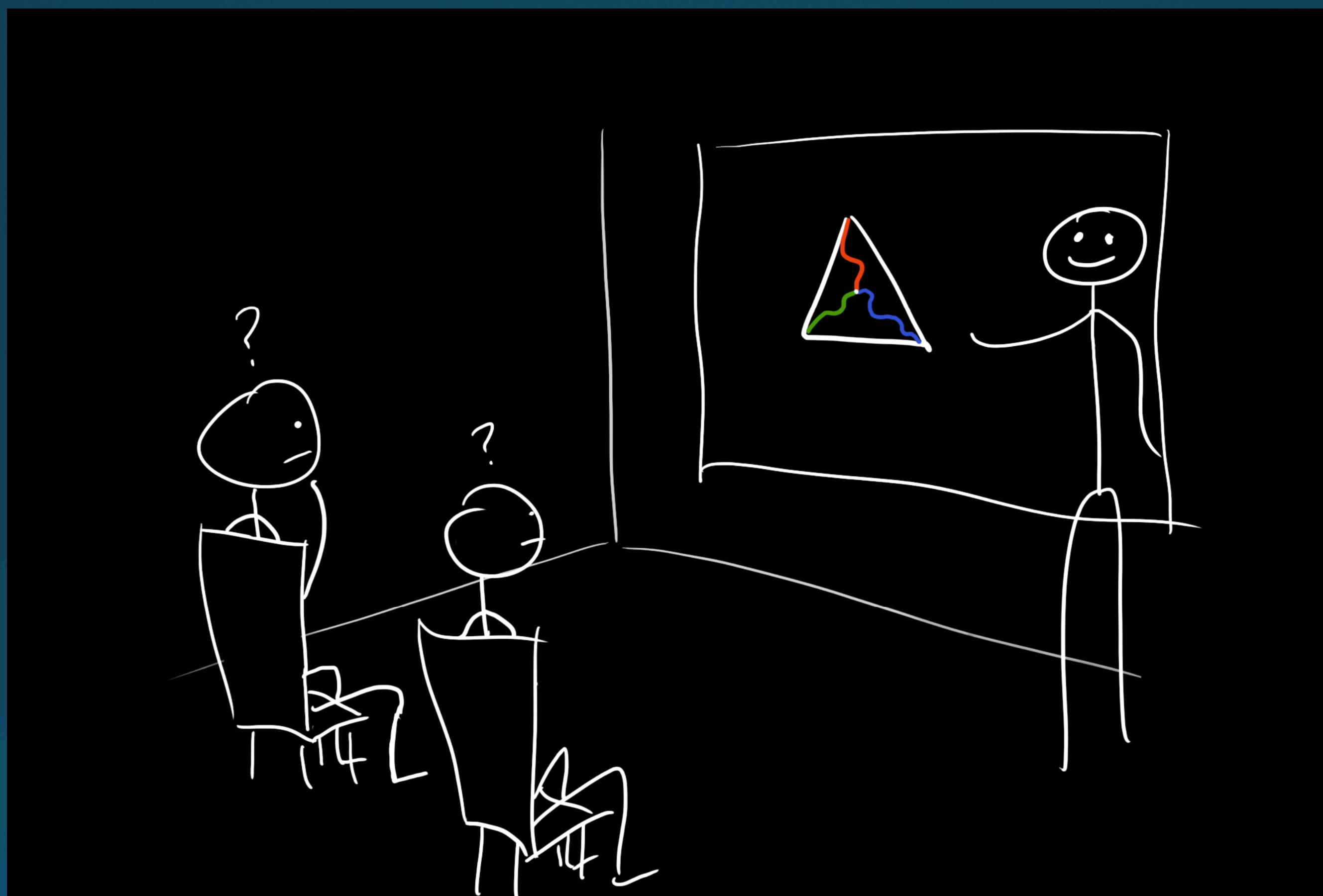
Topological Stabilizer Codes

Surface Code



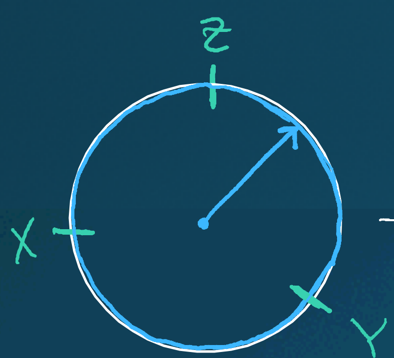
Color Code

No Strings Attached

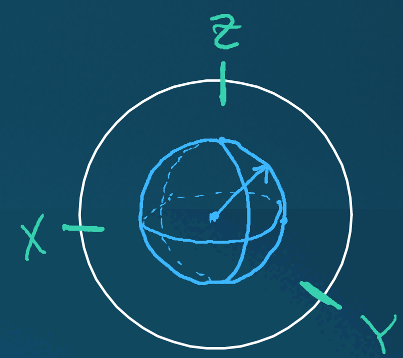


Standard Error Model

- Symmetric depolarizing channel

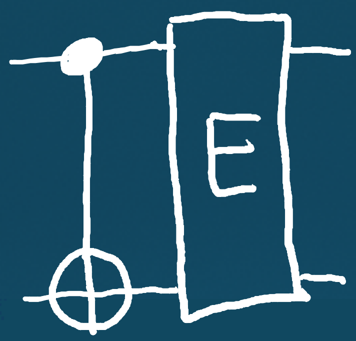


$$\varepsilon(\rho) = (1-p)\,\rho + p\,\frac{I}{2}$$



