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Title: Analytic computation of average energy of neutrons inducing fission

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Introduction and Theory

The objective of this report is to describe how I analytically computed the average energy of neutrons that induce fission in the bare BeRP ball. The motivation of this report is to resolve a discrepancy between the average energy computed via the FMULT and F4/FM cards in MCNP6 by comparison to the analytic results.

The intrinsic efficiency of a detector may be analytically derived using a model, given in Rian's thesis [1], that is based on the probability that a neutron will interact with and be absorbed by the material.

$$\epsilon_{uncollided}(E) = \left(1 - e^{-\Sigma_T(E)x}\right)\frac{\Sigma_a(E)}{\Sigma_T(E)} \tag{1}$$

The model may be made more accurate by adding contributions due to scattering,

$$\epsilon_{collided}(E) = \epsilon_{uncollided}(E) + \epsilon_{scatter\ contribution}(E) \tag{2}$$

where,

$$\epsilon_{scatter\ contribution}(E) = \left[\left(1 - e^{-\Sigma_T(E)x}\right)\frac{\Sigma_s(E)}{\Sigma_T(E)} - \epsilon_{uncollided}(E)\right]\epsilon_{uncollided}(E) \tag{3}$$

and it may be shown that,

$$\epsilon_{collided}(E) = \epsilon_{uncollided}(E)\left[1 + \left(1 - e^{-\Sigma_T(E)x}\right)\frac{\Sigma_s(E)}{\Sigma_T(E)} - \epsilon_{uncollided}(E)\right]^{order} \tag{4}$$

$\Sigma_T(E)$, $\Sigma_a(E)$, and $\Sigma_s(E)$ are energy dependent total, absorption, and elastic scattering macroscopic cross sections, respectively, x is the thickness of the material seen by the neutron, and $order$ is the number of collisions that have occurred. The exponential term represents the probability that a neutron will have an interaction in the material and the ratio of cross sections represents the probability that the interaction will be either an elastic scatter or absorption event. Equation (4) would represent efficiency due to a single elastic scatter if $order = 1$ and several elastic scatters if $order > 1$.

Multiplying Equation (1) by the probability of a neutron being emitted at an energy E results in a model that represents the energy spectrum of neutrons inducing a particular reaction. In the case of Equation (5), the reaction being induced is fission,

$$N_{uncollided}(E) = \left(1 - e^{-\Sigma_T(E)x}\right)\frac{\Sigma_f(E)}{\Sigma_T(E)}\chi(E) \tag{5}$$

where $\chi(E)$ is assumed to be a Watt-fission spectrum,

$$\chi(E) = e^{-\frac{E}{a}} \sinh\left(\sqrt{Eb}\right) \tag{6}$$

where a and b are material dependent constants. Contributions from elastic scattering may be included in Equation (5) in a manner synonymous to Equation (4),

$$N_{collided}^{order}(E) = N_{uncollided}(E)\left[1 + \left(1 - e^{-\Sigma_T(E)x}\right)\frac{\Sigma_S(E)}{\Sigma_T(E)}\chi(E) - N_{uncollided}(E)\right]^{order} \tag{7}$$

The average energy of neutrons inducing fission may be obtained by computing the average of Equation (7).

Methods and Results

Cross section data was obtained from the NNDC website [2]. Induced fission chains in ^{239}Pu in the BeRP ball are driven by spontaneous fission events from ^{240}Pu, and so $\chi(E)$ is modeled as a ^{240}Pu Watt-fission spectrum with $a = 0.996\ MeV$ and $b = 2.842 \frac{1}{MeV}$. Although neutrons originating from induced fissions in ^{239}Pu can induce more fissions in the ^{239}Pu, for simplicity, it is assumed that the ^{240}Pu is the only source of fission inducing neutrons. It is further assumed that, on average, a neutron born in the BeRP ball will see a thickness $x \approx r_{BeRP} = 3.7938\ cm$.

Figure 1 is a plot of the spectrum of energies inducing fission in the BeRP ball for $order = 0, 5, \dots, 25$ with the Watt-fission spectrum for ^{239}Pu from 2 MeV incident neutrons and ^{240}Pu, and Table 1 summarizes the average energies computed.

