SAND2015-8342C

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Calculation of the Dissipated Energy Spectrum from a Fourier Amplitude Spectrum

86^{th} Shock and Vibration Symposium
October 5 – 8, 2015

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Introduction

- Energy spectrum methods have been used and studied for quite some time.
  - Hudson 1956
  - Housner 1959
  - Zahrah and Hall 1984
  - Ordaz, Huerta, and Reinoso 2003
  - Edwards 2007 – 2008
  - And many others

- Most of this work originated in the context of earthquake engineering but its applicability to mechanical shock and vibration is well accepted

Motivation

- Energy spectrum methods have recently been applied to very long duration non-stationary shock and vibration events
  - Analysis of machining operations
- Results in very time consuming calculations
  - Extremely high data sample rates to capture tooling vibrations
  - Long signal durations, minutes or even hours of data
- Traditional methods utilize multiple, repetitive integrations in the time domain
  - Can require significant computational resources
  - One recent example took about a month to calculate

Motivation

- Ordaz, Huerta, and Reinoso proposed a method for calculating the input energy spectra using a smoothed Fourier amplitude spectrum (2003)
  - Fourier methods are typically much faster than temporal methods
- Goal of this work is to derive a similar computational methodology for the dissipated energy spectrum
  - Assume that a similar analogy would be possible
  - Utilize the faster computational efficiency of the Fourier spectrum
  - Make energy spectrum calculations more tractable for analyzing long duration events

Energy Spectrum Introduction

- Input energy spectrum measures energy introduced to a system as a result of a transient insult
  - Must be absorbed, dissipated, or converted to kinetic energy
  - A reasonable measure of damage potential
- Input energy per unit mass in terms of base acceleration $\ddot{x}(t)$

$$E_I = \frac{E_I^*}{M} = -\int_0^Z \ddot{x}(t) dz$$

- The system side of the equation (Zahrah & Hall):

$$E_I^* = \int_0^Z M\ddot{z}(t) dz + \int_0^Z C\dot{z}(t) dz + \int_0^Z Kz(t) dz$$

  - Where M = mass, C = damping coefficient, and K = system stiffness

Energy Spectrum Introduction

- In terms of energy per unit mass gives:

$$E_I = \frac{E_I^*}{M} = \int_0^z \ddot{z}(t)dz + 2\zeta\omega \int_0^z \dot{z}(t)dz + \int_0^z \omega^2 z(t)dz$$

  - Kinetic energy per unit mass

$$E_K = \int_0^z \ddot{z}(t)dz = \int_0^t \ddot{z}(t)\dot{z}(t)dt$$

  - Dissipated energy due to damping per unit mass

$$E_D = 2\zeta\omega \int_0^z \dot{z}(t)dz = 2\zeta\omega \int_0^t \dot{z}(t)^2 dt$$

  - Absorbed energy per unit mass

$$E_A = \int_0^z \omega^2 z(t)dz = \omega^2 \int_0^t z(t)\dot{z}(t)dt$$

Input Energy Calculation

- Ordaz, Huerta, and Reinoso (2003) presented an input energy spectrum calculation using Fourier amplitude spectra

$$\frac{E_I}{M} = -\frac{1}{2\pi}\int_{-\infty}^{\infty} |A(\omega)|^2 H_V(\omega,\Omega,\zeta)\,d\omega$$

  - Since the transfer function is complex but even, it is equivalent to

$$\frac{E_I}{M} = -\frac{1}{\pi}\int_{0}^{\infty} |A(\omega)|^2 \Re[H_V(\omega,\Omega,\zeta)]\,d\omega$$

  - Where $H_V(\omega,\Omega,\zeta)$ is the transfer function of the elastic oscillator:

$$H_V(\omega,\Omega,\zeta) = \frac{-i\omega}{\Omega^2 - \omega^2 + 2i\zeta\omega\Omega}$$

  - The input energy spectrum is a smoothed Fourier amplitude spectrum squared where $\Re[H_V(\omega,\Omega,\zeta)]$ is the smoothing function

