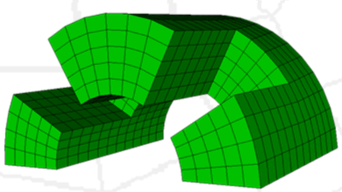
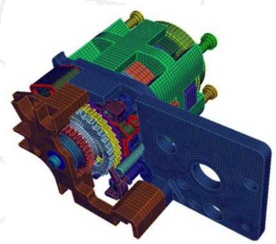
SAND2015-7897C

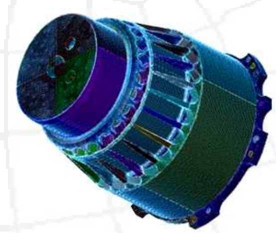
24th International Meshing Roundtable

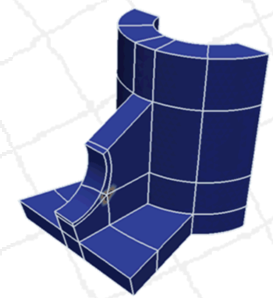
...a U.S. Department of Energy national security laboratory.

Introduction to Quadrilateral and Hexahedral Mesh Generation









Matt Staten

Some slides provided by Steve Owen and Franck Ledoux

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Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.
SAND2015-XXXXX

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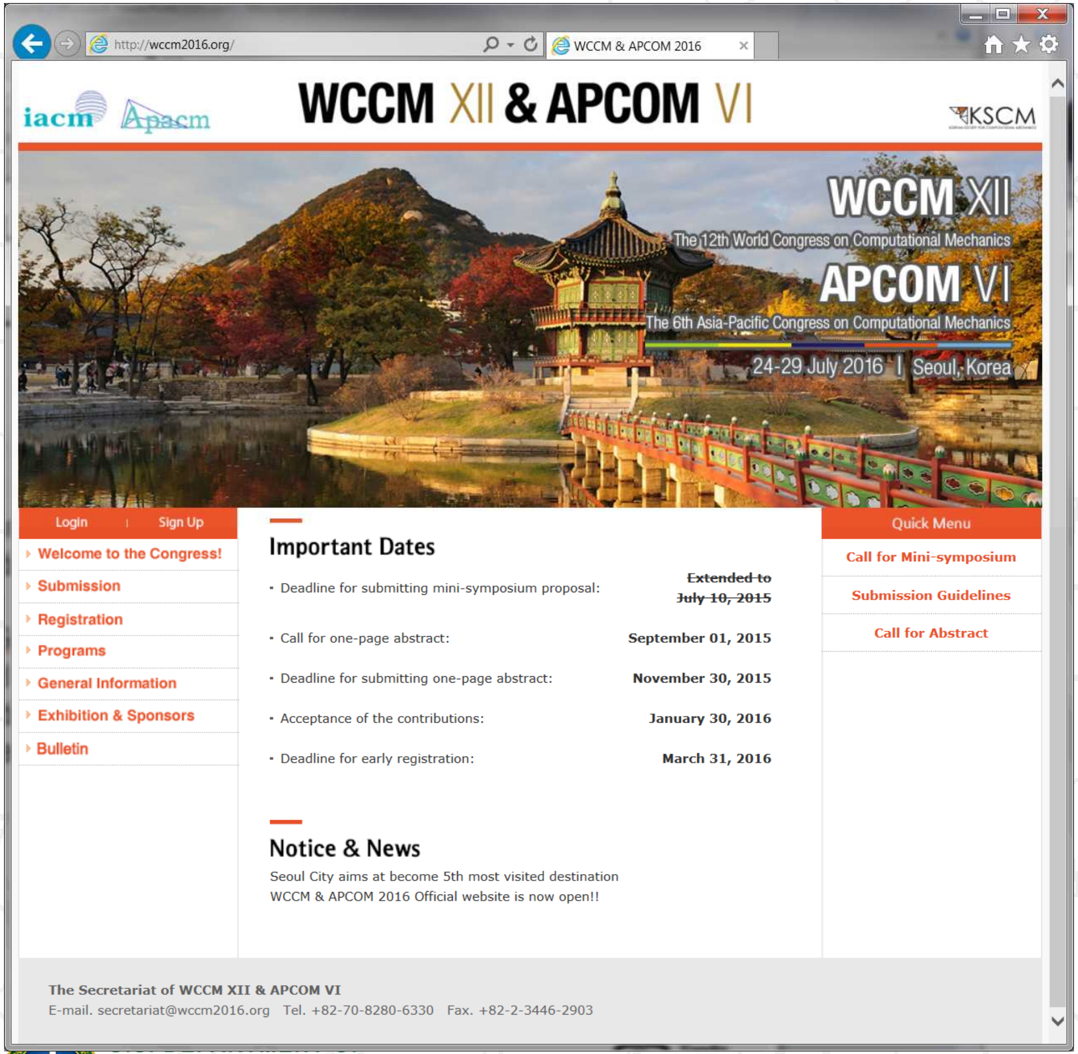
My Background

- **BS/MS Civil Engineering, Brigham Young University, 1996**
- **Ph.D. Computational Sciences and Engineering, Carnegie Mellon University, 2010**
- **Ansys, Inc 1996-1999**
  - Mesh algorithm developer
- **Unigraphics NX (Now Siemens), 1999-2004**
  - Mesh algorithm developer
  - Adams dynamics solver pre/post-processor
- **Sandia National Labs, 2004-present**
  - Cubit team member
  - Mesh Algorithm developer
- **Primary Interests**
  - Quad/hex mesh generation
  - Quad/hex mesh modification
  - Hex dual structure, chord/sheet operations
  - Geometry/Topology

MeshTrends 11 Symposium at WCCM 2016, Seoul, Korea



Consider submitting an abstract and presenting your mesh generation research in MS743 – "Trends in Unstructured Mesh Generation – Mesh Trends 11"

Organizing Committee:

- Matthew Staten
  - Sandia National Laboratories, USA
- Steve Owen
  - Sandia National Laboratories, USA
- Mark Shephard
  - Rensselaer Polytechnic Institute, USA
- Jibum Kim
  - Incheon National University, Korea
- Hang Si
  - Weierstrass Institute for Applied Analysis and Stochastics (WIAS), Germany)

Topics for MeshTrends 11
WCCM 2016

Automatic unstructured mesh generation continues to be a vital technology in computational field simulations. As computing technology continues to advance and modeling requirements become more precise, automatic mesh generation techniques must rise to fulfill ever-increasing and diverse expectations. This symposium is a forum for exploring and synthesizing many of technologies needed to develop a computational grid suitable for simulation.

All abstracts related to geometry and mesh generation for computational simulation are welcome. In this symposium we are soliciting, in particular, advancements and trends from academics and industry in the following areas:

- Mesh generation algorithms: including theoretical foundations and new algorithms for automatic methods for tet, hex and polyhedral methods.
- Parallel and scalable algorithms: including methods for generating and managing mesh and geometry for massively parallel systems.
- Meshing tools and applications: including commercial meshing tools and their application to current problems in industry.
- Multiphysics meshing issues: including tools and methods for managing meshing and geometry for mutiscale, mutliphysics applications.
- Infrastructure and tools for meshing: including APIs and tools for managing and interfacing meshing tools.
- Adaptive meshing tools and applications: including tools and methods for adaptively modifying mesh and geometry based on run-time results or optimization parameters.
- Meshing and geometry for geophysics applications: including geometry and meshing technologies for subsurface modeling and simulation.
- Meshing and geometry for biomedical applications: including geometry and meshing technologies for biomedical applications
- Meshing and CAD Geometry: including tools and methods for properly interacting with CAD geometry, and when needed to characterizing and resolve geometry problems to ensure the reliable generation of controlled meshes.

Survey Paper on Hex Mesh Generation

I recommend this survey paper for people entering the field of hex mesh generation:

J. Sarrate, E. Ruiz-Gironés and X. Roca, "Unstructured and Semi-Structured Hexahedral Mesh Generation Methods," Computational Technology Reviews, Volume 10, 2014, http://www.ctresources.info/ctr/paper.html?id=60

Short Course Outline

- **Tet Meshing Vs. Hex Meshing**
- **Structured vs. Unstructured**
- **Mapping**
- **Paving**
- **Sweeping**
- **Interval Assignment**
- **Dual Representation of a quad/hex mesh**
- **Sheet & Column Operations**
- **Frame Fields Methods**
- **Refinement**
- **Overlay Grid Methods**

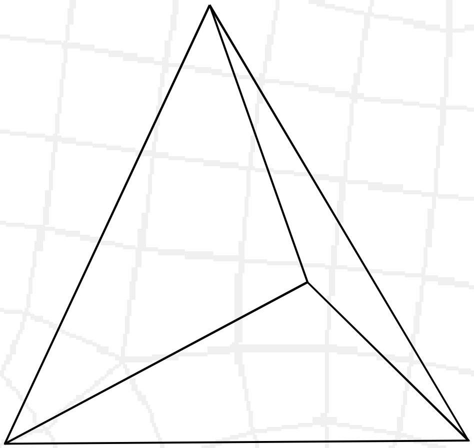
This course is a survey course of the methods I have seen used the most in industry, and some new promising areas of research. I have completely omitted many worthwhile research efforts, including the medial surface methods due only to lack of time to prepare the material.

The age old debate. Do I spend the time to build a hexmesh and possibly get better solution results? Or do I build a tet mesh quickly to get solution results quickly?

HEX VS. TET

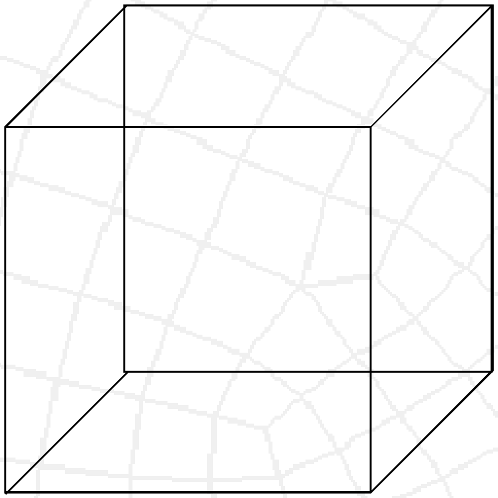
Hexahedra vs. Tetrahedra

Finite element meshes can be generated with either tetrahedral or hexahedral elements. Automatic tetrahedral meshing is generally considered a solved problem, while hexahedral meshing is still an open problem.



Tetrahedra (simplex)
- 4 nodes
- 6 edges
- 4 faces

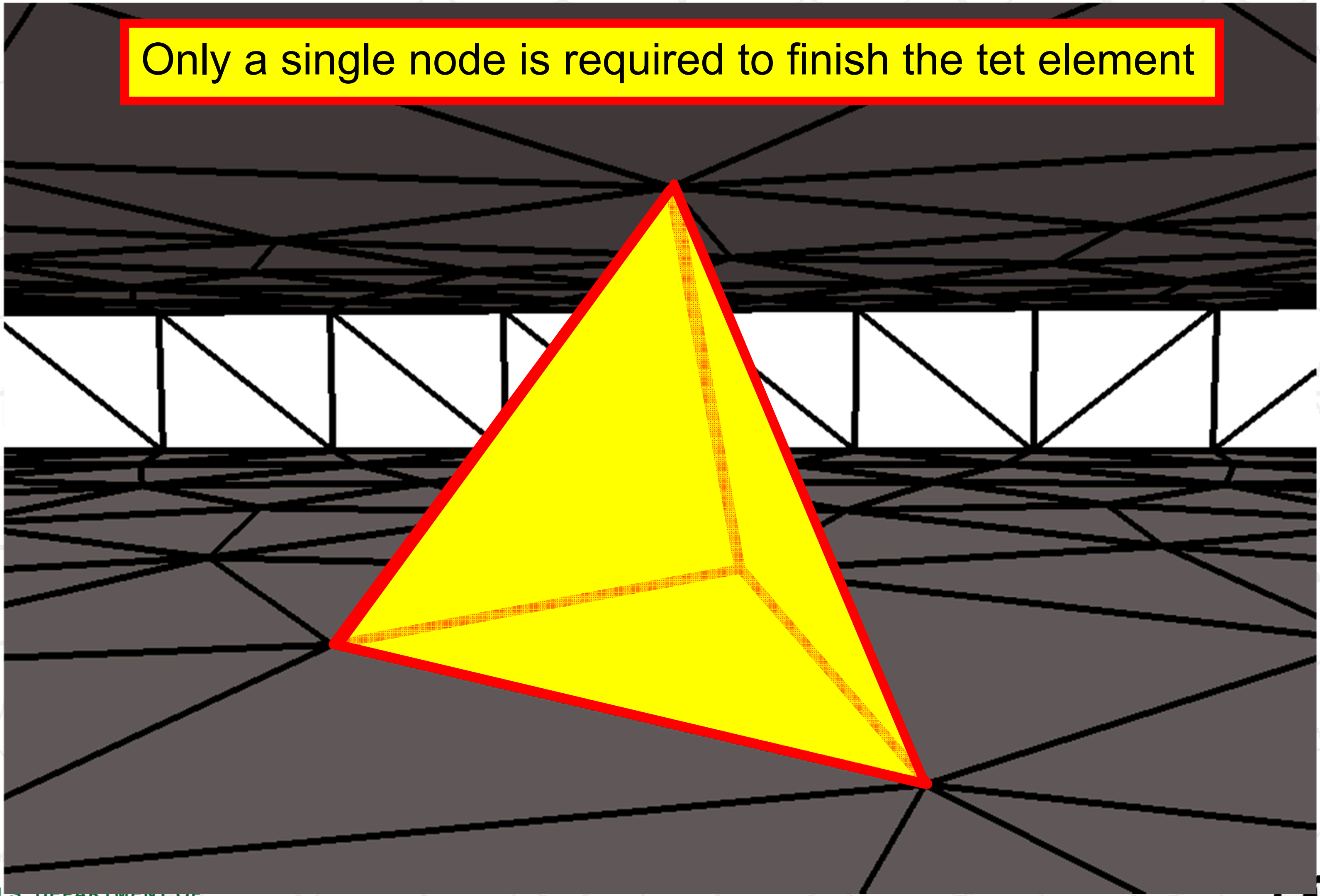
Automated Generation
Locally Modifiable



Hexahedra
- 8 nodes
- 12 edges
- 6 faces

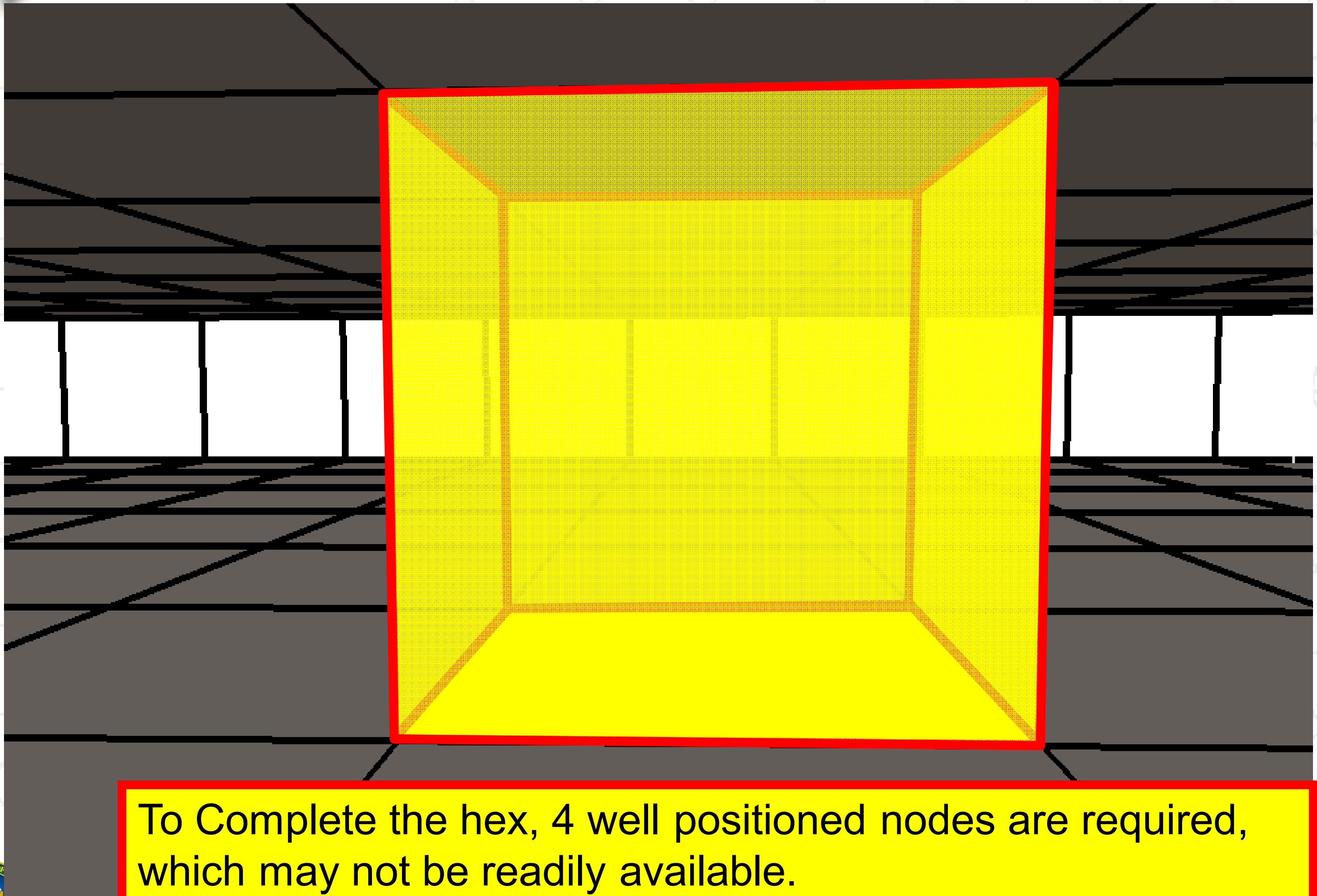
Semi-Automatic & Manual Generation
Constrained Modifications

Advancing Front Element Creation



Matt

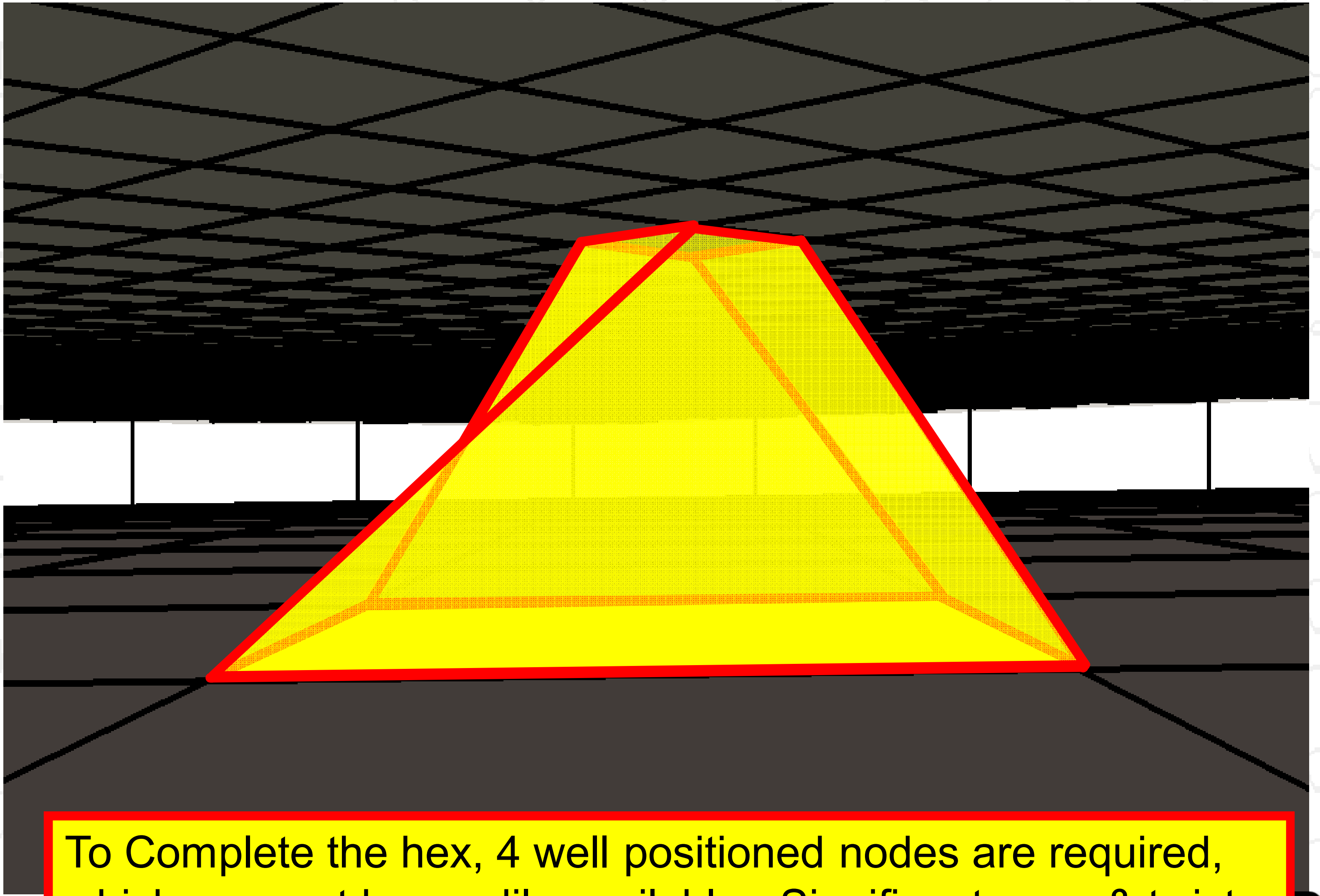
Advancing Front Element Creation



To Complete the hex, 4 well positioned nodes are required, which may not be readily available.

Matt

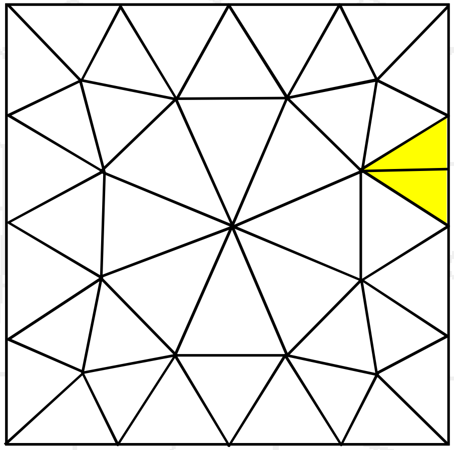
Advancing Front Element Creation



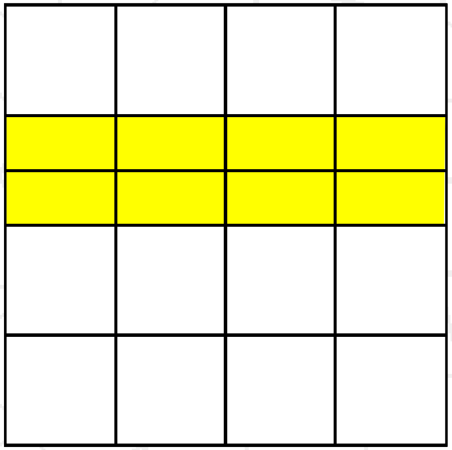
To Complete the hex, 4 well positioned nodes are required, which may not be readily available. Significant warp & twist are often required to build hex elements.

Local Modifications to Meshes

Triangle and tetrahedral meshes can be easily modified locally.



In contrast, to maintain a conforming non-hybrid mesh, small changes to quadrilateral and hexahedral meshes propagate through the mesh due to topology constraints.



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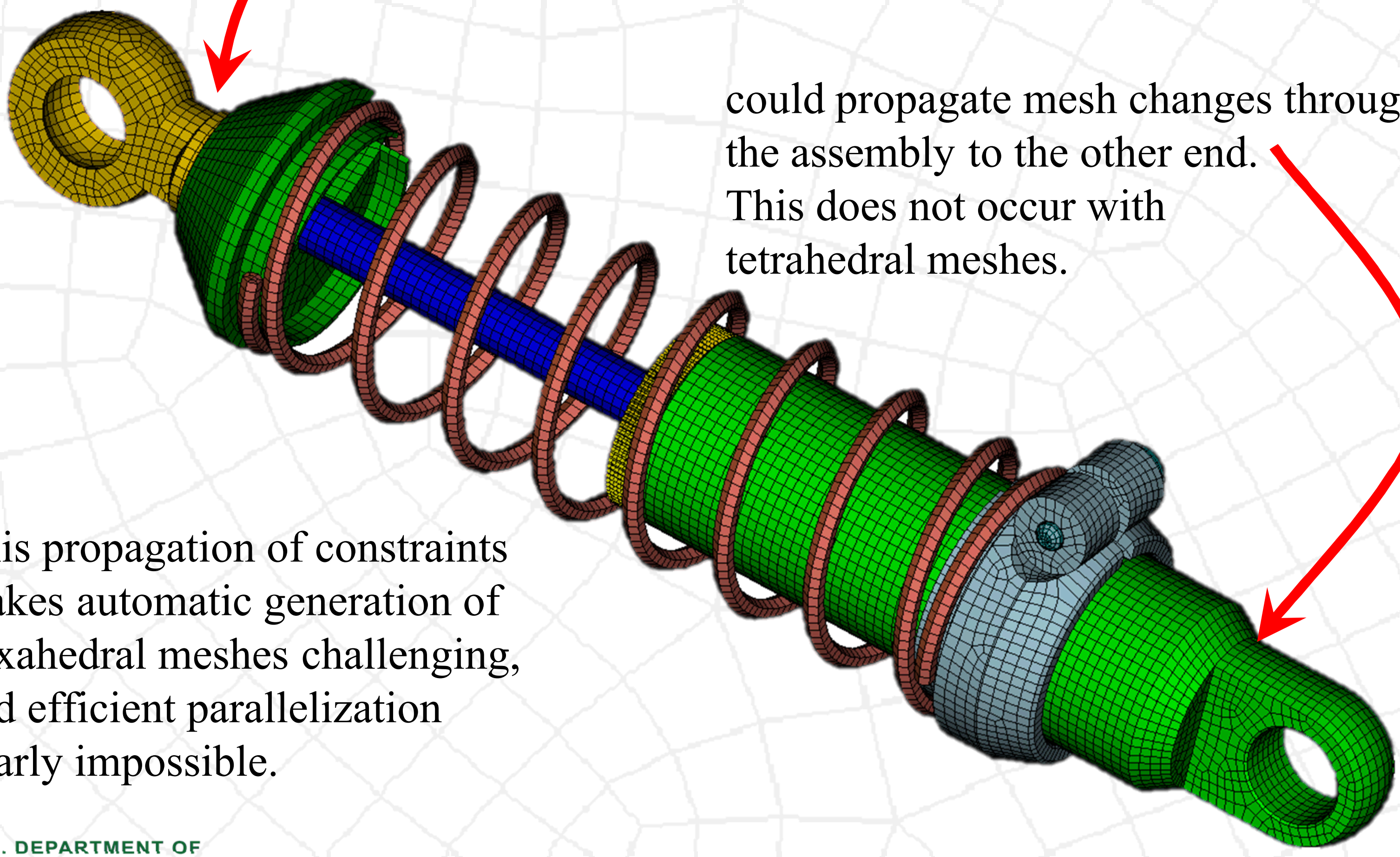
Global Propagation of Hexahedral Constraints on Assembly Models

When generating all-hexahedral conforming mesh through interfaces in an assembly,

a geometric feature here

could propagate mesh changes through the assembly to the other end. This does not occur with tetrahedral meshes.



This propagation of constraints makes automatic generation of hexahedral meshes challenging, and efficient parallelization nearly impossible.

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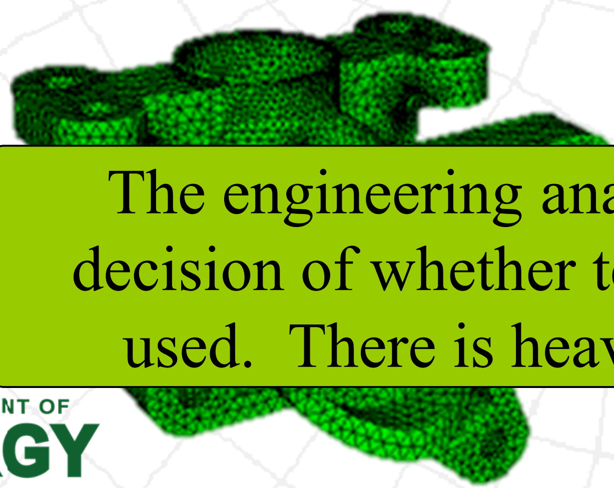
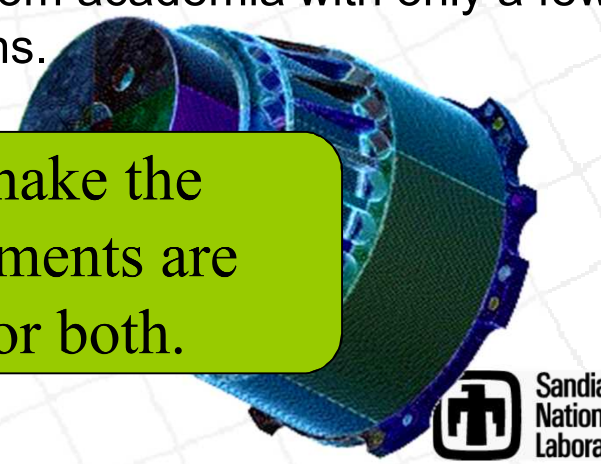
Tet Meshing Vs. Hex Meshing

Tet Meshing
1. Fully Automated, mostly push-button
2. Generate millions of elements in minutes/seconds
3. User time generally minutes/hours
4. Can require 4-10X number of elements to achieve same accuracy as all-hex mesh
5. Tet-Locking phenomenon for linear tet results in stiffer physics
6. Preferred by many academics for mathematic properties in generation

Hex Meshing
1. Partially automated, some manual
2. Can require major user effort/expertise to prepare geometry to accept a hex mesh
3. User time to generate mesh may be typically days/weeks/months
4. Computational methods may prefer or require hex element
5. Preferred by many analysts for solution accuracy
6. Heavily used in industry & national labs. Absent from academia with only a few exceptions.





The engineering analysts must make the decision of whether tet or hex elements are used. There is heavy demand for both.

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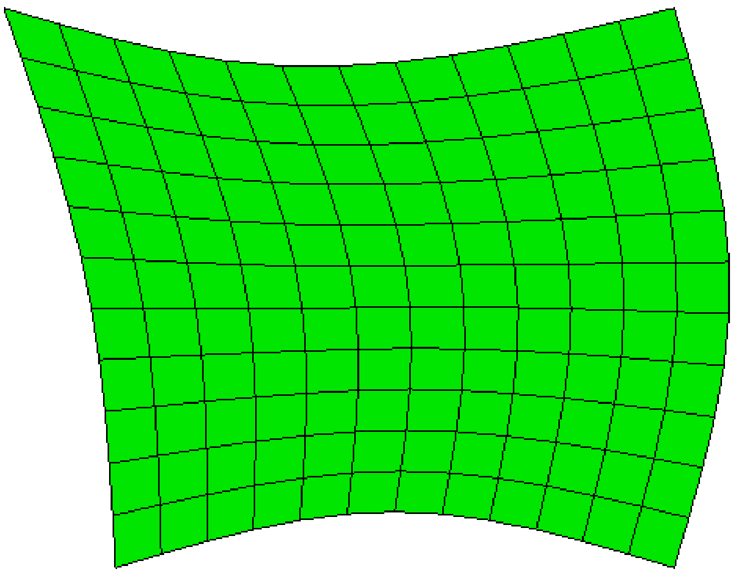
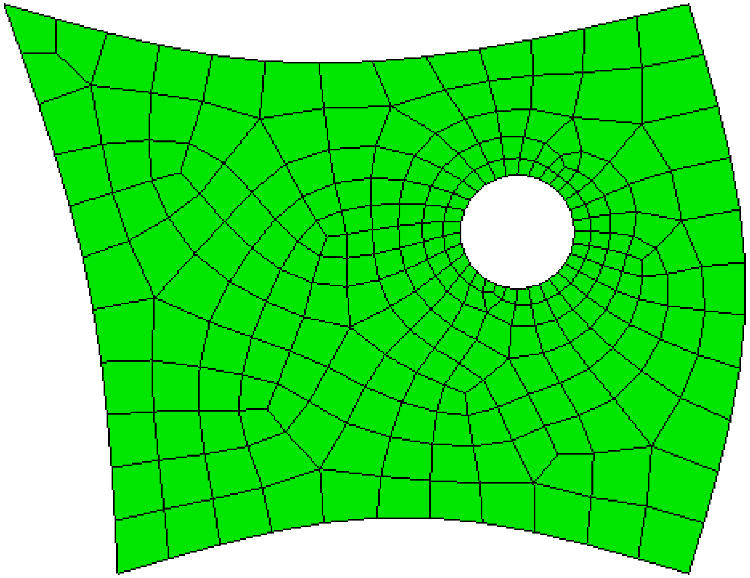
Mapping, Submaping, TFI

STRUCTURED MESHING

Structured vs. Unstructured





Structured

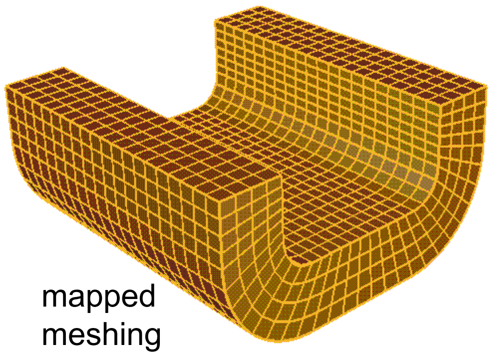
1. Interior node valence is constant.
   ie. number of elements at each interior node=4
2. Meshing algorithm relies on specific topology constraints.
   ie. number of sides=4

Unstructured

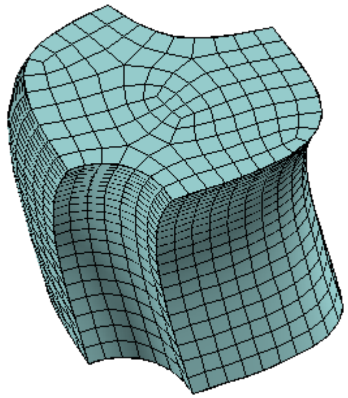
1. Interior node valence varies.
   ie. number of elements at each node=3,4,5…
2. Meshing algorithm applies to arbitrary topology
   ie. number of sides is arbitrary

