

*Exceptional service in the national interest*



# Computational Electromagnetics at Sandia National Laboratories - Current Code Capability

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Sandia National Laboratories – Organization 1352

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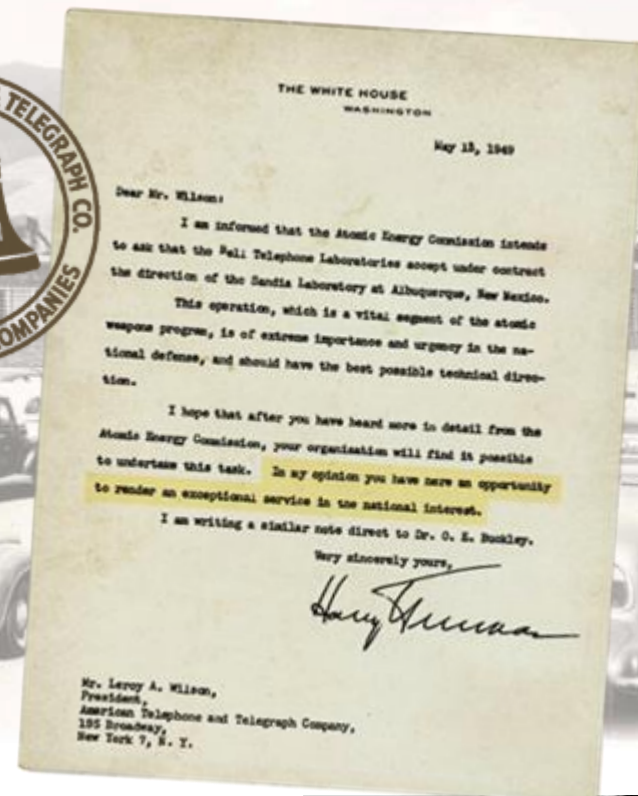
- **Overview of Sandia National Labs**
  - **Electromagnetic Theory Organization**
- **Electromagnetic Environments**
- **Solution Process**
- **Computational Electromagnetics**
  - **Focus on Method of Moments – EIGER**
    - **Thin-slot Algorithm**
    - **Matrix Compression**
- **Conclusions / Future Work**

# Sandia's History

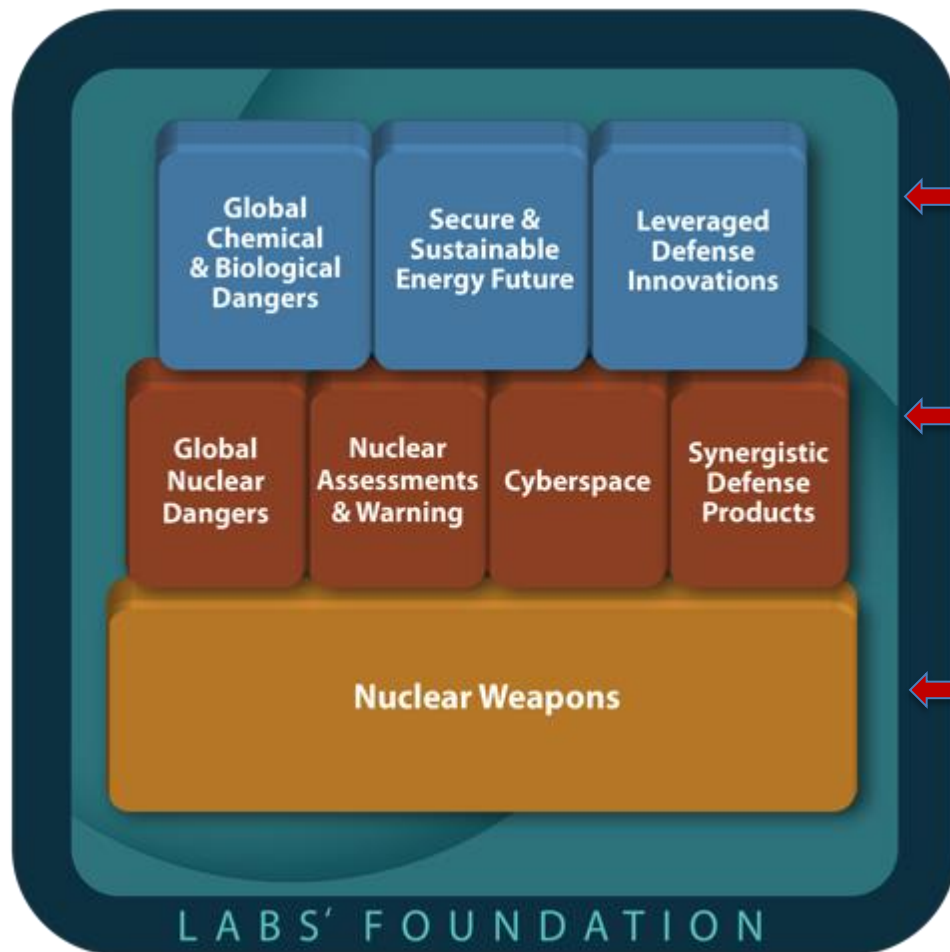
*Exceptional service in the national interest*

- July 1945: Los Alamos creates Z Division
- Nonnuclear component engineering
- November 1, 1949: Sandia Laboratory established

to undertake this task. In my opinion you have here an opportunity to render an exceptional service in the national interest.



# National Security Mission Areas



- Top row: Critical to our national security, these three mission areas leverage, enhance, and advance our capabilities.
- Middle row: Strongly interdependent with NW, these four mission areas are essential to sustaining Sandia's ability to fulfill its NW core mission.
- Bottom row: Our core mission, nuclear weapons (NW), is enabled by a strong scientific and engineering foundation.



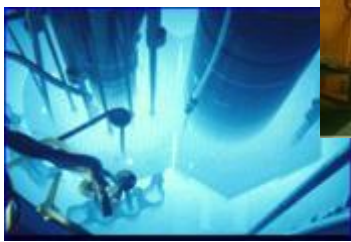
# Sandia's Current Nuclear Weapons Activities

## Warhead Systems Engineering and Integration



*An extensive suite of multi-disciplinary capabilities are required for Design, Qualification, Production, Surveillance, Experimentation / Computation*

## Major Environmental Test Facilities and Diagnostics



Z Machine

Light Initiated High Explosive

Annular core research reactor

Gas  
Transfer  
systems



## Design Agency for Nonnuclear Components

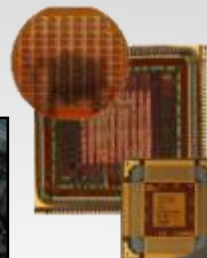


Arming, fuzing, and firing systems

Safety systems

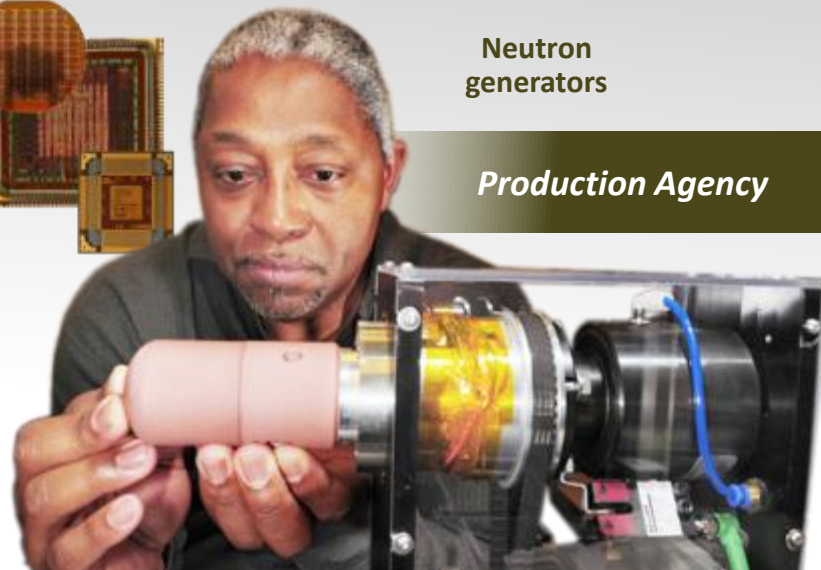


## MESA Microelectronics



Neutron  
generators

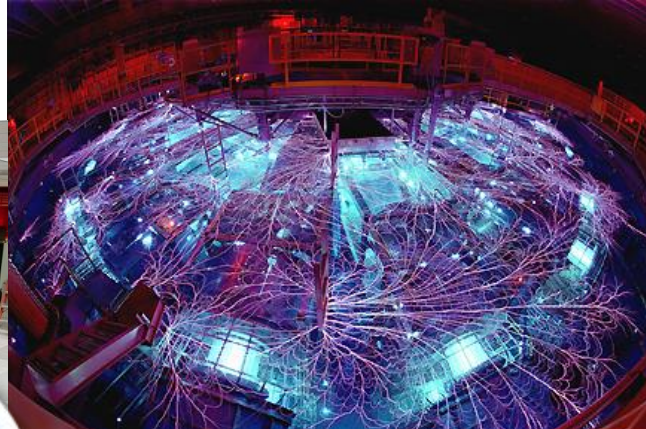
## Production Agency



# Our Research Framework

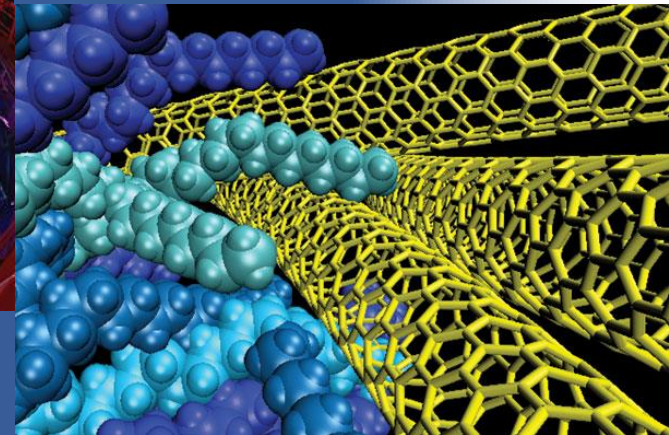
*Strong research foundations play a differentiating role in our mission delivery*