$$\varepsilon(\rho) = (1-p')\,\rho + \frac{p'}{3}(X\rho X + Y\rho Y + Z\rho Z)$$

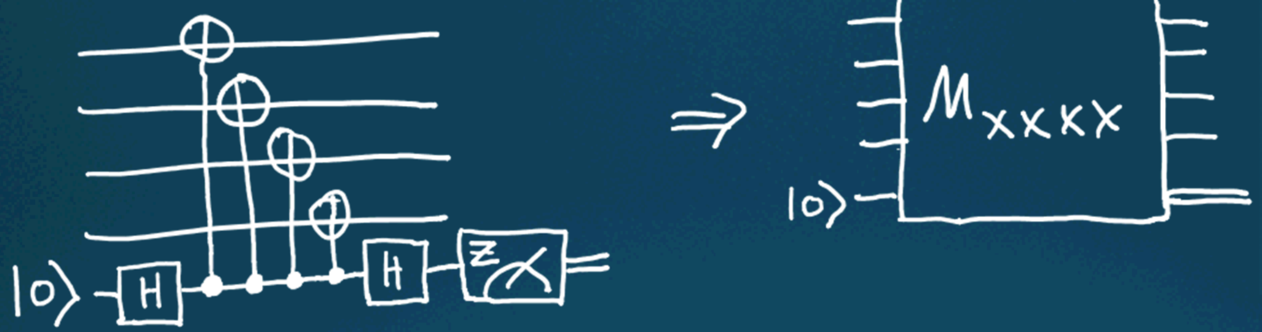
Two Qubit Operations

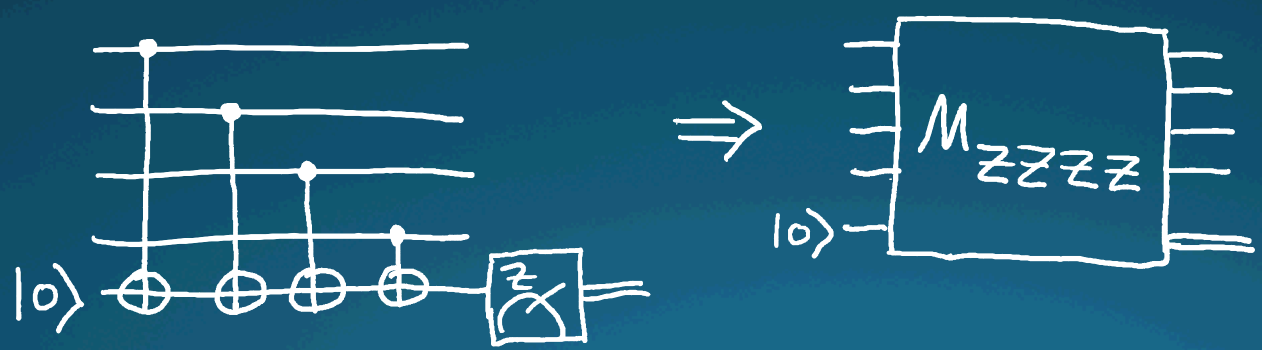


$$1 - p \quad \longrightarrow \quad II$$

$$p \quad \longrightarrow \quad \{IX, IY, IZ, XI, \ldots, ZZ\}$$

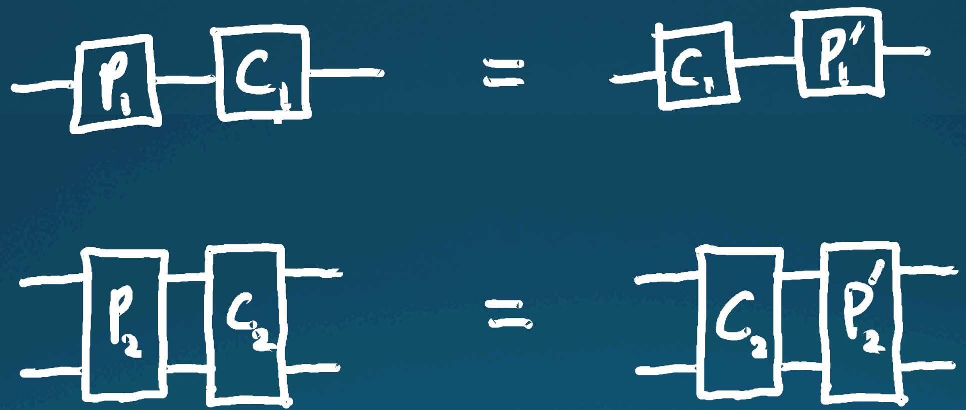
Parity Measurements





Simulation of Standard Choice

- Cliffords allow Paulis to be pushed through



