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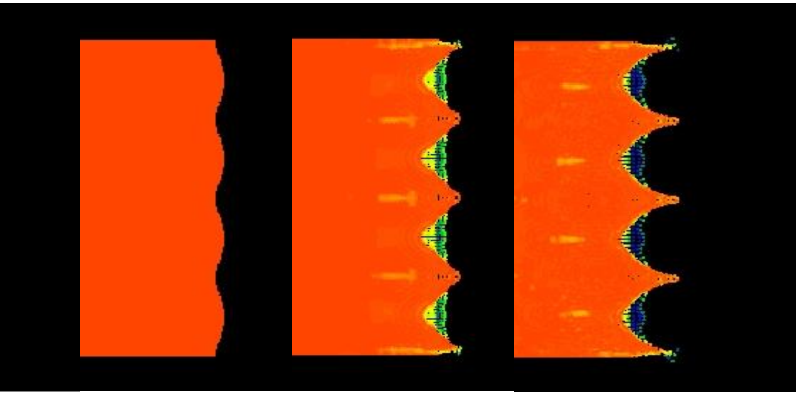

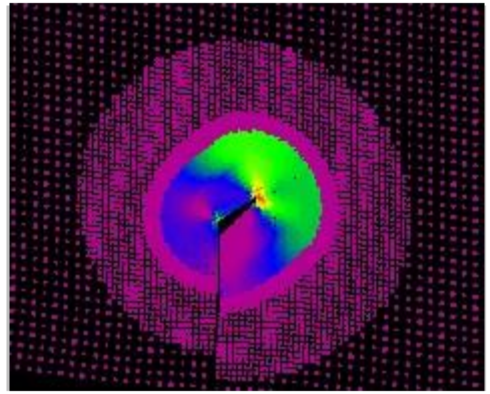

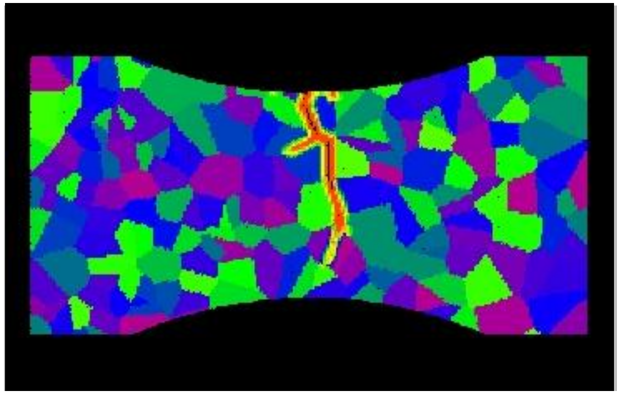

# Peridynamics: What it is, what it does

Stewart Silling

UNM Civil Engineering guest lecture, September 17, 2015

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# Outline

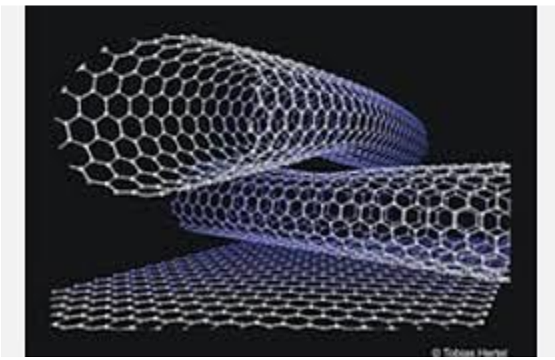
- Purpose of peridynamics
- Personal history of peridynamics
- Capabilities, current research, and opportunities
- Strengths and weaknesses

# What should be modeled as a classical continuum?

- Commercial finite element codes approximate the equations of classical continuum mechanics.
  - Assumes a <u>continuous body</u> under <u>smooth deformation</u>.
  - When is this the right approximation?

$$\nabla \cdot \sigma + b = 0$$

Augustin-Louis Cauchy, 1840
(image: Library of Congress)



Carbon nanotubes (image: nsf.gov)

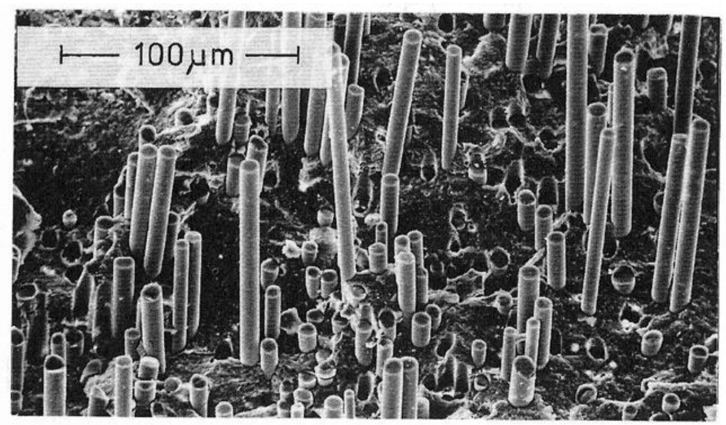


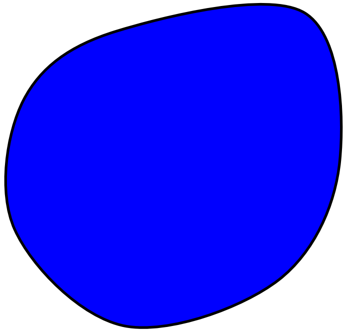

**Figure 11.20** Pull-out: (a) schematic diagram; (b) fracture surface of 'Silceram' glass-ceramic reinforced with SiC fibres. (Courtesy H. S. Kim, P. S. Rogers and R. D. Rawlings.)

Complex failure progression in a composite

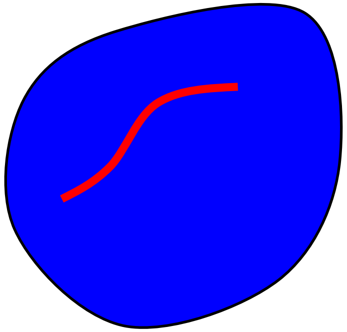
Fragmented glass (image: Washington Glass School)

# Purpose of peridynamics

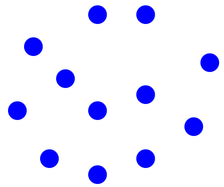
- To unify the mechanics of continuous and discontinuous media within a single, consistent set of equations.



Continuous body



Continuous body
with a defect

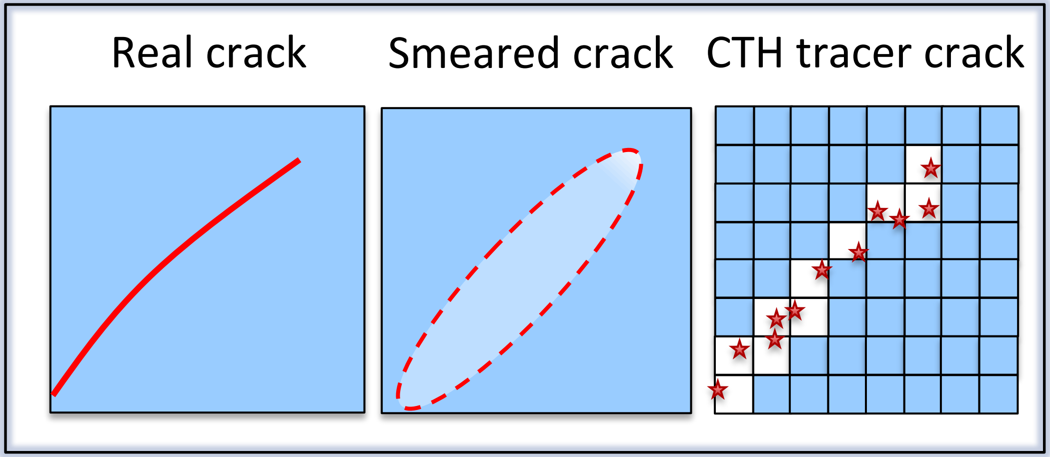


Discrete particles

- Why do this?
  - Avoid coupling dissimilar mathematical systems (A to C).
  - Model complex fracture patterns.
  - Communicate across length scales.

# Personal view of PD history

- 1980's
  - Mathematical theory of singularities in elastic solids
- 1990's:
  - Johnson-Cook, similar models in CTH
  - Tracer crack & shear band model in CTH
  - 1998: "The realization"
- 2000's:
  - Bond-based peridynamics
    - Rush to applications
  - State-based peridynamics
  - Micropolar (Gerstle-Sau)
  - Math, physics foundations
- 2010's:
  - Increasing interest worldwide
  - Sierra, Peridigm
  - Plasticity
  - Address practical issues
    - Wake-up calls
  - Thermodynamics
  - Shock waves
  - Multiscale
  - Multiphysics
  - LS-DYNA

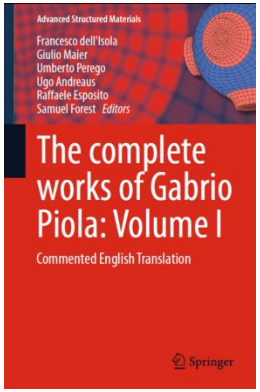



Recognizing the fundamental difficulties in modeling fracture with conventional methods puts us in opposition to conventional views…

# Personal view of PD history: On dealing with criticism

<u>Some things I've learned</u>

- People criticize you because they think it is right to do so.
  –Epictetus (Roman philosopher)
- The people we admire the most are often those who are criticized the most.
- Criticism often contains some truth.
- Don't let critics set your agenda.




In [120] the analysis started by Piola is continued, seemingly as if the author, Silling, were one of his closer pupils: arguments are very similar and also a variational formulation of the presented theories is found and discussed.

$$\Delta_{\mathcal{B}}[(b_m(X) - a(X))\,\delta\chi(X) + (\Delta_{\mathcal{B}}\Lambda(X,\bar{X},\rho)\delta\rho^2\mu(\bar{X})d\bar{X})]\mu(X)dX + \delta W(\partial\mathcal{B}) = 0 \qquad (12\text{bis})$$

By a standard localization argument one easily gets that (12bis) implies

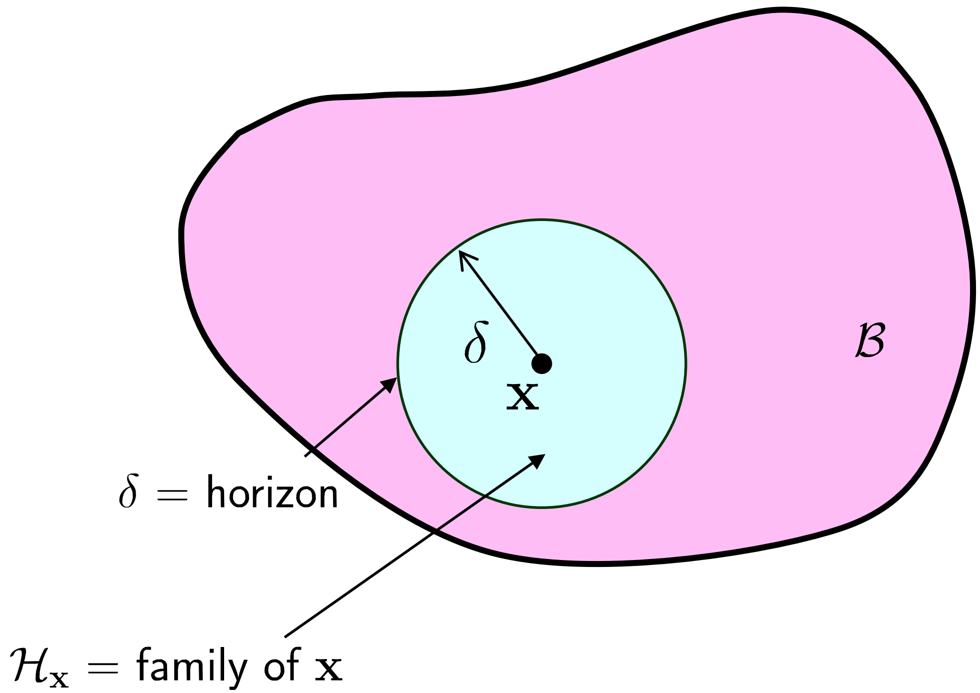
$$a^i(X) = b^i_m(X) + f^i(X) \qquad (\text{N3})$$

which (see also Appendices) is exactly the starting point of modern peridynamics.

