SAND2015-8404C

Gate Set Tomography

10^{-5} error bars and 2 qubits

Robin Blume-Kohout

John K. Gamble, Erik Nielsen, Kenny Rudinger, Peter Maunz

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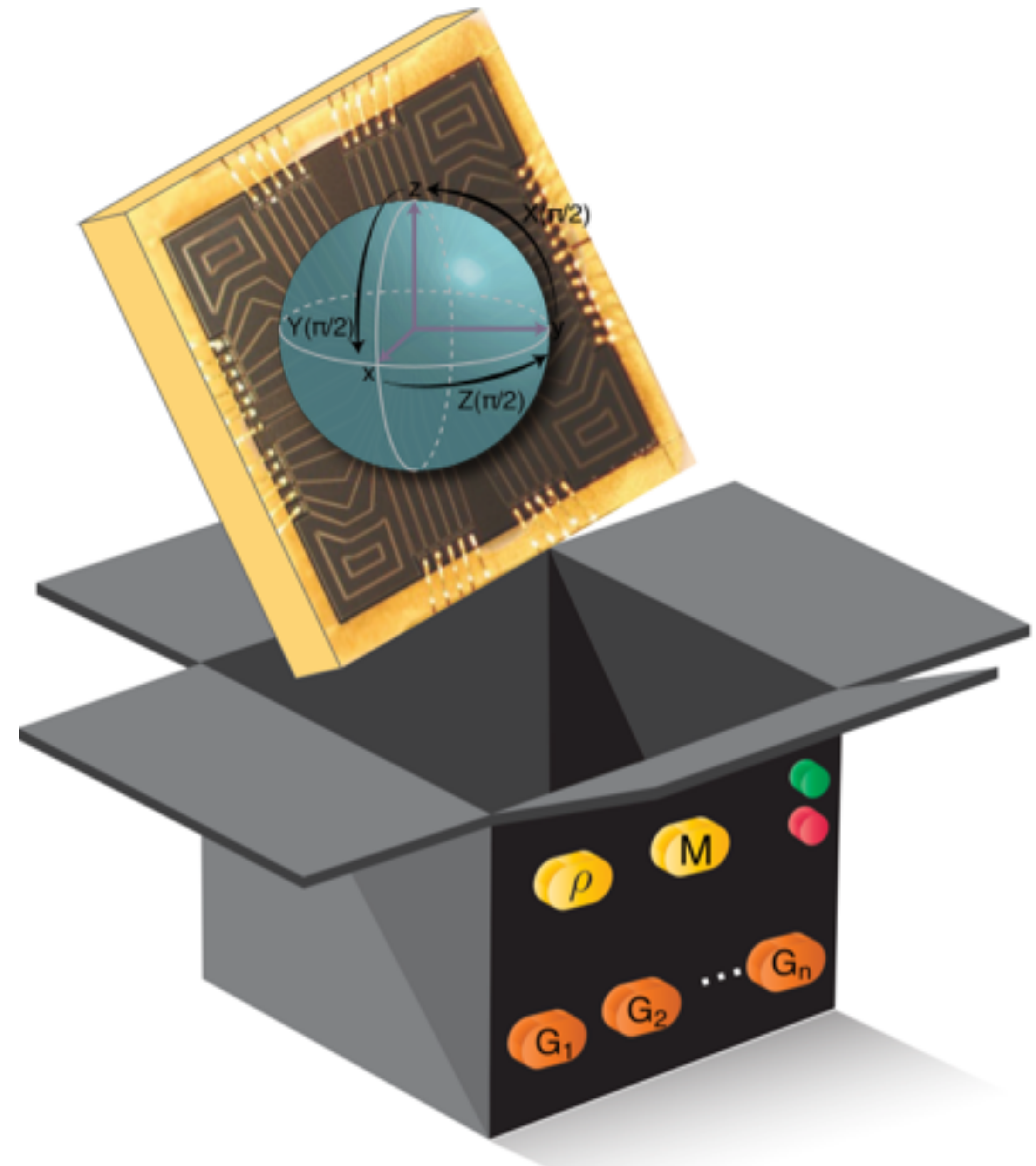
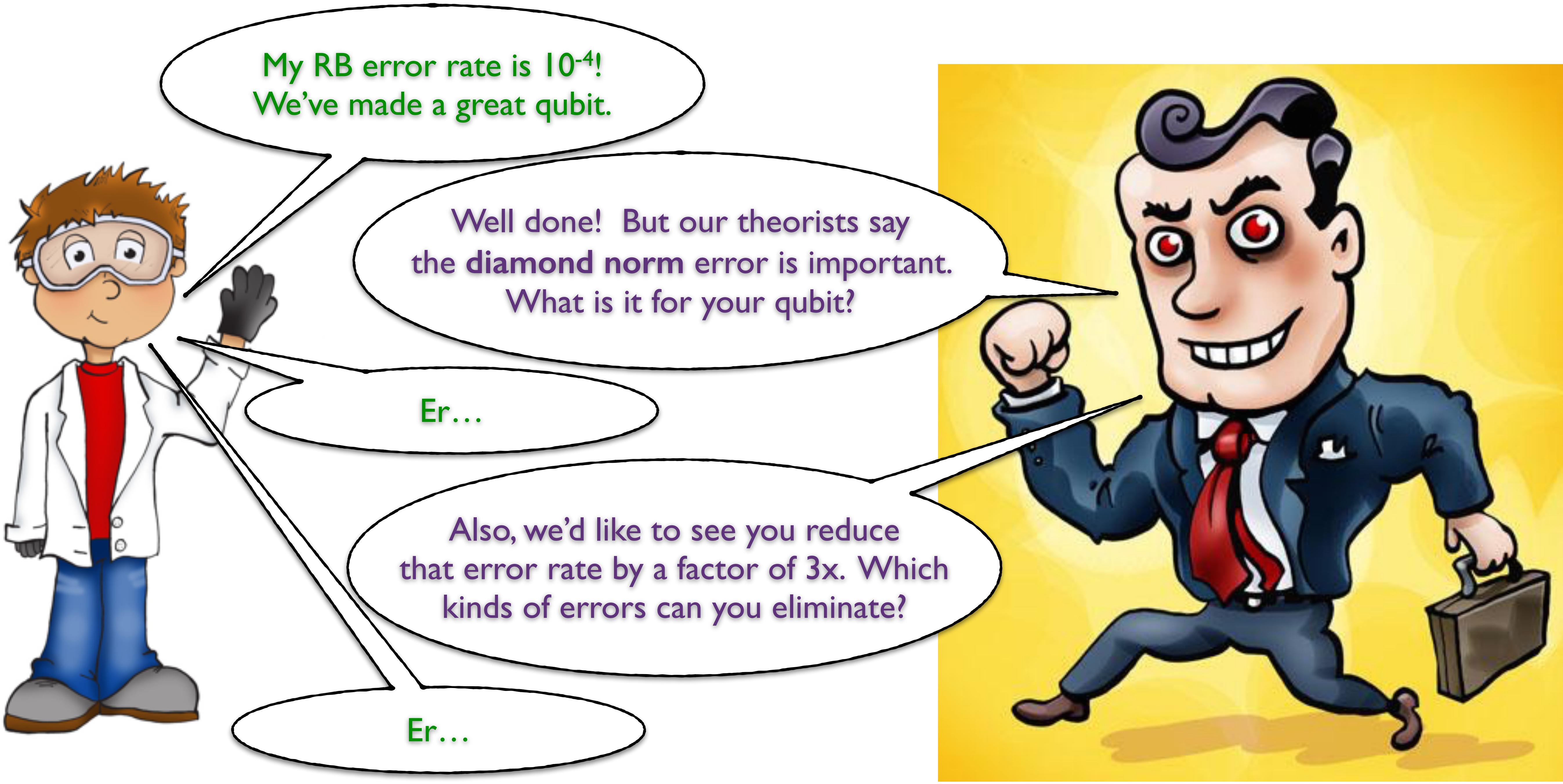
Meet Eric the Experimentalist