## Computing & Information Sciences

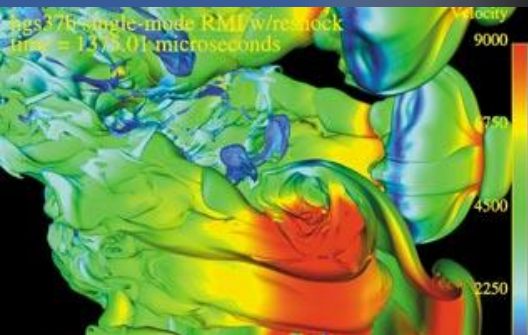


## Radiation Effects & High Energy Density Science

## Materials Sciences

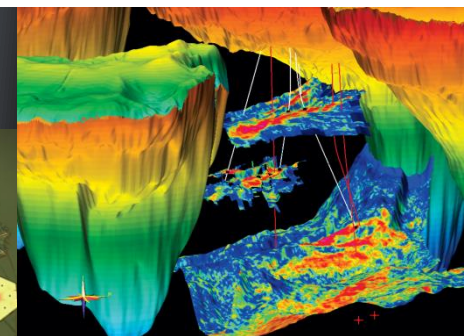
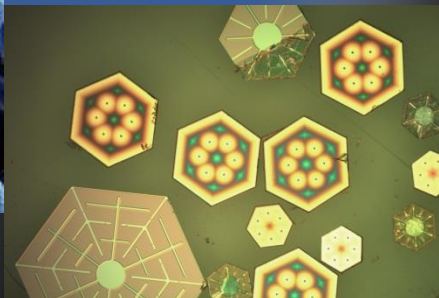


## Engineering Sciences



## Bioscience

## Nanodevices & Microsystems



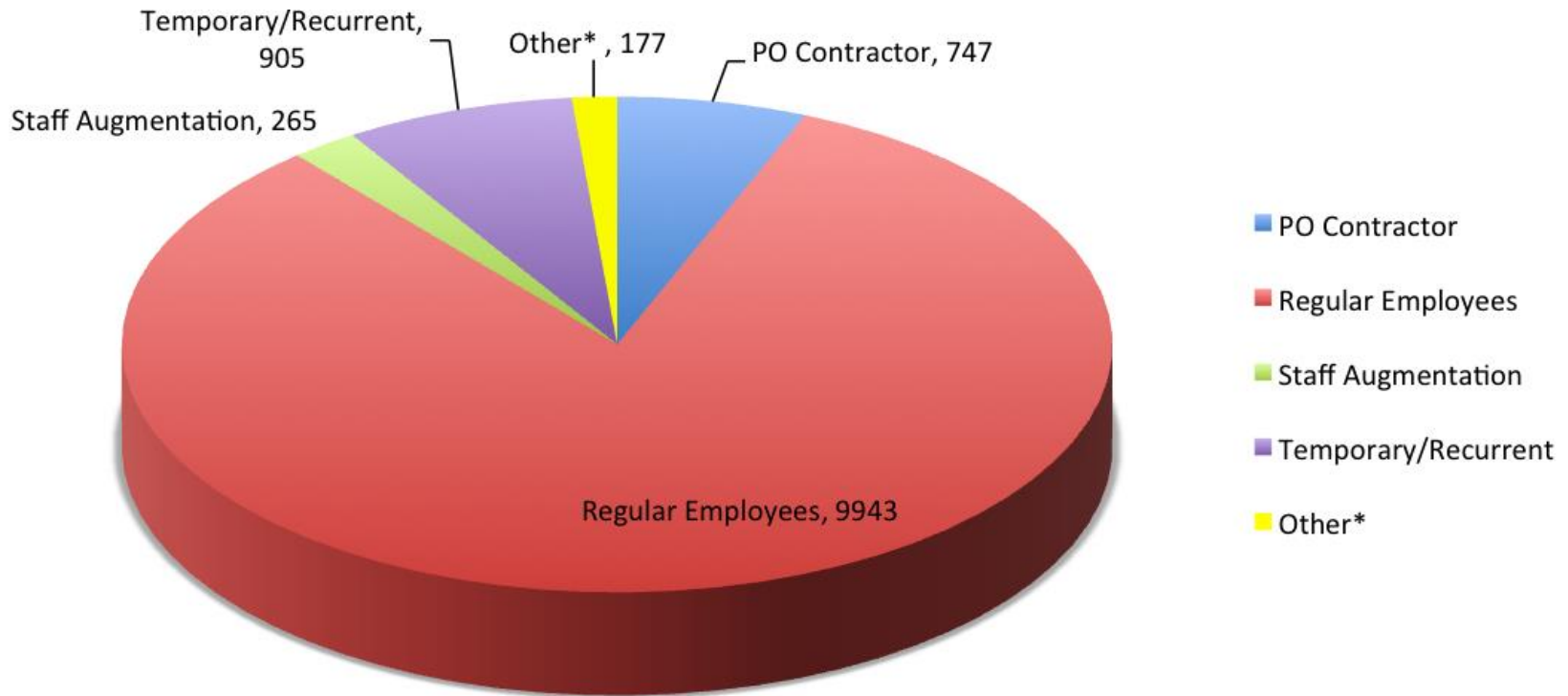
## Geoscience



# Our Workforce

- Total Sandia workforce: 12,037
- Regular employees: 9,943
- Advanced degrees: 5,703

*Data as of August 25, 2014*



# Organization 1352 - Big Picture

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- **What?**
  - Provide high-fidelity, robust, computational tools based on Maxwell's Equations.
  
- **Why?**
  - To aid in weapons qualification in conjunction with experiments.
  - Weapon component and subsystem modeling.
  - In addition can be used to address problems for DoD customers.
  
- **How?**
  - Time-domain finite element formulation.
    - EMPHASIS
  - Frequency domain boundary element formulation.
    - EIGER



# Electromagnetic Environments

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- **Electromagnetic Interference**
  - Radars, etc.
  
- **Lightning**
  - Nearby, direct strike
  
- **System Generated EMP (SGEMP)**
  - High-energy particles produce currents and fields.

- **Geometry to appropriate mesh for analysis.**
  - CUBIT – SANDIA Mesh Software
  
- **Boundary conditions and excitations applied**
  - Computational Electromagnetic Codes in Organization 1352
    - EMPHASIS/EIGER
    - EMPHASIS/NEVADA
    - Solver Technology
      - TRILINOS – SANDIA Solver Technology
  
- **Computational**
  - CIELO - LANL
  - SEQUOIA – LLNL
  - SANDIA Computational Resources

- **RAMSES** (Radiation Analysis, Modeling and Simulation for Electrical Systems) framework includes Emphasis, Xyce, Charon and Sceptre
- **EIGER**
  - Frequency domain, boundary element
- **EMPHASIS**
  - Transient, volumetric finite-element, particle-in-cell
- **QUICKSILVER**
  - Transient, volumetric finite-difference, particle-in-cell (Legacy)

- Time-domain
- Volumetric (space between parts) mesh
  - Unstructured finite-element
  - Structured finite-difference (stair-stepped)
  - Hybrid combination
- Requires truncation of simulation domain
- Formulation results in sparse matrix
  - Limited by ability to generate large mesh



- **Finite-Element Time-Domain (FETD) solver**
  - Full-field (no approximations)
  - Arbitrary geometry subject to meshing limitations
- **Two formulations for field solve**
  - **Unconditionally stable, 2<sup>nd</sup> order Helmholtz**
  - Conditionally stable, 1<sup>st</sup> order Curl-Curl (generalization of structured Finite-Difference Time-Domain (FDTD))
- **Vector finite elements-Edge & Face based**
  - Advantageous field-continuity and boundary properties
  - Divergence-free, avoiding spurious solutions
- **Sub-element algorithms for slots and wires**

# Emphasis – Solution FETD

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- **Newmark-Beta approximation for time derivatives.**
  
- **Implicit solution**
  - **Matrix solve each time step**
    - **Symmetric positive definite**
      - Conjugate gradient can be used
  - **Unconditionally stable**
    - **Theoretically independent of  $\Delta t$**

# Common Emphasis - EIGER Features Sandia National Laboratories

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- General purpose based on Maxwell's equations (3D).
- Full-wave formulations.
- Include separate electrostatic components.
- Massively parallel capable.
- Slot, thin wire, and lumped element models.
  - Sub-cell models
- Wide variety of sources and boundary conditions.

- **Frequency-domain method of moments solution**
  - Steady state solution
  - F90 code – Object Oriented Design
  
- **Boundary element formulation**
  - Mesh surfaces of parts – interface between regions
  
- **Exact radiation boundary condition**
  - Due to Green's function
  
- **Formulation results in dense (fully populated) matrix**
  - Simulations can be limited by available memory
  - Entries are double precision complex



# EIGER – Basic Formulation

$$\bar{\mathbf{E}}^{scatt} = -\left[j\omega\bar{\mathbf{A}} + \nabla\phi\right]$$

$$\bar{\mathbf{A}}(\mathbf{r}) = \mu \int \bar{\mathbf{J}}(\mathbf{r}') g(R) d\mathbf{s}'$$

Surface Current

$$\phi(\mathbf{r}) = \frac{1}{\epsilon} \int \sigma(\mathbf{r}') g(R) d\mathbf{s}'$$

Surface Charge

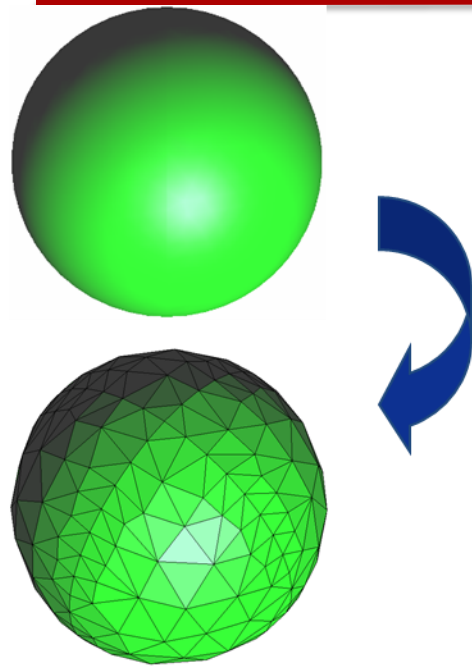
Relationship between current and charge

$$j\omega\sigma = -\nabla \cdot \bar{\mathbf{J}}$$

Free space Green's Function

$g(R)$

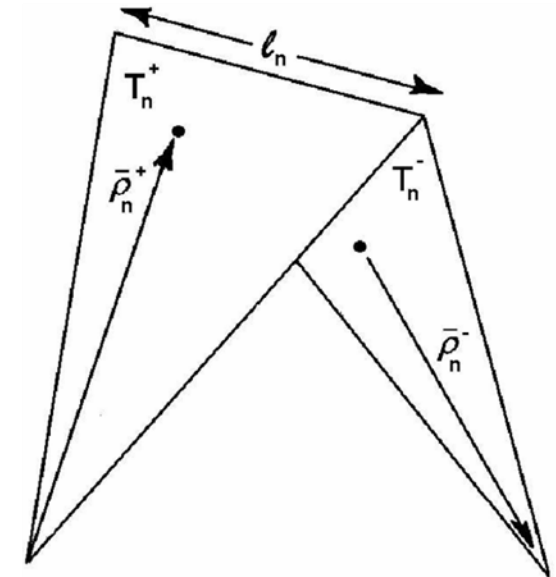
Note : example is for the electric field integral equation



$$\bar{J}(\mathbf{r}) = \sum_{j=1}^N I_j \bar{\mathbf{f}}_j(\mathbf{r})$$

Discretize object

$$\mathbf{f}_n(\mathbf{r}) = \begin{cases} \frac{l_n}{2A_n^+} \rho_n^+ & \mathbf{r} \in T_n^+ \\ \frac{l_n}{2A_n^-} \rho_n^- & \mathbf{r} \in T_n^- \\ 0 & \text{otherwise} \end{cases}$$



Div-conforming Rao-Wilton-Glisson (**RWG**) basis functions

- The integral equation is (on the surface):

$$T(\bar{J}) = \bar{E}_{scatt} \Big|_{\text{tan}} = \bar{E}_{incident} \Big|_{\text{tan}}$$

- The currents are expanded in terms of the Rao-Wilton-Glisson expansion functions ( $\sim 10$  per wavelength) :

$$\bar{J}(r) = \sum_n I_n \bar{f}_n(r)$$

- Test the integral equation with the basis functions:

$$\langle \bar{f}_m, T(\bar{J}) \rangle \Rightarrow \int_{\text{surface}} \bar{f}_m \cdot T(\bar{J}) ds$$

- The integral equation through discretization has become a matrix equation:

$$\bar{\bar{\mathbf{Z}}} \bar{\mathbf{I}} = \bar{\mathbf{V}}$$

- Implications:
  - The matrix  $\mathbf{Z}$  is fully populated – a dense matrix.
  - This method is memory limited.

