

Analytical support for SECANT emitter experiments

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Single-photon sources

- **Quantum dots**
- **Nanocavities**
- **Single-quantum-dot sources**

Other QKD sources

- **Squeezed light**
- **Parametric down conversion**

Experiment on growing highly-uniform quantum-dot samples

Questions:

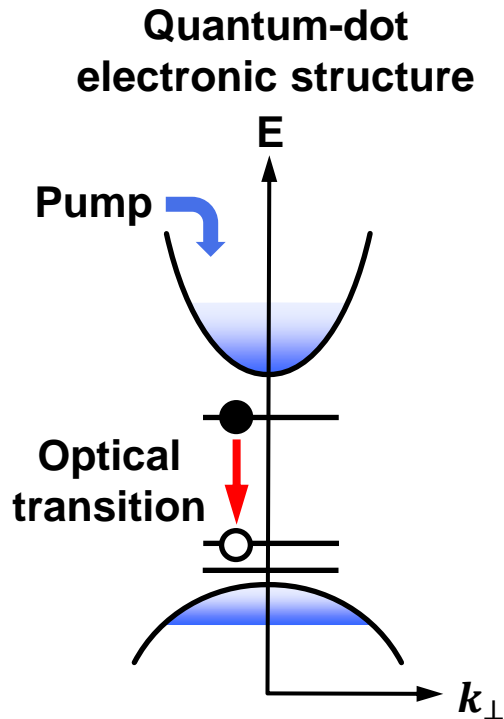
- What does a perfect (homogeneously-broadened) quantum-dot sample look like?
- Is there a way to determine degree of nonuniformity (i.e. inhomogeneous broadening)?

Answer: Yes, by calculating optical response with rigorous description of dephasing (due to Coulomb correlations)



Usually treated as
free parameter

Approach



$$\begin{aligned}
 H = & \sum_{\alpha} \varepsilon_{\alpha}^e c_{\alpha}^{\dagger} c_{\alpha} + \sum_{\beta} \varepsilon_{\beta}^h b_{\beta}^{\dagger} b_{\beta} - \sum_{\alpha} (g_{\alpha} b_{\alpha}^{\dagger} c_{\alpha}^{\dagger} + g_{\alpha}^{*} c_{\alpha} b_{\alpha}) E \\
 & + \frac{1}{2} \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\eta} c_{\sigma} + \frac{1}{2} \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} b_{\alpha}^{\dagger} b_{\beta}^{\dagger} b_{\eta} b_{\sigma} \\
 & - \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} b_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\eta} b_{\sigma} \\
 & + \hbar \sum_{\alpha\beta q} G_q (c_{\alpha}^{\dagger} c_{\beta} + b_{\alpha}^{\dagger} b_{\beta}) (d_q + d_q^{\dagger})
 \end{aligned}$$

$\sum_n c_{\alpha}(R_n) V_{\alpha}(R_n)$ (Light-carrier)
 $\left. \begin{array}{l} \text{Matrix element of } \frac{e^2}{4\pi\epsilon_b|r-r'|} \end{array} \right\}$ (Carrier-carrier)
 (Carrier-phonon)

Heisenberg Picture

$$\frac{\partial}{\partial t} A = -\frac{i}{\hbar} [A, H]$$

Populations and correlations

$$\langle c_{\alpha}^{\dagger} c_{\alpha} \rangle, \langle b_{\alpha}^{\dagger} b_{\alpha} \rangle, \langle c_{\alpha}^{\dagger} b_{\alpha}^{\dagger} \rangle, \langle c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\beta} c_{\alpha} \rangle, \langle b_{\beta}^{\dagger} c_{\alpha}^{\dagger} c_{\alpha} b_{\beta} \rangle, \dots$$

Single particles

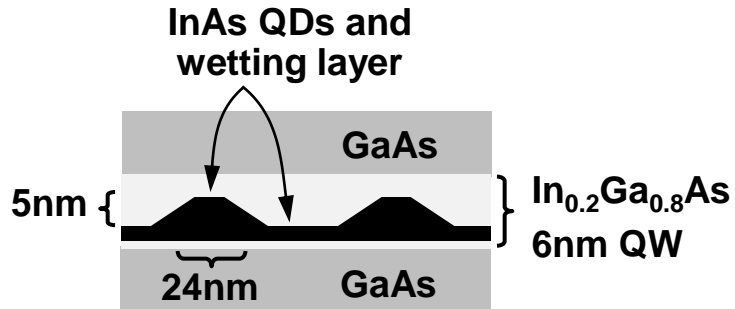
Correlated pairs

Correlated 3-particle clusters

Cluster expansion

$$\langle \hat{N} \rangle = \text{[5 single particles]} + \text{[2 correlated pairs]} + \text{[1 correlated 3-particle cluster]} + \dots$$