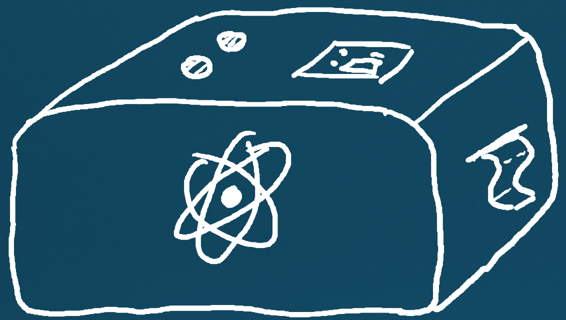
- Logical operations are transversal Paulis, thus logical errors can be determined.

Beyond Depolarizing noise

- Rotation
- Deform principle axes
- Displacement

ρ ρ'

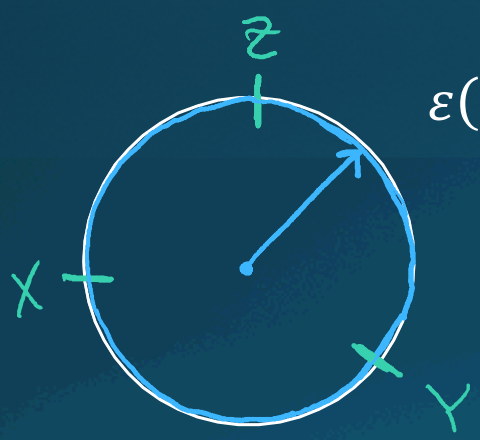
Decoherence



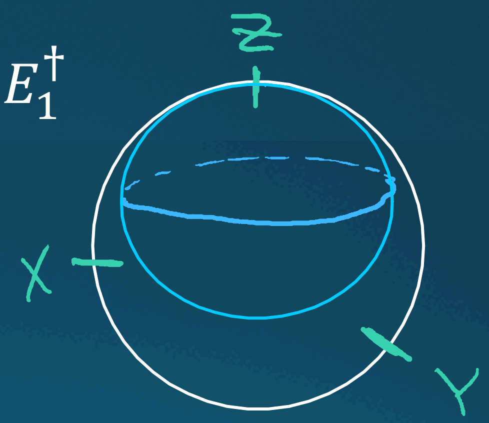
- T1 error - Amplitude Damping
- T2 error - Phase Damping