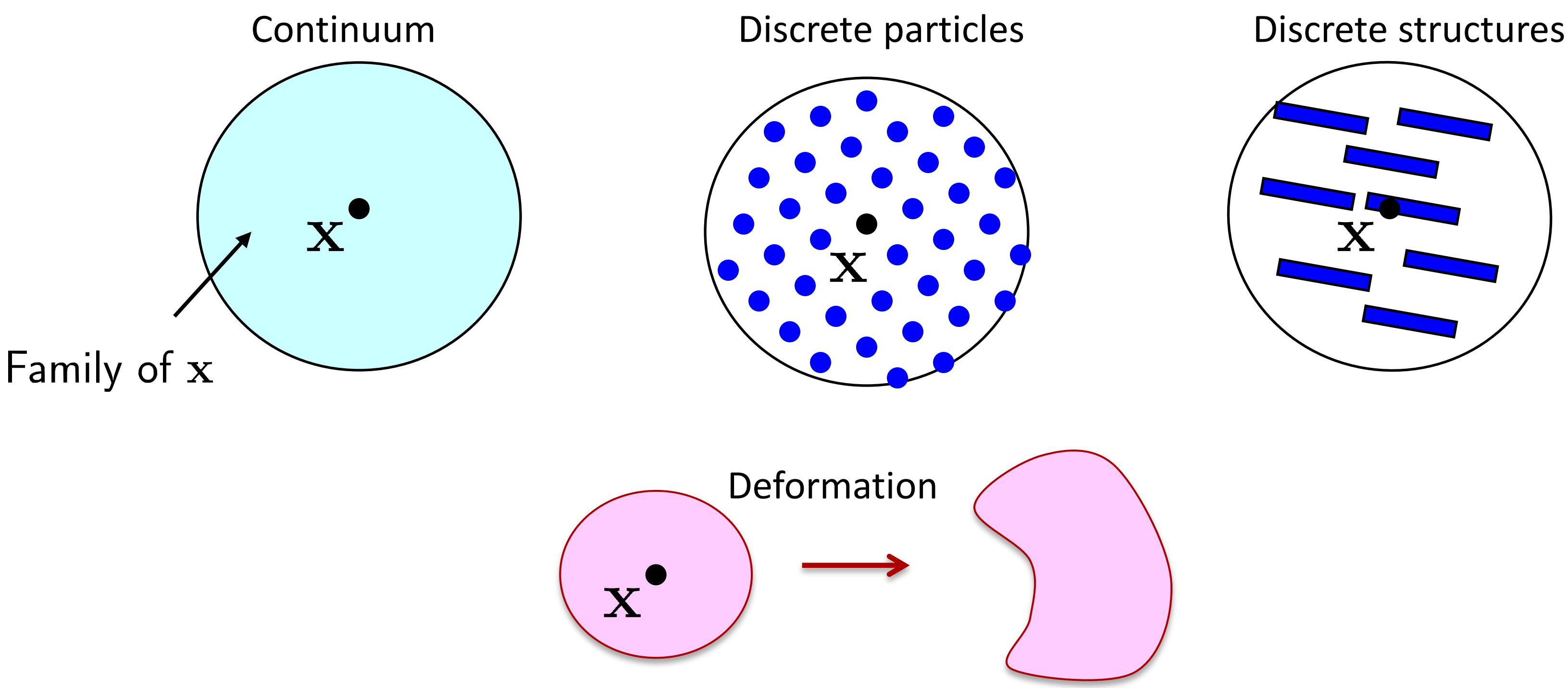
These authors claim that Piola (~1860) should be credited with peridynamics. What is the right response?

# Peridynamics basics: Horizon and family

- Any point $\mathbf{x}$ interacts directly with other points within a distance $\delta$ called the "horizon."
- The material within a distance $\delta$ of $\mathbf{x}$ is called the "family" of $\mathbf{x}$, $\mathcal{H}_{\mathbf{x}}$.



# Point of departure: Strain energy at a point

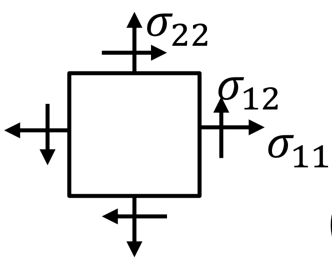


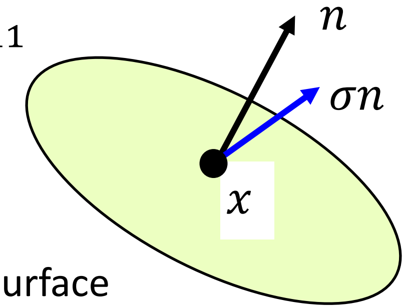

- Key assumption: the strain energy density at $\mathbf{x}$ is determined by the deformation of its family.

# The nature of internal forces

## Standard theory

Stress tensor field
(assumes continuity of forces)



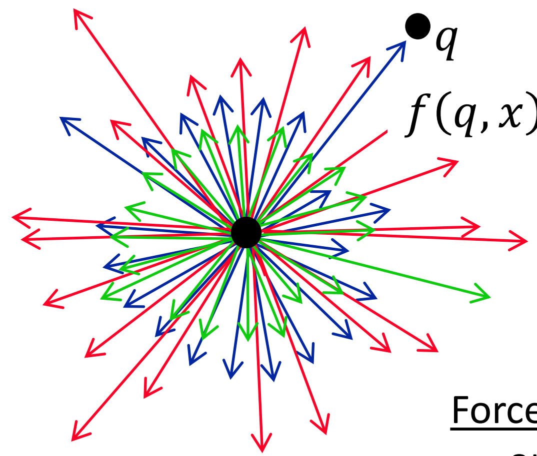

Stress tensor maps surface normal vectors onto surface forces

$$\rho\ddot{u}(x,t) = \nabla \cdot \sigma(x,t) + b(x,t)$$

Differentiation of surface forces

## Peridynamics

Bond forces between neighboring points
(allowing discontinuity)




Force state maps bonds onto bond forces

$$\rho\ddot{u}(x,t) = \int_{H_x} f(q,x)dV_q + b(x,t)$$

Summation over bond forces

# States:
# Objects that keep track of families

- A *state* is a mapping whose domain is a family.

$$\underline{A}\langle\xi\rangle = \text{something}$$

  where $\xi$ is a bond in a family $\mathcal{H}$.

- Famous states: Deformation state...

$$\underline{\mathbf{Y}}[\mathbf{x}]\langle\mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x}) = \text{deformed image of the bond}$$

  Force state...

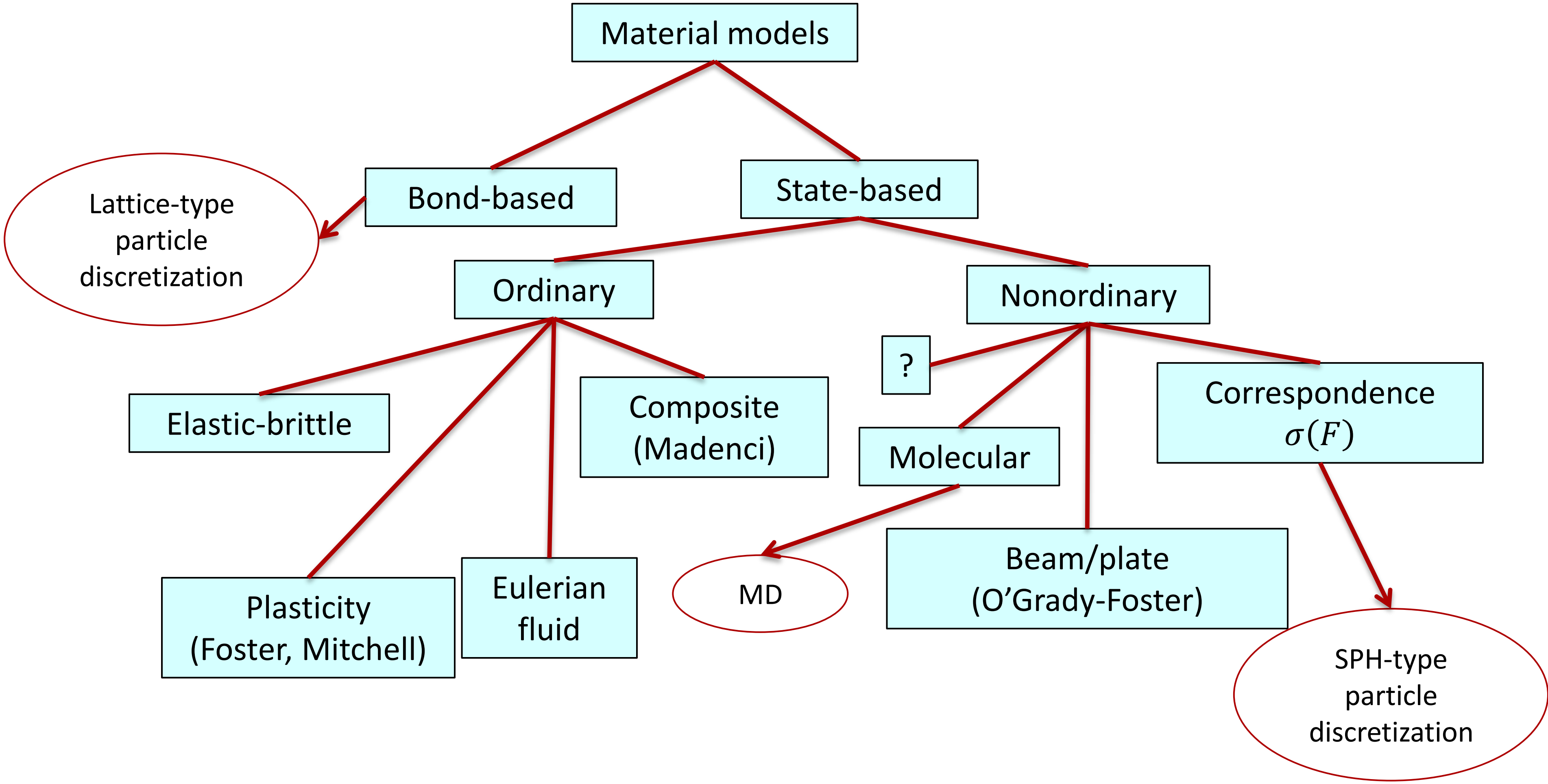
$$\underline{\mathbf{T}}[\mathbf{x}]\langle\mathbf{q}-\mathbf{x}\rangle = \mathbf{t}(\mathbf{q},\mathbf{x}) = \text{force density within a bond}$$

- Dot product of states $\underline{A}$ and $\underline{B}$:

$$\underline{A} \bullet \underline{B} = \int_{\mathcal{H}} \underline{A}\langle\xi\rangle \underline{B}\langle\xi\rangle \, d\xi.$$

# Types of material models

- A material model determines the bond forces in the family according to the deformation of the family.



# Peridynamic vs. local equations

- Peridynamic theory is similar in structure to the local theory but uses nonlocal operators.

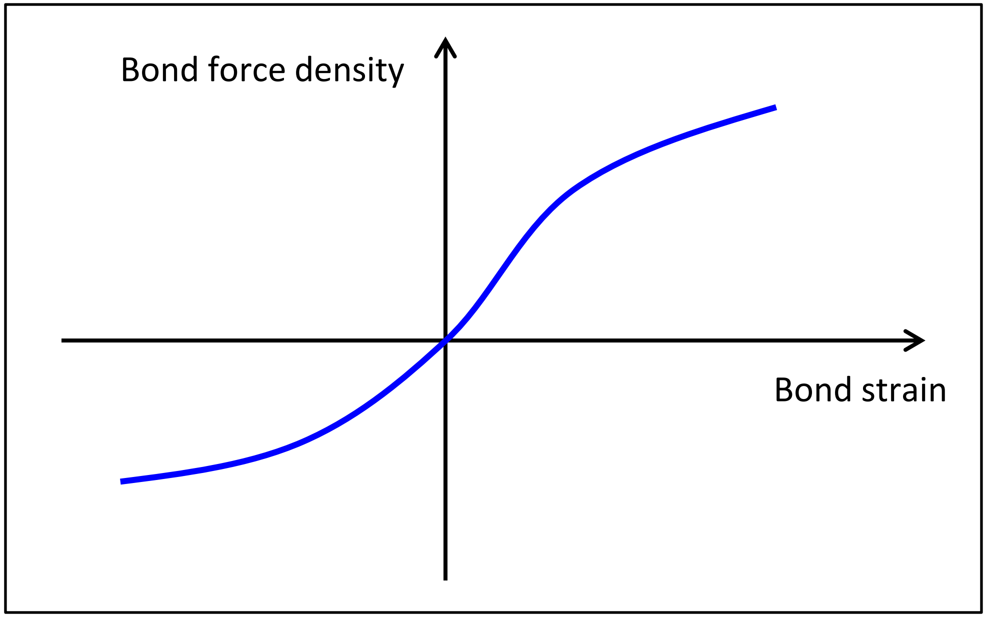
State notation: $\underline{\text{State}}\langle\text{bond}\rangle = \text{vector}$

| *Relation* | *Peridynamic theory* | *Standard theory* |
|---|---|---|
| Kinematics | $\underline{\mathbf{Y}}\langle\mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x})$ | $\mathbf{F}(\mathbf{x}) = \dfrac{\partial \mathbf{y}}{\partial \mathbf{x}}(\mathbf{x})$ |
| Linear momentum balance | $\rho\ddot{\mathbf{y}}(\mathbf{x}) = \int_{\mathcal{H}} \big(\mathbf{t}(\mathbf{q},\mathbf{x}) - \mathbf{t}(\mathbf{x},\mathbf{q})\big)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x})$ | $\rho\ddot{\mathbf{y}}(\mathbf{x}) = \nabla\cdot\boldsymbol{\sigma}(\mathbf{x}) + \mathbf{b}(\mathbf{x})$ |
| Constitutive model | $\mathbf{t}(\mathbf{q},\mathbf{x}) = \underline{\mathbf{T}}\langle\mathbf{q}-\mathbf{x}\rangle, \qquad \underline{\mathbf{T}} = \hat{\underline{\mathbf{T}}}(\underline{\mathbf{Y}})$ | $\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\mathbf{F})$ |
| Angular momentum balance | $\int_{\mathcal{H}} \underline{\mathbf{Y}}\langle\mathbf{q}-\mathbf{x}\rangle \times \underline{\mathbf{T}}\langle\mathbf{q}-\mathbf{x}\rangle\, dV_{\mathbf{q}} = \mathbf{0}$ | $\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$ |
| Elasticity | $\underline{\mathbf{T}} = W_{\underline{\mathbf{Y}}}$ (Fréchet derivative) | $\boldsymbol{\sigma} = W_{\mathbf{F}}$ (tensor gradient) |
| First law | $\dot{\varepsilon} = \underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} + q + r$ | $\dot{\varepsilon} = \boldsymbol{\sigma}\cdot\dot{\mathbf{F}} + q + r$ |

$$\underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} := \int_{\mathcal{H}} \underline{\mathbf{T}}\langle\boldsymbol{\xi}\rangle \cdot \dot{\underline{\mathbf{Y}}}\langle\boldsymbol{\xi}\rangle\, dV_{\boldsymbol{\xi}}$$