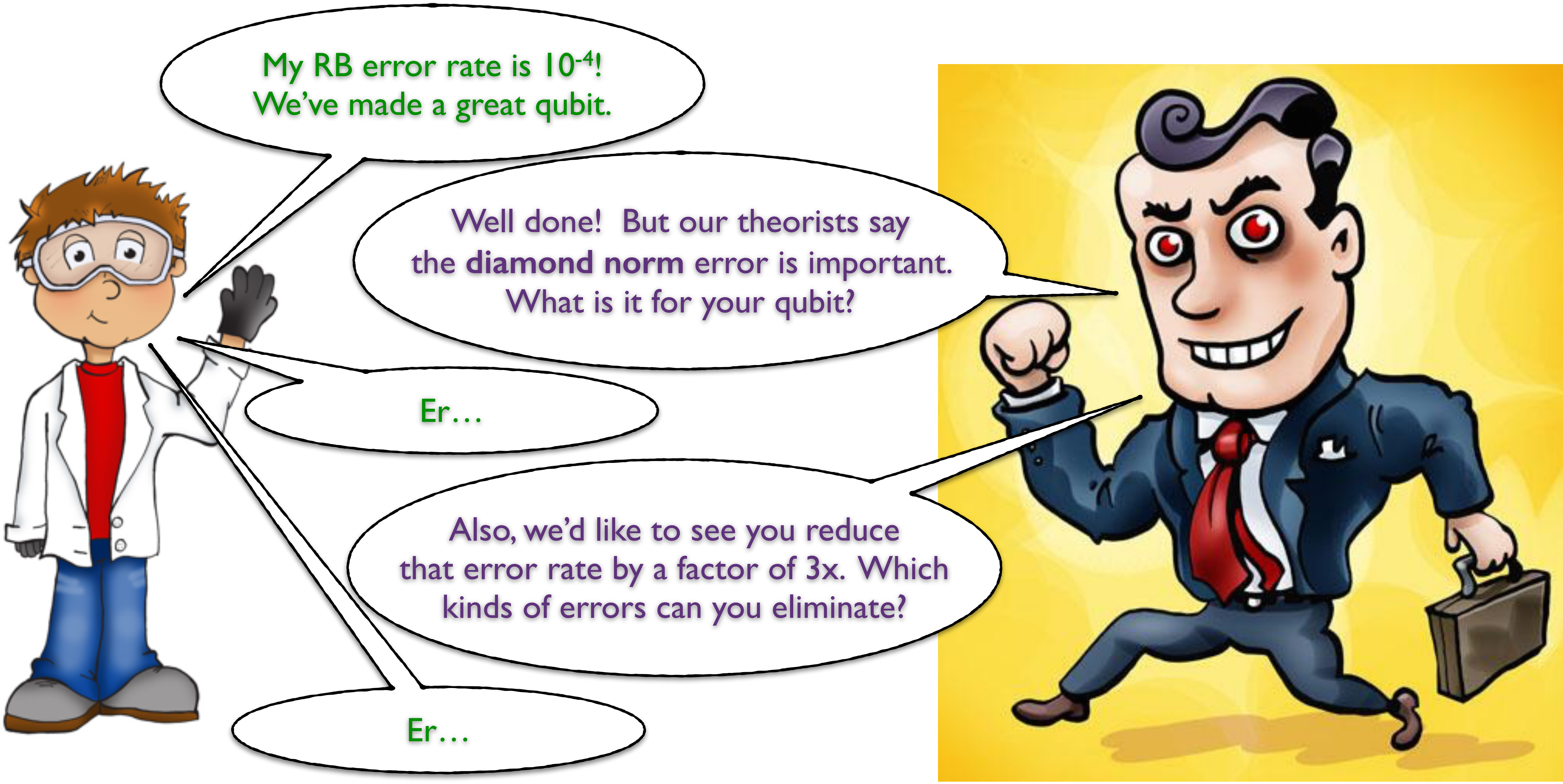


- Eric made a qubit. Now, Eric wants to show his program manager how good it is, and get more well-deserved funding.

Eric does Randomized Benchmarking



Eric does Randomized Benchmarking



- RB didn't tell Eric very much about the *kinds* of error in his qubit. He needs more detailed debugging information!

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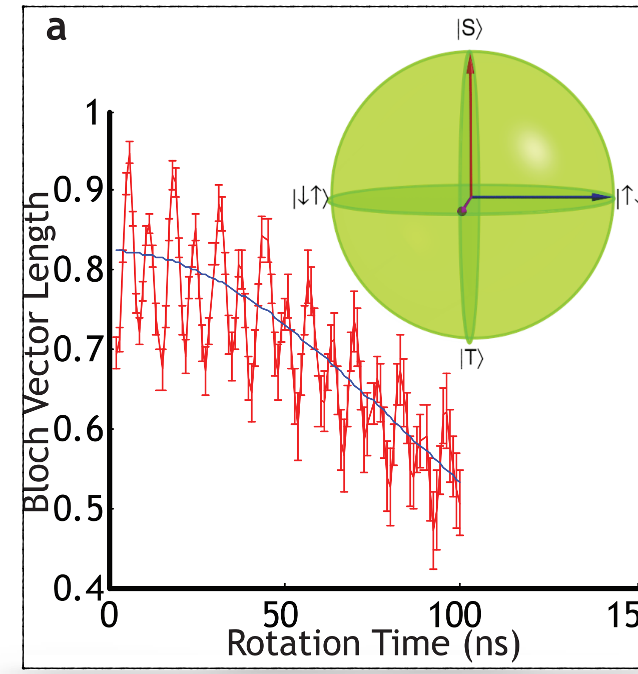
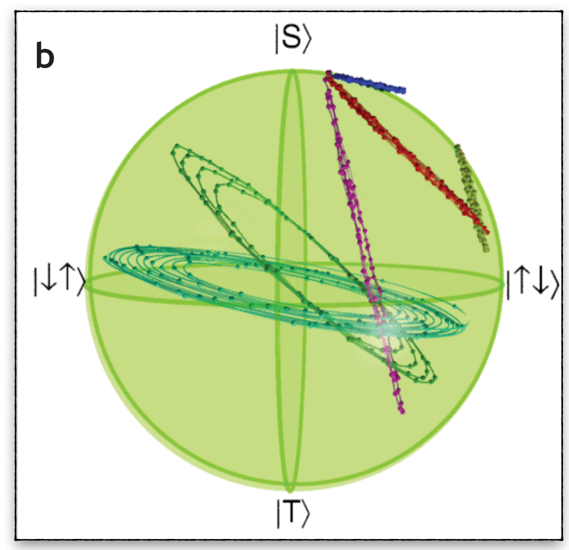
Eric does Process Tomography

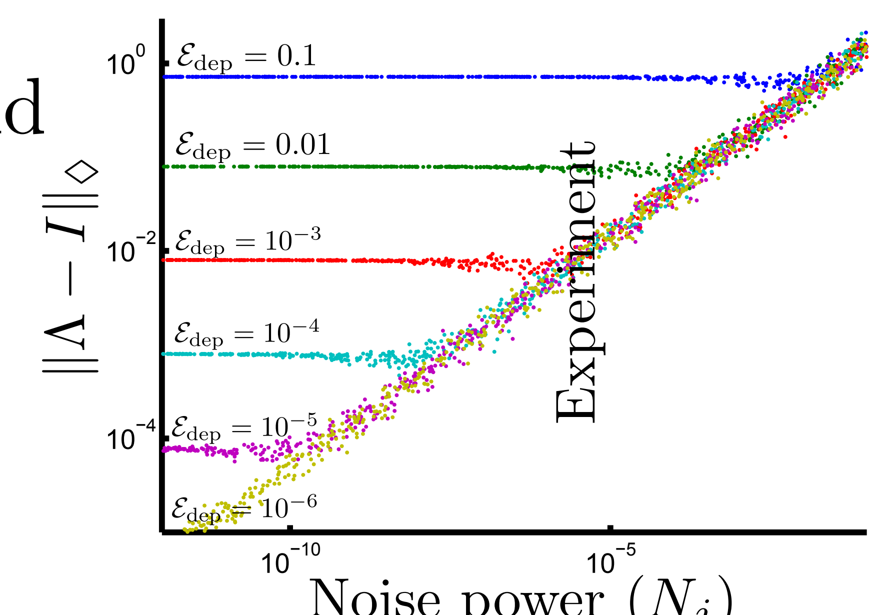
- But when Eric does process tomography at different times, his gate fidelities fluctuate wildly!



- But at least they're high. Like... 105%... 😕.



- Eric reads Merkel et al (PRA 2013) and learns that process tomography is unreliable unless he has perfect gates.



Eric needs Gate-Set Tomography

General GST Properties

- No reliance on pre-calibrated operations (gates, POVMs, etc).
- Unconditional reliability (except due to non-Markovian effects).
- Resource complexity (# of exp'ts, clicks, etc) is only slightly higher than that of process tomography on all gates in the gateset.

Properties of Sandia GST

- Estimates the RB number as accurately as RB itself.
- Estimates *all* gate parameters to high accuracy (including derived quantities, e.g. $\diamondsuit$-norm)
- Usually detects non-Markovian noise/errors/effects.
- Gateset estimate & derived quantities can be equipped with error bars.

Principles of GST

- **Gates are *relational*.** Initial states are prepared using gates, and final measurements are performed using gates.

  Process tomography is not about "How does *this* process transform *these* input states?"

  It is about "How does *this* process relate to *these* other `fiducial' processes?"

- The existence of a *gauge* for gatesets is a direct consequence.

- The probabilities for sufficiently many *gate sequences* (circuits) determine a gateset, up to gauge. These are estimated from data. All ensuing discussion is about: (1) what sequences to measure; and/or (2) how to analyze the data.

"Overkill tomography". IBM 2012-13

- What to measure: all gate sequences of length ≤ 3.

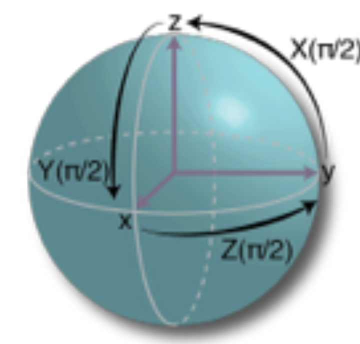
$$P_{ijk} = \langle\langle E|\, G_i G_j G_k \,|\rho\rangle\rangle$$

- Analysis method: maximum likelihood (MLE). Local optimization using target gates as a starting point.

- The Good: first implementation — original idea. Usually worked.

- The Bad: both aspects ad-hoc. Likelihood function known to be non concave (probabilities nonlinear in gates), so hard to optimize reliably.

Linear GST (LGST). Sandia, 2013



- What to measure: specific "fiducial sandwich" sequences of length L ≤ 7.

$$P_{ijk} = \langle\langle E|\{F_i\}G_j\{F_k\}|\rho\rangle\rangle$$

$$\{F_i\} = \{\emptyset, G_x, G_y, G_xG_x, G_xG_xG_x, G_yG_yG_y\}$$

- Analysis method: closed-form linear algebra.

- The Good: incredibly fast, 100% reliable.

- The Bad: Not very accurate. Unweighted linear inversion is statistically unsophisticated. Minimal ability to detect non-Markovian noise.

