Dual Pricing Algorithm in ISO Markets

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***Abstract*—The challenge to create efficient market clearing prices in centralized day-ahead electricity markets arises from inherent non-convexities in unit commitment problems. When this aspect is ignored, marginal prices may result in economic losses to market participants who are part of the welfare maximizing solution. In this essay, we present an axiomatic approach to efficient prices and cost allocation for a revenue neutral and non-confiscatory day-ahead market. Current cost allocation practices do not adequately attribute costs based on transparent cost causation criteria. Instead we propose an ex post multi-part pricing scheme, which we refer to as the Dual Pricing Algorithm. Our approach can be incorporated into current day-ahead markets without altering the market equilibrium.**

***Index Terms*—Centralized day-ahead electricity market, non-convex pricing, power system economics, mixed-integer linear programming.**

I. Introduction

We examine pricing and cost allocation in ISO energy markets with non-convexities (indivisibilities). In the day-ahead and the real-time electricity market, the objective is to find the surplus maximizing dispatch (including unit commitment), where each participant's bid and offer functions are linear with continuous and binary variables. The binary variables in the bids and offers make the market auction non-convex, and the marginal cost of the economically efficient solution cannot guarantee that the total production cost is recovered for all generators. Pricing practice in many ISOs reruns the market relaxing the minimum operating level (MOL). This enables certain generators at MOL to set the price, but also may create an incentive to price chase, which is inefficient because it increases production beyond the optimal dispatch [1].

Non-convex markets may have no single market-clearing equilibrium price and therefore require multi-part prices, first suggested in [2]. Select proposals include using convex hull pricing [3], reducing volatility with a Benders'-like sub-problem [4], zero-sum uplift payments in a quadratic program [5], semi-Lagrangean relaxation [6], and a primal-dual formulation [7]. A thorough comparison can be found in [8], with additional literature in [9]-[11]. Approaches to date either propose a new day-ahead market design for dispatching and pricing—abandoning current practices—or an ex post cost allocation that does not always guarantee revenue neutrality and non-confiscation (see Section II). Here we present an ex post cost allocation that can be easily integrated into ISO market software, and maintains the optimal dispatch solution.

In this letter we propose a dual pricing algorithm (DPA), which is a linear program and incorporates fundamental principles of efficient market design to determine cost allocation. The DPA is transparent and maintains incentive compatibility to prevent economic losses to market participants in the optimal solution. Instead of lost opportunity cost payments to those not selected in the optimal solution, which can cause revenue inadequacy, we prefer to prevent deviations from the optimal dispatch by imposing penalties equal to or greater than the lost opportunity cost. The cost allocation does not alter current dispatching or pricing at the equilibrium, but introduces a multi-part price to cover costs not covered by the LMP alone.

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II. Economic Principles for Cost Allocation

Under the Federal Power Act (FPA), prices must be just and reasonable, and not unduly discriminatory. This implies prices can be discriminatory to achieve economic efficiency. Discriminatory pricing can benefit market efficiency by signaling the need for economic investments. Uniform, non-discriminatory pricing may require that costs are distributed too broadly to provide a meaningful signal in the short-term for efficient operations as well as long-term investments.

At the economic equilibrium, supply and demand are balanced and are supported by market-clearing prices. Then, cost allocation ex post results in a multi-part pricing scheme. Our axiomatic approach requires non-confiscation, i.e., the producers are paid at least their total production cost and consumers pay no more than their willingness to pay. Additionally, the pricing scheme must be revenue neutral, i.e., total revenues paid by the consumers equals the total payments to producers. Revenue neutrality is easily satisfied when we assume infinitely valued consumption, an assumption that leads to false conclusions about revenue adequacy. As such we model price-responsive demand in our analysis.

III. Single-Period, Single-Bus Dual Pricing Algorithm

For simplicity, we present a single period model at a single node. Temporal and network aspects, as well as reserves and other requirements can be added using linear constraints with minor notational changes, and therefore are omitted here.

A. The Unit Commitment Model

The canonical unit commitment problem is:

$$\max\ MS = \sum_{i\in D} b_i d_i - \sum_{i\in G}\left(c_i p_i + c_i^{\mathrm{SU}} z_i\right) \qquad (1a)$$

$$\sum_{i\in D} d_i - \sum_{i\in G} p_i = 0 \qquad \lambda \qquad (1b)$$

$$p_i^{\min} z_i \le p_i \le p_i^{\max} z_i \qquad \forall i\in G \qquad \beta_i^{\max}, \beta_i^{\min} \qquad (1c)$$

$$0 \le d_i \le d_i^{\max} \qquad \forall i \in D \qquad \alpha_i^{\max} \qquad (1d)$$

$$z_i \in \{0,1\} \qquad \forall i\in G \qquad \delta_i \qquad (1e)$$

where b_i is demand i's marginal value of power consumption, c_i is generator i's marginal generation cost, c_i^{SU} is generator i's startup cost, $d_i^{\max}$ is the upper bound on demand i's consumption, and $p_i^{\max}$ and $p_i^{\min}$ are the upper and lower bounds on generator i's output. Variables are demand consumption d_i, generator output p_i, and relaxed unit commitment z_i.