Amplitude Damping

- Energy dissipation



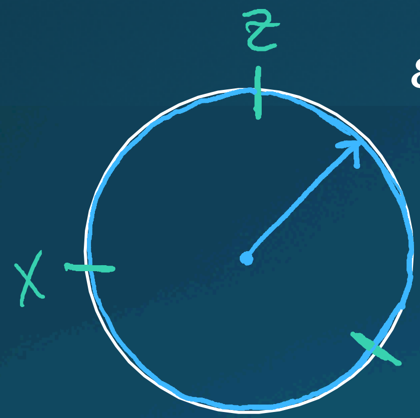
$$\varepsilon(\rho) = E_0\, \rho\, E_0^\dagger + E_1\, \rho\, E_1^\dagger$$



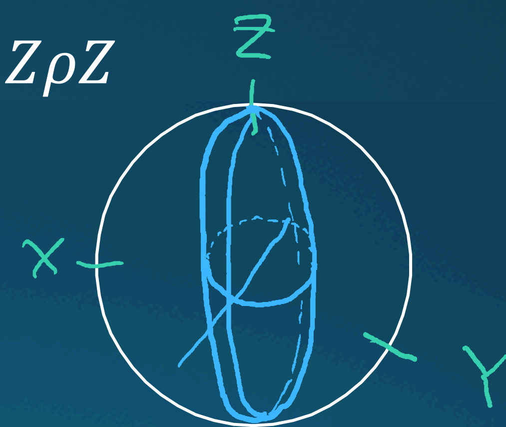
$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix} \qquad E_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}$$

Phase Damping

- Loss of information without loss of energy

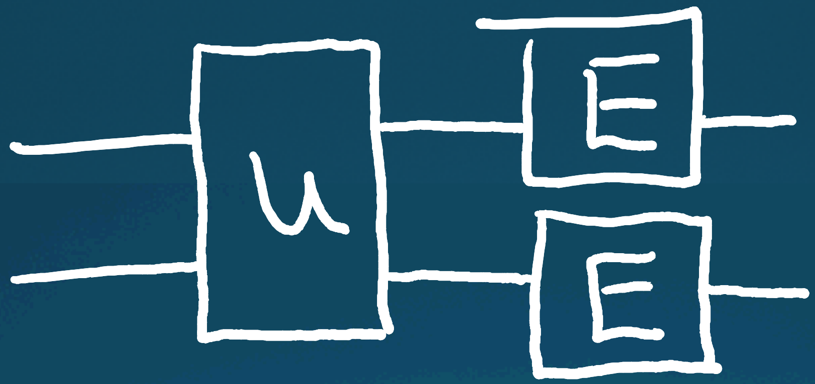


$$\varepsilon(\rho) = (1 - p)\,\rho + p\, Z\rho Z$$



$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{bmatrix} \qquad E_1 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{\lambda} \end{bmatrix}$$

Two Qubit Errors



Approximation

- Pauli Twirling:

$$\varepsilon_{PT}(\rho) = \frac{1}{4} \sum_{P_i \in \mathcal{P}} P_i\, \varepsilon(P_i\, \rho\, P_i)\, P_i$$

- Asymmetric depolarizing channel:

$$\varepsilon(\rho) = (1 - p_X - p_Y - p_Z)\, \rho + p_X\, X\rho X + p_Y\, Y\rho Y + p_Z\, Z\rho Z$$

$$p_X = p_Y = \frac{\gamma}{4} \qquad p_Z = \frac{1 - \sqrt{(1-\gamma)(1-\lambda)}}{2} - \frac{\gamma}{4}$$

[1] Tomita, Svore, arXiv:1404.3747 [2] Ghosh, Fowler, Geller arXiv:1210.5799

Better Modeling

- Gottesman-Knill theorem [1]:
  - Circuits of Clifford gates can be simulated in $O(n^3)$ steps
- Gottesman-Aaronson [2]
  - $O(n^2)$ steps can be used

[1] Gottesman arXiv:quant-ph/9705052
[2] Aaronson,Gottesman arXiv:quant-ph/0406196