Absorbed Energy Calculation

- Hudson (1956) defines absorbed energy equivalent to the potential energy stored in a spring:

$$E_A = \frac{1}{2} K (x - y)^2$$

  - In terms of absorbed energy per unit mass this is:

$$\frac{E_A}{M} = \frac{1}{2} \frac{K}{M} (x - y)^2$$

  - Substituting the system frequency gives:

$$\frac{E_A}{M} = \frac{1}{2} \omega^2 (x - y)^2 = \frac{1}{2} \omega^2 \left(\frac{S_v}{\omega}\right)^2 = \frac{1}{2} S_v^2$$

  - Where S_v is the pseudo-velocity SRS

Dissipated Energy Spectrum

- As stated initially, the goal is to develop a Fourier based method for calculating the dissipated energy spectrum

- All of the input energy must be dissipated in some manner
  - Energy absorbed by the system: E_A
  - Kinetic energy: E_K
  - Energy dissipated through damping: E_D

- What dissipated energy is sought?
  - Energy dissipated during the shock transient $(E_K \neq 0)$
  - Total energy dissipated after the system returns to rest $(E_K = 0)$

Dissipated Energy Calculation

- Dissipated energy given previously by:

$$\frac{E_D}{M} = 2\zeta \int_0^z \omega \dot{z}(t) dz$$

- Which resembles the expression for input energy
  - But in terms of spectrum, $\ddot{x}(t) = \omega \dot{z}(t)$ which implies that:

$$\frac{E_I}{M} = \int_0^z \ddot{x}(t) dz = \int_0^z \omega \dot{z}(t) dz$$

- Therefore

$$\frac{E_D}{m} = 2\zeta(\omega) \frac{E_I}{m}$$

- Dissipated energy during the shock is related to input energy

Dissipated Energy Calculation

- The dissipated energy of most interest is the total of all energy dissipated through damping
  - Occurs at some time after the shock event
  - Reached when kinetic energy goes to zero

- Looking again at the expression for total energy:

$$E_I = E_K + E_D + E_A$$

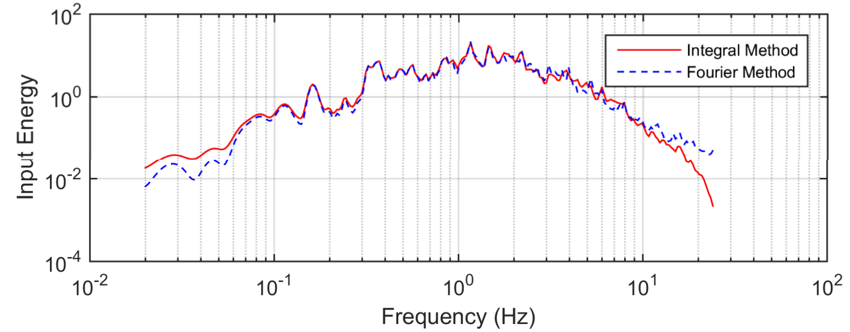
- If $E_K = 0$ then E_D is simply given by:

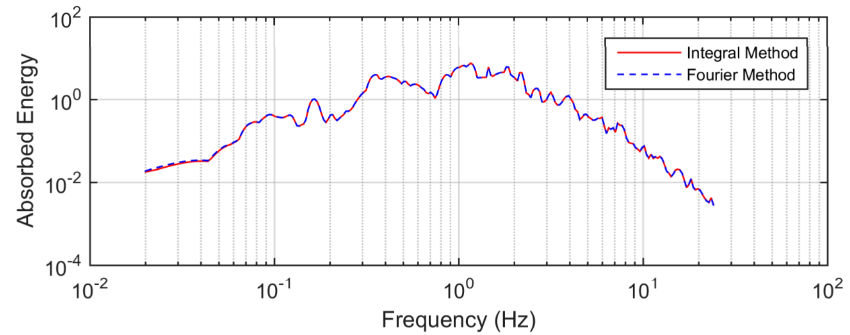
$$E_D = E_I - E_A$$

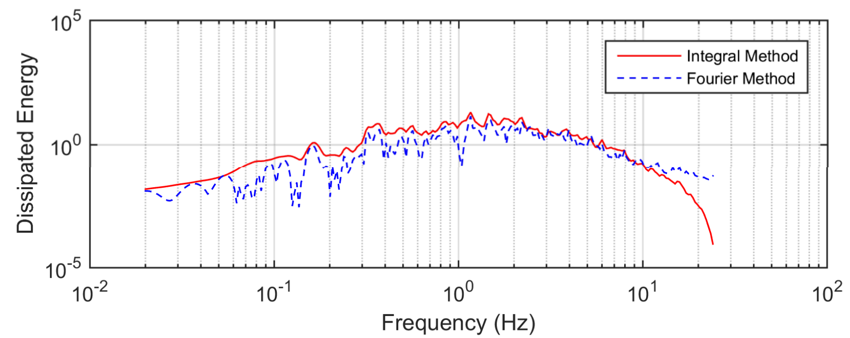
  - Since we have a frequency domain solution for E_I and E_A, we inherently have one for E_D

