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Fully Parameterized Reduced Order Models Using Hyper-Dual Numbers and Component Mode Synthesis

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Outline

- Introduction to Hyper-Dual Numbers
- Using Hyper-Dual Numbers
- Parameter Sensitivity
- Remarks/ Conclusions

What are Dual Numbers?

- Dual numbers: Generalized complex number
  - $\epsilon^2 = 0$ similar to $i^2 = -1$
  - Higher order terms = 0
  - Creates a finite Taylor series expansion
  - $f(x + \delta\epsilon) = f(x) + \delta f'(x)\epsilon$

What are Hyper-Dual Numbers?

- Multi-dimensional expansion of dual numbers
- Similar to quaternions for ordinary complex numbers
- Allows for exact derivatives in higher orders
- Some special considerations in mathematics
- Example of 2^{nd} order, but can be increased in order

Hyper-Dual Math

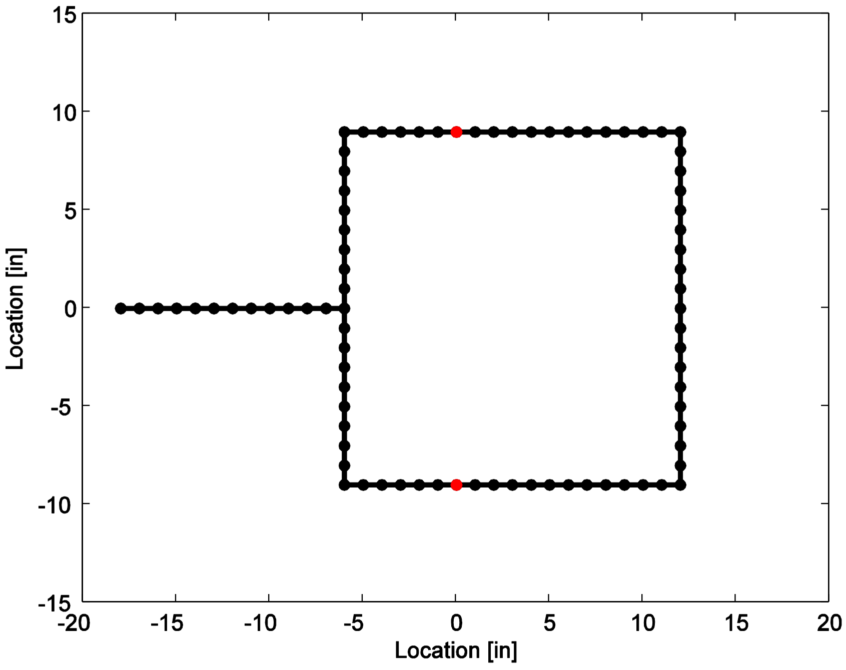
- 2nd Order: $\epsilon_1, \epsilon_2, and\ \epsilon_1\epsilon_2$
- 2 Independent, 1 Dependent
- Treat similar to polynomial or complex
  - $(1 + 1\epsilon_1 + 2\epsilon_2 - 5\epsilon_1\epsilon_2) + (5 - 2\epsilon_1 + 2\epsilon_2 + 5\epsilon_1\epsilon_2) = (6 - 1\epsilon_1 + 4\epsilon_2 + 0\epsilon_1\epsilon_2)$
- Follows associative property
  - $\epsilon_1 * \epsilon_2 = \epsilon_2 * \epsilon_1 = \epsilon_1\epsilon_2$
- Higher order terms are zero
  - $\epsilon_1^2 = \epsilon_2^2 = (\epsilon_1\epsilon_2)^2 = 0$

Using Hyper-Dual Numbers

- Requires slight change in code
  - Programmed into a MatLab class
- Specify parameter directions
  - Young's modulus: $E = E_0 + h_1\epsilon_1 + h_2\epsilon_2 + 0\epsilon_1\epsilon_2$
  - Can have different parameter on each direction
- Run code as normal
- Outputs will have sensitives built-in
  - $F = F_0 + h_1 \frac{\partial F}{\partial E}\epsilon_1 + h_2 \frac{\partial F}{\partial E}\epsilon_2 + h_1 h_2 \frac{\partial^2 F}{\partial E^2}\epsilon_1\epsilon_2$