Structured vs. Unstructured

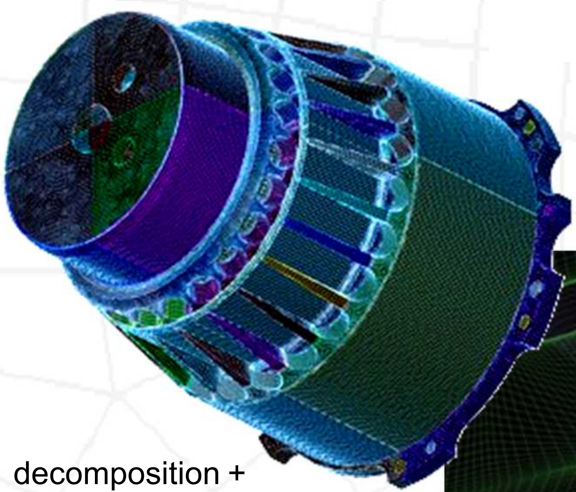
Structured



mapped meshing

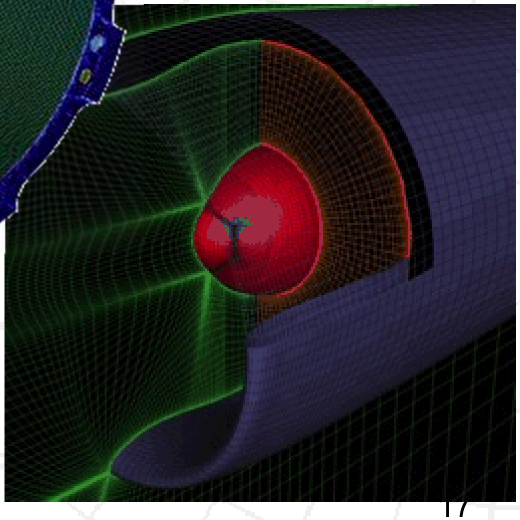


sweeping

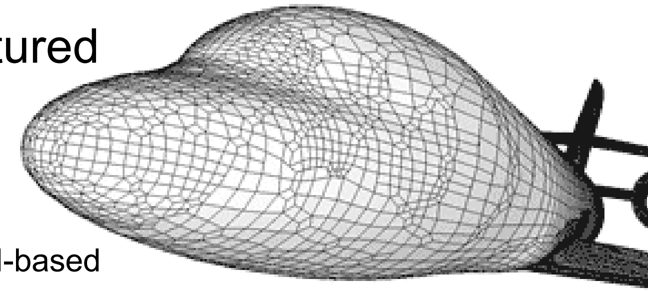


decomposition + sweeping

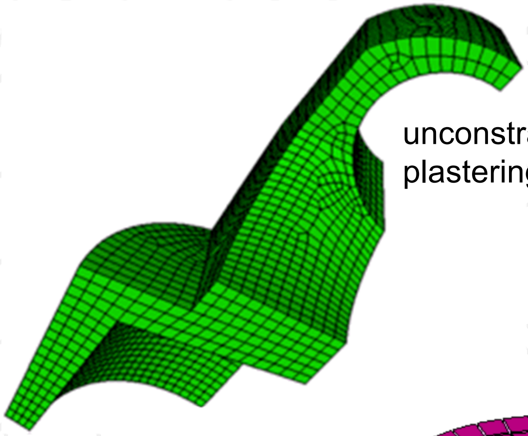
block structured



Unstructured

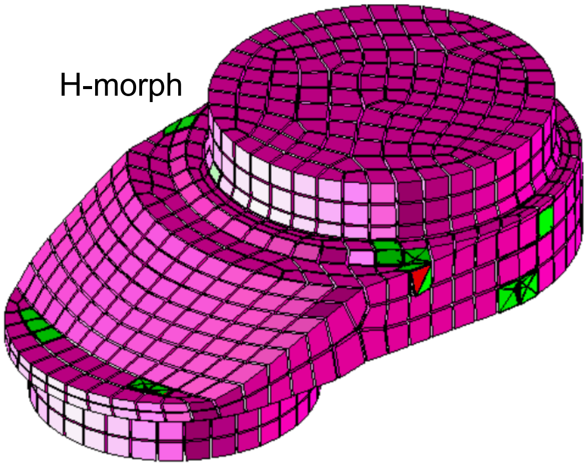


grid-based

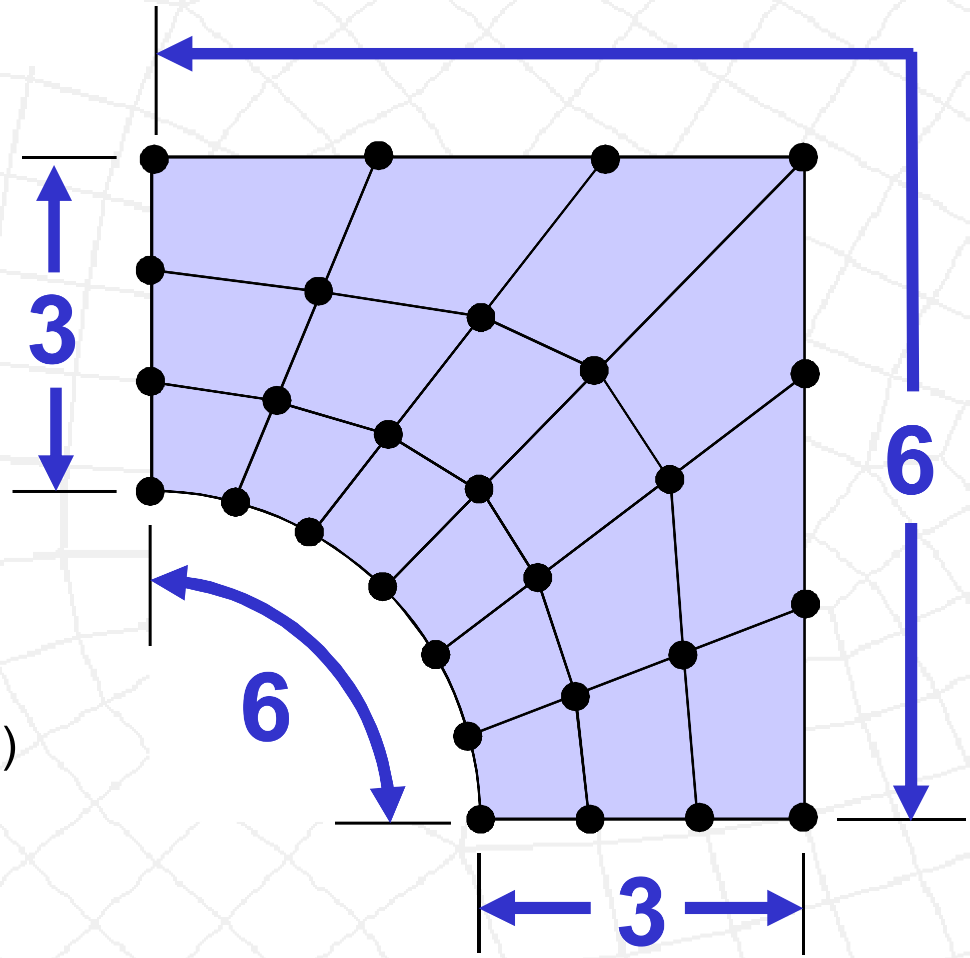


unconstrained plastering

H-morph



Mapped Meshing



Algorithm
- •Trans-finite Interpolation (TFI)
- •maps a regular lattice of quads onto polygon (Thompson,88;99) (Cook,82)

Geometry Requirements
- •4 topological sides
- •opposite sides must have similar intervals

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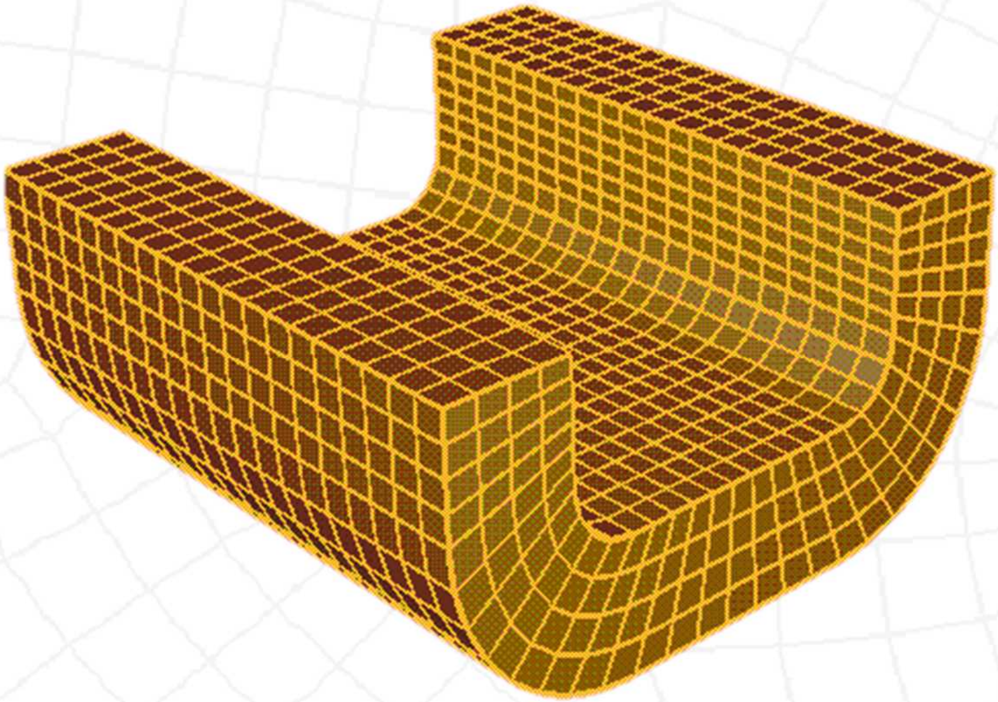
Mapped Meshing

3D Mapped Meshing



Geometry Requirements

- 6 topological surfaces
- opposite surfaces must have similar mapped meshes

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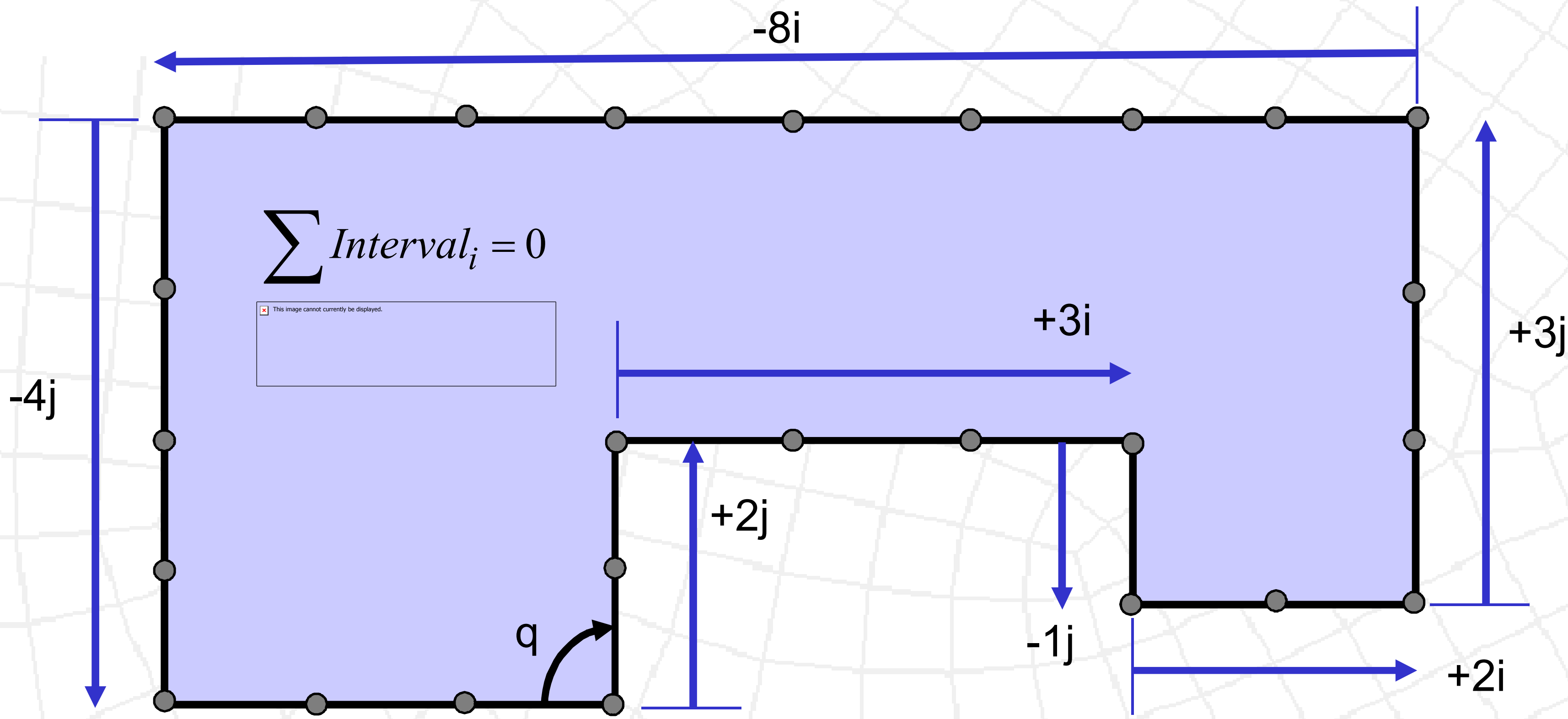
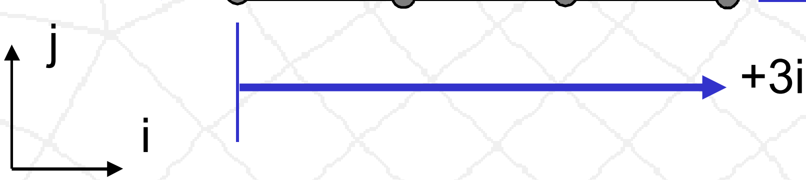
Sub-mapping



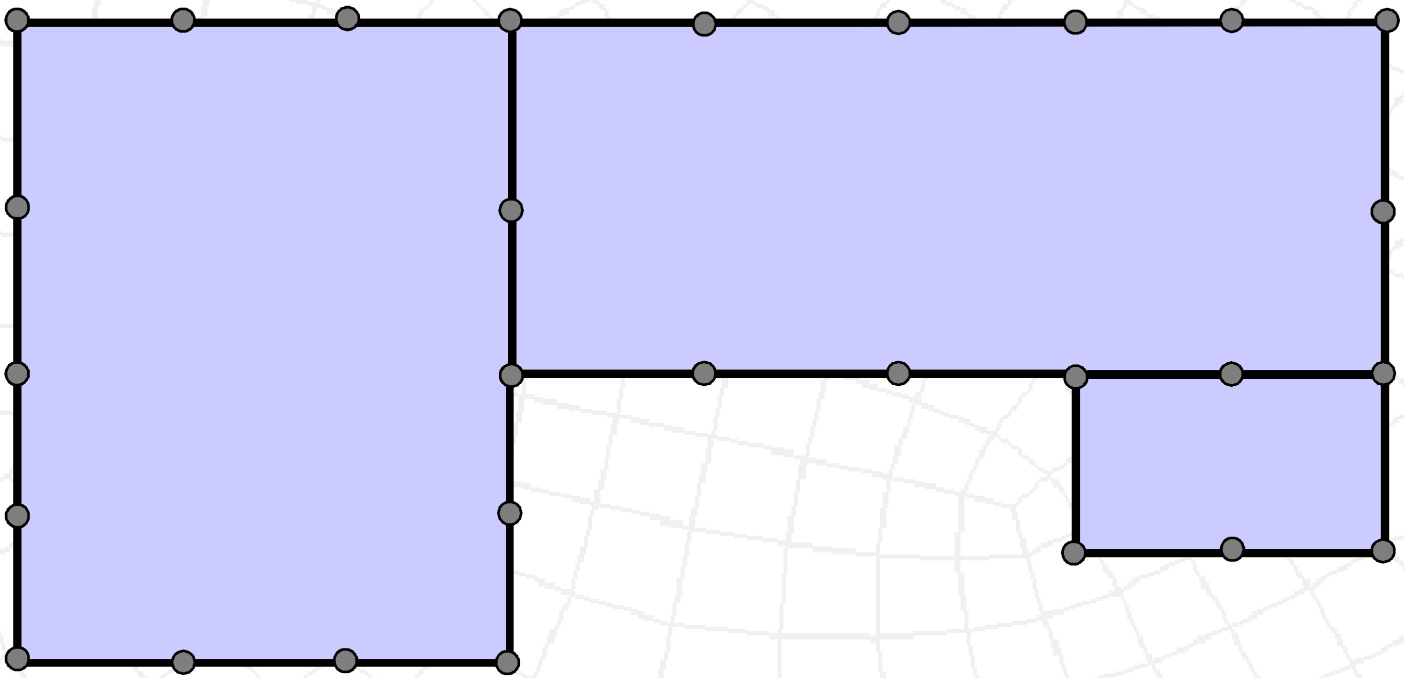


Geometry Requirements

- Blocky-type surfaces (principally 90 degree angles)

White, D R., L. Mingwu, S. Benzley, G. Sjaardema "Automated Hexahedral Mesh Generation by Virtual Decomposition", *Proceedings, 4th International Meshing Roundtable*, Sandia National Laboratories, pp.165-176, October 1995

Sub-mapping



•Automatically decomposes surface into mappable regions based on assigned intervals

(White 95)

White, D R., L. Mingwu, S. Benzley, G. Sjaardema "Automated Hexahedral Mesh Generation by Virtual Decomposition", *Proceedings, 4th International Meshing Roundtable*, Sandia National Laboratories, pp.165-176, October 1995

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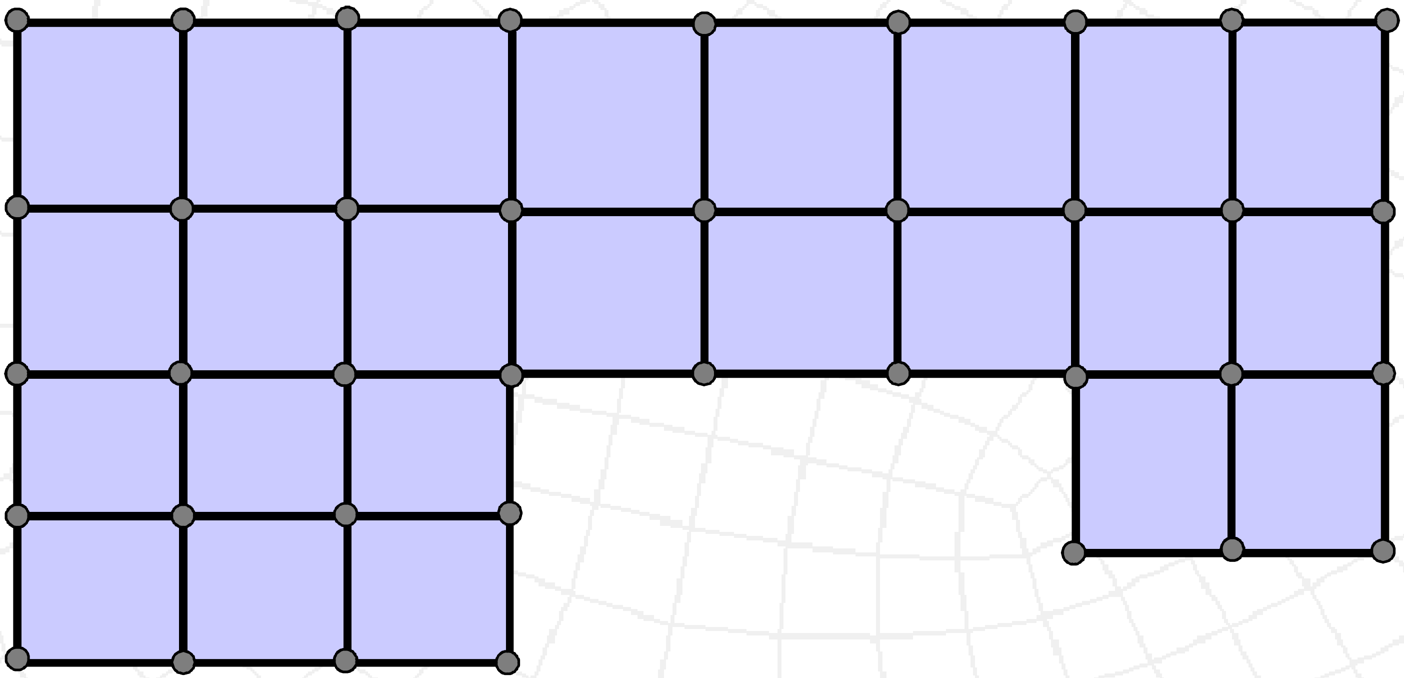

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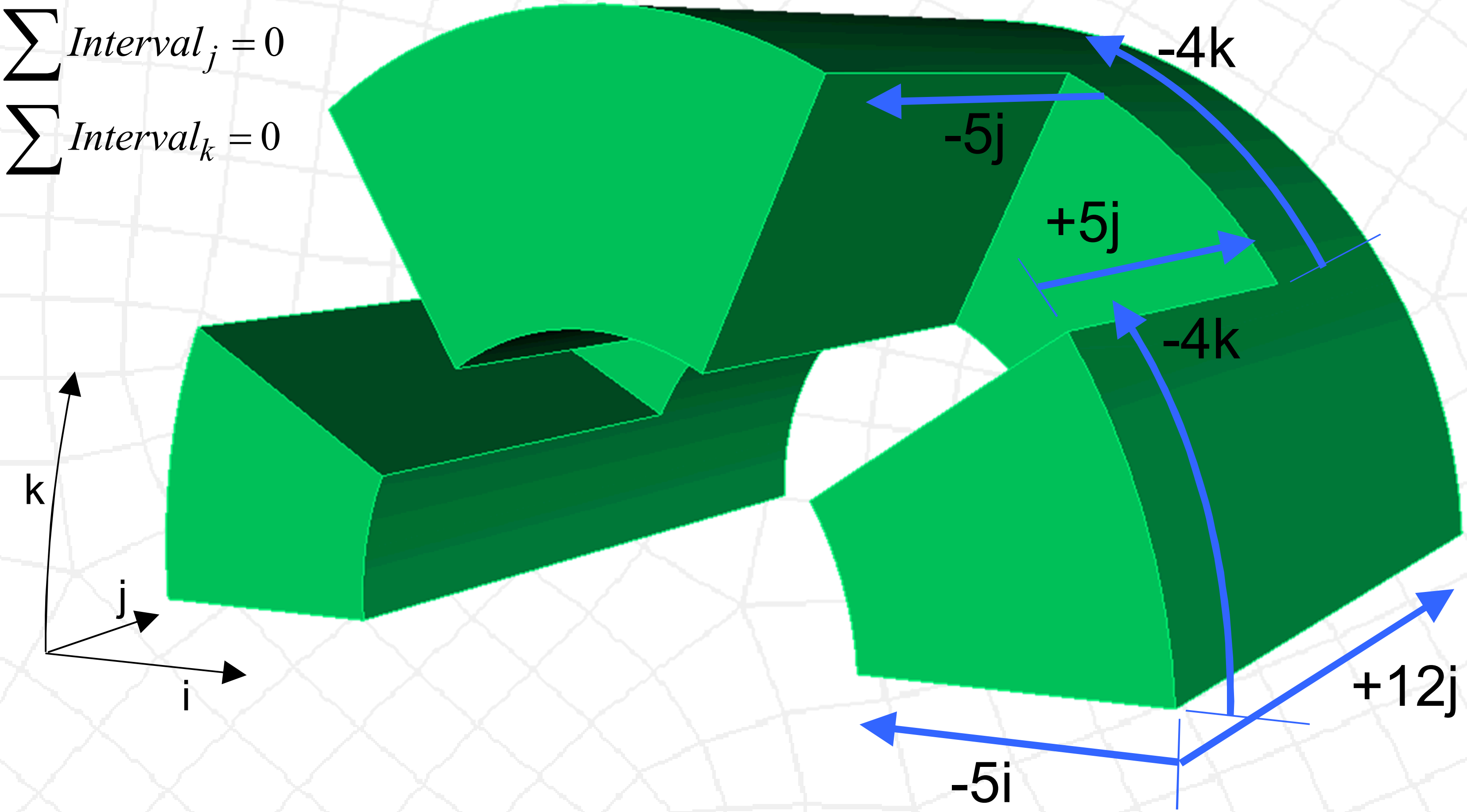
Sub-mapping

$$\sum Interval_i = 0$$

$$\sum Interval_j = 0$$

$$\sum Interval_k = 0$$



Sub-mapping

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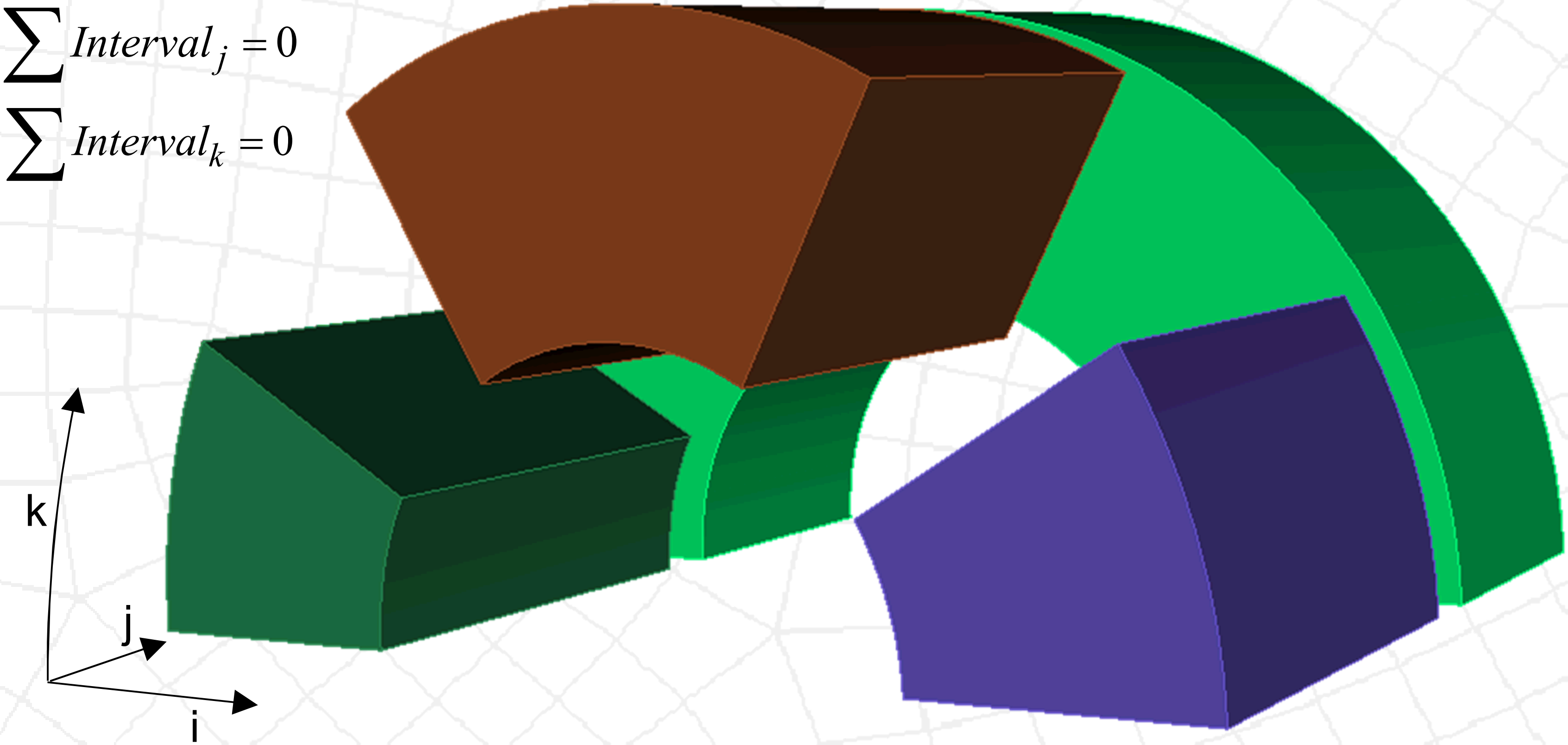

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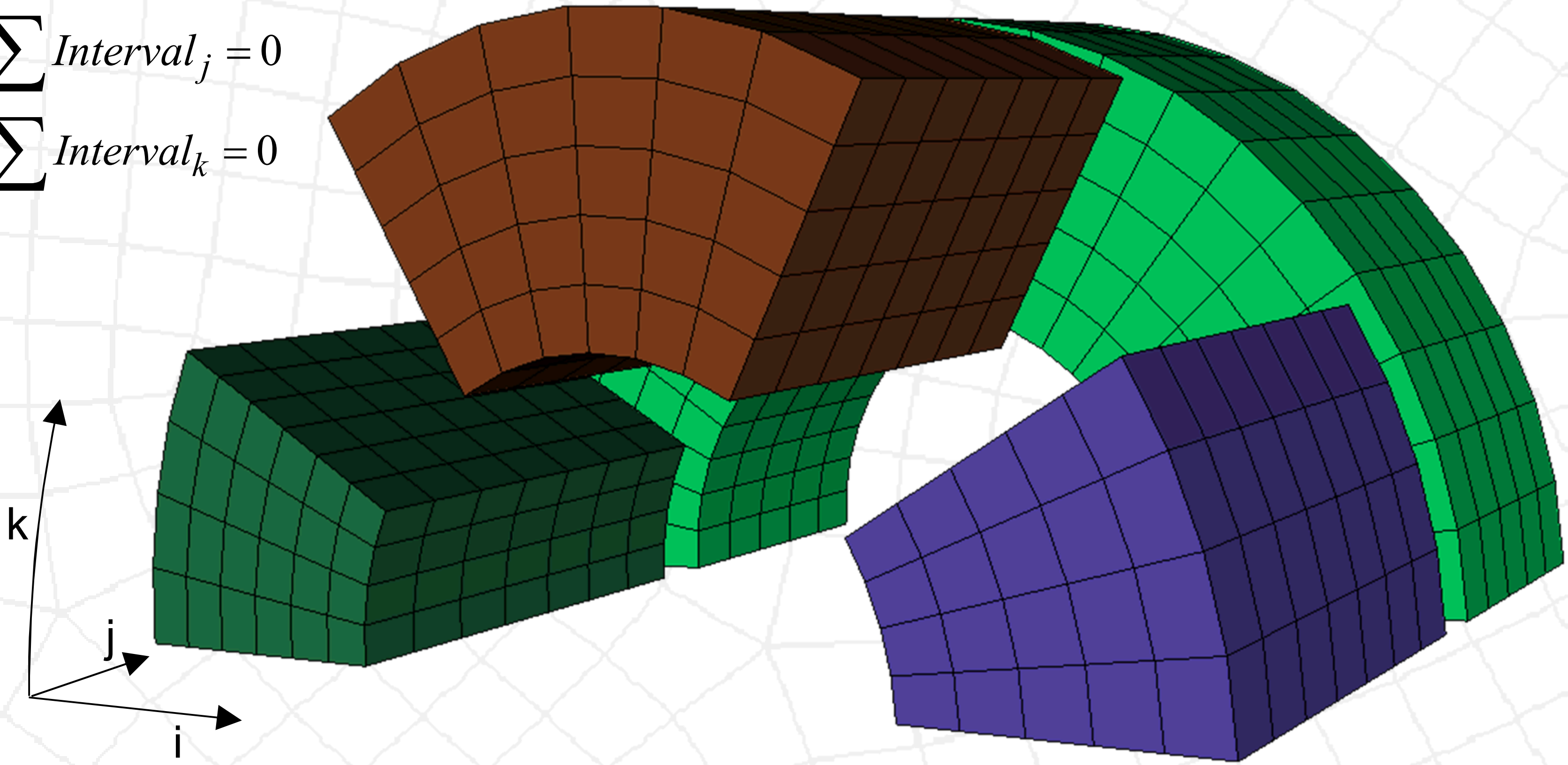

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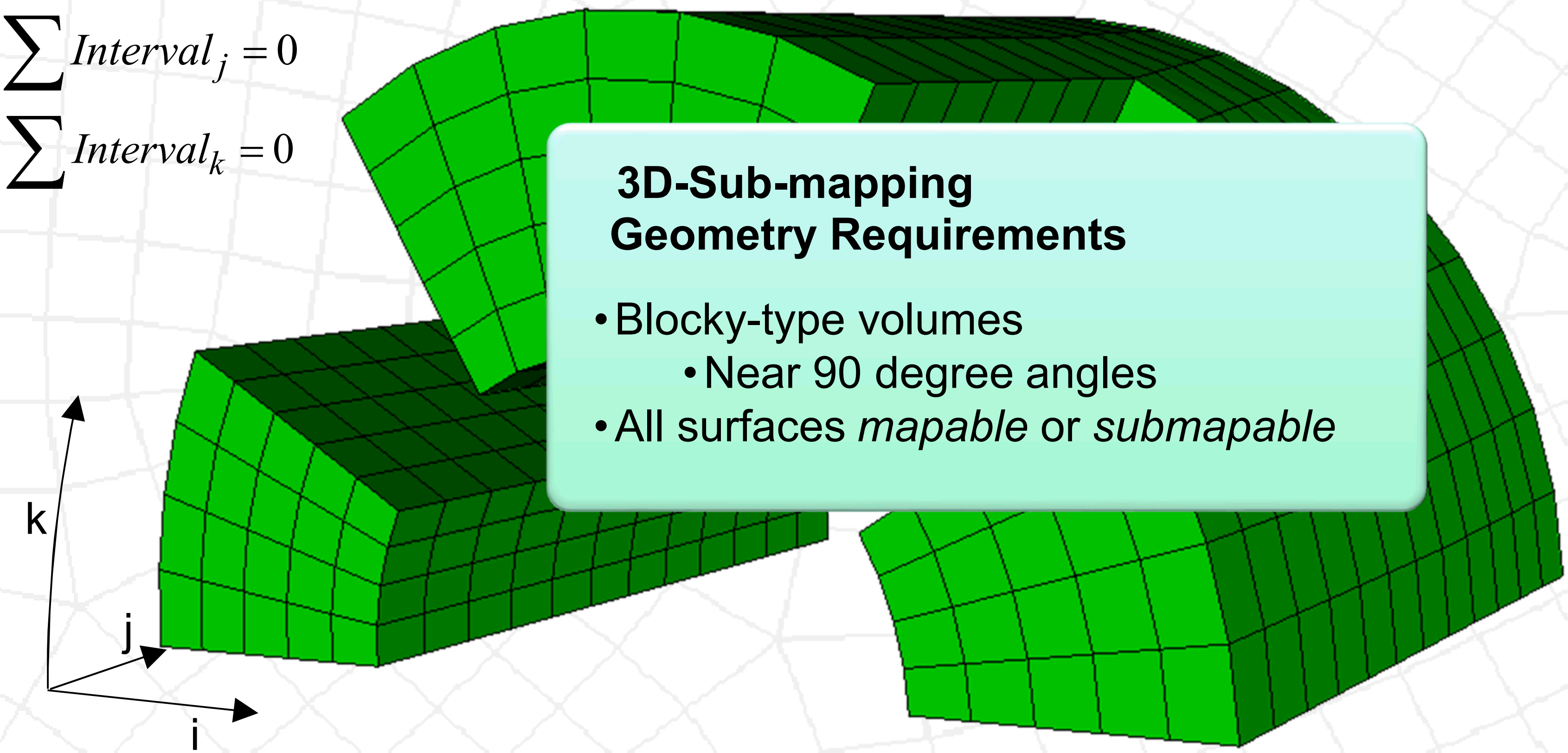


3D-Sub-mapping Geometry Requirements

- Blocky-type volumes
    - Near 90 degree angles
- All surfaces *mapable* or *submapable*

White, D R., L. Mingwu, S. Benzley, G. Sjaardema "Automated Hexahedral Mesh Generation by Virtual Decomposition", *Proceedings, 4th International Meshing Roundtable*, Sandia National Laboratories, pp.165-176, October 1995

Paving and Triangle Combining, and variations/combinations

UNSTRUCTURED SURFACE MESHING

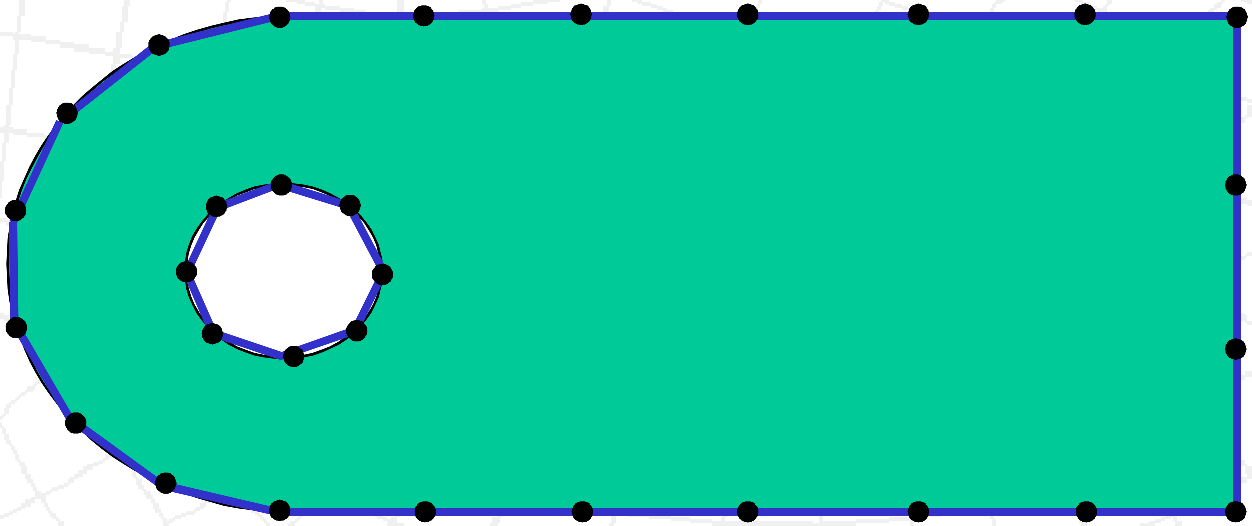
Paving: The First General Unstructured Quad Mesher

Blacker, T and Stephenson, M.B
“Paving: A New Approach to Automated Quadrilateral Mesh Generation",
International Journal for Numerical Methods in Engineering, vol 32, 811-847, 1991

- Advancing front from surface boundary
- Supports large transitions in element size
- Planar and 3D surfaces
- Largely heuristic
- Dozens of variations have been presented through the years
- After 25 years, still used extensively today for industrial mesh generation

Recommended reading for students new to quad/hex mesh generation

Paving



- Advancing Front: Begins with front at boundary
- Forms rows of elements based on front angles
- Must have even number of intervals for all-quad mesh

(Blacker,92)(Cass,96)

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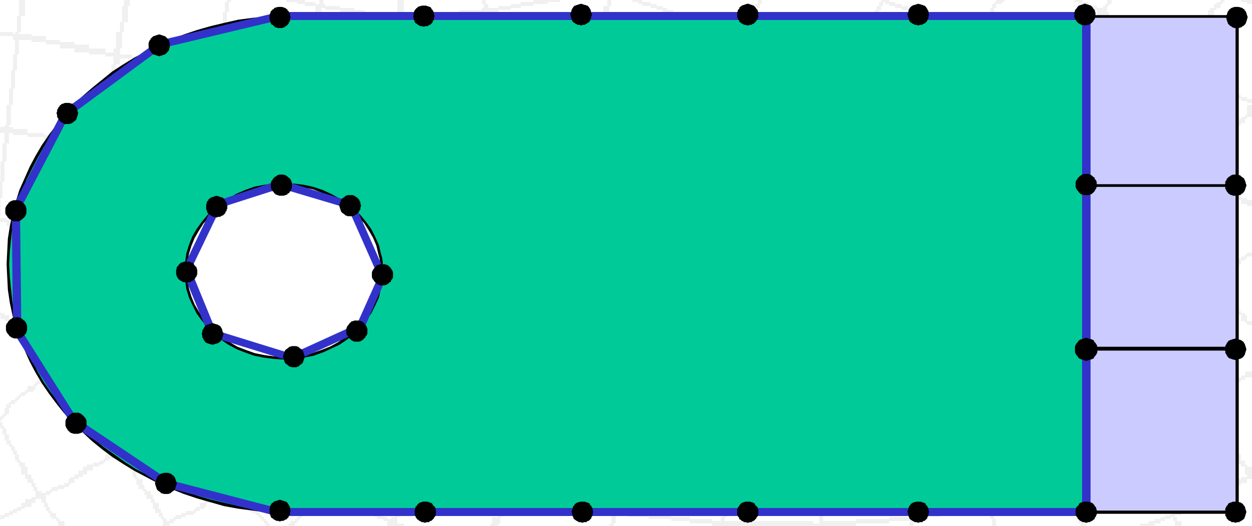
Paving