Energy Spectrum – Earthquake

- 18 May 1940 Imperial Valley N-S earthquake time history
  - Seems to be the most common example in the literature
- Absorbed energy spectrum is identical as expected
- Input energy spectrum is very similar although there does appear to be some slight deviations at the high and low end.
- Dissipated energy spectrum is a good match but more featured.

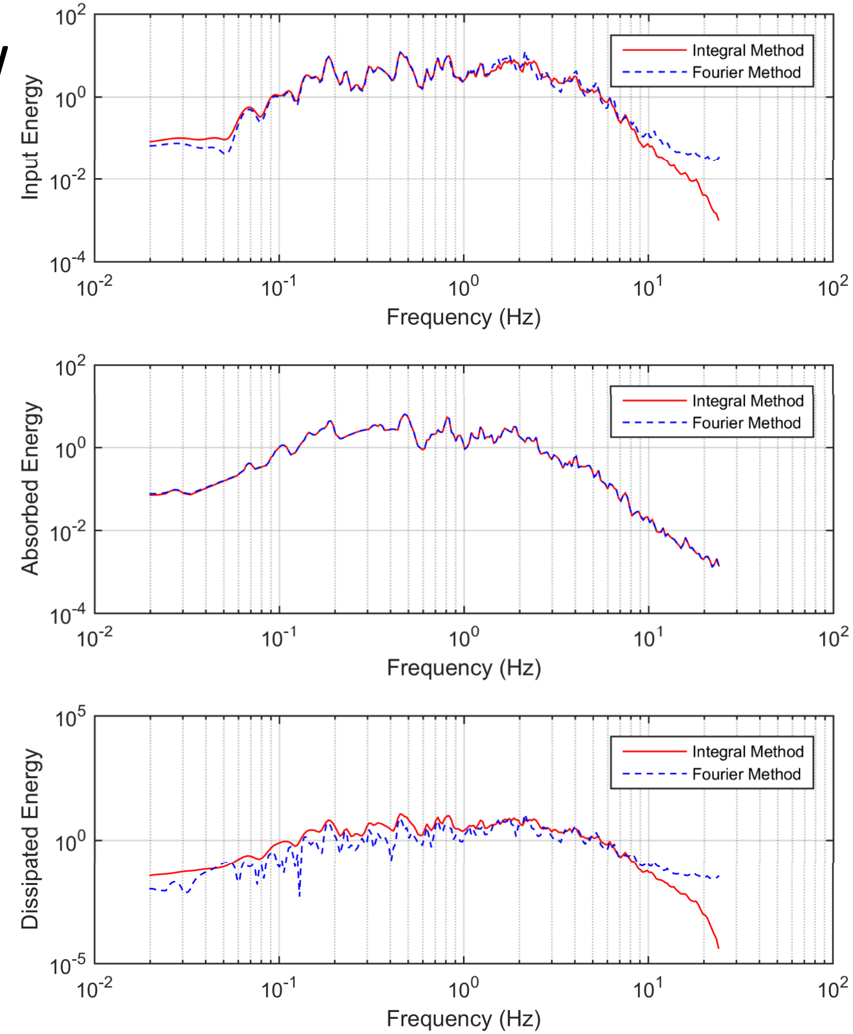






Energy Spectrum – Earthquake

- 18 May 1940 Imperial Valley E-W earthquake time history
- Similar type and quality of response as with the N-S response.



