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Optimization of Compton Source Performance through Electron Beam Shaping

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Abstract. We investigate a novel scheme for significantly increasing the brightness of x-ray light sources based on inverse Compton scattering (ICS) - scattering laser pulses off relativistic electron beams. The brightness of ICS sources is limited by the electron beam quality since electrons traveling at different angles, and/or having different energies, produce photons with different energies. Therefore, the spectral brightness of the source is defined by the 6d electron phase space shape and size, as well as laser beam parameters. The peak brightness of the ICS source can be maximized then if the electron phase space is transformed in a way so that all electrons scatter off the x-ray photons of same frequency in the same direction, arriving to the observer at the same time. We describe the x-ray photon beam quality through the Wigner function (6d photon phase space distribution) and derive it for the ICS source when the electron and laser rms matrices are arbitrary.

INTRODUCTION

Currently there is a strong national need for high quality light sources at hard X-rays [1]. The quality of a light source is characterized by its brightness, or photon density in phase space. Larger brightness corresponds to radiation with a higher degree of coherency and thus permits higher resolution in imaging experiments. Relativistic electron beams are routinely used to generate radiation above optical frequencies. Over the past 50 years light sources based on synchrotron radiation have improved their average brightness by over 15 orders of magnitude [2, 3]. The 3rd and 4th generation light sources produce radiation via magnetic devices known as undulators arrays of alternating permanent magnets which wiggle the electron trajectory. The wavelength of radiation generated by relativistic beams in these devices is on the order of $\lambda_{x-ray} \sim \lambda_u/(2\gamma^2)$. The undulator wavelength λ_u is limited to $\sim 1\ cm$ for practical devices. As a result, generating X-rays with wavelengths < 1 nm requires multi-GeV electron beams. Since the accelerator size and cost scales with energy, the facilities are correspondingly large and expensive.

A large-amplitude electromagnetic wave can also serve to undulate the trajectory of an electron beam. The electron trajectory in the wave field deviates from the straight line motion in a manner similar to that of an undulator, resulting in radiation. Alternatively, this process can be viewed as inverse Compton scattering (ICS) in which photons increase their energy after scattering off the relativistic electrons [4, 5]. Regardless of viewpoint, the end result is the potential to generate hard X-rays using optical wavelength light and 0.1 GeV-range electron beams. X-ray ICS sources have been proposed and demonstrated over the past decade [6, 7, 8]. It produces photons with energies of tens of keV. Estimates for the peak brightness of these sources are close to each other and are on the order of $10^{20} - 10^{22} ph/mm^2/mrad^2/s/0.1\%BW$, more than 10 orders of magnitude below the estimate for hard x-ray free electron lasers (FELs). As a result, ICS sources based on current technology cannot compete with FELs [9]. Significant increases in brightness are needed to make ICS sources more attractive.

The spectrum of a single-particle ICS radiation strongly depends on its energy γ and angle at which it travels in respect to the axis $x' = dx/dz$ (plots (a) and (b) in Fig.1). As a result, angular divergence and energy spread of the electron beam increase the bandwidth $\Delta w/w$ of the backscattered radiation as $(\gamma\Delta x')^2$ and $2\Delta\gamma/\gamma$, respectively [10, 11]. For example, both effects result in the same 1% ICS bandwidth for the following beam parameters: 50 MeV beam having 250 keV energy spread, 10 μm normalized emittance, and 100 μm rms spot size. However, the beam phase space may be conditioned by redistributing electrons in the phase space so that high energy electrons travel at large angles and low energy electrons at small angles. In this case all the electrons emit photons of the same energy in some direction (plot (c) in Fig.1) and ICS peak brightness may reach the limit of a single-electron radiation. Collimation of emitted radiation at this angle would result in a significant increase in ICS source quality compared to the normal, unconditioned case. In this manuscript we find the brightness of an ICS source as a function of the