Long sequence GST. Sandia, 2013-15

- What to measure: Specific "germ power" fiducial sandwich sequences of length $2^L + O(1)$, up to $2^L = 8192$.

$$P_{ijkL} = \langle\langle E| \{F_i\}\{g_j\}^L\{F_k\} |\rho\rangle\rangle$$

$$\{g_j\} = \{G_x, G_y, G_i, G_xG_y, G_xG_xG_iG_y, \text{etc}\}$$

- Analysis method: naive least squares, min-χ^2, or MLE.

- The Good: Heisenberg accuracy. Highly reliable, overcomes likelihood pathologies a la phase estimation (see Kimmel 2015). Provides extensive detection of non-Markovian noise.

- The Bad: complex, finicky, can be slow (1-30 minutes).

State of Play: October 1, 2015

How we implement GST

Identify target gates

Design germ sequences

Design fiducial sequences

Send datafile template to experimental team

Record data in datafile

Do LGST analysis (short sequence data only)

Gauge-optimize LGST estimate & truncate to CP

Iteratively refine estimate by adding L=2,4,8,… data and minimizing χ^2 (scipy.opt)

Refine final estimate by maximizing likelihood.

Compute badness-of-fit

Optimize gauge

Error bars (likelihood-ratio and/or bootstrap)

Compute fidelities, etc.

The Standard Qubit

Operator	Hilbert-Schmidt vector (Pauli basis)	Matrix
ρ_0	$\begin{matrix}0.7071\\0\\0\\0.7071\end{matrix}$	$\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$
E_0	$\begin{matrix}0.7071\\0\\0\\-0.7071\end{matrix}$	$\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$

	Fiducials	
#	**Prep.**	**Measure**
1		
2	Gx	Gx
3	Gy	Gy
4	Gx · Gx	Gx · Gx
5	Gx · Gx · Gx	Gx · Gx · Gx
6	Gy · Gy · Gy	Gy · Gy · Gy

Gate	Superoperator (Pauli basis)	Rotation axis	Angle
Gi	$\begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{pmatrix}$	$\begin{matrix}0\\1\\0\\0\end{matrix}$	0π
Gx	$\begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & -1\\0 & 0 & 1 & 0\end{pmatrix}$	$\begin{matrix}0\\1\\0\\0\end{matrix}$	0.5π
Gy	$\begin{pmatrix}1 & 0 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\\0 & -1 & 0 & 0\end{pmatrix}$	$\begin{matrix}0\\0\\1\\0\end{matrix}$	0.5π

#	**Germ**
1	Gx
2	Gy
3	Gi
4	Gx · Gy
5	Gx · Gy · Gi
6	Gx · Gi · Gy
7	Gx · Gi · Gi
8	Gy · Gi · Gi
9	Gx · Gx · Gi · Gy
10	Gx · Gy · Gy · Gi
11	Gx · Gx · Gy · Gx · Gy · Gy

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Max likelihood vs. Min χ^2

- Data = observed frequencies $f(s_j)$ for various gate sequences.

## Columns = count, total		
{}	0.0	50
Gx	25.0	50
Gy	28.0	50
GxGxGx	18.0	50
GyGyGy	28.0	50
GxGx	50.0	50
GxGy	24.0	50
GxGxGxGx	0.0	50
GxGyGyGy	28.0	50
GyGx	23.0	50
GyGy	50.0	50
GyGxGxGx	21.0	50
GyGyGyGy	0.0	50
GyGxGx	21.0	50
GxGxGxGy	27.0	50
(Gx)^6	50.0	50
GxGxGxGyGyGy	20.0	50

- Gateset predicts probabilities $p(s_j)$.
- Likelihood: $$\mathcal{L} = \prod_j p_j^{Nf_j}(1-p_j)^{N(1-f_j)}$$
- [Semi]-Gaussian approximation:
$$\chi^2 = \sum_j \frac{(p_j - f_j)^2}{p_j}$$
- We use a hybrid method (min-χ^2 with a "final coat" of MLE):
  - The χ^2 function behaves better + faster in SciPy.optimize
  - But Min-χ^2 is *biased* for SPAM estimates (Max-L is not).

Autogenerated GST Reports

- Our software ("PyGSTi") can read in a dataset, analyze it, and generate a comprehensive human-readable report on the gates with a single command.

- Reports run 15-30 pages and contain:

    - Summary of experimental protocol & target gates

    - Estimated gates (including state prep and measurement)

    - Derived quantities (fidelities, diamond norms, rotation angles, rotation axes, SPAM parameter, etc...)

    - Extensive "badness-of-fit" information.

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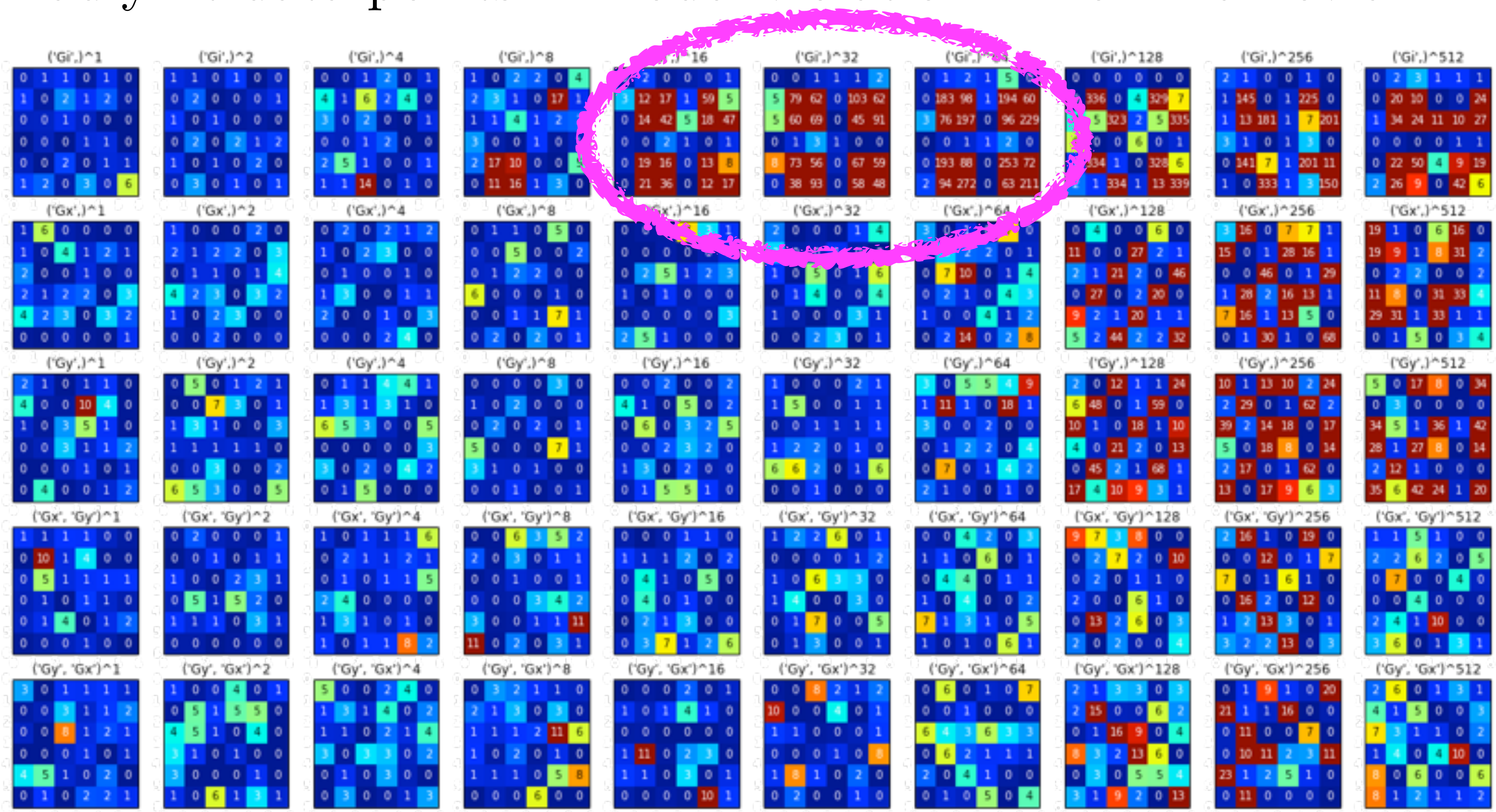
Detecting non-Markovian noise

- Fitting ~30 parameters to ~4500 experiments is *overcomplete.*
  => we have a lot of residual data for *model testing/selection.*

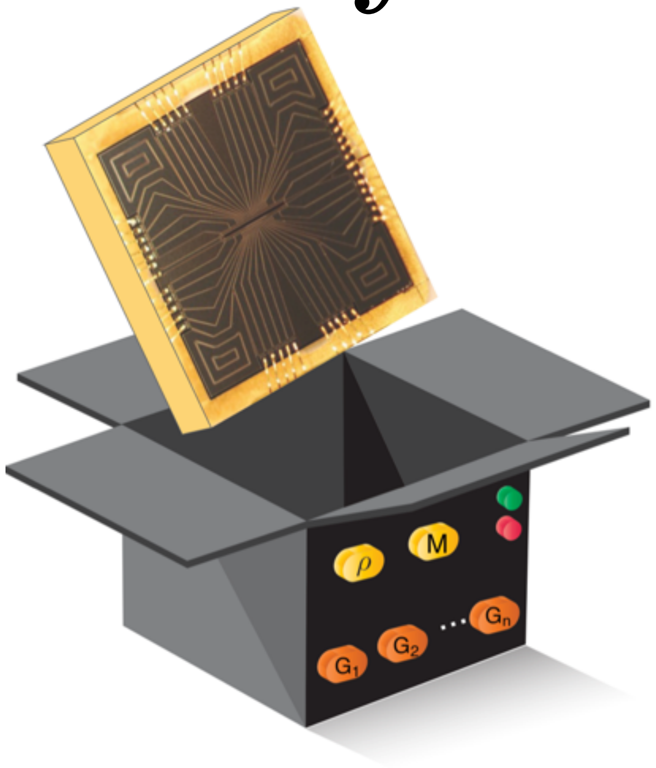
- Badly fit data points = model violation = non-Markovian.