Energy Spectrum – Lathe Operation

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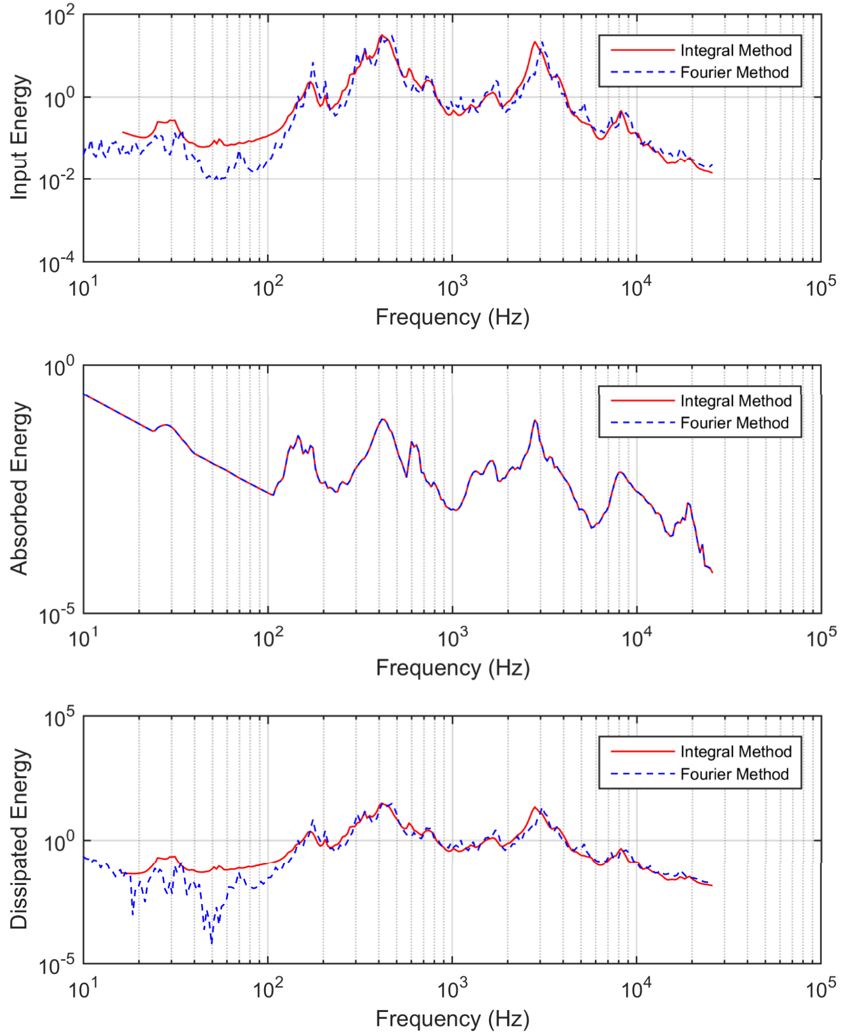
- Energy calculation from a 10min lathe cutting operation
  - Accelerometer mounted on the rotating part
  - Excellent agreement for absorbed energy spectrum
  - Very good agreement with input and dissipated energy spectrum