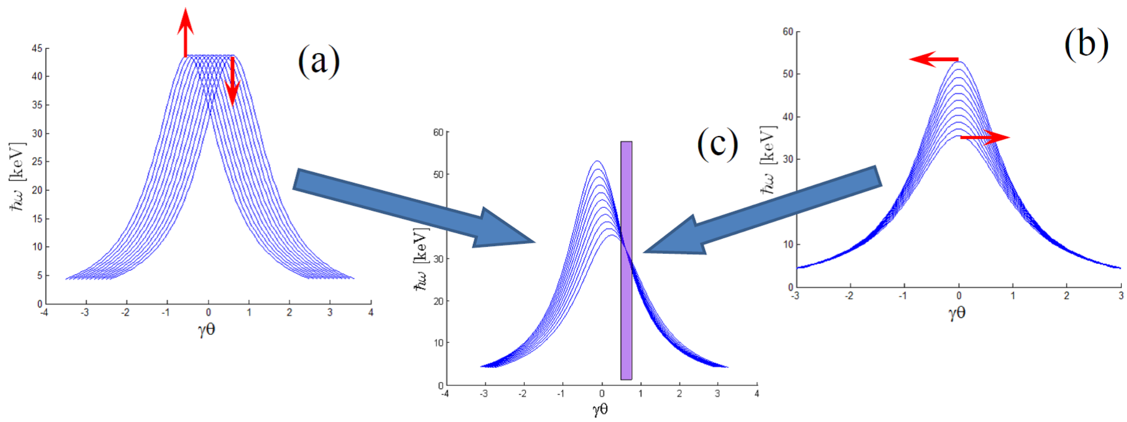

FIGURE 1. ICS spectra emitted by different electrons due to (a) finite angular divergence, (b) finite energy spread, and (c) imposed $x' - \gamma$ correlation in the beam.

6-dimensional electron distribution in the phase space.

6D WIGNER FUNCTION

First, we introduce the 6D Wigner function as an auto-correlation function of the radiated electric field to characterize the distribution of emitted photons in the phase space, based on the Quantum Mechanical Wigner function introduced by Eugene Wigner in 1932 [12]:

$$W_{6d}(\vec{r}_{2d}, t, \vec{k}_{w2d}, w_w; z) = W_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_{rad}(\vec{r}_{2d} + \frac{\vec{\xi}_{2d}}{2}, t + \frac{\xi_t}{2}; z) E^*_{rad}(\vec{r}_{3d} - \frac{\vec{\xi}_{3d}}{2}, t - \frac{\xi_t}{2}; z) e^{i\vec{k}_w \vec{\xi}_{2d} + i w_w \xi_t} d\vec{\xi}_{2d} d\xi_t \qquad (1)$$

This Wigner function is an extension of description introduced by K.J.Kim for describing transverse Brightness of light sources [13]. The description of radiation with full 6D Wigner function is necessary since we are interested in increasing the brightness of a source by narrowing its bandwidth.

SINGLE ELECTRON WIGNER FUNCTION

Radiation of an electron interacting with a plane wave

In the Far Field approximation radiation of a charged particle moving along the trajectory $\vec{r}(t')$ can be found as [13, 14]:

$$\vec{E} = \frac{q}{c^2 R}\left[\vec{n} \times \left[\vec{n} \times \frac{d^2\vec{r}[t'(t)]}{dt^2}\right]\right] \qquad (2)$$

where $R = \sqrt{X^2 + Y^2 + Z^2}$ is the distance to an observer from the origin of the lab frame, $\vec{n} = \vec{R}/\mathrm{R}$ is the unit vector pointing the direction to the observer, q is a particle charge, c is the speed of light, t' is the emitted (retarded) time, and t - observer (advanced) time, which are related through geometric length of the signal propagating from the point where it was emitted to the point where it was detected: $t = t' + \frac{1}{c}\sqrt{\left(\vec{R} - \vec{r}(t')\right)^2}$. The electron trajectory in the field of a plane electromagnetic wave can be found analytically (see, for example [15]). Then the electric field of the scattered radiation can be found using equation (2). Relativistic electron, traveling with the velocity $\vec{\beta_0}c$, scatters of the small amplitude ($a_0 << 1$) incoming wave with the wave-vector ($\vec{k}$) and emits radiation:

$$E_{rad} = \alpha_f E_0 e^{-i\vec{q}(\vec{k})\vec{r}+iq(\vec{k})t} \tag{3}$$

where dimensionless coefficient is defined as $\alpha_f = \frac{1}{2\gamma_0^5}\frac{e^2}{Rmc^2}$. The wave-vector of the emitted radiation is: $\vec{q}(\vec{k}) = q(\vec{k})\vec{n} = \frac{1-\vec{\beta_0}\vec{k}}{1-\vec{\beta_0}\vec{n}} k\ \vec{n}$, .