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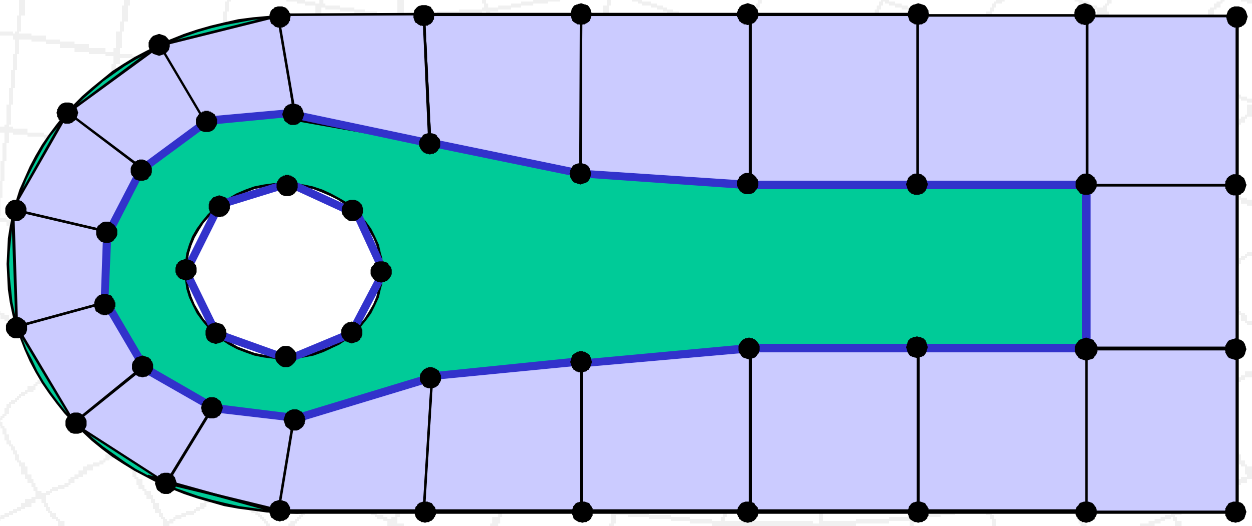

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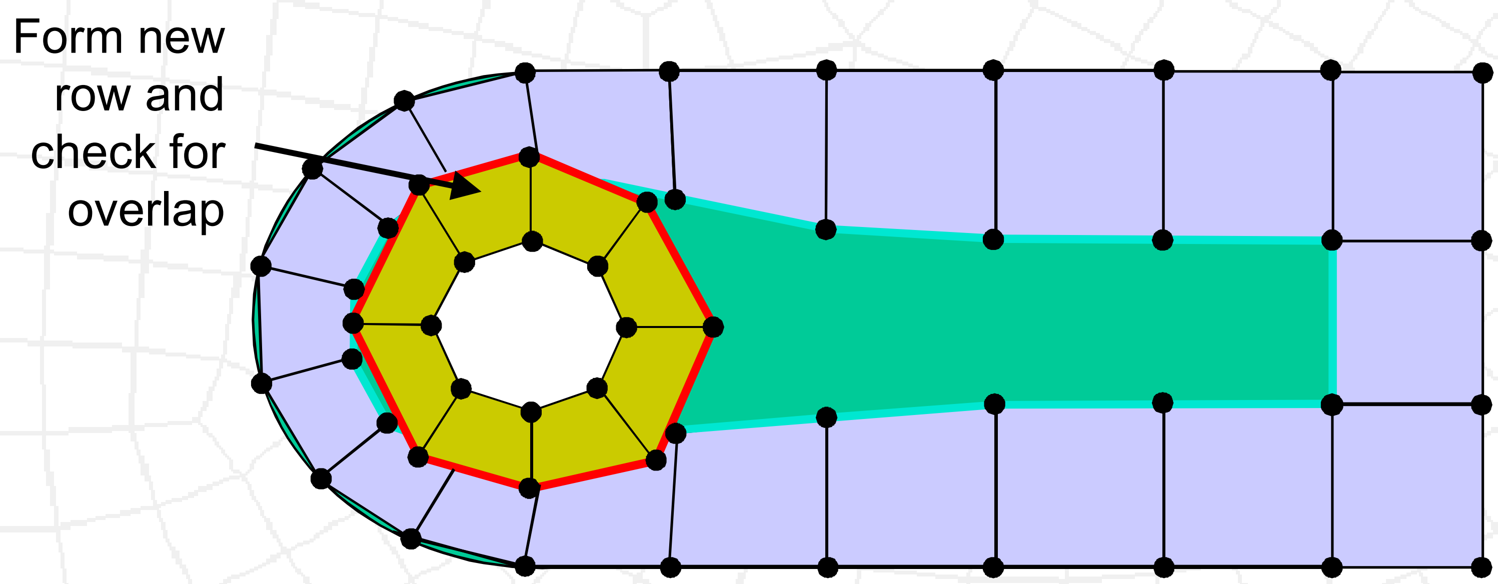
Paving



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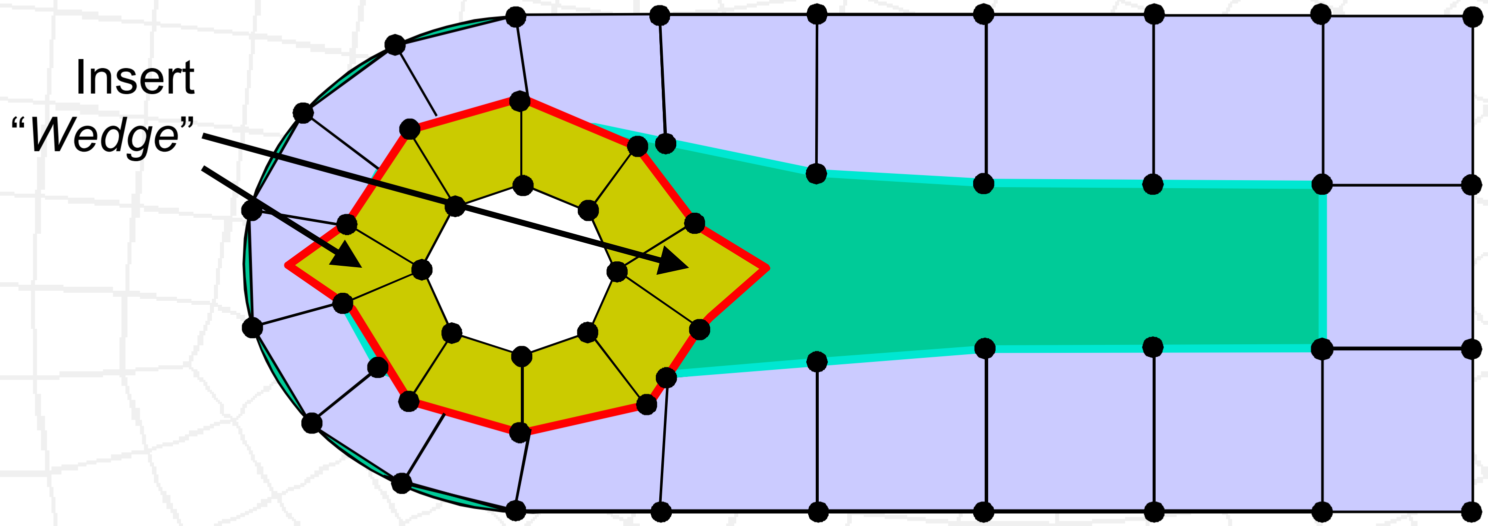

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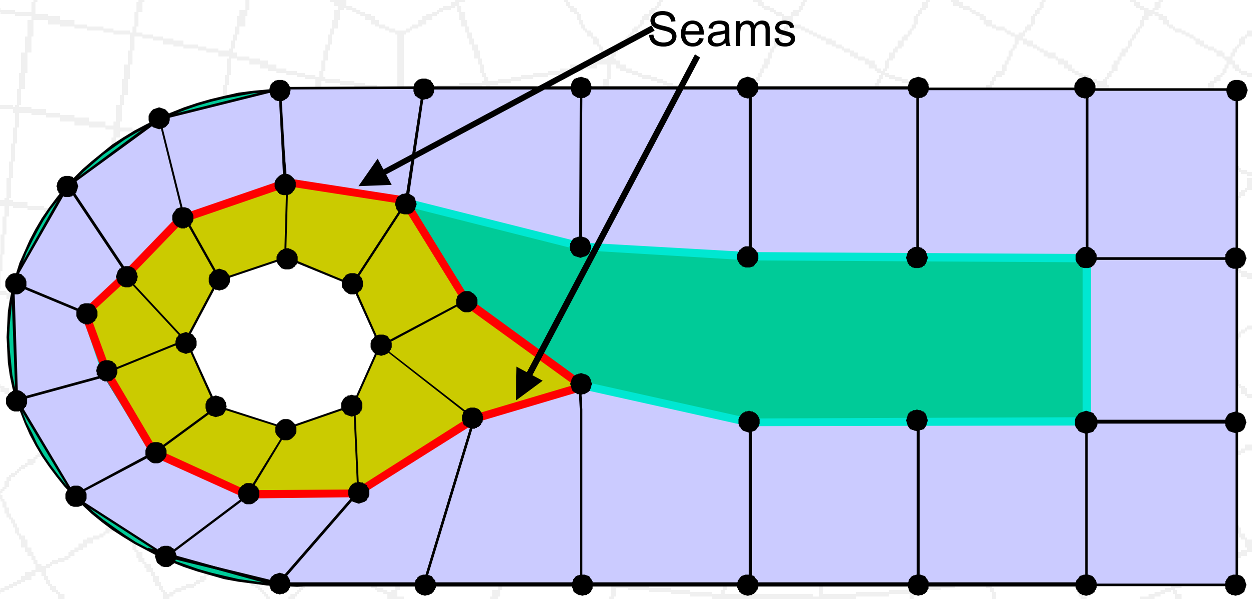

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(Blacker,92)(Cass,96)

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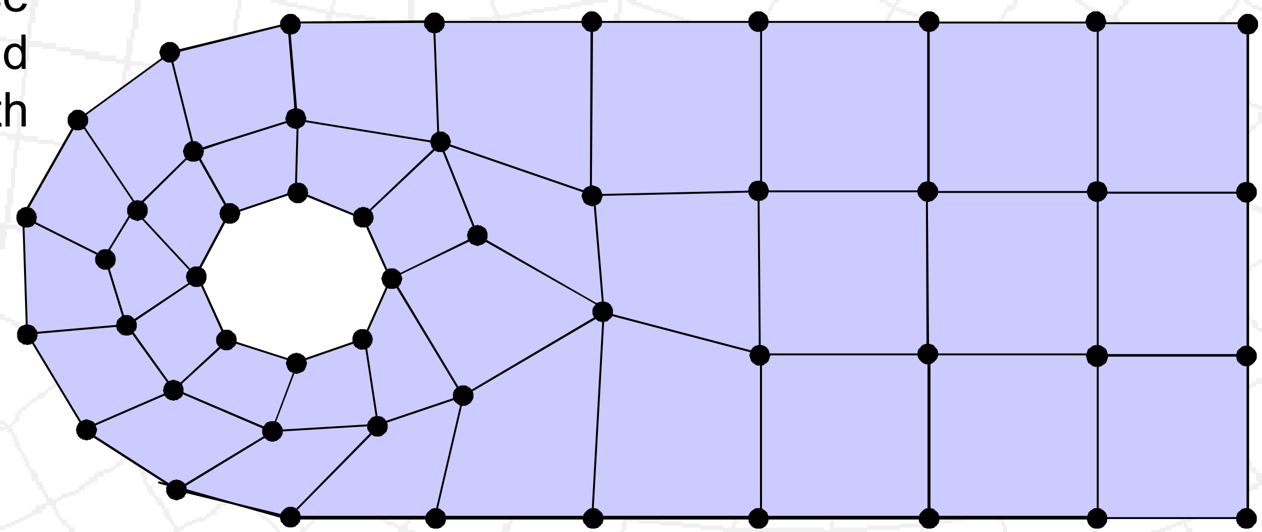
Paving

Close Loops and smooth



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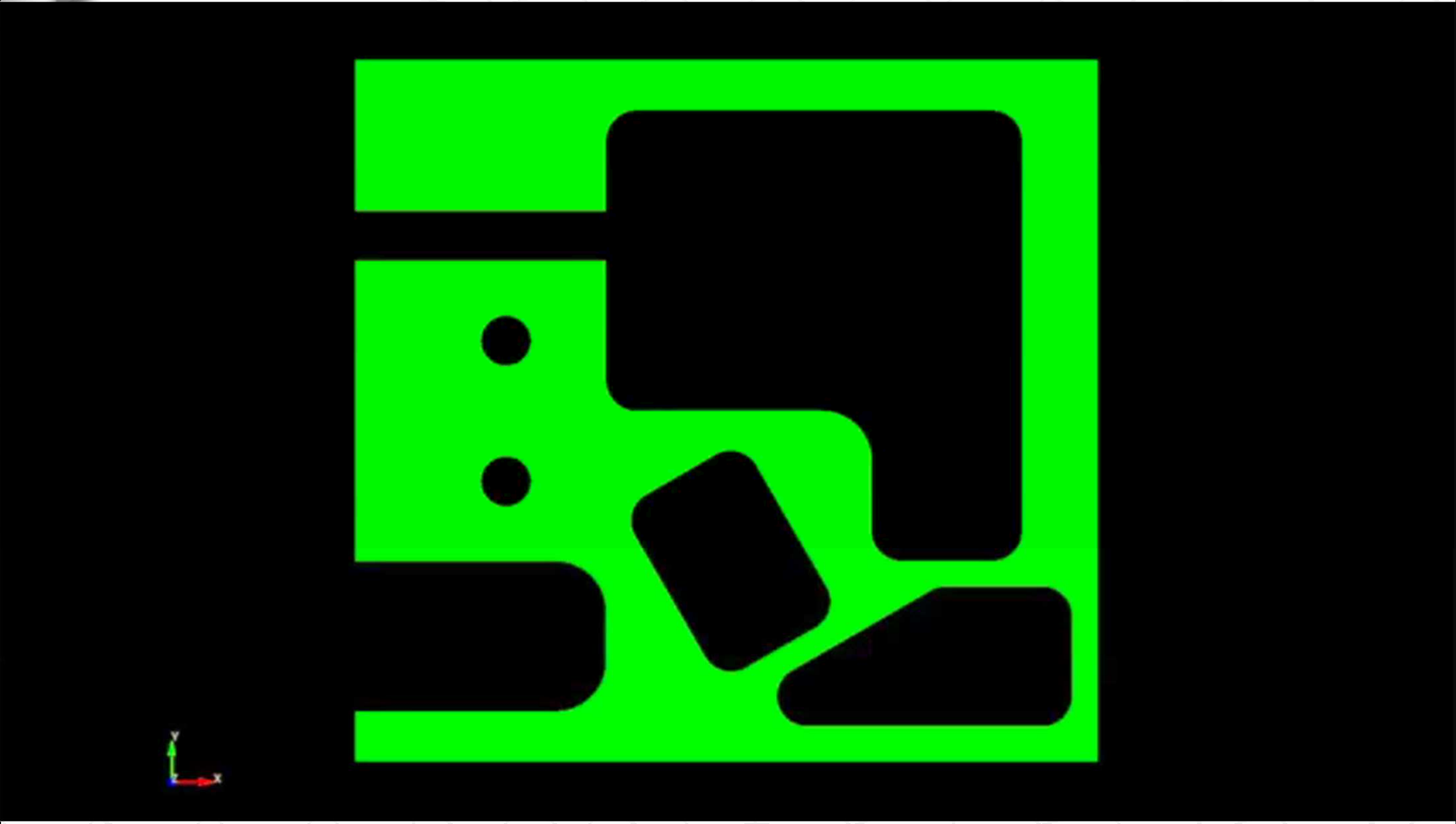
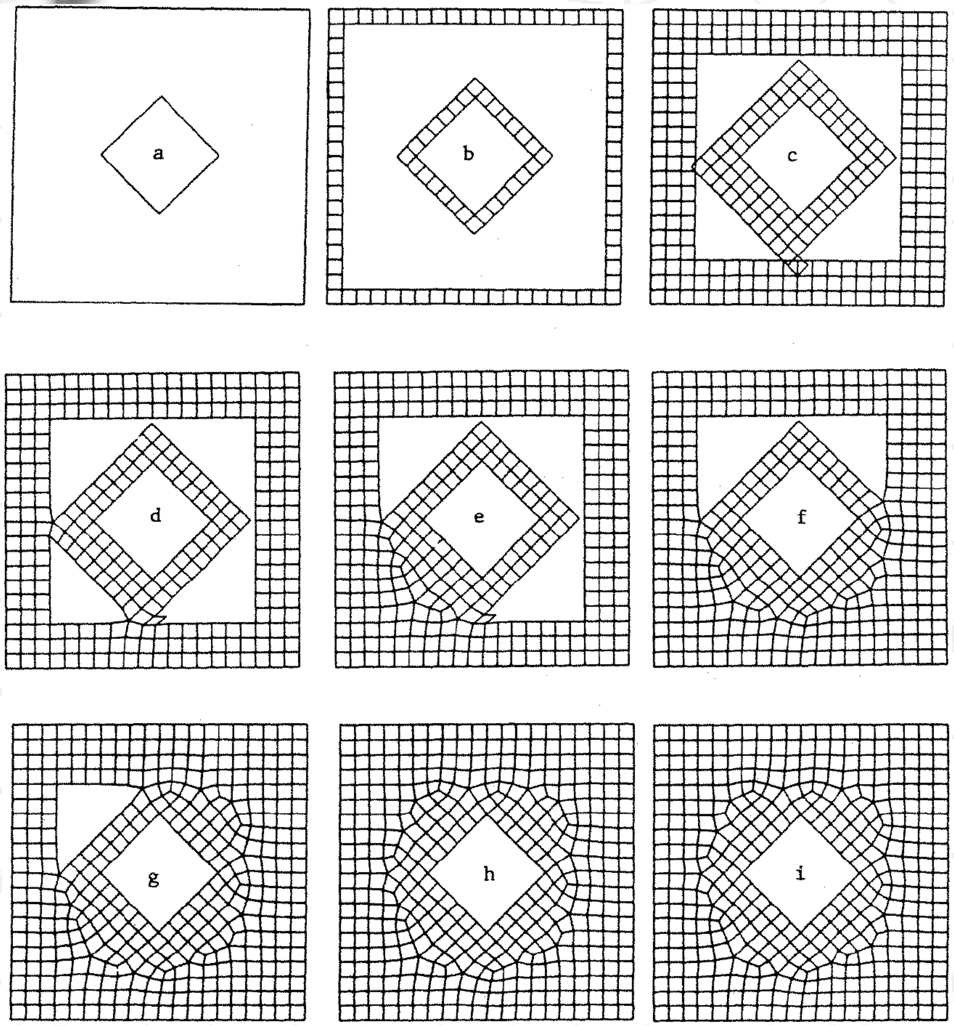
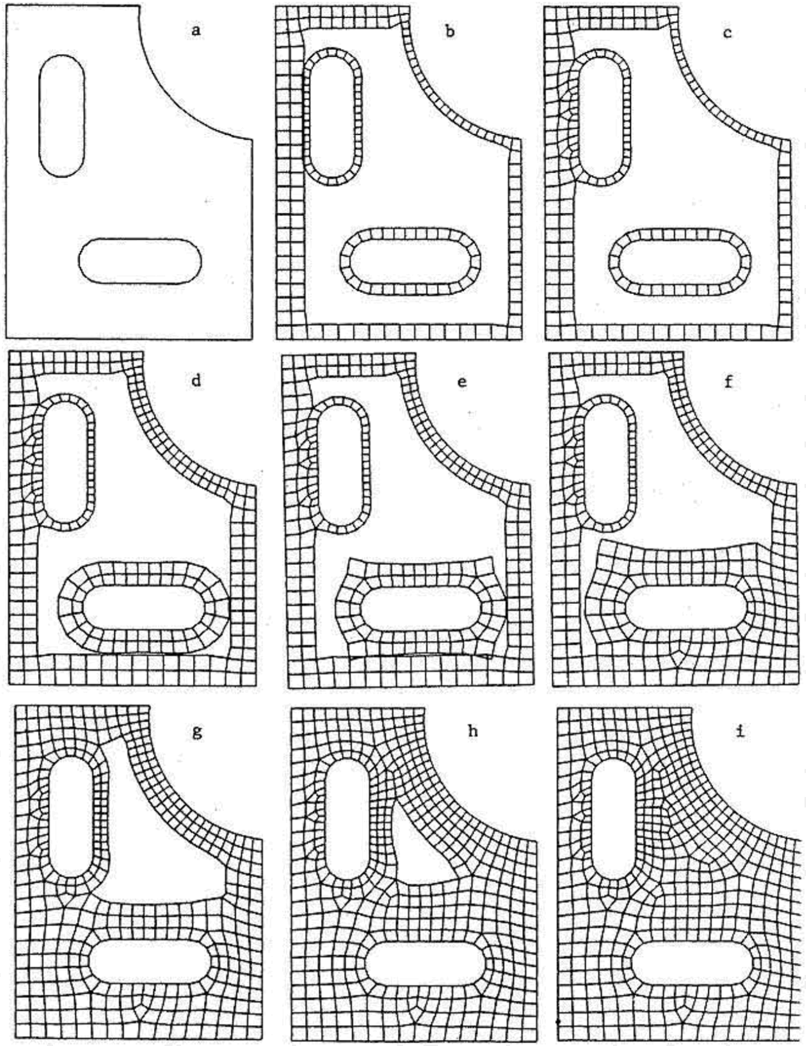
Paving

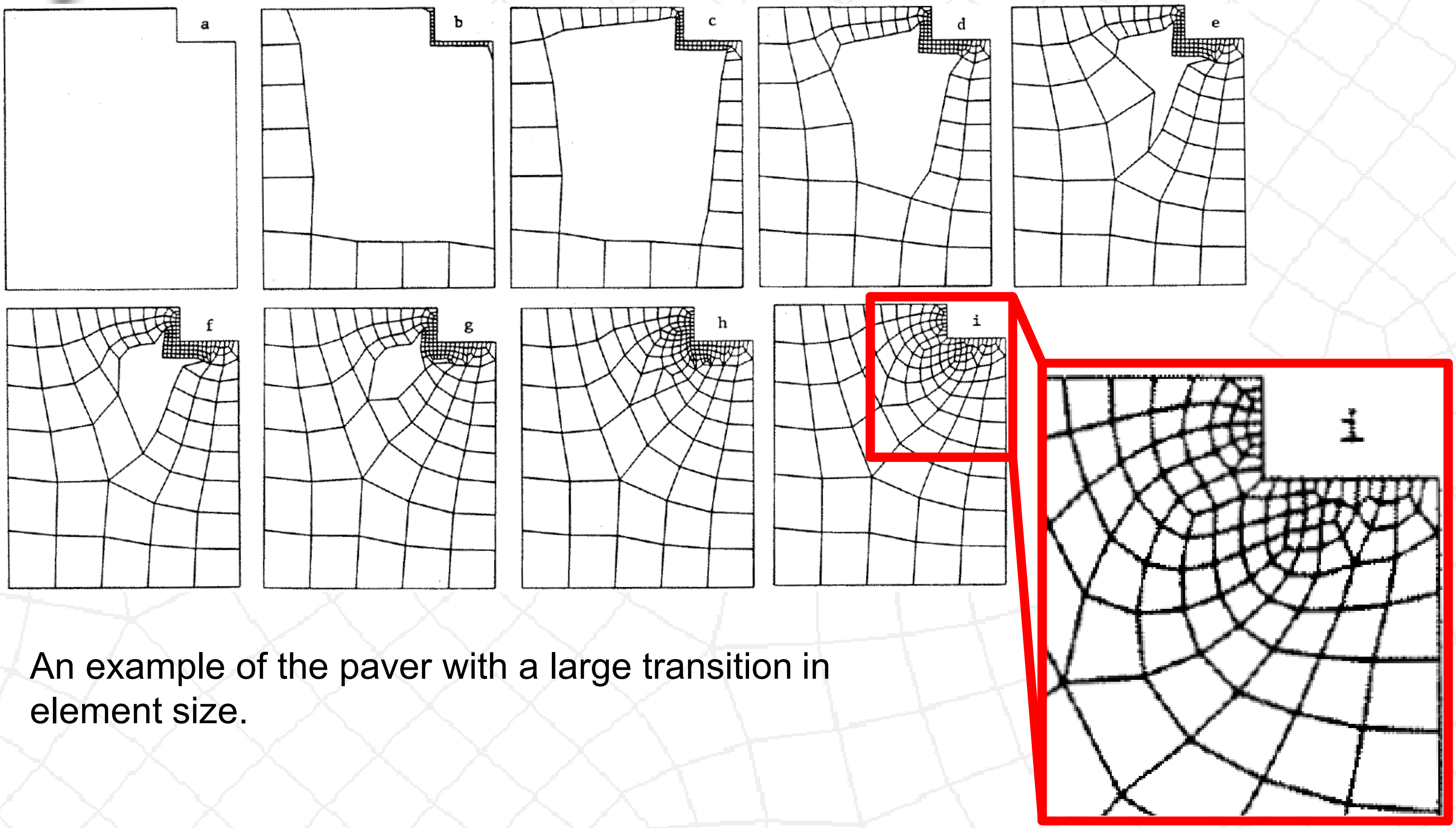


Paving Examples





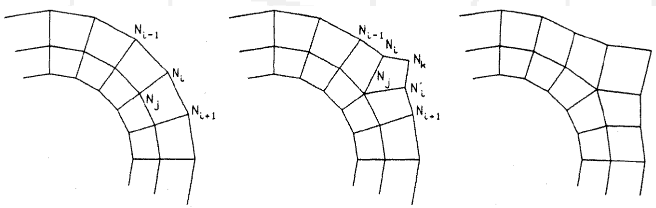
Paving Examples



An example of the paver with a large transition in element size.

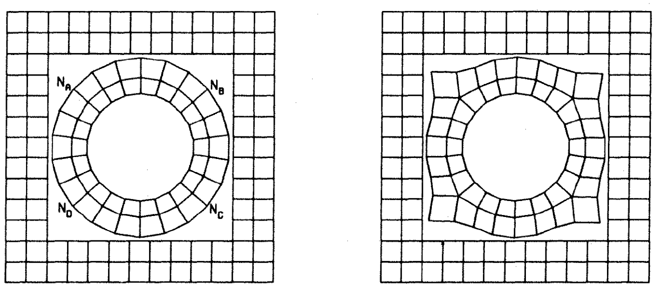
Size Control & Transition in Paving

Inserting wedges to stop element expansion



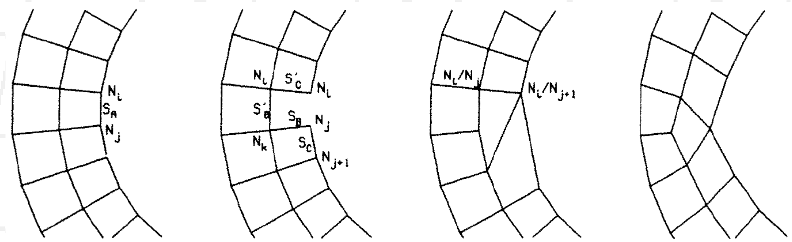
(a) Original Mesh (b) Wedge Insertion (c) Smoothed Mesh

Figure 23. The insertion of a wedge to correct element size expansion



(a) Boundary Before Wedges (b) Boundary After Wedges

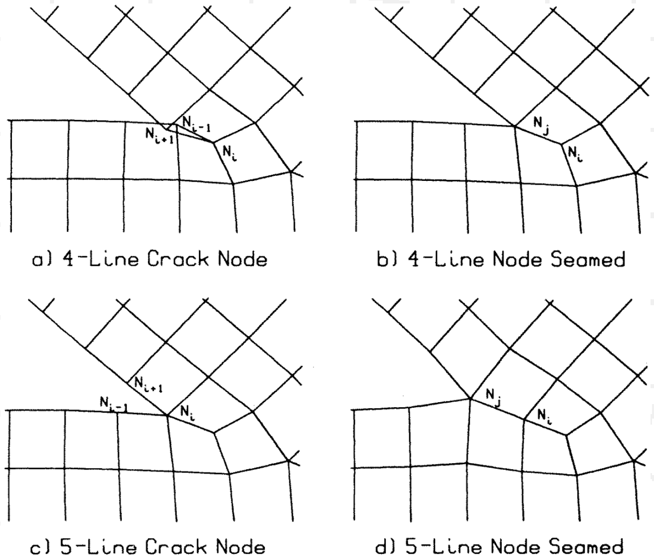
Inserting tucks to stop elements from contracting



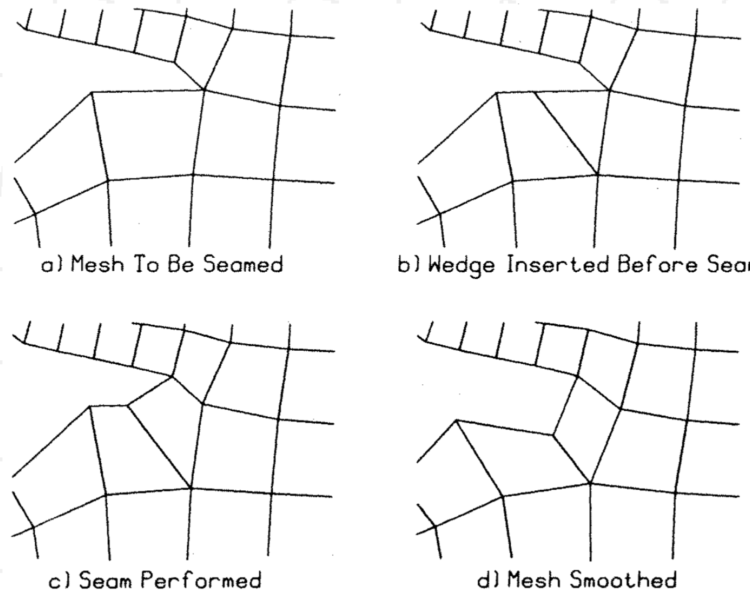
(a) Original Mesh (b) Element Deleted (c) Sides Superimposed (d) Smoothed Mesh

Front Seaming in Paving

Seaming 2 adjacent rows that are about the same size

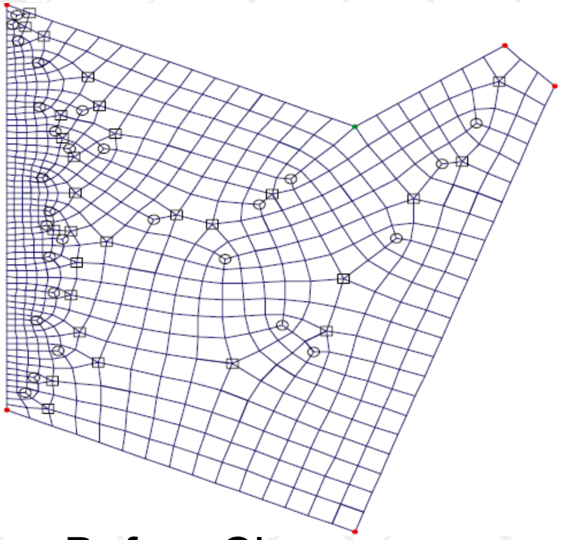


Seaming 2 adjacent rows of different sizes

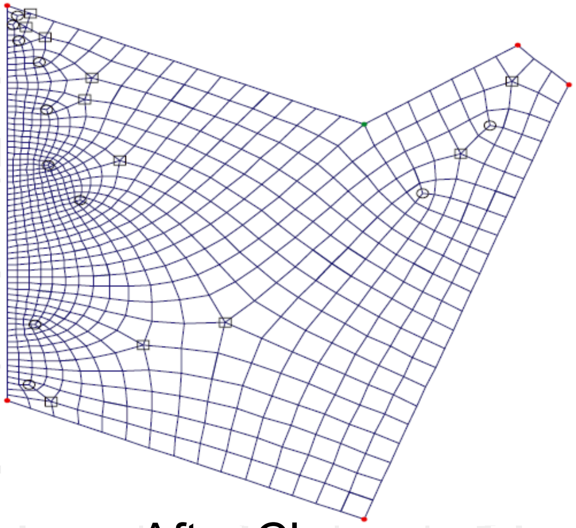


Quad Cleanup

After generating an unstructured quad mesh, with paving, or any other method, you will often want to call a quad "Cleanup" procedure. Cleanup tweaks the element connectivity to reduce the number of "irregular" nodes (i.e. nodes with something other than 4 attached quads).



Before Cleanup



After Cleanup

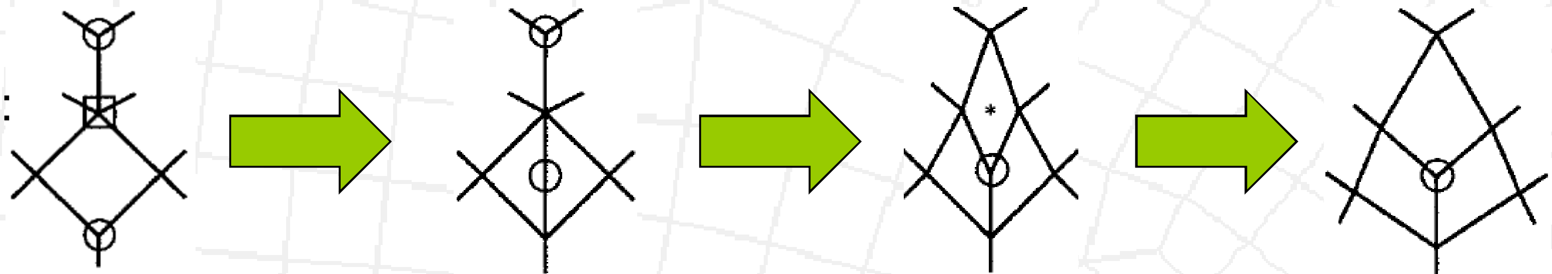
Bunin, Guy, "Non-Local Topological Clean-up", 15th International Meshing Roundtable, 2006.

Canann, S. A. "Topological Improvement Procedures for Quadrilateral Finite Element Meshes," Engineering With Computers 14: 168-177, 1998.

Canann Cleanup Example

Canann looks for specific patterns of irregular nodes adjacent to each other, then performs a recipe of swaps, opens, doublet inserts, and smoothing to locally improve the eliminate irregular nodes.

Example 1:



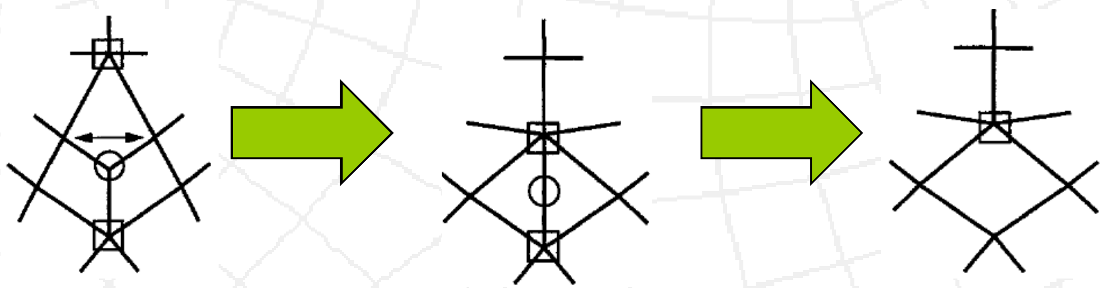
3-5-opposite-3 pattern. Contains 3 irregular nodes.

Insert a 2-valent node.

Open a quad face along 2 edges.

Smooth. Contains 1 irregular node.

Example 2:



5-3-opposite-5 pattern. Contains 3 irregular nodes.

Collapse a quad face.

Remove a doublet.