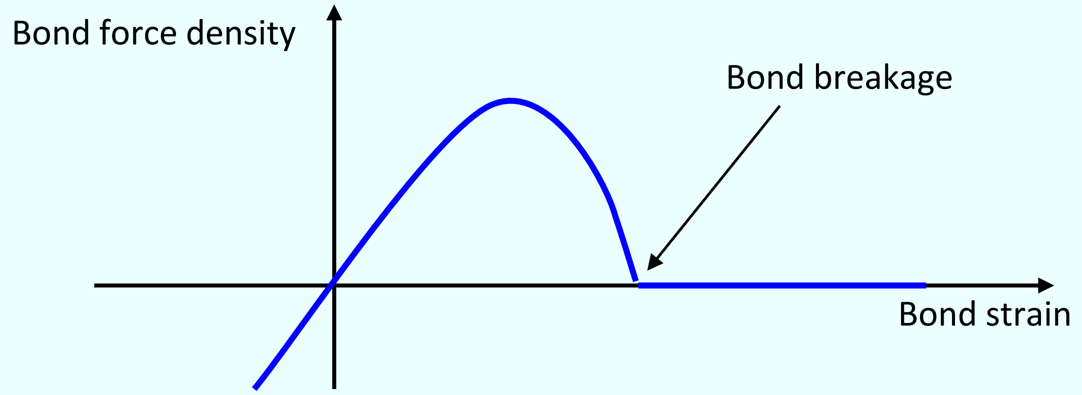
# Bond based materials

- If each bond response is independent of the others, the resulting material model is called bond-based.
- The material model is then simply a graph of bond force density vs. bond strain.
- Main advantage: simplicity.
- Main disadvantage: restricts the material response.
  - Poisson ratio always = 1/4.



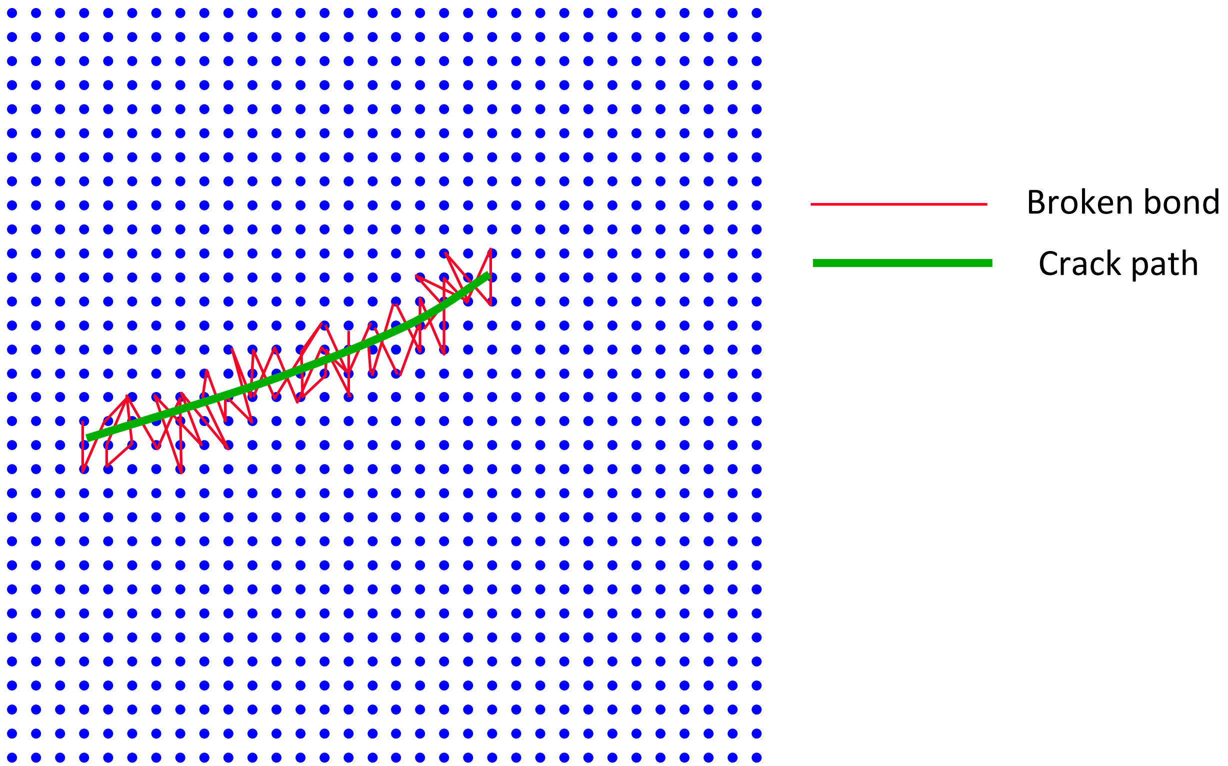
# Damage due to bond breakage

- Recall: each bond carries a force.
- Damage is implemented at the bond level.
  - Bonds break irreversibly according to some criterion.
  - Broken bonds carry no force.
- Examples of criteria:
  - Critical bond strain (brittle).
  - Hashin failure criterion (composites).
  - Gurson (ductile metals).




Critical bond strain damage model

# Bond breakage leads to autonomous crack growth




- When a bond breaks, its load is shifted to its neighbors, leading to progressive failure.

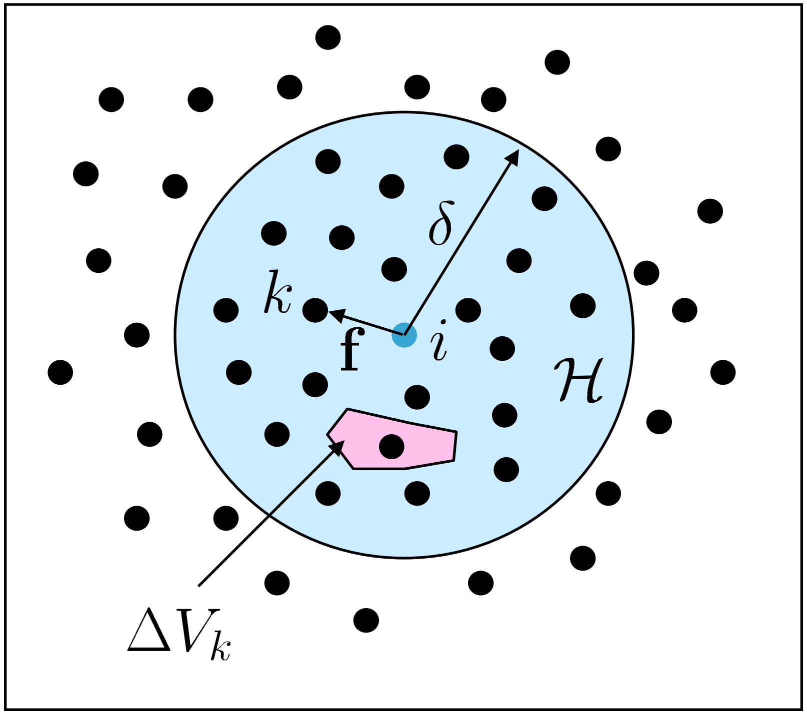
# EMU numerical method

- Integral is replaced by a finite sum: resulting method is meshless and Lagrangian.

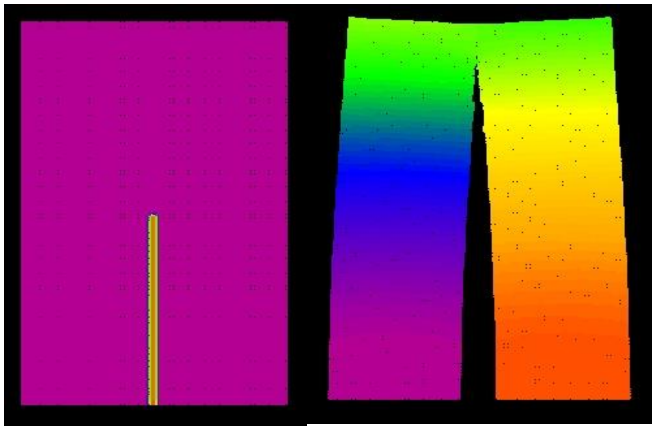
$$\rho \ddot{\mathbf{y}}(\mathbf{x}, t) = \int_{\mathcal{H}} \mathbf{f}(\mathbf{x}', \mathbf{x}, t)\, dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x}, t)$$

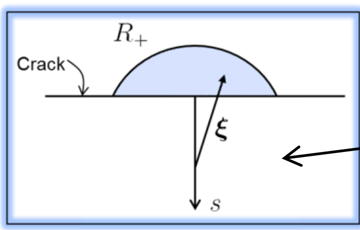
$$\downarrow$$

$$\rho \ddot{\mathbf{y}}_i^n = \sum_{k \in \mathcal{H}} \mathbf{f}(\mathbf{x}_k, \mathbf{x}_i, t)\, \Delta V_k + \mathbf{b}_i^n$$



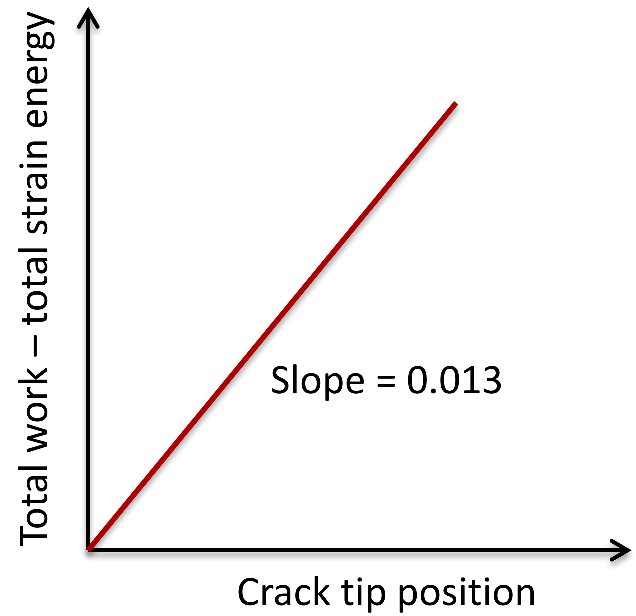
# Constant bond failure strain reproduces the Griffith crack growth criterion






From bond properties, energy release rate should be

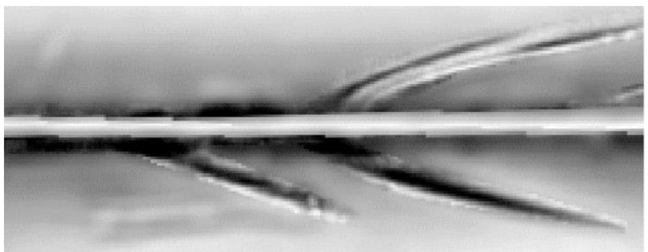
$G = 0.013$



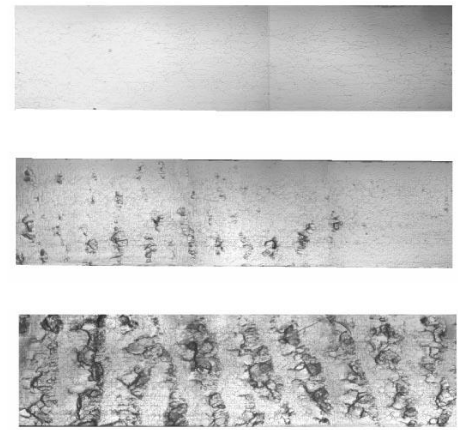

- This confirms that the energy consumed per unit crack growth area equals the expected value from bond breakage properties.

# Dynamic fracture in PMMA

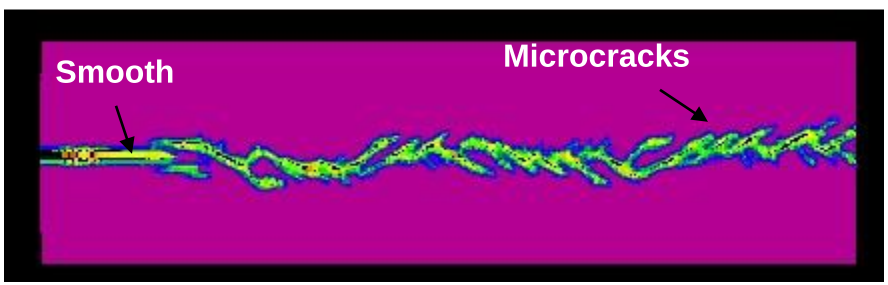
- Peridynamic simulation shows crack surface features related to fracture instability.
- These are difficult to reproduce with standard methods.



