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Approximate Augmentation of Turbulent Law-of-the-Wall by Periodic Free-Stream Disturbances

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Approximate Augmentation of Turbulent Law-of-the-Wall by Periodic Free-Stream Disturbances

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ABSTRACT

We examine the role of periodic sinusoidal free-stream disturbances on the inner law law-of-the-wall (log-law) for turbulent boundary layers. This model serves a surrogate for the interaction of flight vehicles with atmospheric disturbances. The approximate skin friction expression that is derived suggests that free-stream disturbances can cause enhancement of the mean skin friction. Considering the influence of grid generated free stream turbulence in the laminar sublayer/log law region (small scale/high frequency) the model recovers the well-known shear layer enhancement suggesting an overall validity for the approach. The effect on the wall shear associated with the lower frequency due to the passage of the vehicle through large (vehicle scale) atmospheric disturbances is likely small i.e. on the order 1% increase for turbulence intensities on the order of 2%. The increase in wall pressure fluctuation which is directly proportional to the wall shear stress is correspondingly small.

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NOMENCLATURE

Symbols

a Dimensionless model constant

B	Inner aw constant B=5
c	Locally defined constant
C_f	Skin friction $C_f = \frac{2\tau_w}{\rho U^2}$
C_{f_0}	Undisturbed free-stream skin friction
I	Turbulence intensity (absolute value)
K	Clauser turbulent viscosity constant
L	Streamwise length scale
p'	Root Mean Square (RMS) pressure fluctuation amplitude
Re	Reynolds number
Re_s	Stokes Reynolds number: $\mathrm{Re}_s = \frac{\Delta U_\infty \sqrt{\frac{2\nu_w}{\omega}}}{\nu_w}$
Re_x	Streamwise flat plate Reynolds number
Re_θ	Momentum thickness Reynolds number
t	time
u	Streamwise turbulent mean flow
U	Free stream turbulent mean flow velocity
U^+	Inner law velocity free stream U/v^*
u'	Root Mean Square (RMS) streamwise velocity fluctuation amplitude
u^+	Inner law velocity u/v^*
v'	Root Mean Square (RMS) cross-stream velocity fluctuation amplitude
v^*	Friction velocity $v^* = \sqrt{\frac{\tau_w}{\rho}}$
x	Streamwise spatial coordinate
y	Cross-stream spatial coordinate
y^*	y/δ
y^+	Cross-stream inner law length scale $y^+ = \frac{yv^*}{\nu_w}$

Greek

δ	Boundary layer thickness
δ^+	Boundary layer thickness inner law length scale $\delta^+ = \frac{\delta v^*}{\nu_w}$
ΔU_∞	Pulsatile flow streamwise velocity amplitude
η	Pulsatile flow similarity independent variable: $\eta = y\sqrt{\frac{\omega}{\nu_{eff}}}$
κ	Von Karman constant κ=0.41
$\widetilde{\kappa}$	Pulsatile flow modified Von Karman constant
ν	Kinematic viscosity
ω	Pulsatile flow frequency
ρ	Density
τ	Shear stress
θ	Momentum thickness

Subscripts/Superscripts

eff	Turbulent effective
inc	Incompressible
FT	Fully Turbulent
max	Maximum
os	Laminar-turbulent pressure “over-shoot”
rms	Root Mean Square (RMS)
s	Steady
turb	Turbulent
t, tran	Transition
vehicle	Reentry vehicle
w	Wall
∞	Steady free-stream constant

I. INTRODUCTION

To this point, we have modeled reentry vehicle boundary layer induced loading assuming a steady (or more properly slowly varying) approach and captured stochastic (atmospheric) perturbations using a direct pressure loading approach. This decoupled modeling with associated superposition provides a reasonable approach however there is value in attempting to ascertain the couple effect of disturbance on the boundary layer loading.

The effect of free-stream turbulence on skin friction is well known e.g. Simonich and Bradshaw (1978), Meier and Kreplin (1980), and Blair and Werle (1980), Kondjoyan (2002). These results suggest a significant, i.e. as much as (20-30%) increase in wall shear due to free stream turbulence. Correlations are available to estimate this increase as a function of turbulence intensity, e.g. the result by Simonich and Bradshaw $\frac{C_f}{C_{f\_0}} = 1 + 2I$ where C_f is the skin friction, C_{f_0} is the unenhanced skin friction and I is the turbulence intensity. See DeChant (2015) for additional discussion.

Though the effect of free-stream turbulence characterized by length scales that are the order of the boundary layer thickness or smaller and frequencies that are correspondingly large, is well understood, the effect of low-moderate frequency large scale free-stream disturbances is not readily available. Indeed we suggest that large scale disturbances of this type are perhaps best modeled as unsteady pulsatile free-stream disturbances where a temporal frequency is specified. There is little direct information regarding this type of model regarding skin friction so we propose to test the result by considering it in the small scale/high frequency limit. The classical free-stream turbulence skin friction enhancement should be recovered in the high frequency limit. With some sense of the viability of the model, the behavior of the lower frequency encounter can then be estimated.

We have a particular interest in the skin friction since wall pressure fluctuation root-mean-square values are well correlated with the mean wall shear. $p_w' \propto \tau_w$ The proportionality "constant" varies considerably range from 2-6. Moreover, the "constant value" is likely Reynolds number dependent as suggested by Farabee and Casarella (1991), Bernardini and Pirozzoli (2011) and DeChant (2015) as suggested in table 1.

