DOE award DE-SC0010025 - Boston University
Topics in Theoretical Physics

DOE-BU-10025

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Date: September 30, 2016.

Period Covered: May 1, 2013 to June 30, 2016.

Accomplishments, by CoPI.

Richard Brower

Lattice Gauge Theory for Beyond the Standard Model

Appelquist, Brower, Fleming, Osborn, Rebbi, Vranas and associated post-doctoral fellows formed the Lattice Strong Dynamics (LSD) collaboration (`http://www.yale.edu/LSD`) in 2007 aimed at adapting lattice field theory to explore possible non-perturbative dynamics in the Higgs sector beyond the standard model (BSM) [1].

1. Strong dynamics approaching the conformal window.

In the LSD collaboration, Brower began investigating how electroweak symmetry breaking might be explained by replacing the elementary Higgs by a strong gauge theory extension to the Standard Model. One possible mechanism to avoid conflict with precession electroweak data is called "walking technicolor" in which it is conjectured that a new dynamics transpires as the number (N_F) of light mass "techni-quarks" is increased approaching the critical value for the IR conformal fixed point. With an ensemble of SU(3) Domain Wall lattices with $N_f = 2$, $N_f = 6$ and $N_f = 10$ dynamical flavors for a range of Fermion masses approaching the chiral limit, computations were made for coefficients in the effective chiral Lagrangian, the chiral condensate, $\langle\overline{\psi}\psi\rangle/F_\pi^3$, responsible for mass generation in extended technicolor models, the S-parameter and the low lying spectra. In broad terms the enhancement in the condensate exceeded the expectation of naive scaling and the S parameter was reduced [30].

Additional phenomenological implications such as the determination of WW scattering near threshold [2] are being extended at present. Also lattice field theory is an effective tool for the investigation of new strong dynamics capable of producing candidates for Dark Matter [3]. The LSD collaboration is studying a range of baryonic Dark Matter models to set limits for detection with initial results in Phys. Rev. D [31] and Phys. Rev. Lett. [54].

In the preliminary spectral studies for the 8-flavor theory, there is also a hint of a parametrically low mass 0^{++} scalar in the neighborhood of 500 GeV which is low compared to the broad sigma in QCD but still well above the Higgs boson mass of 126 GeV. It is possible that the negative contributions coming from the top-quark loop could drive this mass to smaller values, within the ballpark of the measured Higgs mass. However much more extensive investigations are need to see if any such new strong dynamics can provide a phenomenologically viable alternative to the elementary Higgs. New approaches, which we describe next, should be explored to investigate these interesting theories with or near to an IR conformal fixed point.

2. IR Conformal Field Theory.

Brower in collaboration with Oliver Wietzel, Anna Hasenfratz and Evan Weinberg have developed one new approach with the introduction of 12 staggered flavors, separating masses into 4 light and 8 variable masses [32, 33]. The advantage is that in the limit where all 12 flavors are massless, there is the expectation of an IR conformal fixed point. Indeed extensive studies have been carried out that show that as the 8 flavors are reduce the spectrum undergoes a dramatic change. An expanding window of slow evolution (i.e. walking) is observed and the spectrum obeys the hyperscaling relation of a mass deformed conformal theory [34]. The the quantitative investigation of UV complete non-SUSY strongly coupled IR conformal theories may open up new options for BSM phenomenology.

Lattice Field Theory on Curved Riemann Manifolds

Lattice gauge theory on hypercubic lattices [1] provides a powerful *ab initio* approach to strongly coupled field theories in flat Euclidean space, $\mathbb{R}^D$. However there is important non-perturbative physics that would benefit from the extension of lattice field theory methods to more general curved Riemann manifolds. One example is a recent proposal to implement radial quantization for conformal field theories [6, 35, 39]. This requires replacing the flat Euclidean manifold, $\mathbb{R}^D$, by the cylinder, $\mathbb{R} \times \mathbb{S}^{D-1}$, which represents the boundary of Anti-de Sitter space AdS^{D+1} in global coordinates. Other examples include *ab initio* calculations of the c and a terms, tests of AdS/CFT duality, quantum criticality in condensed matter and perhaps quantum physics near blackholes.

1. Quantum Finite Elements

In 2012, Brower, Fleming and Neuberger [6] suggested one possibility based on lattice radial quantization applied to the Wilson-Fisher conformal fixed point for the 3D Ising model. In spite of some quantitative success, small defects were observed in the restoration of full conformal symmetry in the continuum limit. To address this defect, Brower, Cheng and Fleming [35, 39] replaced the Ising model by its universally equivalent ϕ^4 theory and began to explore Finite Element Methods (FEM). They found that one needs to develop a new FEM formalism to renormalize the simplicial lattice Lagrangian, referred to below as the **Quantum Finite Elements (QFE)** method. Brower, George Fleming and Chung-I Tan in collaboration with students Andrew Gasbarro (Yale), Timothy Raben (Brown) and Evan Weinberg (BU) are proposing a step by step program to attack this problem. Initial progress is encouraging as reported at Lattice 2015 in Kobe, Japan [36] in the session on Theoretical Development (July 17, 2015), at the KITP in Santa Barbara (Aug. 21, 2015) and at three workshops in the Higgs Centre in Edinburgh (Jan 7-8, 2015; June 27-30,2016; Aug 1-3,2016).

The goal of QFE is to develop a non-perturbative lattice quantum field theory formalism capable of converging in the continuum, as the UV cut-off is taken to infinity, for any renormalizable theory on a smooth Riemann manifold. For Euclidean field theory, this would provide a non-perturbative formulation of classic papers from the 1990 of the renormalizability of quantum field theories on curved manifolds. This is an ambitious project but some concrete step have been taken in the last year.

2. Simplicial Dirac Fermions

For both bosonic and Fermionic fields, new Finite Elements Methods (FEM) have been developed that show convergence to the exact continuum for the spectrum. For bosons this is based essentially on classical theoretical framework of FEM. However the Dirac Fermion is a new and interesting challenge. The spinor probes the underlying geometry of the manifold through its vierbein and spin connection. From the perspective of Regge Calculus [37], the vierbein and spin connection are sufficient to define a simplicial manifold in the Einstein-Cartan formulation of lattice gravity [38]. In a soon to be published article [40] an explicit algorithm is define for fixing the lattice vierbein and spin connection designed to converge to any target smooth Riemann manifold $(\mathcal{M}, g)$. The rate of convergence for this simplicial Dirac Fermion representation on $\mathbb{S}^2$ is tested in comparison with the exact continuum theory. In addition the 2D simplicial Majorana Fermion is shown to converge to the analytical result for 2-point and 4-point correlation functions for the c = 1/2 minimal model conformal field theory on $\mathbb{S}^2$ projective sphere.