Recent Advances @ Sandia
(a.k.a. "The point of this talk")

1. We tomographed a qubit really really really precisely.

- Trapped-ion (Yb+) qubit in Peter Maunz's lab (March 2015)
- 3 gates ($X_{\pi/2}$, $Y_{\pi/2}$, Idle). 6 fiducials. 11 germs. L = 1,2,4,...,8192. 4657 sequences. 50 counts/each.



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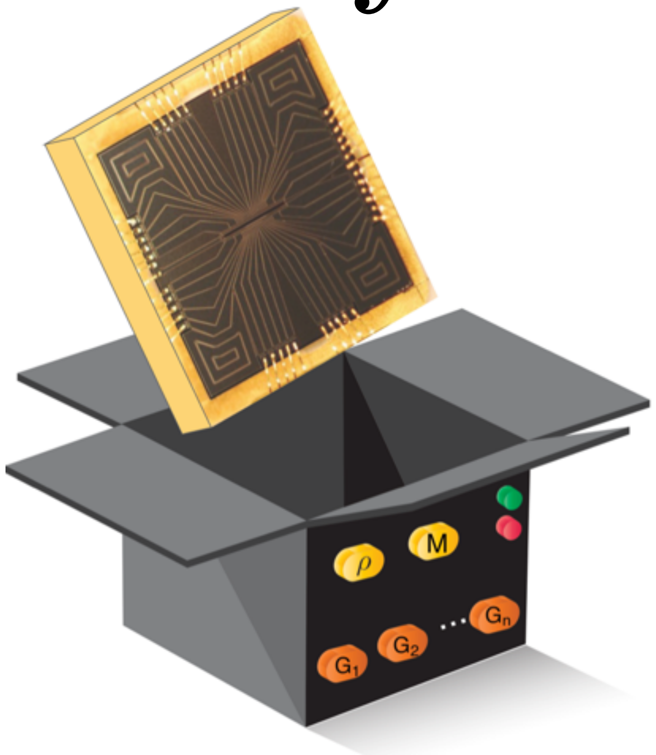

$$\widehat{E} = \begin{bmatrix} 0 & 0 \\ 0 & 0.9929 \end{bmatrix} \pm \begin{bmatrix} 0.6 & 2 \\ 2 & 0.6 \end{bmatrix} \times 10^{-3}$$

$$\widehat{\rho} = \begin{bmatrix} 0.9957 & (2+4i)\times 10^{-3} \\ (2-4i)\times 10^{-3} & 4.3\times 10^{-3} \end{bmatrix} \pm \begin{bmatrix} 0.4 & 2 \\ 2 & 0.4 \end{bmatrix} \times 10^{-3}$$

$$P_{\text{spam}} = (4.2 \pm 0.5)\times 10^{-3}$$

1. We tomographed a qubit really really really precisely.

- Trapped-ion (Yb+) qubit in Peter Maunz's lab (March 2015)

$$\widehat{G}_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.999932 & -6\times10^{-5} & 1\times10^{-5} \\ 6\times10^{-6} & 3\times10^{-5} & 0.999891 & 2\times10^{-5} \\ 0 & -3\times10^{-5} & -6\times10^{-5} & 0.999900 \end{bmatrix} \pm \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0.7 & 1.3 & 1.2 \\ 0.4 & 1.3 & 0.9 & 1.3 \\ 0.5 & 1.2 & 1.3 & 0.9 \end{bmatrix} \times 10^{-5}$$

$$\widehat{G}_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.999946 & 5\times10^{-5} & 0 \\ 0 & 2\times10^{-5} & 5\times10^{-5} & -0.999904 \\ 0 & 4\times10^{-5} & 0.999904 & 6\times10^{-5} \end{bmatrix} \pm \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0.7 & 1.6 & 1.5 \\ 0.6 & 1.4 & 1.7 & 0.7 \\ 0.6 & 1.5 & 0.7 & 1.7 \end{bmatrix} \times 10^{-5}$$

$$\widehat{G}_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2\times10^{-5} & 2\times10^{-5} & 0.999876 \\ 0 & -2\times10^{-5} & -0.999962 & 0 \\ 0 & 0.999876 & 3\times10^{-5} & -5\times10^{-5} \end{bmatrix} \pm \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.6 & 1.6 & 1.5 & 0.7 \\ 0.4 & 1.5 & 0.6 & 1.5 \\ 0.5 & 0.7 & 1.6 & 1.7 \end{bmatrix} \times 10^{-5}$$

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$$1 - F(\widehat{G}, G_{\text{ideal}}) = (6.9 \pm 0.3)\times 10^{-5}$$
$$\|\widehat{G} - G_{\text{ideal}}\|_\diamond = (7.9 \pm 0.9)\times 10^{-5}$$

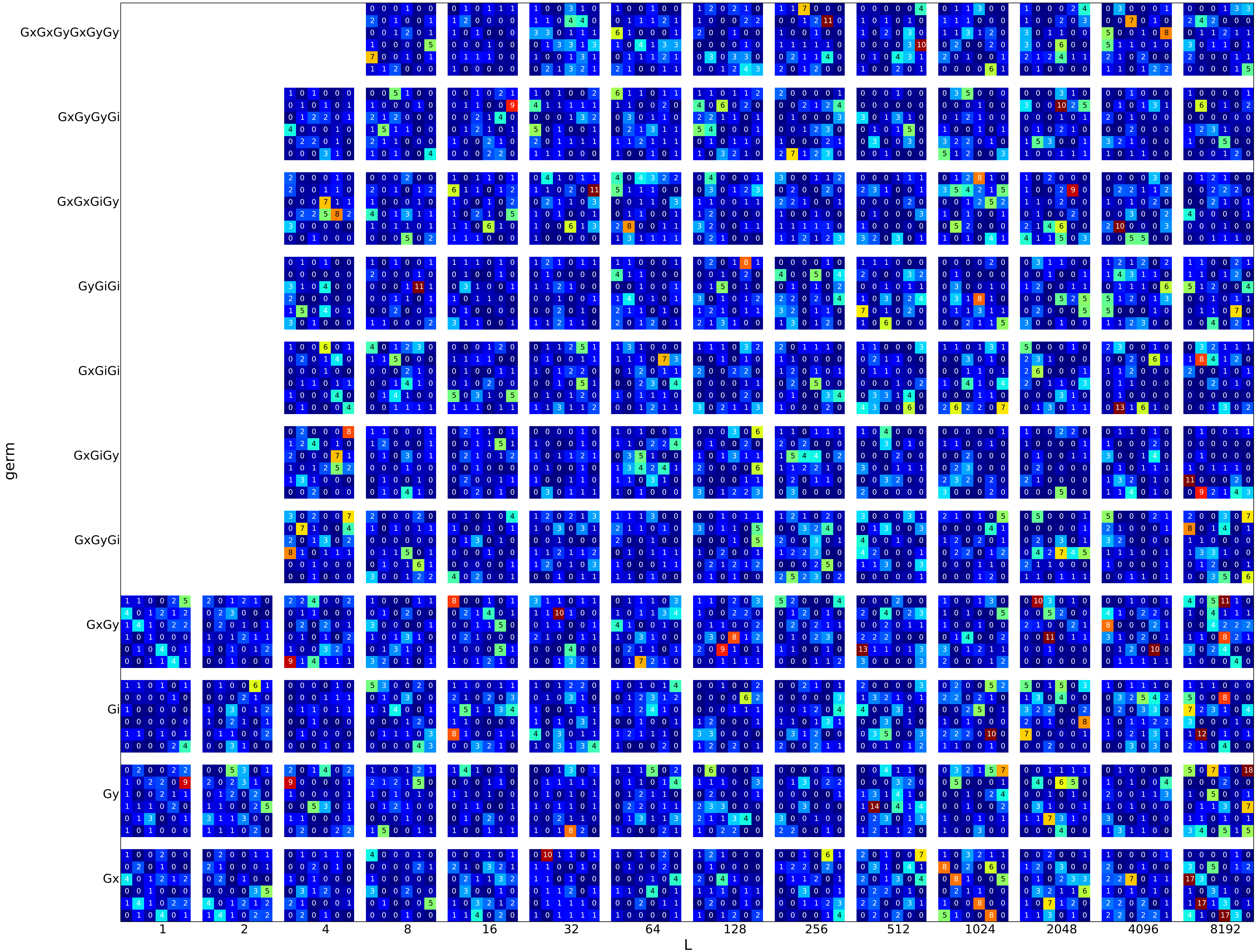
$$\widehat{G_x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.999946 & 5\times10^{-5} & 0 \\ 0 & 2\times10^{-5} & 5\times10^{-5} & -0.999904 \\ 0 & 4\times10^{-5} & 0.999904 & 6\times10^{-5} \end{bmatrix}$$

$$1 - F(\widehat{G}, G_{\text{ideal}}) = (6.1 \pm 0.4)\times 10^{-5}$$
$$\|\widehat{G} - G_{\text{ideal}}\|_\diamond = (7.0 \pm 1.3)\times 10^{-5}$$

$$\widehat{G_y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2\times10^{-5} & 2\times10^{-5} & 0.999876 \\ 0 & -2\times10^{-5} & -0.999962 & 0 \\ 0 & 0.999876 & 3\times10^{-5} & -5\times10^{-5} \end{bmatrix}$$

$$1 - F(\widehat{G}, G_{\text{ideal}}) = (7.1 \pm 0.4)\times 10^{-5}$$
$$\|\widehat{G} - G_{\text{ideal}}\|_\diamond = (8.1 \pm 1.3)\times 10^{-5}$$