Stabilizer Simulation

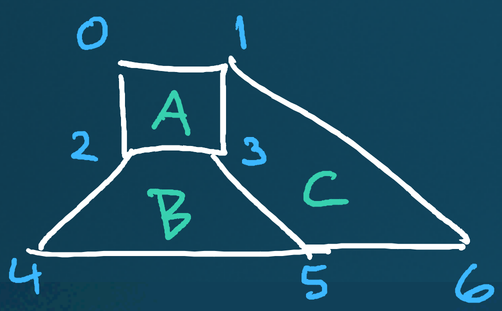
$$S|\Psi\rangle = +|\Psi\rangle$$

$$U|\Psi\rangle = US|\Psi\rangle = USU^{\dagger}\ U|\Psi\rangle$$

$\boldsymbol{M_S}$**:** Projects onto subspace of $+S$ or $-S$

$$\frac{I \pm S}{2}$$

Improve By Using Sparsity



$$\begin{array}{c c}
 & \begin{array}{cccccccccccccccc} & X_0 & X_1 & X_2 & X_3 & & & & Z_0 & Z_1 & Z_2 & Z_3 & & & \end{array} \\
\text{Checks} \left\{ \right. & \left[\begin{array}{c|ccccccc|ccccccc}
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1
\end{array}\right]
\end{array}$$

$\oplus$ signs

$Y: \; X \to -X$
$\quad\; Z \to -Z$
$\quad\; Y \to Y$

$$s' = s \oplus X \oplus Z$$

zrow[(0, 1, 2, 3), ( ), (2, 3, 4, 5), ( ), (1, 3, 5, 6), ( )]

Likewise for X...

zcolumn[(1), (1, 5), (1, 3), (1, 3, 5), (3), (3, 5), (5)]

Likewise for X...

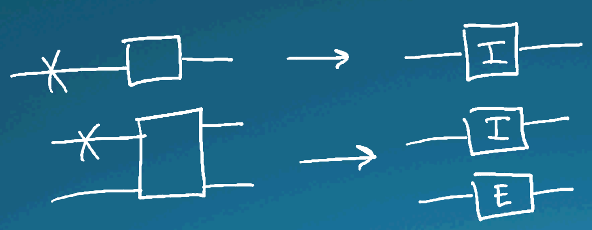
What Does this Buy Us?

- More accurate approximations
  - Clifford noise (24 C_1 and 11,520 C_2 elements)
  - Measurement-like errors
    - $E_0 = \sqrt{p}|0\rangle\langle 0| \quad E_1 = \sqrt{p}|0\rangle\langle 1|$
    - Measurement followed by conditioned gate
  - Allows amplitude damping to be modelled more accurately
- Other errors
  - Replace gates with other Cliffords

[1] Magesan et al. arXiv:1206.5407, [2] Puzzuoli, et al. arXiv:1309.4717
[3] Gutiérrez et al. arXiv:1207.0046

Other Errors

- Unitary rotation (e.g. global magnetic field)
- Bursts
- Non-local two qubit error
- Other space and time correlations
- Non-uniform error model
- Leakage



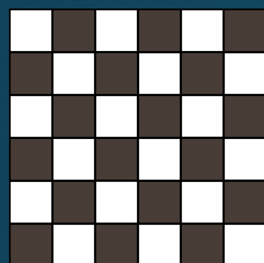
Experiment-Informed Error

- Tomography and state simulation
  - Process matrix
  - Cliffords and other simulatable errors

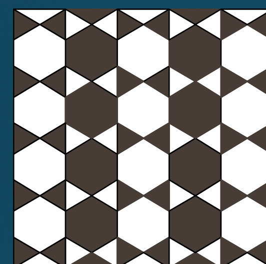
    $\Rightarrow$Approximate more accurately
    new (more realistic) error models