3. Counter Terms for UV divergences

The next, even more critical step, is to introduce simplicial counter terms to account correctly for the UV divergence. Unlike $\mathbb{R}^D$, there is no uniform simplicial lattice refinement scheme, so the effective lattice cut-off is position dependent. For example on a sphere, the finest uniform discretization of $\mathbb{S}^2$ and $\mathbb{S}^3$ are the 20-cell icosahedron and the 600-cell tetraplex respectively. Consequently one must introduce (finite scheme dependent) counter terms. In a soon be competed paper, one example is fully explored. For the 2D ϕ^4 theory on a the Riemann sphere [41] the one loop lattice diagram fixes the counter term, allowing a very accurate test of the continuum at the Wilson-Fisher conformal fixed point. Agreement with exact continuum result for the two and four point correlation functions is established to high precision. Currently this method is being extended to the 3D Ising CFT. The method appears general enough for any super renormalizable theory opening up a large range of interesting conformal field theories, but extensions to 4D lattice gauge theory on Riemann manifolds is an on going research topic.

Application of AdS/CFT to Diffractive Higgs Production

Brower has had a long interest in String Theory and the formulation of the QCD string. The discovery by Maldacena of exact String/Gauge duality through the AdS/CFT mechanism now allows calculation in some QCD-like theories at strong coupling. The challenge is to extract generic features applicable to QCD. Brower, in collaboration with Matt Strassler, Chung-I Tan and Joel Polchinski initiated the investigation of high energy scattering processes using AdS/CFT duality. This Brower-Polchinski-Strassler-Tan (BPST) Pomeron kernel (at $t = 0$), exhibits the same diffusive character as the BFKL weak coupling result with diffusion in $k_\perp$ for the gluons replaced by diffusion in the 5th co-ordinate r.

Using this formalism, Brower and collaborators developed a simple selfconsistent holographic framework to provide phenomenologically compelling estimates for diffractive cross sections at the LHC [42] as well as diffractive Higgs production by AdS Pomeron fusion[8]. More recently, this work has been extended to the suppression (the "survival probability") due to multiple Pomeron exchange in the eikonal approximation and to higher order calculation of the *Strong Coupling Expansion for the Conformal Pomeron/Odderon Trajectories* [44] and to parameterizing the effects of breaking conformal invariance by a IR deformation of AdS space to include the effects of confinement.

Multi-scale Algorithms and USQCD Service

Algorithmic research is an integral part of lattice field theory deriven by and contributing to a deeper understanding of the physics. As lattice field theory gains more power to resolve finer scales, the impact of multi-scale algorithms inevitably increases in importance, very likely radically transforming lattice field theory software. A salient example was the discovery of an adaptive multigrid solver applied to the Wilson quarks in lattice QCD [45] resulting in a 10x -25x increase in efficiency for light quarks. Current work continues to extend multigrid solvers for staggered Dirac fermions on GPU hardware in collaboration with Clark (NVIDIA), Strelchenko (FNAL) and Evan Weinberg (BU). New projects underway are the investigation of multi-scale Hybrid Monte Carlo (HMC) algorithm to accelerate equilibration [46] of large lattices with Endres, Detmolt, Pochinsky (MIT) and Orginos (W&M) and accelerating topological evolution with Peter Boyle (Edinburg) and Evan Weinberg (BU). These are difficult problems with high risk and high potential.

Brower continues to serve in a number of leadership roles in the US Lattice Field Theory. For example he is the chair of the USQCD Software Committee and a member of the Executive Committee. As the SciDAC Project Co-Director for Computation, he is responsible for co-ordinating software and algorithm development. He is an NVIDIA CUDA Fellow tasked with co-ordinating with two of his former postdoctoral fellows, Mike Clark and Ron Babich, who are now full time engineers at NVIDIA.

Andrew Cohen

Lorentz violation and the Higgs boson.

Cohen has shown that up to dimension four there is a unique Lorentz-violating operator (consistent with all the Standard Model symmetries) involving the Higgs boson. This operator modifies the maximal propagation speed of the physical Higgs boson and leads to significant changes in the physics of both the Higgs boson and the weak interactions. By considering these modifications Cohen has obtained stringent bounds on this maximal propagation speed. Surprisingly even though not many Higgs bosons have been observed this speed (which is very close to the speed of light) is now the best determined property of the Higgs boson. The analysis and subsquent limits differs slightly depending on whether the Higgs is superluminal or subluminal.

In the subluminal case, sufficiently high energy particles that couple to the Higgs may lose energy through emission of physical Higgs particles. While this process is kinematically forbidden if the Higgs maximal speed is the same as that of the high energy particle, it allowed should these speeds differ. The Lorentz-violating nature of this difference allows this effect to grow as the energy of the primary particle increases. Thus the best limit of this kind comes from examining the highest energy particles that couple to the Higgs. The highest energy particles of this character are protons in cosmic rays, with energies as high as $E \sim 10^{20}$ eV. Higgs emission is allowed when the difference in Higgs and proton maximal speeds is greater than the square of the Higgs mass divided by the energy of the proton M_H^2/E^2. The proton couples to the Higgs through the quark Yukawa couplings as well as through gluon couplings to the Higgs induced by loops. Since these couplings involve quarks and gluons, the proton-Higgs coupling involves the strong interactions and is inherently non-perturbative. Nevertheless Cohen showed that they can be reliably calculated using the parton model and the known parton distribution functions for the proton. The result of this computation is a rapid energy loss due to Higgs emission, and inevitably such high energy protons would never be observed. This constrains the maximal speed difference between the proton and the Higgs to be less than parts in 10^{18}! Since the proton is known to have a maximal speed equal to the speed of light to parts in 10^{20} this establishes that the maximal Higgs speed is the speed of light to exquisite precision.

The superluminal Higgs case is somewhat trickier, and somewhat more interesting. In this case it is the Higgs that can decay through Lorentz-violating interactions. However the Higgs is so short-lived due to its conventional interactions that such effects are unobservably small. But the superluminal

Higgs case also modifies the electroweak symmetry breaking part of the weak gauge-boson propagators. This in turn leads to anomalous features of the weak interactions of lighter particles such as pions and muons. As before the presence of Lorentz-violation causes these effects to grow with the energies involved, and cosmic rays again provide the most stringent bounds.

Pions decay through the weak interaction with a form factor that is well-described by the tree-level W propagator. The Lorentz-violating modification of the W propagator from the superluminal Higgs therefore leads to modification of pion decay and of the pion lifetime. Most notably the pion lifetime does *not* increase according to the usual relativistic time-dilation factor as the pion energy increases. Also, the decay at high energies is not helicity suppressed, and a high energy pion would decay about half the time to electrons and half the time to muons. Muon decay is similarly altered leading to a sort muon lifetime at high energies.

The decay length of high energy charged pions and muons is measured in cosmic rays showers. The shower morphology and the detection probability for muons at the ground are both sensitive functions of the pion and muon lifetimes at high energy. Even the gross features of cosmic ray showers would be radically altered if these lifetimes are shortened as would be the case with a superluminal Higgs. A superluminal Higgs maximal speed greater than the speed of light by parts in 10^{14} would lead to easily detected changes in the behavior of cosmic ray showers.

The bounds described here can likely be improved (especially the superluminal case) by a more detailed study of cosmic ray shower morphology. More interestingly these kinds of effects have not been fully exploited as a sensitive probe to physics at high energies and Lorentz violation. Work in this direction is on-going.