Laser field approximation: 3D

Including effects of the transversally limited driver laser beam is typically done under assumption of a plane wave with imposed transverse profile. Such an approximation results in same answer for the Wigner function as it was obtained for a time-limited plane wave. However, this approximation is not correct in regimes where the laser beam is focused down to the wavelength since the off-axis plane waves in such a beam propagate at different directions. Often strong focusing is required to reach high amplitudes of the laser field. The correct approximation would be to present laser radiation as a continuous combination of the plane waves, find the radiation scattered by single electron from each of them and obtain the total radiation of the single electron as a superposition of the emitted fields. Then one can find the single electron Wigner function:

$$W_{6d}(\vec{r}_{3d}, \vec{k}_{w3d}) \sim \int_{-\infty}^{\infty} d\vec{k'} \int_{-\infty}^{\infty} d\vec{k} E_{k_0}(\vec{k}) e^{-i\vec{q}(\vec{k})\vec{r}+iq(\vec{k})t} E_{k_0}(\vec{k'}) e^{-i\vec{q}(\vec{k'})\vec{r}+iq(\vec{k'})t} \int_{-\infty}^{\infty} d\vec{\xi}_{3d} e^{-i(\frac{\vec{q}(\vec{k})+\vec{q}(\vec{k'})}{2} - \vec{k}_w)\vec{\xi}_{3d}} \tag{4}$$

Integrating over $\vec{\xi}_{3d}$ and taking into account that $\vec{q}(\vec{k})$ direction is independent from $\vec{k}$ we obtain:

$$W_{6D} \sim \delta\left(\frac{n_x}{n_z}k_{w_z} - k_{w_x}\right)\delta\left(\frac{n_y}{n_z}k_{w_z} - k_{w_y}\right)\int_{-\infty}^{\infty} d\vec{k'} E_{k_0}(\vec{k'}) e^{-i\vec{q}(\vec{k'})\vec{r}+iq(\vec{k'})t} \int_{-\infty}^{\infty} d\vec{k} E_{k_0}(\vec{k}) e^{-i\vec{q}(\vec{k})\vec{r}+iq(\vec{k})t} \delta\left(\frac{q(\vec{k}) + q(\vec{k'})}{2} n_z - k_{w_z}\right) \tag{5}$$

This expression works for any type of upcoming laser field with arbitrary $E_{k_0}(\vec{k'})$ and, in principle, can be evaluated numerically. For the Gaussian envelope $E_{las}(\vec{r}; 0) = E_0 e^{-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}} e^{ik_{z_{las}} z}$ at t=0 , we find $E_{k_0}(\vec{k}) = E_0\sigma_x\sigma_y\sigma_z e^{-\frac{k_x^2\sigma_x^2}{2} - \frac{k_y^2\sigma_y^2}{2} - \frac{(k_z+k_{z_{las}})^2\sigma_z^2}{2}}$ and in case of cylindrical symmetry ($\sigma_x = \sigma_y \equiv \sigma_\perp$) one can get the final answer as a 1D unidimensional integral:

$$W_{6D} \sim e^{4k_{w_N} n_z^2 \alpha^2 (1+k_{w_N}(-1+\alpha^2))} n_z^2 \alpha^4 \int_{-\infty}^{\infty} dk_N (k_N^2 - 4k_{w_N}^2) e^{k_N^2 n_z^2 \alpha^2 (-1+\alpha^2)} Erfc[k_N^+] Erfc[k_N^-] e^{-2ik_N r_N} \tag{6}$$

where $k_N^{+/-} = \frac{n_z(2+(\pm k_N - 2k_{w_N})(1-2\alpha^2))}{2\sqrt{2}}$ are arguments of the complementary error function, $k_{w_N} = \frac{k_{w_z}(1-\vec{\beta}\vec{n})}{k_{z_{las}}(1+\beta_z)}$ and $r_N = \frac{(r-ct)k_{z_{las}}(1+\beta_z)}{1-\vec{\beta}\vec{n}}$ are the normalized Wigner wave-vector and space-time coordinate respectively, $n_z = \sigma_z k_{z_{las}} = 2\pi\sigma_z/\lambda$ is the number of longitudinal oscillations in the driver pulse, and $\alpha = \sigma_\perp/\sigma_z$ is the transverse and longitudinal size ratio. This integral can be simply evaluated numerically for an arbitrary set of parameters $\sigma_x,\ \sigma_z$. In the Fig. 2 a) and 2 b) we demonstrate 3D plots of the Wigner function for the case of focusing the laser beam down to 3 and 10 wavelengths, respectively, with large longitudinal size. We find that for the strong focusing case Wigner function oscillates to the negative values and cannot be accurately described by a single Gaussian mode, while for the case of $\sigma_x \geq 10\lambda$ a single mode Gaussian approximation works fine. In the case of $\sigma_x >> \sigma_z$ it can be shown that the result exactly matches the one derived for the homogeneous time-limited plane wave, while in the case of a moderate strong focusing characteristic scales in k- and r- space of a single Gaussian mode as well as central k-vector (k_{rad}) have to be corrected by numerically evaluated coefficients ($\alpha_{3D},\ \mu_{3D},\ \eta_{3D}$):