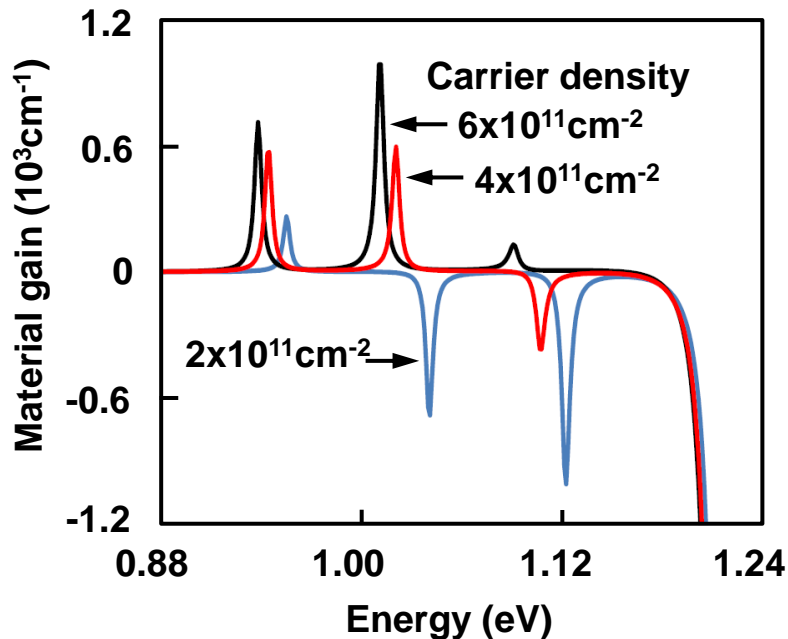
Calculated quantum-dot optical response



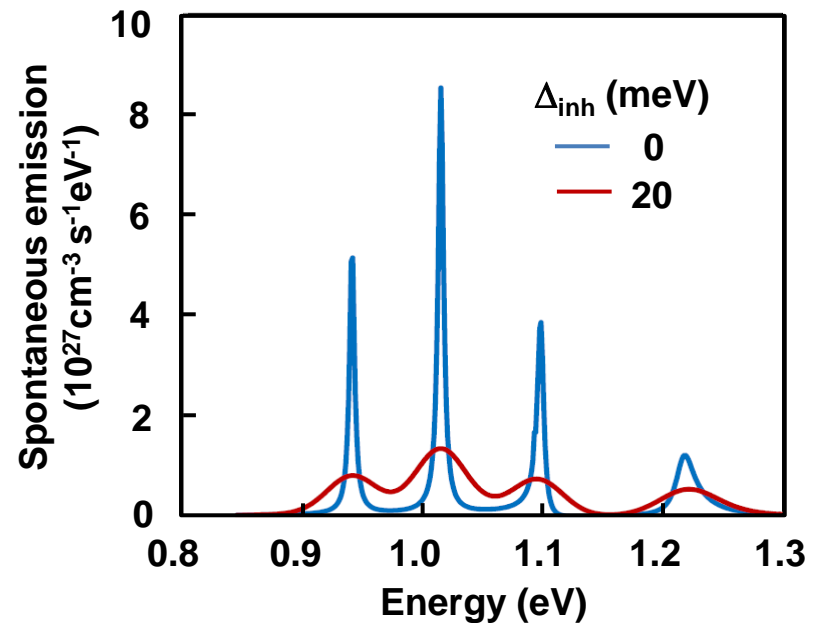
$$N_{\text{dot}} = 5 \times 10^{10} \text{cm}^{-2}$$

Perfect sample

Homogeneously-broadened
absorption & gain

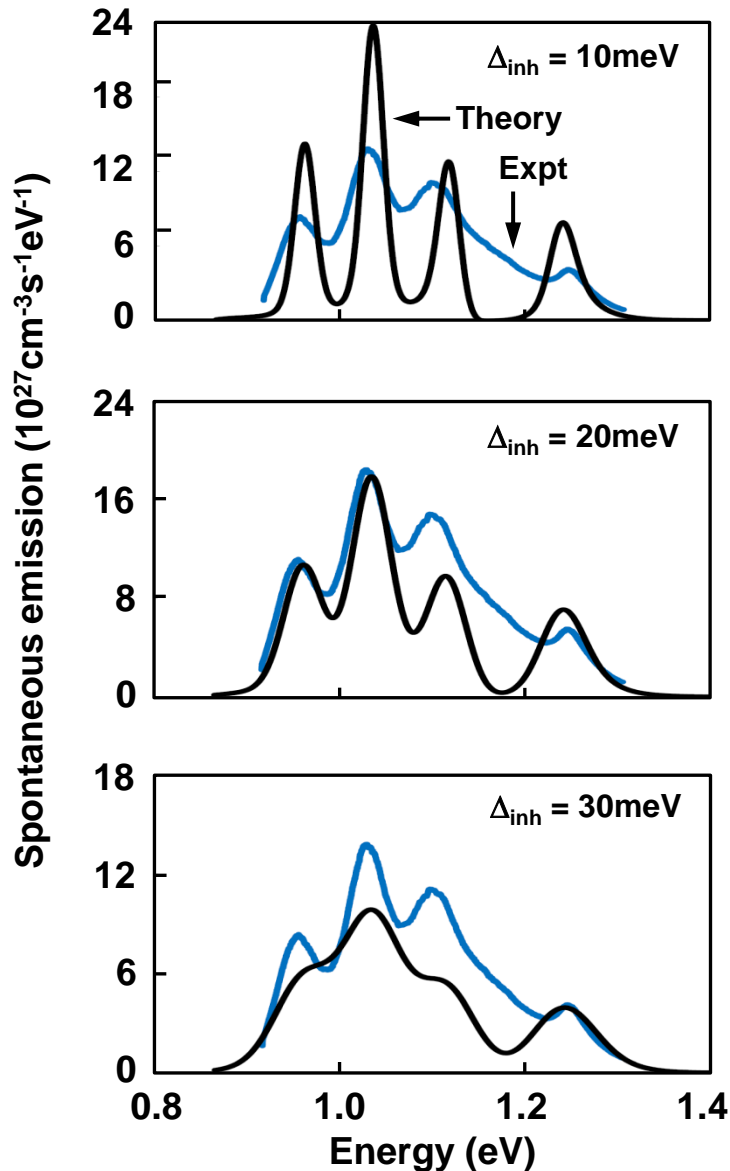


Spontaneous emission and
effect of QD nonuniformity

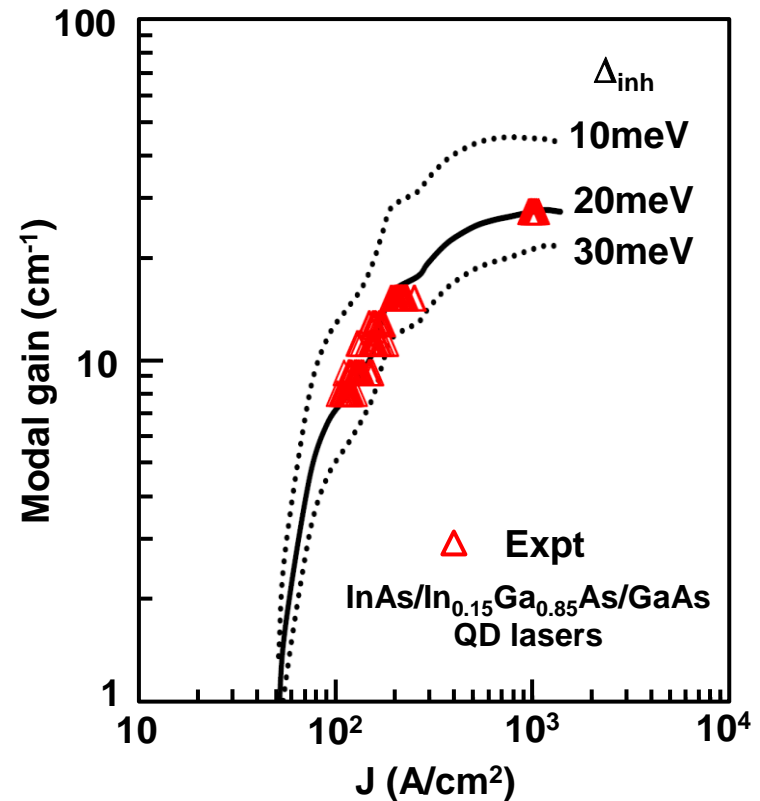


Extraction of inhomogeneous broadening

From spontaneous emission





From lasers



Chow, Liu, Gossard and Bowers, 'Extraction of inhomogeneous broadening and nonradiative losses in InAs quantum-dot lasers,' (submitted APL)