Example Problem

- Theory tested on planar frame
- 3 different parameters
  - Young's Modulus
  - Mass Density
  - Appendage Length
- ROMs using Craig-Bampton



Example Parameters

- Sweep ±75% of nominal value
  - Resolution 0.1%
- One a single code evaluation
- Varied order on Taylor series
- 20 Fixed Interface Modes

Example Procedure

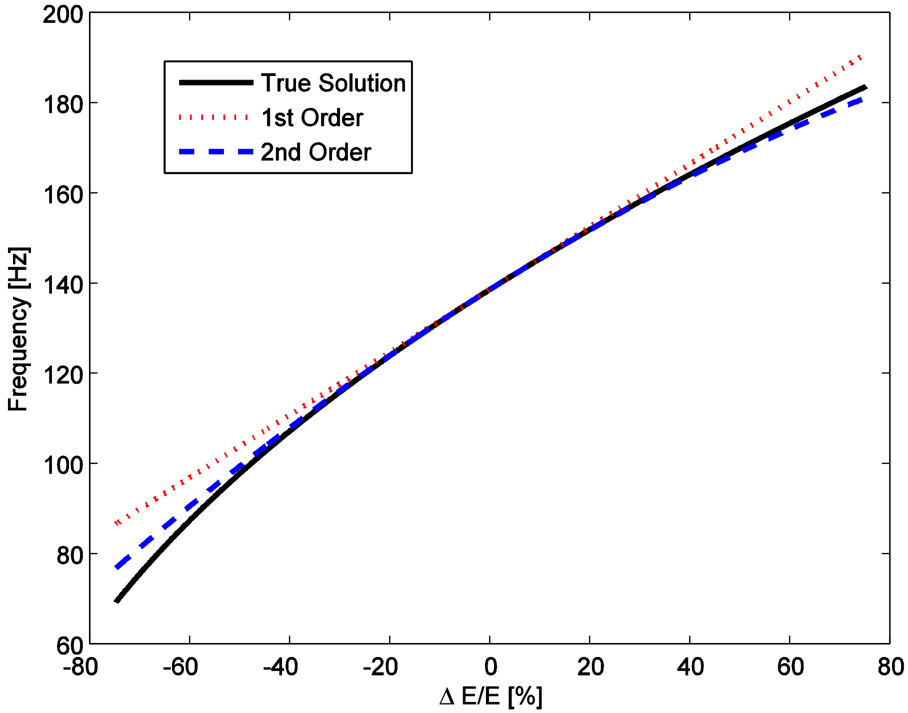
- Define Hyper-Dual numbers in FE Code
- Create high-fidelity model
- Create Craig-Bampton ROMs
- Assemble using CMS
- Calculate natural frequencies

- Results in parameterized natural frequencies

Young's Modulus Sensitivity

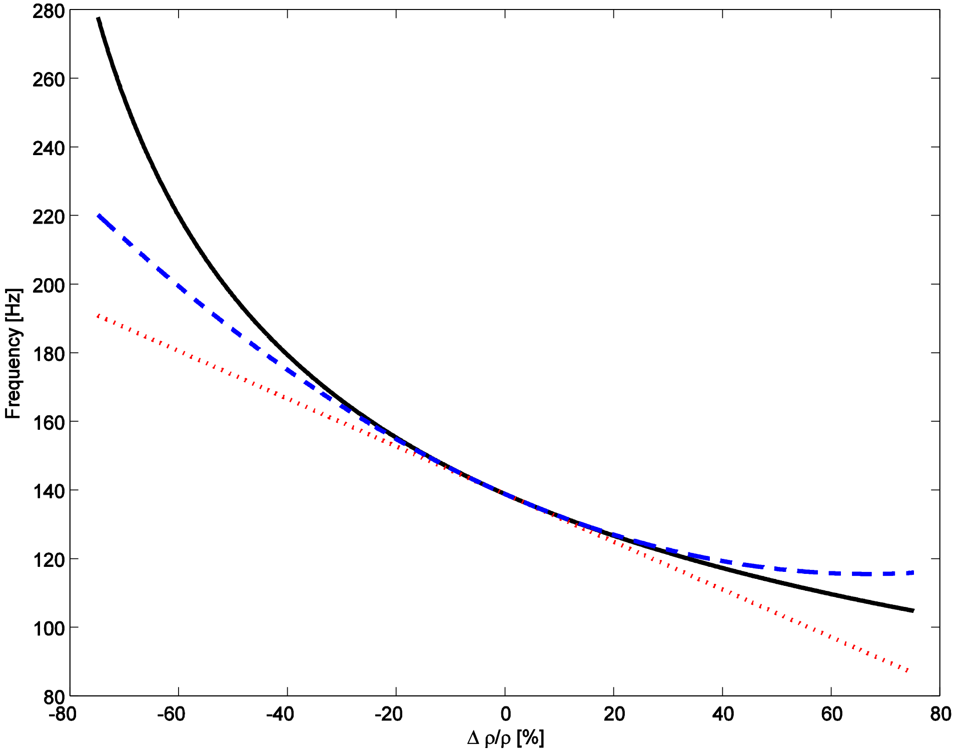
- Compared 1st and 2nd order Taylor series expansion