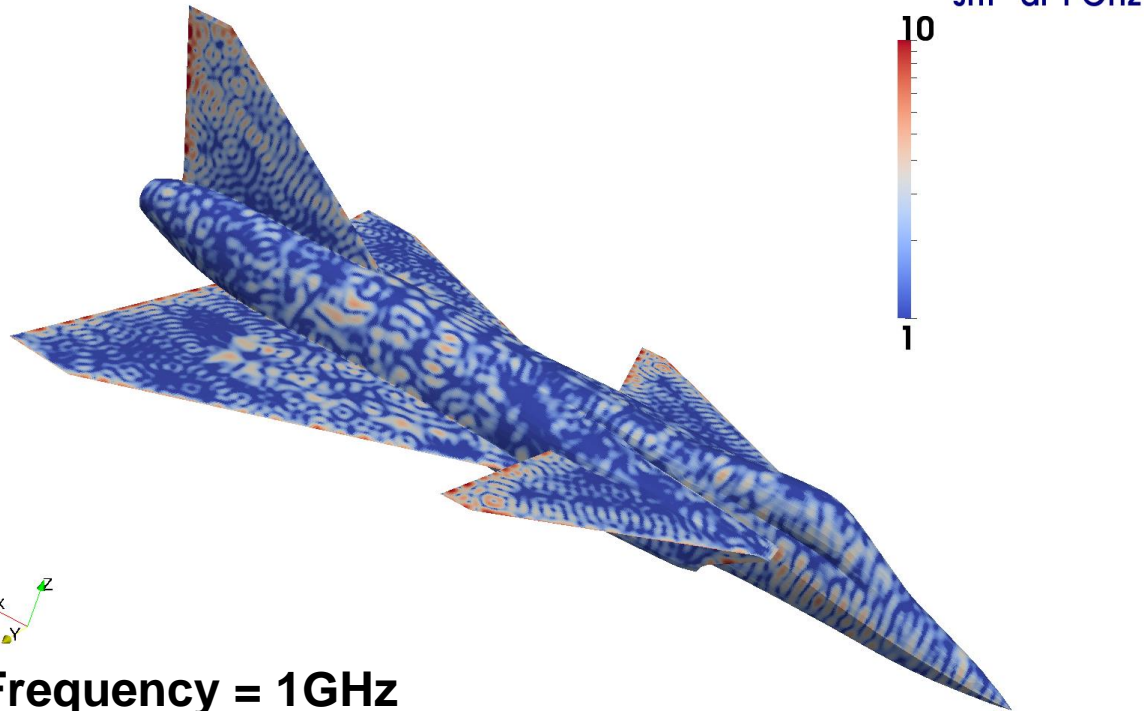


- The boundary element code EIGER:
  - Validated with:
    - Measurements (slot in a box)
    - Analytical solutions (sphere)
  - Used for weapon qualification
  - Additional customers
- The method is memory limited.
  - Limits size of the problem ( with respect to frequency)
- **Path Forward -> Compression techniques:**
  - Relaxes the memory limit issue.
    - Increases the size of the problem (with respect to frequency) that now can be solved.

# Results – External Problem

(Direct Solve on CIELO)

**External Problem**  
**VFY 218 (50.6 ft. length)**



**Frequency = 1GHz**  
**930,000 Unknowns Run on 10240 Processors**  
**Memory Requirement 13.8 TBytes**

**Direction of Incident Field**

# Direct Solve Information

Geometry	Max Frequency (GHz)	Unknowns	Solve Time (per frequency)	Number of Processors	Flop Rate (Gflops per Processor)
#1	5.	389756	13529	1600	7.3
#1	12.	1855082	65875	40000	6.5
#2	16.5	2474989	83575	80000	6
#3	2.	226647	7271	640	6.7
#3	12.5	858826	33375	7600	6.7
#3	12.5	858826	30364	8000	6.9

- This modeling feature enables the incorporation of potential penetration points on a structure that couple fields into a cavity without gridding the slot explicitly.
- Based on research by Warne and Chen.
  - Slot is modeled by a wire (carrying magnetic current) whose effective radius depends of the depth and width of the slot.
    - Note the length of the slot  $\gg$  depth, width
  - Incorporated into EIGER, EMPHASIS, and used by other investigators.
  - Validated
    - Compared to analytic and experimental results.

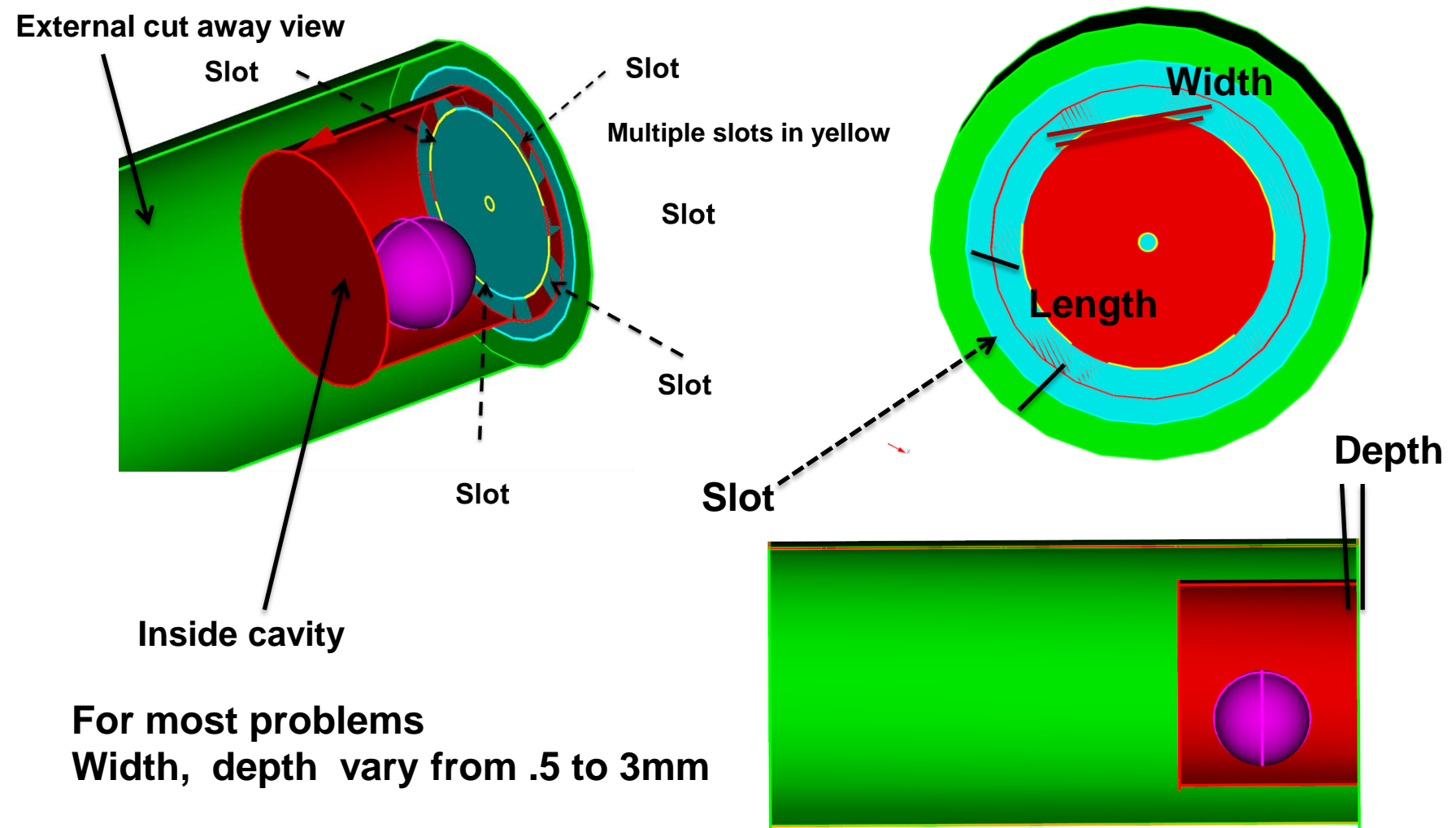
- **Key features**

- Integral equation for the exterior surface current and slot current (magnetic current)
- Integral equation for the interior surface current and slot current (magnetic current).
- Two contributions
  - Green's function
  - Non-Green's function

- **Implications**

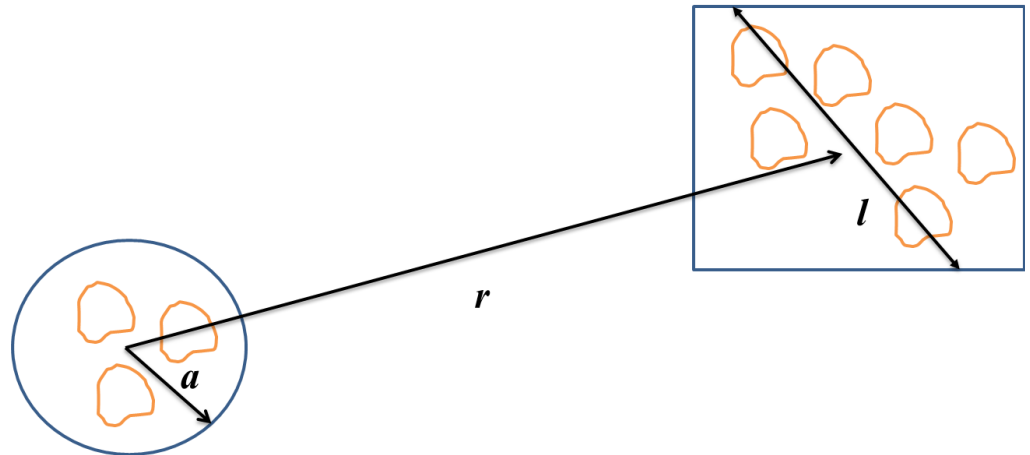
- The exterior unknowns do not interact with the interior unknowns.
- Coupling of the exterior to the interior is through the slot contribution.
  - Matrix has blocks with zero elements – no coupling.