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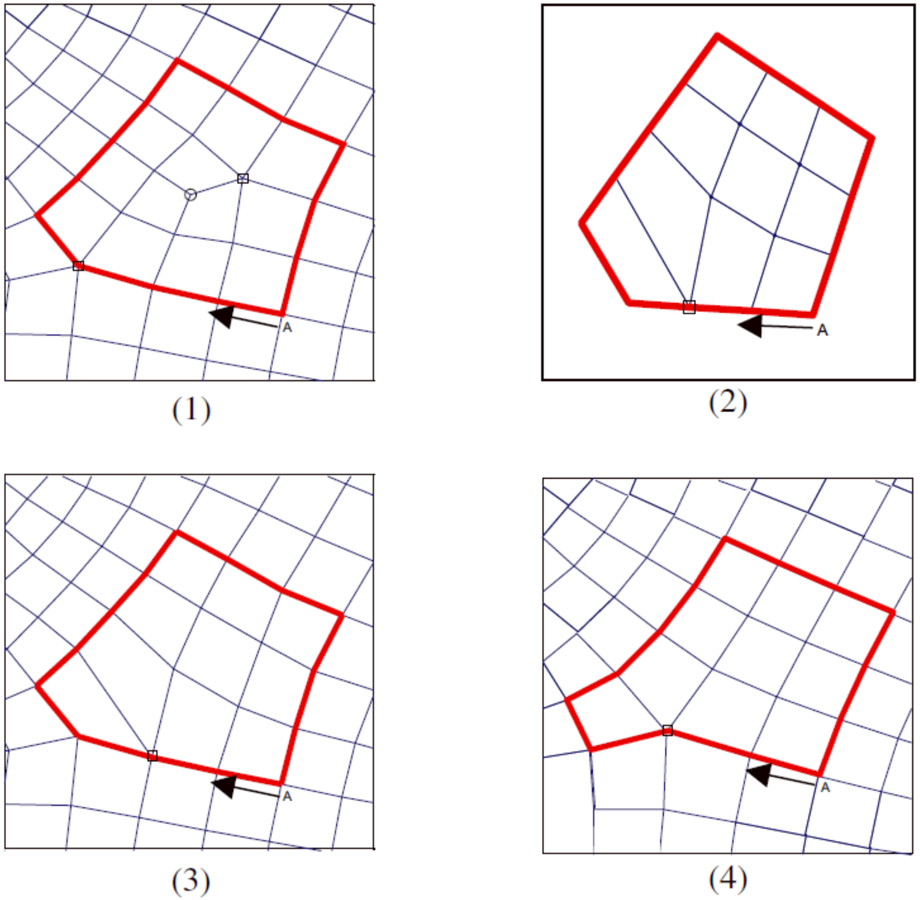
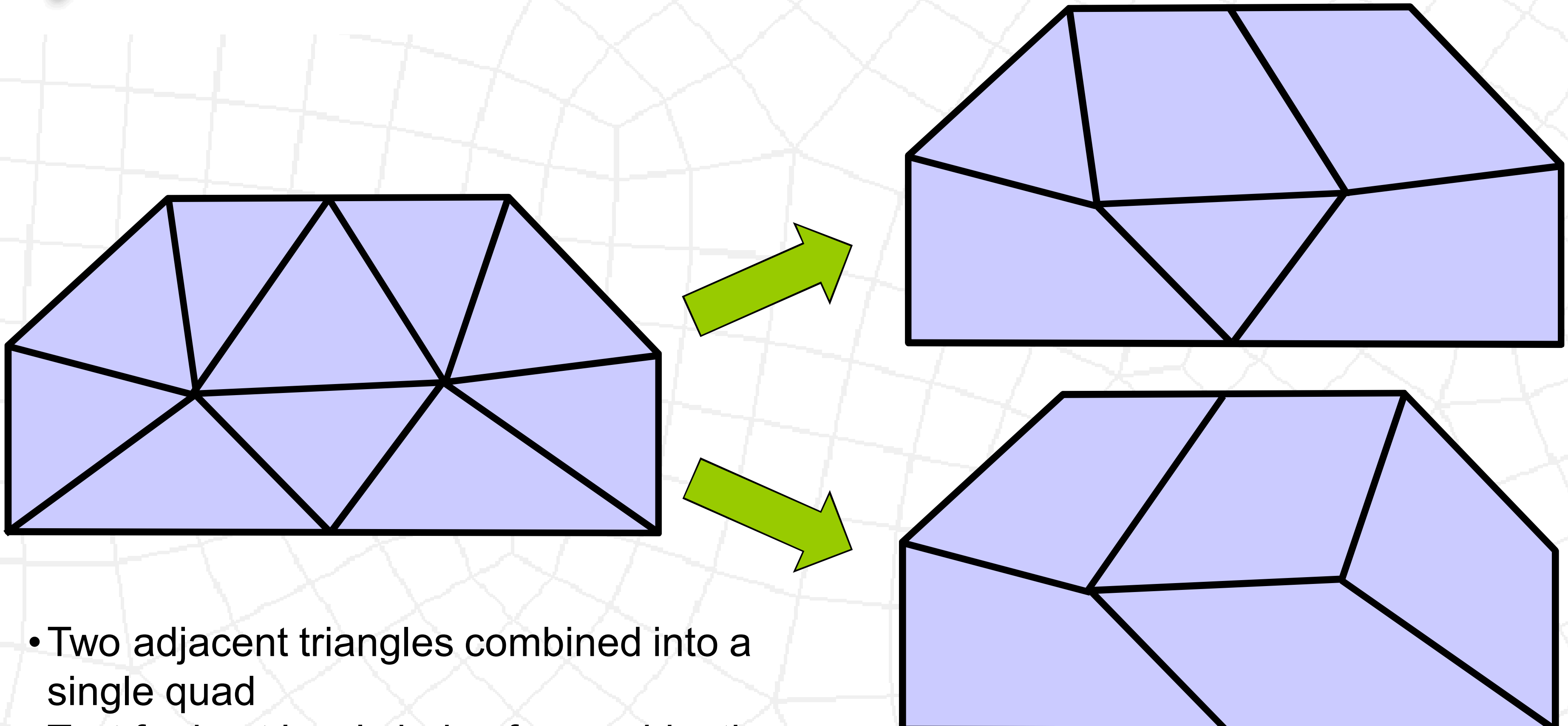
Bunin Cleanup Example

Bunin finds several "irregular" nodes close to each other, surrounds them with several layers of quad elements. Then this patch of quads is replaced with a pattern which minimizing the number of irregular nodes inside the loop.



Triangle Merging



- Two adjacent triangles combined into a single quad
- Test for best local choice for combination
- Triangles can remain if attention is not paid to order of combination

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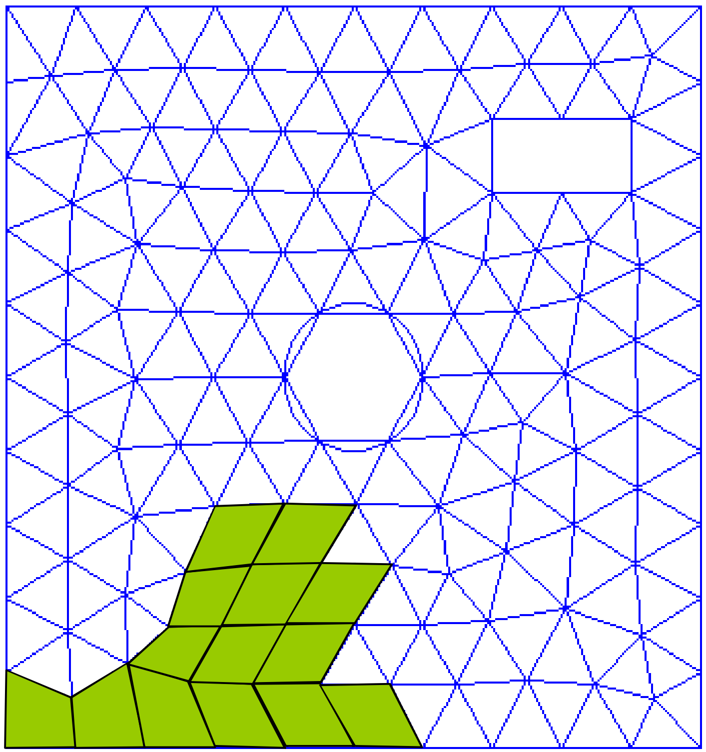
Directed Triangle Merging
Lee, Lo, 1994

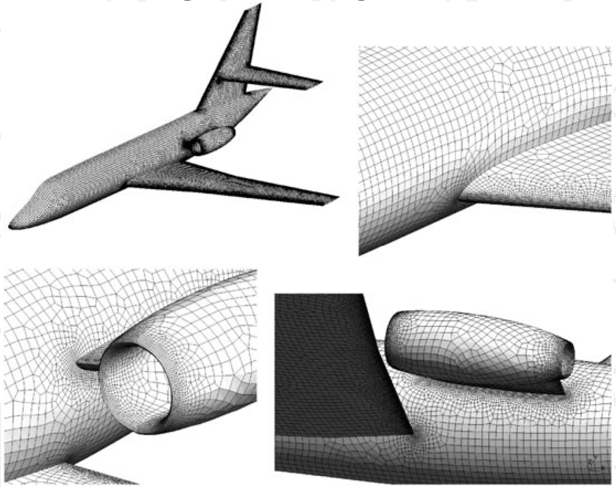
- Merging begins at a boundary
- Advances from one set of triangles to the next
- Attempts to maintain even number of intervals on any loop
- Can produce all-quad mesh
- Can also incorporate triangle splitting
- (Lee and Lo, 94)



Blossom Quad

A newer triangle combination algorithm

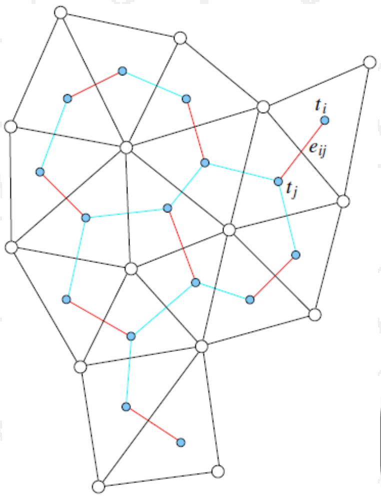
- Uses Blossom algorithm to optimize triangle combinations
- Uses the infinity norm during node placement to maximize right triangles in triangle mesh, maximizing quad quality.
- Open Source, http://www.geuz.org/gmsh/



References:

J.F. Remacle et. al., “A Frontal Delaunay Quad Mesh Generator Using L^∞ Norm,” International Journal for Numerical Methods in Engineering, 94:494-512, 2013.

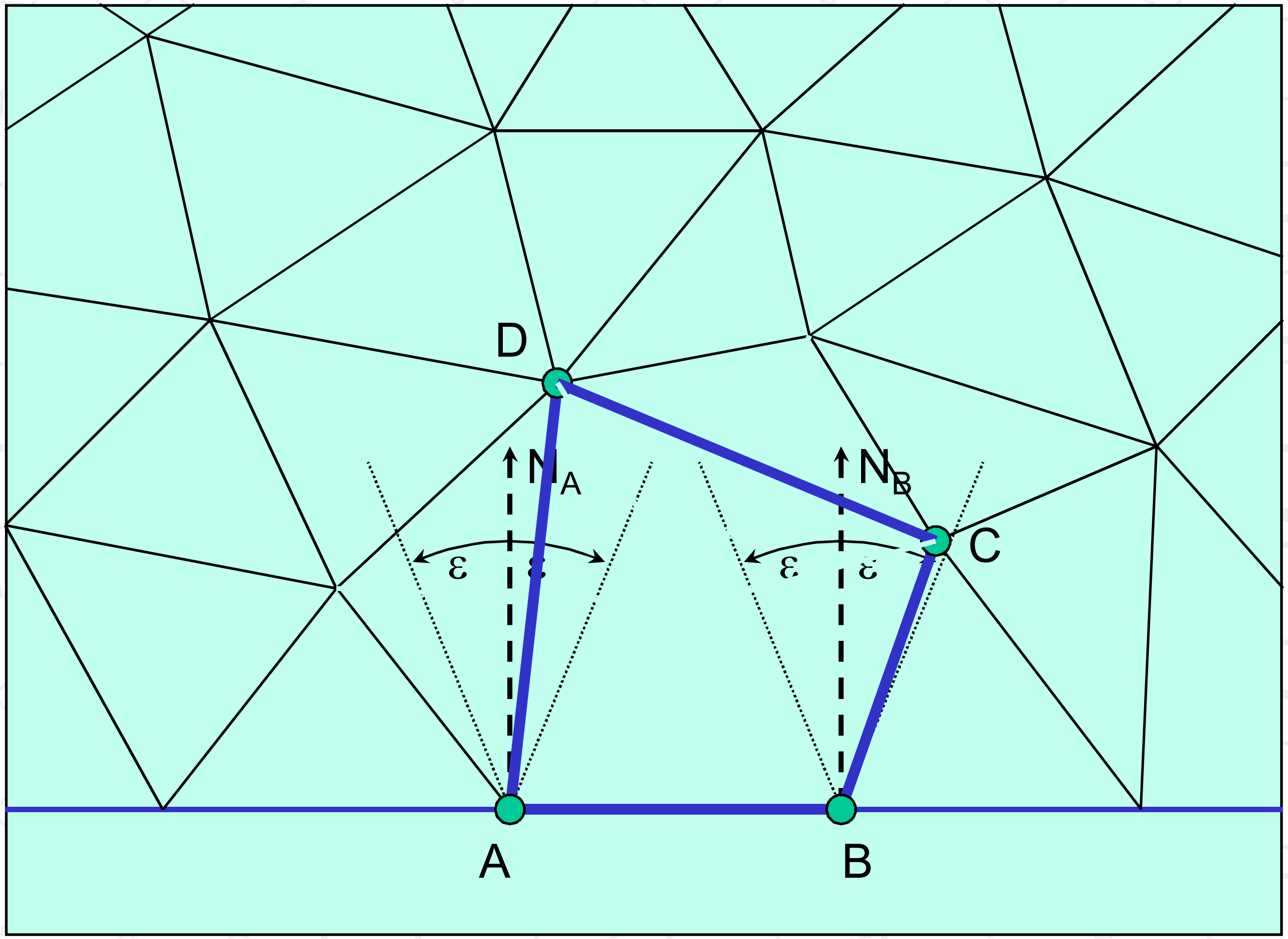
J.F. Remacle et. al., “Blossom-Quad: Anon-uniform quadrilateral mesh generator using a minimum-cost perfect-matching algorithm,” International Journal for Numerical Methods in Engineering, 89:1102-1119, 2012.



Q-Morph, Merges
Paving & Triangle Combining

Triangle Merge with local transformations

- Uses an advancing front approach
- Local swaps applied to improve resulting quad
- Any number of triangles merged to create a quad
- Attempts to maintain even number of intervals on any loop
- Produces all-quad mesh from even intervals
- (Owen, 99)



Q-Morph

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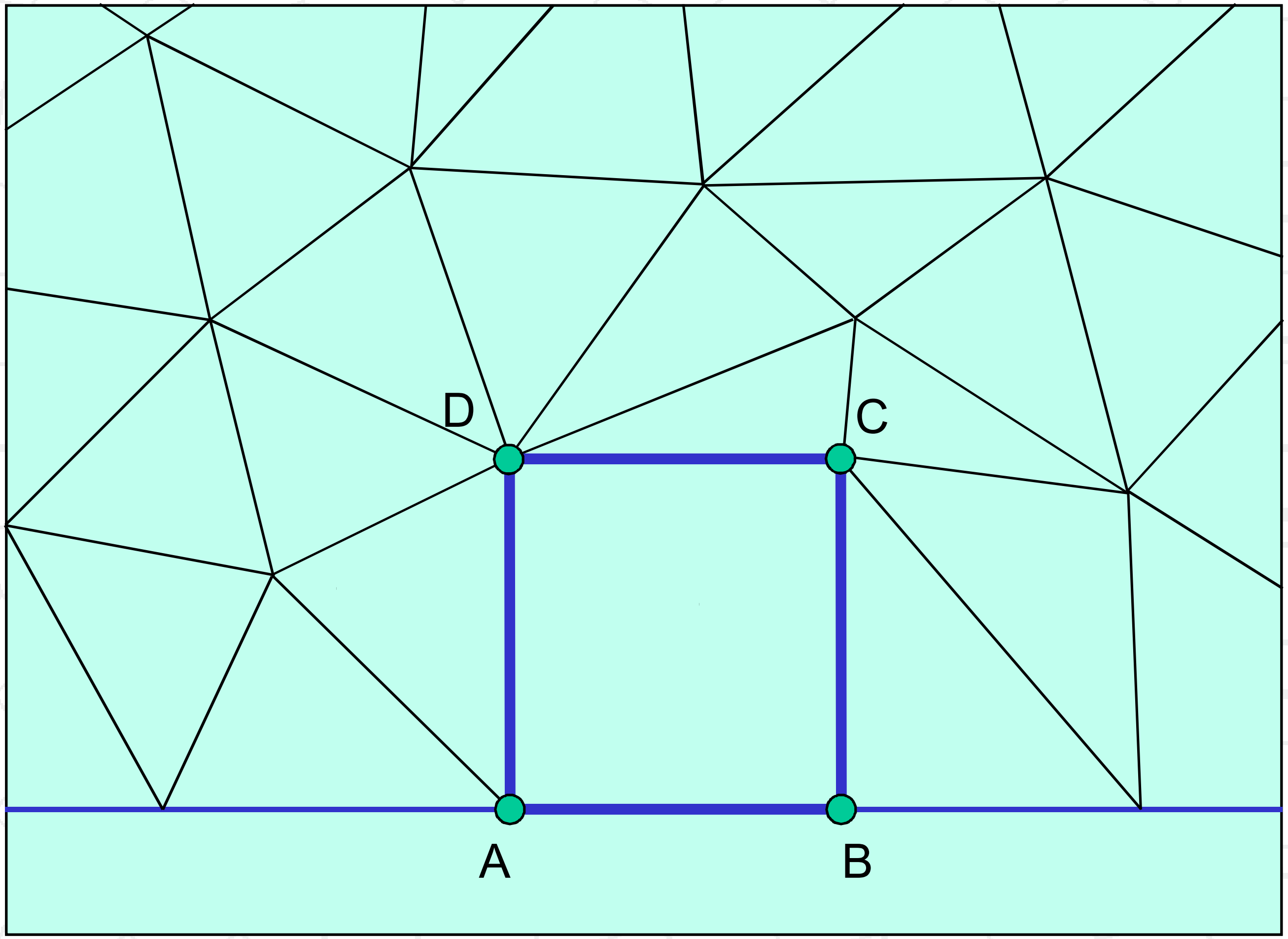
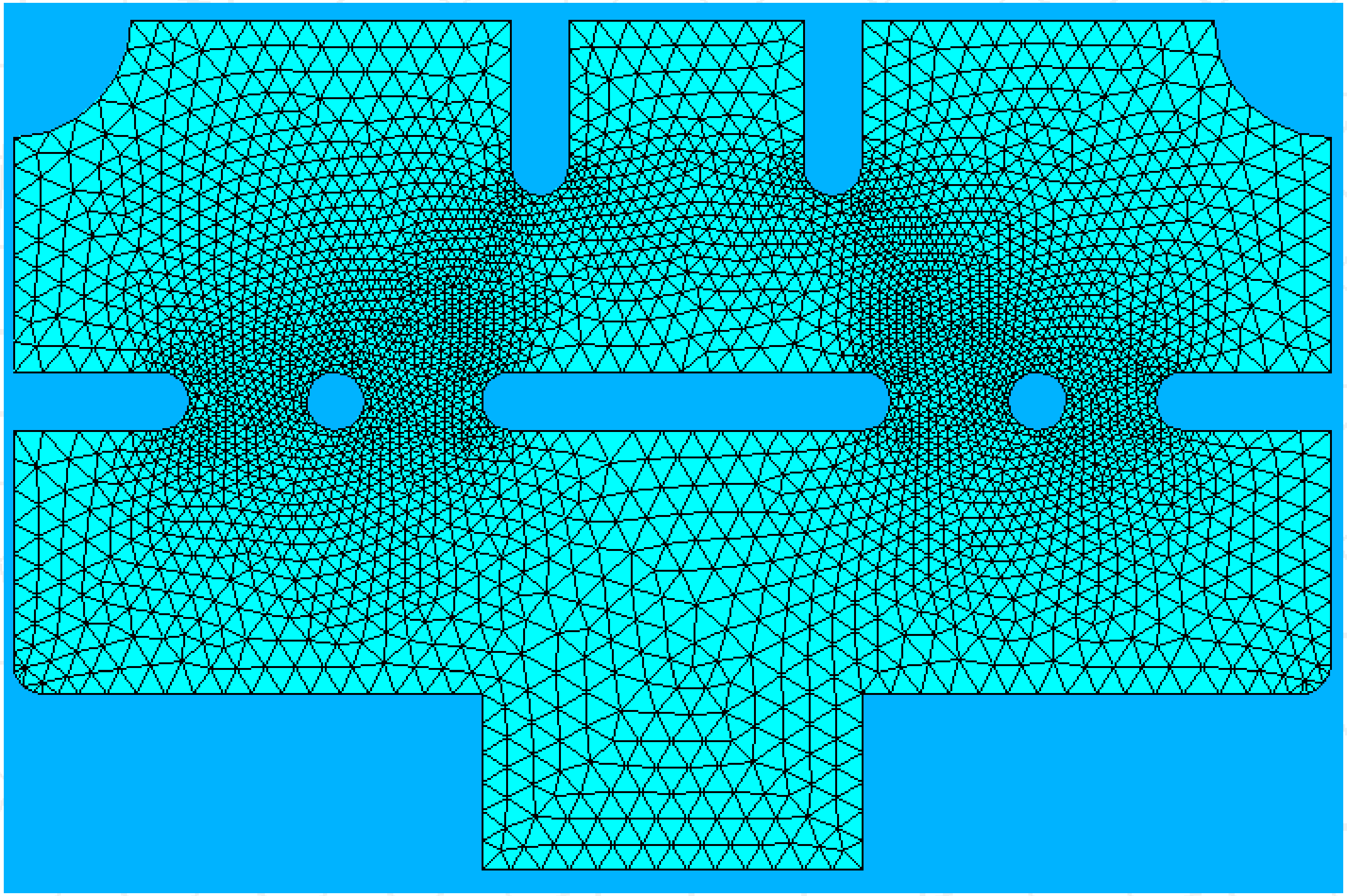
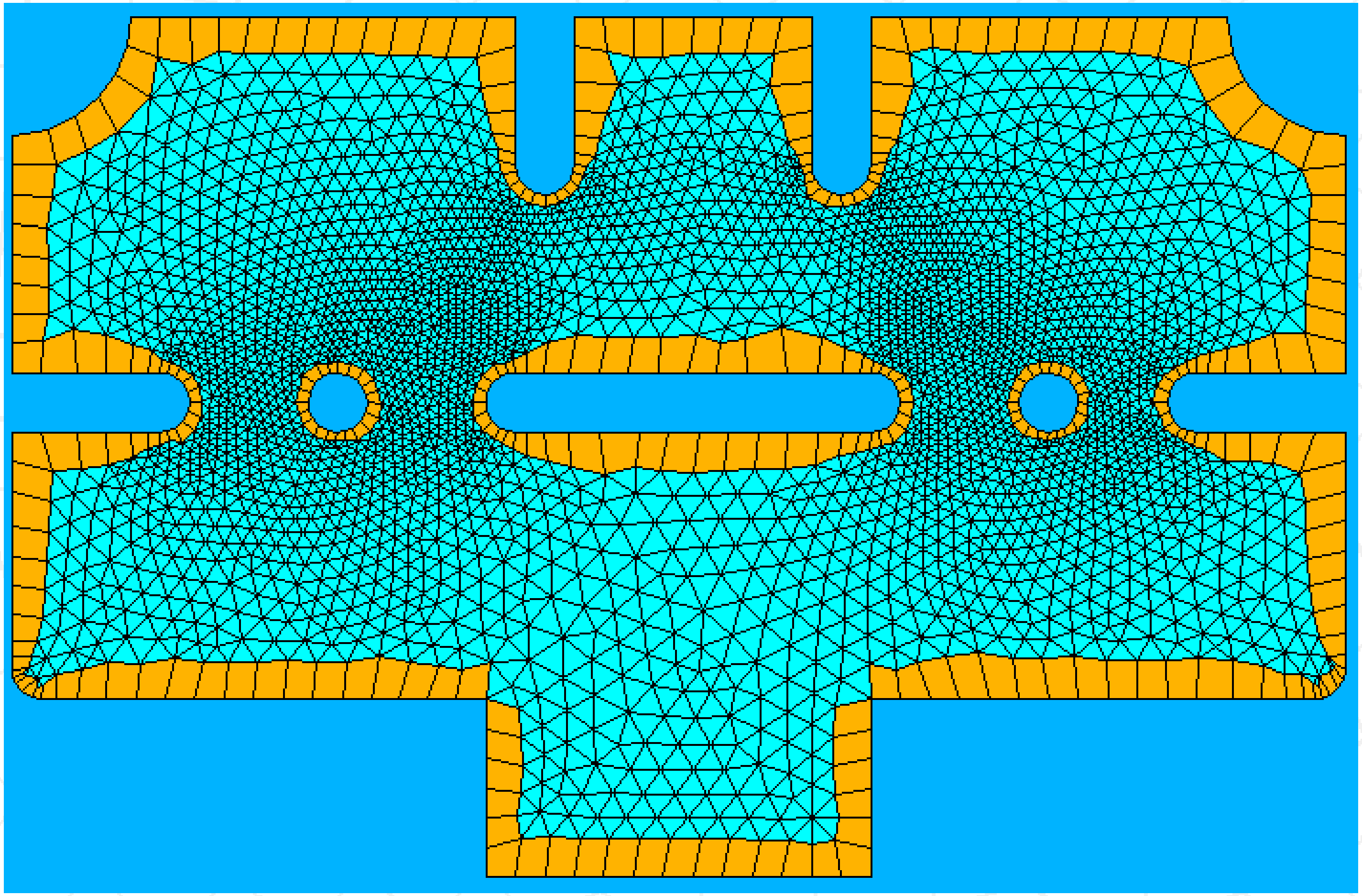


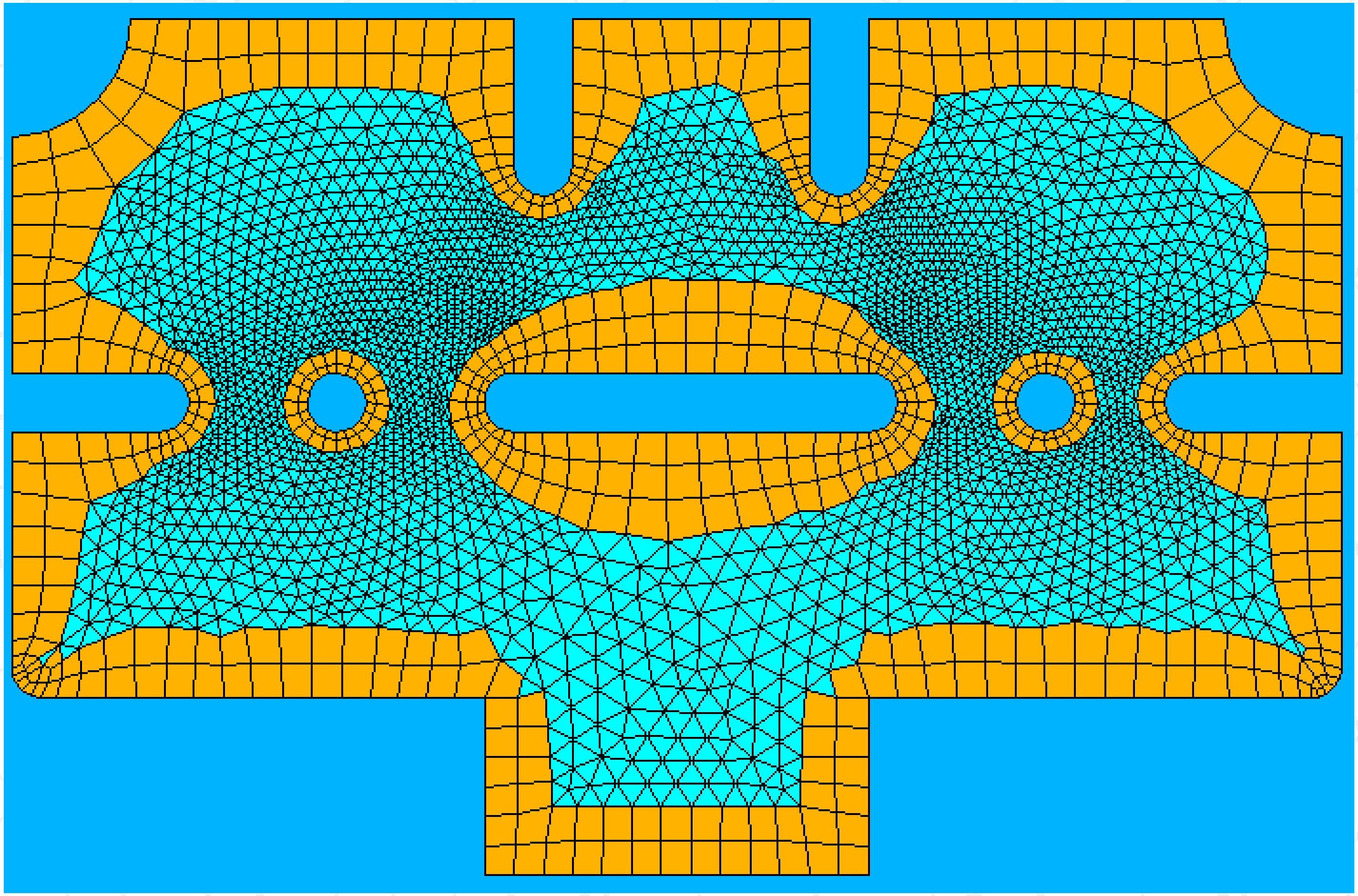
Q-Morph



Q-Morph



Q-Morph



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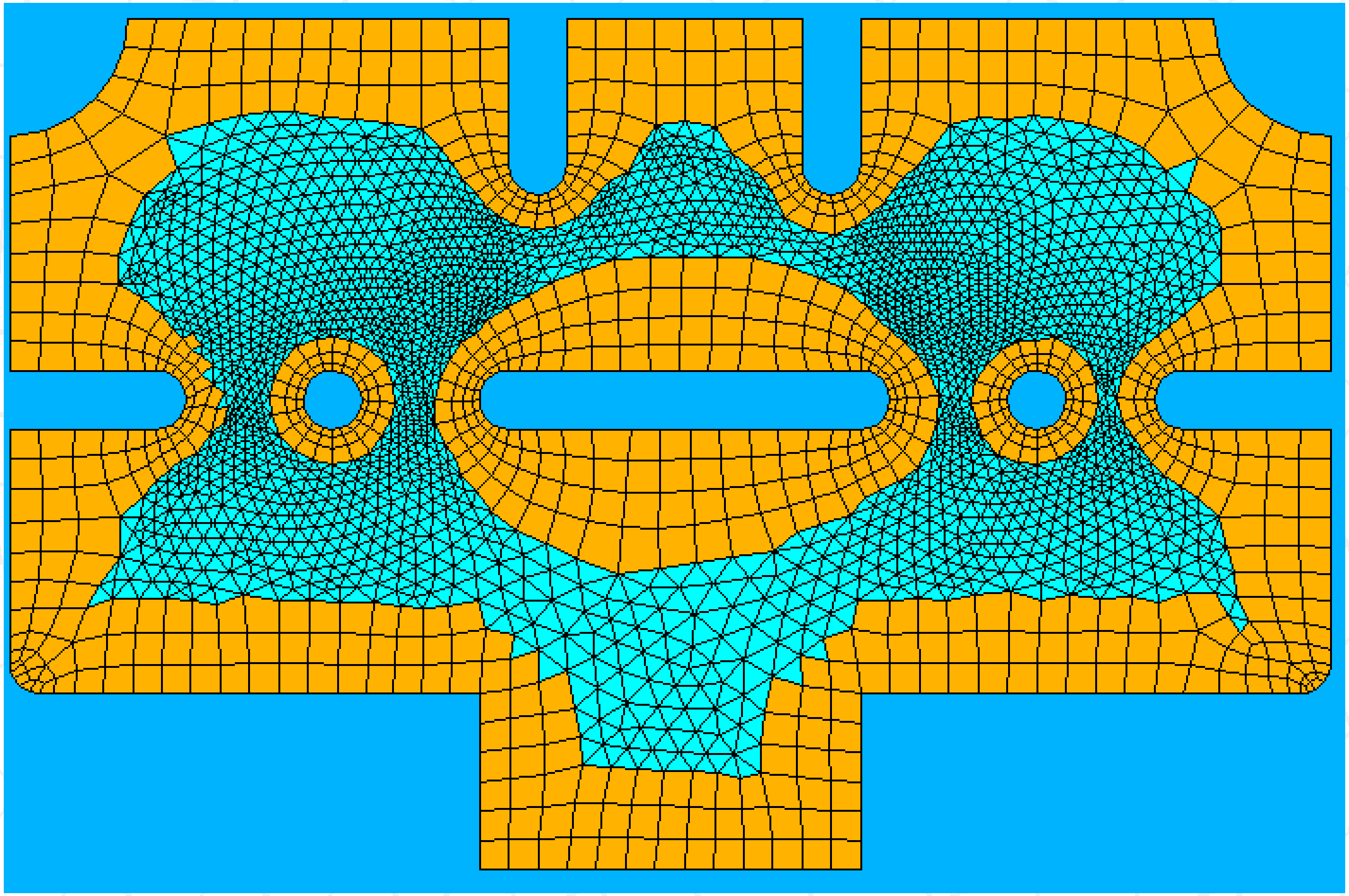
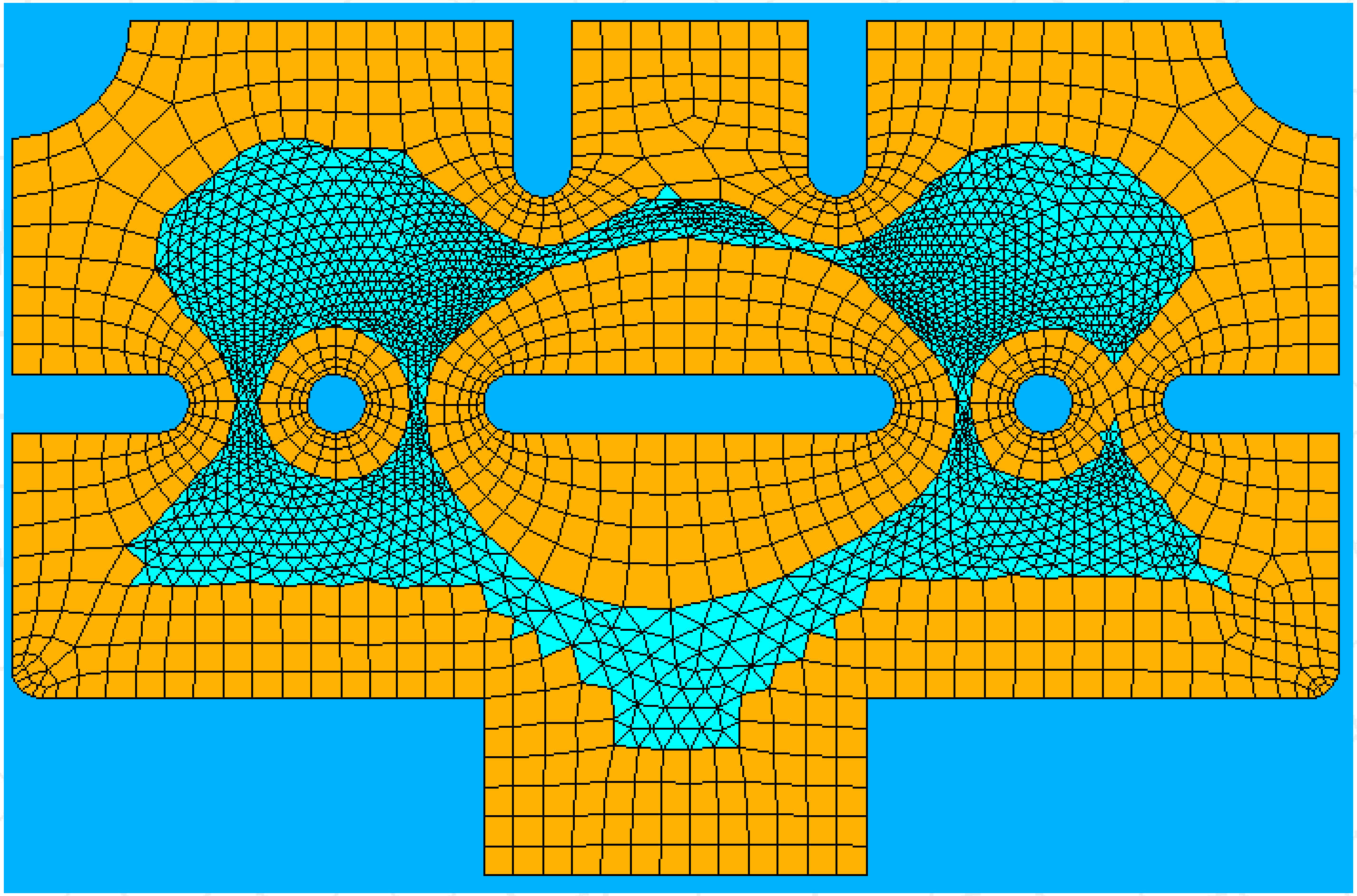