$$W_{1e}^{(6D)} \sim \delta\left(\frac{n_x}{n_z}k_{w_z} - k_{w_x}\right)\delta\left(\frac{n_y}{n_z}k_{w_z} - k_{w_y}\right) e^{\frac{-(r-ct)^2}{\sigma_{rad}}} e^{\frac{-(k_{w_z}/n_z - k_{rad_{3D}})^2}{\sigma_{k_{rad}}}}. \tag{7}$$

where $k_{rad_{3D}} = \alpha_{3D}\frac{k_{z_{las}}(1+\beta_z)}{1-\vec{\beta}\vec{n}},\ \sigma_{rad} = \mu_{3D}\frac{\sigma_z(1-\vec{\beta}\vec{n})}{(1+\beta_z)}, \sigma_{k_{rad}} = \frac{\eta_{3D}}{\sigma_{rad}}$.

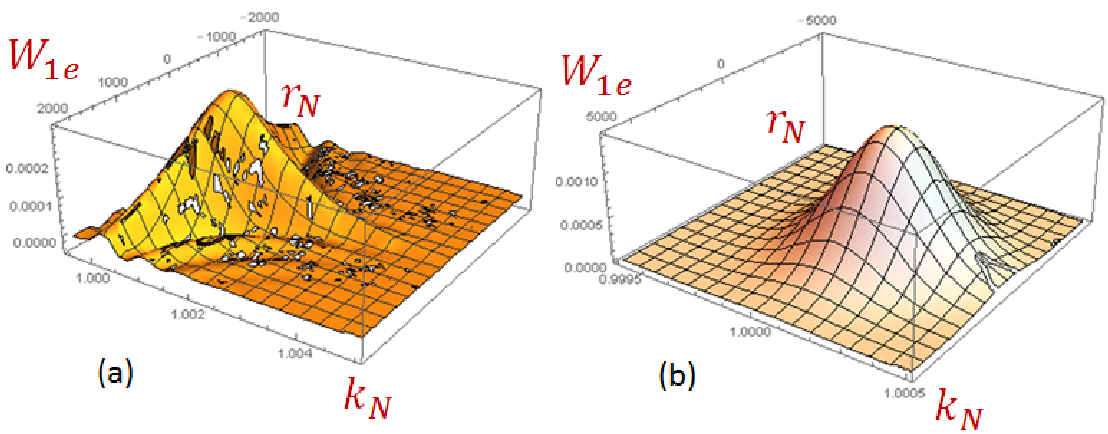

FIGURE 2. Wigner function dependence from normalized k- vector and normalized $r-ct$ in case of the longitudinal size $\sigma_z = 853\lambda$ and transverse sizes: of $\sigma_\perp = 3\lambda$ (a) and $\sigma_\perp = 10\lambda$ (b) - possible parameter for the ICS source at Fermi Lab [16]

WIGNER FUNCTION OF RADIATION EMITTED BY ELECTRON BEAM

Convolution theorem

Under the approximation that all electrons radiate incoherently the total Wigner function of scattered radiation can be found as the convolution of the single-electron Wigner function derived before with the electron distribution function in 6D phase space:

$$W_{beam} \sim \int_{-\infty}^{+\infty} W_{1e}(\vec{\zeta}_e, \vec{\zeta}_{ph}) f(\vec{\zeta}_e) d^6\vec{\zeta}_e \tag{8}$$