# Thin-Slot Parameters



- These are techniques that no longer store the full matrix but a lower rank version of the matrix.
- Based on work by Bucci and Francescetti
  - “On the Degrees of Freedom of Scattered Fields” IEEE AP, July 1989

$$N_{dof} = \frac{4la}{r\lambda}$$



- **Fast Multipole Method (FMM)**
  - Compression achieved through Green's function simplification:
    - Factorization
    - Use of the addition theorem
    - Diagonalization
    - Results in low-rank approximation of matrix blocks
  
- **Adaptive Cross Approximation (ACA)**
  - Compression achieved:
    - Low-rank approximation of matrix blocks.
    - Done on the fly
      - Compressed matrix blocks never fully populated.
    - Since the process only operates on matrix blocks it is independent of Green's function simplification.

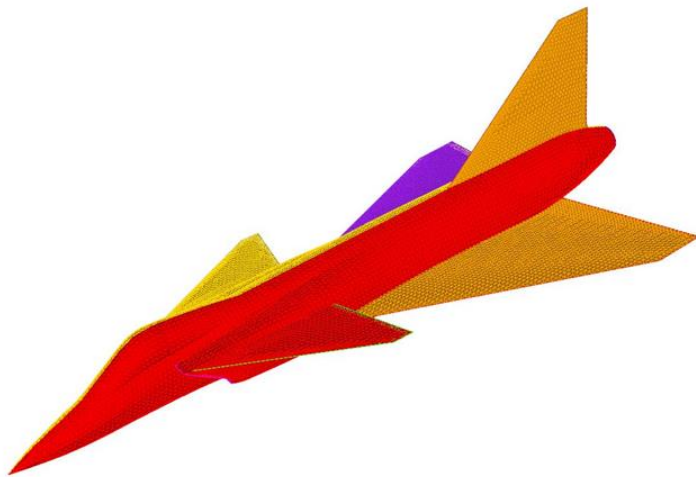


# Compression Techniques

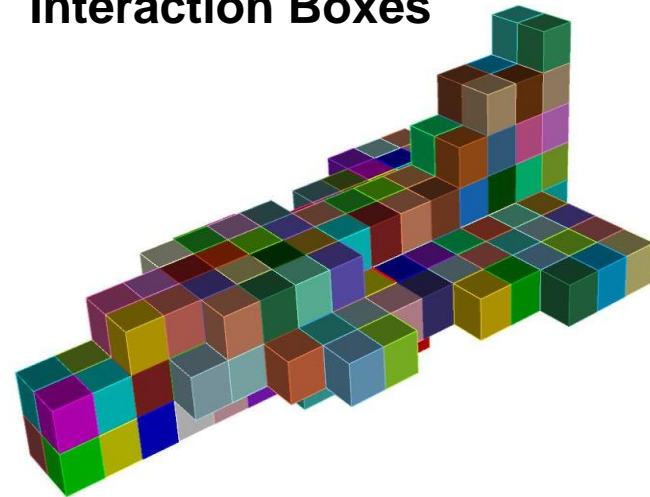
- Identification of all matrix blocks
  - Discretized object (meshed) is encased in a oct-tree structure

VFY 218

Meshed Object



Interaction Boxes



All compression techniques use this step in the solution process

# ACA Matrix Compression

- Each box contains elements with current unknowns on the elements.
  - Can be compared to a 1-level fast multipole algorithm
- 2 boxes interact to form a matrix block.
- The distance between boxes, size of the boxes, and wavelength determine if a reduced or low-rank approximation can be used.
  - **Not all blocks can be compressed.**
    - **Compression criterion :**
      - Distance between the center of boxes  $> 2 * (\text{box radius})$

# ACA Matrix Compression

- The matrix  $\bar{\bar{Z}}$  is given by:

$$\bar{\bar{Z}} = \sum_{j=1}^{MOM\_blocks} Z_j^{mom} + \sum_{i=1}^{COM\_blocks} \tilde{Z}_i^{com}$$

***MOM\_Blocks*** – *Moment method matrix blocks (full matrix blocks)*

***COM\_Blocks*** – *Compressed matrix blocks (low-rank approximation)*

- Approximate matrix description:

$$\tilde{Z}^{m \times n} = U^{m \times r} V^{r \times n} = \sum_{i=1}^r u_i^{m \times 1} v_i^{1 \times n}$$

- The key step is the determination of the sub-matrices  $u$  and  $v$ .

# Solution of the Compressed System

- The matrix equation to be solved is :

$$\bar{\bar{\mathbf{Z}}} \bar{\mathbf{I}} = \bar{\mathbf{V}}$$

- The matrix is not completely available but is stored as:

$$\bar{\bar{\mathbf{Z}}} = \sum_{j=1}^{MOM\_blocks} \mathbf{Z}_j^{mom} + \sum_{i=1}^{COM\_blocks} \tilde{\mathbf{Z}}_i^{com}$$

- Therefore a iterative solution approach needs to be used.
  - Generalized Minimum residual method(GMRES)
    - Saad and Schultz 1986
  - Transpose Free Quasi Minimum Residual (TFQMR)
    - Freund 1993

- **The Iterative solution technique of choice is the TFQMR method.**
  - Based on heuristic numerical experiments performed on electromagnetic problems.
  - Extended for use on parallel platforms.
- **On a parallel machine each processor does not have all the matrix blocks – they are partitioned on different processors for load balancing and memory balancing.**
  - No processor can have more or less than one block than any other processor.
  - Processors have both MOM and COM blocks.

## Using the TFQMR Method

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- In all iterative methods a matrix vector product is needed during the solution process.
  - This is performed in parallel (each processor has a portion of the compressed and MOM blocks).
- In the original algorithm (used here) the residual norm is not available.
  - However an estimate is computable.
  - The convergence curves show two values
    - The normalized initial residual norm
    - The estimate to the norm.
- A solution tolerance of  $5 \times 10^{-3}$  was used in all problems.
  - Will affect accuracy.

# VFY-218 Compression Results

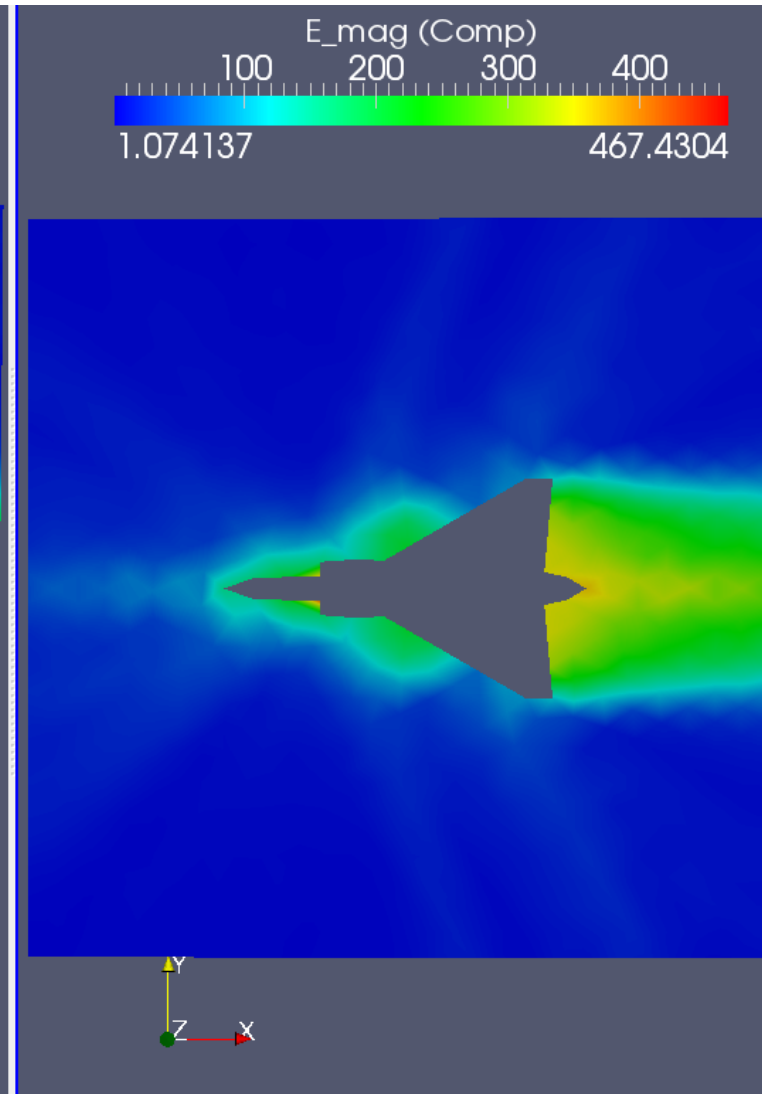
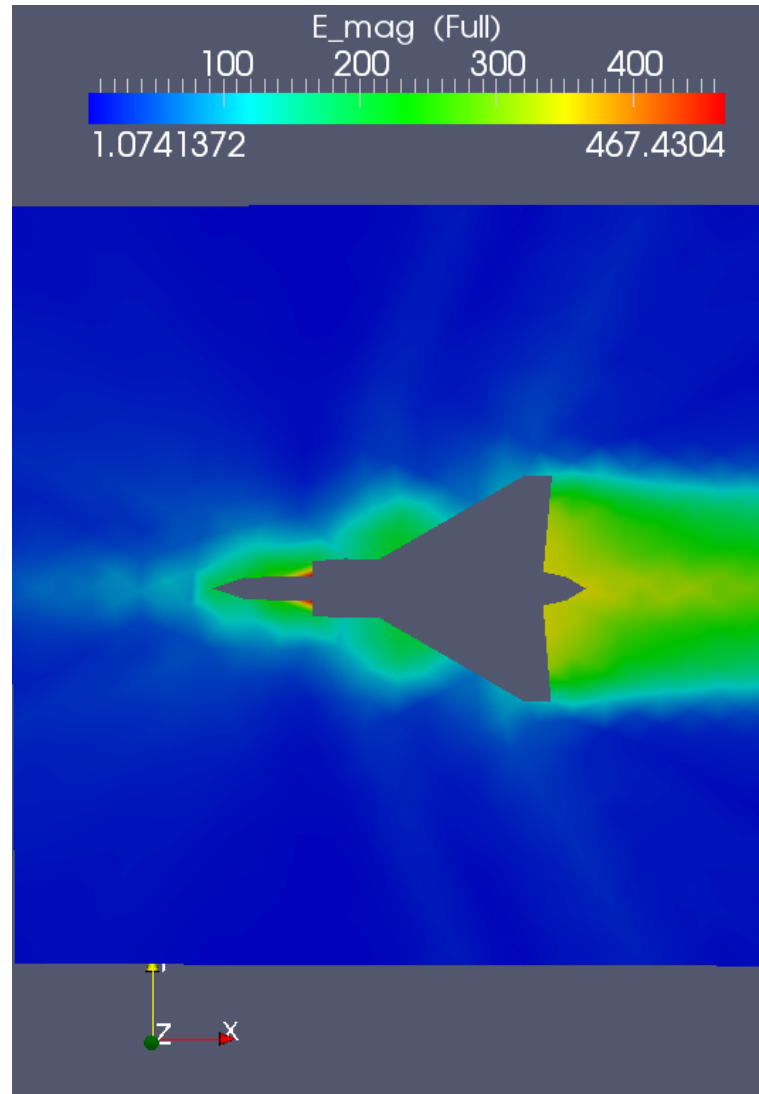
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- 15 meter long aircraft.
- Frequency 1 GHz
- Number of unknowns 934128
  - 2500 iterations
  - 256 Processors
  - 70,826 sec.
- Epsilon 4.e-02
- Memory
  - Full matrix  $16 * (872)$  GBytes
  - Compressed  $16 * (19 + 7.7)$  GBytes
  - ~ 97 % compressed.