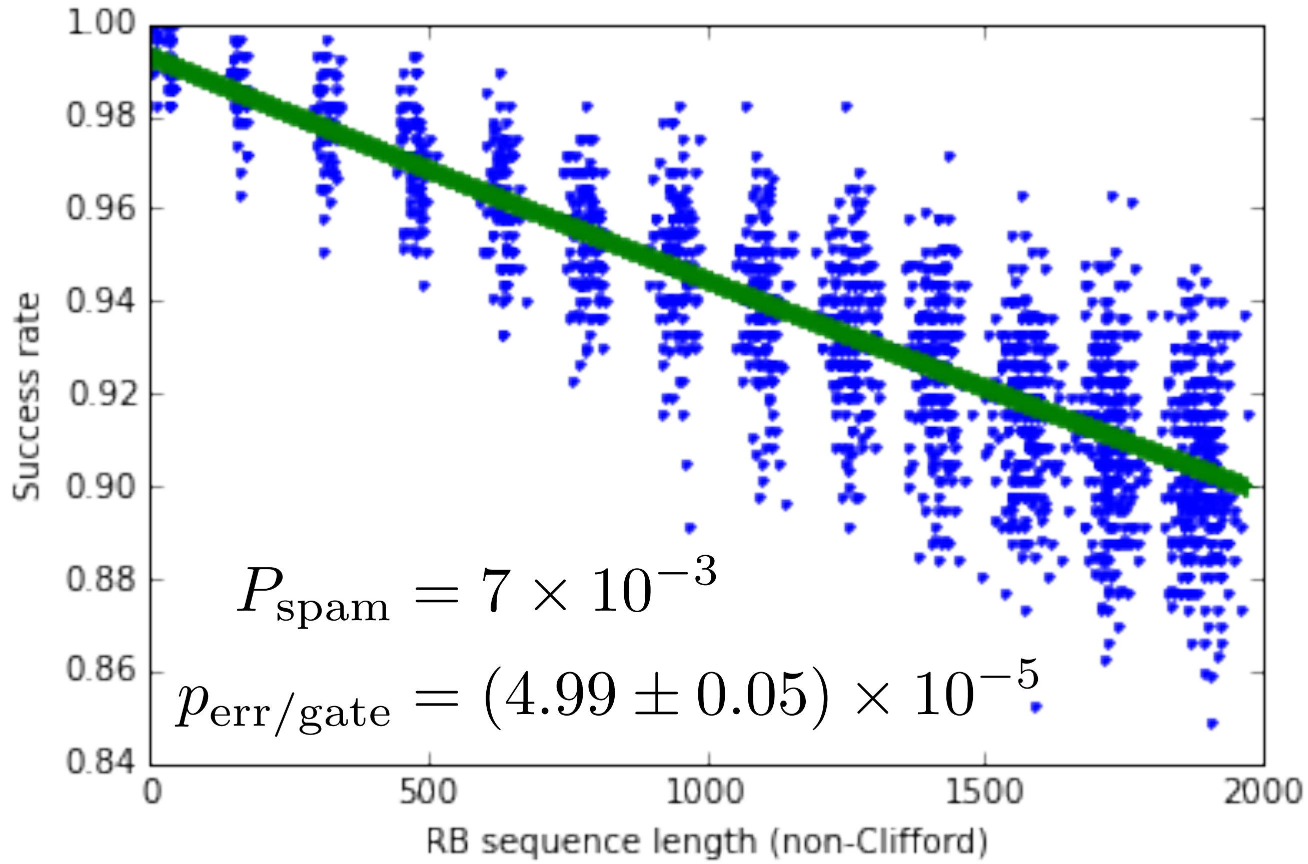
Gates are *really* Markovian!



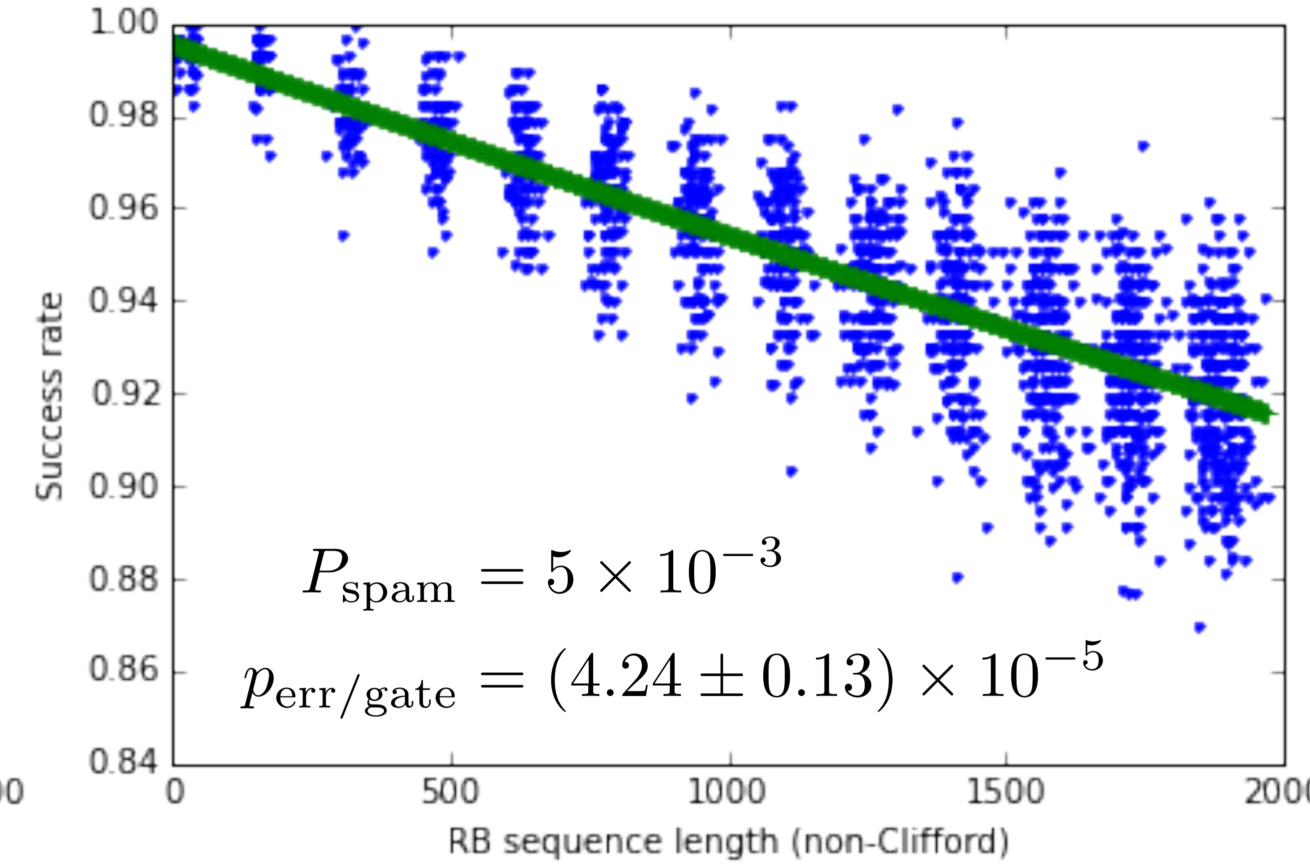
RB vs. GST

- Peter did RB experiments at the same time. We used GST to predict what the RB experiments should look like.

Experimental RB



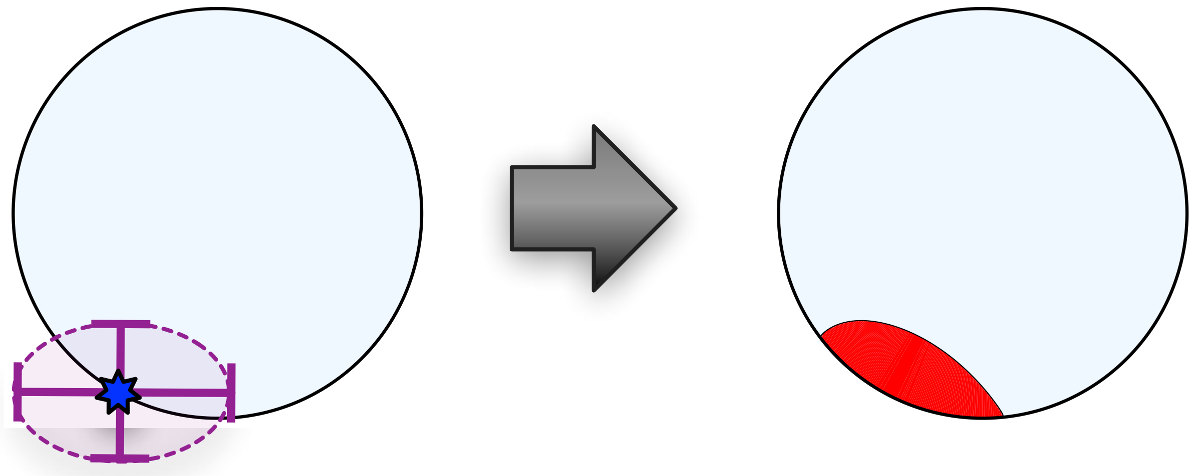
GST Prediction



2. We put error bars on it

"All the single theorists... if you like it, put error bars on it!"