Q-Morph



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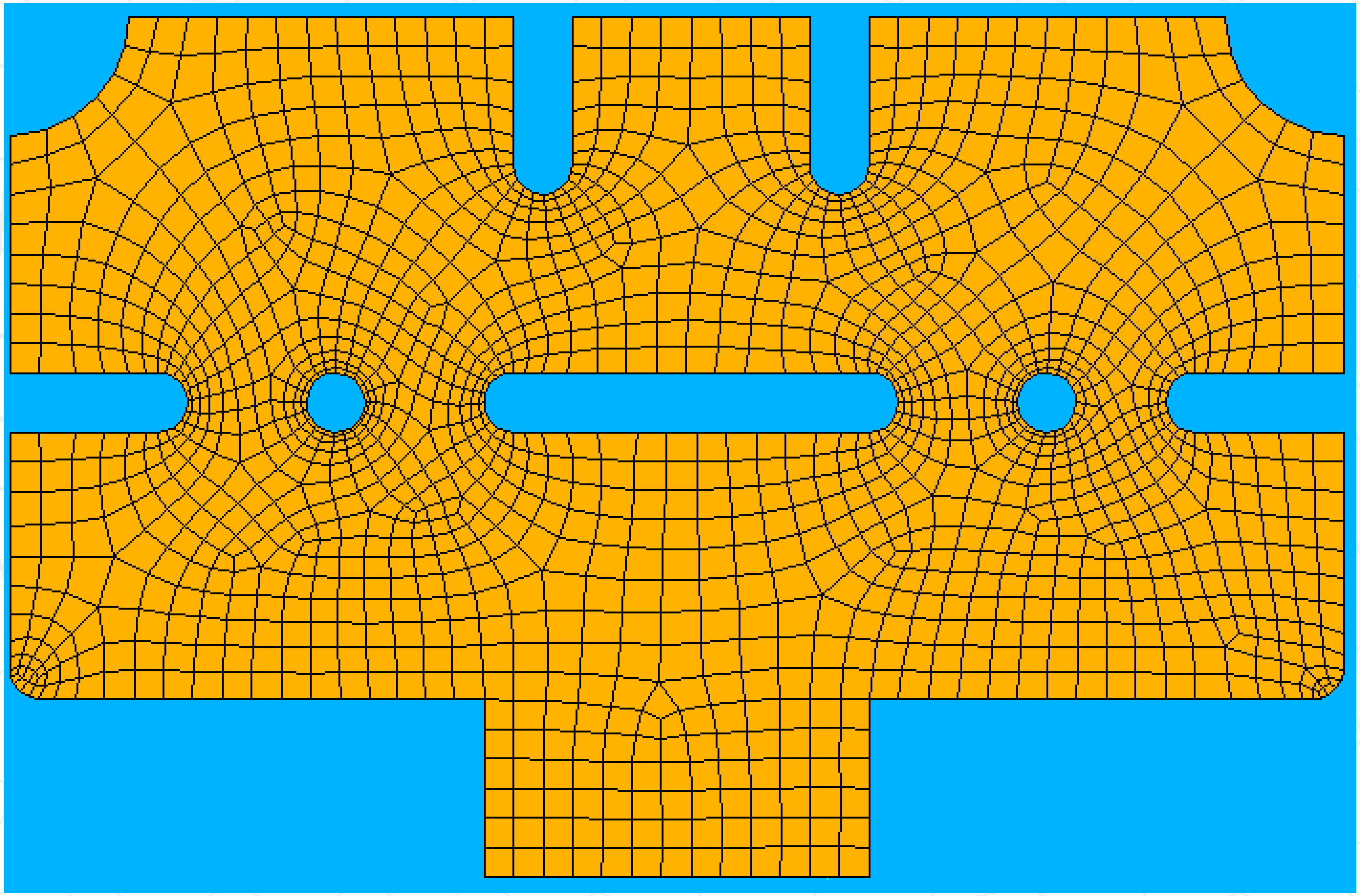
Q-Morph



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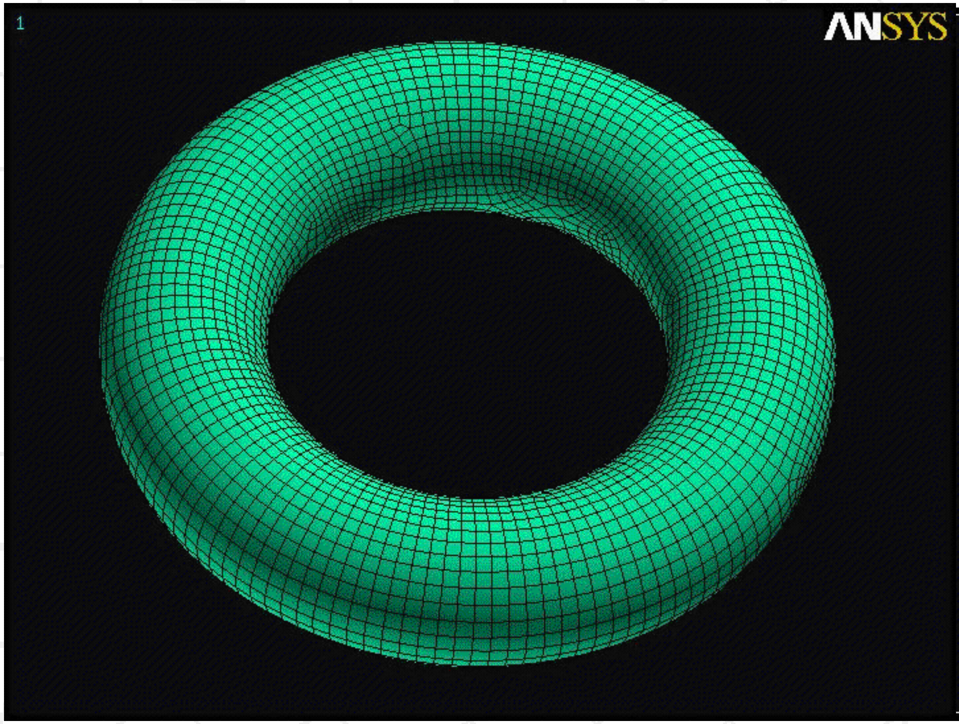
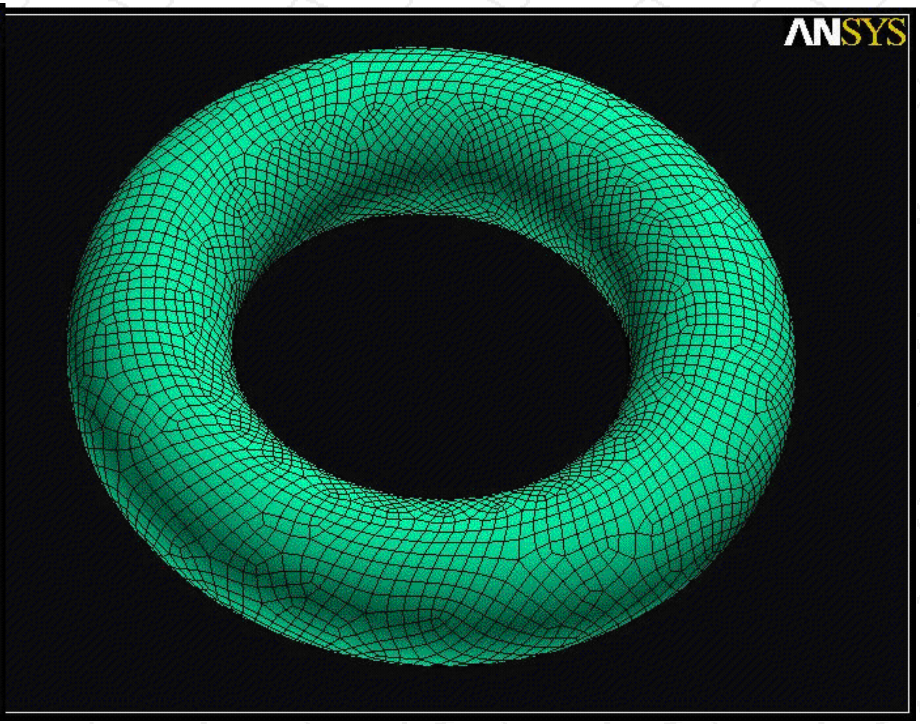
Q-Morph



Q-Morph



Lee, Lo Triangle Combine Method

1-to-1, Many-to-1, Many-to-Many, Partition and Swee

SWEEPING

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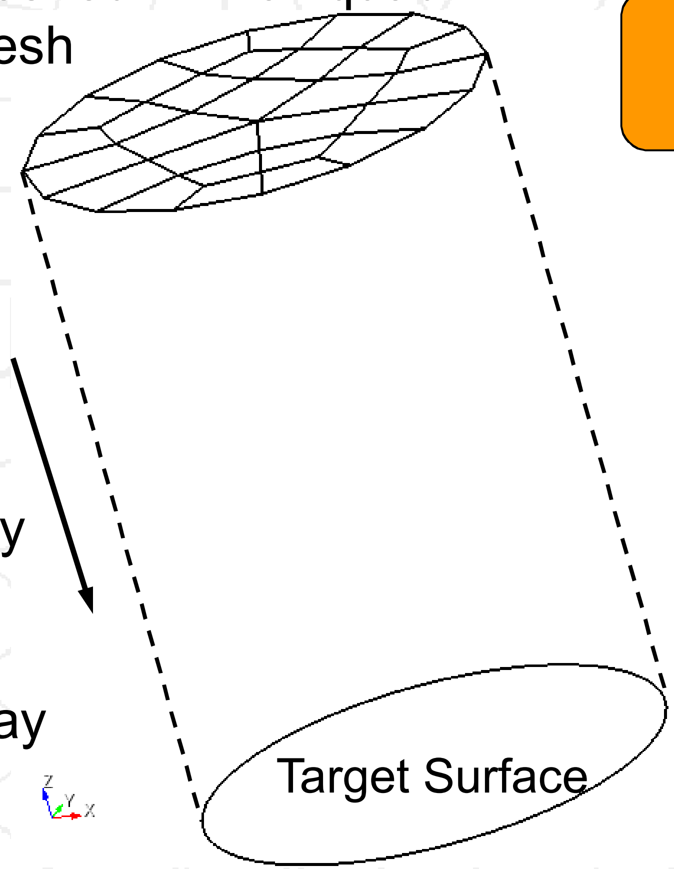
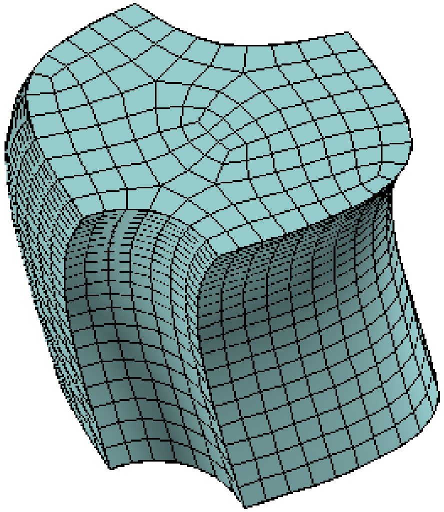
Sweeping

Source surface is meshed with all quad mesh

1-to-1 sweepable

Sweep Direction





2.5D

Source/Targets may be non-planar.

Sweep direction may be non-linear.

Matt Staten

Sweeping

Sweeper Input

- Source surface meshed with quad mesh
- Linking surfaces meshed with a structured mapped mesh.
- Optionally meshed target with quads matching source.

Output

- Optionally mesh on target surface
- Interior node locations
- Hex connectivity

Geoemtry Characteristics

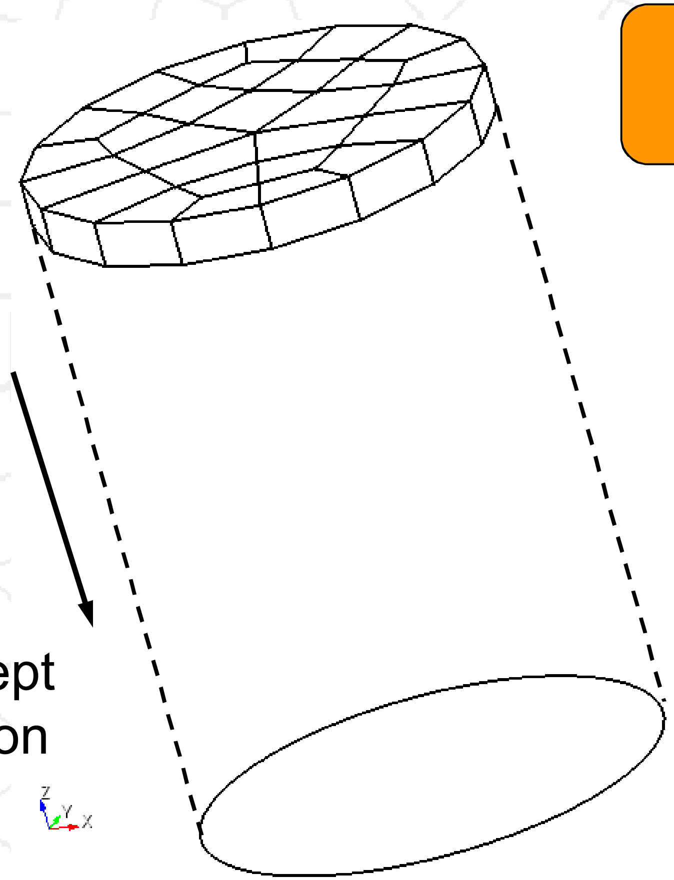
- 2.5D
- Source/Targets may be non-planar.
- Sweep direction may be non-linear.

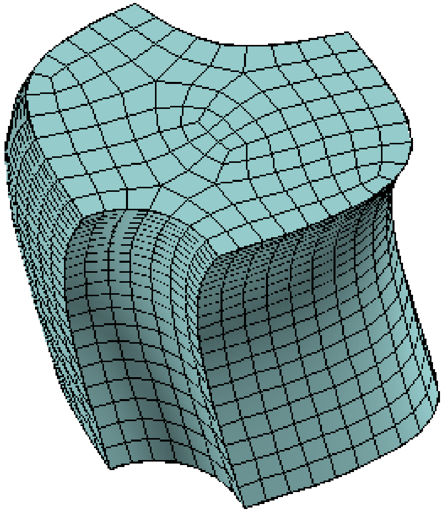
Sweeping

1-to-1 sweepable

Sweep Direction

Source mesh is swept along sweep direction towards the targets.





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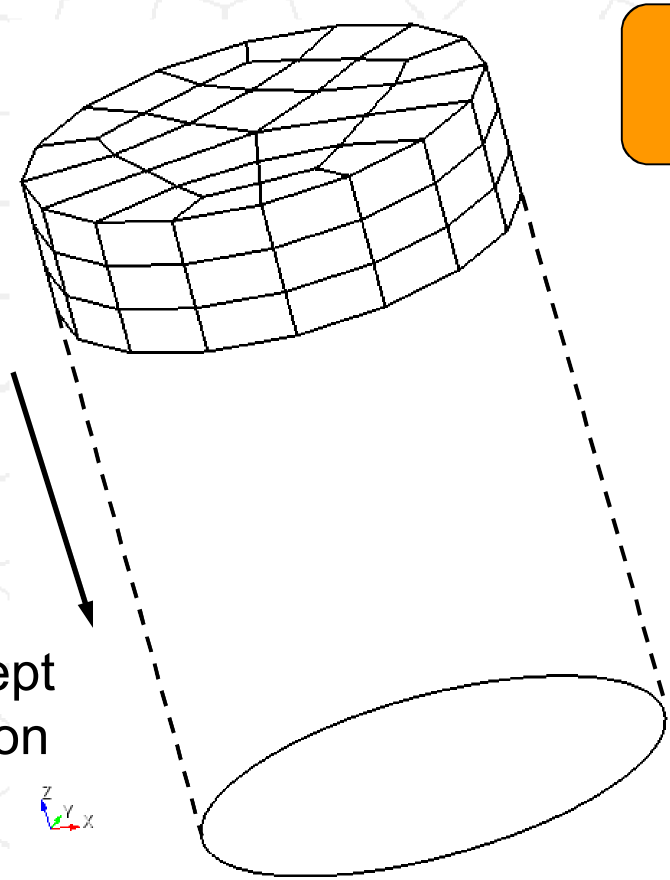
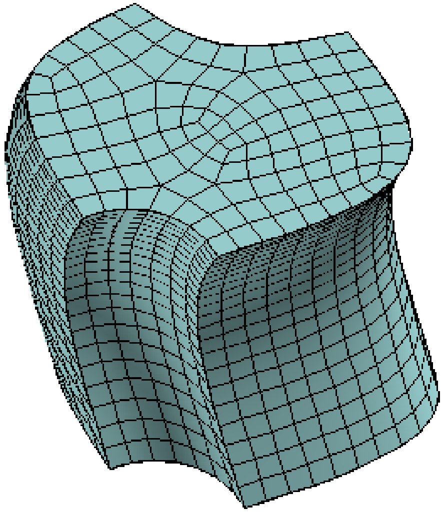
Sweeping

1-to-1 sweepable

Sweep Direction



Source mesh is swept along sweep direction towards the targets.



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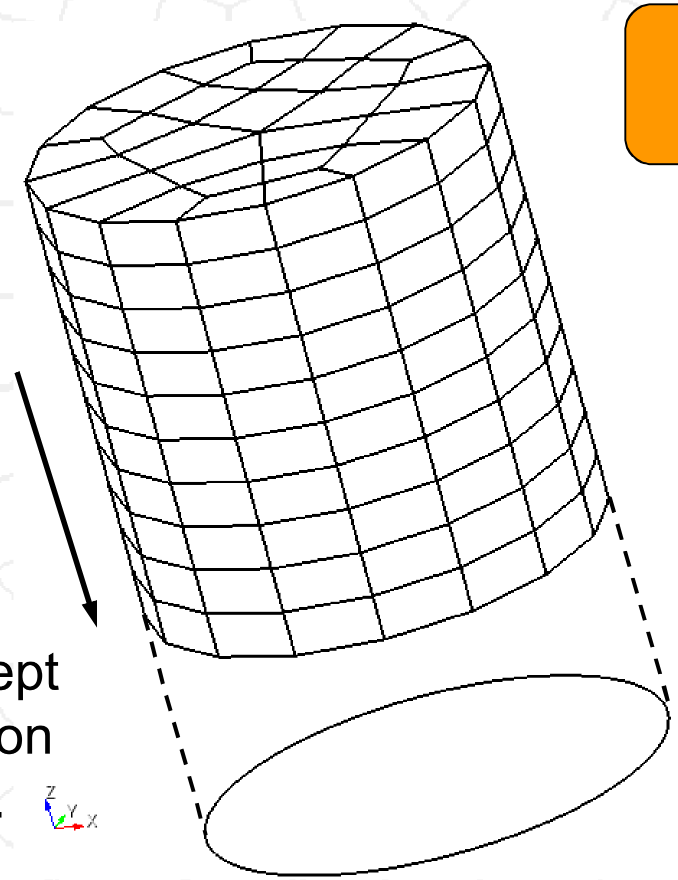
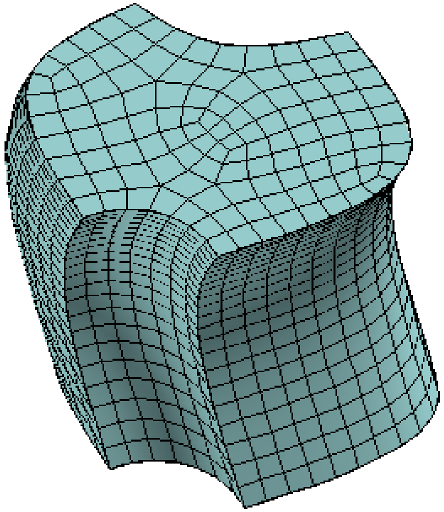
Sweeping

1-to-1 sweepable

Sweep Direction

Source mesh is swept along sweep direction towards the targets.





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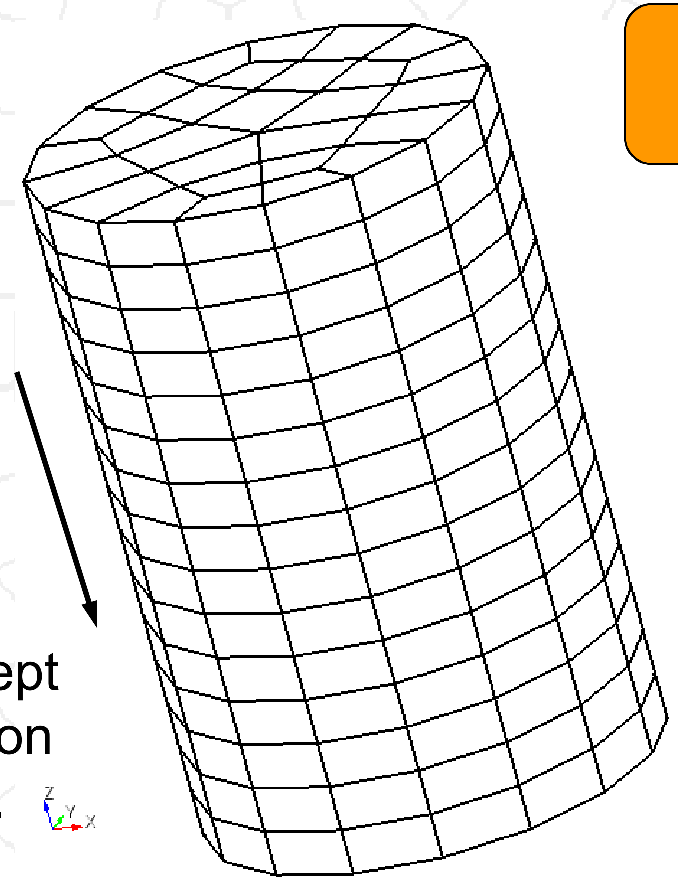
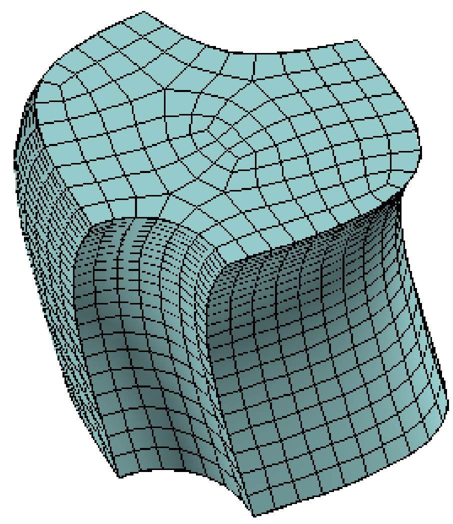
Sweeping

Sweep Direction



Source mesh is swept along sweep direction towards the targets.

1-to-1 sweepable



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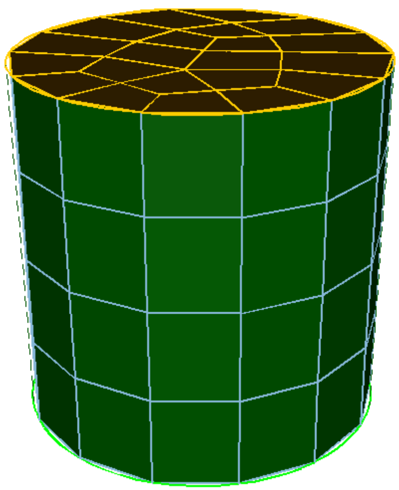
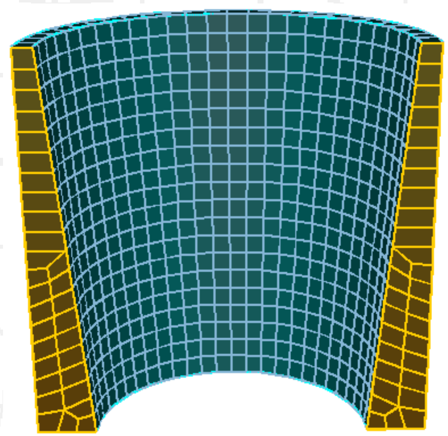
Sweeping

Typical one-to-one sweeps

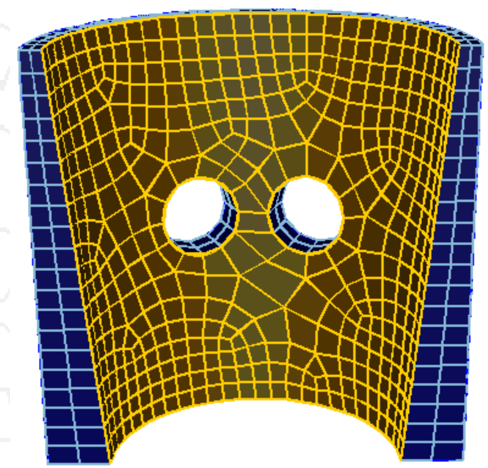
translation

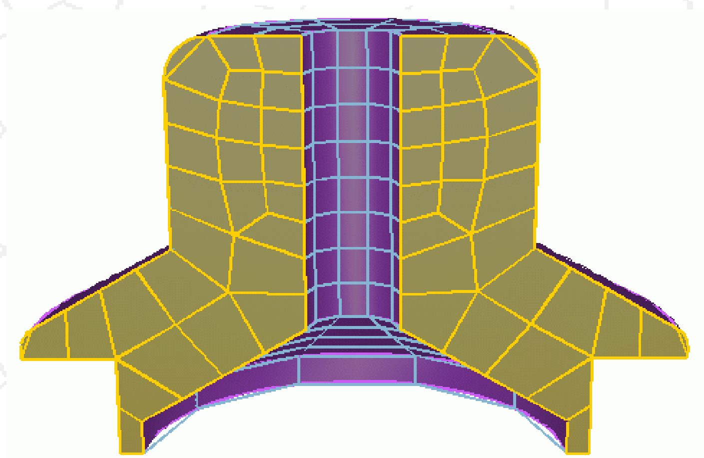


rotation

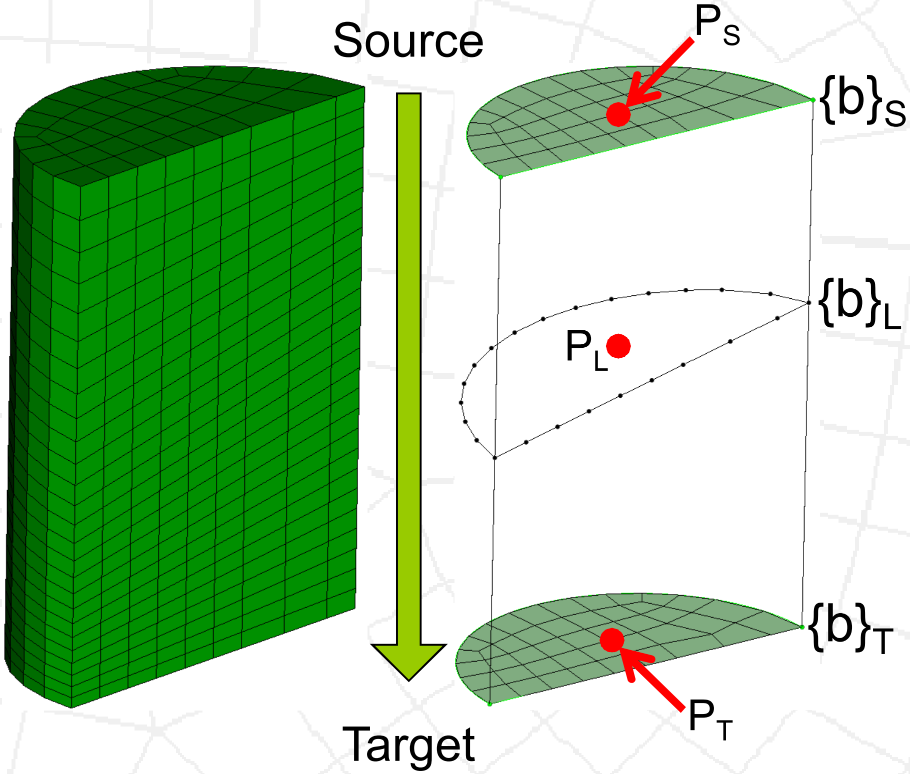


inside-out





One-to-one-Sweeping



Known: P_S, P_T, $\{b\}_S$, $\{b\}_T$, $\{b\}_L$
Unknown: P_L

Solve for $\boldsymbol{M_S}$: $\{b\}_S * \boldsymbol{M_S} = \{b\}_L$
Solve for $\boldsymbol{M_T}$: $\{b\}_T * \boldsymbol{M_T} = \{b\}_L$

Solve for SP_L: $SP_L = P_S * \boldsymbol{M_S}$ + boundary correction
Solve for TP_L: $TP_L = P_T * \boldsymbol{M_T}$

$P_L = w_S * SP_L + w_T * TP_L$

How to find w_S and w_T?

We parameterize each 'rib' (0 to1) along sweep path, each node on a rib gets a t value, which is ratio along sweep path. We then do inverse distance squared weighted interpolation of t values to compute w_S and w_T.

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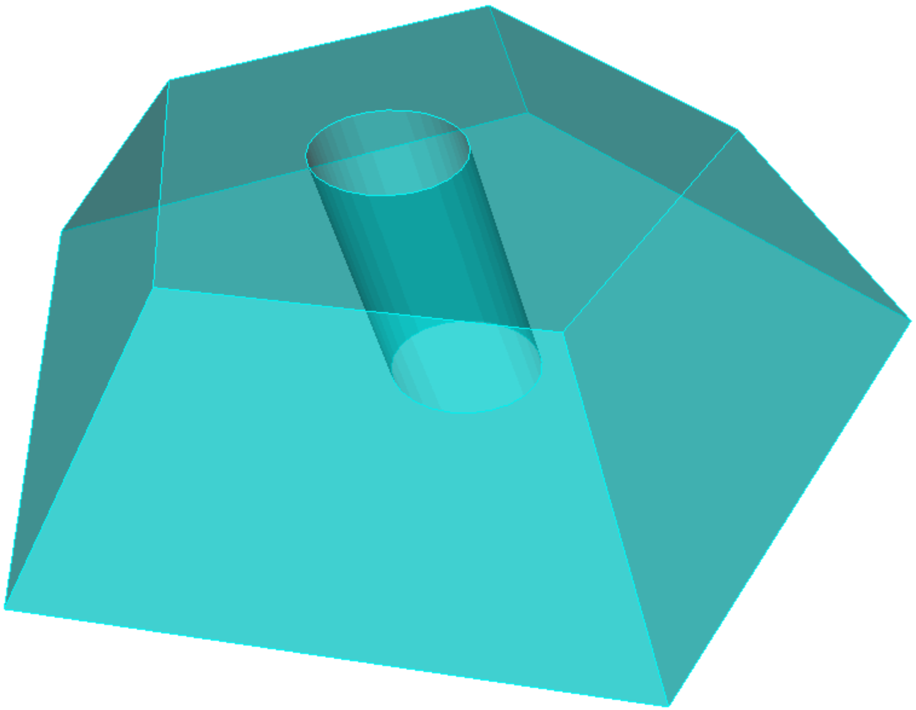
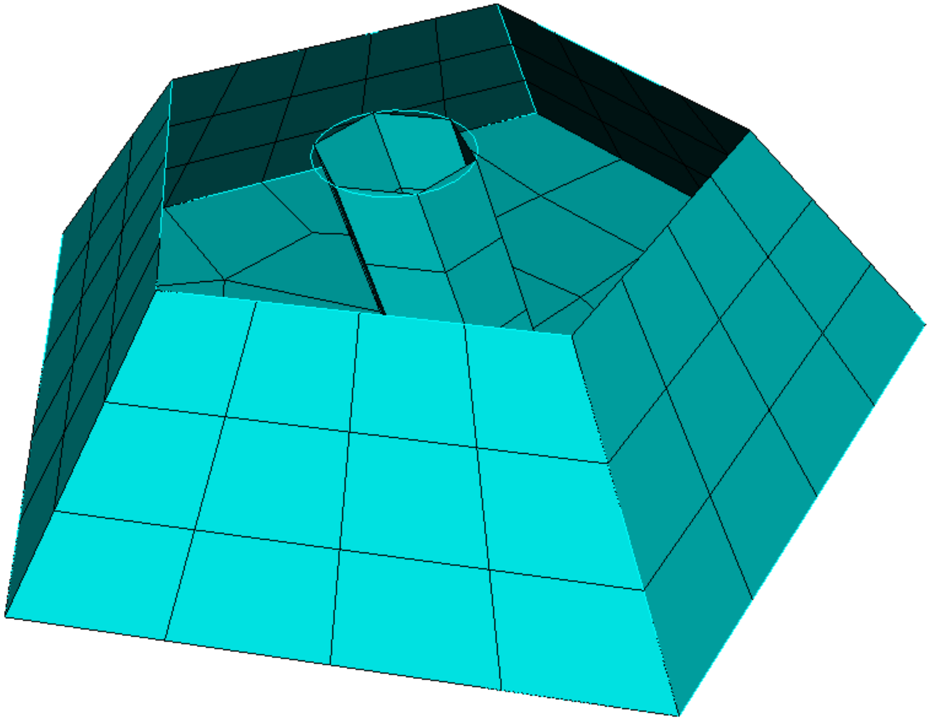
One-to-one-Sweeping, Finding M_S & M_T

An input volume that is 2.5D, with source and all wall faces meshed.

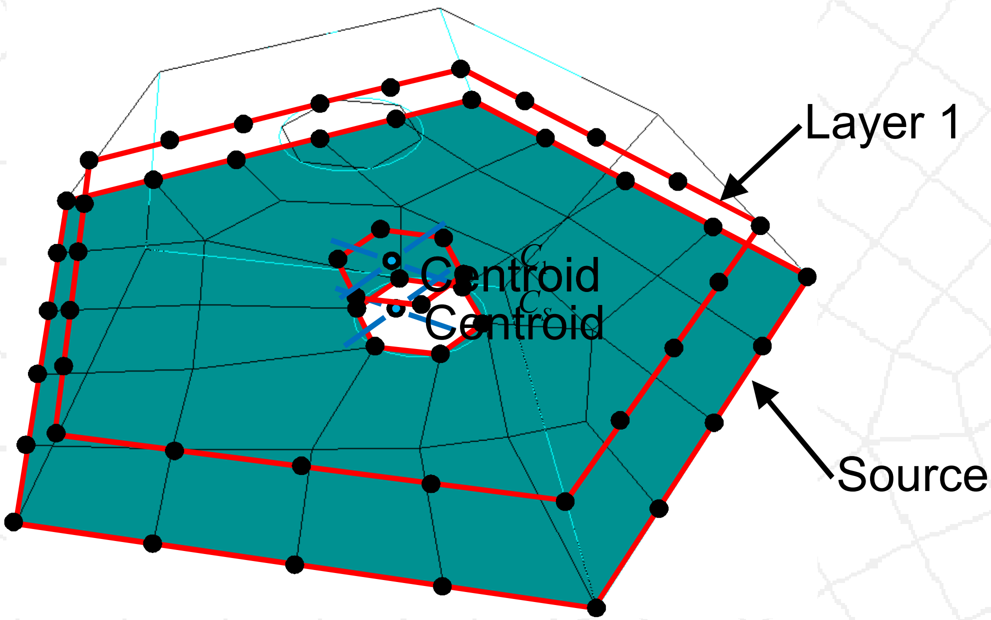




One-to-one-Sweeping, Finding M_S & M_T

M_S & M_T are 3X3 matricies computed as a least-squares approximation of an affine mapping.

$$\{b\}_S * \boldsymbol{M_S} = \{b\}_L$$

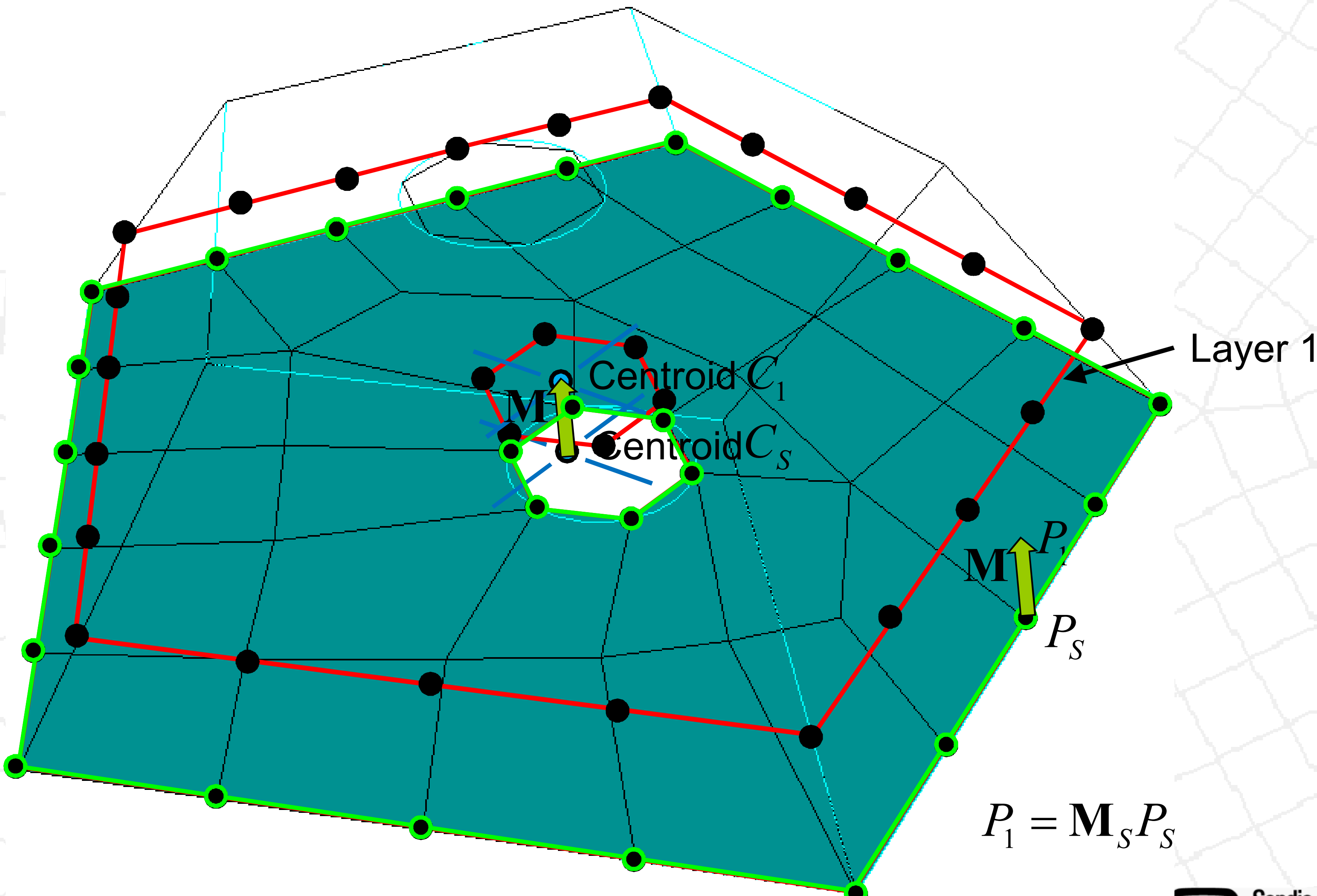


References:

P.M. Knupp, "Next-Generation Sweep Tool: A Method for Generation All-Hex Meshes on Two-and-One-Half Dimensional Geometries," Proceedings of the 7th International Meshing Roundtable, 1998.