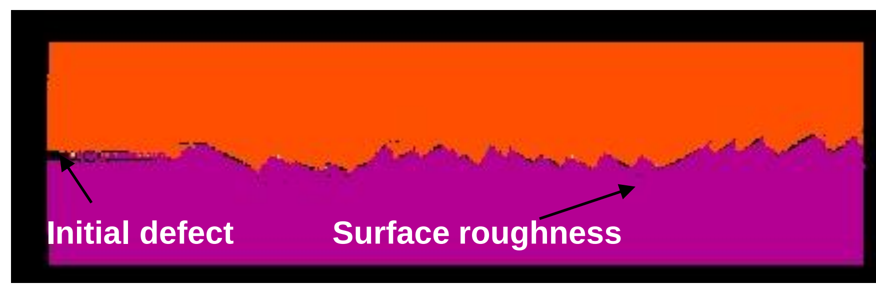
Microbranching



Mirror-mist-hackle transition*




EMU damage

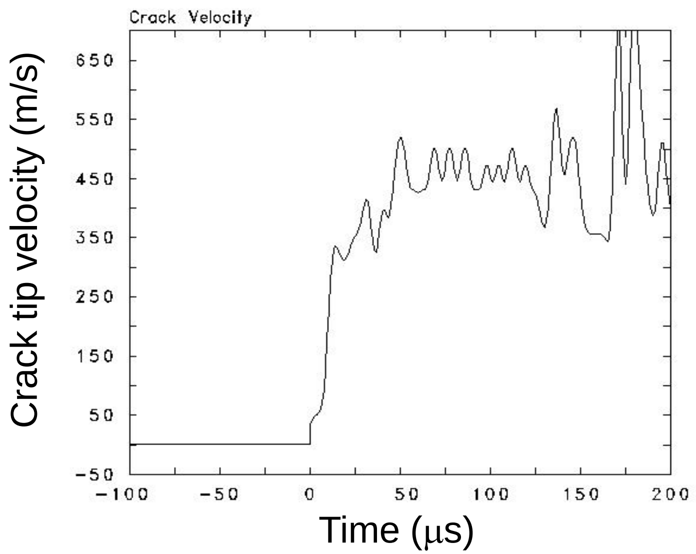



EMU crack surfaces

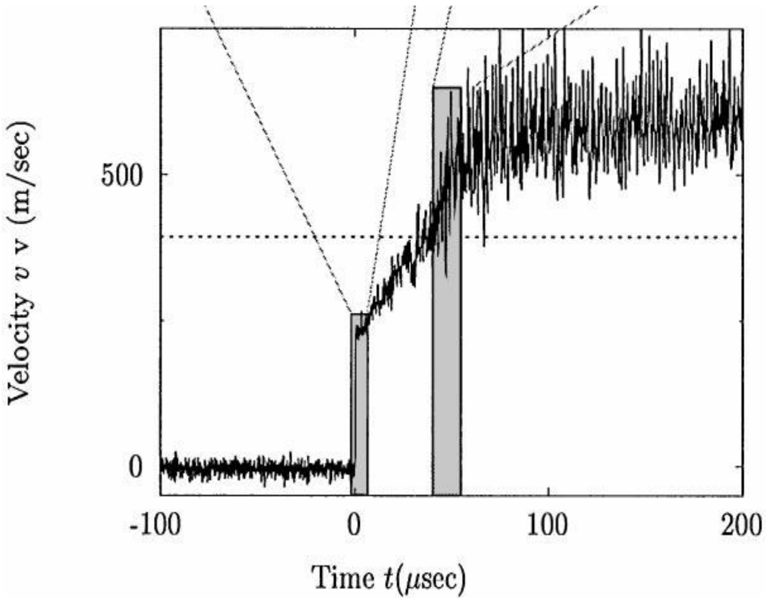
* J. Fineberg & M. Marder, *Physics Reports* 313 (1999) 1-108

# Dynamic fracture in PMMA, ctd: Crack tip velocity

- Simulation reproduces the main features of dynamic crack velocity history.




EMU



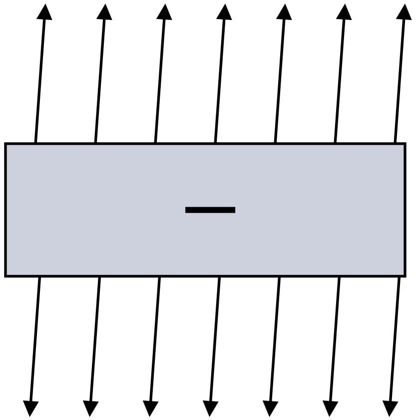

Experiment*

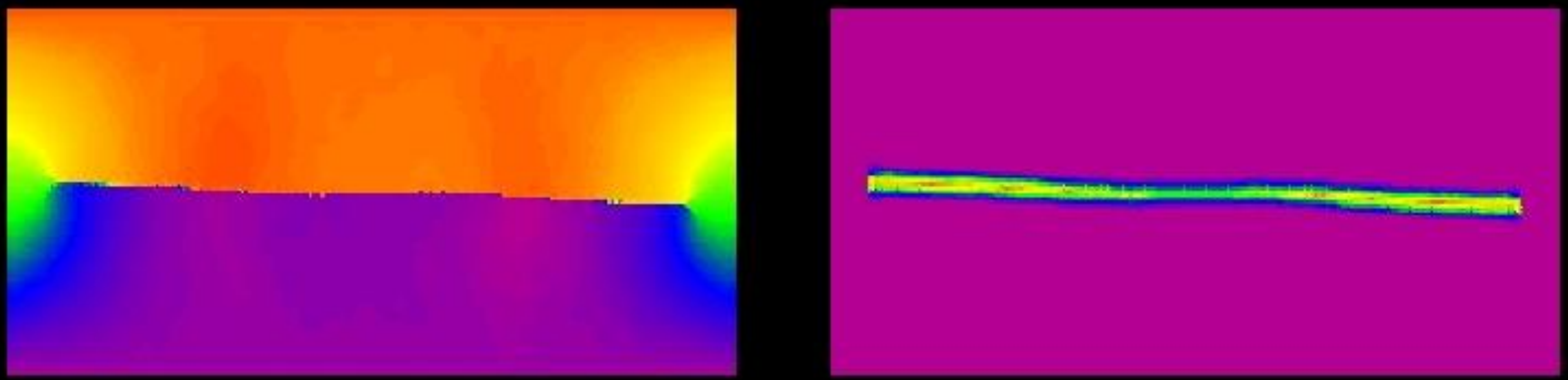
* J. Fineberg & M. Marder, *Physics Reports* 313 (1999) 1-108

# Benefits of autonomous fracture: Mesh effects are small

- Plate with a pre-existing defect is subjected to prescribed boundary velocities.
- These BC correspond to mostly Mode-I loading with a little Mode-II.

$$\dot{\varepsilon} = (0.25\mathrm{s}^{-1}) \begin{bmatrix} 0 & 0.1 \\ 0 & 1 \end{bmatrix}$$

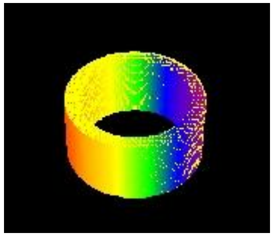




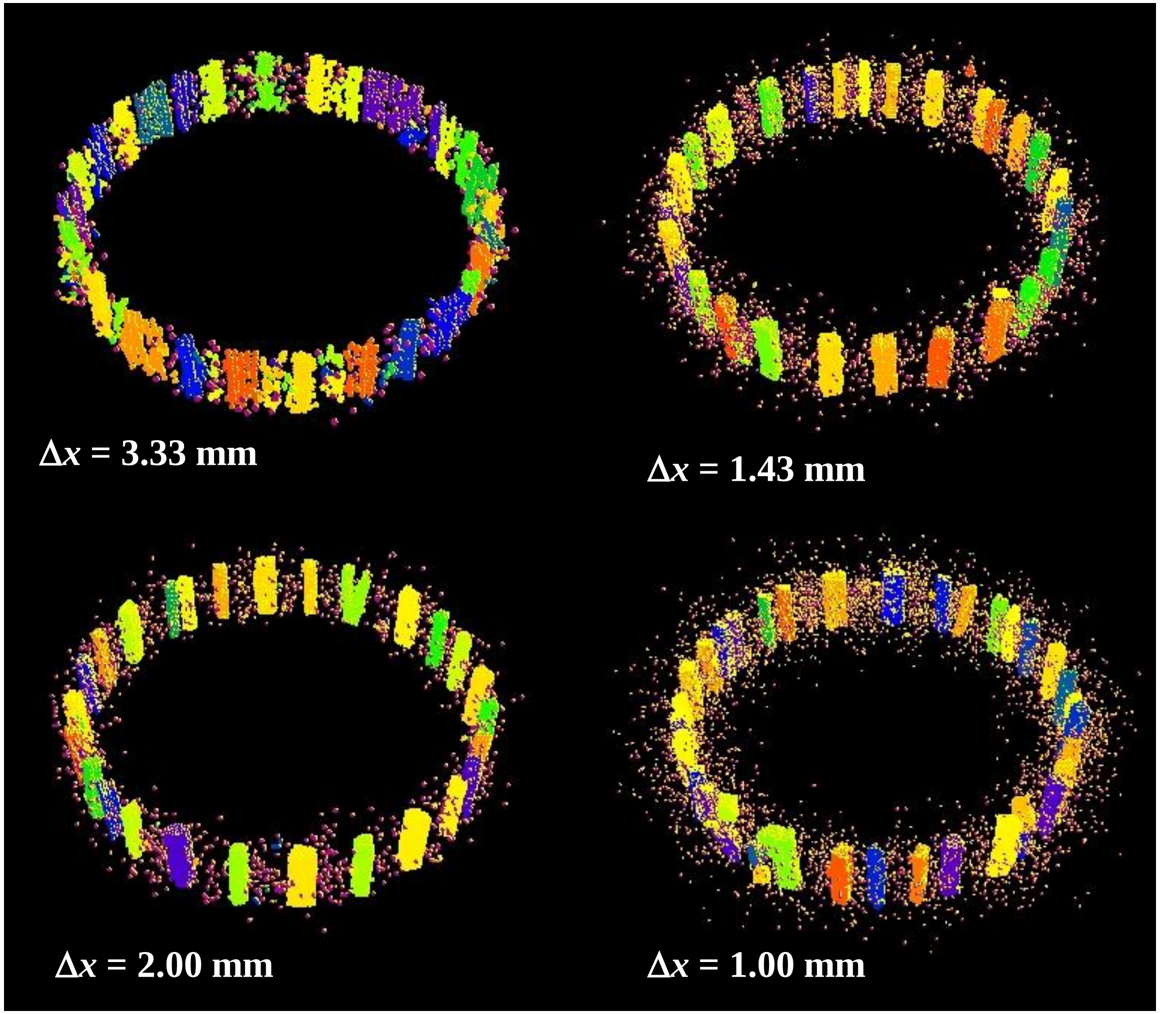
Contours of vertical displacement

Contours of damage

# Fragmentation is not strongly dependent on mesh spacing



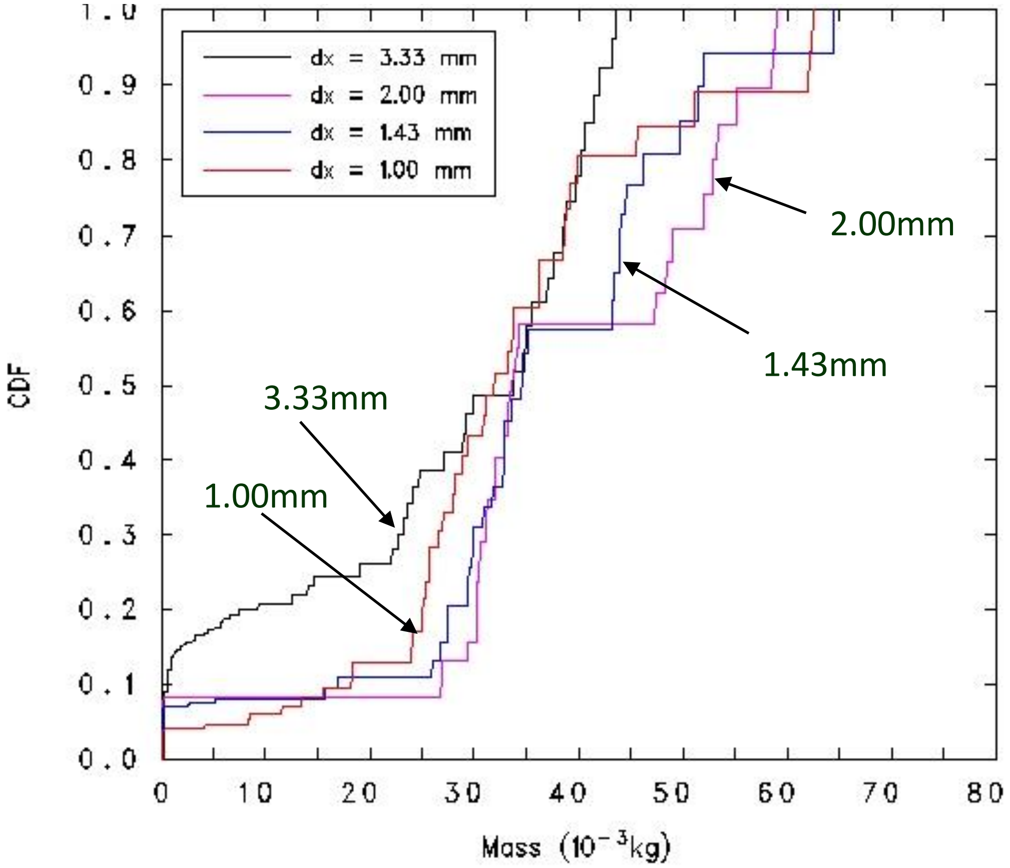
Brittle ring with initial radial velocity