Researcher	Model ($\frac{p_w'}{\tau_w}$)	Value $Re_x=1x10^8$
Farabee and Casarella (1991)	$\frac{p_w'}{\tau_w}=\sqrt{6.5+1.86\ln(\frac{\delta^+}{333})}$	3.90
Bernardini and Pirozzoli (2011)	$\frac{p_w'}{\tau_w}=\sqrt{2.27\ln(\delta^+)-7.36}$	4.05
DeChant (2005)	$\frac{p_w'}{\tau_w}=1.828(\delta^+)^{0.0681}$	3.74

Table 1. Root-mean-square wall pressure expressions

For our purposes it is likely sufficient to describe the RMS wall pressure to mean wall shear as approximately four, i.e. $p_w'\approx 4\tau_w$. These paragraphs suggest that modeling the effect of unsteady fluctuation encounters on the wall shear stress will provide a first order loading estimate.

II. ANALYSIS

A. Pulsatile Flow

A simplified modeling approach for free-stream fluctuation encounter is considered here. The fluctuation is presumed to take the form: $U(x,t)=U_\infty+\Delta U_\infty\cos(\omega t)$. Our focus is the wall shear stress expression and we can therefore restrict our modeling efforts to:

$$\left(\frac{\partial u}{\partial t}-\frac{\partial U}{\partial t}\right)=\frac{\partial}{\partial y}\left(v^*\kappa y\frac{\partial u}{\partial y}\right) \tag{1}$$

Where we have (consistent with traditional near wall modeling) we have neglected the convective acceleration terms. See DeChant (2015) for a related analysis.

We are interested in a near wall model and will construct expressions that fundamentally retain steady turbulent wall layer behavior, specifically, local law of the wall profiles. To achieve this goal, we will use an iteration procedure rather than the more classical perturbation series. Let's then consider the outer-law dominated unsteady flow:

$$\frac{\partial u}{\partial t}-\frac{\partial U}{\partial t}=\frac{\partial}{\partial y}\left(\nu_{eff}\frac{\partial u}{\partial y}\right) \tag{2}$$

Where ν_{eff} is a turbulent effective viscosity. We will require that it be a constant or at most a function of the streamwise local "x". "Obvious" choices would include: $\nu_{eff}=KU_{\infty}\delta$, $\nu_{eff}=C\frac{U_{\infty}}{v^{*}}\nu_{w}$ where:

K	=	Clauser constant 0.016,
δ	=	local (steady) boundary layer thickness
ν_w	=	wall molecular viscosity
v^*	=	friction velocity
U_∞	=	steady free-stream velocity

To solve equation (2) we propose that a solution takes the form:

$$u\approx u_{s}(y)+\Delta U_{\infty}\cos\omega t-\Delta U_{\infty}{}'f(\eta)\cos(\omega t-a\eta)\quad;\quad\eta=y\sqrt{\frac{\omega}{\nu_{eff}}} \tag{3}$$

Substitution of this expression into equation (3) yields an expression in terms of $\cos(\omega t-a\eta)$ and $\sin(\omega t-a\eta)$. Demanding that both the cosine and sine expressions are individually satisfied yields two linear equations for f(η) of the form:

$$2a\frac{df}{d\eta}+f=0\rightarrow f=\exp(-\frac{1}{2a}\eta) \tag{4}$$

and

$$\frac{d^{2}f}{d\eta^{2}}-a^{2}f=0\rightarrow f=\exp(-a\eta) \tag{5}$$

Obviously, these two expressions can represent the same functional behavior by choosing "a" such that

$$\frac{1}{2a}=a\rightarrow a=\frac{\sqrt{2}}{2} \tag{6}$$

which yields $a=\frac{\sqrt{2}}{2}$. We can thus, estimate that the (unsteady) term is now:

$$\begin{aligned}u&=\Delta U_{\infty}\left(\cos\omega t-\exp(-\frac{\sqrt{2}}{2}\eta)\cos(\omega t-\frac{\sqrt{2}}{2}\eta)\right)\\&=\Delta U_{\infty}\left(\cos\omega t-\exp(-\sqrt{\frac{\omega}{2\nu_{eff}}}y)\cos(\omega t-\sqrt{\frac{\omega}{2\nu_{eff}}}y)\right)\end{aligned} \tag{7}$$

The steady portion of equation (3) $u_s(y)$ would be governed by $0=\frac{\partial}{\partial y}\left(\nu_{eff}\frac{\partial u_s}{\partial y}\right)$. Though we are unlikely gain a useful estimate for laminar from this type of expression since it is only valid near the wall (we would need something like $U_\infty\frac{\partial u_s}{\partial x}=\frac{\partial}{\partial y}\left(\nu_{eff}\frac{\partial u_s}{\partial y}\right)$) there is still value in considering laminar flow for the unsteady portion. Laminar flow with $\nu_{eff}=\nu_w$ yields wall shear (Arpaci and Larsen, (1984)) as:

$$\begin{aligned}\frac{\tau_w}{\rho}&=-\nu_{eff}\left.\frac{\partial u}{\partial y}\right|_{y=0}\\&=-\nu_{eff}\left.\frac{\partial u_s}{\partial y}\right|_{y=0}+\nu_{eff}\Delta U_\infty\sqrt{\frac{\omega}{\nu_{eff}}}\cos(\omega t+\frac{\pi}{4})\end{aligned}\tag{8}$$

Where we have used: $\cos(\omega t+\frac{\pi}{4})=\frac{\sqrt{2}}{2}(\cos(\omega t)-\sin(\omega t)$. This expression is not terribly useful in this form, however, since the problem of interest involves turbulent flow.

