Theory of the Electron Sheath and Presheath

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Conventional Wisdom: Electron Sheaths Collect a Random Flux of Electrons

Global Current Balance

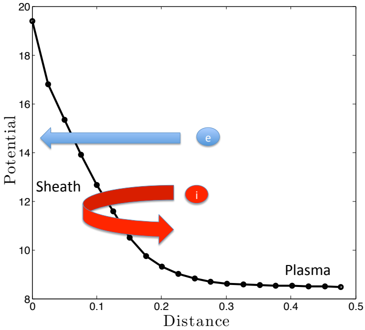
Global current balance requires that for a monotonic electron sheath

$A_E/A_w < \sqrt{2.3m_e/M_i}$ [1]

Conventional Wisdom

Understanding of the electron sheath is from probe theory

- Flux collected by an electron sheath is the random flux [2,3]
- The electron velocity distribution function is a truncated Maxwellian at the sheath edge [4]
- The electron sheath equivalent of the Bohm criterion is trivially satisfied $\implies$ no need for presheath [5,6]



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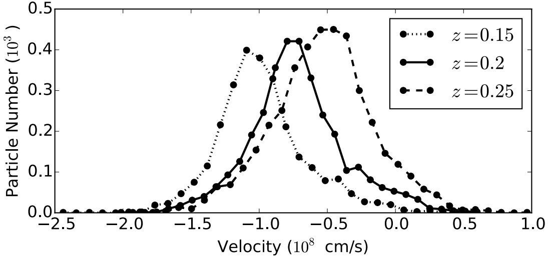
Do electron sheaths collect a random flux of electrons? Simulations and experiments suggest NO!

- Experiments show that an electron sheath causes density gradients over large scales (Talk NR3.00003 in this session)



PIC simulation results show that the EVDF can be modeled as a flowing maxwellian in the electron sheath and presheath. The sheath edge is at $z \approx 0.25$ cm.

- The electron VDFs are flow shifted, but where do they get their velocity?
- Reconsider the electron sheath plasma interface $\Longrightarrow$ Consider a presheath

Fluid Model for the Electron Sheath Plasma Interface

For the purposes of modeling the presheath and sheath edge, consider a model that:

- Assumes that the plasma is generated at a rate proportional to the density

$$\frac{dn_e}{dz} = \nu_s n_e$$

- Ions are assumed to obey a Boltzmann density

$$n_i = n_o \exp(-e\phi/T_i)$$

- Models electrons with the momentum equation

$$V_e \frac{dV_e}{dz} = -\frac{e}{m_e}E - \frac{T_e}{m_e n_e}\frac{dn_e}{dz} - V_e(\nu_R + \nu_s)$$

Electron Sheath Analog of the Bohm Criterion

- **Sheath Criterion**: At the sheath edge $\rho \approx 0$ and $\left| d\rho/d\phi|_{\phi=\phi_0} \right| > 0$
- This can be rewritten as

$$\sum_s q_s \frac{dn_s}{dz} \leq 0$$

- Using the continuity equation (no source)

$$\sum_s q_s \frac{n_s}{V_s} \frac{dV_s}{dz} \leq 0$$

- Inserting the momentum equation

$$\boxed{V_e \geq \sqrt{\frac{T_e + T_i}{m_e}} \equiv v_{eB}}$$

- V_e is achieved through a flow shift, but how is such a large flow generated?

Pressure Drives the Electron Flow

- Quasineutrality implies

$$\frac{dn_e}{dz} = -en_i E/T_i$$

- The momentum equation shows the pressure term is T_e/T_i larger than the electric field term

$$V_e \frac{dV_e}{dz} = -\frac{e}{m_e} E - \frac{T_e}{m_e n_e}\left(-\frac{en_i E}{T_i} \right) - V_e \nu$$

- In the electron presheath the electric field causes a density (pressure) gradient. The acceleration of the flow velocity is dominated by the pressure gradient. This is significantly different from the ion presheath.