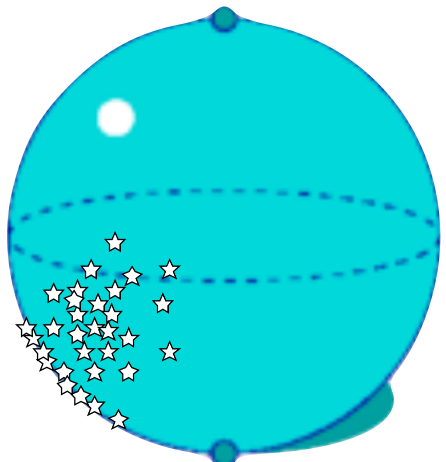
- Error bars have been a perennial challenge for tomography.
- Traditionally, a positivity constraint is essential to achieve high accuracy...
  ...but it's hard to assign error bars that respect this constraint.



- Also, assigning error bars is just plain tricky and hard.
- We've equipped GST with two kinds of error bars: *bootstrap* and *likelihood ratio confidence regions*.
- **Key**: GST is so accurate that positivity isn't really relevant!

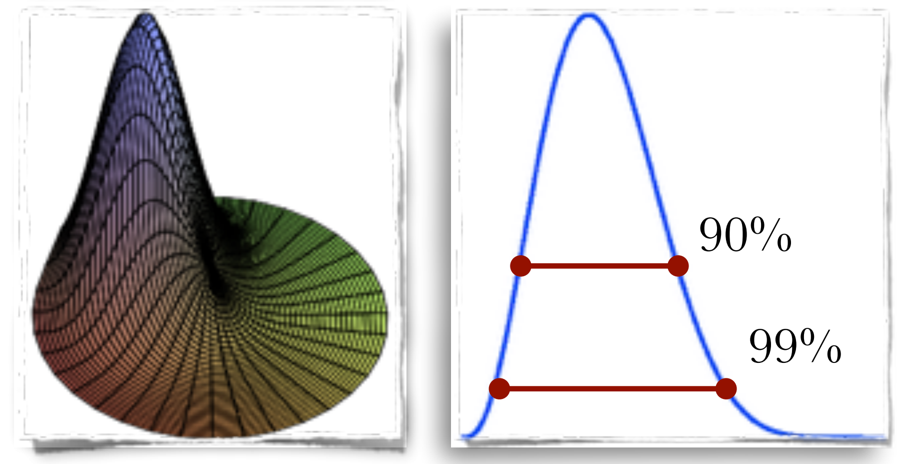
More about the bootstrap

- The error bars shown previously were obtained using the [parametric] bootstrap.
- I've probably yelled at you for bootstrapping before! ?????
- The bootstrap is wildly unreliable *when used with a biased estimator*. MLE state/process tomography is biased because of the boundary.

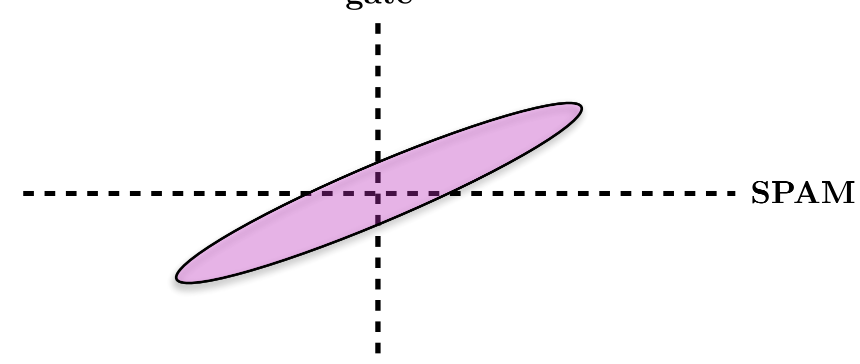


- We *only* use unbiased estimators in GST (it's so accurate that gates are usually CP. If they aren't, we just accept it.)
- Ultra-long sequences mean likelihood function is ~Gaussian.

Likelihood ratio confidence regions



- Better method: use the shape of the likelihood function near its maximum to assign a confidence region. — arxiv:1202.5270



- We've implemented this method, but are still fixing bugs: gauge freedom mixes SPAM error bars (large) with gate error bars (small).

- Prelim results: **SPAM error bars** $\pm\{4\times10^{-4}\ldots3\times10^{-3}\}$

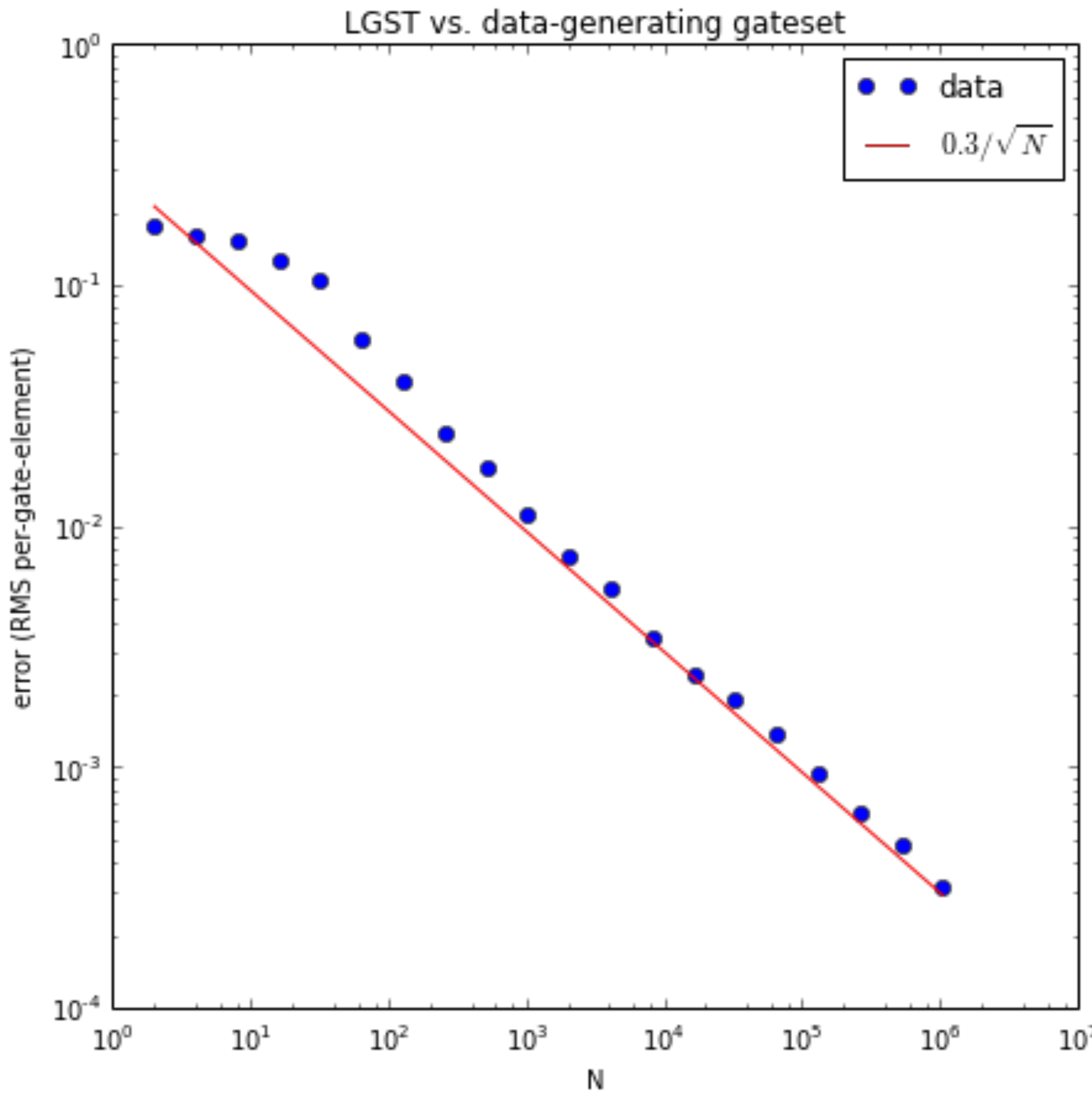
  Gate matrix error bars $\pm\{3\times10^{-6}\ldots3\times10^{-5}\}$

3. We did 2 qubits at the same time

- 2-qubit GST presents some challenges:
  - At least 5 gates required for a complete gate set
  - Each gate is 16x16 => 256 parameters
  - Roughly **1250** parameters to estimate (vs ~50 for 1 qubit).
  - Does anybody know how to interpret results?
- We have implemented *simulations* of 2-qubit and "biqubit" (symmetric subspace) *linear* GST.
- We have also simulated *long sequence* GST on a biqubit up to L=8.
- And we just did *experimental* GST on a biqubit up to L=8.