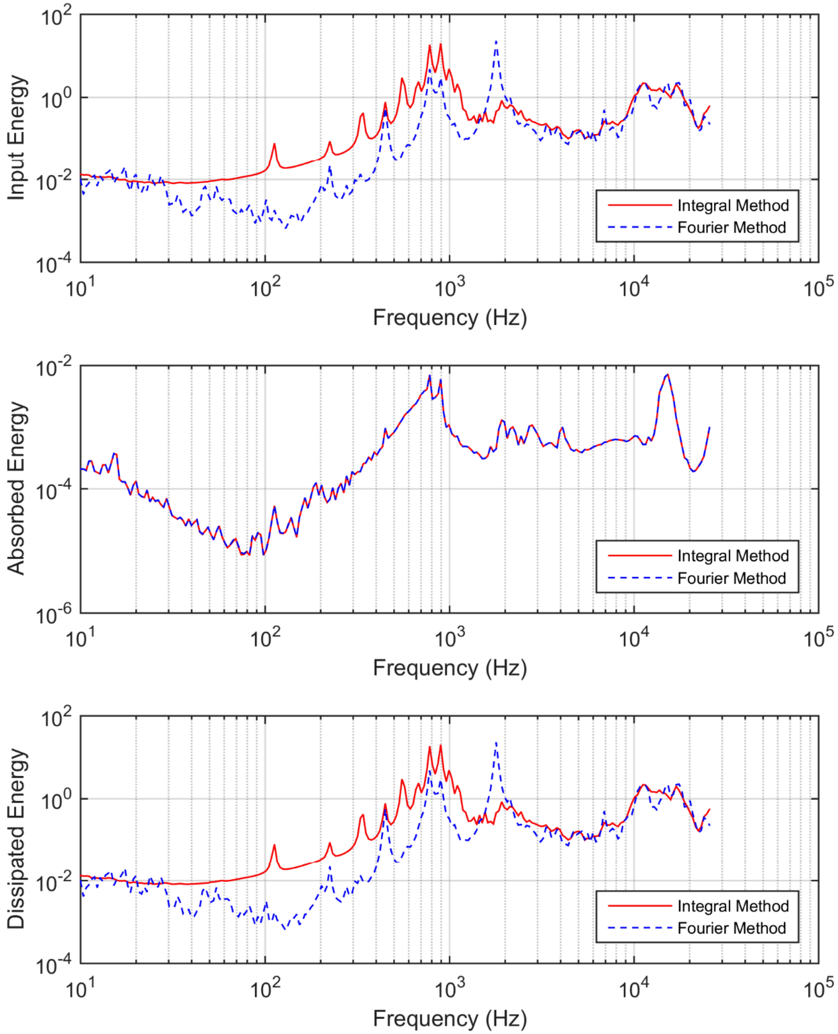
- Calculation time comparison
  - Integral method: 41.7 min
  - Fourier method: 12.6 min



Energy Spectrum – Mill Operation

- Energy calculation from a 6min milling machine operation
  - Accelerometer mounted on part
  - Excellent agreement for absorbed energy spectrum
  - Not very good agreement with input and dissipated energy spectrum

- Calculation time comparison
  - Integral method: 49.7 min
  - Fourier method: 14.8 min