Presheath

- The quasineutrality condition with the momentum and continuity equation combine to give a mobility limited flow equation

$$V_e = -\mu_e \left(1 - \frac{V_e^2}{v_{eB}^2}\right) E$$

where $\mu_e = e(1 + T_e/T_i)/[m_e(\nu_R + 2\nu_s)]$ is the electron mobility

- The quasineutral limit of Poisson's Eq with Boltzmann ions gives

$$\phi = -\frac{T_i}{e} \ln\left(\frac{v_{eB}}{V_e}\right)$$

- These two combine to give

$$\frac{dz}{dV_e} = \frac{v_{eB}^2 - V_e^2}{\nu V_e^2},$$

which can be solved analytically for constant mean free path ($\nu = V_e/l$) and constant collision frequency ($\nu = v_{eB}/l$) cases

Sheath is Larger in the Presence of Electron Flow

- Combining the momentum and continuity equations and integrating

$$\left(\frac{V_e}{v_{eB}}\right) - 2\log\left(\frac{V_e}{v_{eB}}\right) = \frac{2e\phi}{T_e} + 1$$

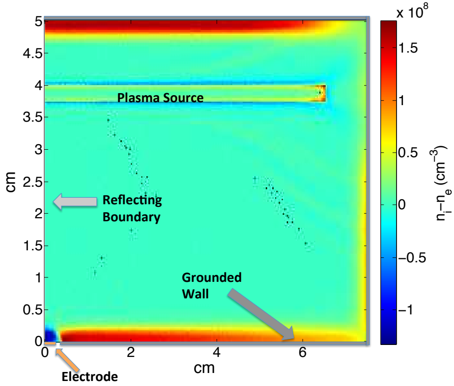
- For large V_e pressure contributions are negligible
- Within the sheath flux conservation $(n_e(\phi)V_e(\phi) = n_o v_{eB})$ is obeyed approximately. Neglecting the ion density, and integrating Poisson's equation twice:

$$\frac{z}{\lambda_{D_e}} = 0.79\left(\frac{e\Delta\phi}{T_e}\right)^{3/4}$$

- The numerical constant is typically assumed to be 0.32 $\Longrightarrow$ Sheath is twice as thick as previously thought

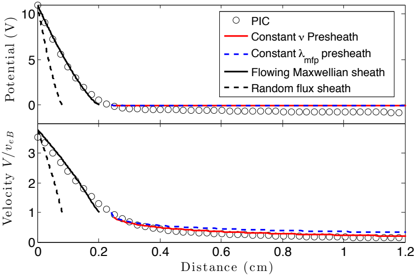
Where to Look Next: Simulations

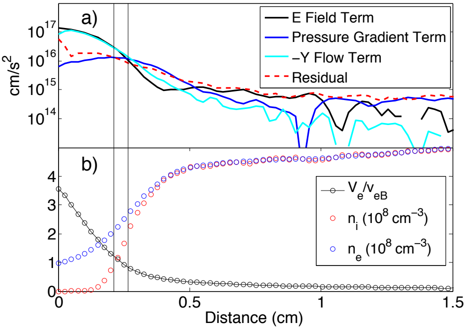
- ALEPH: an electrostatic particle-in-cell code with direct simulation Monte Carlo collisions
  - Reduced scale compared to experiments: 5 cm x 7 cm
  - $A_E/A_w \approx 0.011$
  - a 0.25 cm electrode was separated from the grounded walls by a .2 cm dielectric
  - The typical cell size was $0.7\lambda_{De}$
  - Timestep of $10^{-4}\mu s$ resolved the electron plasma frequency
  - Included ion neutral collisions
  - $n_e \approx 5 \times 10^8$cm^{-3}, $P = 1$mTorr, $T_e =\approx 2$ eV, $T_i \approx 0.05$ eV



Model vs PIC

- The flowing Maxwellian picture correctly predicts the sheath length
- A constant collision frequency model accurately predicts the flow velocity for a presheath length of $\ell = 0.3$ cm
- The electron flow achieves the required velocity near the sheath edge (as predicted by the sheath thickness)
- The portion of the presheath immediately before the sheath is pressure driven
- Residuals in the momentum equation could possibly be due to electron scattering off waves