Simulations (2-qubit LGST)

- 5 gates: $X_{\pi/2}\otimes I$, $Y_{\pi/2}\otimes I$, $I\otimes Y_{\pi/2}$, $I\otimes X_{\pi/2}$, CNOT)
- 36 x 36=1296 "fiducial pairs" x (5+1) operations = 5700 distinct experiments
- With N=1000 ($6\mathrm{x}10^6$ total "clicks") we get **≈**1% accuracy.
- If we wanted to get 0.1% accuracy using LGST, we would need to do N=130,000 counts/experiment $\Rightarrow 10^9$ clicks. Not feasible! 👎



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Simulations (2-qubit LSGST)

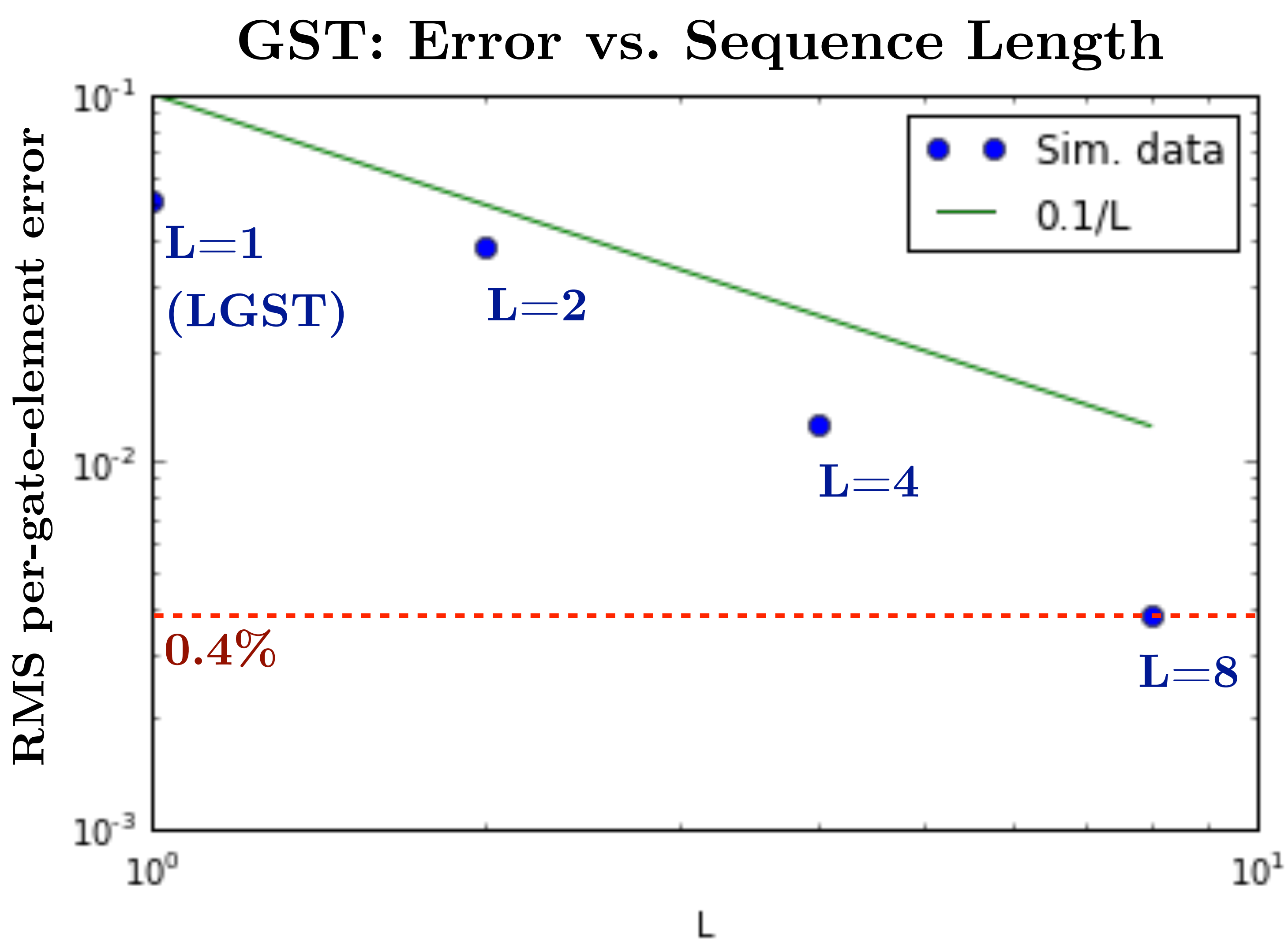
- 5 gates: $X_{\pi/2}\otimes$I, $Y_{\pi/2}\otimes$I,
  I$\otimes Y_{\pi/2}$, I$\otimes X_{\pi/2}$, CNOT)

- 16 x 10 = 160 "fiducial pairs"
  71 germs
  3528 distinct experiments
  (N.B. we threw out about 85% of experiments — "compressed GST")

- At L=1, error is 4%.

- At L=8, error is 0.4%

- Consistent with 1/L scaling.



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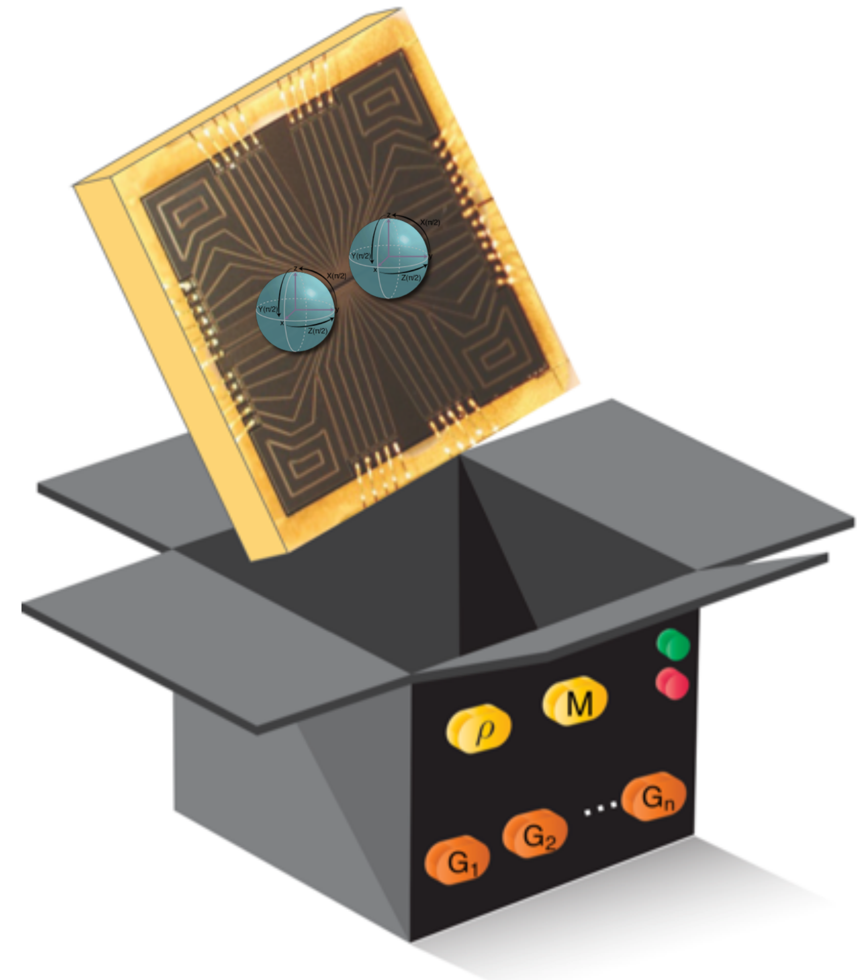
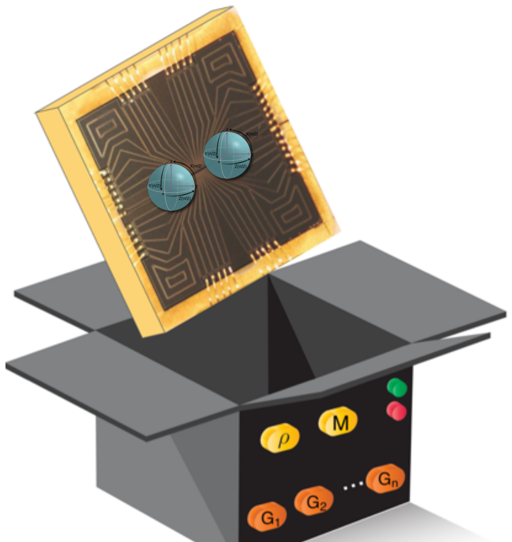
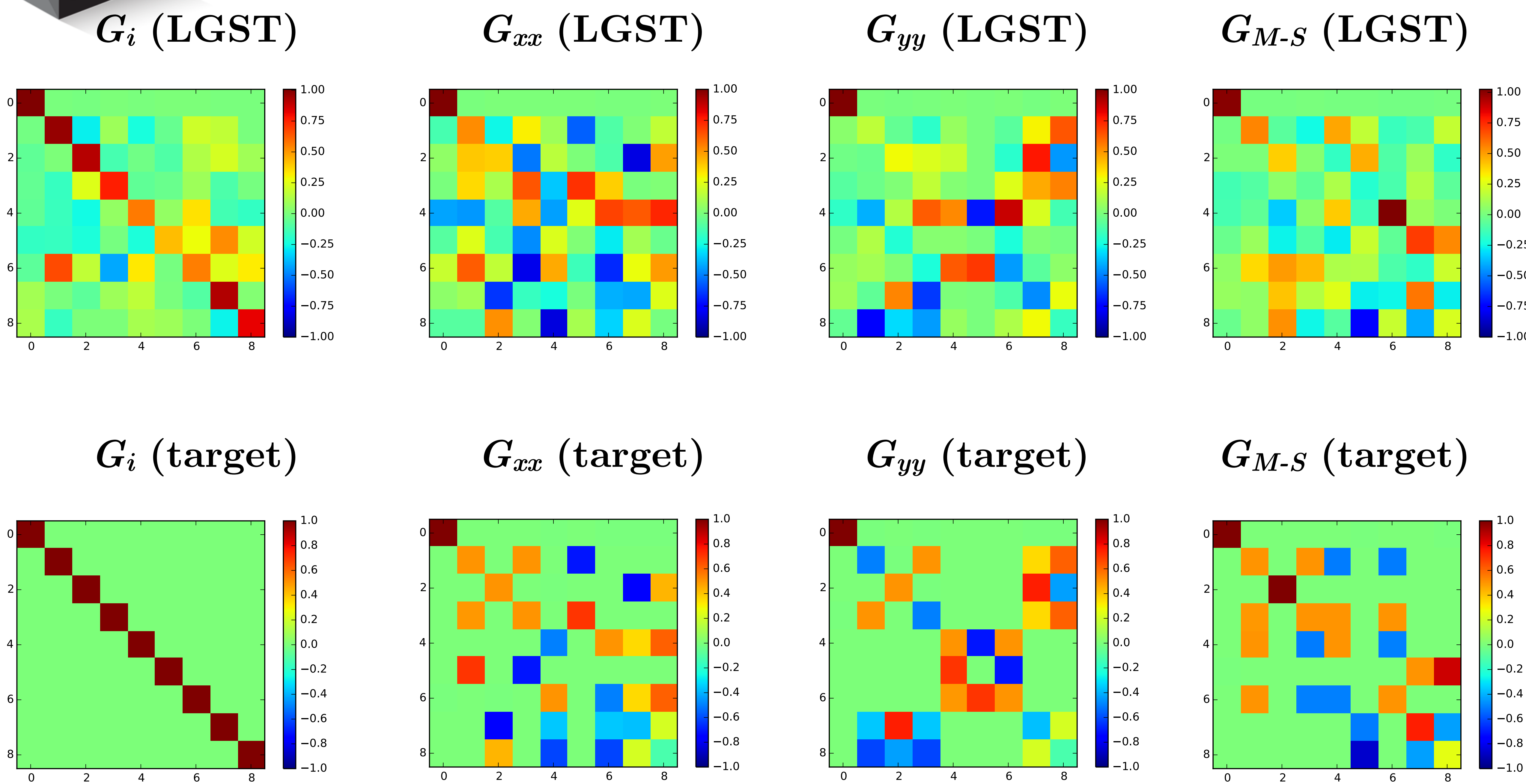
Biqubit Experiment