B. Pulsatile Inner Law

Let's then consider equation (2) $\frac{\partial u}{\partial t}-\frac{\partial U}{\partial t}=\frac{\partial}{\partial y}\left(\nu_{eff}\frac{\partial u}{\partial y}\right)$ where we formally integrate with respect to "y" to give:

$$\int_0^\infty(\frac{\partial u}{\partial t}-\frac{\partial U}{\partial t})dy+\frac{\tau_w}{\rho}=v^*\kappa y\frac{\partial u}{\partial y}\tag{9}$$

Certainly to proceed, we need to be able to estimate the LHS of equation (9). If we use our unsteady estimate for velocity $u=u_s(y)+\Delta U_\infty\left(\cos\omega t-\exp(-\frac{\sqrt{2}}{2}\eta)\cos(\omega t-\frac{\sqrt{2}}{2}\eta)\right)$ i.e. equation (9) we find that:

$$\int_0^\infty(\frac{\partial u}{\partial t}-\frac{\partial U}{\partial t})dy=\frac{\sqrt{2}}{2}\Delta U_\infty\sqrt{\nu_{eff}\omega}\cos(\omega t+\frac{\pi}{4})\tag{10}$$

We can then write:

$$\frac{\sqrt{2}}{2}\frac{\Delta U_\infty}{\kappa v^* y}\sqrt{\nu_{eff}\omega}\cos(\omega t+\frac{\pi}{4})+\frac{v^*}{\kappa y}=\frac{du}{dy}\tag{11}$$

Which we can integrate to give:

$$\begin{aligned}\frac{u}{v^*} &= \frac{1}{\kappa}\left(\frac{\sqrt{2}}{2}\frac{\sqrt{\nu_{eff}\omega}\Delta U_\infty}{v^{2*}}\cos(\omega t+\frac{\pi}{4})+1\right)\ln(y)+c(x,t) \\ &= \frac{1}{\kappa}\left(\frac{\sqrt{2}}{2}\frac{\sqrt{\nu_{eff}\omega}\Delta U_\infty}{v^{2*}}\cos(\omega t+\frac{\pi}{4})+1\right)\ln(\frac{yv^*}{\nu_w})+C(x,t)\end{aligned} \qquad (12)$$

Traditionally for a steady flow we would modify the constant in equation (12) to include $\frac{1}{\kappa}\ln(y)+c=\frac{1}{\kappa}\ln(\frac{yv^*}{\nu_w})-\frac{1}{\kappa}\ln(\frac{v^*}{\nu_w})+C$, however, it is less clear how one should modify the associated constant, i.e. should there be a net unsteady effect? Moreover, the constant certainly must honor the wall boundary condition which for steady flow would simply imply that $u^+=\frac{u}{v^*}=\frac{1}{\kappa}\ln(\frac{yv^*}{\nu_w})+B$ By way of guidance simulation and measurements described in Ribas and Deschamps (2004) suggest that flow within the laminar sublayer is independent of the pulsatile behavior which implies that the wall function C(x,t) should be C=B=5.0

Using this constant we can then write the pulsatile flow modified law-of-the-wall as:

$$u^+=\frac{1}{\kappa}\left(\frac{\sqrt{2}}{2}\frac{\Delta U_\infty\sqrt{\nu_{eff}\omega}}{v^{2*}}\cos(\omega t+\frac{\pi}{4})+1\right)\ln y^+ + B \qquad (13)$$

The steady law-of-the-wall model is well understood but the scaling associated with the unsteady term requires additional consideration. Following Arpaci and Larsen the laminar wall friction can be shown to be:

$$\frac{\tau'_{w\_lam}}{\rho}=\nu_w\Delta U_\infty\sqrt{\frac{\omega}{2\nu_w}}\cos(\omega t+\frac{\pi}{4}) \qquad (14)$$

If we intend to ignore the particular temporal behavior associated with a disturbance we can easily replace the periodic function by the associated root-mean square magnitude as: $\left|\left(\frac{\Delta U_\infty}{U_\infty}\right)^2\mathrm{Re}_s^{-1}\cos(\omega t+\frac{\pi}{4})\right|\rightarrow\left(\frac{\Delta U_\infty}{U_\infty}\right)^2\mathrm{Re}_s^{-1}$. The magnitude associated with the unsteady term is easily assessed but the sign is less readily apparent since either sign is plausible using the pulsatile external flow. We suggest, however, that the net effect of external disturbances will be associated with a favorable pressure gradient implying that we should utilize a negative sign for the unsteady term which permits us to write:

$$u^{+} = \frac{1}{\kappa}\left(1 - \left(\frac{\nu_{eff}}{\nu_{w}}\right)^{1/2}\left(\frac{\tau_{w\_lam}}{\tau_{w\_turb}}\right)\left|\frac{\tau'_{w\_lam}}{\tau_{w\_lam}}\right|\right)\ln y^{+} + B \tag{15}$$

Let's estimate the size of the terms in equation (15). Using the Clauser constant (White (2006)) to estimate the effective viscosity we can write:

1. $\left(\frac{\nu_{eff}}{\nu_{w}}\right)^{1/2} \approx 0.127\,\mathrm{Re}_{\delta}^{1/2} = 0.05\,\mathrm{Re}_{x}^{3/7}$
2. $\frac{\tau_{w\_lam}}{\tau_{w\_turb}} \approx 25\,\mathrm{Re}_{x}^{-5/14}$

So that $\left(\frac{\nu_{eff}}{\nu_{w}}\right)^{1/2}\left(\frac{\tau_{w\_lam}}{\tau_{w\_turb}}\right) \approx O(2)\,\mathrm{Re}_{x}^{1/14}$.

Of more interest is the ratio between the steady laminar and periodic shear magnitudes: $\frac{\nu_{w}\Delta U_{\infty}\sqrt{\frac{\omega}{2\nu_{w}}}}{U_{\infty}^{2}} = \left(\frac{\Delta U_{\infty}}{U_{\infty}}\right)^{2}\mathrm{Re}_{s}^{-1}$ where Re_s is the Stokes Reynolds number $\mathrm{Re}_{s} = \frac{\Delta U_{\infty}\sqrt{\frac{2\nu_{w}}{\omega}}}{\nu_{w}}$. Notice that for low frequency, $\mathrm{Re}_s \gg 1$ and that for high frequency $\mathrm{Re}_s \ll 1$.

