A Nonlocal Strain Measure for Digital Image Correlation

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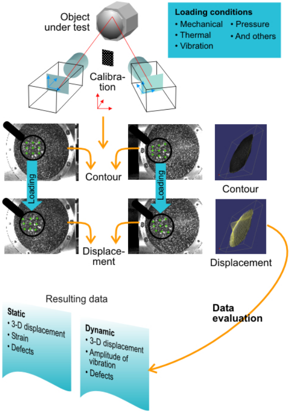
Digital Image Correlation (DIC)

- DIC is a full-field image analysis method based on grey value digital images
- Determine the deformation, strain of an object subjected to a load
- Courtesy Dantec Dynamics



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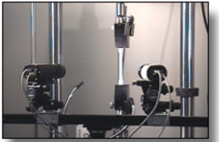
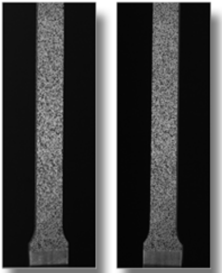
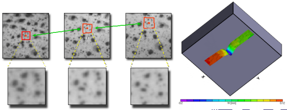
Basic DIC process

- Two cameras photograph a "speckled" dogbone subjected to a load
- A speckling (or contrast) occurs by spraying a mist of black paint over a white dogbone
- DIC tracks the speckles by comparing the sequence of photos, or images
- Extract deformation, strain
- Courtesy Correlated Solutions







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DIC Strain

- Two standard approaches for generating strain from image displacement data; (1) finite differences (2) polynomial fitting; both are problematic
- A Novel Class of Strain Measures for Digital Image Correlation, (with Phil Reu, Dan Turner), *Strain: A journal of experimental mechanics* (2015) demonstrates that the nonlocal strain $\tilde{\nabla}$ leads to a robust approximation of the strain with excellent signal to noise ratio (patent pending)
- Nonlocal strain exploits an integral approximation—uses points in a region instead of coordinate directions (finite difference) and doesn't over-smooth as in polynomial filtering

Nonlocal derivative in 1D

$$\frac{d}{dx}u(x) = -\int_{\mathbb{R}} u(x')\,\frac{d}{dx'}\delta(x'-x)\,\mathrm{d}x'$$

assuming $\frac{d}{dx}u$ is continuous at x

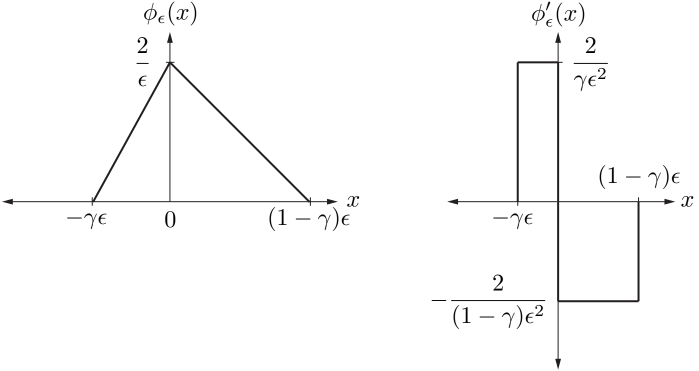
$$\approx -\int_{\mathbb{R}} u(x')\,\alpha_{\epsilon}(x'-x)\,\mathrm{d}x'$$

where

$$\int_{\mathbb{R}} \alpha_{\epsilon}(x'-x)\,\mathrm{d}x' = 0$$

The approximation does not assume that u is differentiable at x!

Depiction of the nonlocal derivative in 1D



$$\phi_\epsilon(x) \approx \delta(x) \text{ and } \phi'_\epsilon(x) \approx \delta'(x)$$

Beyond one dimension

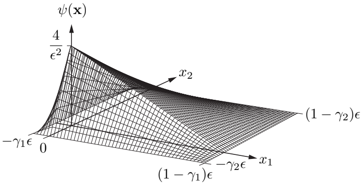
$$\tilde{\nabla}\,\mathbf{u}(\mathbf{x}) \stackrel{\text{def}}{=} -\int_{\mathbb{R}^3} \mathbf{u}(\mathbf{x}') \otimes \boldsymbol{\alpha}_\epsilon(\mathbf{x}' - \mathbf{x})\,\mathrm{d}\mathbf{x}'$$

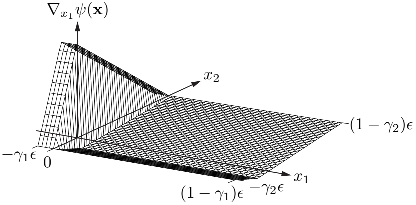
where

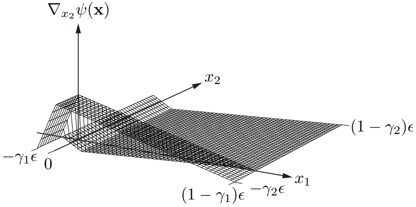
$$\int_{\mathbb{R}^3} \boldsymbol{\alpha}_\epsilon(\mathbf{x}' - \mathbf{x})\,\mathrm{d}\mathbf{x}' = \mathbf{0} \qquad \Big(\boldsymbol{\alpha}_\epsilon(\mathbf{x}', \mathbf{x}) = -\boldsymbol{\alpha}_\epsilon(\mathbf{x}, \mathbf{x}')\Big)$$