Naturalness and new physics

Cohen has reconsidered naturalness of existing models of EW symmetry breaking in light of the results from run I of the LHC. Many researchers view these results, especially the absence of new states beyond those of the Standard Model, as an indication that naturalness as a guiding principle for uncovering the physics of EW symmetry breaking is being significantly challenged by experiment. It has become common to argue that all models of EW symmetry breaking are necessarily finely-tuned at the 5% level or more.

While it is (mostly) true that existing models of EW symmetry breaking are tuned at this level, the nature of this tuning is not common among all models. For theories of EW symmetry breaking that realize the Higgs as a pseudo-Nambu-Goldstone Boson (such as Little Higgs, Composite Higgs, Twin Higgs, etc.) this fine tuning arises in fundamentally different ways. Models such as composite Higgs and Twin Higgs do not naturally obtain an EW Higgs vev that is lighter than the composite scale f. In particular in these models the parameter $(v/f)^2$ sets the size of this fine tuning. Conventional power counting suggests that $f \sim 4\pi v$ which would correspond to fine tuning at more than the percent level. Composite Higgs and Twin Higgs evade this by lowering the scale f. However the new physics associated with the scale f gives rise to new physics contributions (both new particles and precision EW corrections). Therefore f cannot be made too low. The strategy in these models is to compromise between fine tuning and SM corrections. The best that has been achieved is $(v/f)^2 \sim .05$ and this explains the 5% fine tuning. It seems unlikely that these models can evade this constraint.

Little Higgs models are fundamentally different. They try to realize a *natural* EW vev by imposing a complicated set of approximate symmetries. While successful in this regard, existing models implementing the Little Higgs mechanism nevertheless are currently finely tuned also at the 5% level. However this fine tuning arises entirely from the limits on the mass of the "top partner", the new colored fermion that is responsible for canceling the top quark's large contributions to the Higgs mass. The current limits on a new colored fermion of this sort from the LHC is more than 800 GeV. As the mass of this new particle is increased, the removal of the top quark renormalzation of the Higgs mass becomes less effective, and fine tuning is required to keep the Higgs mass low. In existing models this fine tuning depends on the partner mass as $m_T^2 \ln \Lambda^2/m_T^2$. where Λ is the scale of new physics, typically $4\pi f$. With a partner mass about 1 TeV this gives the 5% fine tuning quoted above.

However the presence of the logarithm in this is far from inevitable. In the original little Higgs models the quadratic sensitivity to the new physics scale Λ was removed and the logarithmic sensitivity remained. This was largely to keep the models simple and not for any fundamental reason. At the time these models were introduced the constraints on a top partner mass were significantly less severe and even the logarithmic sensitivity did not indicate fine tuning.

Now that the data has pushed the mass of the top partner into the TeV range, the logarithmic sensitivity to the high scale has become a problem. However Cohen has built models that remove this logarithmic sensitivity and leave only a finite contributions proportional to the top partner mass m_T^2. These models are only slight modifications to existing little Higgs models, typically requiring one additional top partner (either an EW singlet or doublet). This reduces the fine tuning from a few percent to 50%; that is, no fine tuning at all. These models are still phenomenologically viable without fine tuning. The finite contribution becomes finely tuned when the top partner mass rises to 3 or 4 TeV. Unfortunately (or fortunately for model builders) this mass is beyond the current reach of the LHC.

Little Higgs and Neutral Naturalness

With Tao Liu and YingYing Li (HKUST) Cohen is constructing a new model of natural EW symmetry breaking by combining ideas of neutral naturalness with the little Higgs mechanism. They have succeeded in crafting a modification of the littlest Higgs in which the Higgs mass and vev are protected via the litte Higgs mechanism except for the top quark sector. In this sector they introduce a color neutral top partner which removes sensitivity of the Higgs mass and vev to short distance top quark physics. This is implemented by an approximate Z_2 symmetry in the top quark Yukawa coupling. Unlike other theories of neutral naturalness, the Higgs is mostly protected by the little Higgs mechanism, and consequently the full structure of theories such as the Twin Higgs or orbifold Higgs are unnecessary. Only the top Yukawa is treated differently. The result is a theory like the littlest Higgs, with significantly relaxed constraints on the top partner since it is not colored.

It remains to construct a T-parity invariant version of the theory in order to keep the scale f low, which is necessary to eliminate fine tuning. This work is currently under way.

A model for the LHC diboson excess

With M. Schmaltz and M. Buen Abad Najar Cohen examined the phenomenology of a charged heavy vector resonance, a W'. Run I of the LHC produced hints of the existence of such a particle with a mass near 1.9 TeV decaying into SM vector boson pairs as well as dijets. While Run II has not yet confirmed nor ruled out such a resonance, it has yielded interesting new constraints on models attempting to explain the putative signal. In many such models the new charged vector boson is an isospin singlet and there is potential conflict with the electroweak precision test, in particular the T parameter.

Cohen et al examined general models with an iso-singlet W' in light of these constraints. Many previous models are in significant conflict with these constraints. Generally these models lead to large dijet cross sections, and these large cross sections are in moderate tension with the Run I results and in serious conflict with the Run II limits on excess dijets. By incorporating fermion mixing Cohen et al are able to evade these constraints and produce models that can successfully explain the Run I excess without conflict with Run II results.

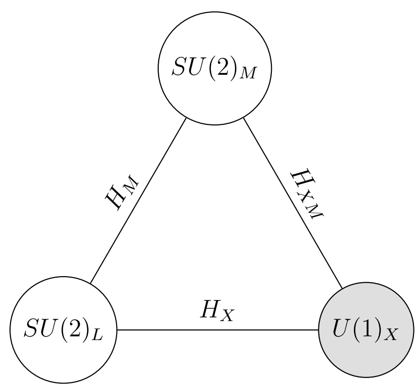

Figure 1: Theory space diagram representing the bosonic field content for the successful iso-singlet W' models.

Especially intriguing is the general analysis of electroweak precision constraints, which are dominated by the T parameter. Generally the iso-singlet W' contributes negatively to the T parameter, while the precision data has a small preference for a positive value (T is defined to be zero in the Standard Model). A Z' accompanying the W' will also contribute to the T parameter, and can not only cancel the W' contribution but can also lead to a better fit to T than the Standard Model. All this requires a Z' that is not too heavy, since Z' contributions to the T parameter decouple for large Z' mass.

Cohen et al perform a global fit to the relevant LHC Run I and Run II as well as precision electroweak data. They find that an excellent fit can be obtained with a Z' mass in a narrow window, $2.7 < M_{Z'} < 3.1$ TeV. This is especially intriguing in light of the reported observation by CMS of a di-electron event with invariant mass of 2.9 TeV. The likelihood of such an event arising from background is quite small.

Sheldon Glashow

1. Glashow has been continuing his investigations of the flavor problem: the search for a deeper understanding of the masses and mixings of leptons, including the three known neutrino states, the three charged leptons, and others states that may exist. Of course, he would hope that the missing connections would not be independent of those yet-unknown relations having to do with uark masses and mixings.