Using these expressions we can write:

$$u^{+} \approx \frac{1}{\kappa}\left(1 - 2\,\mathrm{Re}_{x}^{1/14}\left(\frac{\Delta U_{\infty}}{U_{\infty}}\right)^{2}\mathrm{Re}_{s}^{-1}\right)\ln y^{+} + B \tag{16}$$

Equation (16) provides a high frequency periodic modification to the law-of the wall.

Equation (16) is a modified law-of-the-wall which can be used to estimate a periodic flow sensitive skin friction expression. To do this, we consider the inner variable derivation for flat plate skin friction. A classically useful model based upon inner variables is developed by White (2004). In this derivation one solves the approximate momentum equation:

$$\rho u\frac{\partial u}{\partial x} \approx \frac{\partial \tau}{\partial y} \tag{17}$$

where τ is assumed to be the turbulent stress only. Utilizing the inner variables: $u = v^* u^+$ where v^* is the friction velocity $v^* = \left(\frac{\tau_w}{\rho}\right)^{1/2}$. Note that the wall shear and the thereby friction velocity are functions of "x", and $\frac{\partial}{\partial y} = \frac{\partial}{\partial y^+}\frac{\partial y^+}{\partial y} = \frac{v^*}{\nu_w}\frac{\partial}{\partial y^+}$ since $y^+ = \frac{v^* y}{\nu_0}$, Thus equation (17) can be written:

$$v^* \frac{dv^*}{dx}(u^{+2}) \approx \frac{v^*}{\rho \nu_w}\frac{\partial \tau}{\partial y^+} \tag{18}$$

Following White we now integrate between the wall and the boundary layer thickness (in terms of inner variables) as:

$$\frac{dv^*}{dx}\int_0^{\delta^+} u^{+2} dy^+ \approx -\frac{1}{\rho \nu_0}\tau_w = -\frac{v^{*2}}{\nu_w} \tag{19}$$

The kernel of the integral in equation (19) $\int_0^{\delta^+} u^{+2} dy^+$ is where the unsteady behavior associated with equation (16) is utilized.

The basic structure of equation (16) is essentially consistent with the traditional steady law-of-the-wall $u^+ = \frac{1}{\tilde{\kappa}}\ln y^+ + B$ except the $\ln y^+$ coefficient is modified as $\tilde{\kappa} = \kappa\left(1 - 2\,\mathrm{Re}_x^{1/14}\left(\frac{\Delta U_\infty}{U_\infty}\right)^2 \mathrm{Re}_s^{-1}\right)^{-1}$ where κ=0.41 and B=5.0. Note that for small disturbances we can write: $\tilde{\kappa} \approx \kappa\left(1 + 2\,\mathrm{Re}_x^{1/14}\left(\frac{\Delta U_\infty}{U_\infty}\right)^2 \mathrm{Re}_s^{-1}\right)$ The integral is computed as:

$$\int_0^{\delta^+} u^{+2} dy^+ = \frac{\delta^+}{\kappa}\left((\ln \delta^+)^2 + 2(B-1)\ln \delta^+ + (B^2\tilde{\kappa}^2 - 2\tilde{\kappa} + 2)\right) \tag{20}$$

This expression appears to be of rather limited value since it contains the inner law boundary layer thickness, however we can eliminate δ^+ using the law of the wall evaluated at y=δ, i.e. $U^+ = \frac{U}{v^*} = \frac{1}{\tilde{\kappa}}\ln \delta^+ + B$ so we have:

$$\frac{1}{\tilde{\kappa}}\ln\delta^{+} = U^{+} - B$$

$$\delta^{+} = \exp(\tilde{\kappa}(U^{+} - B)) = \frac{\exp\tilde{\kappa}(U^{+})}{\exp(\tilde{\kappa}B)} \tag{21}$$

So that the integral becomes:

$$\int_{0}^{\delta^{+}} u^{+2}dy^{+} = \frac{\exp\tilde{\kappa}(U^{+})}{\exp(\tilde{\kappa}B)}\left(\tilde{\kappa}(U^{+} - B)^{2} + 2(B-1)(U^{+} - B) + (B^{2}\tilde{\kappa}^{2} - 2\tilde{\kappa} + 2)\right) \tag{22}$$

The form of the integral in equation (22) suggests that it will be convenient to work in the variable U^{+} rather than v^{*} itself. Notice that by using $v^{*} = \frac{U}{U^{+}}$; $\frac{dv^{*}}{dx} = -\frac{U}{U^{+2}}\frac{dU^{+}}{dx}$ can readily rewrite equation (19) as:

$$\frac{U}{U^{+2}}\frac{dU^{+}}{dx}\int_{0}^{\delta^{+}} u^{+2}dy^{+} = \frac{1}{\nu_{0}}\frac{U^{2}}{U^{+2}}$$

$$\frac{dU^{+}}{d\,\mathrm{Re}_{x}}\int_{0}^{\delta^{+}} u^{+2}dy^{+} = 1 \tag{23}$$

Equation (23) is a first order ODE that can be solved for U^{+} as a function of Re_x and once we have solved for U^{+} one has immediate access to the skin friction since $U^{+} = \left(\frac{2}{C_{f}}\right)^{1/2} \rightarrow C_{f} = \frac{2}{U^{+2}}$.