$$\delta = 3\Delta x$$

# Fragment distribution converges

Cumulative distribution function for 4 grid spacings

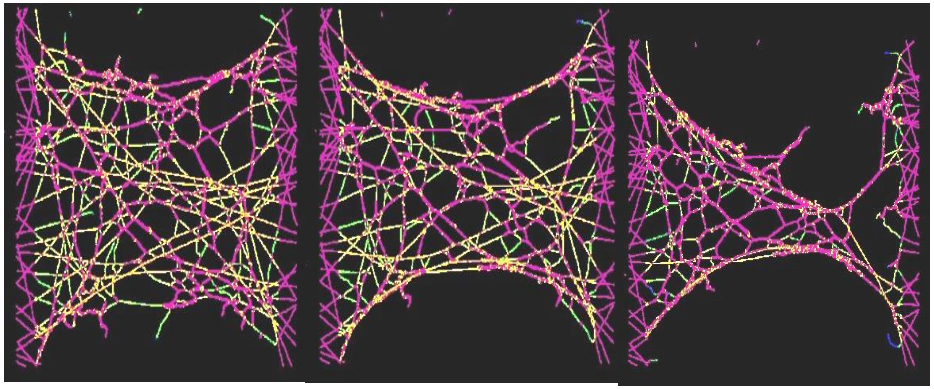



| $\Delta x$ (mm) | Mean fragment mass (g) |
|---|---|
| **3.33** | **27.1** |
| **2.00** | **37.8** |
| **1.43** | **35.9** |
| **1.00** | **33.5** |

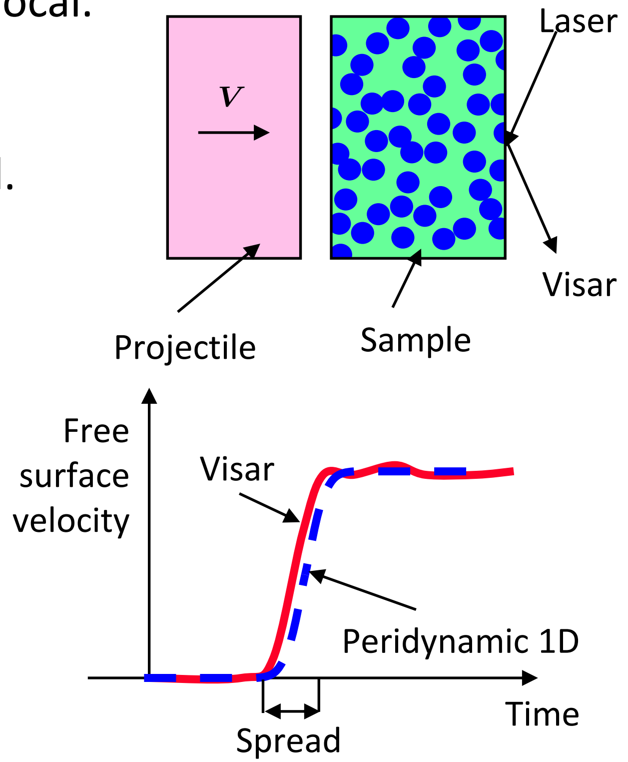
Solution appears essentially converged

# Importance of nonlocality

- Peridynamics is consistent with all laws of classical physics.
- It uses nonlocal interactions between material points.
  - The Cauchy theory is local.
  - Locality is often mistakenly assumed to be a law of physics.
- Molecular scale, nanoscale interactions are always nonlocal.
- Complex fluids are nonlocal.
- Any heterogeneous medium is nonlocal.
- Any discretized model of the local equations is nonlocal.



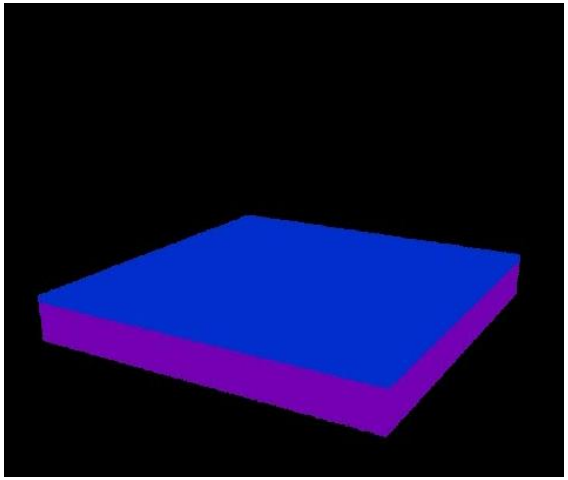
Peridynamic model of a nanofiber membrane
(F. Bobaru, Univ. of Nebraska)



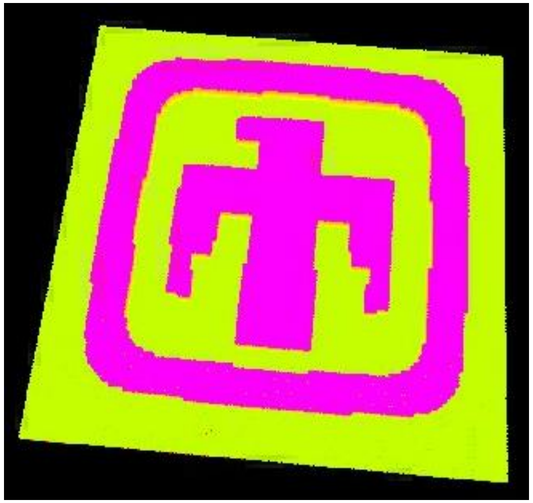

Local model would predict zero spread.

# Method reveals subtleties in the mechanics of thin structures

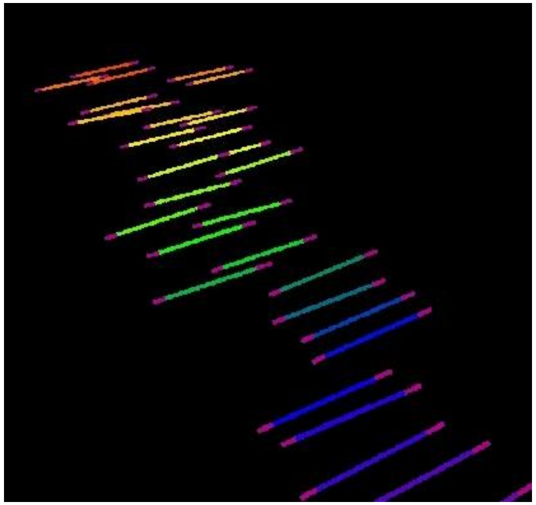
- Autonomous crack growth and long-range forces are crucial.



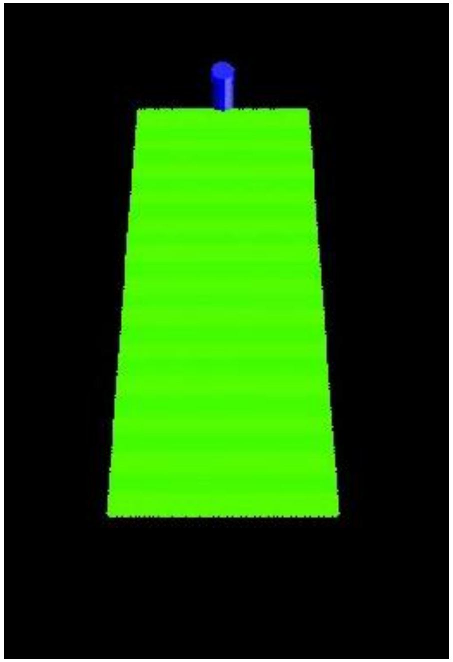
Membrane decohesion



Aging



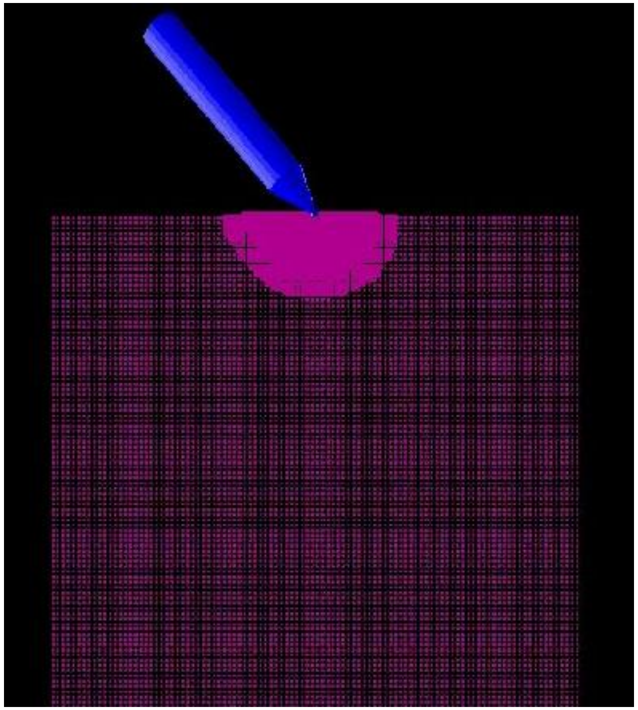
Self assembly



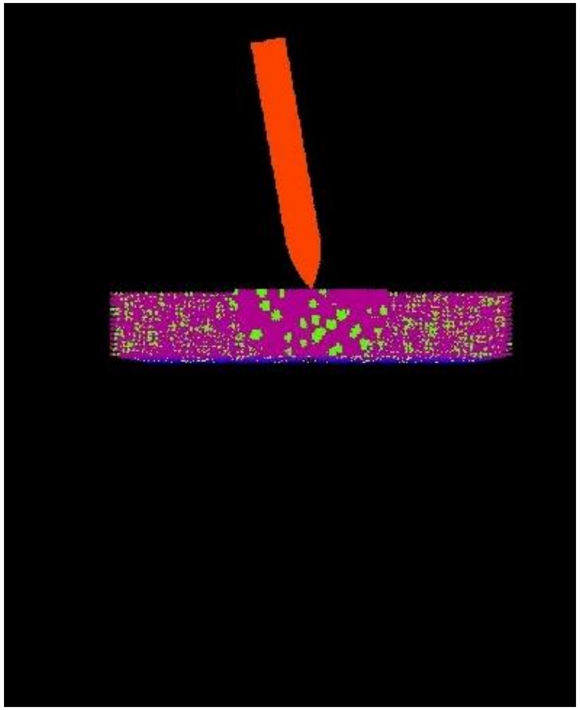
Oscillatory crack path

# Examples: Impact and penetration (JMP)

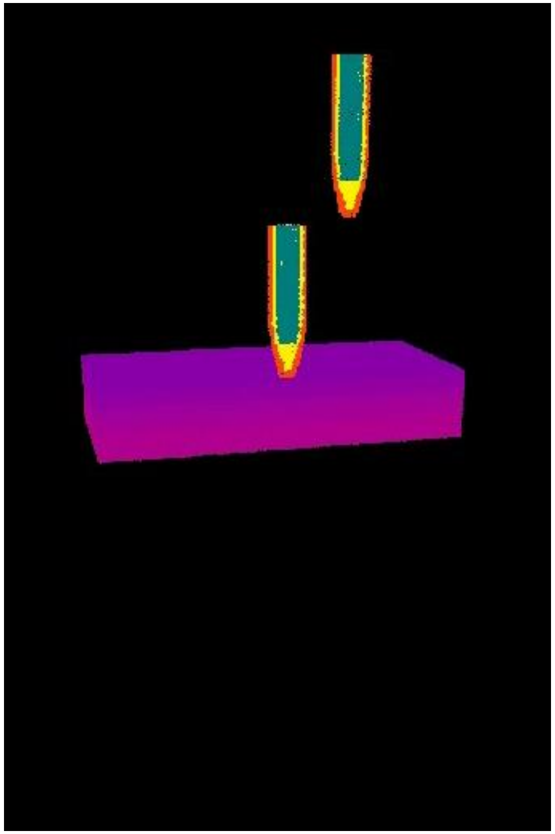
VIDEOS



Ricochet from heterogeneous target



Tail slap in a deformable penetrator



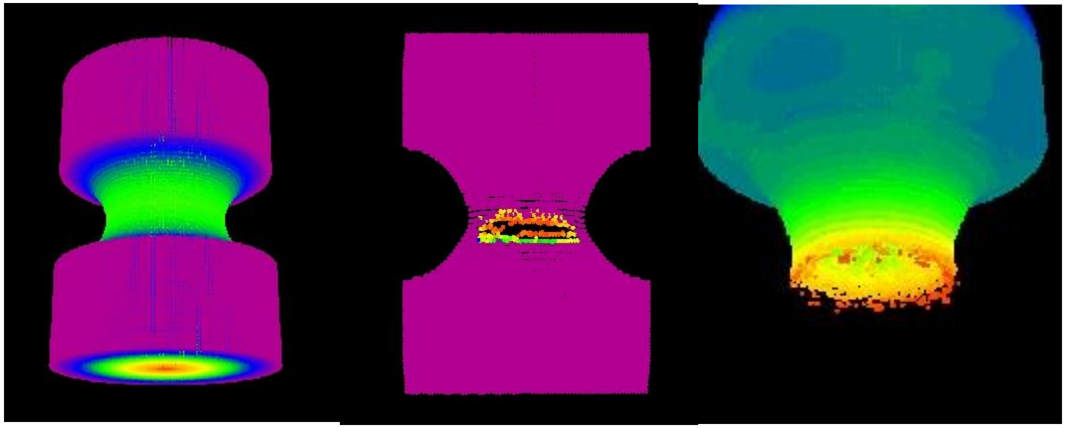
Small arms multihit

# Ductile failure in peridynamics

- Many problems in engineering involve failure of ductile metals.
  - But peridynamics has mostly been applied to brittle fracture.
- Incorporate Wellman's (1500) Tearing Parameter Model (TPM).
  - TPM captures the main effect (tensile pressure) in a simple way.
  - TPM was fairly successful in the X-prize (FEM + element death) .
  - Peridynamics avoids the need for element death.