2. The experimental high-energy frontier is, and for some time will remain at the large hadron collider LHC. We have seen the discovery of what appears to be a Higgs Boson, but as yet we have seen nothing at all in the way of'beyond-the-standard-model' physics. Of course, this situation may change when the LHC reopens at a somewhat greater collision energy. With baited breath (but folded hands) Glashow awaits possible new experimental discoveries at this frontier.

3. There are many ways to explore beyond the standard model, such as and not limited to: searches for electric dipole moments of elementary particles, constraints from astrophysicists (who are closing in on the sum of neutrino masses), neutrinoless double beta decay, CP violation in the neutrino sector (The big question being not its value but whether it is observably large or not), or the search for certain (but by no means all!) rare decay modes of elementary particles, in which connection see below:

Why does Glashow dream about a high inensity muon source? Glashow believes that much can be learned from a dedicated tripartite search for the following three muonic processes: (i) Radiative decay ($\mu \to e + \gamma$), (ii) Electronic decay ($\mu \to e + e + \overline{e}$) and (iii) Muon conversion ($\mu + N \to e + N$). These three processes are closely related: if any one of them is found to proceed at an observable rate, then it is very likely that all three can be found at rates possibly greater, but not more than two orders of magnitude smaller. Furthermore, it is straighforward to devise specific models (involving new heavy leptons, dileptons, leptoquarks or whatever) in which any one of these decay processes has the largest branching ratio. Thus, the acquisition of three data points: the rates of each of the three suppressed modes, would provide us with clean and otherwise inaccessible hints of b.s.m. physics lying at energies well beyond the reach of the LHC. Glashow would hope that American planning for the future of high energy physics would include this project.

Emmanuel Katz

Holography, strong dynamics, and non-perturbative methods

Our ability to construct models of electroweak physics and to understand the strong interactions of QCD are limited by our ignorance of strong dynamics and the challenges inherent to non-perturbative phenomena. Thus, there can be interesting possibilities for physics beyond the standard model that we are missing. Development of new techniques for studying field theory in the strong coupling regime is therefore desirable.

In the past, Katz has developed holographic models to capture properties of the lowest QCD resonances [115][116]. More recently, Together with S. Sachdev, E. Sorensen and W. Witczak-Krempa, Katz has also used holography to connect Euclidian lattice data to dynamical quantities at finite temperature like the diffusion constant [117]. The idea is to use the lattice data and the OPE to construct the holographic background, with finite temperature modeled by a horizon. The expectation value of operators (vevs) in the presence of finite temperature calculated on the lattice then contributes to thermodynamic quantities. Interestingly, the contribution of high dimension operators to these quantities is quite suppressed unless the vevs grow very rapidly with dimension. This suggests a possible systematic approximation in obtaining these quantities from lattice data for certain strongly coupled theories.

Holography has also motivated Katz to consider a novel technique for quantizing and solving a field theory. The basic observation is that the mass of a bulk field in a holographic model, M_{bulk} grows with the scaling dimension, Δ of the dual primary operator as $M_{bulk} \sim \Delta$. Once conformal symmetry is broken, therefore, the naive expectation is that high scaling dimension operators will decouple from the low-energy sector of the theory (as bulk fields can be integrated out). The most likely case would be power law decoupling (i.e. as $\sim \frac{1}{\Delta^p}$), however for modes well localized in the bulk geometry L. Fitzpatrick, J. Kaplan, and L. Randall, and Katz have argued that the decoupling can be exponential [13].

The above observation allows for a new way to study RG flows from a known UV CFT (typically a free theory) to a strongly coupled IR theory. Namely, one constructs a discrete basis for the Hilbert space of the field theory, with each basis vector corresponding to a state of given conformal Casimir (i.e. the CFT dual to to a given bulk field). Working in light-cone quantization, where the vacuum is naively trivial (although there are many important subtleties), the light-cone Hamiltonian is then expressed in this basis. In order to calculate, one truncates the discrete basis to states with primary operators of dimension below a certain Δ_{max} (or correspondingly Casimirs below a certain number). The Hamiltonian is then diagonalized to yield the spectrum of the theory. Physical quantities are thus expected to have errors of order $\sim 1/\Delta_{max}^p$, where p depends on the quantity and the detailed dynamics.

Together with BU students Y. Xu and G. Marques-Tavares, Katz first used the above technique to solve 2D large N QCD with an adjoint fermion [14]. Here the basis for the Hilbert space is related to the quasi-primary operators of UV free fermionic theory. They found very good agreement with previous DLCQ numerical methods, and verified that the decoupling of high dimension operators occurs very rapidly (possibly exponentially for the lowest states). This implies that fairly accurate analytic expressions can be given for the the parton wave functions of the lowest lying states.

Following the above work, Y. Xu, G. Marques-Tavares, and Katz have tested the above conformal basis approach at small N [118]. They found, for example, that for a 2D version of QCD with three colors and one quark flavor, the convergence of the basis is just as efficient. Thus 2D QCD can be solved numerically, with the spectrum and wave functions of the lowest lying states well approximated analytically due to the rapid convergence.

Most recently, together with BU postdocs Z. Khandaker and M. Walters, Katz has extended the conformal basis approach to 3D scalar theories[28]. The key observation is that it is possible to regulate the inner product of basis states in a manner that both preserved Lorentz invariance and the important part of the conformal structure. In some way, the regulator is much like adding a soft UV and IR cutoff in AdS. Using this scheme the authors were able to reproduce known perturbative results as well as Large-N non-perturbative for the spectrum and the Kallen-Lehmann spectral density of of various operators along an RG-flow. Knowing these densities allows one to calculate dynamical correlation functions in real time which are a challenge for standard lattice methods.

Katz plans to explore this technique further for theories containing fermions and gauge bosons in both 3D and 4D. One attractive feature of the method is that it can be attempted with theories which suffer from ”sign problems” on the lattice, as well as with SUSY theories for which lattice implementations are currently unknown.

Exploration of non-equilibrium field theory

Non-equilibrium dynamics play an increasing role in condensed matter systems and may also be relevant for the physics of the early universe. Together with M. Kolodrubetz and A. Anatoli Polkovnikov, Katz has studied dynamical trapping which can occur when a classical field dials a parameter which takes a quantum system across a quantum phase transition [119]. An example, would be the inflaton field coupling to the Higgs mass. In such cases, the entropy production of quantum modes near the critical point can trap the classical field there (in an expanding universe the trapping is of course temporary, but might have interesting consequences). Interestingly, the parametrics of this trapping can be determined completely in terms of the critical exponents of the quantum system. To check this intuition, Katz and collaborators studied the trapping in a particular 2D spin system numerically and found agreement with the expected scaling behavior. It would be interesting to extend this study to higher dimensions and to more complicated systems.

Dark Matter

The nature of dark matter (DM) remains a mystery. Direct detection experiments offer the promise that this mystery will be solved in the near future. Indeed, there has been some excitement in the past few years over various reported anomalies in these experiments. Though no firm evidence for DM has been found, in light of the anomalies, there was a realization that the DM interaction with nucleons can be more complicated then perviously considered. Indeed, there are well motivated models where the interaction of nucleons with dark matter can depend on momentum and/or velocity. For example, visible matter is mostly neutral, and the interaction of light with atoms is momentum dependent. It is thus quite conceivable that the dark matter particle is equally complex, resulting in momentum dependent interactions with nucleons.