The form of the integral in equation is complex and can be simplified. For the steady flow problem with $\tilde{\kappa} = \kappa$ where κ=0.41 and B=5.0, White suggests that $\int_{0}^{\delta^{+}} u^{+2}dy^{+} \approx 8\exp(0.48U^{+})$. White actually uses the more complete Spalding's law-of-the-wall to estimate the form of the integral, but the difference between the current log-law and the Spalding's closure is small. Since White's (2006) approximate result is only valid for the steady problem it cannot be used here, however, examination of the terms in equation (22) suggests that:

$$\frac{\exp(\tilde{\kappa}U^{+})}{\exp(\tilde{\kappa}B)}\left(\tilde{\kappa}(U^{+} - B)^{2} + 2(B-1)(U^{+} - B) + (B^{2}\tilde{\kappa}^{2} - 2\tilde{\kappa} + 2)\right) \approx \frac{3}{2}U^{+2}\frac{\exp(\tilde{\kappa}U^{+})}{\exp(\tilde{\kappa}B)} \tag{22}$$

We plot this result for $\tilde{\kappa} = \kappa$ where κ=0.41 and B=5.0

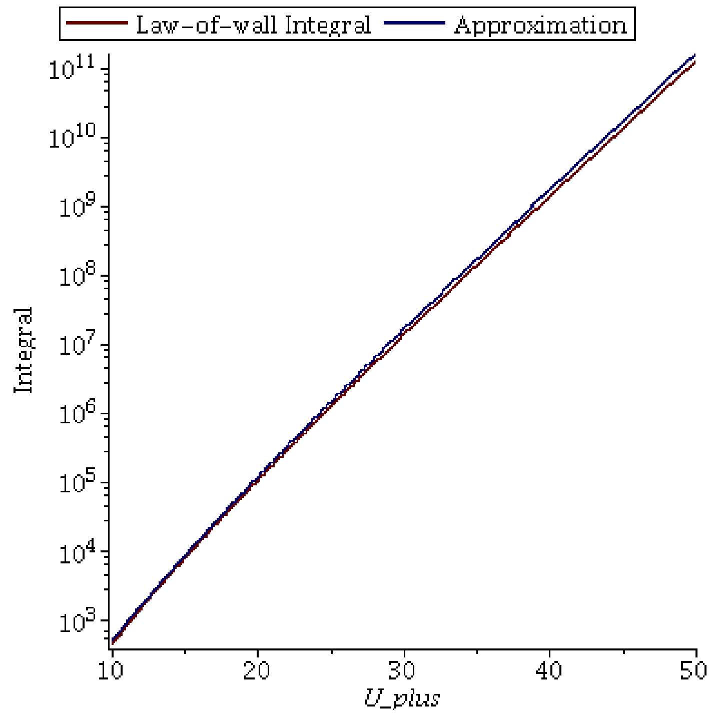


Figure 1. Comparison between full integral expression and the approximation expressed in equation (22).

Comparison between full integral expression equation (22) and the approximation $\frac{3}{2}\tilde{\kappa}U^{+2}\frac{\exp\tilde{\kappa}(U^{+})}{\exp(\tilde{\kappa}B)}$ for $\tilde{\kappa}=\kappa$ where κ=0.41 and B=5.0

Though this approximate integral $\frac{3}{2}\tilde{\kappa}U^{+2}\frac{\exp(\tilde{\kappa}U^{+})}{\exp(\tilde{\kappa}B)}$ can utilized in equation (23) there is value (which will become apparent subsequently) in following White in attempting to approximate the integral by $\frac{3}{2}\tilde{\kappa}U^{+2}\frac{\exp(\tilde{\kappa}U^{+})}{\exp(\tilde{\kappa}B)}\approx C_0\exp(\tilde{\kappa}C_1U^{+})$. An examination of White's result for $\tilde{\kappa}=\kappa$ where κ=0.41 and B=5.0 suggests that $\frac{3}{2}\tilde{\kappa}U^{+2}\frac{\exp(\tilde{\kappa}U^{+})}{\exp(\tilde{\kappa}B)}\approx\frac{3}{2}\tilde{\kappa}\frac{(10)^2}{\exp(\tilde{\kappa}B)}\exp(1.17\tilde{\kappa}U^{+})$ will provide a reasonable comparison since $\frac{3}{2}\tilde{\kappa}\frac{(10)^2}{\exp(\tilde{\kappa}B)}\approx 7.9$ thus recapitulating Whites approximate expression.

Utilizing the approximate integral form $\frac{150\tilde{\kappa}}{\exp(\tilde{\kappa}B)}\exp(1.17\tilde{\kappa}U^{+})$ in equation (23) we can write the separable first order differential equation:

$$\frac{U}{U^{+2}}\frac{dU^{+}}{dx}\int_{0}^{\delta^{+}}u^{+2}dy^{+}=\frac{1}{\nu_0}\frac{U^2}{U^{+2}}$$

$$\frac{150\widetilde{\kappa}}{\exp(\widetilde{\kappa}B)}\int\exp(1.17\widetilde{\kappa}U^{+})dU^{+}=\mathrm{Re}_x \tag{23}$$

The integral $\int U^{+2}\exp\widetilde{\kappa}(U^{+})dU^{+}$ follows as:

$$\frac{U}{U^{+2}}\frac{dU^{+}}{dx}\int_{0}^{\delta^{+}}u^{+2}dy^{+}=\frac{1}{\nu_0}\frac{U^2}{U^{+2}}$$

$$\frac{128.2}{\exp(\widetilde{\kappa}B)}\exp(1.17\widetilde{\kappa}U^{+})=\mathrm{Re}_x \tag{24}$$

This equation in terms of U^+ can be solved as:

$$U^{+}=\frac{1}{1.17\widetilde{\kappa}}\ln\left(\frac{1}{128.2}\exp(\widetilde{\kappa}B)\,\mathrm{Re}_x\right) \tag{25}$$

Using the relationship for $C_f=\frac{2}{U^{+2}}$ we can write:

$$C_f=\frac{2(1.17)^2\widetilde{\kappa}^2}{\ln^2\left(\frac{1}{128.2}\exp(\widetilde{\kappa}B)\,\mathrm{Re}_x\right)}=\frac{2.74\widetilde{\kappa}^2}{\ln^2\left(0.0078\exp(5\widetilde{\kappa})\,\mathrm{Re}_x\right)} \tag{26}$$