# Compression Results VFY-218

Magnitude of the near field full and  
compressed matrix solution.

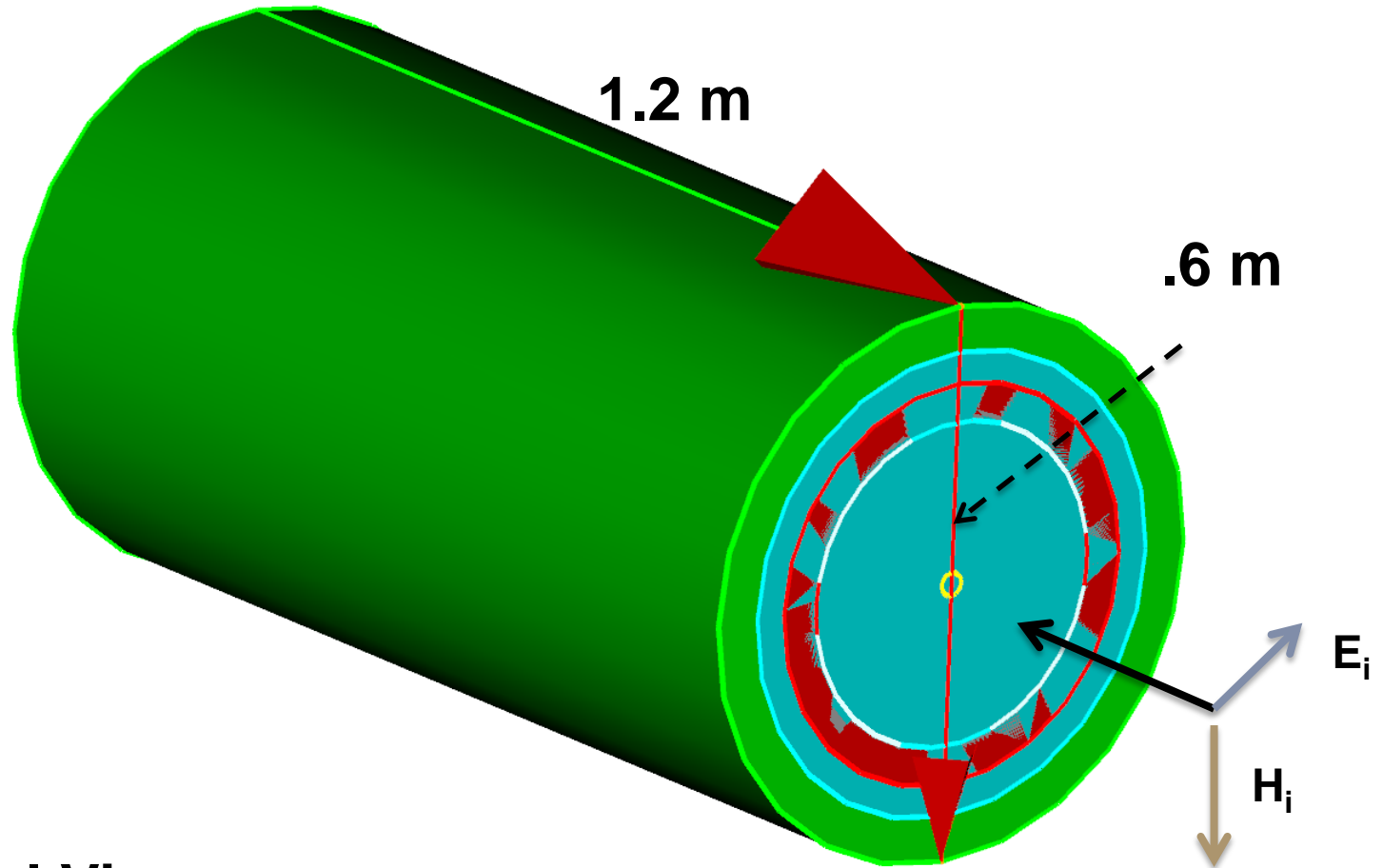


# Compression applied to an object with slots

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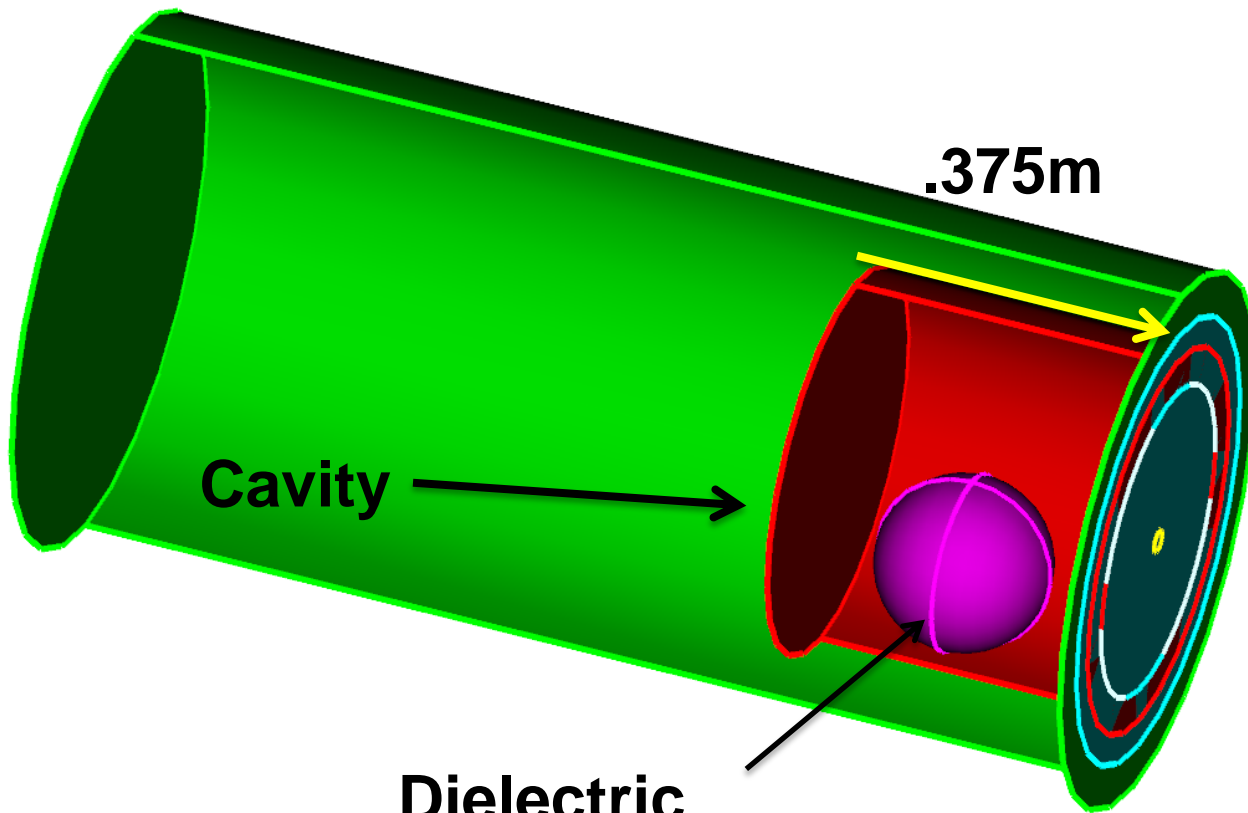
- Referred to as D\_cavity.
- A number of different mesh densities considered.
  - Increases the useful upper frequency limit for the model.
- Contains essential features to exercise the compression algorithm on an problem with slots.