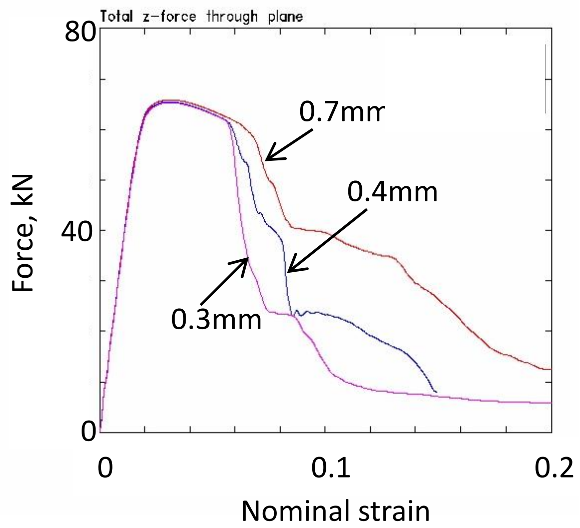
Cup & cone failure
(R. W. Hertzberg)

Emu+TPM simulation of a notched tensile test in 6061 Al



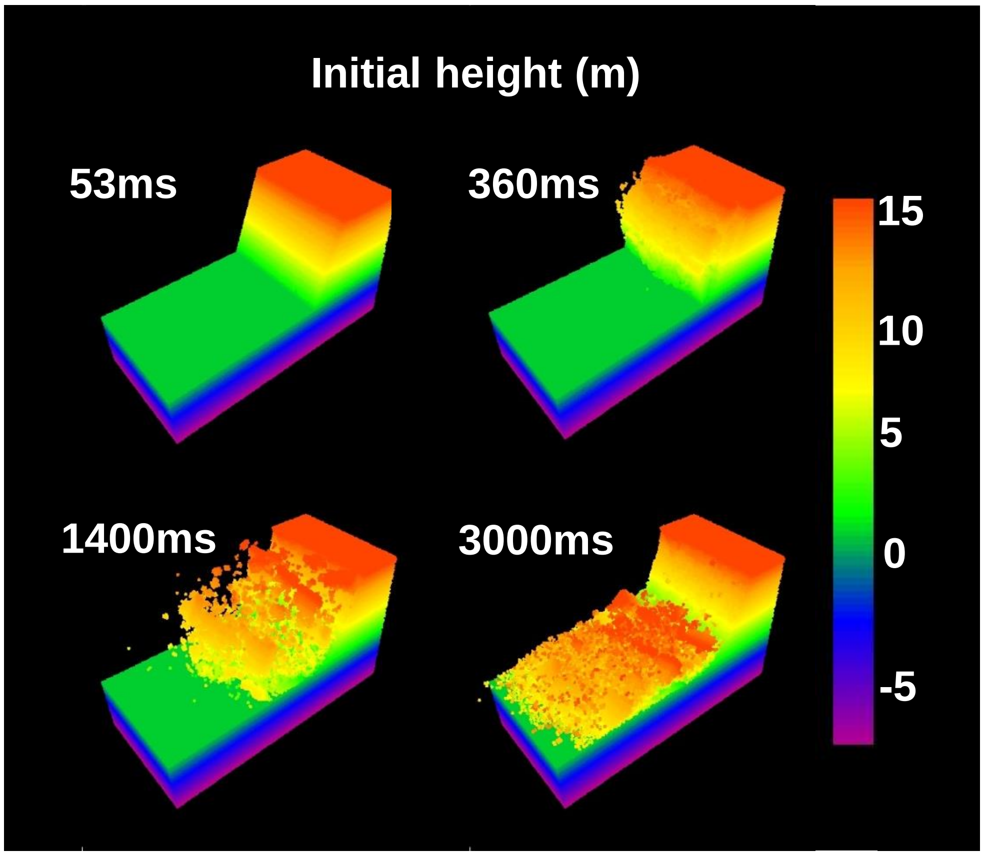
Initial | Failure starts in the interior | Curved fracture surface

Convergence



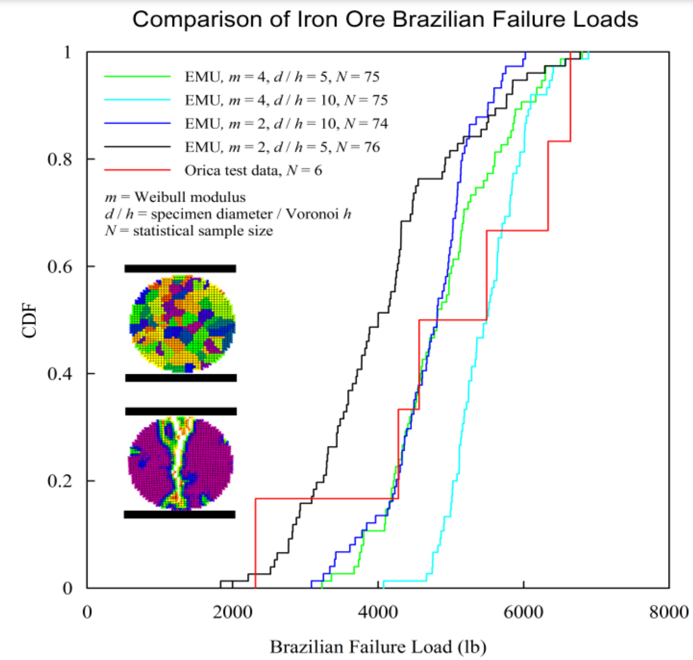
# Bench blasting

- Peridynamics correctly reproduces fragment size and velocity distributions in rock blasting (Orica USA Corp).




Predicted motion of fragments at four times

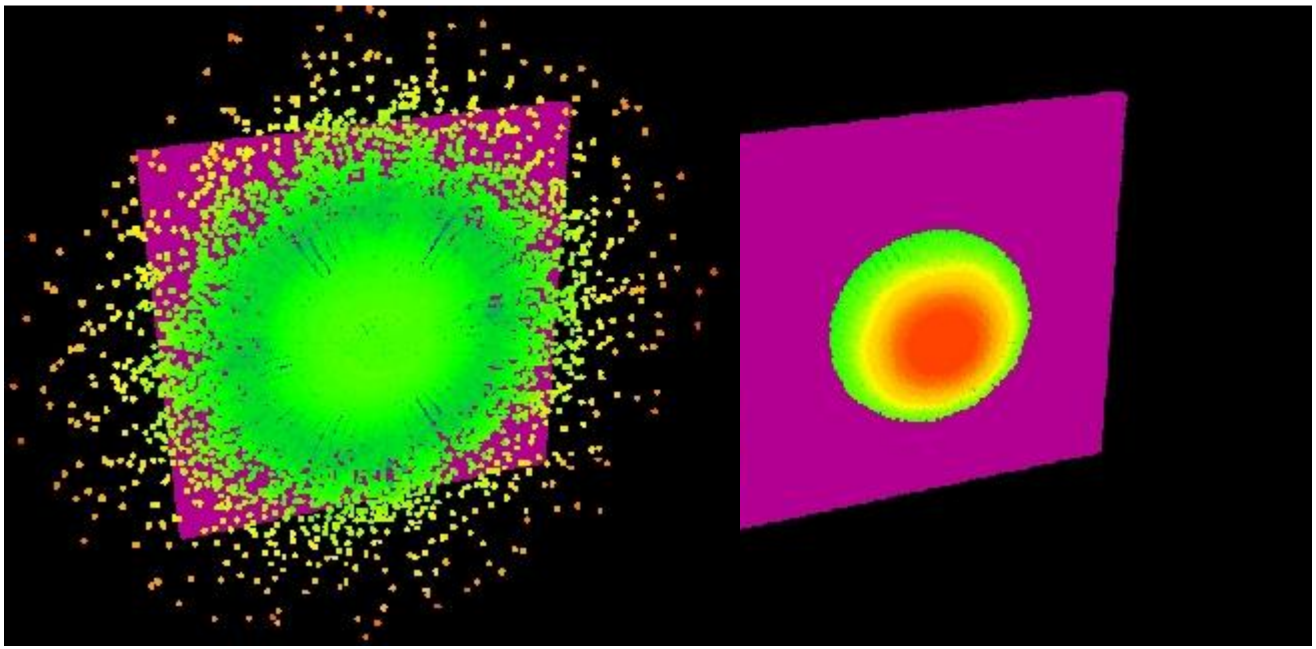
Actual bench blasting




J. Bishop & J. Holland (1500)

# Fluid-structure interaction: Bird strike

- Bird simulant (gelatin) vs. heavy plate
- A material model that includes Eulerian fluid response and Lagrangian bond forces helps reduce the “spray” that is sometimes seen with SPH.



PD – Fluid only

PD – Fluid + bond forces

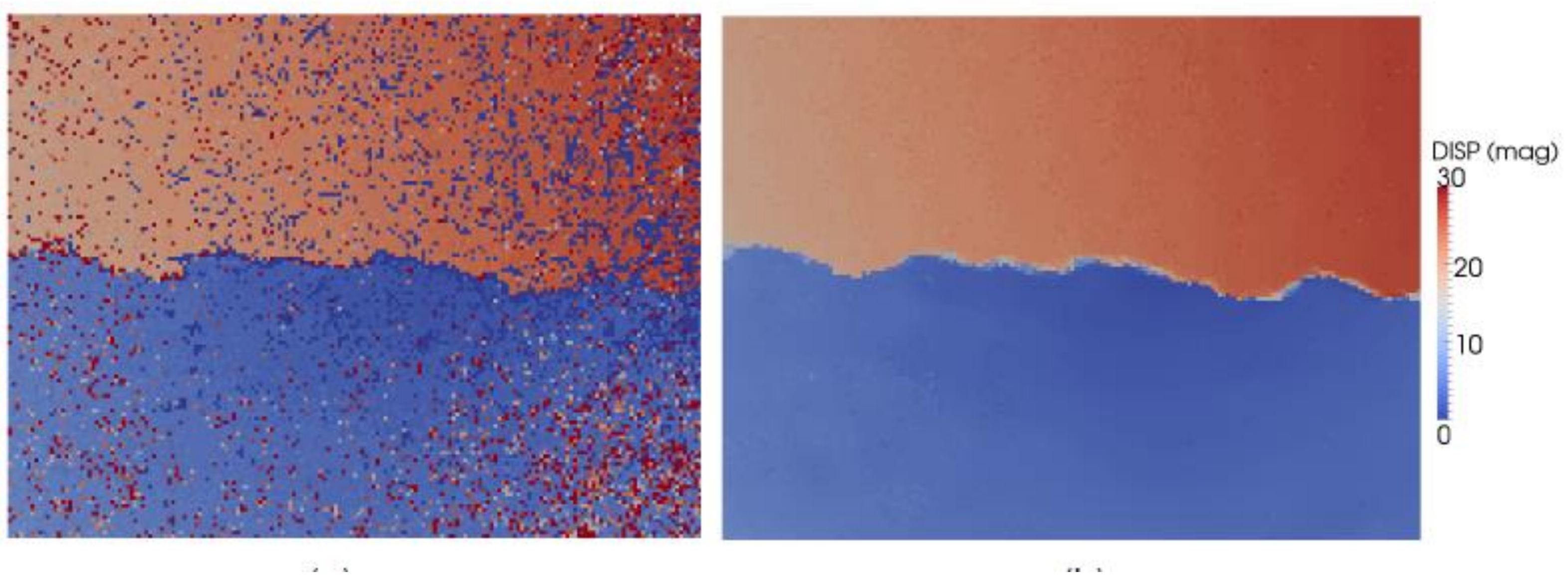
Test - LG 997

SPH

Olivares, NIS Document 09-039 (2010)

# DIC data processing algorithm based on peridynamics*

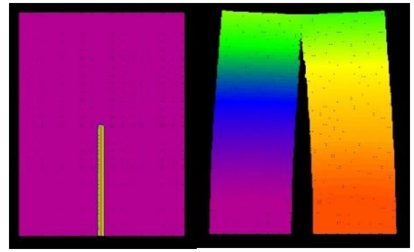
- A new algorithm using peridynamics helps interpret Digital Image Correlation data when discontinuities are present.
- Reference: D. Turner, "Peridynamics-Based Digital Image Correlation Algorithm Suitable for Cracks and Other Discontinuities." J. Eng. Mech. (2014)

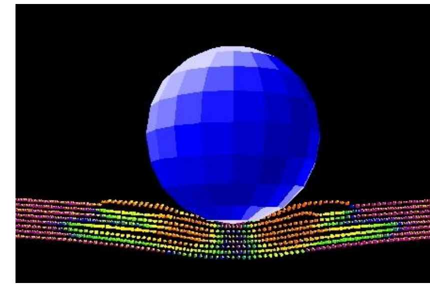


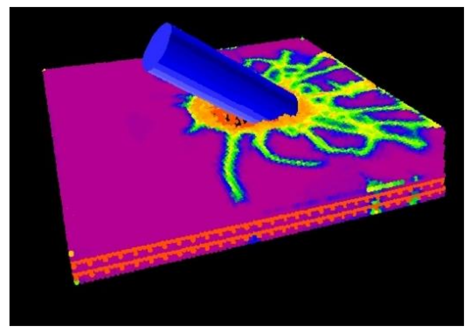

*Dan Turner and Rich Lehoucq.