Where we recall that $\widetilde{\kappa}=\kappa\left(1-10\left(\frac{\Delta U_\infty}{U_\infty}\right)^2\mathrm{Re}_s^{-1}\right)^{-1}\approx\kappa\left(1+10\left(\frac{\Delta U_\infty}{U_\infty}\right)^2\mathrm{Re}_s^{-1}\right)$. Let's examine this expression for $\widetilde{\kappa}=\kappa$ and B=5 so as to write: $C_f=\frac{0.460}{\ln^2\left(0.06\,\mathrm{Re}_x\right)}$. This expression compares well with White's steady flow classical flat plate formula with is written: $C_f=\frac{0.455}{\ln^2\left(0.06\,\mathrm{Re}_x\right)}$

With access to the effect of a disturbance within the skin friction model via equation (26) it is of particular importance to us is to assess the size of the term: $\left(\frac{\Delta U_\infty}{U_\infty}\right)^2 \mathrm{Re}_s^{-1}$ where Re_s is the Stokes Reynolds number $\mathrm{Re}_s^{-1} = \frac{\sqrt{2}}{2}\frac{\sqrt{\omega \nu_w}}{\Delta U_\infty}$ whereby: $2\,\mathrm{Re}_x^{1/14}\left(\frac{\Delta U_\infty}{U_\infty}\right)^2 \mathrm{Re}_s^{-1} = \sqrt{2}\,\mathrm{Re}_x^{1/14}\left(\frac{\Delta U_\infty}{U_\infty}\right)\frac{\sqrt{\omega \nu_w}}{U_\infty}$.

C. Disturbance Frequency Estimate

The size of the frequency term $\frac{\sqrt{\omega \nu_w}}{U_\infty}$ is obviously an essential part of the problem. The size of the frequency term follows from the external forcing and the local or near wall viscous boundary layer flow field. For true pulsatile or periodic flow, the frequency is externally imposed and is well characterized while for encounters with free-stream disturbances of various scales it is less well known.

Here we consider a family of velocity scales and length scales that permit estimation of the dimensionless frequency term $\frac{\sqrt{\omega \nu_w}}{U_\infty}$. In table 2 we present a family of length and velocity scales associated with external, outer boundary layer and inner variable boundary layer behavior. Length scale range from a maximum of vehicle scale O(1m) to a small scale of laminar sublayer O(1E-7 m). Velocity scales follow from a maximum of free-stream O(1000 m/s) to a minimum of the friction velocity $v^* = \frac{\sqrt{2}}{2} C_f^{1/2} U_\infty \approx O(100 m/s)$. Frequency estimates follow using a velocity scale divided by the length scale. A very wide range of scales is possible. Via the equilibrium assumption, large scale disturbances must transmit their energy through to Kolmogorov scale (Tennekes and Lumley (1972)) which will imply a large frequency $\left(\frac{\delta \nu_w^3}{U_\infty^3}\right)^{-1/4} U_\infty = O(E9)$ and a relatively large dimensionless frequency $\frac{\sqrt{\omega \nu_w}}{U_\infty} \approx O(1) - O(1E-1)$. This large frequency estimate is consistent with the well-known k-ω turbulence model "rough wall". boundary condition (See Wilcox 2002) where $\omega_{wall} = \frac{2500 \nu_w}{k_r^2}$

where "k" is an effective roughness length scale. Wilcox recommends that $k^+ = 5 \rightarrow k = 5\frac{\nu_w}{v^*} \approx \frac{5\sqrt{2}}{\sqrt{C_f}}\frac{\nu_w}{U_\infty}$. Using this value we estimate that: $\omega_{wall} = O(\frac{100C_f U_\infty^2}{\nu_w}) \approx O(1E9)$

Length	Velocity	Frequency ω	$\frac{\sqrt{\omega\nu_w}}{U_\infty}$	$\frac{\sqrt{\omega\nu_w}}{U_\infty}$ Order of Magnitude*	Comment
$L_{vehicle}$	U_∞	$\frac{U_\infty}{L_{vehicle}}$	$\mathrm{Re}_{L\_vehicle}^{-1/2}$	O(1E-4)	Vehicle transit
δ	U_∞	$\frac{U_\infty}{\delta}$	$\mathrm{Re}_{\delta}^{-1/2}$	O(1E-3)	Outer B.L.
$\frac{\nu_w}{U_\infty}$	U_∞	$\frac{U_\infty^2}{\nu_w}$	1	1	Laminar sublayer
$\left(\frac{\delta\nu_w^3}{U_\infty^3}\right)^{1/4}$	U_∞	$\left(\frac{\delta\nu_w^3}{U_\infty^3}\right)^{-1/4} U_\infty$	$\mathrm{Re}_{\delta}^{-1/8}$	O(1E-1)	Kolmogorov
δ	v^*	$\frac{v_w^*}{\delta}$	$\left(\frac{v^*}{U_\infty}\right)^{1/2} \mathrm{Re}_{\delta}^{-1/2}$	O(1E-5)	Inner/Outer transition
$\frac{\nu_w}{U_\infty}$	v^*	$\frac{v^* U_\infty}{\nu_w}$	$\left(\frac{v^*}{U_\infty}\right)^{1/2}$	O(1E-2)	Log Law
$\left(\frac{\delta\nu_w^3}{U_\infty^3}\right)^{1/4}$	v^*	$\left(\frac{\delta\nu_w^3}{U_\infty^3}\right)^{-1/4} v^*$	$\left(\frac{v^*}{U_\infty}\right)^{1/2} \mathrm{Re}_{\delta}^{-1/4}$	O(1E-3)	Inner law Kolmogorov

Table 2.. Estimates for vehicle turbulent boundary layer frequency scales. Estimated magnitudes follow for νw=1x10-4 m2/s, U∞=1000 m/s, Lvehicle=1m and Rex=1E8.

We will examine two classes of frequency behavior: (1) Flow dominated by basically isotropic free-stream turbulence with high frequency content that is dominated by the smallest possible viscous scale. This behavior is the classical grid generated free-stream boundary layer interaction problem for which we have significant data which can be used to substantiate the analytical development of the model and (2) flow dominated by the passage of a reentry vehicle through atmospheric disturbances, the problem of particular interest here, but where we have little data with which to ascertain the validity of the estimates.