- Experiment performed 9/2015 with 2 Yb^{+} qubits trapped in a SNL surface trap.
- Individual addressing not yet possible — all gates are symmetric (XX, YY, Molmer-Sorenson).
  => only symmetric subspace (d=3) is accessible. Call it a "biqubit".
- **Good**: full (L=8) GST is feasible, ran in a few hours.
  Bad: gates not stable => systematic errors
  => only LGST makes sense



Experiment Results



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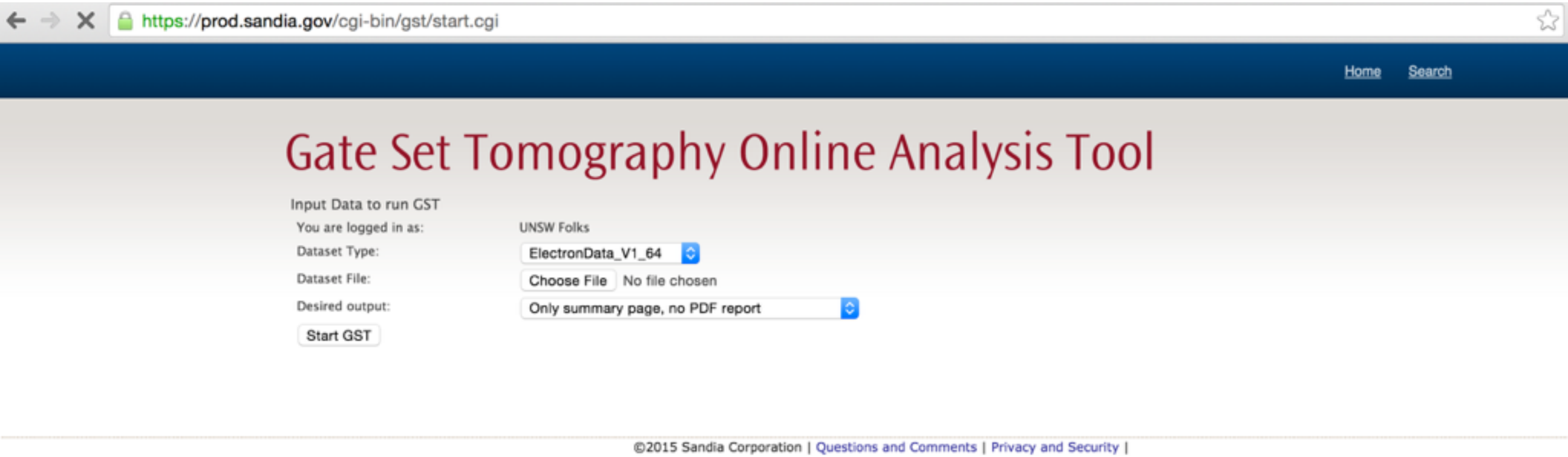
Now, do GST
in the privacy of your own home!



Now, do GST
in the privacy of your own home!

Home Search

Gate Set Tomography Online Analysis Tool

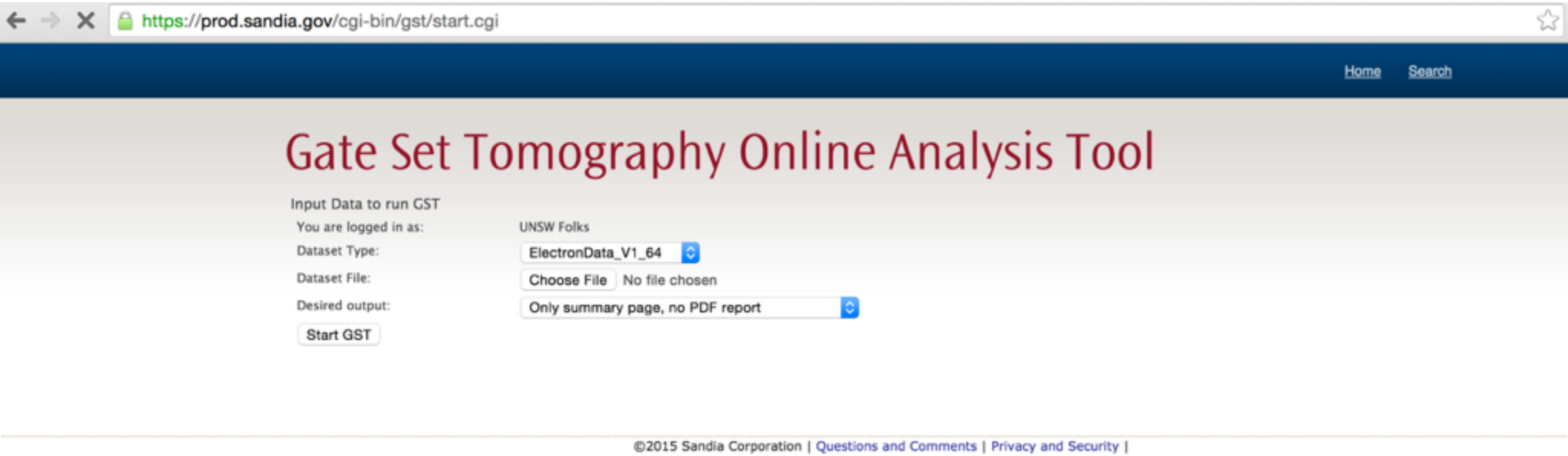
GST Status

```
LGST: Singular values of I_tilde (truncating to first 4 of 6) =
[ 4.24975293  0.833907    0.75160394  0.73493365  0.19956877  0.12781025]

--- LGST ---

--- Iterative LSGST: Beginning iter 1 of 9 : 92 gate strings ---
--- Least Squares GST ---
 Sum of Chi^2 = 69.0899 (92 data params - 56 model params = expected mean of 36)

--- Iterative LSGST: Beginning iter 2 of 9 : 92 gate strings ---
--- Least Squares GST ---
 Sum of Chi^2 = 69.0899 (92 data params - 56 model params = expected mean of 36)

--- Iterative LSGST: Beginning iter 3 of 9 : 168 gate strings ---
--- Least Squares GST ---
 Sum of Chi^2 = 157.953 (168 data params - 56 model params = expected mean of 112)

--- Iterative LSGST: Beginning iter 4 of 9 : 441 gate strings ---
--- Least Squares GST ---
 Sum of Chi^2 = 455.977 (441 data params - 56 model params = expected mean of 385)

--- Iterative LSGST: Beginning iter 5 of 9 : 817 gate strings ---
--- Least Squares GST ---
 Sum of Chi^2 = 870.698 (817 data params - 56 model params = expected mean of 761)

--- Iterative LSGST: Beginning iter 6 of 9 : 1201 gate strings ---
--- Least Squares GST ---
 Sum of Chi^2 = 1263.19 (1201 data params - 56 model params = expected mean of 1145)

--- Iterative LSGST: Beginning iter 7 of 9 : 1585 gate strings ---
```

GST Best Estimate
Gateset Spam:

Operator	Hilbert-Schmidt vector (Pauli basis)	Matrix
rho_0	0.7957	1.2451 0.0053e^(i1.9)

Now, do GST
in the privacy of your own home!



Expanded functionality coming soon (Nov. 1)

Email rjblume@sandia.gov to get started.