Analytical support for SECANT emitter experiments

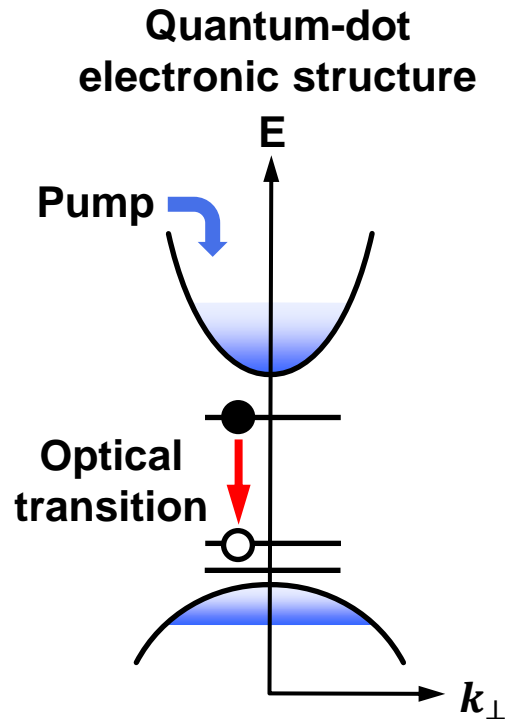
Single-photon sources

- Quantum dots
- Nanocavities 
- Single-quantum-dot sources 

Other QKD sources

- Squeezed light
- Parametric down conversion

Approach (quantized electrons, classical optical field)



$$\begin{aligned}
 H = & \sum_{\alpha} \varepsilon_{\alpha}^e c_{\alpha}^{\dagger} c_{\alpha} + \sum_{\beta} \varepsilon_{\beta}^h b_{\beta}^{\dagger} b_{\beta} - \sum_{\alpha} (g_{\alpha} b_{\alpha}^{\dagger} c_{\alpha}^{\dagger} + g_{\alpha}^{*} c_{\alpha} b_{\alpha}) E \\
 & + \frac{1}{2} \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\eta} c_{\sigma} + \frac{1}{2} \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} b_{\alpha}^{\dagger} b_{\beta}^{\dagger} b_{\eta} b_{\sigma} \\
 & - \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} b_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\eta} b_{\sigma} \\
 & + \hbar \sum_{\alpha\beta q} G_q (c_{\alpha}^{\dagger} c_{\beta} + b_{\alpha}^{\dagger} b_{\beta}) (d_q + d_q^{\dagger})
 \end{aligned}$$

$\sum_n c_{\alpha}(R_n) V_{\alpha}(R_n)$
 $\frac{e^2}{4\pi\epsilon_b |r - r'|}$

Carrier-carrier
Carrier-phonon

Heisenberg Picture

$$\frac{\partial}{\partial t} A = -\frac{i}{\hbar} [A, H]$$

Populations and correlations

$$\langle c_{\alpha}^{\dagger} c_{\alpha} \rangle, \langle b_{\alpha}^{\dagger} b_{\alpha} \rangle, \langle c_{\alpha}^{\dagger} b_{\alpha}^{\dagger} \rangle, \langle c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\beta} c_{\alpha} \rangle, \langle b_{\beta}^{\dagger} c_{\alpha}^{\dagger} c_{\alpha} b_{\beta} \rangle, \dots$$

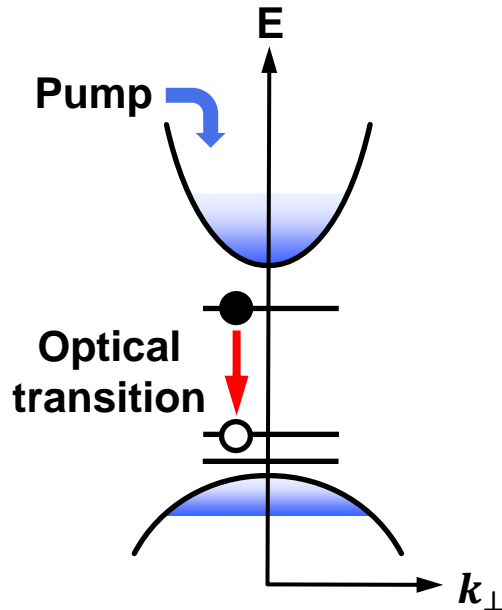
Cluster expansion

Single particles Correlated pairs Correlated 3-particle clusters

$$\langle \hat{N} \rangle = \text{[diagram of 5 single particles]} + \text{[diagram of 2 correlated pairs]} + \text{[diagram of 3 correlated particles]} + \dots$$

Approach (quantized electrons and optical field)

Quantum-dot
electronic structure



$$H = \sum_{\alpha} \varepsilon_{\alpha}^e c_{\alpha}^{\dagger} c_{\alpha} + \sum_{\beta} \varepsilon_{\beta}^h b_{\beta}^{\dagger} b_{\beta} + \hbar \omega \left(a^{\dagger} a + \frac{1}{2} \right) \quad \text{Single-particle}$$

$$- \hbar \sum_{\alpha} (g_{\alpha} b_{\alpha}^{\dagger} c_{\alpha}^{\dagger} a + g_{\alpha}^{*} a^{\dagger} c_{\alpha} b_{\alpha}) \quad \text{Light-carrier}$$

$$g_{\alpha} = \sqrt{\frac{v}{\hbar \epsilon_b V}} W(R_{QD}) \sum_n c_{\alpha}(R_n) V_{\alpha}(R_n)$$

+ Carrier-carrier and carrier-phonon interactions

Heisenberg Picture

$$\frac{\partial}{\partial t} A = -\frac{i}{\hbar} [A, H]$$

Populations and correlations

$$\langle c_{\alpha}^{\dagger} c_{\alpha} \rangle, \langle b_{\alpha}^{\dagger} b_{\alpha} \rangle, \langle c_{\alpha}^{\dagger} b_{\alpha}^{\dagger} a \rangle, \langle a^{\dagger} a \rangle, \langle a^{\dagger} a^{\dagger} a a \rangle, \langle c_{\alpha}^{\dagger} c_{\alpha} a^{\dagger} a \rangle, + \dots$$