III. RESULTS

A. Small Scale, High Frequency, Isotropic Grid Induced Free-Stream Turbulence

"Small scale" isotropic free-stream turbulence is usually generated by grids upstream of the flow test section and is primarily characterized by a turbulence intensity, which for our purposes is given by $\left(\frac{\Delta U_\infty}{U_\infty}\right)$. The smallest frequency associated with the free-stream can be estimated from $\omega_{grid} = \frac{U_\infty}{\lambda}$ where λ is the Taylor micro-scale length scale (Roach 1987). For grid induce turbulence the Taylor microscale is related to grid mesh diameter "d" and the distance from the grid. A more useful result for a free flight vehicle is to correlate the Taylor micro-scale to the associated boundary layer thickness as: $\lambda \approx \delta \, \mathrm{Re}_\delta^{-1/2}$. The associated Kologorov scale $\left(\frac{\delta \nu_w^3}{U_\infty^3}\right)^{-1/4} U_\infty$ with the length scale λ would them imply that the dimensionless frequency is $\frac{\sqrt{\omega \nu_w}}{U_\infty} = \mathrm{Re}_\delta^{-1/16}$. Thus, one can estimate: $\frac{\tilde{\kappa}}{\kappa} \approx 1 + \sqrt{2}\left(\frac{\Delta U_\infty}{U_\infty}\right)$ where we estimate that $\frac{\mathrm{Re}_\delta^{1/14}}{\mathrm{Re}_\delta^{1/16}} = O(1)$. The magnitude of the external disturbances is rather more easily estimated with $\left(\frac{\Delta U_\infty}{U_\infty}\right) \approx I$ where "I" is the turbulence intensity. Using the closure suggested by we can readily estimate the increase in skin friction associated with free-stream turbulence. We plot:

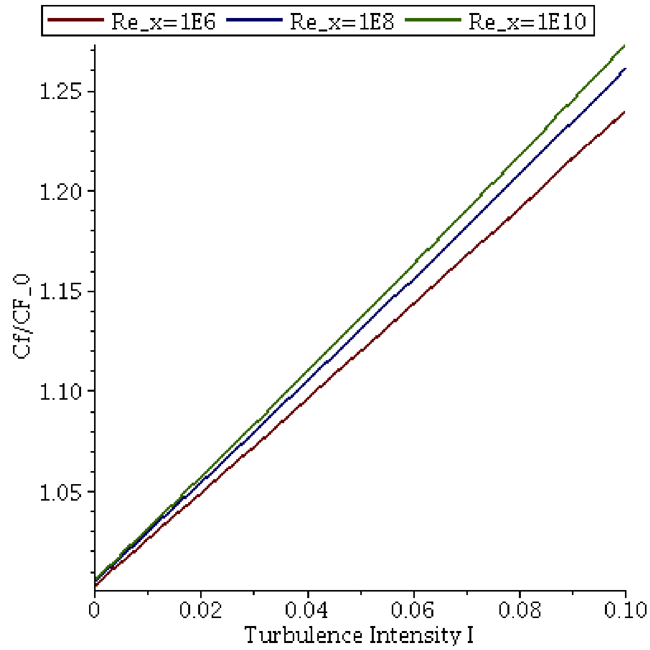


Figure 2. Skin friction enhancement due to free stream turbulence using equation (26)

Figure 2. provides a depiction of the skin friction enhancement due to free stream turbulence using equation (26) with $C_f = \dfrac{2.74\tilde{\kappa}^2}{\ln^2\left(0.0078\exp(5\tilde{\kappa})\,\mathrm{Re}_x\right)}$ and $\tilde{\kappa} \approx \kappa(1+\sqrt{2}I)$. Due to the importance of estimating the enhancement of skin friction (and by the Reynolds analogy) heat transfer associate with free-stream turbulence a number of researchers have proposed analytical and semi-empirical based expressions (Kondjoyan et. al. 2002) to estimate the skin friction enhancement. Simonich and Bradshaw (1978) provide a simple (and well regarded) classical model as:

$$\frac{C_f}{C_{f\_0}} = 1 + 2I \tag{27}$$

Where $C_{f\_0} = C_f(I=0)$. A similar model discussed by Meier and Kreplin (1980) takes the form:

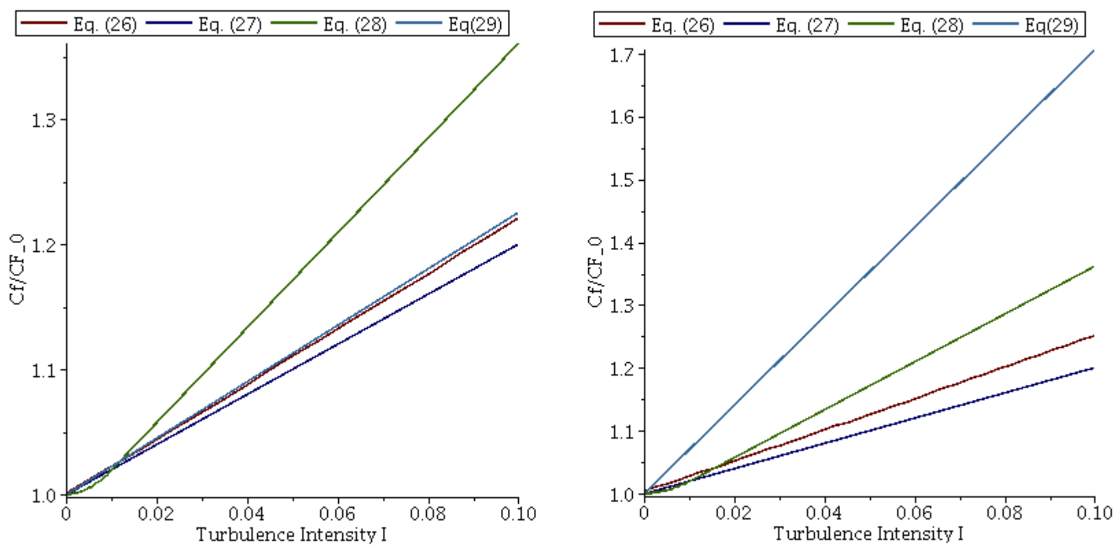
$$\frac{C_f}{C_{f\_0}} = \begin{cases} 190I^2 & 0 < I < 0.01 \\ 3.8I - 0.019 & I > 0.01 \end{cases} \tag{28}$$