# Geometry D\_cavity



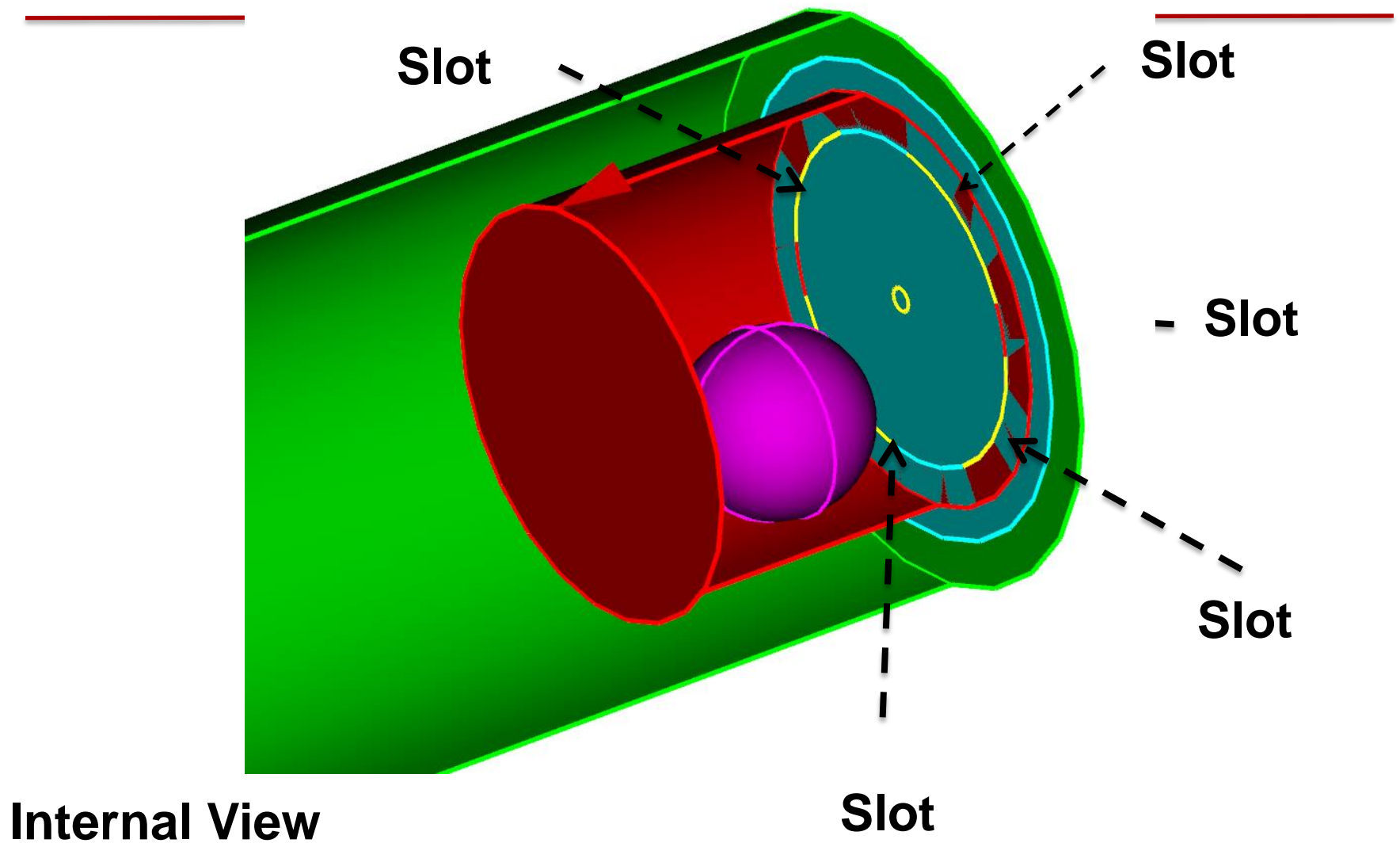
External View

# Geometry D\_cavity



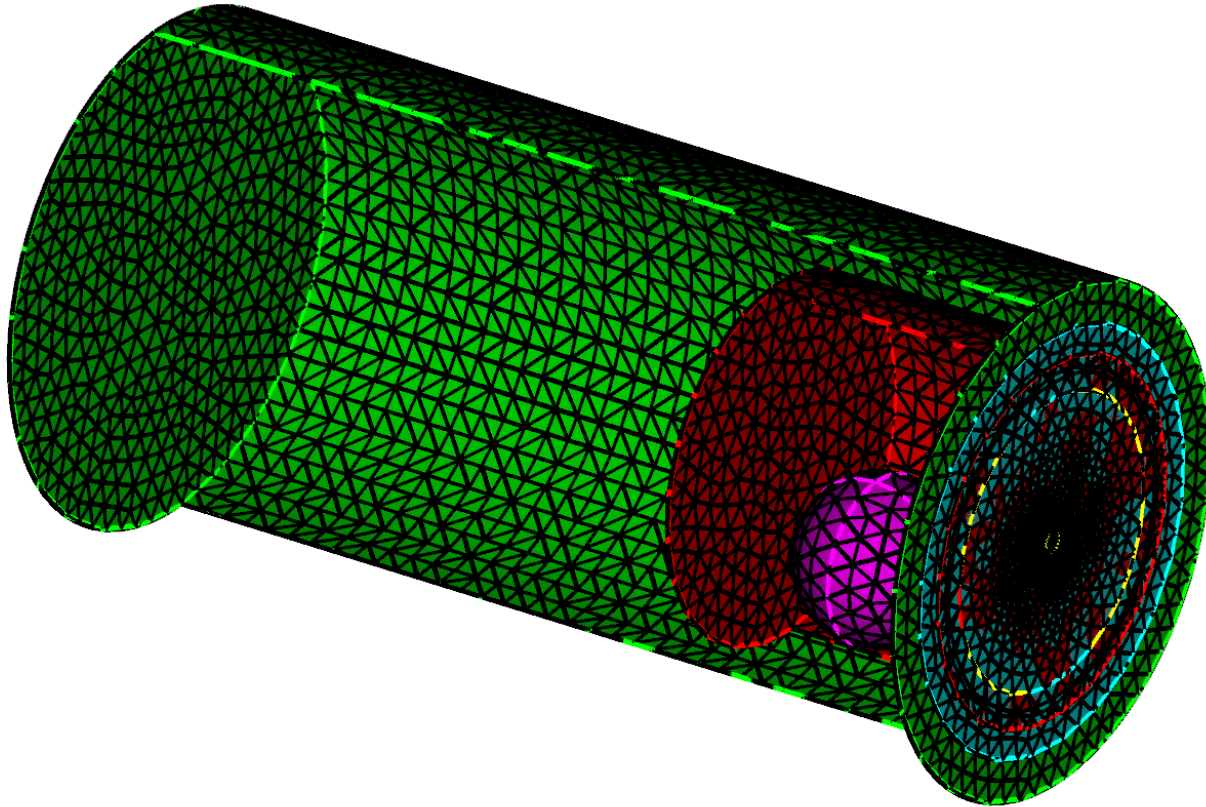
Internal View

# Geometry D\_cavity



# Geometry D\_cavity with Mesh

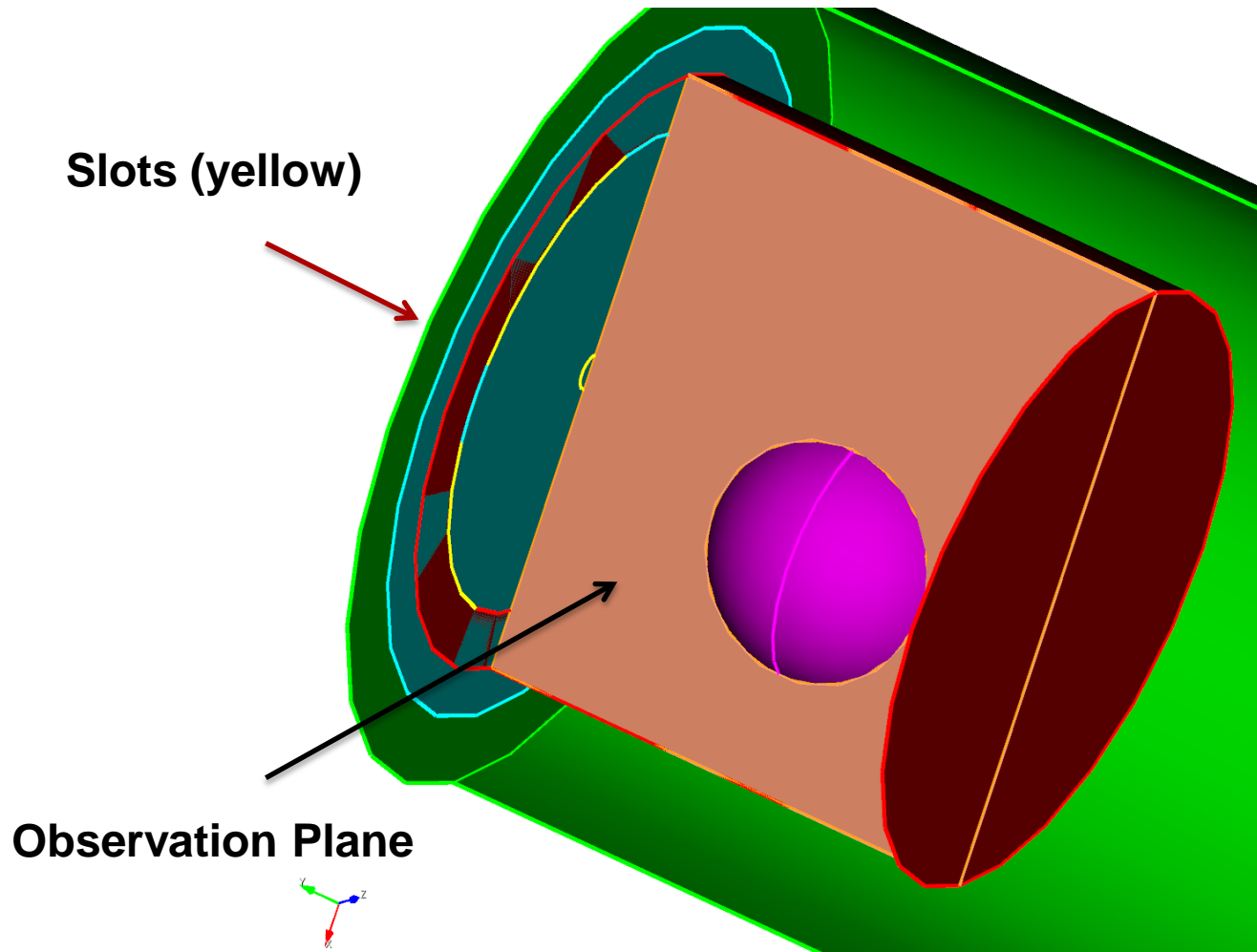
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**Mesh 1 – 10090 elements    ---    15565 unknowns**

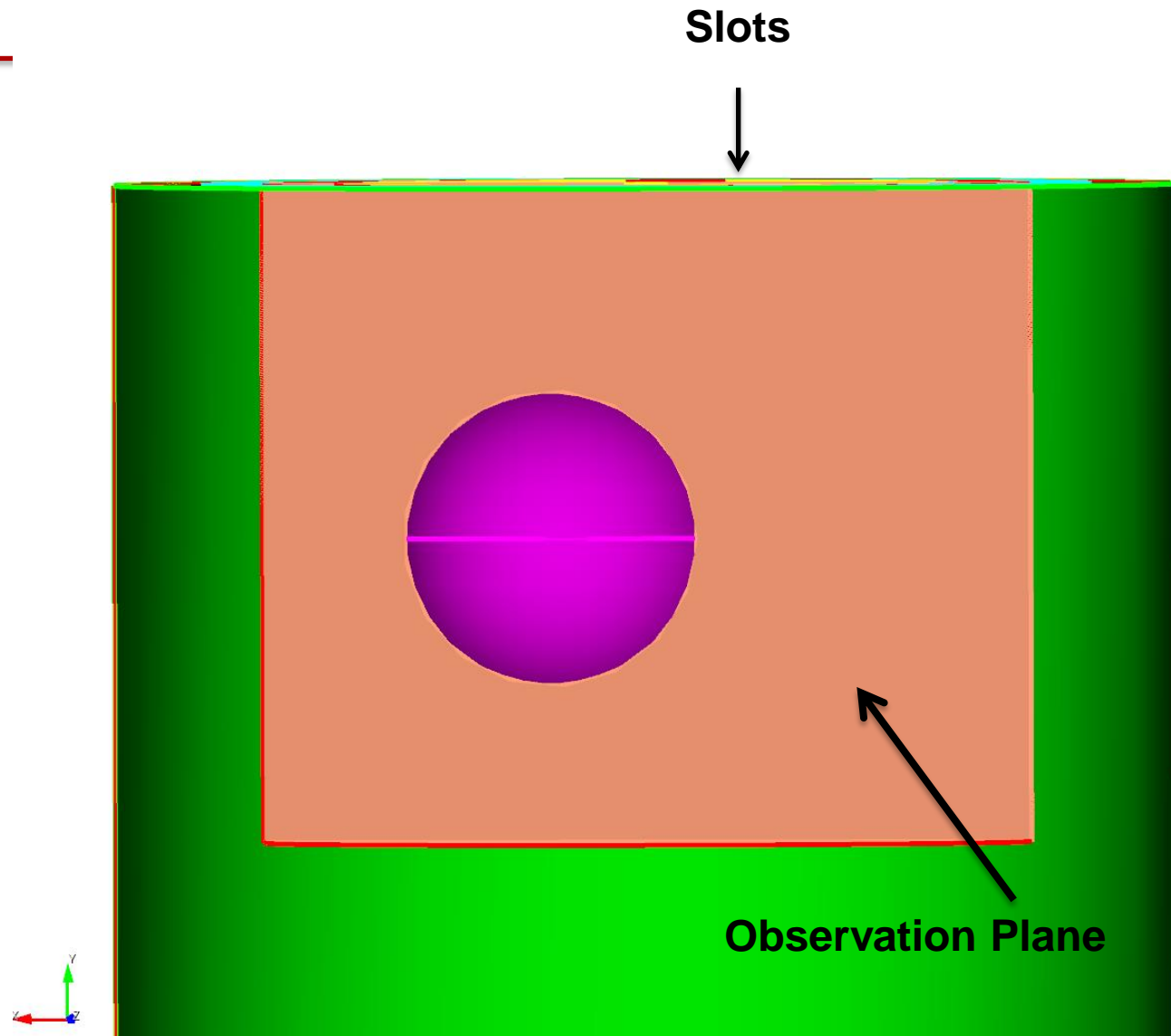
- 
- The magnitude of the scattered electric field will be considered.
  - This field value will be calculated on planes both inside the cavity and outside the cavity.
  - Because of the proximity of these observation points to the object these are near field quantities.