- 2nd order is almost exact for up to 75% change



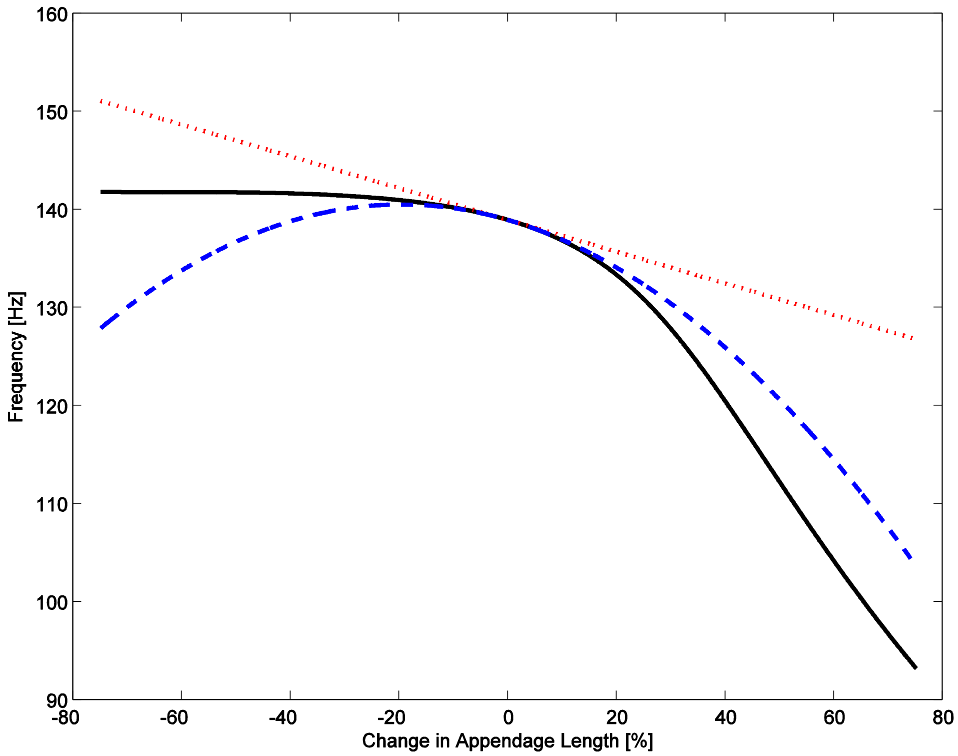
Mass Density Sensitivity

- Compared 1st and 2nd order Taylor series expansion
- Requires more orders to accurately predict



Appendage Length Sensitivity

- Compared 1st and 2nd order Taylor series expansion
- Very non-linear behavior
- Does not require re-meshing system



Expanding the Accuracy Range

- Finite difference gives a decent accuracy over a range of values
- Hyper-Dual gives an exact derivative at a single point

- Why not take the best of both and combine together?

Remarks

- Hyper-Dual numbers allow for exact sensitivities with a single evaluation
- Implemented in Matlab and Sierra/SD
- Can calculate geometric changes
- Methods can be used as a surrogate model for uncertainty propagation

Special Thanks

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- Questions?