Together with L. Fitzpatrick, W. Haxton, N. Lubbers, and Y. Xu, Katz has suggested a model independent framework by developing a non-relativistic effective theory (EFT) describing DM nucleon interactions[17] which accommodates possible momentum dependence. With minimal assumptions, they found that there are nine non-relativistic operators (up to second order in the momentum transfer of the collision) needed to describe possible dark matter interactions with nucleons. These interactions are general in that they apply for dark matter of any spin. In principle, any measurement can be converted into a bound on the coefficients of these operators, including possible interference effects. In a sense, these operators are thus the dark matter equivalent of the precision electroweak observables. Interestingly, some of these nine operators lead to completely novel nuclear responses, previously not considered in the direct detection literature. These include two separate ways of coupling to spin, a new contribution sensitive to the angular momentum of unpaired nucleons in the nucleus, as well as a new coupling to the product of spin and angular momentum. This enlarges the set of possibilities for signals at direct detection experiments. Since then, L. Fitzpatrick, W. Haxton, Y. Xu, and Katz have performed an analysis of this new parameter space [18]. Most recently, the SuperCDMS collaboration have performed a dedicated study of the EFT operators and the implications of this broader approach for the next-generation direct detection experiments[120].

Katz and Fitzpatrick plan to continue exploring this EFT. Some interesting questions are whether in the future, once we are faced with the neutrino background, some EFT interactions provide a sufficiently different signal so that they can be identified despite the background. Also, whether or not there is a novel nuclear response that can occur for coherently scattering DM, where the DM structure can be resolved and the derivative expansion of the EFT is not an appropriate truncation.

So-Young Pi

Topics in Quantum Field Theory

So-Young Pi has been working on topological effects in quantum field theory, cosmology and general relativity. Past few years Pi investigated topological effects in two spatial dimensions (2D), such as fractional charge and Majorana bound states, as realized in condensed matter physics, conformally invariant systems (AdS/CFT) and local conformal invariance (Weyl invariance) in cosmological models.

Topological Effects in Quantum Field Theories Introduction

Last several years, together with C. Chamon, R. Jackiw and students at Boston University, Pi has investigated topological properties of two-dimensional (2D) systems, such as fermion fractionalization in graphene-like systems [63, 64, 65] and Majorana fermion bound states [66, 67], which appear in the surface state of a topological insulator [68] due to the proximity effect of a superconductor.

Interesting, exotic phenomena in graphene is due to the fact that the Fermi energy crosses two Dirac points. Electrons in graphene, obeying a linear dispersion relation at the two Dirac points, behave like massless relativistic particles. The left and right moving chiral electrons at the two Dirac points are described by a Dirac-type (4×4) matrix equation. Emergence of graphene opened the door to fractional fermions in 2D, three decades after the first discovery of fermion fractionalization in 1D and 3D. Particle physics field theorists learned that the fermion doubling in graphene is the key for modeling 2D fractional fermions.

On the other hand, in order to discuss possible fermion fractionalization Pi and her collaborators have assumed *Kekulé* lattice dimerization, which provides fermion mass term (both homogeneous and inhomogeneous) by connecting the two Dirac points. But Kekulé dimerization has not been observed so far and graphene's gaplessness poses a critical challenge to both theoretical and practical applications, as the development of graphene-based semiconductors is predicated on the ability to induce a gap. Recent research by Pi and her collaborators, described below, concludes their work on fermion fractionalization in graphene-like systems.

Driven graphene with topological properties

In a recent Letter [19] Pi and her collaborators considered the interesting possibility of inducing a gap in graphene by using non-equilibrium technique. Non-equilibrium quantum systems constitute a natural frontier in physics that is only beginning to be probed by theory and experiment. Non-equilibrium methods can be used to study and control condensed matter systems [69]. Particularly exciting is the possibility of engineering the properties of novel materials, like graphene, by driving them out of equilibrium. Pi and her collaborators gave a striking theoretical example of such materials design from non-equilibrium steady states: a tunable gap in monolayer graphene is generated by exciting a particular optical phonon. They have shown that the system reaches a steady state whose transport properties are the same as if the system had a static electronic gap, controllable by the driving amplitude. Moreover, the steady state displays topological phenomena: there are chiral edge currents, which circulate a fractional charge e/2 per rotation cycle, with frequency set by the optical phonon frequency.

Generalized energy and time-translation invariance in a driven system

In a more recent paper [22], Pi and her collaborators studied theoretical understanding of driven systems. Progress in the study of such systems has been achieved using the Floquet formalism [70], but certain aspects of this approach are not well understood. The apparent simplicity of Floquet theory belies certain conceptual difficulties. In particular, because of periodic time-dependence one can only use concepts of quasi-energies. However, since they are only well defined modulo frequency, there is no way of defining a lowest quasi-energy state. Consequently, there is no notion of the ground state of a driven system in Floquet theory.

In this paper, Pi and her collaborators studied an exactly solvable model where such questions have clear answers. They considered the steady state reached by a system of Dirac fermions in graphene when coupled to a heat bath of acoustic phonons in the presence of a rotating Kekulé mass term. This problem can be solved exactly both with and without Floquet theory. Without Floquet theory, one can solve it by a mapping to a time-independent system via an axial gauge transformation, which preserves all transport properties. They showed that the gauge-equivalence of the two problems implies that the quasi-energies of Floquet theory correspond to a continuous symmetry of the full time-dependent Lagrangian.

Local Conformal Invariance in Cosmology

Field theoretic models that possess Weyl invariance (local conformal invariance) are the focus of present day attention in cosmology. Recently some cosmologists suggested in a series of papers that Weyl invariant dynamics in the conformally coupled scalar-tensor theory can assist in unravelling various cosmological issues, including inflation [94, 95].

In view of the claimed importance of the above Weyl invariance, Jackiw and

Pi [96] investigated the dynamical role of Weyl symmetry in scalar-tensor theory used in the above cosmological models. In their paper they assert that Weyl invariance, as used in these models, has no dynamical role. At best, a possible calculational convenience may be achieved.

It is common practice in recent papers on conformal inflationary models [94], to view the Weyl invariant tensor-scalar Lagrangians as describing gauge theories presented in the so-called "Jordan" frame. Gauge fixing brings them to "Einstein" frame, with one less minimally coupled scalar field, and local conformal symmetry is spontaneously broken due to gauge fixing. Thus the authors call these models "conformal inflationary models". However, since there is no gauge potential in the model, a better description is that the Einstein-Hilbert theory does not arise from gauge fixing but from a redefinition of dynamical variables [97, 98, 99]. Explicitly, in the Weyl invariant theory with one scalar field, after a field redefinition, $g_{\alpha\beta}^{Jordan}\varphi^2 = g_{\alpha\beta}^{Einstein}$, the spurion field φ in Jordan frame disappears and the Einstein-Hilbert action, which clearly lacks local conformal symmetry, emerges [100, 101]. Setting $\varphi = 1$, which resembles choosing the "unitary gauge" in the gauge theory of the standard model, leads to Einstein-Hilbert action. However, this is not spontaneous symmetry breaking.