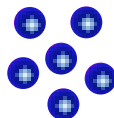
Single particles

Correlated pairs

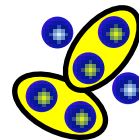
Correlated 3-particle clusters

Cluster expansion

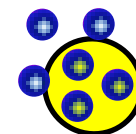
$$\langle \hat{N} \rangle =$$



+



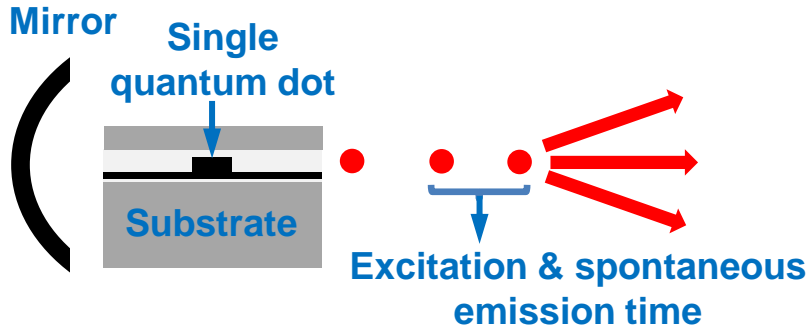
+



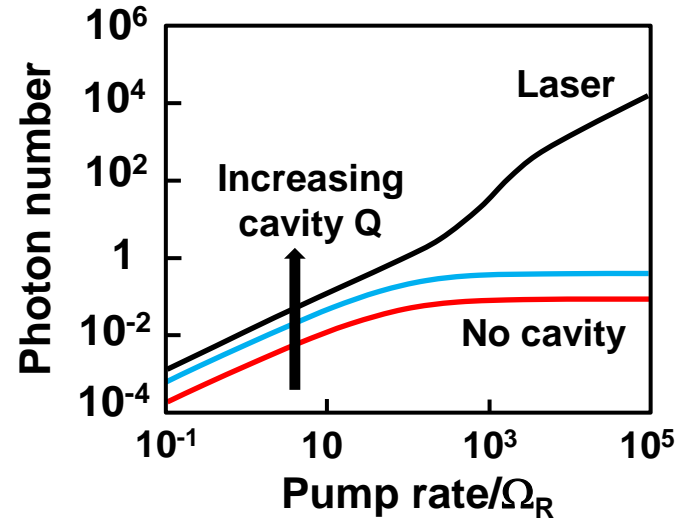
+ ...

Single-photon source

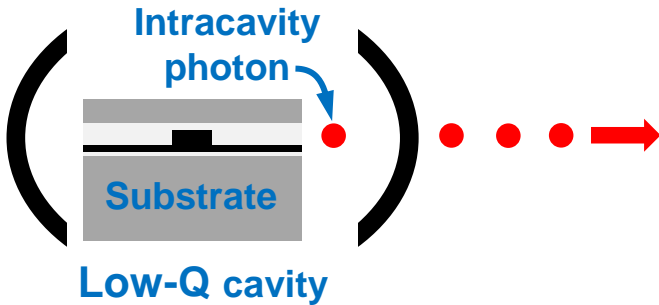
Error-free but slow



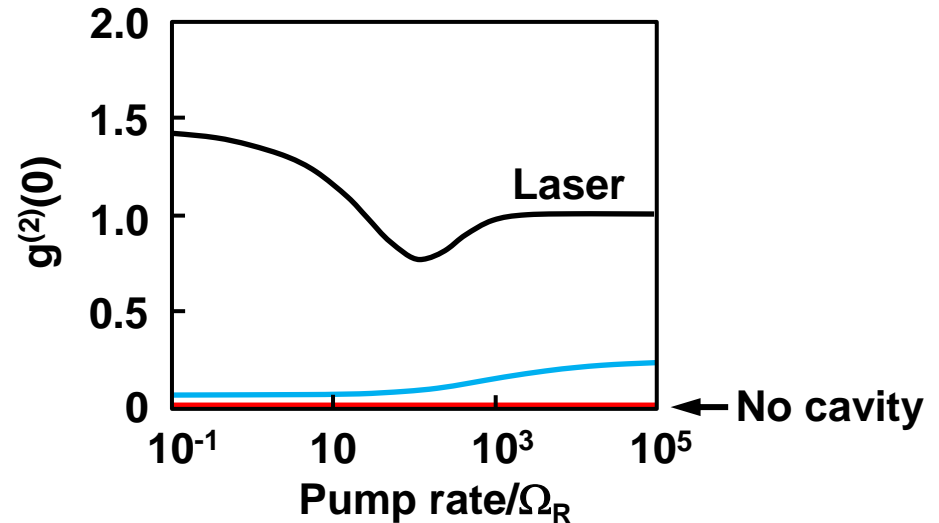
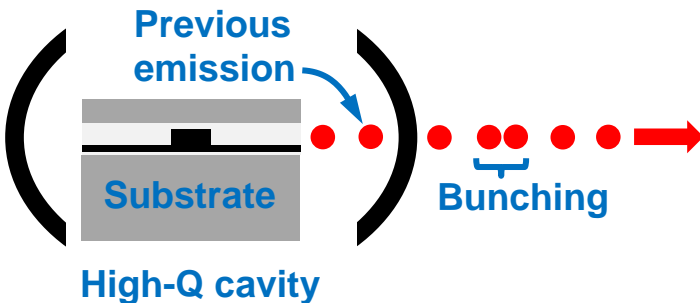
From cavity-QED model