Summary: Electrons are Accelerated in a Presheath

- The velocity distribution function at the sheath edge of an electron sheath is a flowing Maxwellian
- The flow moment satisfies an electron sheath analog of the Bohm criterion $V_e \geq \sqrt{\frac{T_e+T_i}{m_e}}$
- This velocity is achieved in a presheath where the electron flow is driven by pressure gradients
- Under these conditions the sheath is twice as thick as previously assumed
- These are in agreement with the PIC simulations

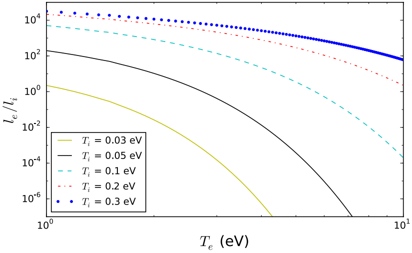
Extra Slides

Presheath Scaling

- In this case the collision frequencies are different, using $\nu_s = n_g K_s$ the ratio of presheath length scales is

$$\frac{l_e}{l_i} = \frac{v_{eB}}{c_s}\frac{\nu_i}{\nu_e} = \sqrt{\frac{m_i}{m_e}\left(1+\frac{T_i}{T_e}\right)}\frac{K_i}{K_e} \tag{1}$$

- For helium using the total momentum scattering cross section for electrons [7] and the charge exchange and elastic cross sections [8] for ions



[7] Phelps database, www.lxcat.net, retrieved on september 30, 2015. [8] W. H. Cramer and J. H. Simons, JChPhys 26, 1272 (1957)

Failure of the Conventional Picture

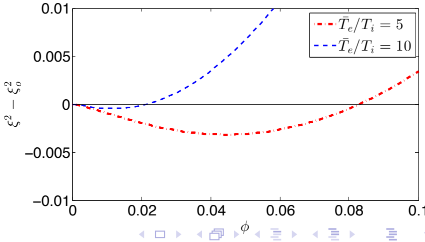
- $$f_e(v) = \frac{\bar{n}_e}{\pi^{3/2}\bar{v}_T^3} e^{-v^2/\bar{v}_T^2} \Theta(v_y - v_{\parallel,c}) \quad (2)$$

  Here Θ is the Heaviside function, the parameters $\bar{n}_e$ and $\bar{v}_T = \sqrt{\frac{2\bar{T}_e}{m}}$ are functions of location in the sheath, and the truncation velocity throughout the sheath is given by conservation of energy $v_{\parallel,c} = \sqrt{\frac{2e\Delta\Phi}{m_e}}$, where $\Delta\Phi = \Phi - \Phi_{plasma}$
- The square of the electric field can be solved using the density moment of Eq. (1) along with the Boltzmann relation for the ion density $n_i = n_o \exp(-e\Phi/T_i)$, and Poisson's equation:

$$\xi^2 = \xi_o^2 + 2\left[2\sqrt{\frac{\phi}{\pi}} - 1 + e^{\phi}\mathrm{erfc}(\sqrt{\phi}) - \frac{T_i}{\bar{T}_e} + \frac{T_i}{\bar{T}_e} e^{-\phi\bar{T}_e/T_i}\right], \quad (3)$$

where $\xi = eE\lambda_{D_e}/\bar{T}_e$ and$\phi = e\Delta\Phi/\bar{T}_e$

- $|\xi_o|$ must be greater than zero
- inconsistent with no E field at sheath edge



Instabilities

- The dielectric response for a plasma where the electrons are Maxwellian with flow V_e and stationary Maxwellian ions is

$$\epsilon(\mathbf{k},\omega) = 1 - \frac{\omega_{pe}^2}{k^2 v_{T_e}^2} Z'(\xi_e) - \frac{\omega_{pi}^2}{k^2 v_{T_i}^2} Z'(\xi_i) \tag{4}$$

where $\xi_e = \frac{\omega - \mathbf{k}\cdot\mathbf{V_e}}{k v_{T_e}}$ and $\xi_i = \frac{\omega}{k v_{T_i}}$