Jackiw and Pi have examined the dynamical role of Weyl symmetry by determining its Noether symmetry current. Although Weyl symmetry has a long history, the associated current in these theories has not been previously studied. They applied Noether's first theorem [102] and obtained vanishing current.

The Noether procedures always leave current formulas ambiguous up to identically conserved superpotentials. This is because one is extracting an expression from its divergence. An extension of Noether's theorem, called the "Second Theorem," [103] establishes that the current associated with a local symmetry is always a superpotential. They applied Noethers second theorem and regained the previous result: vanishing current — a trivial superpotential. The peculiarity of vanishing current of Weyl symmetry deserves further investigation.

Claudio Rebbi

Lattice Gauge Theory Simulations for Electroweak Symmetry Breaking

The main focus of Claudio Rebbi's research during the grant period has been the use of lattice methods to the study of models of electroweak symmetry breaking based on strong dynamics. In 2007 Rebbi and several other US colleagues established the "Lattice Strong Dynamics (LSD) Collaboration"[1] to pursue non-perturbative studies of strongly interacting theories likely to produce observable signatures at the Large Hadron Collider (LHC). A large component of Rebbi's current research has been conducted on projects of the LSD collaboration.

During our last funding period the LSD collaboration has worked on the following:

A study of the infrared behavior of an SU(2) theory with six massless Dirac fermions in the fundamental representation. Using the Schrödinger functional theory the running of the coupling constant was calculated non perturbatively. No evidence was found of an infrared critical point within the extensive range of couplings accessible to the simulation, showing that the model either undergoes chiral symmetry breaking or has an infrared fixed point at very strong coupling [5].

The development of a technique for calculating the topological susceptibility in cases where topology tunneling transition are rare. This work was published in [48].

A detailed study of the SU(3) theory with eight fundamental domain wall fermions was carried out and presented in [49]. The results favor spontaneous chiral symmetry breaking, although they leave room for the hypothesis of mass-deformed infrared conformality. This can be an indication that the model is very close to the conformal window and warrants further studies of this system. For this reason the LSD collaboration has embarked in a detailed, large statistics study of the same model, but using this time staggered fermions. The staggered discretization, although it does not preserve the full chiral symmetry of the continuum, maintains one degree of chiral symmetry in the zero mass limits, which guarantees the presence of massless

[1]Current membership: Xiao-Yong Jin, James Osborn (ANL), Richard Brower, Claudio Rebbi, Evan Weinberg (BU), Meifeng Lin (BNL), Anna Hasenfratz(UColorado), Ethan Neil (UColorado and RIKEN BNL), Joseph Kiskis (UCDavis), Oliver Witzel (UEdinburgh), Evan Berkovitz, Enrico Rinaldi, Chris Schroeder, Pavlos Vranas (LLNL), Graham Kribs (UOregon), Sergey Syritsyn (RIKEN BNL), David Schaich (Bern), Tom Applequist, George Fleming, Andrew Gasbarro (Yale).

Goldstone boson if chiral symmetry is spontaneously broken. On the other hand it is much less computationally demanding and thus permits the collection of data which mass higher statistical significance, which is particularly important for the calculation of the mass of the singlet scalar excitations that requires the calculation of disconnected correlators. Results obtained so far have presented in [55]. The investigation is still in progress.

The formulation and analysis of models of dark matter based on strong dynamics. In a first study he LSD collaboration calculated the electromagnetic form factors of electroweak-neutral, composite dark matter baryons for a three color theory with 2 and 6 degenerate fermions in the fundamental representation. The results, together with the experimental results from XENON100, excluded dark matter candidates of this type with masses below 10 TeV [3].

The study of an innovative model of composite dark matter based on the gauge group SU(4). In an SU(4) theory baryons are bound states of four fermions and, as such, are bosons. The constituents carry electroweak charges, but the model can escape the limits from direct detection experiments, because the neutral composite has only higher-dimensional interactions suppressed by the confinement scale. One still needs to worry about electromagnetic interactions, for example the magnetic dipole moment interaction considered in [3]. Composite bosonic dark matter theories based on SU(N) with even N do not have dipole moment interactions if the stable dark matter particle carry no spin, nor charge radius interaction in presence of a custodial symmetry. The leading interactions include Higgs exchange, and the electromagnetic polarizability interaction. Lattice calculations of the dark matter mass spectrum and of the effective Higgs-dark matter interaction were performed by extensive lattice simulations assuming that that the constituent fermions have both vector-like masses as well as electroweak symmetry breaking masses. Then the effective Higgs interaction determines the fraction of the constituents' mass which originates from electroweak symmetry breaking. Comparisons with the results of the LUX experiment led to rather stringent bounds on the effective Higgs interaction strength. In particular composite dark matter theories with fermion masses purely from electroweak symmetry breaking appeared strongly disfavored [47].

The detailed study of a model of "Stealth Dark Matter": a composite baryonic scalar of an SU(N) strongly-coupled theory with even N≥4. Specializing to

SU(4), the collaboration analyzed the constraints coming from dark meson decay, electroweak precision measurements, basic collider limits, and spin-independent direct detection scattering through Higgs exchange. A lower bound of 300GeV for the dark baryon mass was obtained from the requirement that the lightest dark meson was not observable at LEP II. [53] The collaboration also calculated the polarizability for the lightest baryons in SU(3) and SU(4). The results were used to estimate the scattering cross section of the "Stealth" baryon finding that it should be detectable in the 200-700GeV mass range. [54].

The LSD collaboration meets in weekly conference calls in which Rebbi has been an assiduous participant. Rebbi's contributions to the collaboration efforts have included: input on lattice calculations based on his long standing expertise; an analysis of patterns of symmetry breaking in models with color SU(2); the study of the effects that the limited topology transitions may have on the simulations; a detailed analysis of the spectrum of scalar excitations in the SU(3) theory with 8 fermion flavors, making use also of an interactive fitting program developed by Rebbi which allows for the running-time visualization of the data and fits; suggesting techniques to overcome problems encountered in the vacuum subtraction of the disconnected correlator; mentoring the doctoral research work BU's student Evan Weinberg, who received his Ph.D. in August 2015.