Cavity enhancement Directionality and Purcell



Too much cavity

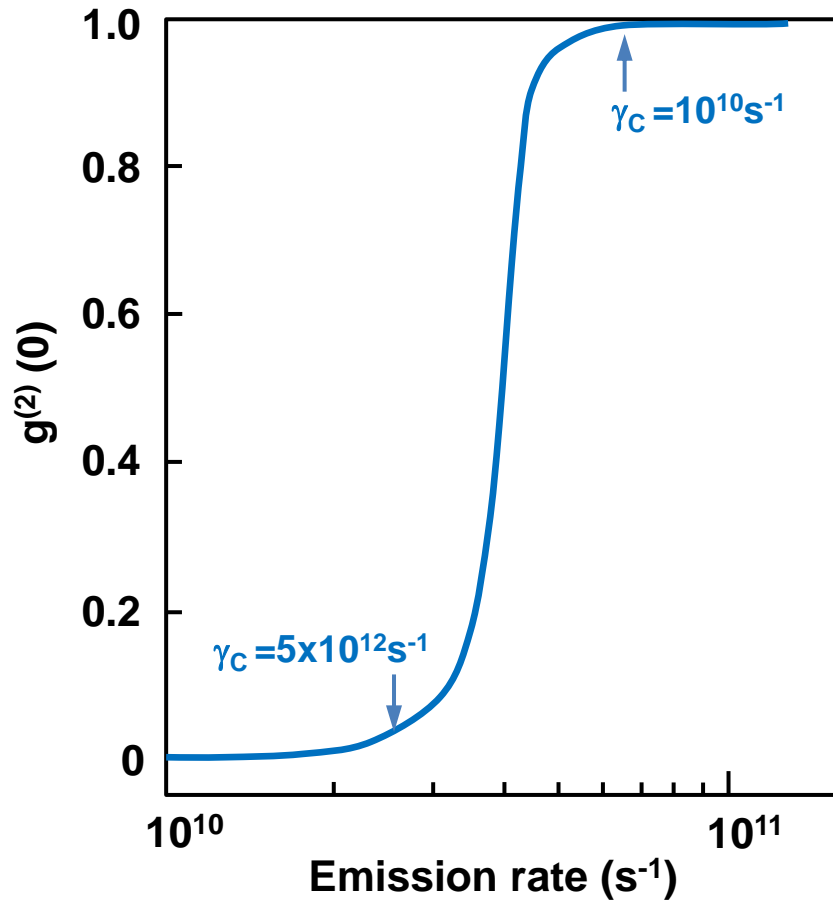


What is the right Q?

Fundamental limit to efficiency, rate and error?

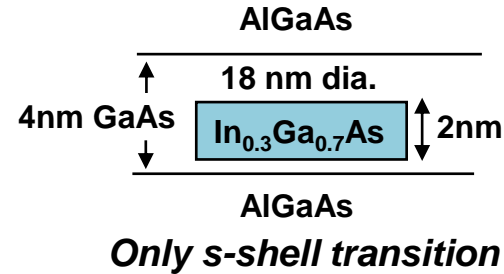
Single-photon purity and emission rate

$g^{(2)}(0)$ vs. emission rate
(by increasing cavity-Q)



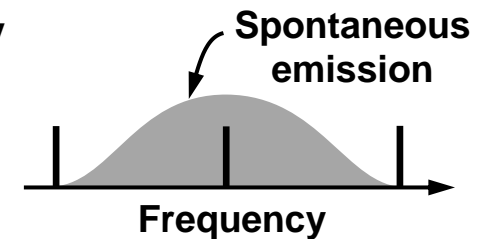
Cavity-enhanced rate $\sim 10^9 \text{ s}^{-1}$ (expt)

① Shallow quantum dot



② Nanocavity

$$\beta = \frac{\gamma_l}{\gamma_{sp}} = 1$$



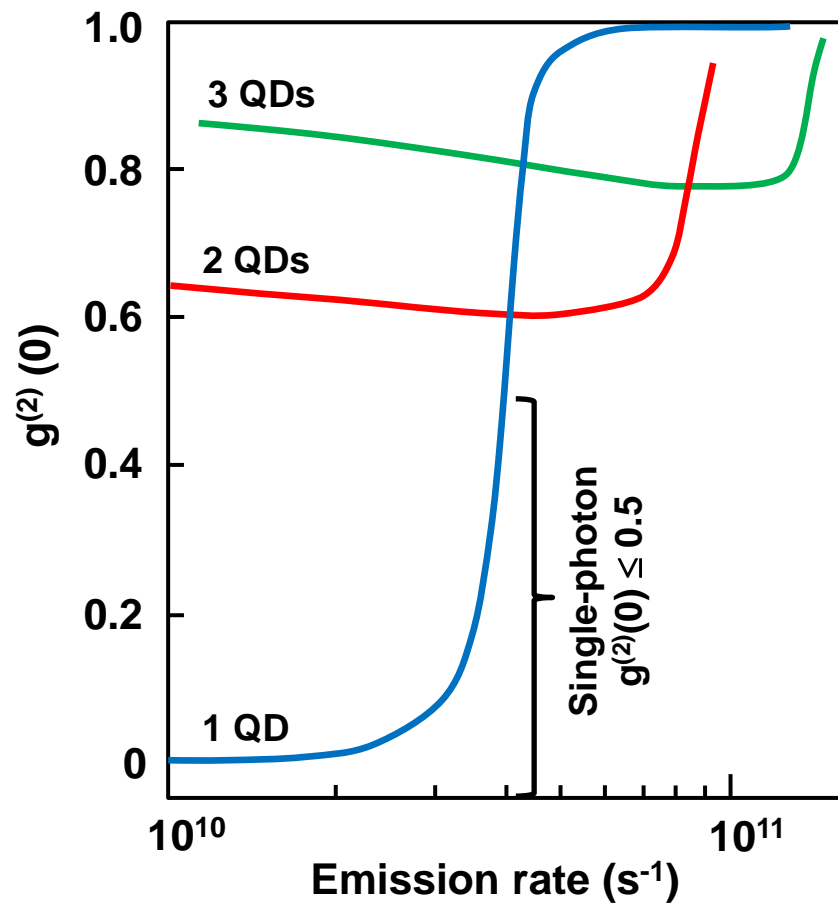
All emission into single resonator mode

③ Scaling with electron-light coupling

$$\underbrace{\frac{v}{\hbar \epsilon_b V}}_{\text{Mode volume}} \underbrace{W(R_{QD})}_{\text{Confinement factor}} \underbrace{\sum_n C(R_n) V(R_n)}_{\text{Electron-hole envelope overlap}}$$