# Data Results Observation Plane





# Data Results Observation Plane



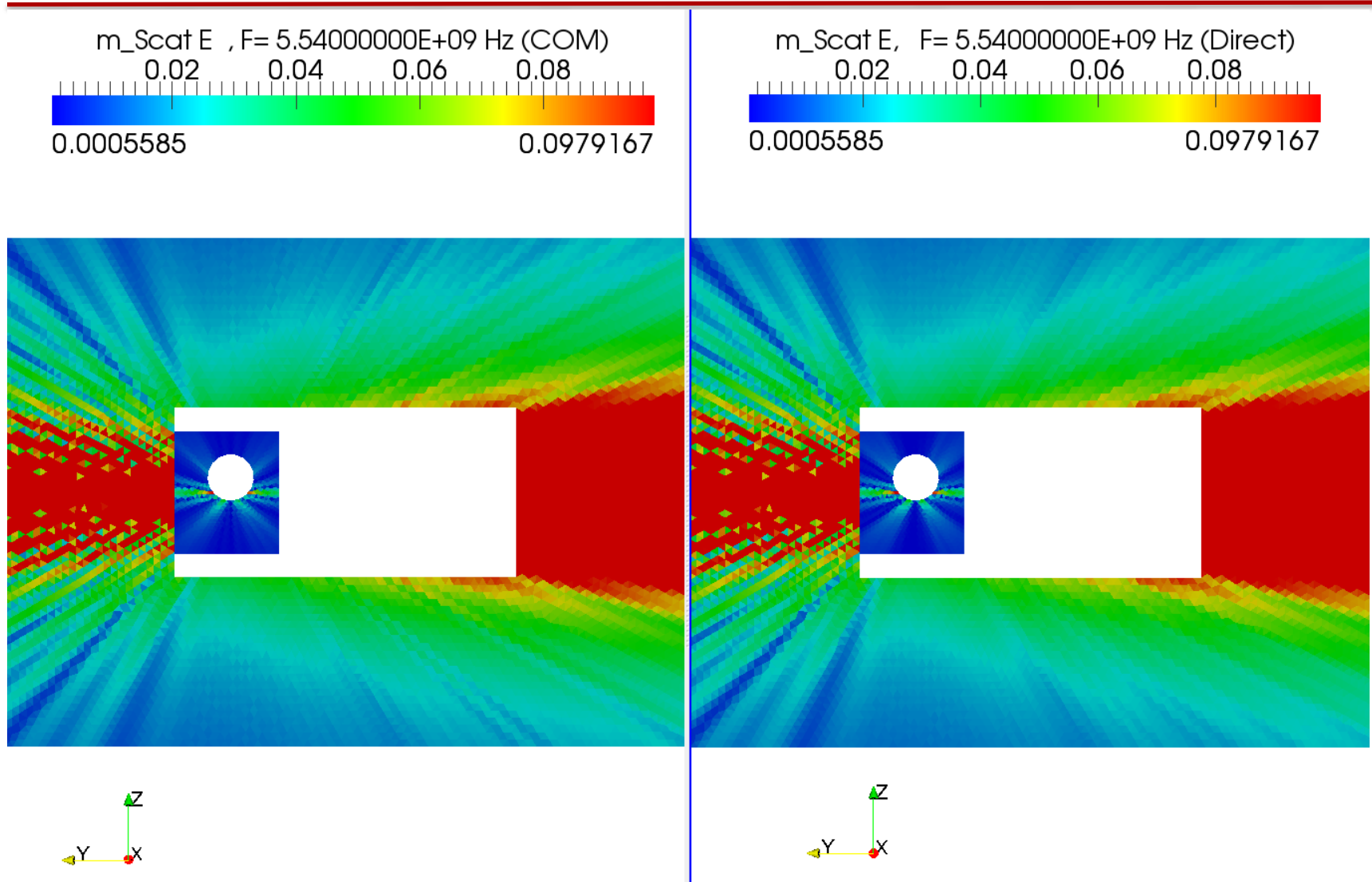
# Results - Mesh 3 D\_cavity

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- Object 1.2 m in length
- Frequency 5.5 GHz
- Number of Unknowns      247604
- Epsilon 3.1e-03
- Memory
  - Full matrix      16\*(61) GBytes
  - Compressed      16\*(36.8 + .2) Gbytes
  - ~40% compressed.

# Results - Mesh 3 D\_cavity

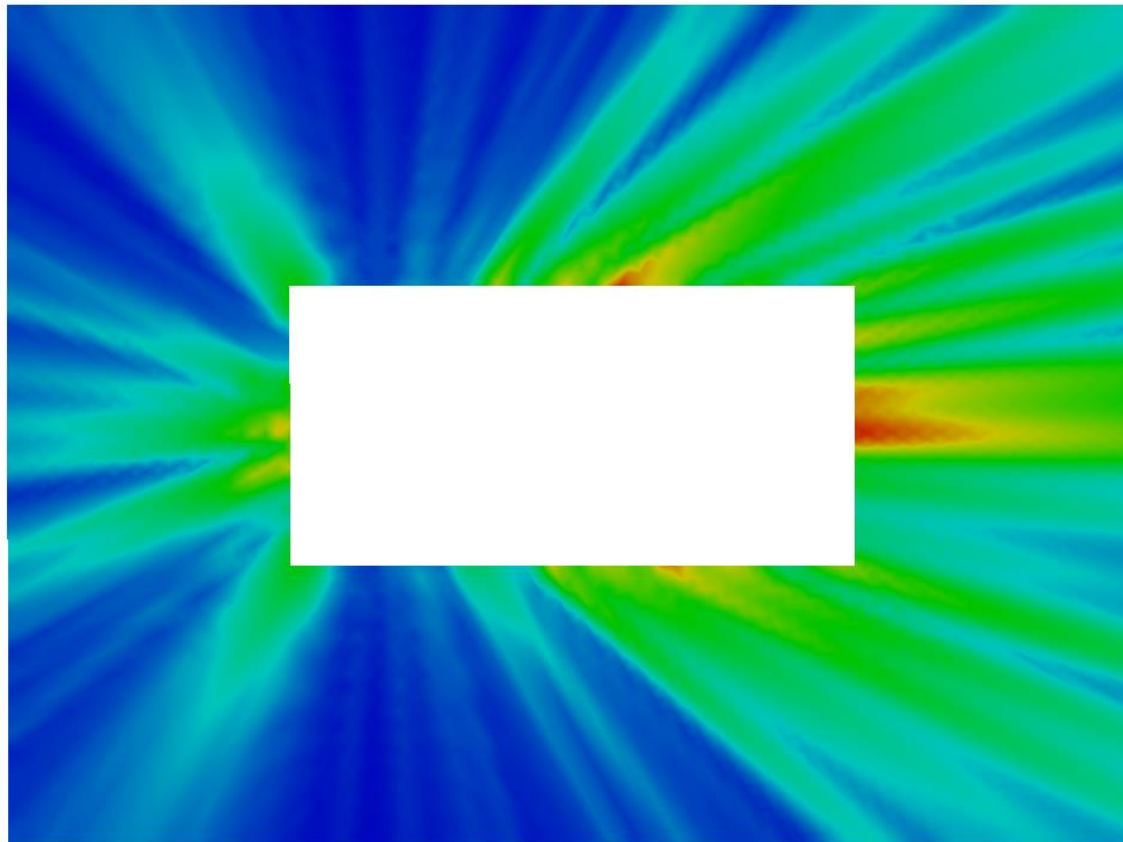
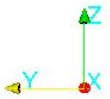
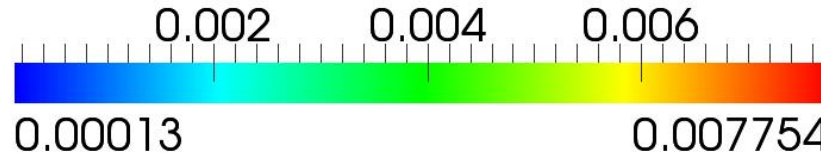
## Magnitude of Scattered Field