A Study of Near Conformal Behavior with Four Light Flavors and Eight Flavors of Variable Mass

Many theories for beyond the standard model physics rely on a large separation of scales between the infrared (IR) and ultraviolet (UV) domains [56, 57, 58, 32, 59, 60]. This leads to a "walking" gauge coupling and provides a dynamical mechanism for electroweak (EW) symmetry breaking, avoiding unnaturally large tuning of the Higgs mass, while satisfying stringent EW precision measurement constraints. Such a separation of scales is expected to occur if the UV dynamics is theory is dominated by an IR fixed point, as will happen with a sufficiently large number of flavors. In order to study the transition between a confining IR domain, with spontaneous chiral symmetry, and an UV regime governed by an IR fixed point, Brower, Anna Hasenfratz, Rebbi, Evan Weinberg, and Oliver Witzel embarked in the lattice investigation of an SU(3) theory with four light fermions and eight fermions of heavier, variable mass[2], the general idea being that by varying the mass of the heavy fermions one could interpolate between the QCD-like four flavor theory and the twelve flavor SU(3) theory which is widely believed to have an IR fixed point. Rebbi and collaborators performed extensive simulations of the model at a fixed gauge coupling $\beta = 4.0$, using 4 different values of the heavy mass $\widehat{m}_h = 0.05$, 0.06, 0.08 and 0.100, and 4 to 6 $\widehat{m}_\ell$ values for each $\widehat{m}_h$ (the $\widehat{m}$ notation is used to indicate quantities in lattice units.) Using the gradient flow to set the scale for the various simulations, they found that the coupling constant exhibited walking and that the range of walking increased, indeed, as the heavy mass was lowered, approaching the 12 flavor theory. They also calculated the spectrum of light-light fermion composites, inclusive of the mass of the isosinglet scalar meson which requires the notoriously difficult calculation of disconnected contributions to the correlator [50, 52, 51, 33, 32].

A very interesting result was that, contrary to walking range of the coupling constant which gradually increased as the heavy fermion mass was lowered, once the masses were measured in the appropriate scale, the spectrum of composite states did not depend, separately, on the heavy fermion masses, but only on the ratios $\widehat{m}_\ell/\widehat{m}_h$. Hasenfratz, Rebbi and Witzel went on to calculate also the masses of the heavy-heavy composites, which also appear to depend only on the ratios $\widehat{m}_\ell/\widehat{m}_h$, and gave an interpretation of the results in terms of hyperscaling in the neighborhood of an infrared fixed point. Figure 2 shows the results obtained for the spectrum of various states, from which hyperscaling is apparent. These latest results have been presented in a paper submitted to PRL [34]. They give strong evidence for hyperscaling in the spectrum and, by inference, to the presence of an IR fixed point in the 12 flavor SU(3) theory. There is some analogy between $\widehat{m}_h$ in the model and the bare coupling constant g in QCD: near the fixed point the physical values of masses and other observables do not depend on them, changing their value only varies the domain of scaling (in QCD lowering g increases the high energy cut-off, here lowering $\widehat{m}_h$ increases the domain of walking.) The values of the observables will, however, depend on the model, and if the theory of electrweak symmetry breaking is to be found in a strong dynamics framework, experiment and theory will decide which is the correct model.

Additional research

[2]The specific choice, 4 and 8, of the number of flavors is due to technical aspects of the work, which uses the staggered formulation of lattice fermions.

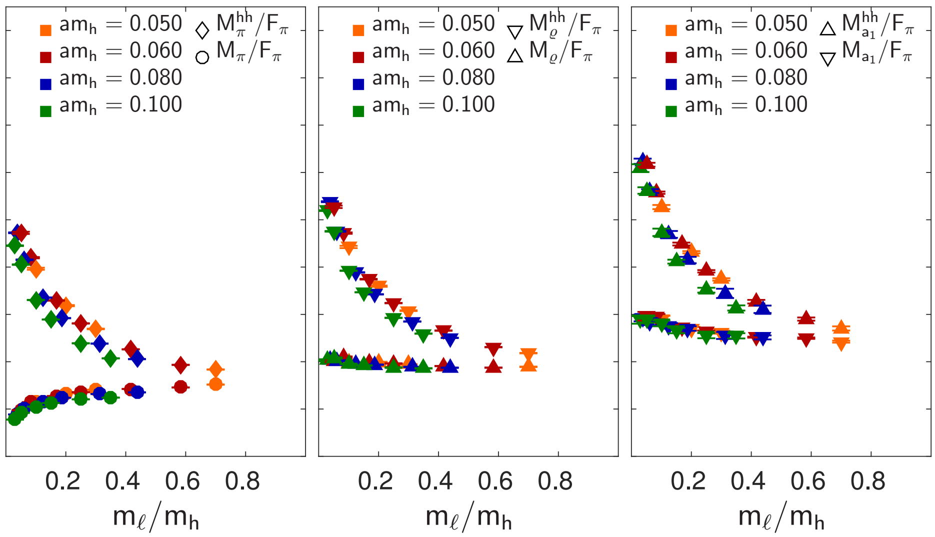

Figure 2: Results for the masses of pseudoscalar, vector and axial-vector states (denoted here by π, ρ and a_1 for convenience) in units of the pseudoscalar decay constant (denoted by F_π), plotted as functions of $\widehat{m}_\ell/\widehat{m}_h$. Circles (diamonds) are used for the light-light (heavy-heavy) states. The fact that data obtained with widely different heavy fermion masses fall on the same curves gives strong evidence for hyperscaling.

In addition to the above, Rebbi has studied algorithms to overcome topological barriers in lattice simulations, the use of the analytic continuation of correlators into the complex momentum plane to obtain information on the spectrum, the applicability of an approximation similar to the QCD quenched approximation to simulations of graphene, and other topics. These efforts are in progress and have not yet led to published results.

Publications. Please see the bibliography for the following published papers: [5], [48], [49], [55], [3], [47], [53], [54], [32], [50], [52], [51], [33], [34]

Martin Schmaltz

The research performed by Schmaltz with support from this grant was closely related to experiments, in particular the LHC and cosmological probes of dark matter. The following paragraphs describe each separate research project which lead to at least one publication.

1. New physics and lepton flavor violation in semi-leptonic B meson decays.

Flavor changing neutral current $|\Delta B| = |\Delta S| = 1$ processes are sensitive to possible new physics at the electroweak scale and beyond, providing detailed information about flavor, chirality and Lorentz structure. In 2014, the LHCb collaboration announced a 2.6σ deviation in the measurement of $R_K = \mathcal{B}(\bar{B} \to \bar{K}\mu\mu)/\mathcal{B}(\bar{B} \to \bar{K}ee)$ from the standard model's prediction of lepton universality. Schmaltz and collaborator Hiller [121] identified dimension six operators which could explain this deviation and studied constraints from other measurements. They found that vector and axial-vector four-fermion operators with flavor structure $\bar{s}b\bar{\ell}\ell$ can provide a good description of the data. Tensor operators cannot describe the data. Pseudo-scalar and scalar operators only fit the data with some fine-tuning. Hiller and Schmaltz pointed out that these operators can be further probed with the $\bar{B} \to \bar{K}ee$ angular distribution. Currently available data appears to point towards $C_9^{\mathrm{NP}\mu} = -C_{10}^{\mathrm{NP}\mu} < 0$, an $SU(2)_L$ invariant direction in parameter space supported by R_K, the $\bar{B} \to \bar{K}^*\mu\mu$ forward-backward asymmetry and the $\bar{B}_s \to \mu\mu$ branching ratio, which is currently allowed to be smaller than the standard model prediction. Hiller and Schmaltz also showed that there is a general method for constructing models with leptoquarks which can generate any of the chiral vector or axial-vector dimension six operators. As an example, they constructed two models which can explain the FCNC data and give predictions for the LHC and rare decays.