Concern: Extraneous quantum dots

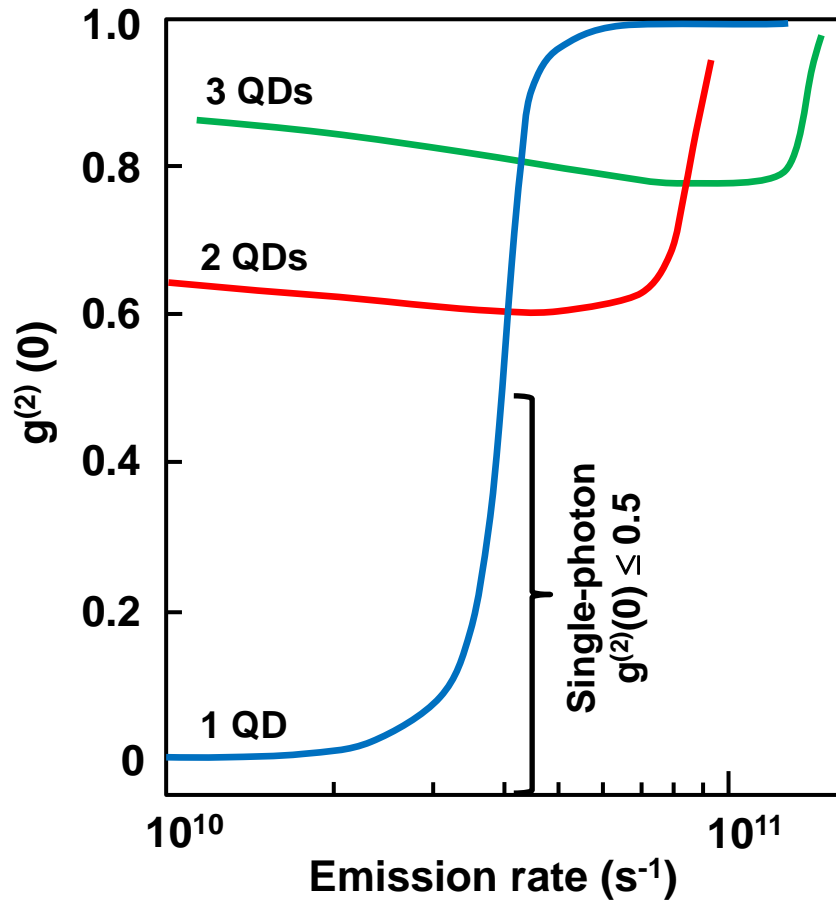
$g^{(2)}(0)$ vs. emission rate
(by increasing cavity-Q)



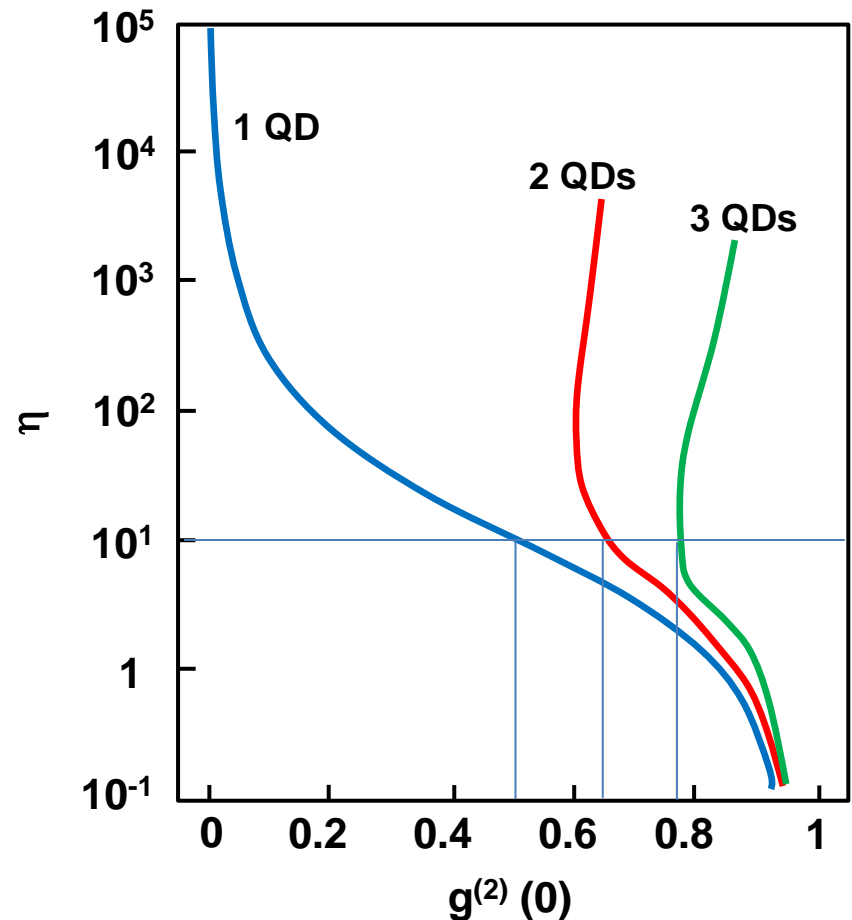
Concern: $g^{(2)}(0)$ as measure of error

Single-photon purity: $\eta = \frac{\text{Single-photon emission probability}}{\text{Multi-photon emission probability}}$

$g^{(2)}(0)$ vs. emission rate
(by increasing cavity-Q)