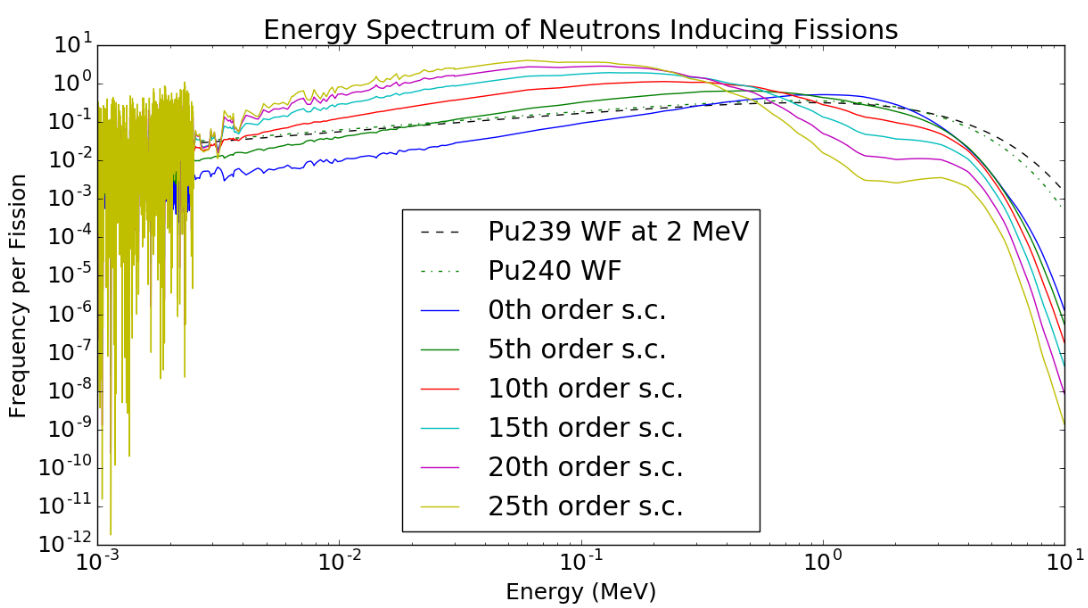

Figure 1 – Spectrum of energies inducing fission in the BeRP ball for several scattering orders.

Table 1 – Average energy of neutrons inducing fission in the BeRP ball for several scattering orders.

Order	Average Energy (MeV)
0	8.67E-01
5	5.34E-01
10	3.03E-01
15	1.86E-01
20	1.31E-01
25	1.02E-01

Conclusions

The average energies in Table 1 are in better agreement with the average energies given for the F4/FM card in [3], which lists the average energy of neutrons inducing fission for the BeRP ball as 5.76E-01 MeV, than that given for the FMULT card, which lists the average energy as 1.93 MeV. The energies in Table 1 decrease, which is physically meaningful because neutrons lose energy as they scatter. Figure 1 indicates that as more scattering occurs, a larger fraction of the neutron population has energies in the epithermal to thermal range than the fast range, which is indicative of neutrons losing energy to scattering. Although it is unlikely that fast neutrons will experience more than a few collisions in the BeRP ball due to their mean free path at that energy, Figure 1 suggests that the model adequately captures the elastic scattering contributions i.e. the spectrum becomes more thermalized as more collisions occur. The relative agreement between the average energies computed via the analytic technique given here and the F4/FM card suggest that the FMULT card result may either be in error or reporting a different quantity than anticipated.

References

[1] Rian Mustafa. "A New High Energy Resolution Neutron Transmission Detector at the Gaerttner LINAC Center and Isotopic Molybdenum Total Cross Section Measurements in the keV-Region". Ph.D. Dissertation submitted to Rensselaer Polytechnic Institute July 2013.

[2] Brookhaven National Laboratory. National Nuclear Data Center: Sigma – Evaluated Nuclear Data File (ENDF) Retrieval & Plotting. Accessed Aug. 12th, 2016. < http://www.nndc.bnl.gov/sigma/>

[3] Alexander Clark. "Using MCNP6 to estimate fission – inducing neutron energies, neutron multiplicity statistics, and neutron multiplication for the BeRP ball". LA-UR-…