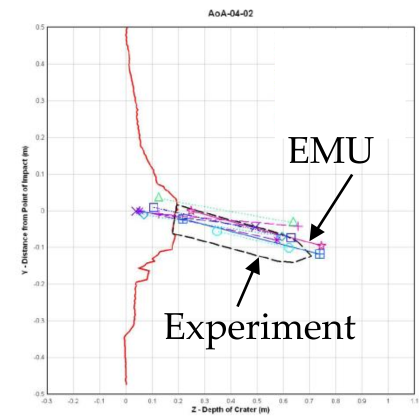
# Examples of validation for peridynamics

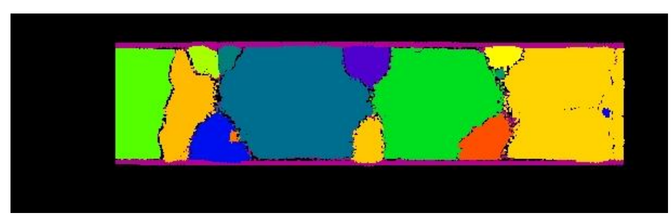
- Single crack brittle energy balance
- 3-point bend test
- Dynamic fracture
  - Crack growth velocity
  - Trajectory
  - Branching
- Impact into concrete and aluminum
  - Residual velocity
  - Penetration depth
  - Crater size
- Fatigue
  - S-N curves for aluminum and epoxy
  - Paris law curves for aluminum
- Composite impact, damage, and fracture
  - Delaminations (compare NDE)
  - Residual strength in OHC, OHT
  - Stress concentration profile in OHT
  - Bird strike loading
  - Lamina tensile fracture





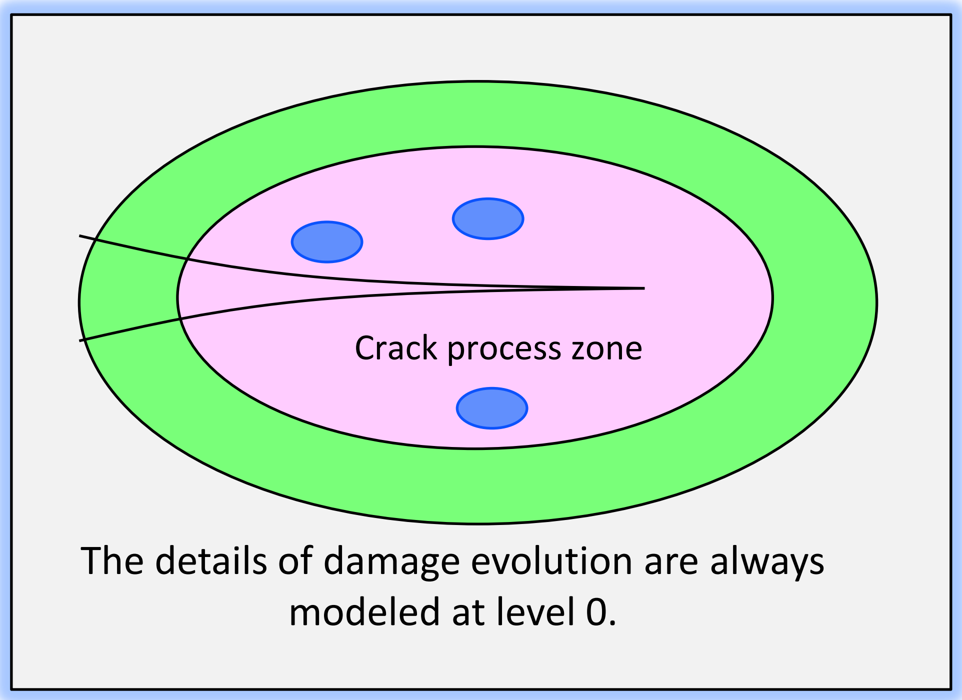




# Concurrent multiscale method for defects

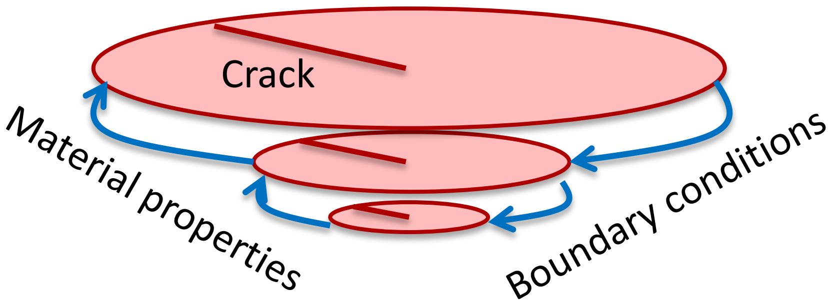
- Apply the best practical physics at the smallest length scale (near a crack tip).
- Scale up hierarchically to larger length scales.
- Each level is related to the one below it by the same equations.
  - Any number of levels can be used.
- Adaptively follow the crack tip.

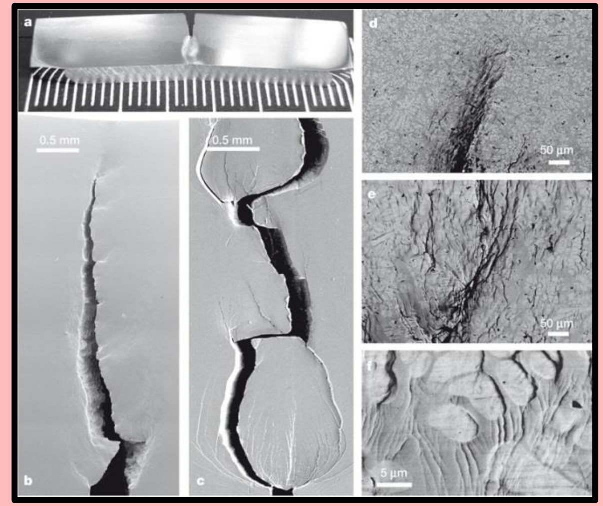


# Multiscale peridynamics helps to reveal the structure of brittle cracks

Sandia National Laboratories

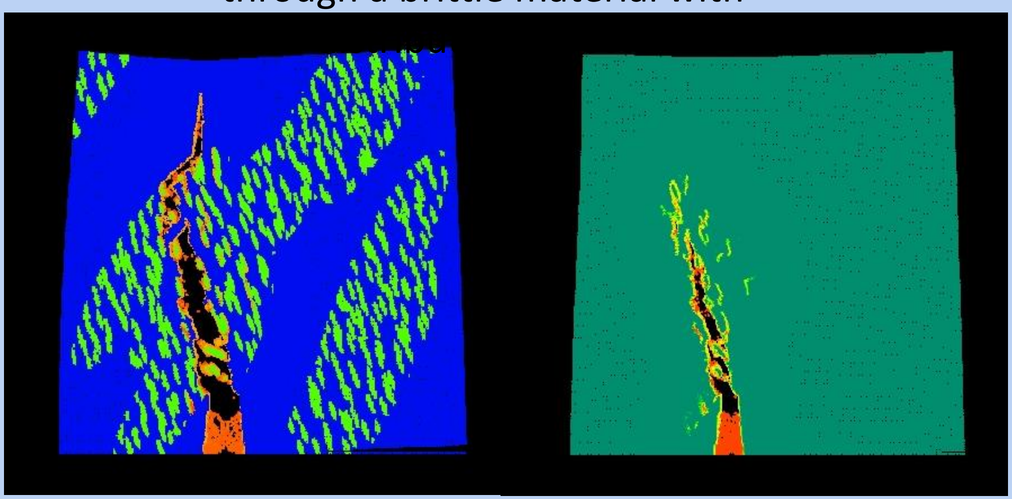
- Material design requires understanding of how morphology at multiple length scales affects strength.
- This is a key to material reliability.



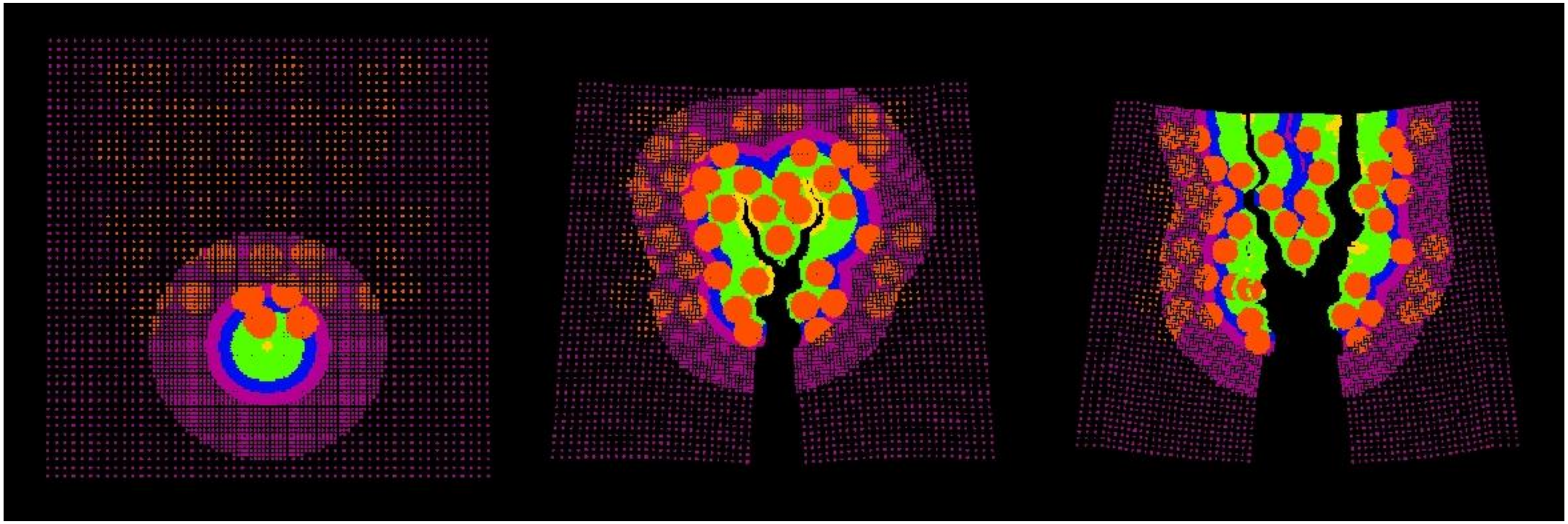

Metallic glass fracture (Hofmann et al, Nature 2008)

Multiscale model of crack growth through a brittle material with

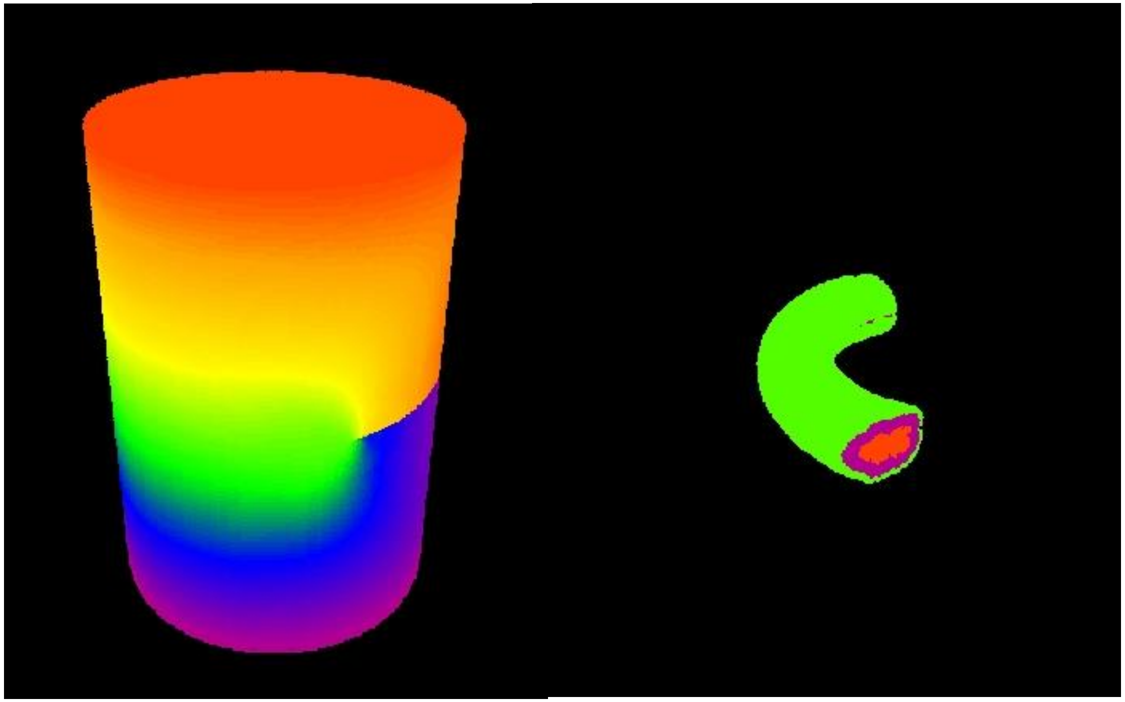


# Branching in a heterogeneous medium

- Multiscale method tracks crack growth between randomly placed hard inclusions.