An empirical model proposed by Blair and Werle (1980) explicitly includes Reynolds number information in the formulation as:

$$\frac{C_f}{C_{f\_0}} = 0.98 + 1.92\left(\frac{\mathrm{Re}_\theta}{1000}\right)^{0.4} \tag{29}$$

where Re_θ is the momentum thickness Reynolds number. We can relate Re_θ to Re_x as: $\mathrm{Re}_\theta \approx 0.015\,\mathrm{Re}_x^{6/7}$.

Figure 2 compares the skin friction enhancement using equation (26), and the three models, i.e. equations (27), (28) and (29) for Re_x=1E5 (low) and Re_x=1E7. We see from figure 2 that for the low (transitional) Reynolds number that the current model, the Simonich and Bradshaw expression and the Blair and Werle expressions are in good agreement. The Meier and Kreplin expression tends to over-predict the skin friction enhancement. At the higher (and more practical) Reynolds number Re_x=1E7 the explicit Reynolds number dependence associated with equation (29) $\dfrac{C_f}{C_{f\_0}} = 0.98 + 1.92\left(\dfrac{\mathrm{Re}_\theta}{1000}\right)^{0.4}$ is not viable for Reynolds numbers larger than Re_x=5E5.



(a) Re_x=1E5 (b) Re_x=1E7

Figure 3. Comparison between analytical estimate via equation (26) and the classical/semi-empirical models equations (27), (28) and (29) for (a) Rex=1E5 and (b) Rex=1E7. Equation (29) while useful for Rex=1E5 is not applicable for the higher Reynolds number.

Figure 3. suggests that derived expression, i.e. equation (26) is viable to provide estimates for skin friction enhancement due to free-stream turbulence which suggests that this approach may be viable for large scale free-stream effects.

B. Large Scale Free-Stream Disturbances

Our focus is on the boundary layer skin friction enhancement effect associated with the flight of reentry vehicles through atmospheric disturbances. Utilizing this approach for the large scale problem may be possible by examining frequency estimates consistent with the transit of the vehicle through large scale disturbances. A simple estimate of frequency, say, $\omega_{vehicle} = \frac{U_\infty}{L_{vehicle}} = O(1E3)Hz$ would seem appropriate. Let's examine some very rough estimates for the terms in this expression:

ν_w	=	$1x10^{-4}$ m^2/s
U_∞	=	1000 m/s
$L_{vehicle}$	=	1m

We immediately see that the vehicle scale is much larger than the Taylor scale: $\frac{\lambda}{L_{vehicle}} \propto \frac{\delta}{L} \propto \mathrm{Re}_x^{-9/7} <<< 1$

As before, the external excitation, regardless of the source will ultimately be dissipated via the Kolmogorov scales and frequencies. Hence an essential computation will be the Kolmogorov frequency as estimated using $\left(\frac{L_{vehicle}\nu_w^3}{U_\infty^3}\right)^{-1/4} U_\infty$ that implies that $\frac{\sqrt{\omega\nu_w}}{U_\infty} \propto L_{vehicle}^{-1/8} \propto \mathrm{Re}_x^{-1/8}$. For the notional value of Re_x=O(1E8) we expect that $\frac{\sqrt{\omega\nu_w}}{U_\infty} \propto O(\frac{1}{10})$. This value is an order of magnitude smaller than the homogeneous turbulence behavior discussed previously. Nonetheless, this effect could be important suggesting that we should examine atmospheric free-stream turbulence intensity magnitudes. A survey by Reidel and Sitzmann (1998) suggest that so called clear air turbulence intensity is less than 0.1% while flights through observable clouds can be very large with fluctuation on the order of 1-10%. In all cases, these intensities were measured using subsonic aircraft where the associated free-stream velocities were from 100-200 m/s. For a notional velocity of 1000 m/s we could expect that clear air turbulence intensities may be on the order of 0.02% while observable clouds are less than 2%

A basic upper bound estimate for passage of a high speed vehicle through observable clouds suggests that I=O(2%) and $\frac{\sqrt{\omega \nu_w}}{U_\infty} \propto O(\frac{1}{10})$ whereby we estimate that $\frac{C_f}{Cf_0} \approx 1.01$. This suggests that large scale/external disturbances may induce a relatively small increase in wall pressure and may be safely ignored for a preliminary modeling effort.

IV. CONCLUSIONS

A classical inner law model for wall friction has been modified to include the effect of a periodic sinusoidal free-stream for turbulent boundary layers. This model serves a surrogate for the interaction of flight vehicles with atmospheric disturbances. Consistent with experience, the skin friction expression that is derived suggests that free-stream disturbances can cause enhancement of the mean skin friction. Considering the influence of grid generated free stream turbulence in the laminar sublayer/log law region (small scale/high frequency) the model recovers the well-known shear layer enhancement suggesting an overall validity for the approach. The effect on the wall shear associated with the lower frequency due to the passage of a reentry vehicle through large (vehicle scale) atmospheric disturbances is likely small i.e. on the order 1% increase for turbulence intensities on the order of 2%. The increase in wall pressure fluctuation which is directly proportional to the wall shear stress is shown to be correspondingly small.

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