B. Post Unit Commitment Dual Problem

Problem (1) is a MILP; when (1e) is replaced with $z_i = z_i^*$, (1) becomes a linear program with the corresponding dual:

$$\min RC = \sum_{i\in D} d_i^{\max}\alpha_i^{\max} + \sum_{i\in G} z_i^*\delta_i \qquad (2a)$$
$$\lambda + \alpha_i^{\max} \geq b_i \qquad \forall i \in D \qquad d_i \qquad (2b)$$
$$-\lambda + \beta_i^{\max} - \beta_i^{\min} \geq -c_i \qquad \forall i \in G \qquad p_i \qquad (2c)$$
$$\delta_i - p_i^{\max}\beta_i^{\max} + p_i^{\min}\beta_i^{\min} = -c_i^{\mathrm{SU}} \qquad \forall i \in G \qquad z_i \qquad (2d)$$
$$\alpha_i^{\max}, \beta_i^{\max}, \beta_i^{\min} \geq 0 \qquad \forall i \in D \cup G \qquad (2e)$$

The dual constraints (2c) and (2d) show that a generator's profits from LMP (λ) payments alone may not be enough to cover startup costs, thus the need for multi-part pricing.

C. Dual Pricing Algorithm (DPA)

We formulate the single-period, single-bus DPA model, for which we define the sets $GUP = \{i|i \in G, \delta_i^* < 0\}$, $G^+ = \{i|i \in G, p_i > 0\}$, $D^+ = \{i|i \in D, d_i > 0\}$, and $D^0 = \{i|i \in D, d_i = 0\}$. If $GUP = \emptyset$, then the outcome is non-confiscatory and no ex post cost allocation is required. However, if $GUP \neq \emptyset$, then we introduce a dual modification to the LMP (λ^{DPA}) and associated uplift payments can be derived through solving the dual pricing problem (3) below.

$$\min \sum_{i\in D} d_i^* u_i^{\mathrm{pd}} + \sum_{i\in G} p_i^* u_i^{\mathrm{p}} \qquad (3a)$$
$$\sum_{i\in D^+} d_i^*\left(u_i^{\mathrm{pd}} - u_i^{\mathrm{cd}}\right) + \sum_{i\in G^+} p_i^*\left(u_i^{\mathrm{p}} - u_i^{\mathrm{c}}\right) = 0 \qquad (3b)$$
$$\Psi_i = d_i^*\left(b_i - \lambda^{\mathrm{DPA}} + u_i^{\mathrm{pd}} - u_i^{\mathrm{cd}}\right) \qquad \forall i \in D^+ \qquad (3c)$$
$$\Pi_i = p_i^*\left(\lambda^{\mathrm{DPA}} - c_i + u_i^{\mathrm{p}} - u_i^{\mathrm{c}}\right) - c_i^{\mathrm{SU}} \qquad \forall i \in G^+ \qquad (3d)$$
$$\lambda^{\mathrm{DPA}} \geq b_i \qquad \forall i \in D^0 \qquad (3e)$$
$$\Psi_i, \Pi_i \geq 0 \qquad \forall i \in D^+ \cup G^+ \qquad (3f)$$
$$u_i^{\mathrm{p}}, u_i^{\mathrm{c}}, u_i^{\mathrm{pd}}, u_i^{\mathrm{cd}} \geq 0 \qquad \forall i \in D^+ \cup G^+ \qquad (3g)$$

where u_i^{pd} and u_i^{cd} are uplift payments and charges, respectively, Ψ_i and Π_i are the demand value and generator profit functions, respectively, and λ^{DPA} is the new LMP.

Summing (3c) over *D* and (3d) over *G*, we maintain market surplus. Uplift payments and charges are balanced through the revenue neutrality condition in (3b), and non-confiscation in (3f). The optimal solution to the DPA is revenue neutral, i.e., all revenues are distributed to cover all incurred costs, and therefore revenue adequate. As formulated above, the DPA is always feasible and may have multiple optimal solutions. Additional constraints can condition the resulting prices.

IV. Example

Considering a one period example with the data provided below, the optimal dispatch is Generator A = 40 MW and Generator B = 90 MW with λ = \$60/MWh, resulting in Generator B operating at a \$500 loss. The DPA determines the modified LMP to be λ^{DPA} = \$65.56/MWh, which recovers Generator B's startup costs by charging an additional \$5.56/MWh to both buyers. Since the new LMP is above the bid of Buyer 2, Buyer 1 pays an additional \$1.37/MWh (u_1^{pd}) making Buyer 2 break even by receiving a payment of \$4.56/MWh (u_2^{cd}). The λ^{DPA} better reflects the incremental cost of serving load and the resulting settlement leaves Generator B and Buyer 2 at a break-even point, Generator A with increased profits, and Buyer 1 with decreased additional value.

TABLE 1. GENERATOR COSTS

Gen	Marginal cost (\$/MWh)	Startup cost (\$)	Min (MW)	Max (MW)
A	40	500	0	40
B	60	500	10	200

TABLE 2. DEMAND FUNCTION

Buyer	Value (\$/MWh)	Max demand (MW)
1	100	100
2	61	30

V. Conclusion

Due to the non-convexities in markets, uplift payments may be necessary to satisfy non-confiscation. Some uplift cost allocation mechanisms spread the burden of payment over customers, without considering who benefits. We propose the DPA which is an ex post pricing scheme that reallocates the optimal market surplus using incremental costs. This approach can be easily extended to multiple time horizons with more operational constraints, e.g., reserve requirements. The DPA is a linear program, and can be easily incorporated into current ISO software and solved quickly. Through specific conditioning, the prices can reflect preferences such as minimizing the relative deviation from the dispatch LMP. The DPA guarantees non-confiscation of demand bids and generator supply offers, and a revenue neutral outcome. Actual implementation of the DPA would first require full-scale ISO validation and a stakeholder review process. Further work can compare the DPA to other practical approaches such as ELMP and extend the algorithm to a transmission network.

VI. References

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