In principle, one can numerically evaluate the 6D integral, if laser field and electron beam distribution are arbitrary. However, we can rewrite the expression, in case of a Gaussian distribution of the electron beam described its 6D Σ-matrix as:

$$W_{beam} = \sqrt{\pi}^{-6}\sqrt{Det[\Sigma^{-1}]}\int_{-\infty}^{+\infty} W_{1e}(\vec{\zeta}_e, \vec{\zeta}_{ph}) e^{-\vec{\zeta}_e^{\,T}\Sigma^{-1}\vec{\zeta}_e} d^6\vec{\zeta}_e \tag{9}$$

Moreover, the expression in (7) can be presented in the similar to the electron distribution form by representing each delta-function as a limit of Gaussian function with its standard deviation (δ_i) approaches to "*zero*":

$$W_{beam} = W_0 \frac{1}{\sqrt{\pi}\delta_x} \frac{1}{\sqrt{\pi}\delta_y} e^{-(\vec{\zeta}_{ph}^{(i)}-\vec{\zeta}_e)^T M_0 (\vec{\zeta}_{ph}^{(i)}-\vec{\zeta}_e)} \tag{10}$$

where $\vec{\zeta}_{ph}^{(i)} = Drift^{-1}\vec{\zeta}_{ph}^{(f)}$ is the 6D vector of a photon in the phase space at the moment it was emitted, M_0 -is the characteristic matrix for the single electron radiation Wigner function, describing emitted photons distribution similar to Σ^{-1}-matrix describing the electron distribution in the phase space. This expression looks similar to the 4D case studied by K. J. Kim [13], but here $\vec{\zeta}_{ph} - \vec{\zeta}_e$ cannot be expressed in symplectic coordinates due to the specific feature of the ICS process: $w_{rad} \sim \gamma^2$. If photon ($\vec{\zeta}_{ph}^{T} = \{\Delta x, \frac{\Delta k_{w_x}}{k_{w_{z0}}}, \Delta y, \frac{\Delta k_{w_y}}{k_{w_{z0}}}, \Delta z, \frac{\Delta k_{w_z}}{k_{w_{z0}}}\}$) and electron ($\vec{\zeta}_e^{\,T(s)} = \{x_e, \Delta\beta_x, y_e, \Delta\beta_y, z_e, \Delta\beta_z\}$) coordinates expressed in the position-momentum symplectic representation, then the transformation from the symplectic coordinates for an electron to ones which were used in (10): $\vec{\zeta}_e = \Omega\vec{\zeta}_e^{\,(s)}$ is described by the matrix Ω, where all elements are similar to identity matrix but Ω_{66} is substituted with $2\gamma_0^2$.

Peak Brightness

We find the Wigner function of radiation scattered by the electron beam as a 6-dimensional form:

$$W_{beam} = W_0\, lim_{\delta_x,\delta_y \longrightarrow 0}\left\{\frac{\sqrt{Det[\Sigma^{-1}]}}{\delta_x\delta_y\sqrt{Det[M]}} e^{-(\vec{\zeta}_{ph}^{(i)})^T (M_0 - M_0 M^{-1} M_0)\vec{\zeta}_{ph}^{(i)}}\right\} \tag{11}$$

where $M = M_0 + \Sigma^{-1}$. The limit should exist for any $\vec{\zeta}_{ph}$, which means the expression before the exponent should converge itself, and if it does, it defines the peak brightness of the source. We have verified numerically that it does exist for an arbitrary electron Σ-matrix and keep working on finding it analytically as a 6-dimensional form.

CONCLUSION

We have demonstrated that the Brightness of an ICS source can be represented as a Wigner function in the 6D photon phase space and found it as a 6D Gaussian form of an arbitrary electron phase space distribution described by 6D $\Sigma - matrix$. We found that the convolution theorem proved in 4D for the on-axis observer fails in 6D for an arbitrary direction to the observer in symplectic coordinates for a photon and an electron simultaneously, but can be expressed in non-symplectic coordinates for, at least, one of them. We have studied different approximations for the driver laser field and found that in the case of focusing to its theoretical limit ($\sim \lambda$) the Wigner function of a single electron oscillates to negative values and cannot be represented by a single Gaussian mode. In the case of a moderate focusing we found that standard 1D model with assumption of homogeneous field in the transverse plane can work but will require evaluation of correcting coefficients numerically from the laser beam parameters. Our next step will be to find an analytical 6D form expression for the electron beam radiation Wigner function and find the electron phase space distribution, which delivers maximum of the peak brightness for an ICS source.

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