X. Roca, J. Sarrate, A. Huerta, "Mesh Projection Between Parametric Surfaces," Communications in Numerical Methods in Engineering, 22:591-603, 2006.

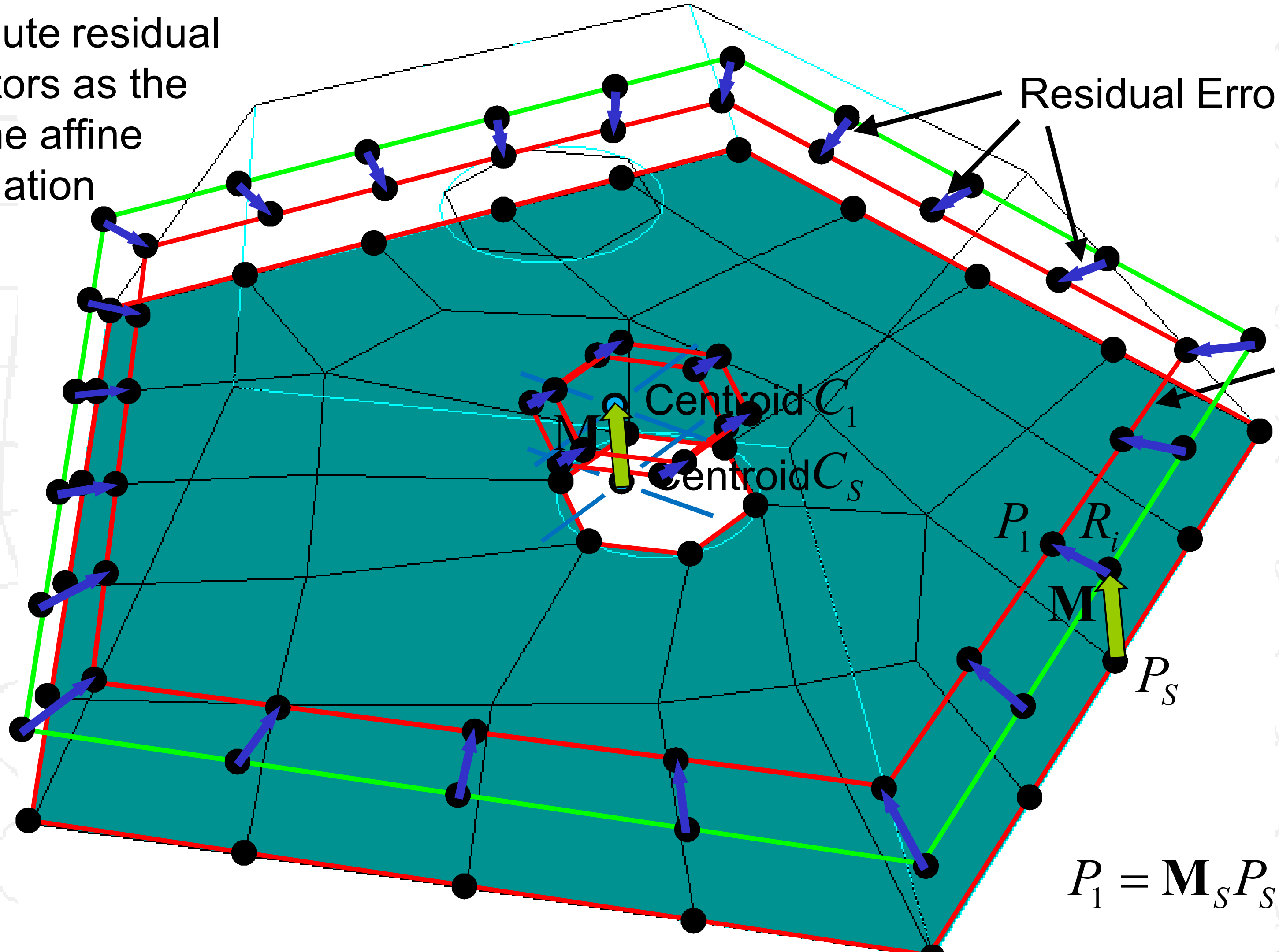
Weighted Residual Method



$$P_1 = \mathbf{M}_S P_S$$

Weighted Residual Method

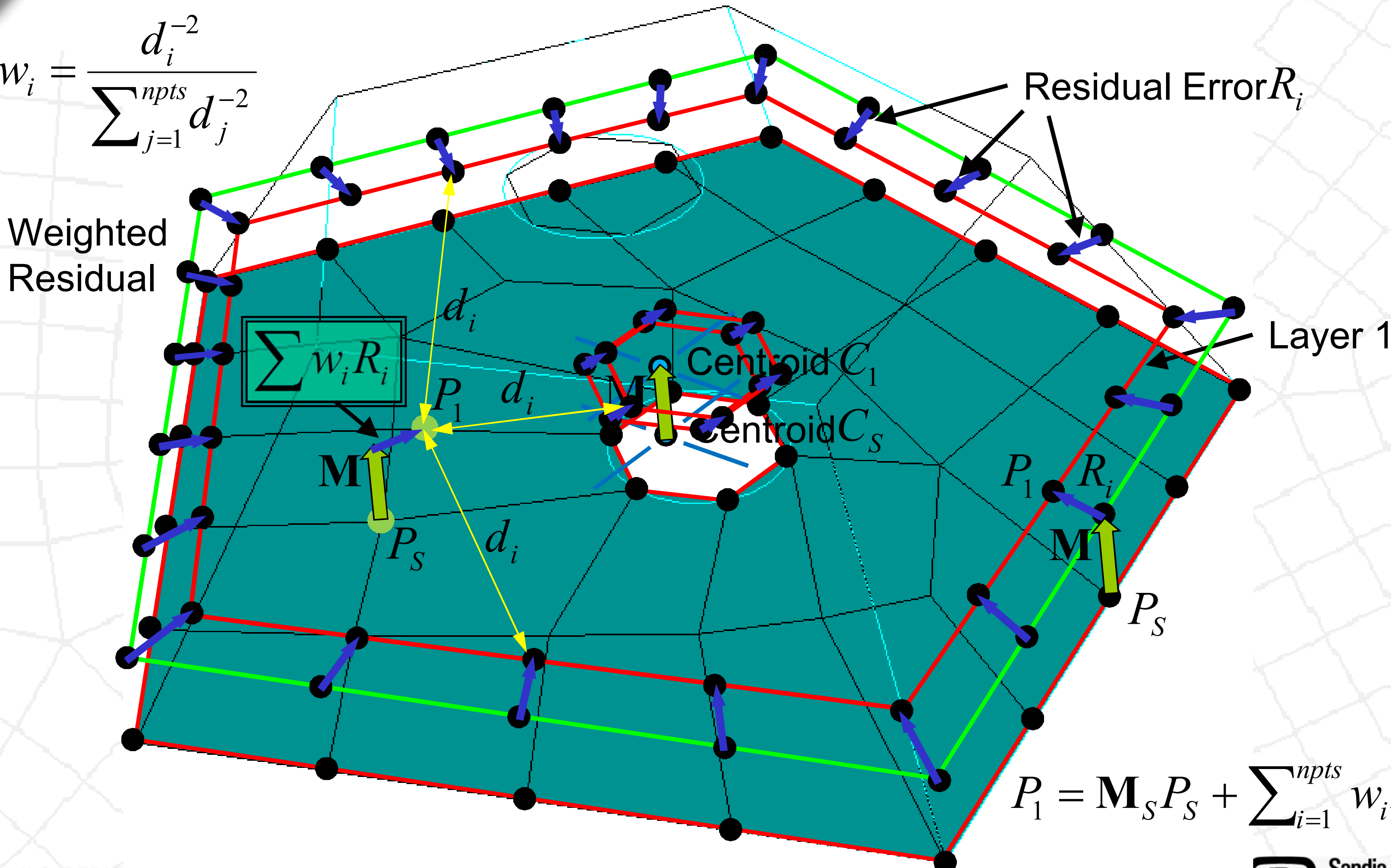
We compute residual error vectors as the error in the affine transformation



$$P_1 = \mathbf{M}_S P_S$$

Weighted Residual Method

$$w_i = \frac{d_i^{-2}}{\sum_{j=1}^{npts} d_j^{-2}}$$



$$P_1 = \mathbf{M}_S P_S + \sum_{i=1}^{npts} w_i R_i$$

Other References

Different approaches to one-to-one-sweeping

Cai, Shengyong and Timothy J. Tautges
"Robust One-to-One Sweeping with Harmonic S-T Mappings and Cages", *22nd International Meshing Roundtable*, Springer-Verlag, pp.1-18, October 13-16 2013

Staten, Matthew L., Scott A. Canann, and Steve J. Owen
"BMSWEEP: Locating Interior Nodes During Sweeping", *7th International Meshing Roundtable*, Sandia National Labs, pp.7-18, October 1998

M.A. Scott, M.N. Earp, S.E. Benzley
“Adaptive Sweeping Techniques", 14th *International Meshing Roundtable*, 2005

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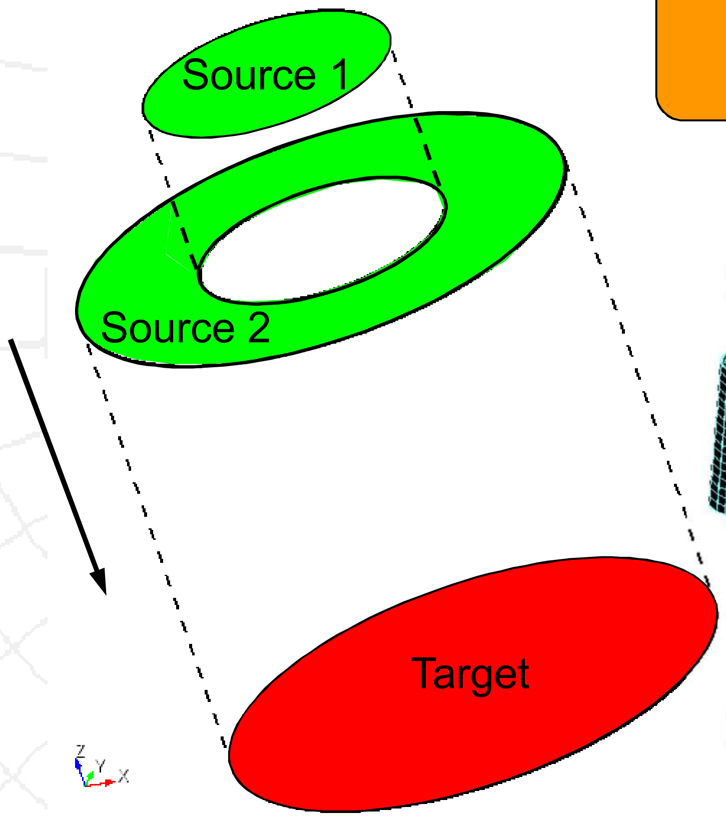
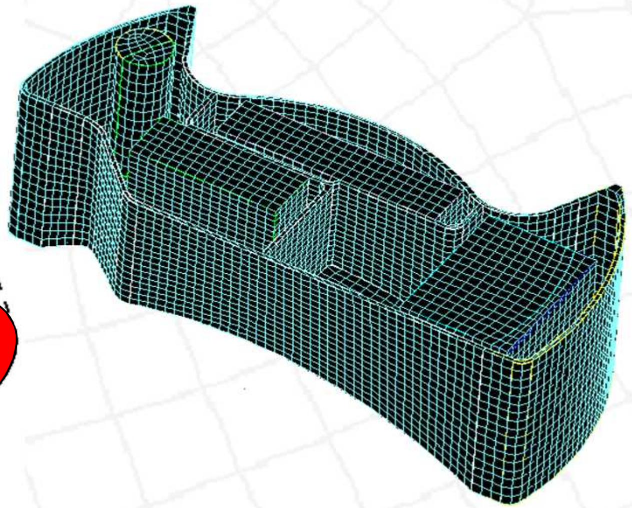
One-to-many Sweeping

n-to-1 sweepable

Sweep Direction





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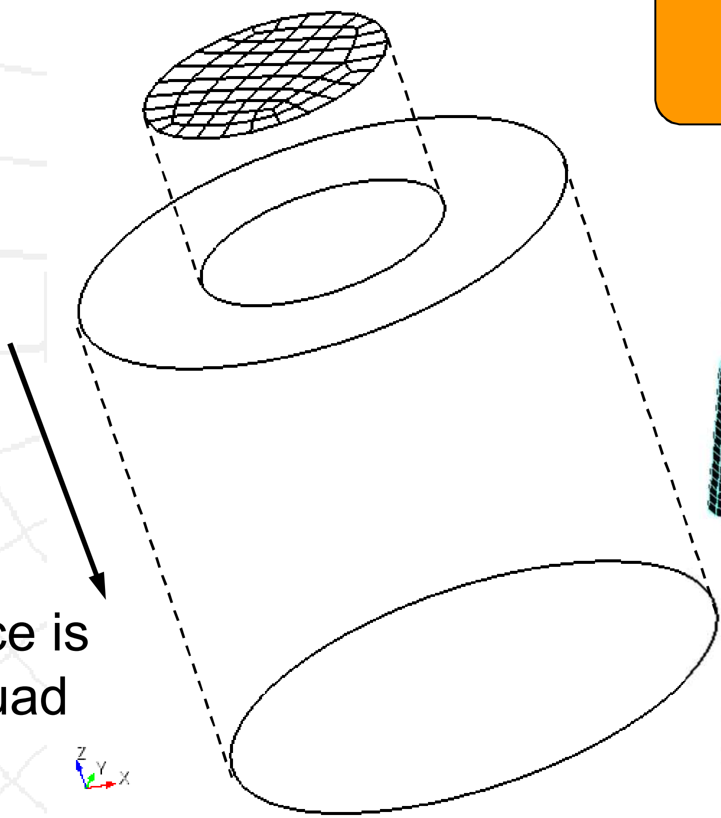
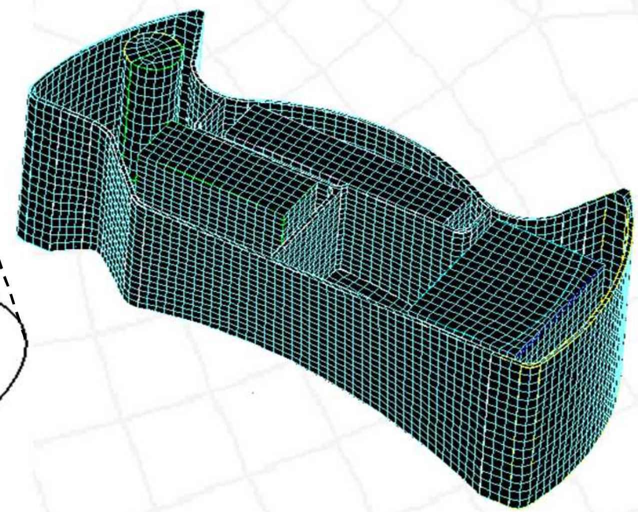
One-to-many Sweeping

n-to-1 sweepable

Sweep Direction



First source surface is meshed with all quad mesh



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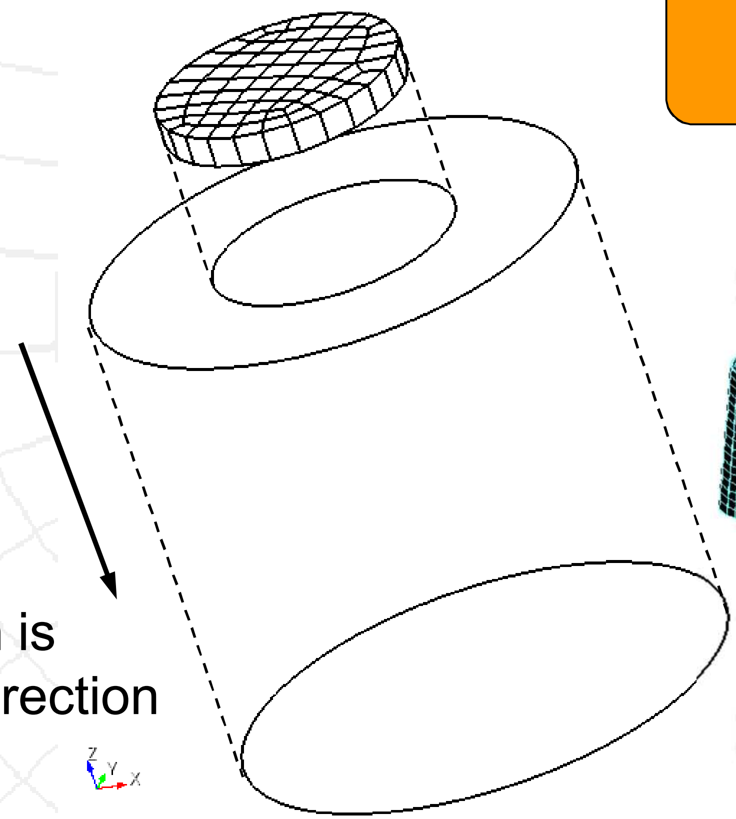
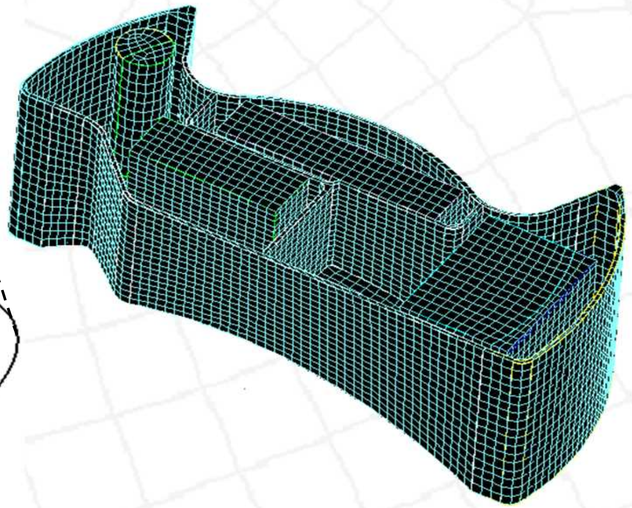
One-to-many Sweeping



First source mesh is swept in sweep direction until it meets with another source

n-to-1 sweepable



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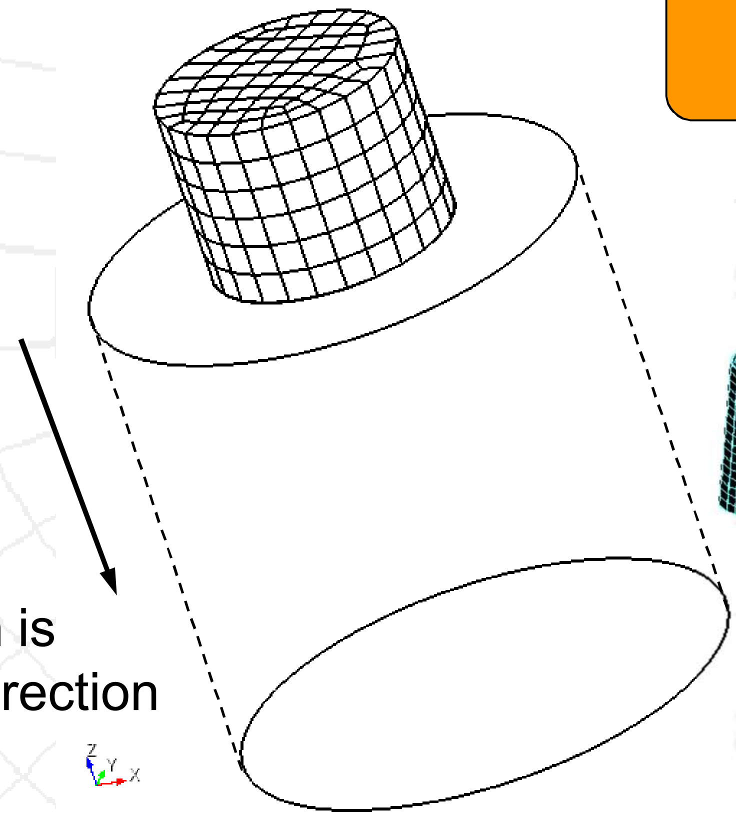
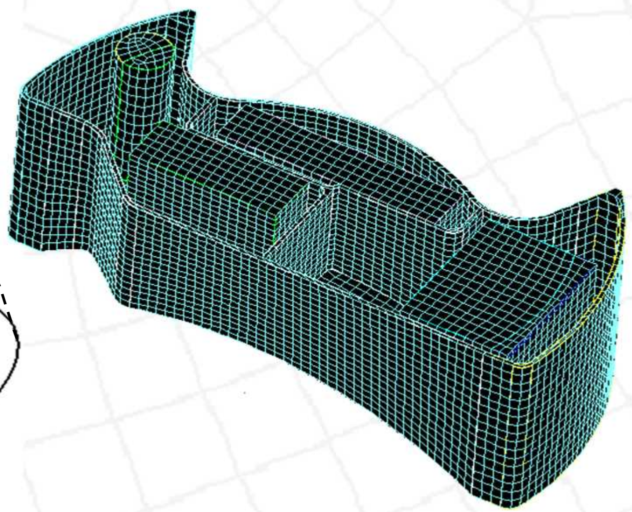
One-to-many Sweeping

n-to-1 sweepable

Sweep Direction





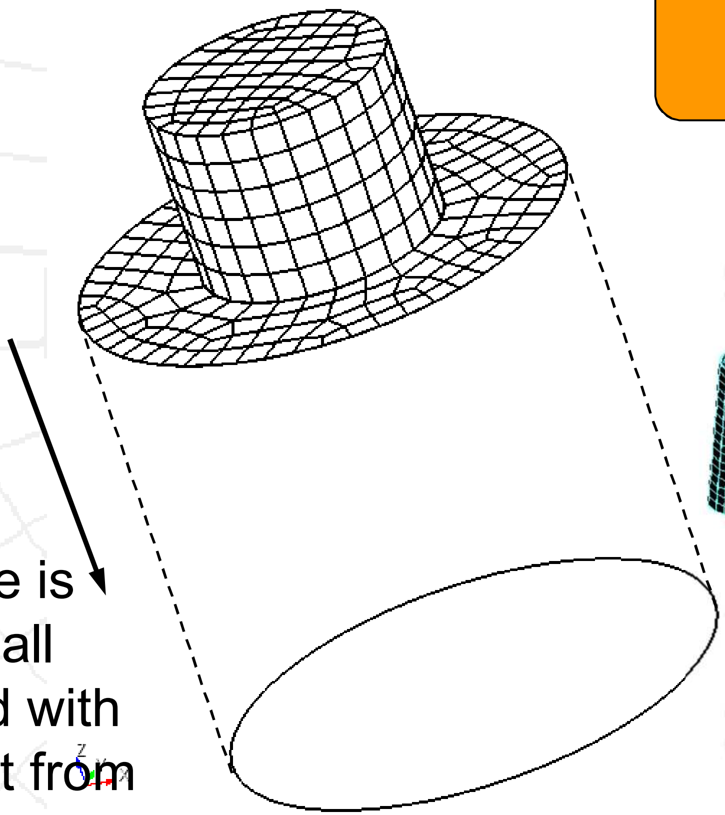
First source mesh is swept in sweep direction until it meets with another source

Matt Staten

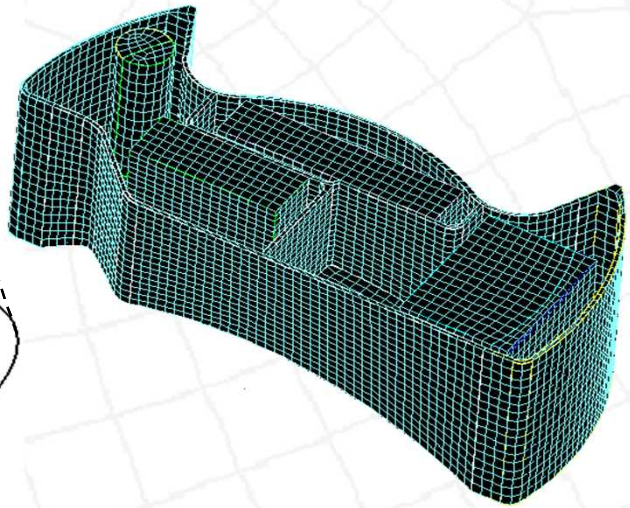
One-to-many Sweeping

n-to-1 sweepable

Sweep Direction



The second source is then meshed with all quads and merged with quads being swept from the first source.



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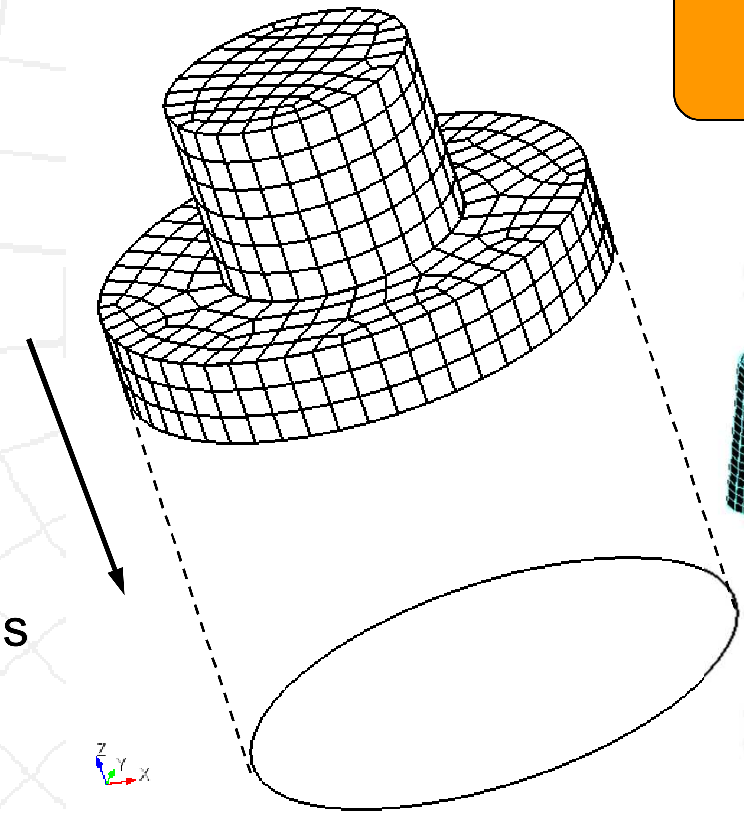
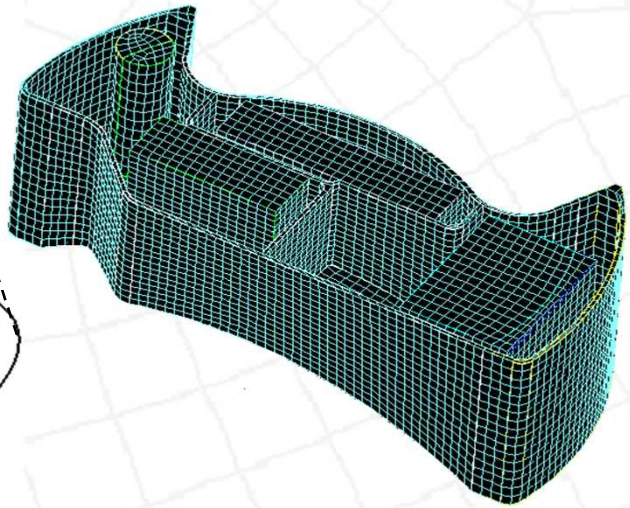
One-to-many Sweeping

Sweep Direction

Combined mesh is swept towards a single target.



n-to-1 sweepable



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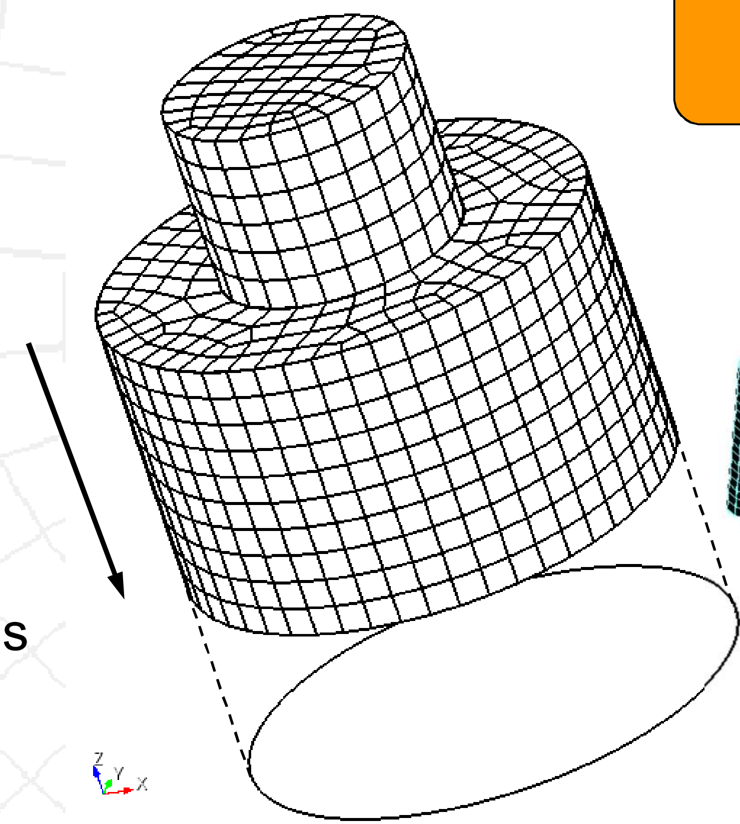
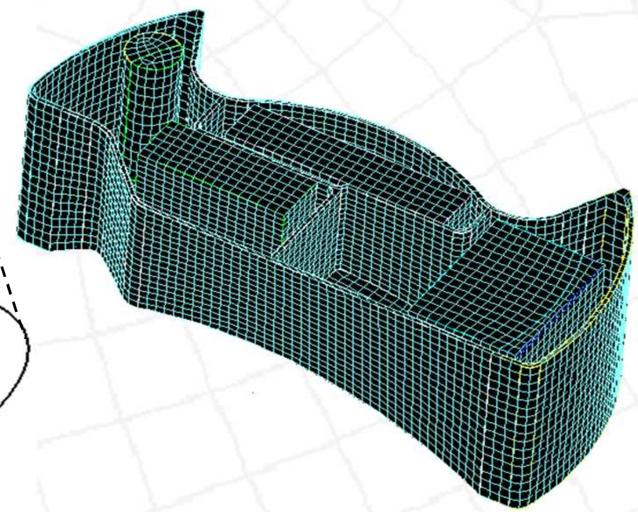
One-to-many Sweeping

n-to-1 sweepable

Sweep Direction





Combined mesh is swept towards a single target.

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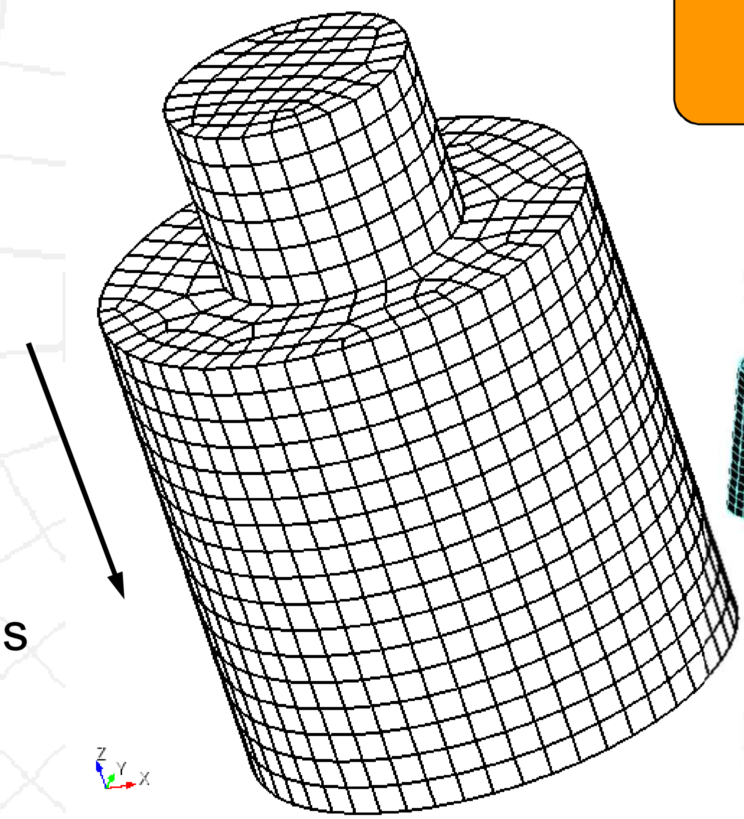
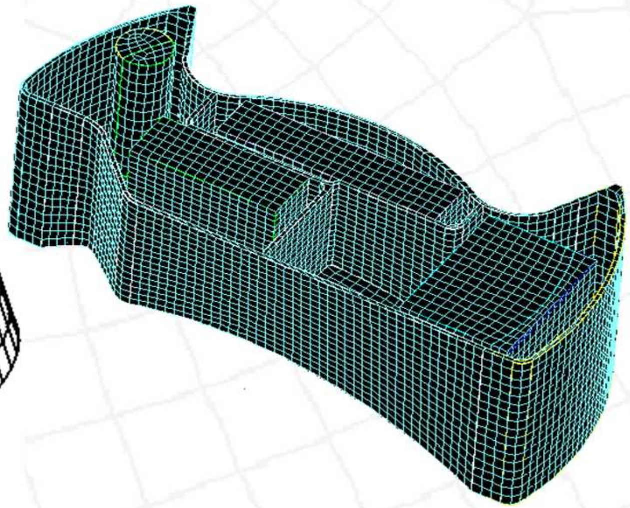
One-to-many Sweeping

n-to-1 sweepable

Sweep Direction



Combined mesh is swept towards a single target.



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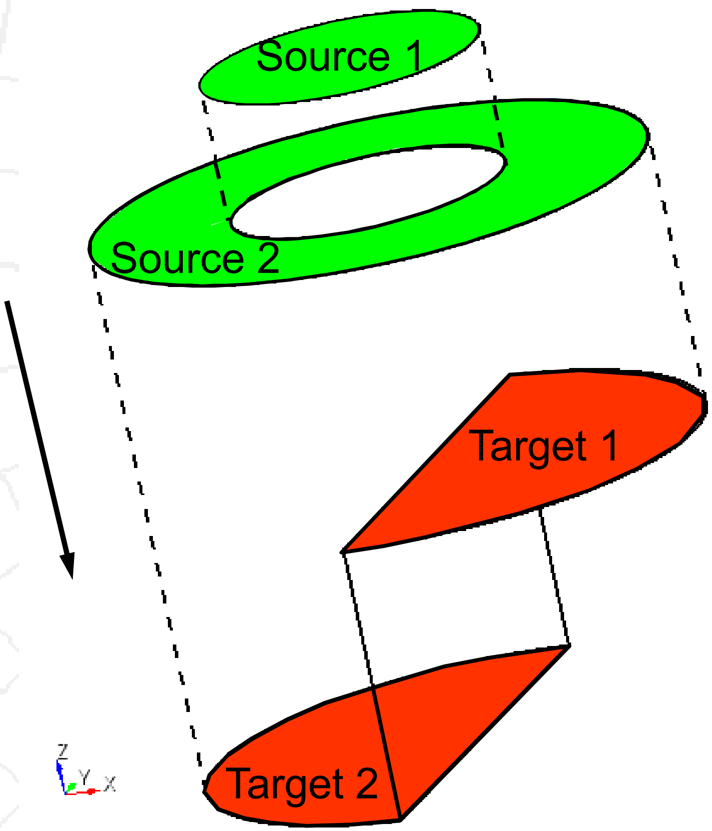
Many-to-many Sweeping

n-to-m sweepable
Multi-Sweep

Sweep Direction



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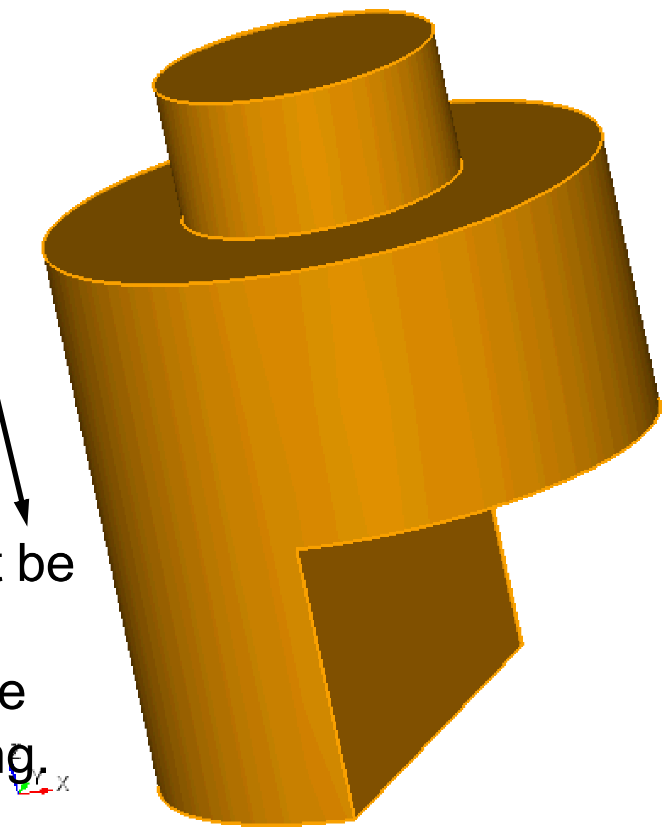
Many-to-many Sweeping

n-to-m sweepable
Multi-Sweep

Sweep Direction

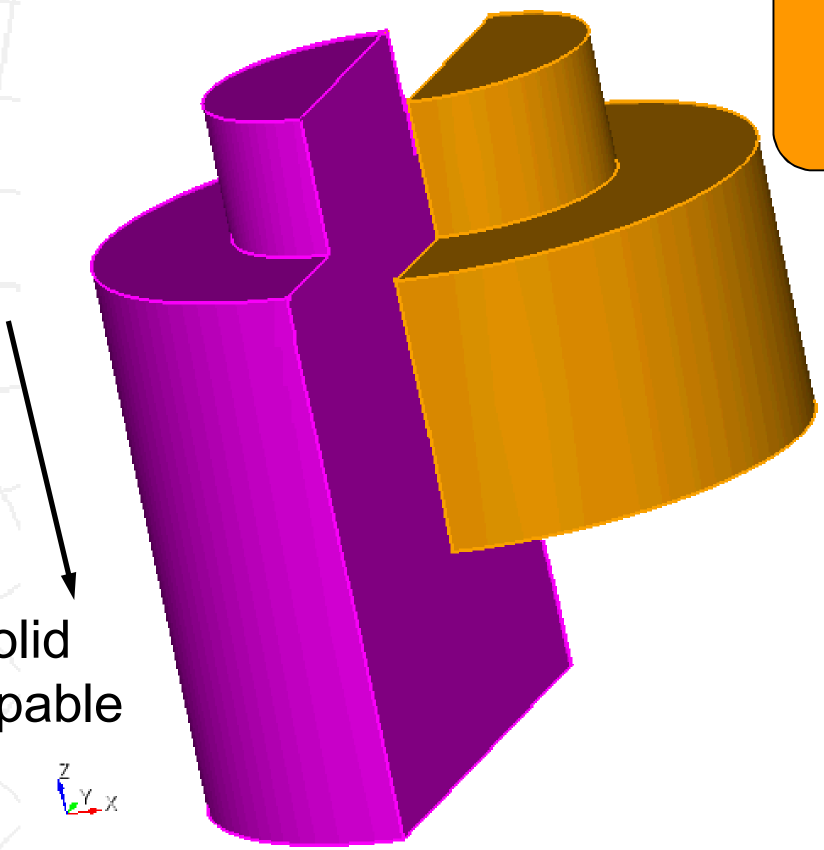


The footprints of the multiple targets must be imprinted onto the sources. This is done with virtual partitioning.