In an extension of the previous work, Hiller and Schmaltz pointed out that ratios of branching fractions of semileptonic B decays, $(B \to H\mu\mu)$ over $(B \to Hee)$ with $H = K, K^*, X_s, K_0(1430), \phi, \ldots$ are sensitive probes of lepton universality. In the Standard Model, the underlying flavor changing neutral current process $b \to s\ell\ell$ is lepton flavor universal. However models with new flavor violating physics above the weak scale can give substantial non-universal contributions. The leading contributions from such new physics can be parametrized by effective dimension six operators involving left- or right-handed quarks. In [122], Hiller and Schmaltz showed that in the double ratios R_{X_s}/R_K, R_{K^*}/R_K and R_ϕ/R_K the dependence on new physics coupling to left-handed quarks cancels out. Thus a measurement of any of these double ratios is a clean probe of flavor non-universal physics coupling to right-handed quarks. They also point out that the observables R_{X_s}, R_{K^*}, $R_{K_0(1430)}$ and R_ϕ depend on the same combination of Wilson coefficients and therefore satisfy simple consistency relations which can be tested at LHCb and future

B-factories.

2. A model for the LHC diboson excess.

The first run of the LHC at 8 TeV energy showed hints of a new resonance with mass near 1.9 TeV decaying into electroweak gauge boson pairs as well as into dijets. While Run 2 has neither confirmed nor ruled out such a resonance, it has yielded new constraints on models attempting to explain these decays. Additionally in W' models where this new resonance is a charged vector boson that is a weak isospin singlet there is the potential for conflict with the electroweak precision T parameter. Schmaltz and collaborators [123] constructed two variants of a W' resonance model that provide an excellent fit to both Run 1 (8 TeV) and Run 2 (13 TeV through 2015) data, as well as electroweak precision measurements. The model also predicts a neutral vector boson, a Z', with mass close to 3 TeV. This Z' is compatible with the intriguing Run 2 observation of a dielectron pair with invariant mass of 2.9 TeV at CMS.

3. Experimental constraints on the coupling of the Higgs boson to electrons.

In the Standard Model, the coupling of the Higgs boson to electrons is real and very small, proportional to the electron mass. New physics could significantly modify both real and imaginary parts of this coupling. In [124] Schmaltz and collaborators discussed experiments which are sensitive to the Higgs-electron coupling and derived the current bounds on new physics contributing to this coupling. The strongest constraint follows from the ACME bound on the electron electric dipole moment (EDM). Schmaltz and collaborators calculated the full analytic two-loop result for the electron EDM for the first time, and showed that it bounds the imaginary part of the Higgs-electron coupling to be less than 1.7×10^{-2} times the standard model electron Yukawa coupling. Deviations of the real part are much less constrained. Schmaltz and collaborators determined the bounds from Higgs decays, resonant Higgs production at electron colliders, Higgs mediated $B \to e^+e^-$ decays, and the anomalous magnetic moment of the electron. They found that the current strongest constraint comes from $h \to e^+e^-$ decays at the LHC, bounding the coupling to be less than ~ 600 times the SM Yukawa coupling. Important improvements can be expected from future EDM measurements as well as from resonant Higgs production at a next-generation high-luminosity e^+e^- collider.

4. Dark matter and dark radiation from a non-Abelian dark sector.

In the area of Astro-Particle Physics and Cosmology, Schmaltz, Marques-Tavares and Buen-Abad (current graduate student at BU) constructed a model of dark matter in which the dark matter is charged under a dark non-Abelian gauge group with weakly coupled dark gluons [125]. They showed the dark gluons contribute to the radiation background of the universe and that dark matter can scatter off this dark radiation background. The most important effect of this scattering is to suppress the power spectrum of density perturbations in the dark matter fluid. This is interesting because current measurements of density perturbations at large scales are in disagreement with predictions in the standard ΛCDM model. In addition, the energy density in the dark radiation that the model predicts contributes to the expansion rate of the universe. Recent measurements of this expansion rate, the Hubble parameter, show a discrepancy between the measured value and predictions in the ΛCDM model. This discrepancy can also be explained in the new model proposed by Schmaltz and collaborators by virtue of the new dark radiation: the energy density in the dark radiation contributes to the expansion rate of the universe in the early universe (which is measured very precisely with the cosmic microwave background) but it has a negligible effect on the expansion measured directly today.

Schmaltz, Marques-Tavares and Lesgourgues further explored the predictions of this model for precision cosmology [126]. In particular they performed a global fit of the non-Abelian dark matter and dark radiation model to the Planck 2015 CMB temperature correlation spectrum, measurements of the baryon acoustic oscillation scale, supernova measurements of the current Hubble expansion rate, and measurements of large scale structure from galaxy surveys. The precision fit confirmed that the non-Abelian dark matter model can provide a significantly better fit than the standard ΛCDM model. The best fit was found for a dark radiation component that has an overall energy density of a fraction of a neutrino, $\Delta N_{eff} \sim 0.3$, and a non-vanishing scattering rate between dark matter and dark radiation. The improvement in the fit is significant with the new minimum preferred by over 3σ over ΛCDM (i.e. zero dark radiation and zero scattering rate).

In performing the fit Schmaltz and collaborators discovered that there is a second region in parameter space that also significantly - and unexpectedly - reduces the χ-squared of the fit. This second region corresponds to very small values of $\Delta N_{eff} < 0.1$ with somewhat larger dark matter - dark radiation scattering rates. This scattering is sufficiently strong to make the two fluids tightly coupled, effectively giving rise to a small pressure for the dark matter fluid. A manuscript reporting on these results and the precision fit to cosmological data is in preparation.

5. Higgs mass naturalness and scale invariance in the UV.

There have been suggestions in the literature that electroweak symmetry breaking in the Standard Model may be natural if the Standard Model merges into a conformal field theory at short distances. In such a scenario the Higgs mass would be protected from quantum corrections by scale invariance. In order for the Standard Model to merge into a conformal field theory at least one new ultraviolet (UV) scale is required at which the couplings turn over from their usual Standard Model running to the fixed point behavior. In [127], Schmaltz and collaborators Skiba (Yale) and Marques Tavares showed that the Higgs mass is sensitive to such a turn-over scale even if there are no associated massive particles and the scale arises purely from dimensional transmutation. They demonstrated the sensitivity to the turnover scale explicitly in toy models and argued for it on general grounds. A conclusion of this work is that if scale invariance is responsible for Higgs mass naturalness, then the transition to CFT dynamics must occur near the TeV scale with observable consequences at colliders. In addition, the UV fixed point theory in such a scenario must be interacting because logarithmic running near a free fixed point constitutes hard breaking of scale invariance and spoils the Higgs mass protection.

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All of the above articles have authors supported by various grants and so acknowledge other grants and funding agencies beyond our DOE grant. Also, the papers published by Katz acknowledge the support he received from a DOE Career award.

Junior personnel trained under this grant.

This award supported the training of 5 postdoctoral associates: Siavosh Behbahani, Michael Cheng, Zuhair Khandker, Matt Walters and Oliver Witzel.

Four graduate students were supported by this award, three of whom received PhDs during the grant period: Manuel Buen-Abad, Gustavo Marques-Tavares (PhD 2015), Evan Weinberg (PhD 2015), Yiming Xu (PhD 2014).

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