Other Lattices

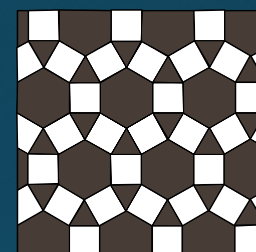
Surface codes



4.4.4.4

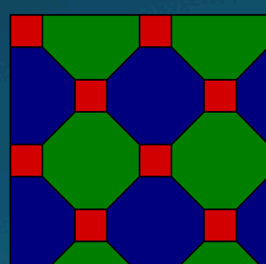


3.6.3.6

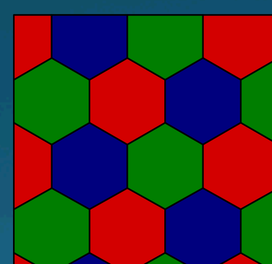


3.4.6.4

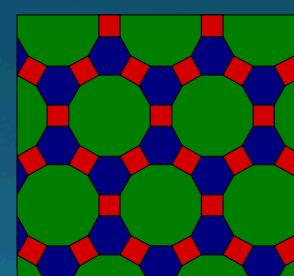
Color codes



4.8.8



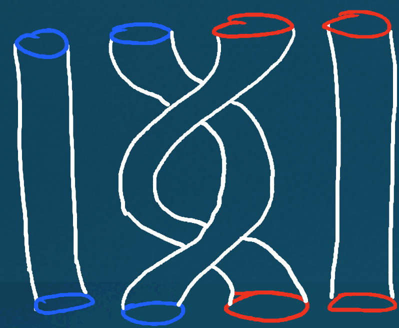
6.6.6

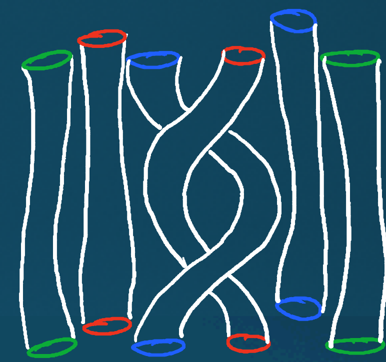


4.6.12

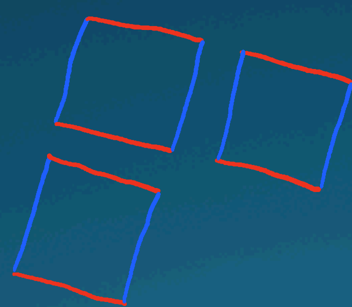
Beyond Memory

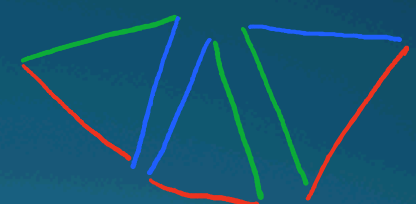
Defects





Lattice Surgery





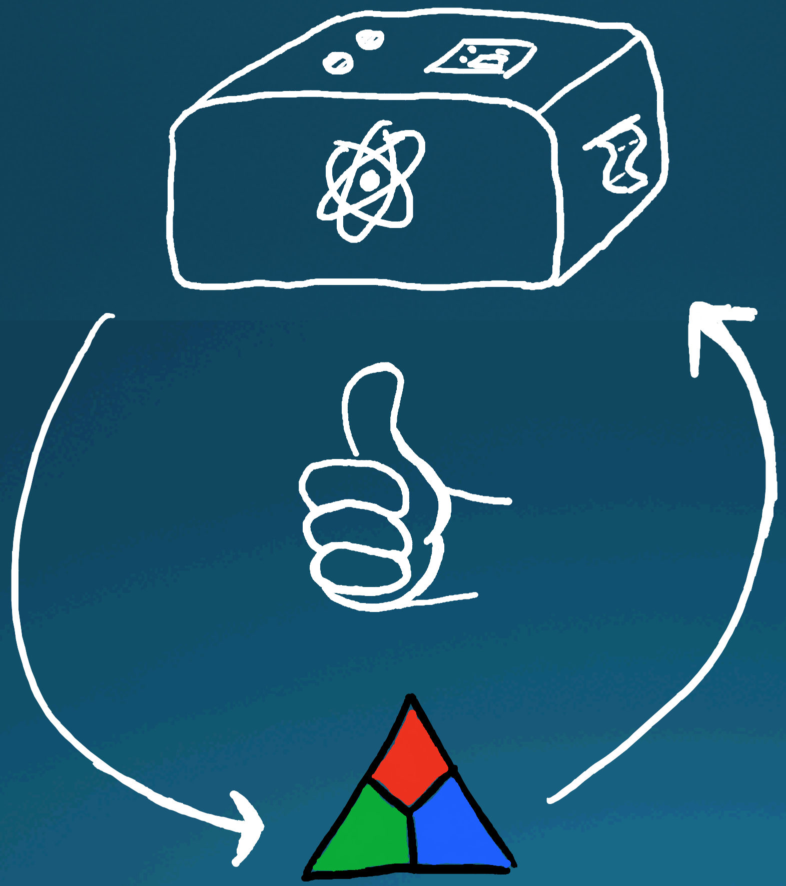
Horsman *et al.*, NJP **14**, 123011 (2012).

arXiv: 1407.5103

Dealing With Error

- Modify decoders
- Different parity measurement circuitry
- Alter logical operations and other tweak to lattice

QECC and Experiment



The End