Many-to-many Sweeping

n-to-m sweepable
Multi-Sweep

Sweep Direction

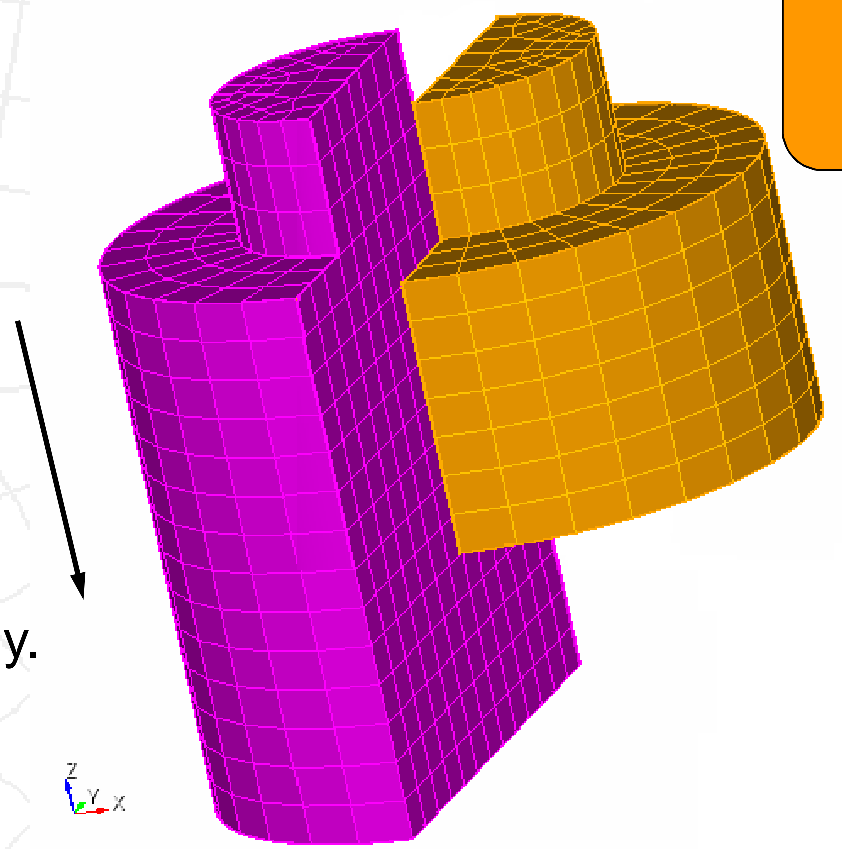


Virtual partitioning decomposes the solid into n-to-one sweepable sub-volumes.

Many-to-many Sweeping

n-to-m sweepable
Multi-Sweep

Sweep Direction

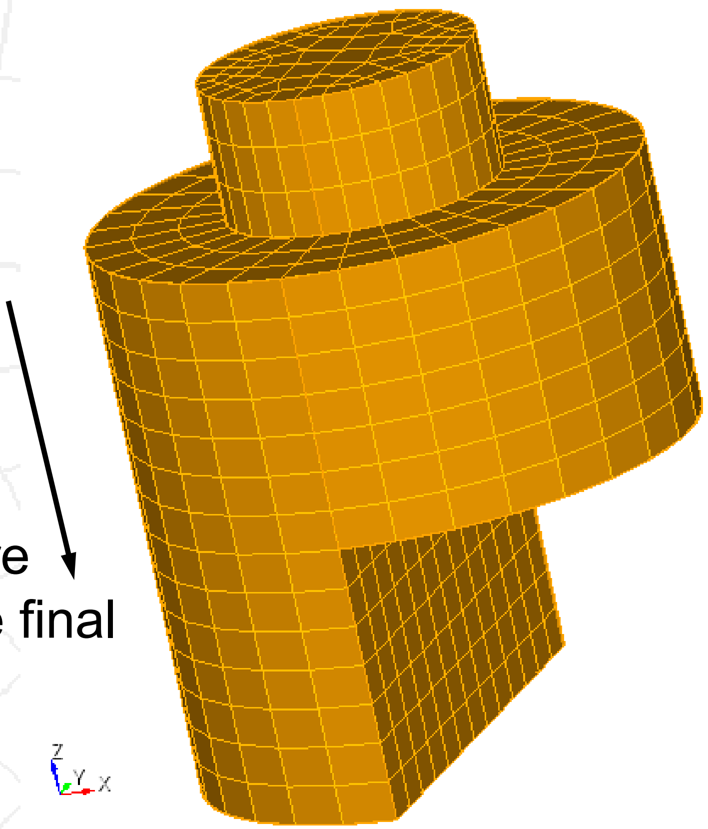


Each partition is meshed individually.

Many-to-many Sweeping

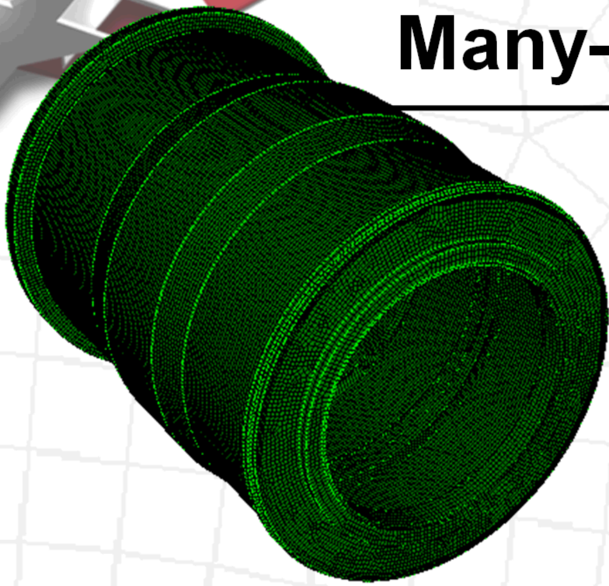
n-to-m sweepable
Multi-Sweep

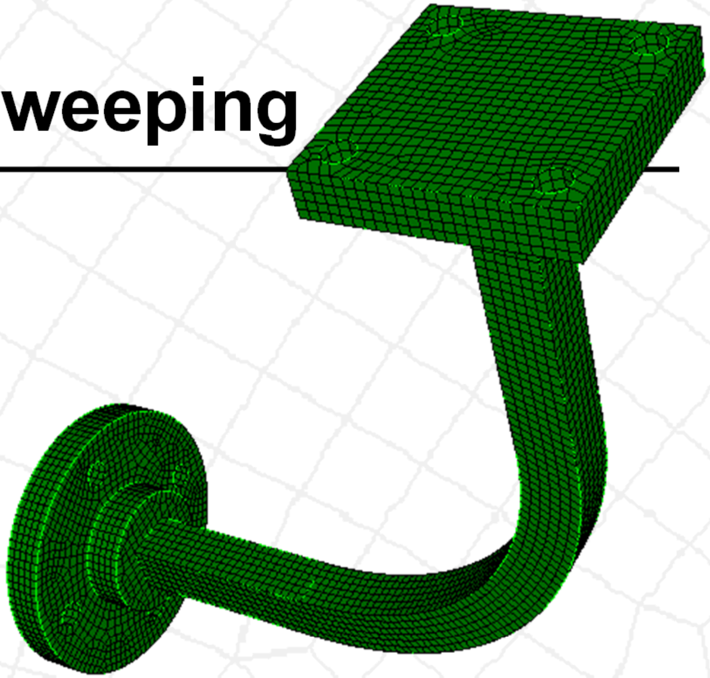
Sweep Direction



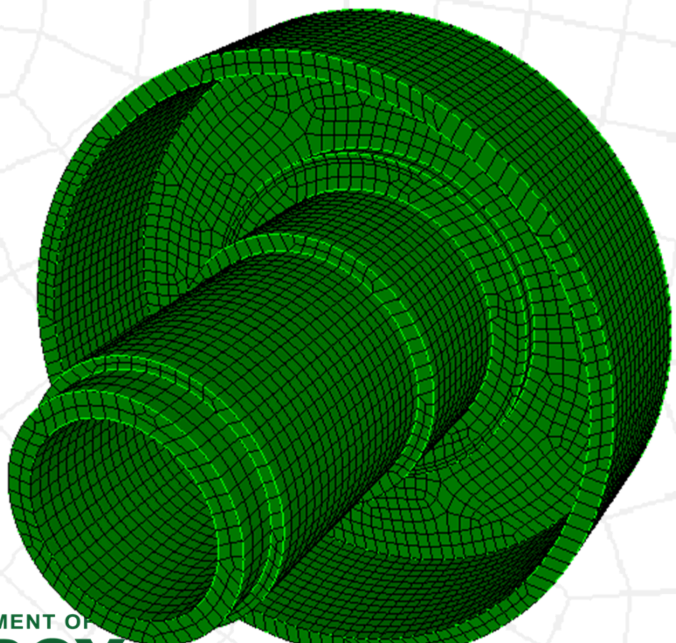
Virtual partitions are deleted leaving the final mesh.

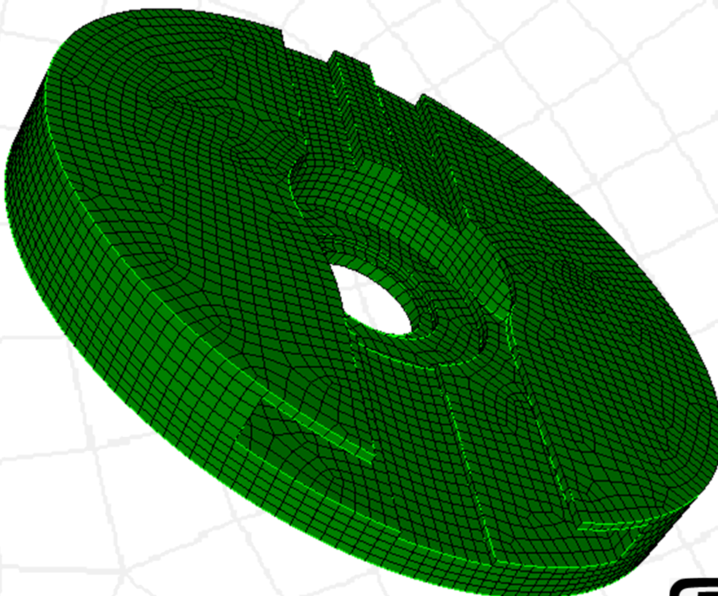
Many-to-many Sweeping





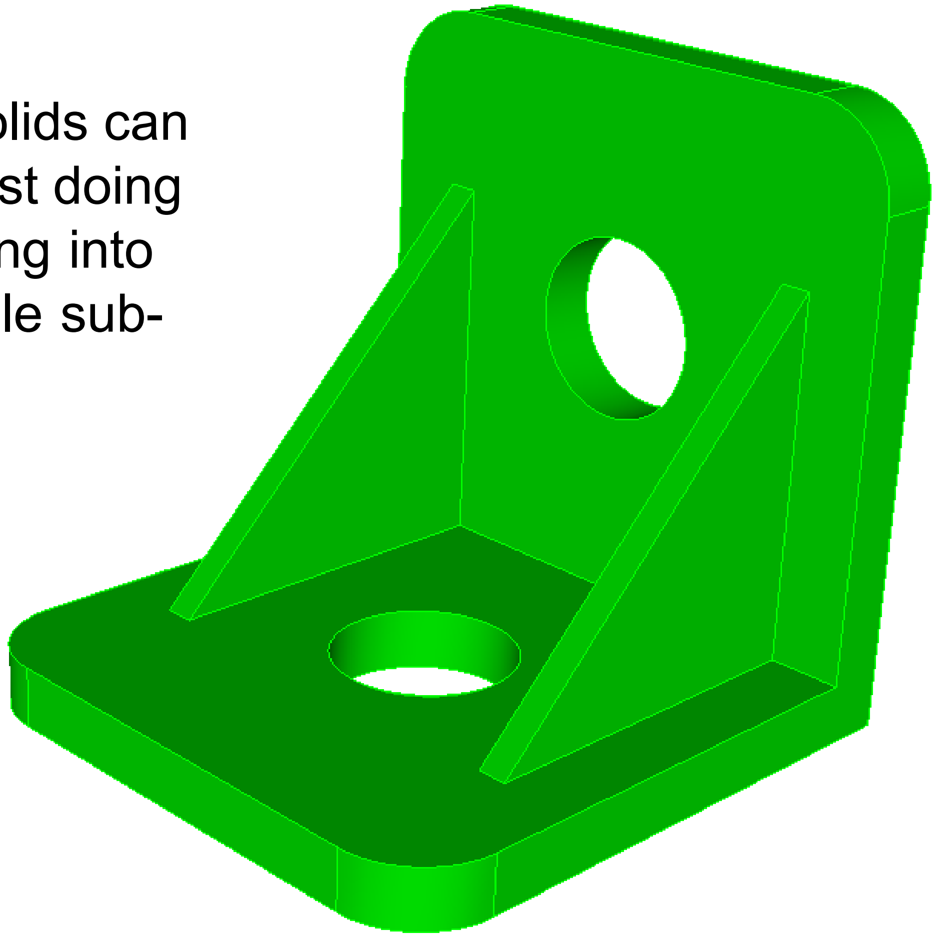
Examples of Many-to-many sweeping with CUBIT





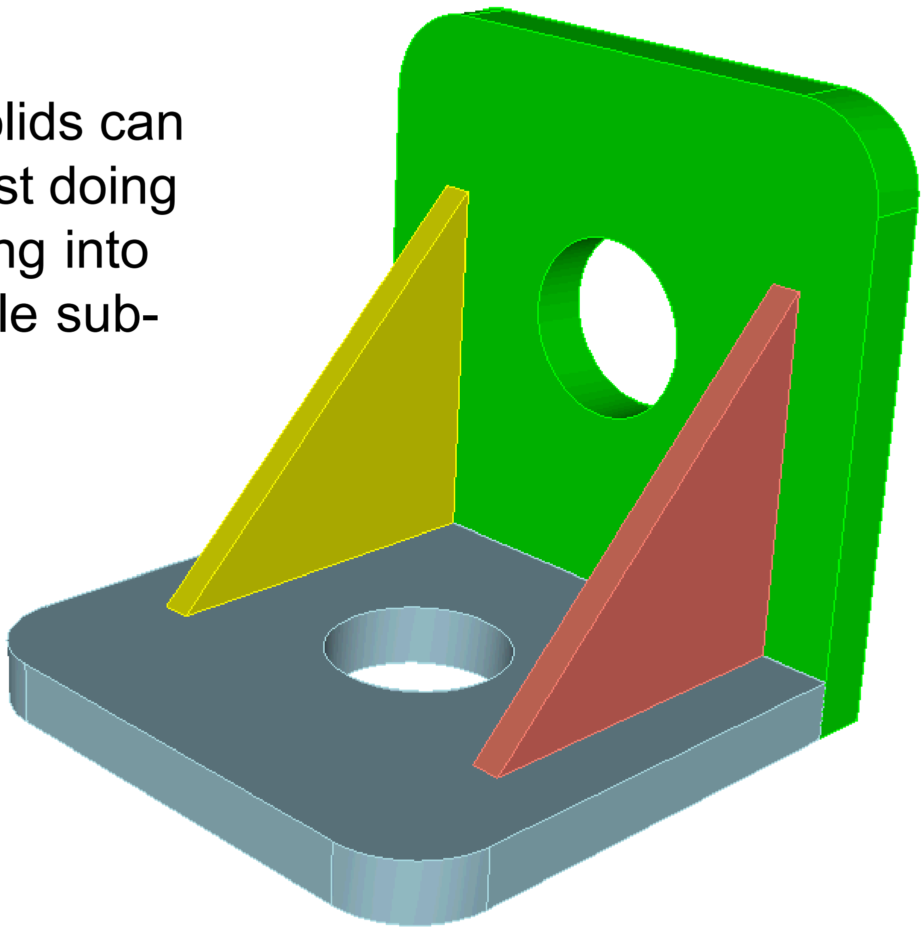
Partition & Sweeping

More complex solids can be meshed by first doing manual partitioning into several sweepable sub-solids.



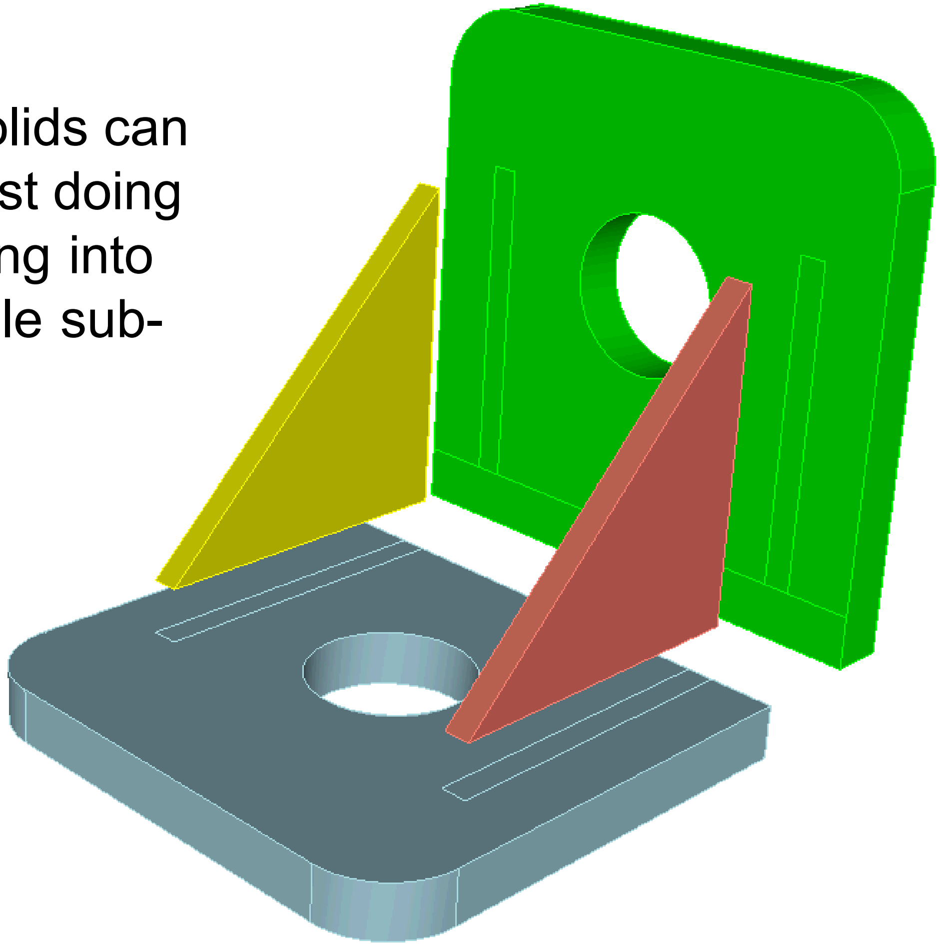
Partition & Sweeping

More complex solids can be meshed by first doing manual partitioning into several sweepable sub-solids.



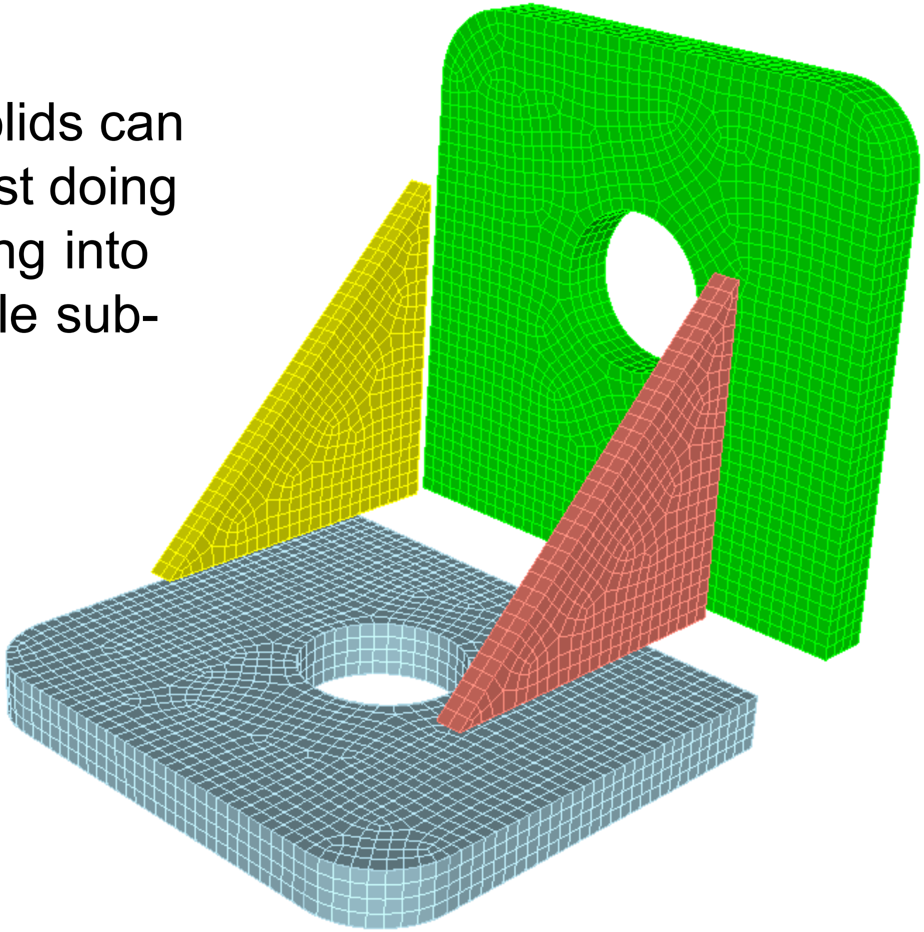
Partition & Sweeping

More complex solids can be meshed by first doing manual partitioning into several sweepable sub-solids.



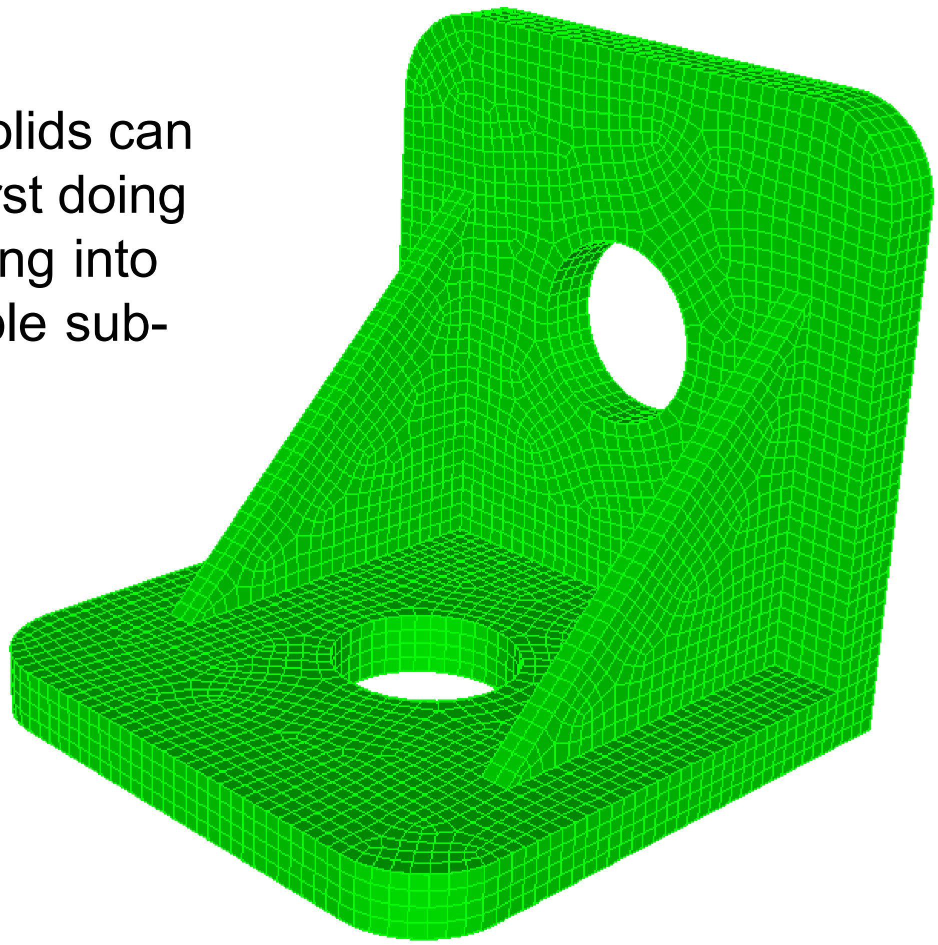
Partition & Sweeping

More complex solids can be meshed by first doing manual partitioning into several sweepable sub-solids.

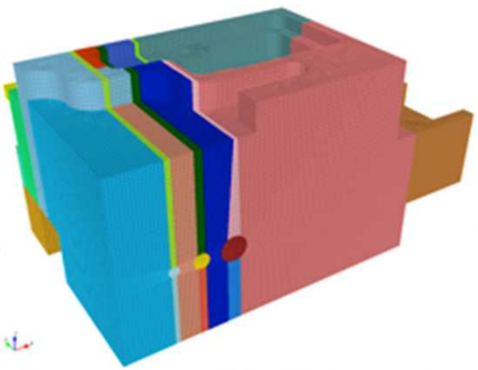


Partition & Sweeping

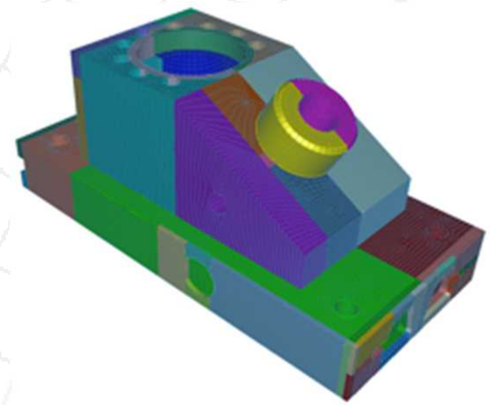
More complex solids can be meshed by first doing manual partitioning into several sweepable sub-solids.

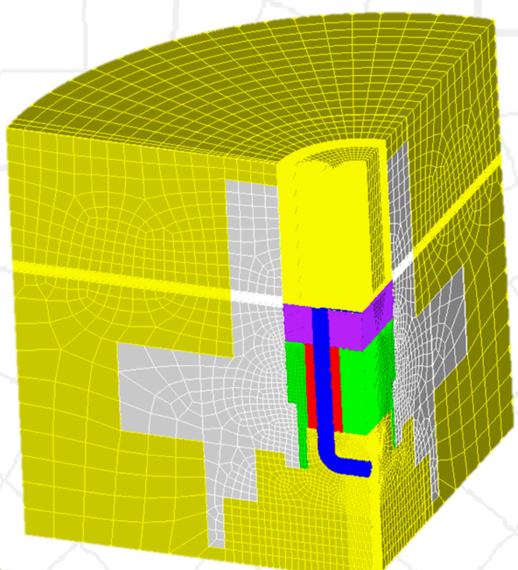


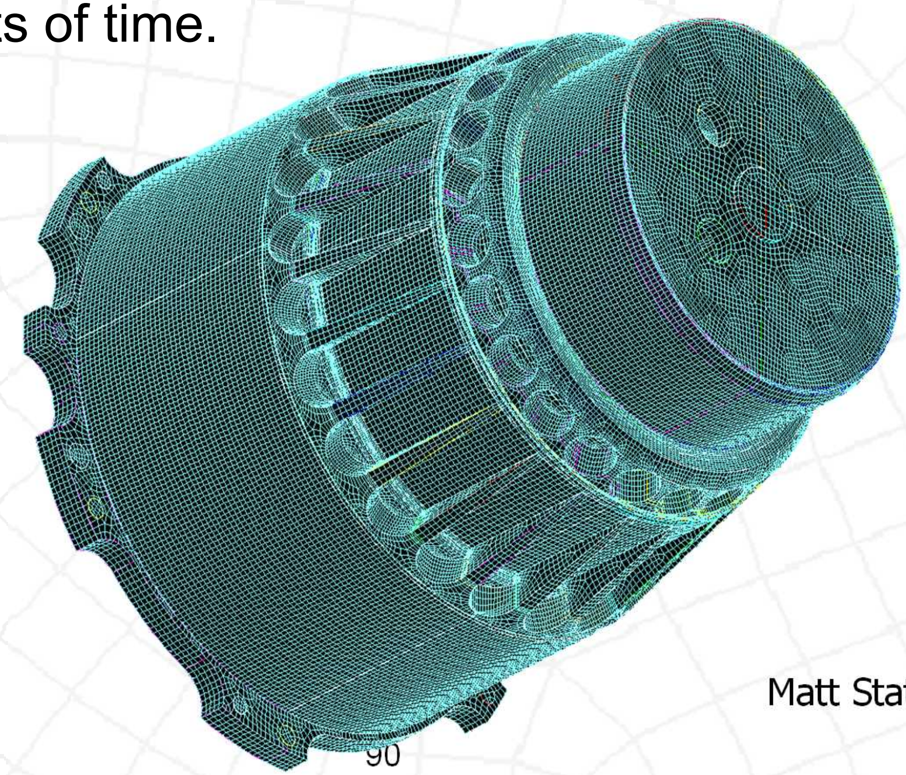
Partitioning & Sweeping Very Complex Solids

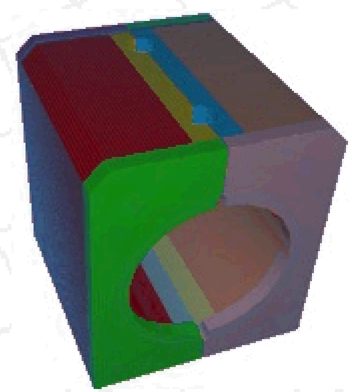


Any geometry, regardless of complexity, can be meshed by first decomposing it into sweepable sub-solids. Decomposition step of complex solids requires tedium, experience, and creativity and often lots of time.