- $\tilde{\nabla}\,\mathbf{u}$ is dimensional-less when $\mathbf{u}$ has dimensions of length
- $\tilde{\nabla}\,(\mathbf{A}\,\mathbf{x} + \mathbf{c}) = \mathbf{A}\tilde{\nabla}\,\mathbf{x} = \mathbf{A}$ when $\mathbf{A}$ and $\mathbf{c}$ are a matrix and a vector

Tensor product of nonlocal 1D







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Nonlocal strain

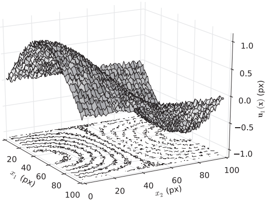
- Deformation $\mathbf{y}(\mathbf{x}, t) = \mathbf{x} + \mathbf{u}(\mathbf{x}, t)$
- Nonlocal deformation gradient $\tilde{\mathbf{F}} \stackrel{\text{def}}{=} \mathbf{I} + \tilde{\nabla}\mathbf{u}$
- Nonlocal strain $\tilde{\mathbf{E}} \stackrel{\text{def}}{=} \frac{1}{2}\left(\tilde{\mathbf{F}}^T\tilde{\mathbf{F}} - \mathbf{I}\right)$
- If $\mathbf{y}(\mathbf{x}) = \mathbf{R}\,\mathbf{x} + \mathbf{c}$ with $\mathbf{R}^T\mathbf{R} = \mathbf{I}$, $\det\mathbf{R} = 1$

$$\tilde{\mathbf{E}} = \mathbf{0}$$

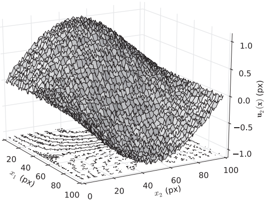
- Final step: Use quadrature rule to link pixel image data to integral operators

Example

Displacement vector field

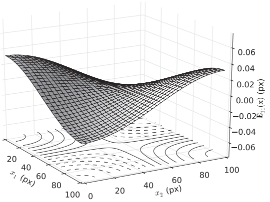


(a)

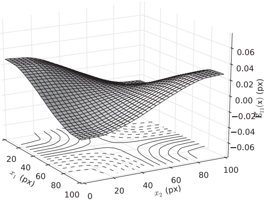


(b)

True strain (no high frequency portion) & Nonlocal Strain



(a)



(b)

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Related work

- Corresponding deformation gradient tensor $\bar{\mathbf{F}}$ (Silling, Epton, Weckner , Xu, Askari 2007)
- $\tilde{\nabla}$ is an instance of the weighted nonlocal $\mathcal{D}_\omega^*$ operator (Du, Gunzburger, Lehoucq, Zhou 2013)
- Why does it work so well?
  1. Using all the nearby information instead of restricting to coordinate axes (as in a finite difference approach)
  2. According to Buades, A., Coll, B., and Morel, J. *Image denoising methods. A new nonlocal principle* (2015), fine scale structures are not filtered along with white noise

Outline

- Digital image correlation (DIC)
- Nonlocal approximation to the strain
- Nonlocal strain can be used to develop constitutive relations
- Let's consider *Nonlocal thermodynamic restrictions*, paper in progress with Ph.D student Carlos Garavito (UMN math)

Kinematics

derivative	integral
$\nabla \mathbf{u}$	$\tilde{\nabla} \mathbf{u}$
$\mathbf{F} = \nabla \mathbf{u} + \mathbf{I}$	$\tilde{\mathbf{F}} := \tilde{\nabla} \mathbf{u} + \mathbf{I}$
$\frac{1}{2}\left(\mathbf{F}^T \mathbf{F} - \mathbf{I}\right)$	$\frac{1}{2}\left(\tilde{\mathbf{F}}^T \tilde{\mathbf{F}} - \mathbf{I}\right)$

Compare with the peridynamic deformation state $\underline{\mathbf{Y}}$

$$\begin{aligned} \underline{\mathbf{Y}}\langle \mathbf{x}' - \mathbf{x} \rangle &= \mathbf{y}'(\mathbf{x}', t) - \mathbf{y}'(\mathbf{x}, t) \\ &= \mathbf{u}(\mathbf{x}', t) + \mathbf{x}' - \left(\mathbf{u}(\mathbf{x}, t) + \mathbf{x}\right) \end{aligned}$$

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Alternative Derivation of Nonlocal Balance Laws