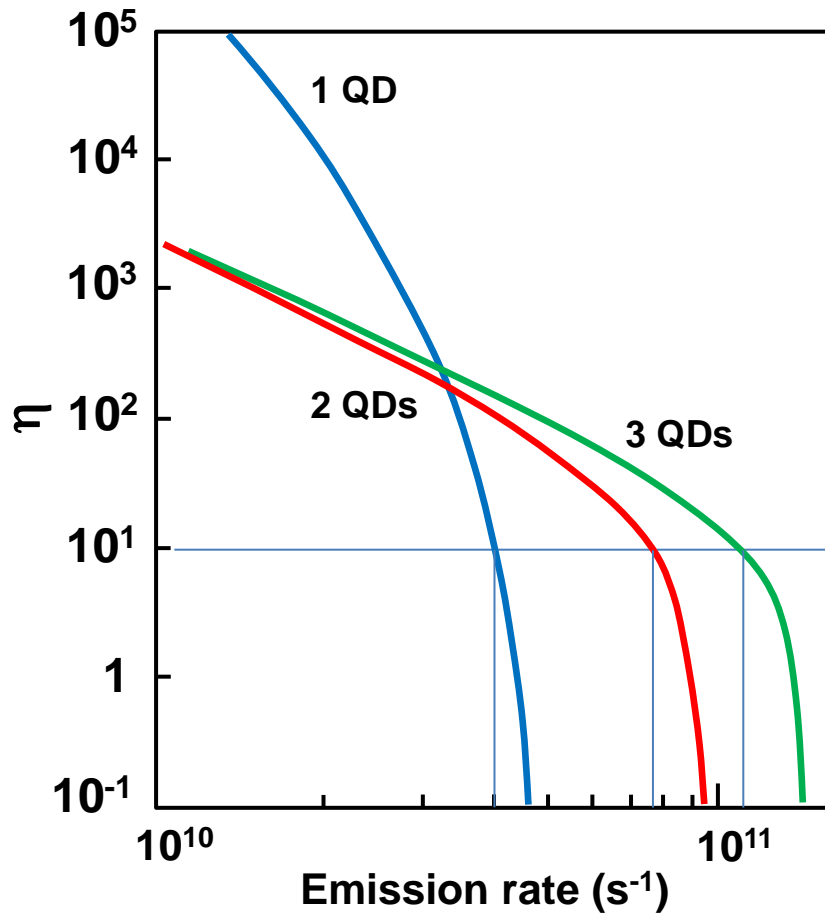
$g^{(2)}(0)$ fails



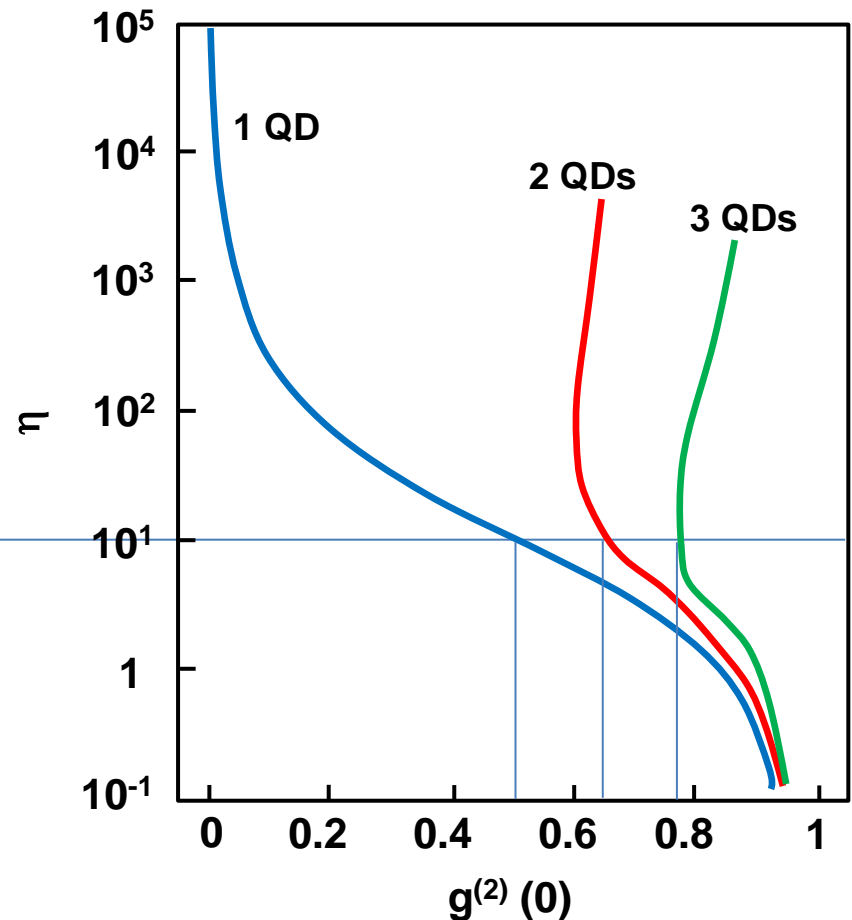
Full photon statistics vs. $g^{(2)}(0)$

Single-photon purity: $\eta = \frac{\text{Single-photon emission probability}}{\text{Multi-photon emission probability}}$

Purity vs. emission rate
(by increasing cavity-Q)



$g^{(2)}(0)$ fails



Analytical support for SECANT emitter experiments

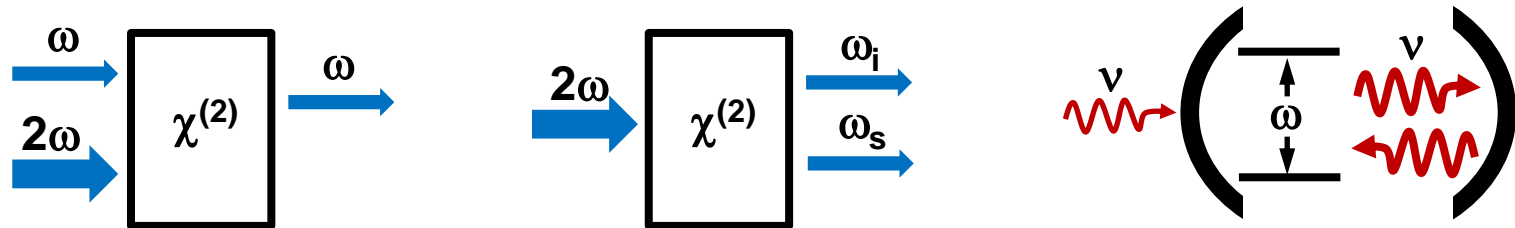
Single-photon sources

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Other QKD sources

- Squeezed light
- Parametric down conversion

On-going



Cluster expansion: $\langle \hat{N} \rangle =$

Single particles	Correlated pairs	Correlated 3-particle clusters
	+	+
		...

Before :

$$\langle c_{\alpha}^{\dagger} c_{\alpha} \rangle, \langle b_{\alpha}^{\dagger} b_{\alpha} \rangle$$

$$\langle a^{\dagger} a \rangle, \langle c_{\alpha}^{\dagger} b_{\alpha}^{\dagger} a \rangle$$

Now :

$$\langle c_{\alpha}^{\dagger} c_{\alpha} \rangle, \langle b_{\alpha}^{\dagger} b_{\alpha} \rangle$$

$$\langle a \rangle, \langle a^{\dagger} \rangle, \langle c_{\alpha}^{\dagger} b_{\alpha}^{\dagger} \rangle$$

Before :

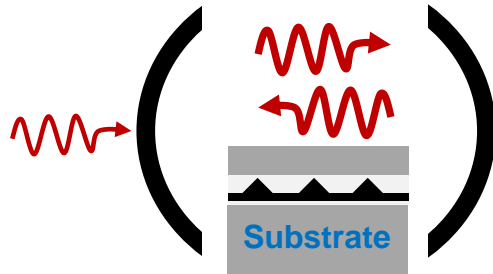
$$\langle a^{\dagger} a^{\dagger} a a \rangle = 2 \langle a^{\dagger} a \rangle \langle a^{\dagger} a \rangle + \delta \langle a^{\dagger} a^{\dagger} a a \rangle$$