# Results - Mesh 3 D\_cavity

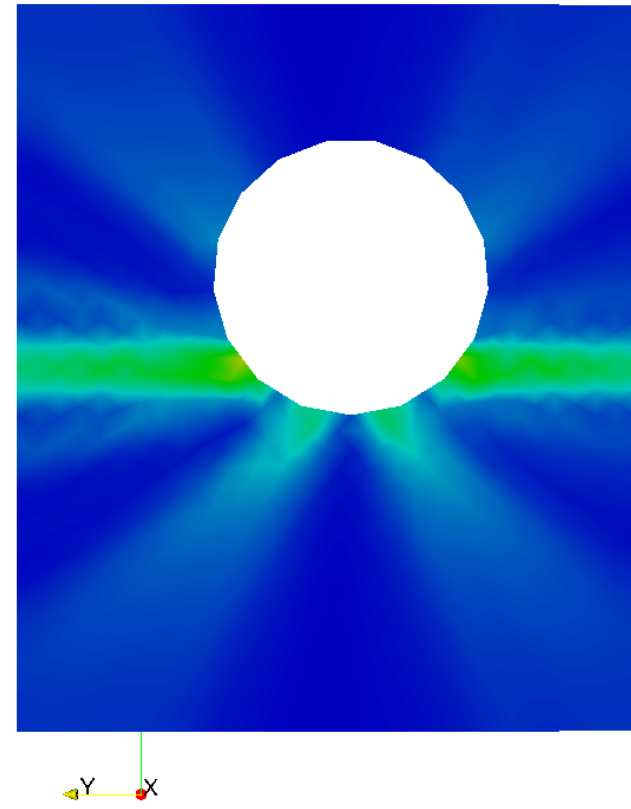
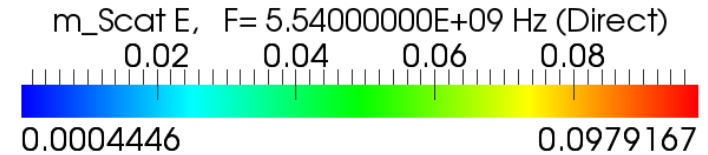
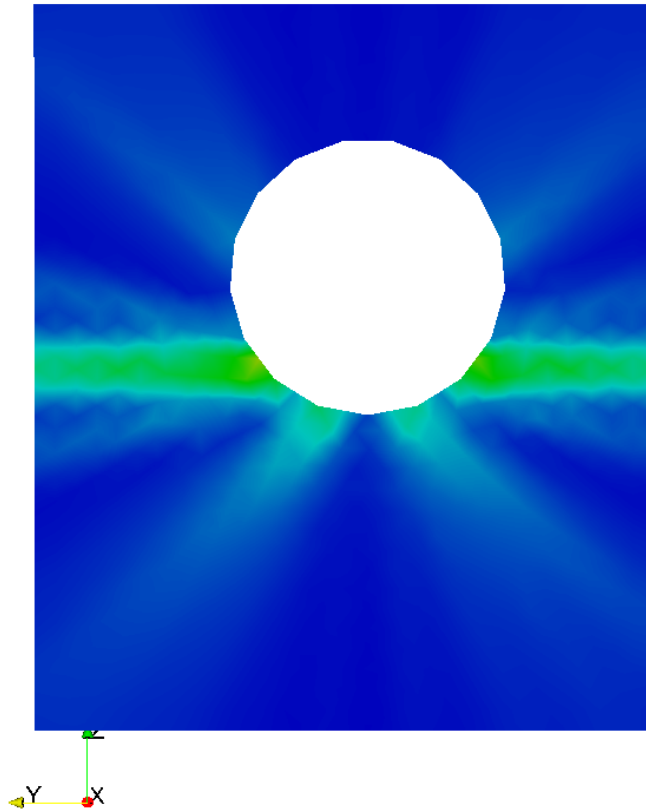
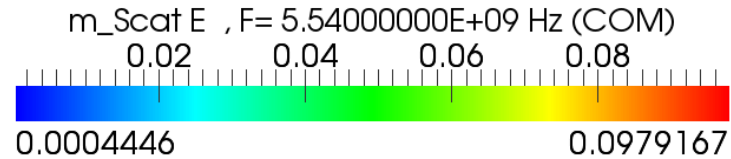
Magnitude of Scattered Field Difference between direct and compressed matrix solutions

m\_Scat E, F= 5.54000000E+09 Hz Difference



# Results - Mesh 3 D\_cavity

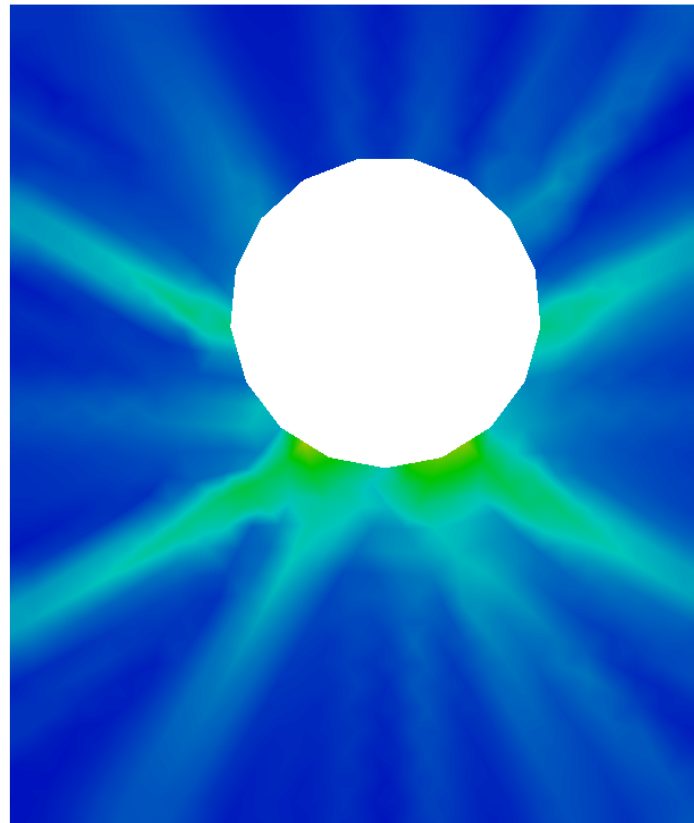
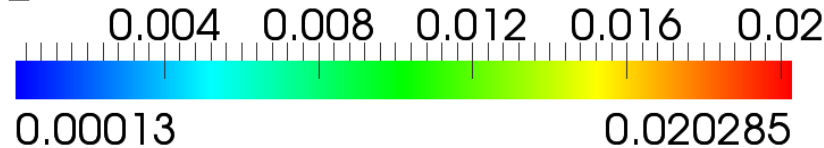
## Magnitude of Scattered Field



# Results - Mesh 3 D\_cavity

Magnitude of Scattered Field Difference between direct and compressed matrix solutions

m\_Scat E, F= 5.54000000E+09 Hz Difference



- Definition

$$2 - norm = \frac{Sqrt(\sum_{point\ s} abs(error\_x)^2 + abs(error\_y)^2 + abs(error\_z)^2)}{Sqrt(\sum_{point\ s} abs(e\_x)^2 + abs(e\_y)^2 + abs(e\_z)^2)}$$

# Error Norms

## 2-Norm

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### 247604 Unknown Problem

Location	2-Norm
Interior_x	.23
Interior_z	.2
Exterior_x	5.33e-03
Extrerior_z	5.26e-03

**Solution tolerance 5.e-03**



- **The matrix compression has been successfully integrated in EIGER.**
  - For parallel machines
  - With iterative solver
- **The viability of the technique has been demonstrated on a diverse group of problems.**
  - Exterior problems
  - Problems with external geometry connected through slots.
    - Uses the thin-slot formulation already integrated in EIGER

# Future Work - Compression

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- **Improve the load balancing of the matrix:**
  - For the MOM blocks, by the block size not just by block number.
  - Use preprocessing to generate block matrix structure.
- **Improve solution time by reducing the iteration count**
  - Preliminary work performed by Matt Bettencourt on preconditioning revealed:
    - Standard methods ILU, Diagonal preconditioning will fail
    - Use Sparse Approximate Inverse (SAI)
    - Applied it to the two smaller problems discussed earlier.
    - Defined the algorithm to implement and tested it in MATLAB.
- **Continue testing on problems of interest to Sandia.**
  - Verify and quantify errors for a robust implementation.

- **Implement alternative compression techniques**
  - Fast Multipole Method (FMM)
  - Direct solve / ACA
- **Implement cable models and interface to the EMPHASIS suite.**
  - External field to pin voltage.
- **Investigate hybrid techniques**
  - High- frequency approximations with full wave solvers.
- **Continue to validate the EMPHASIS suite**
  - Comparisons to measurements.

# Future Work - Continued

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- **1352 collaborates with Universities for the advancement of our computational capabilities.**
  - **Contracts**
  - **Post-docs**
  - **Summer employment**
  
- **The codes developed in 1352 will be repackaged for Next Generation Computing Platforms (NGP).**

# Matrix Compression Backup Slides

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## Definitions

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$$Z(I_1, :)$$

**Row 1 of matrix  $Z$**

$$Z(:, J_1)$$

**Column 1 of matrix  $Z$**

$$\|Z\|^2 = \|Z\|_F^2 = \sum_{i=1}^m \sum_{j=1}^n |z_{ij}|^2$$

**Frobenius Norm squared of matrix  $Z$**

# ACA Matrix Compression

## Algorithm - Initialization

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Compute the first row of the matrix.  $Z(I_1, :)$

Find the maximum element of the row.  $\max_j |Z(I_1 :, j)| = J_1$

Normalize the row by this element.  $v_1 = Z(I_1, :) / Z(I_1, J_1)$

The vector  $u$  is defined.  $u_1 = Z(:, J_1)$

The first contribution to the  
Frobenius Norm squared is calculated.  $\|\tilde{Z}^1\|^2 = \|u_1\|^2 \|v_1\|^2$

Find the largest row value in the column  
 $J_1$  – is different from the previous row  
used.  $\max_i |Z(i :, J_1)| = I_2 \neq I_1$

# ACA Matrix Compression

## Algorithm – k'th Iteration

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1) Calculate next row and compute approximate matrix row.

$$\tilde{R}(I_k, :) = Z(I_k, :) - \sum_{l=1}^{k-1} (u_l)_{I_k} v_l$$

2) Find the maximum element of the row.

$$\max_j |\tilde{R}(I_k, j)| = J_k$$

3) Normalize the row by this element.

$$v_k = \tilde{R}(I_k, :) / \tilde{R}(I_k, J_k)$$

4) Adjust approximate column and compute  $u_k$ .

$$\tilde{R}(:, J_k) = Z(:, J_k) - \sum_{l=1}^{k-1} (v_l)_{J_k} u_l$$

$$u_k = \tilde{R}(:, J_k)$$

5) Compute the update to the Frobenius Norm.

$$\|\tilde{Z}^{(k)}\|^2 = \|\tilde{Z}^{(k-1)}\|^2 + 2 \sum_{j=1}^{k-1} |u_j^T u_k| \cdot |v_j^T v_k| + \|u_k\|^2 \|v_k\|^2$$

6) Find the next maximum row index.

$$\max_i |\tilde{R}(i, J_k)| = I_{k+1}$$



# ACA Matrix Compression

## Algorithm – Convergence

- The test for convergence is (Step 5 in the previous slide) :
  - Epsilon chosen by the user

$$\left\| \mathbf{u}_k \right\| \cdot \left\| \mathbf{v}_k \right\| \leq \varepsilon \left\| \tilde{\mathbf{Z}}^{(k)} \right\|$$

- The computation of a low-rank approximation to the matrix is complete.
  - Note that this was done by row and column – the full matrix was not computed and reduced.
  - The number of elements to store for this matrix is  $(n + m) \times r$ , instead of  $m \times n$ .