- Frame indifference of the absorbed and supplied *power expenditures* (invariant under rotations + translations)

$$w_{\mathrm{abs}}(\Omega) := \int_{\Omega}\int_{\mathbb{R}^3} \mathbf{t}\cdot(\mathbf{v}'-\mathbf{v})\,dV'dV \quad \text{“absorbed power”}$$

$$w_{\mathrm{sup}}(\Omega) := \int_{\Omega}\int_{\mathbb{R}^3\setminus\Omega} (\mathbf{t}\cdot\mathbf{v}'-\mathbf{t}'\cdot\mathbf{v})\,dV'dV + \int_{\Omega}\mathbf{b}\cdot\mathbf{v}\,dV$$

“supplied power”

$$\mathbf{v}' = \mathbf{v}(\mathbf{x},t) = \dot{\mathbf{u}}(\mathbf{x},t)$$

- Resulting power expenditures are additive over disjoint sub domains
- w_{abs} and w_{sup} introduced by Silling and Lehoucq 2010

Nonlocal Balance Laws

$$\int_\Omega \mathcal{D}\,\boldsymbol{\Psi}\,dV + \int_\Omega (\mathbf{b} - \rho\ddot{\mathbf{u}})\,dV = \mathbf{0} \quad \text{Linear momentum}$$

$$\int_\Omega \int_{\mathbb{R}^3} (\mathbf{y}' - \mathbf{y}) \times \boldsymbol{\Psi}\alpha\,dV'dV = \mathbf{0} \quad \text{Angular momentum}$$

$$\dot{\mathcal{E}}(\Omega) - \underbrace{\int_\Omega \int_{\mathbb{R}^3} \mathbf{t}\cdot(\mathbf{v}' - \mathbf{v})\,dV'dV}_{w_{\text{abs}}(\Omega)\ \text{“absorbed power”}} = 0 \quad \text{Energy balance}$$

$$\boldsymbol{\Psi}\,\boldsymbol{\alpha} := \left(\mathbf{t}\otimes\boldsymbol{\alpha}\right)\boldsymbol{\alpha} = \mathbf{t}\,, \quad \boldsymbol{\alpha}\cdot\boldsymbol{\alpha} = 1\,, \quad \boldsymbol{\alpha}(\mathbf{x}',\mathbf{x}) = -\boldsymbol{\alpha}(\mathbf{x},\mathbf{x}')$$

$$\boxed{\mathcal{D}\,\boldsymbol{\Psi} = \int_{\mathbb{R}^3} (\boldsymbol{\Psi} + \boldsymbol{\Psi}')\alpha\,dV' = \int_{\mathbb{R}^3} (\mathbf{t} - \mathbf{t}')\,dV'}$$

Entropy imbalance

$$\int_{\Omega} \frac{1}{\theta} \left(\rho \, \dot{\psi} - \int_{\mathbb{R}^3} \boldsymbol{\Psi} : \mathcal{D}^* \, \mathbf{v}' \, dV' \right) dV \leq 0$$

ψ represents the *free energy*

$$\mathcal{D}^* \, \mathbf{v}' = (\mathbf{v}' - \mathbf{v}) \otimes \boldsymbol{\alpha}$$

where $\mathbf{v}' = \mathbf{v}(\mathbf{x}, t) = \dot{\mathbf{u}}(\mathbf{x}, t)$ and recall

$$\boldsymbol{\Psi} \, \boldsymbol{\alpha} := \big(\mathbf{t} \otimes \boldsymbol{\alpha}\big) \, \boldsymbol{\alpha} = \mathbf{t} \, , \quad \boldsymbol{\alpha} \cdot \boldsymbol{\alpha} = 1 \, , \quad \boldsymbol{\alpha}(\mathbf{x}', \mathbf{x}) = -\boldsymbol{\alpha}(\mathbf{x}, \mathbf{x}')$$

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Coleman-Noll procedure in the nonlocal theory

Suppose the free energy ψ depends on $\tilde{\mathbf{F}} = \tilde{\nabla}\mathbf{u} + \mathbf{I}$

$$\frac{d}{dt}\psi(\tilde{\mathbf{F}}) = \frac{\partial \psi}{\partial \tilde{\mathbf{F}}} : \dot{\tilde{\mathbf{F}}}$$

Energy imbalance for nonlocal elasticity

$$\int_\Omega \frac{1}{\theta} \left(\underbrace{\rho \frac{\partial \psi}{\partial \tilde{\mathbf{F}}} : \dot{\tilde{\mathbf{F}}}}_{\text{power}} - \underbrace{\int_{\mathbb{R}^3} \mathbf{\Psi} : \mathcal{D}^* \mathbf{v}' \, dV'}_{\text{absorbed power}} \right) dV \leq 0$$

This imbalance can be satisfied in two ways

Two allowable constitutive relations

1. The first alternative explains how to incorporate classical constitutive models in the nonlocal theory.

$$\mathbf{P}(\tilde{\mathbf{F}}) = \rho \frac{\partial \psi}{\partial \tilde{\mathbf{F}}}$$

   correspondence-like model is a *thermodynamic* analogue of Silling et. al. (2007).

2. The second alternative is *strictly nonlocal*

- Both classes of peridynamic constitutive relations derived from entropy balance
- This work generalizes a thermodynamic basis provided by Silling and Lehoucq (2010) by using a nonlocal analogue of the deformation gradient