Now :

$$\langle a^{\dagger} a^{\dagger} a a \rangle = \langle a^{\dagger} \rangle^2 \langle a \rangle^2 + 4 \langle a^{\dagger} \rangle \langle a \rangle \delta \langle a^{\dagger} a \rangle + \langle a^{\dagger} \rangle^2 \delta \langle a a \rangle + \langle a \rangle^2 \delta \langle a^{\dagger} a^{\dagger} \rangle$$

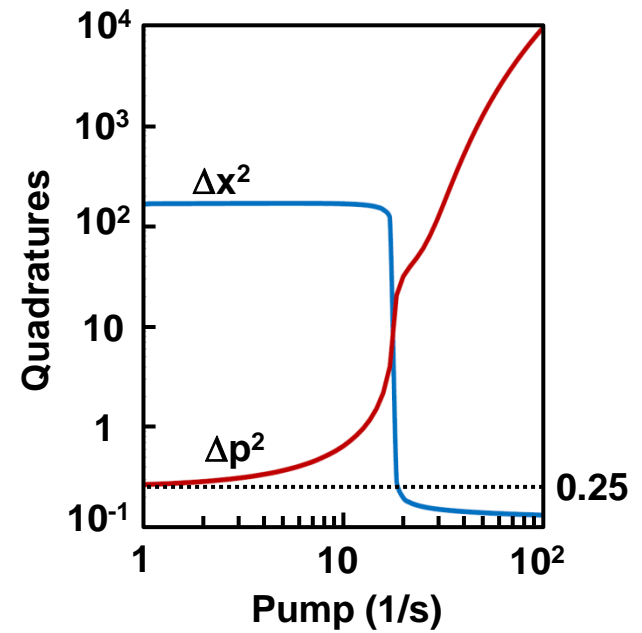
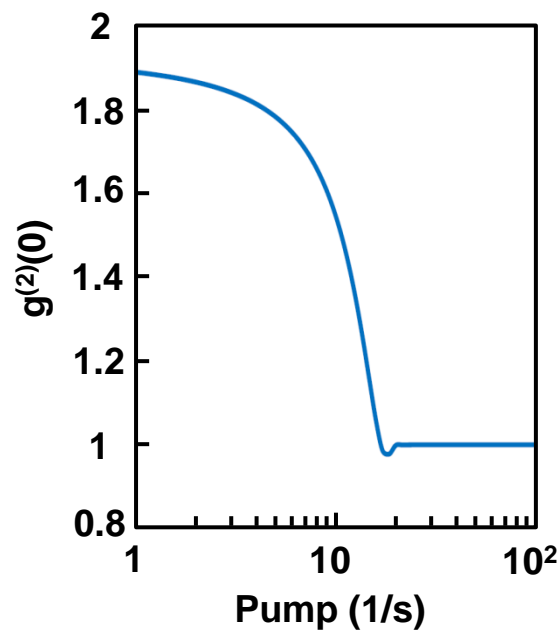
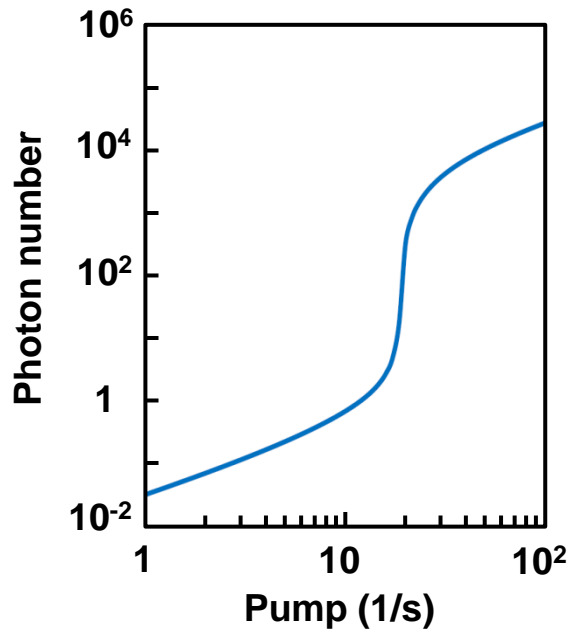
$$+ 2 \delta \langle a^{\dagger} a \rangle \delta \langle a^{\dagger} a \rangle + \delta \langle a^{\dagger} a^{\dagger} \rangle \delta \langle a a \rangle$$

Preliminary result



$$\Delta x^2 = \frac{1}{4} [1 + 2\delta \langle a^\dagger a \rangle + 2 \operatorname{Re} (\delta \langle a^2 \rangle)]$$

$$\Delta p^2 = \frac{1}{4} [1 + 2\delta \langle a^\dagger a \rangle - 2 \operatorname{Re} (\delta \langle a^2 \rangle)]$$



SECANT related publications

Single-photon (quantum dots, nanocavities and single-QD sources)

Chow, Jahnke 'On the physics of semiconductor quantum dots for applications in lasers and quantum optics,' PROGRESS IN QUANTUM ELECTRONICS **37** 109-184 (2013)

Chow, Jahnke, Gies 'Emission properties of nanolasers during the transition to lasing' LIGHT-SCIENCE & APPLICATIONS **3** e201 (2014)

Gies, Jahnke, Chow 'Photon antibunching from few quantum dots in a cavity' PHYSICAL REVIEW A **91** 061804(R) (2015)

Chow 'Are Nonclassical Light Sources Lasers?' IEEE JOURNAL OF SELECTED TOPICS IN QUANTUM ELECTRONICS **19** 1503402 (2013)

Chow, Liu, Gossard and Bowers 'Extraction of inhomogeneous broadening and nonradiative losses in InAs quantum-dot lasers' (submitted APPLIED PHYSICS LETTERS)

TA13282 'Higher photon-flux in single-photon emission using a few emitter system' (2014)

Beyond single-photon sources (squeeze-light, parametric down conversion)

Lingnau, Chow, Scholl, Ludge 'Feedback and injection locking instabilities in quantum-dot lasers: a microscopically based bifurcation analysis,' NEW JOURNAL OF PHYSICS **15** 093031 (2013)

Michael, Chow, Schneider 'Group-velocity slowdown in a double quantum dot molecule' PHYSICAL REVIEW B **88** 125305 (2013)

Michael, Chow, Schneider, 'Microscopic model for intersubband gain from electrically pumped quantum-dot structures,' PHYSICAL REVIEW B **90** 165302 (2014)

Chow, Straatsma, Anderson, 'Numerical model for atomtronic circuit analysis,' PHYSICAL REVIEW A **92** 013621 (2015)