Energy Spectrum – Mill Operation

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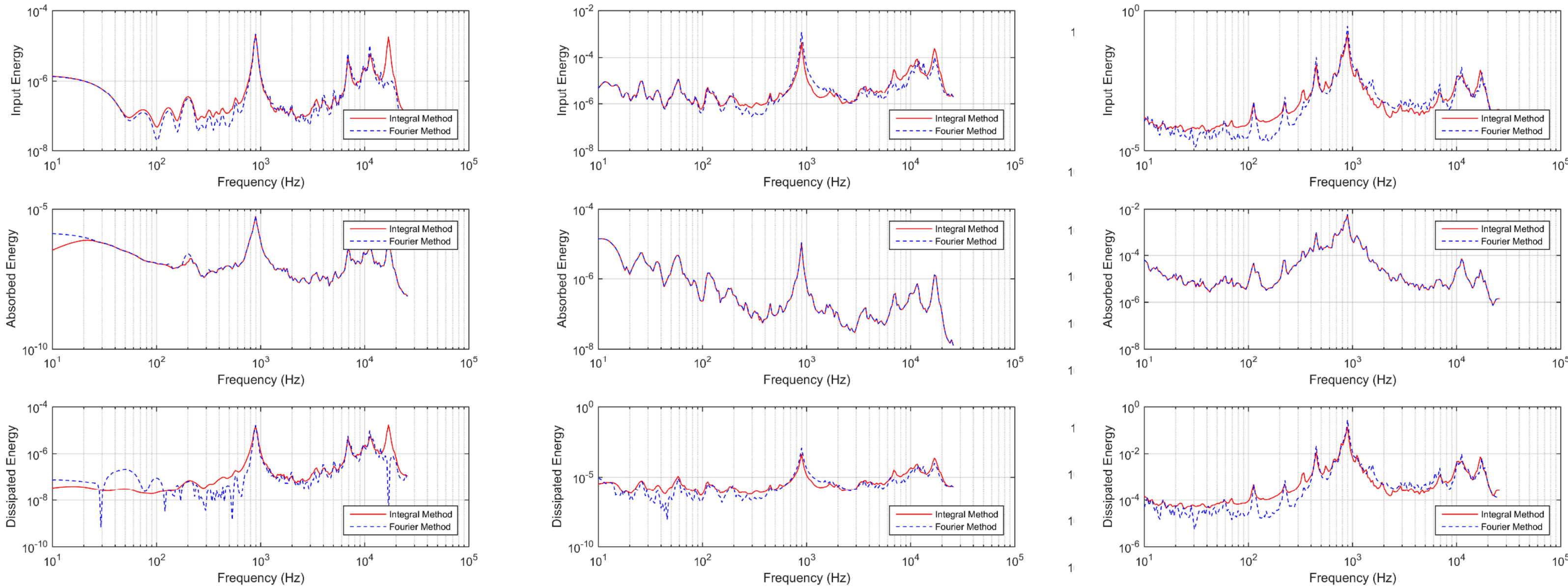
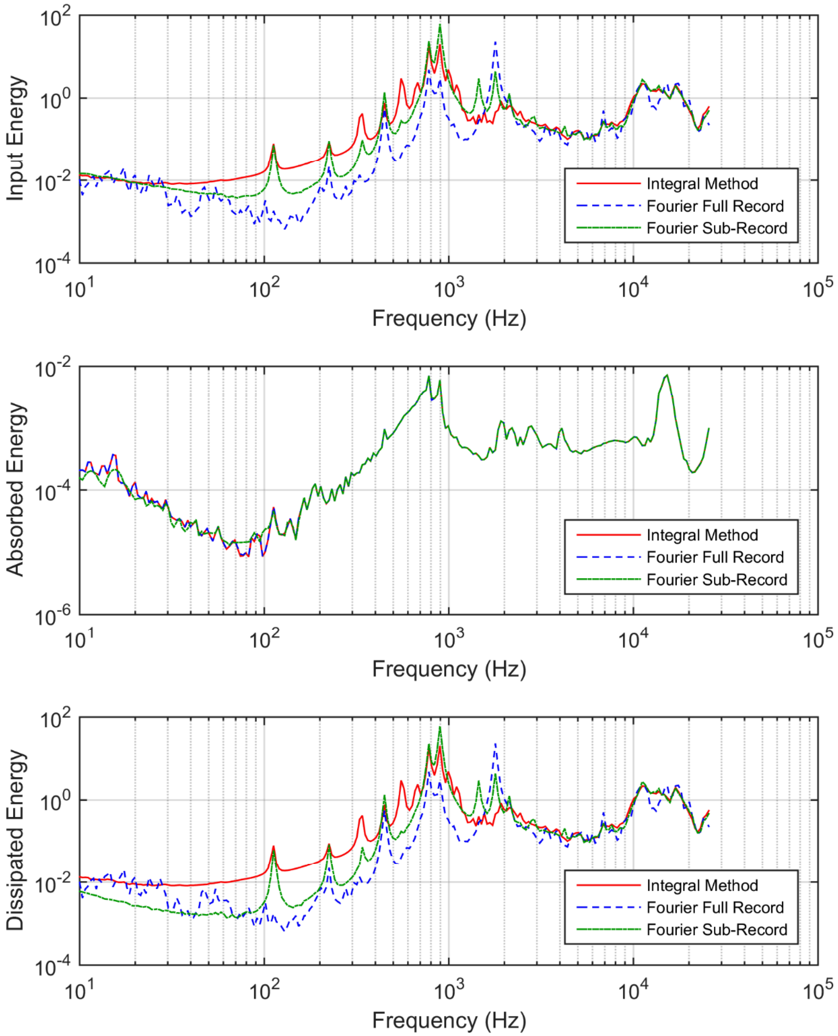
- Is it possible to get better agreement between the two methods for this operation?
  - First try to figure out where the comparison breaks down
  - Repeated the calculation using a sub-sets of the data for different record lengths (total record length is > 77 million data points)



2^{12} points 2^{16} points 2^{20} points

Energy Spectrum – Mill Operation

- Results obtained by dividing the record into approximately 1700 sub-records of 2^{16} points each and combining the results
- Calculation time comparison
  - Integral method: 49.7 min
  - Fourier full record: 14.8 min
  - Fourier sub-records: 7.62 min
- Which is right?
  - They are both numerical methods on a very long data set.



Conclusion

- A method for calculated the dissipated energy spectrum is presented which is based on Fourier analysis
- New method gives results that are comparable with the previously used direct time integration methods
- New method is approximately 3 – 6 times faster than the baseline method