# Failure of a glass rod in tension

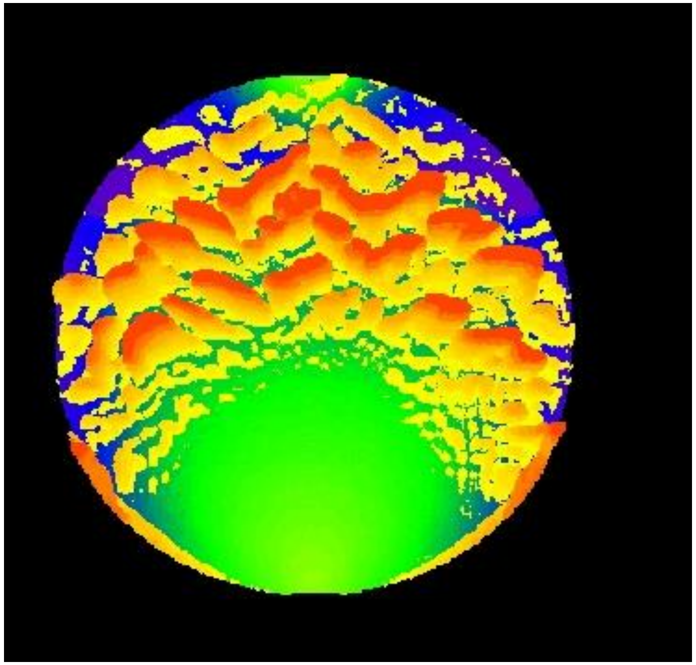


Level 1 displacement　　Level 0 surrounds the crack front

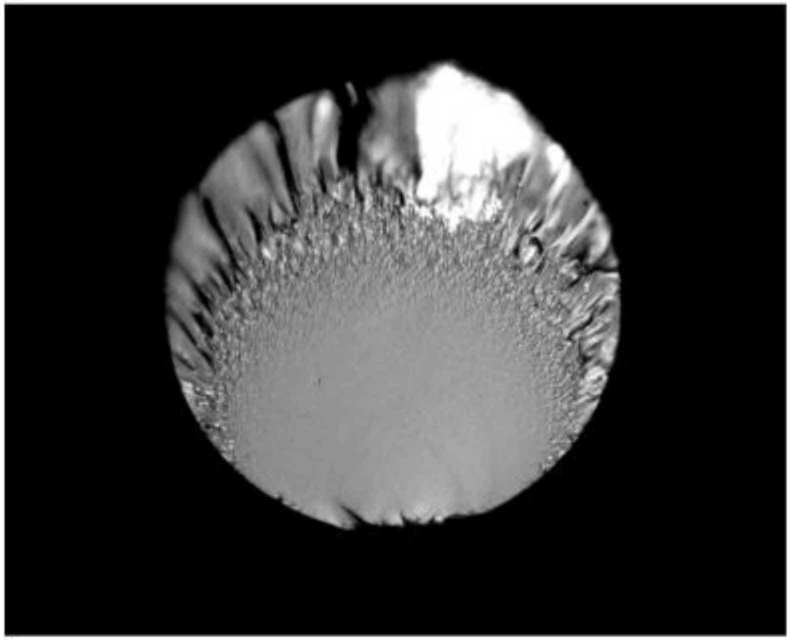
- Level 1 multiscale.
- 20,000,000 level 0 sites (most are never used).
- Level 0 horizon is 25μm.

# Mirror-mist-hackle

- Model predicts roughness and microbranches that increase in size as the crack grows.
- Transition radius decreases as initial stress increases – trend agrees with experiments.



3D peridynamic model



Fracture surface in a glass optical fiber
(Castilone, Glaesemann & Hanson, Proc. SPIE (2002))

# Fatigue model

- The fatigue model specifies how the remaining life of each bond depends on the loading.

$$\frac{d\lambda}{dN}(N) = -A\varepsilon^m$$

where $A$ and $m$ are constants and $\varepsilon$ is the cyclic bond strain.

- The constants are calibrated separately for phases I and II (nucleation and growth).

# Time mapping permits very large $N$

- We can avoid modeling each cycle explicitly.
- Define the *loading ratio* by

$$R = \frac{s^-}{s^+} \qquad \Longrightarrow \qquad \varepsilon = |s^+ - s^-| = |(1-R)s^+|.$$

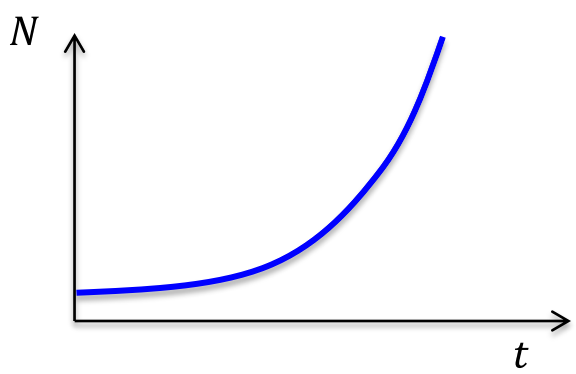
- Map $t$ to $N$:

$$N = e^{t/\tau}$$

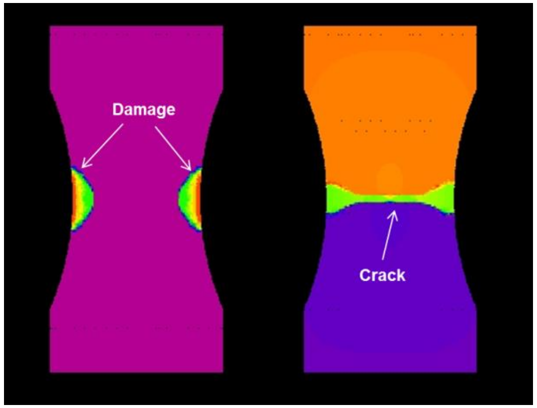
  where $\tau$ is a constant chosen according to convenience.

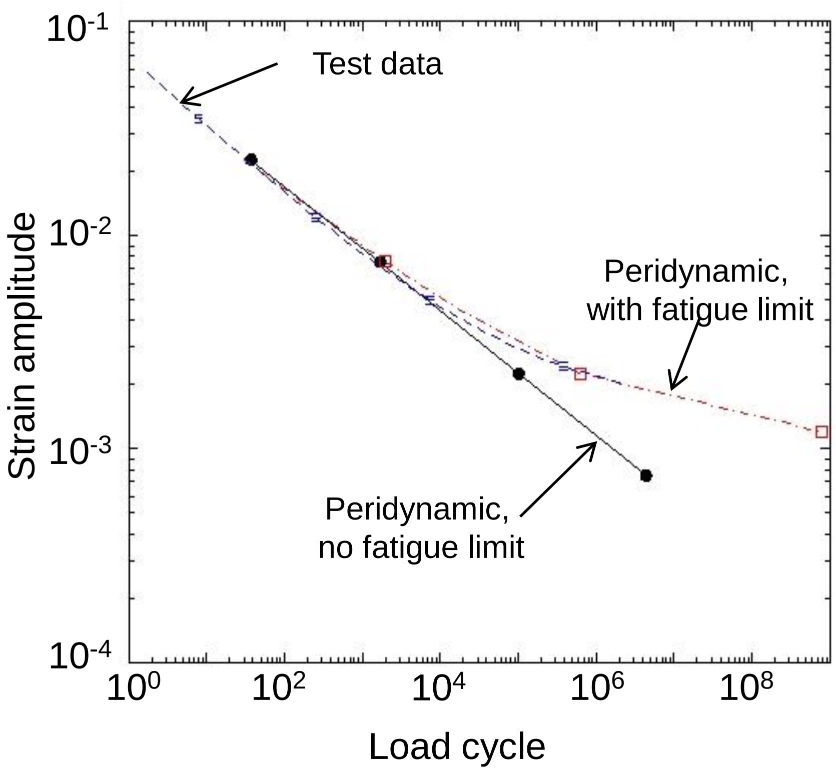
- Fatigue model in terms of $t$ instead of $N$:

$$\frac{d\lambda}{dt} = \frac{d\lambda}{dN}\frac{dN}{dt} = \frac{-|1-R|AN}{\tau}|s^+|^m.$$



# Fatigue nucleation in aluminum alloy




- Model with a fatigue limit:

$$\frac{d\lambda}{dN}(N) = -A\Big(\max(0, \varepsilon - \varepsilon_\infty)\Big)^m$$

Test data: T. Zhao and Y. Jiang. Fatigue of 7075-T651 aluminum alloy. International Journal of Fatigue, 30 (2008)834-849.

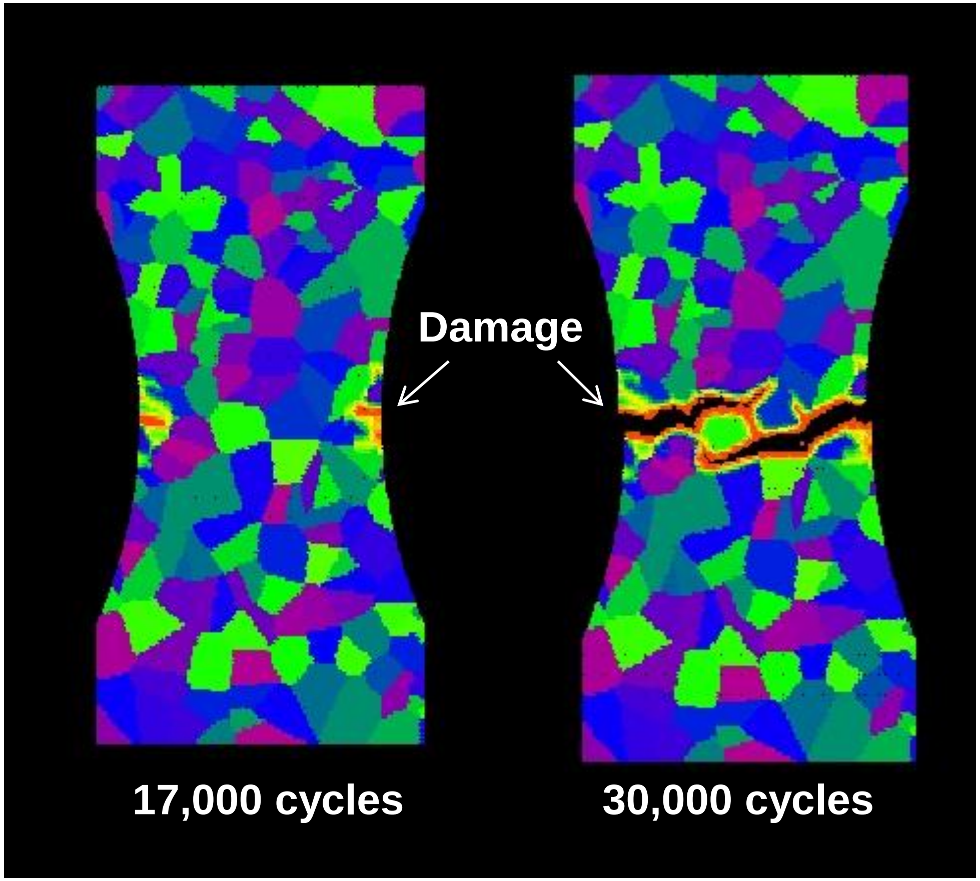
# Mesoscale:
# Fatigue cracks at grain boundaries

- Recall the peridynamic fatigue model: For a given bond,

$$\lambda(0) = 1, \qquad \frac{d\lambda}{dN} = -A\varepsilon^m$$

- Set:

$A = 5$ for bonds within a grain
$A = 50$ for bonds between grains




Fatigue crack growth between grains represented as Voronoi cells

# Peridynamics strengths and weaknesses

**Strengths**

- Offers potentially great generality in fracture modeling.
  - Cracks nucleate and grow spontaneously.
  - Cracks follow from the basic field equations.
- Any material model from the local theory can be used.
  - Plus a lot more!
- Compatible with molecular scale long-range forces.
  - MD is a special case.
  - Cauchy theory is a limiting case.
- Length scale can be exploited for multiscale modeling.

**Weaknesses**

- Slow due to many interactions.
  - Local-nonlocal coupling will help.
  - Need smarter integration methods.
- Surface effects
  - Correction methods are available, none totally satisfactory.
  - PALS material model will help solve this.
- Boundary conditions are different from the local theory.
- Particle discretization has known limitations.
  - FE methods are under development.