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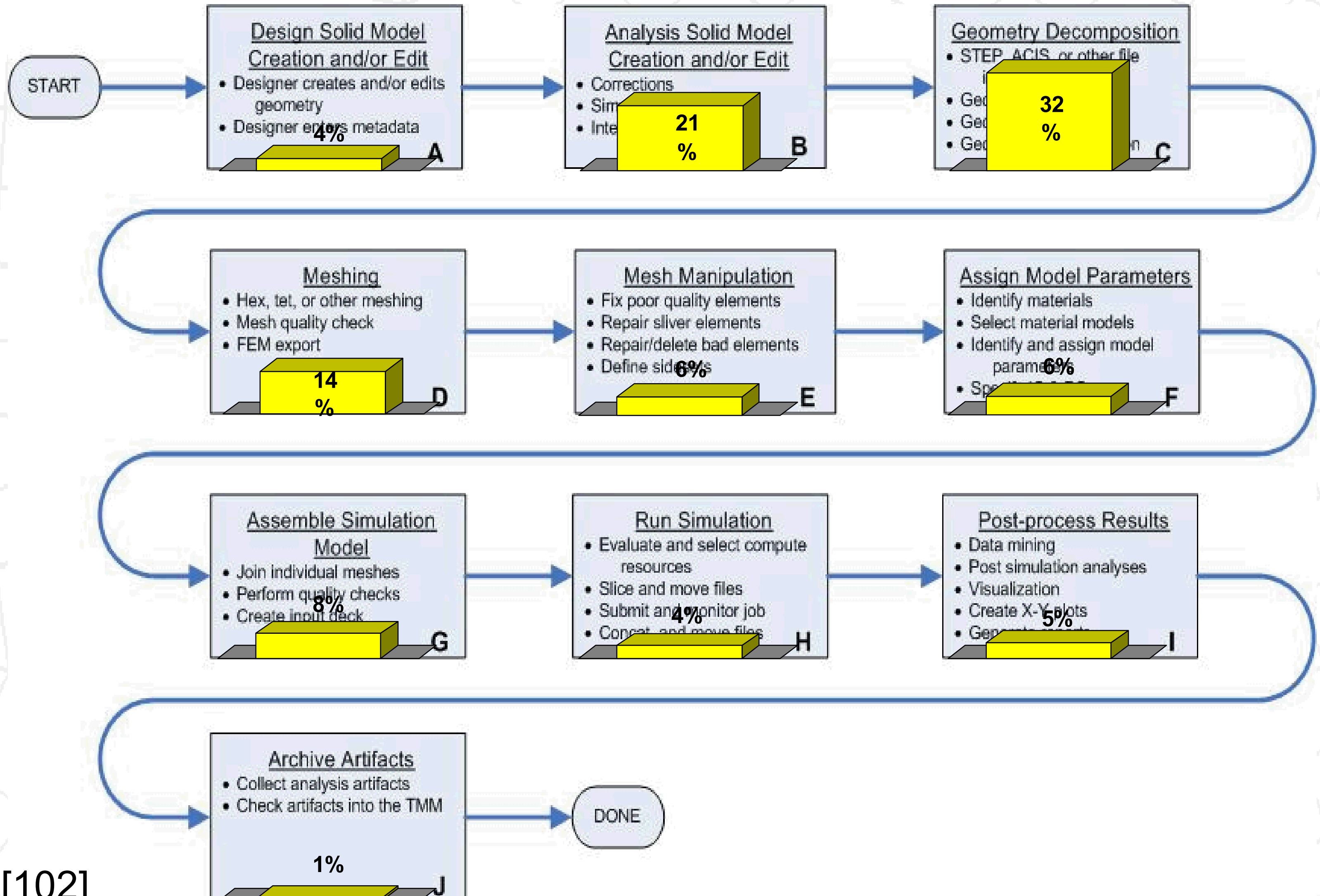
Engineering Analysis with Geometry Conforming Hexahedral Elements



Owen [102]

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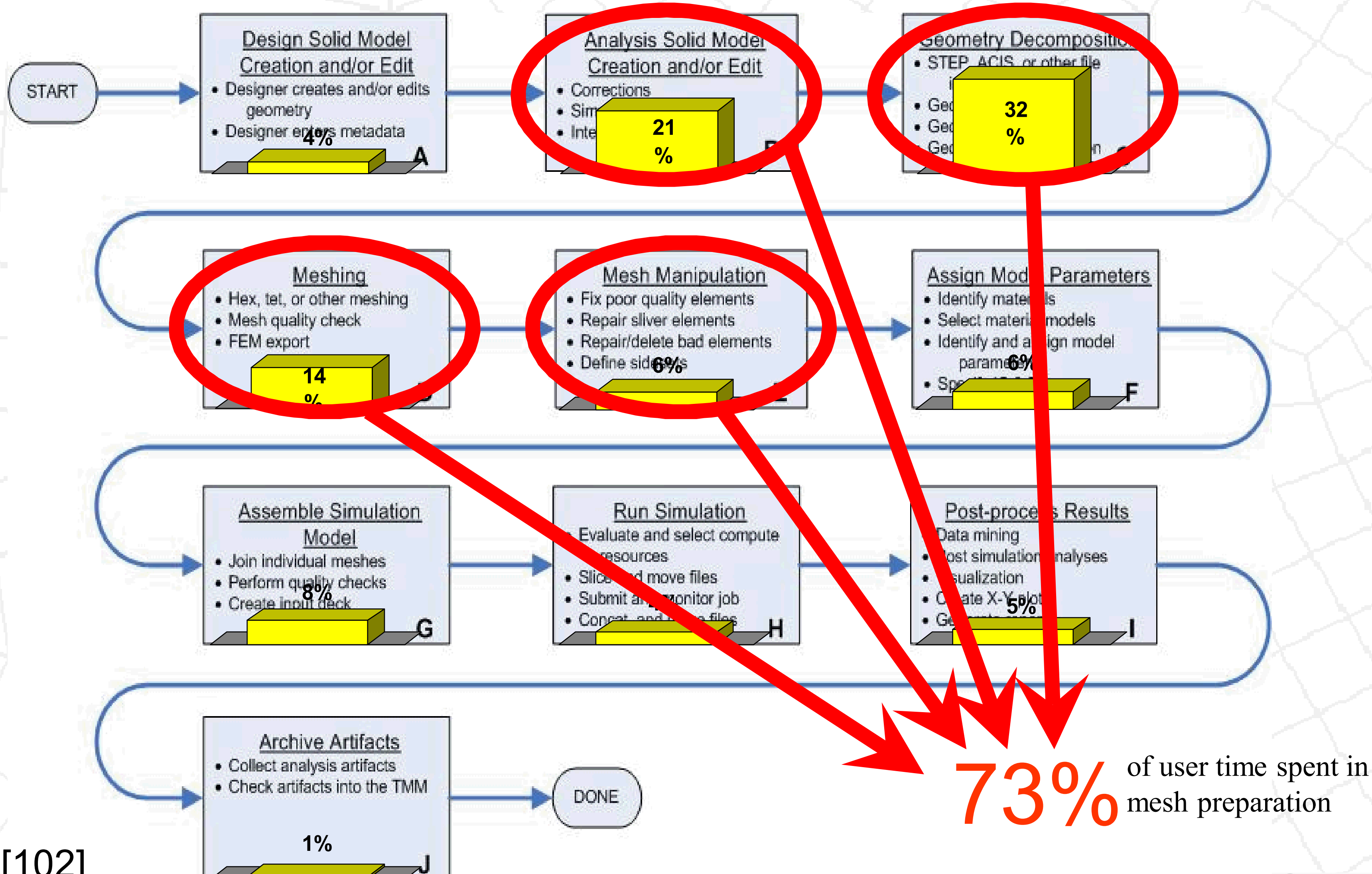
Engineering Analysis with Geometry Conforming Hexahedral Elements



Owen [102]

Other References

Other References to many-to-many sweeping

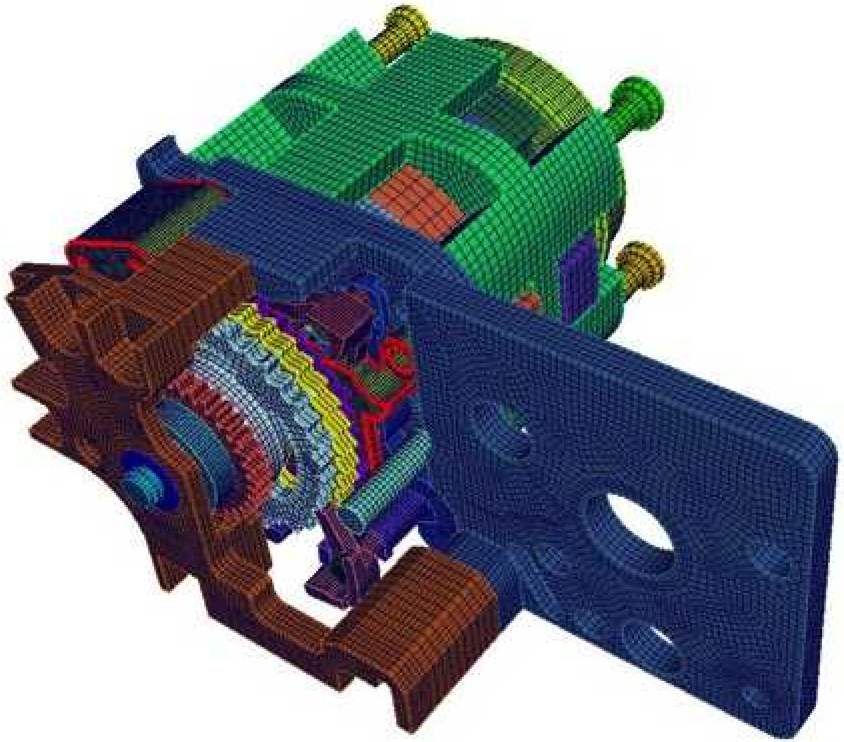
T. D. Blacker
"The Cooper Tool", 5th *International Meshing Roundtable*, October 1996

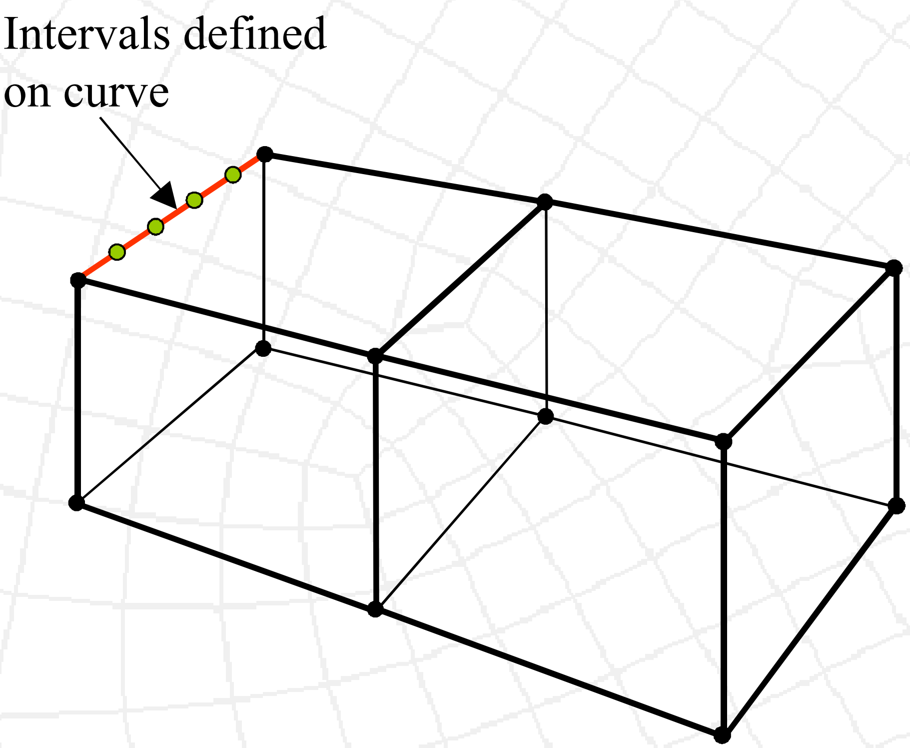
D. R. White, S. Saigal, S.J. Owen,
"CCSweep: automatic decomposition of multi-sweep volumes," Engineering With Computers, 15:212-218, 1999

The propagation of Hex/Quad constraints

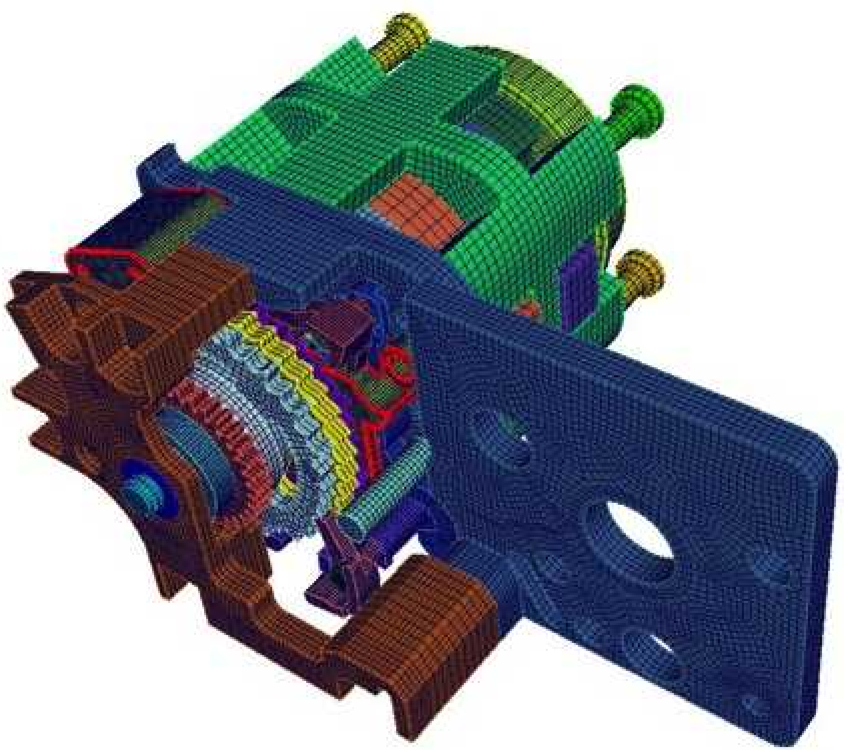
INTERVAL ASSIGNMENT

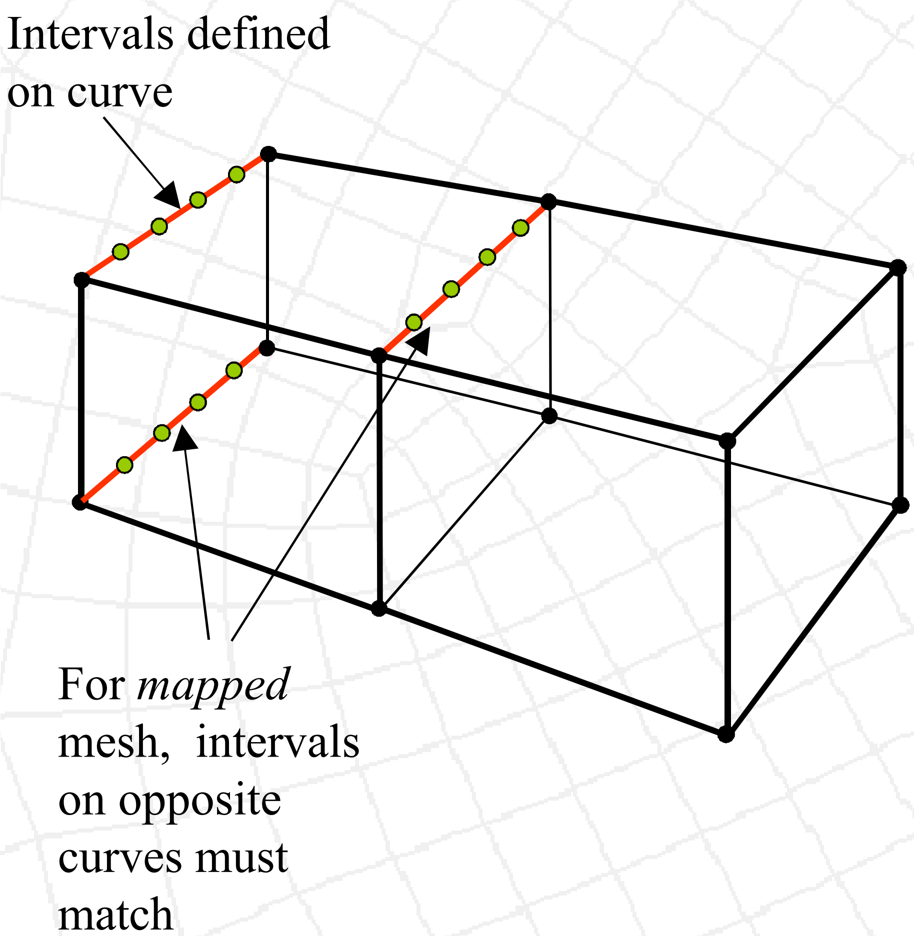
Interval assignment



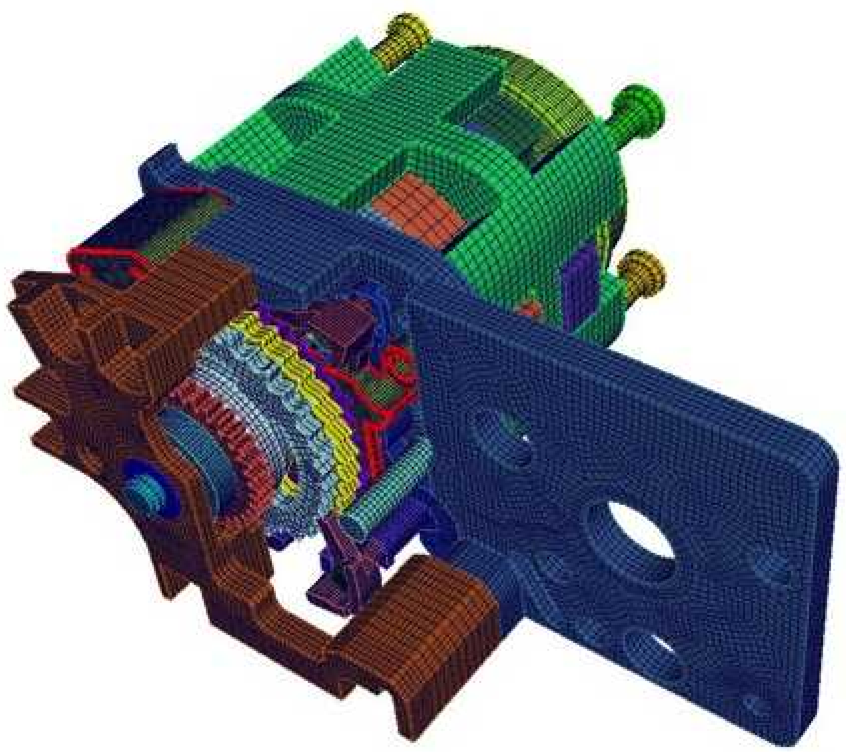


Interval assignment

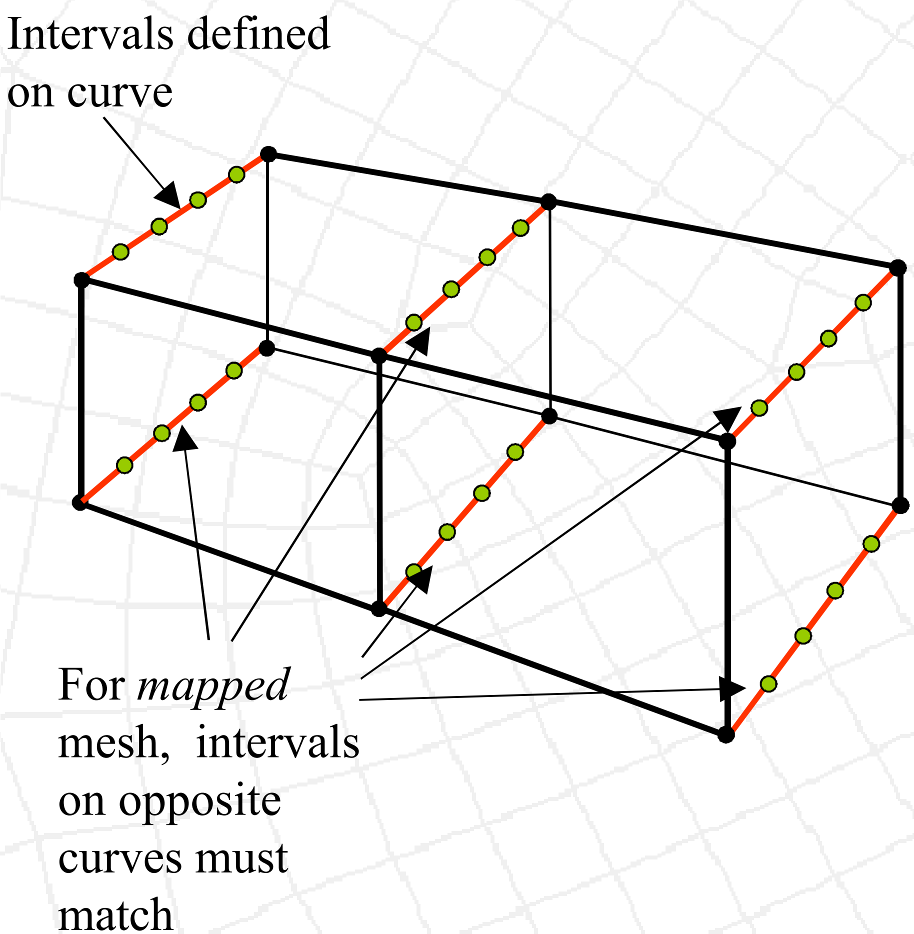




Interval assignment



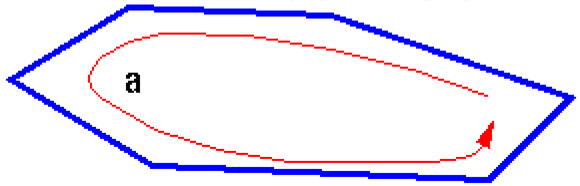
Intervals must propagate through model based on meshing scheme



Interval assignment

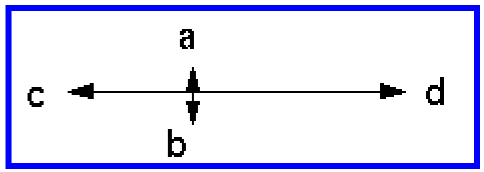
Pave

$\sum a = 2k$, k int.
sum-even



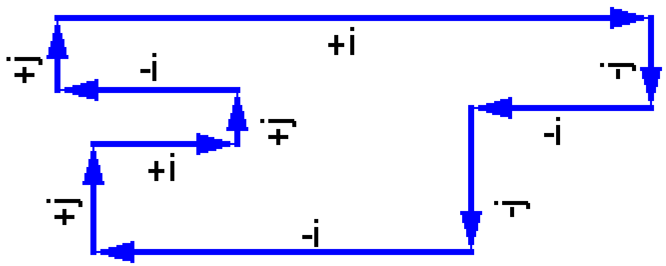
Map

$\sum a = \sum b$
$\sum c = \sum d$



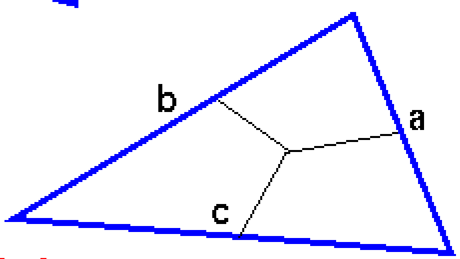
Submap*

$\sum +i = \sum -i$
$\sum +j = \sum -j$



Tri-map

$a + b > c + 2$
$a + c > b + 2$
$b + c > a + 2$
$a + b + c = 2k$, k int.
Each curve has integer # of intervals
Note: "sides" are composite



Mixed Integer Linear Program (MILP)

Objective Function: weighted differences between assigned and goal intervals

Algorithm: iteratively solve relaxed LP
All-integer solution

Interval Assignment References

Mitchell, Scott A.
"High Fidelity Interval Assignment", *Proceedings, 6th International Meshing Roundtable*, Sandia National Laboratories, pp.33-44, October 1997

Mitchell, Scott A.
"Simple and Fast Interval Assignment Using Nonlinear and Piecewise Linear Objectives", *22nd International Meshing Roundtable*, Springer-Verlag, pp.203-221, October 13-16 2013

http://lpsolve.sourceforge.net/

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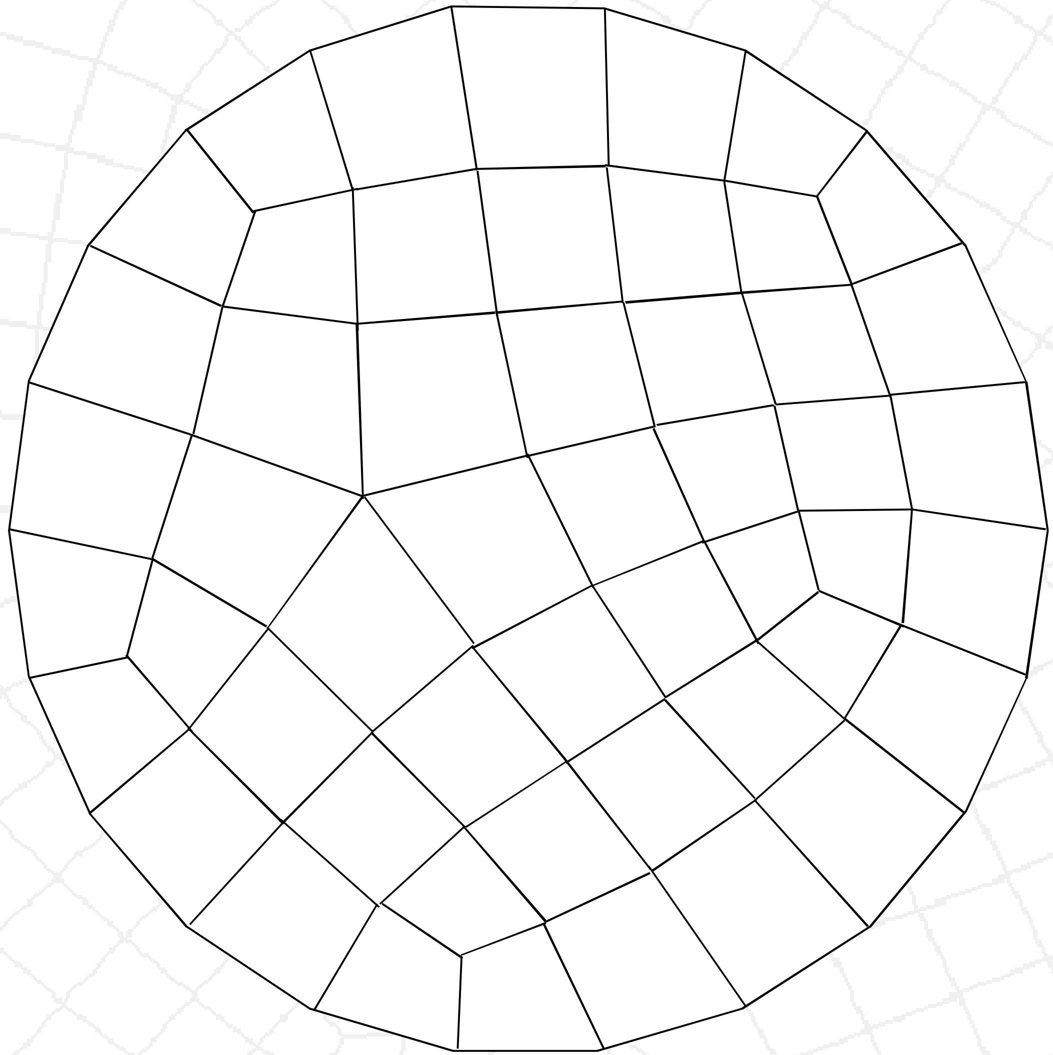
Chords, Sheets, Pillowing, Dicing, Hex Mesh Modification

DUAL OF QUAD/HEX MESHES

Quadrilateral Dual Representation

The elemental representation of a mesh, composed of elements, edges, and nodes, is known as the *primal*.

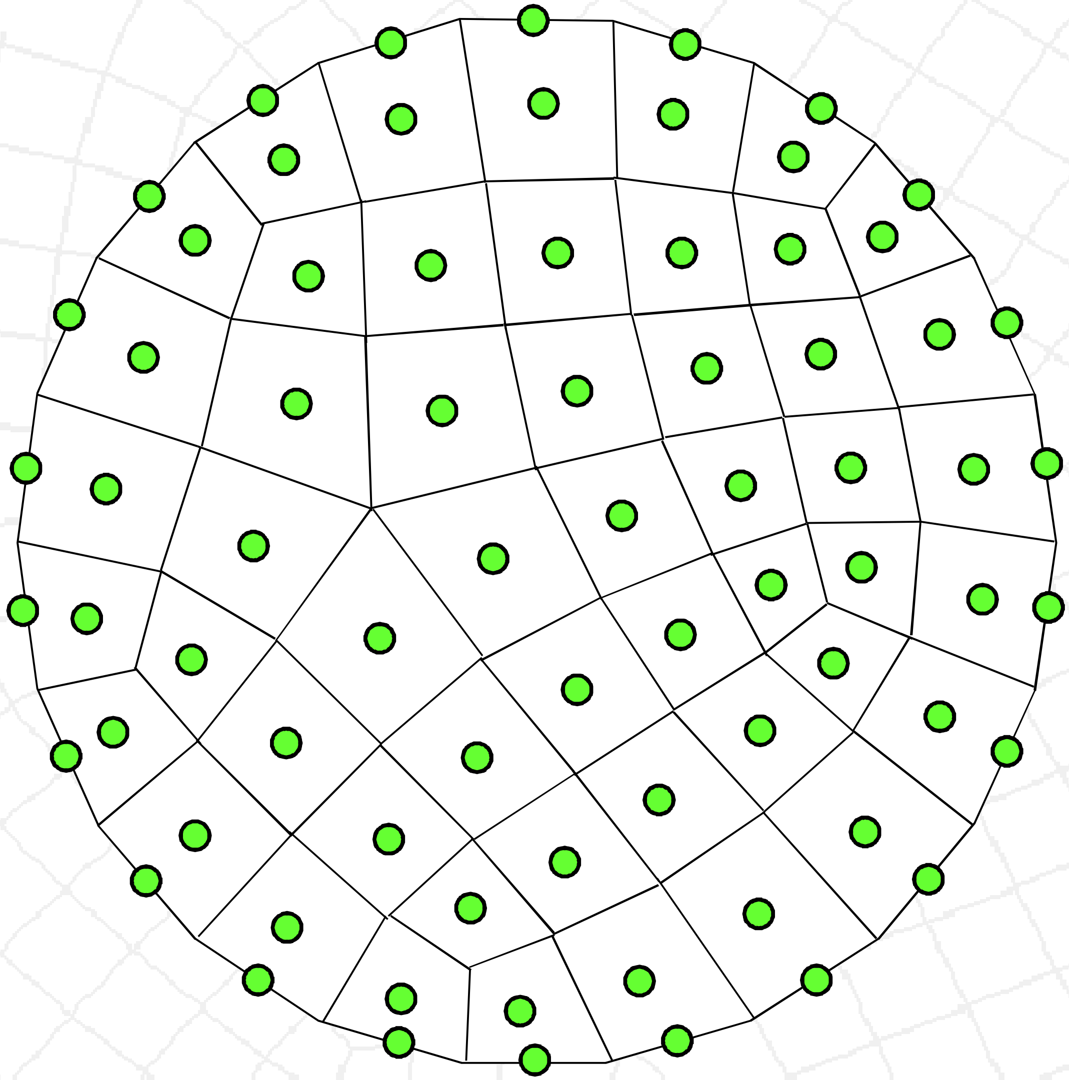
Quadrilateral meshes have a *dual* representation, similar to the voroni skeleton of a triangular delaunay mesh.



Quadrilateral Dual Representation

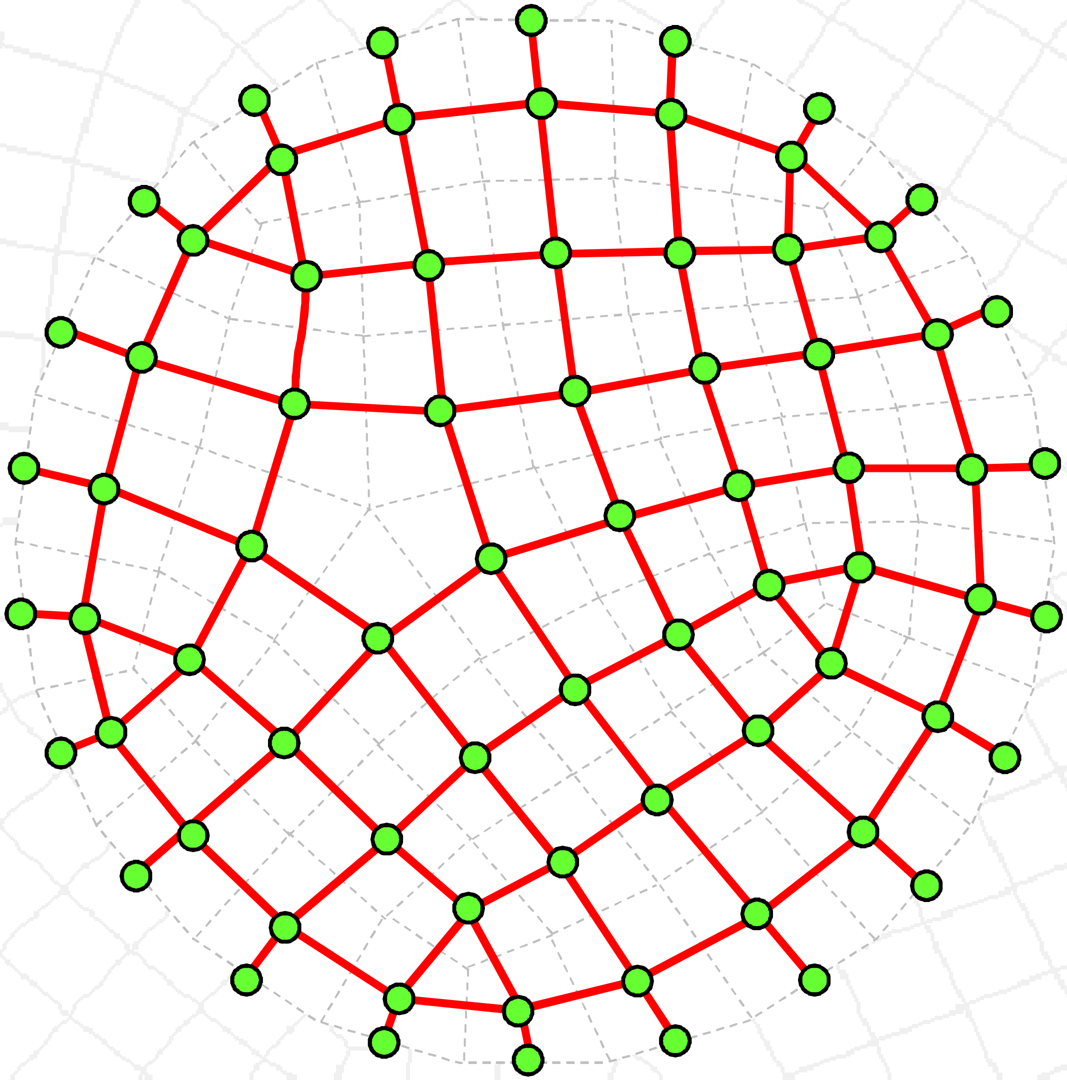
A dual vertex, v_i, is defined at the centroid of each quadrilateral element.

A dual vertex is also placed at the centroid of every boundary edge.



Quadrilateral
Dual Representation

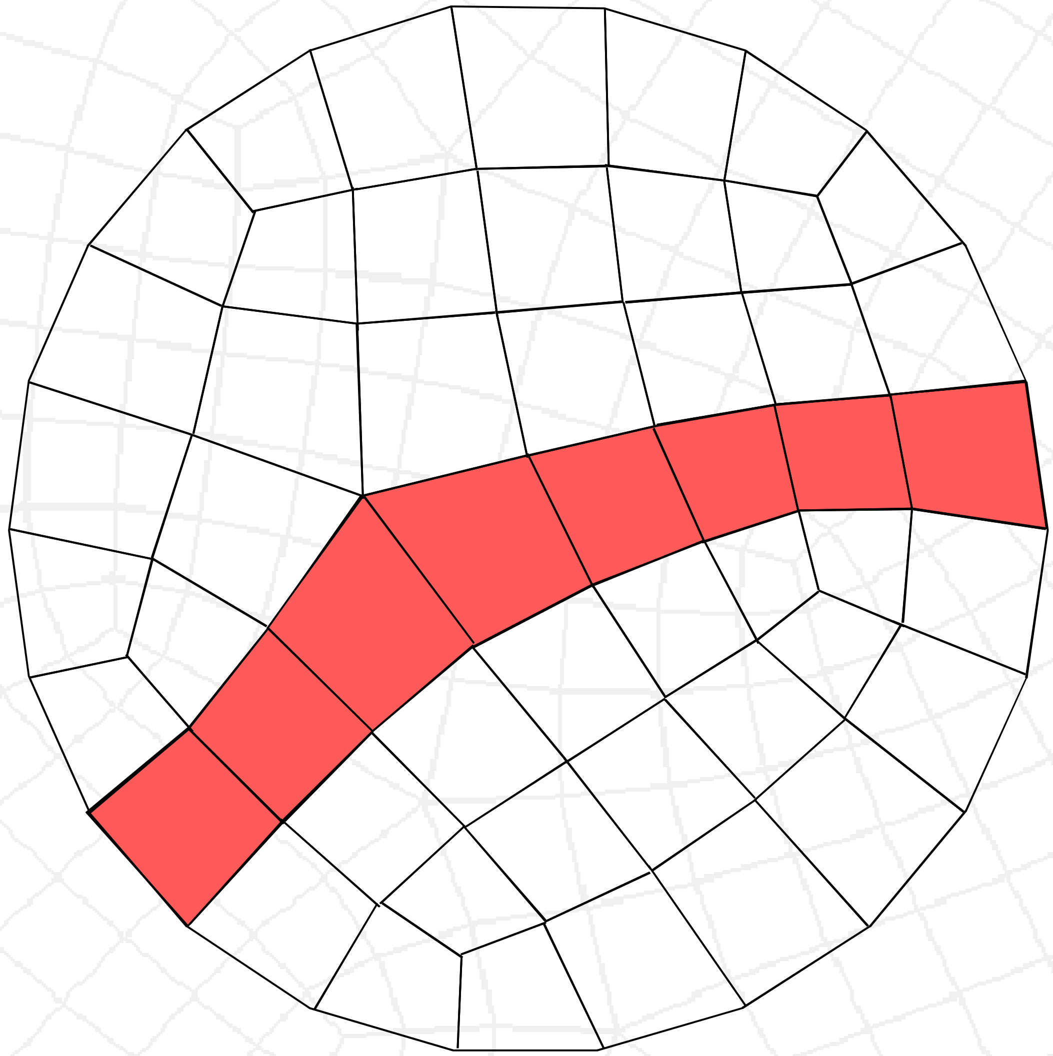
Connecting the dual vertices through adjacent elements creates the edges of the dual.



Quadrilateral Dual Representation

Quadrilateral meshes have an inherent row structure. The red quads illustrate one row.

Each row corresponds to one dual chord.



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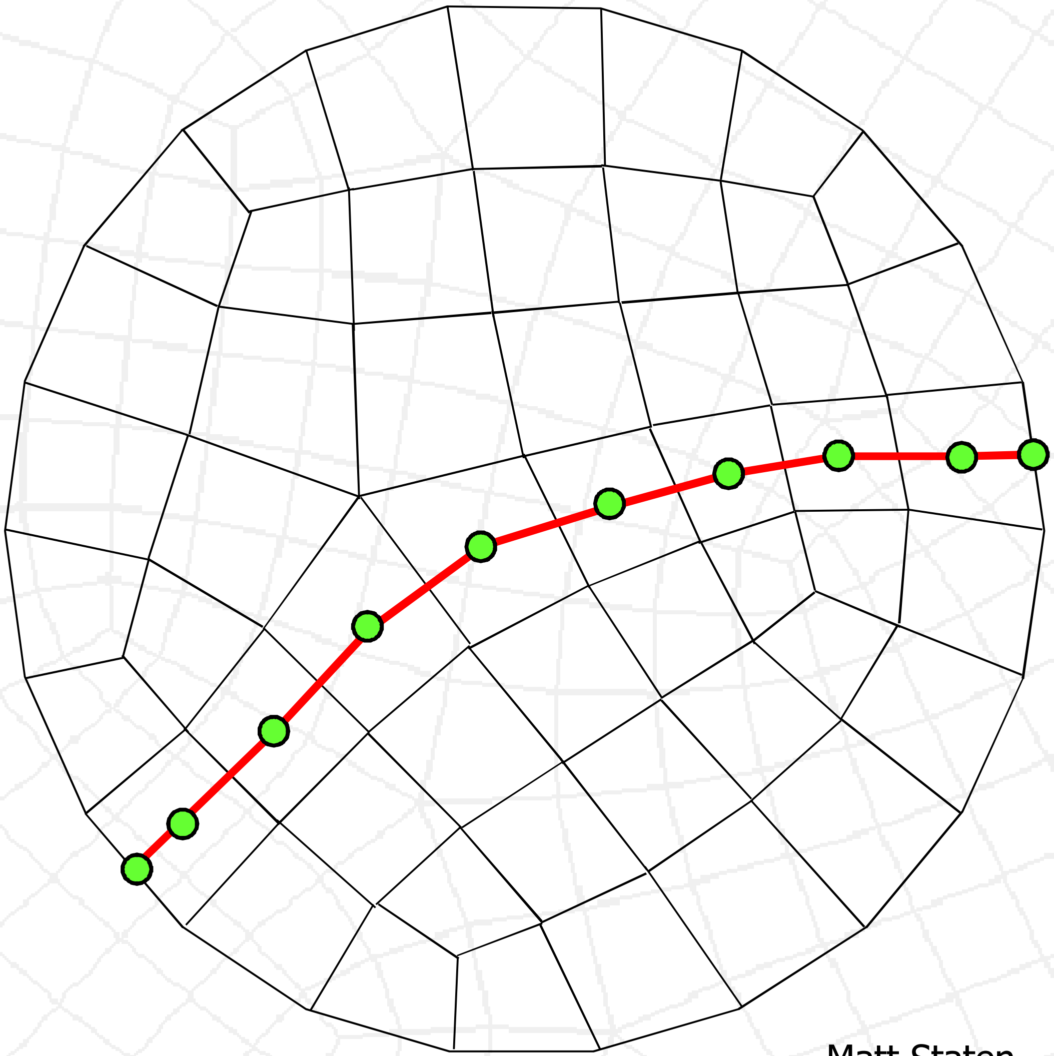
Quadrilateral Dual Representation

The set of all dual edges which connect quads in each row forms a dual chord, c_i.



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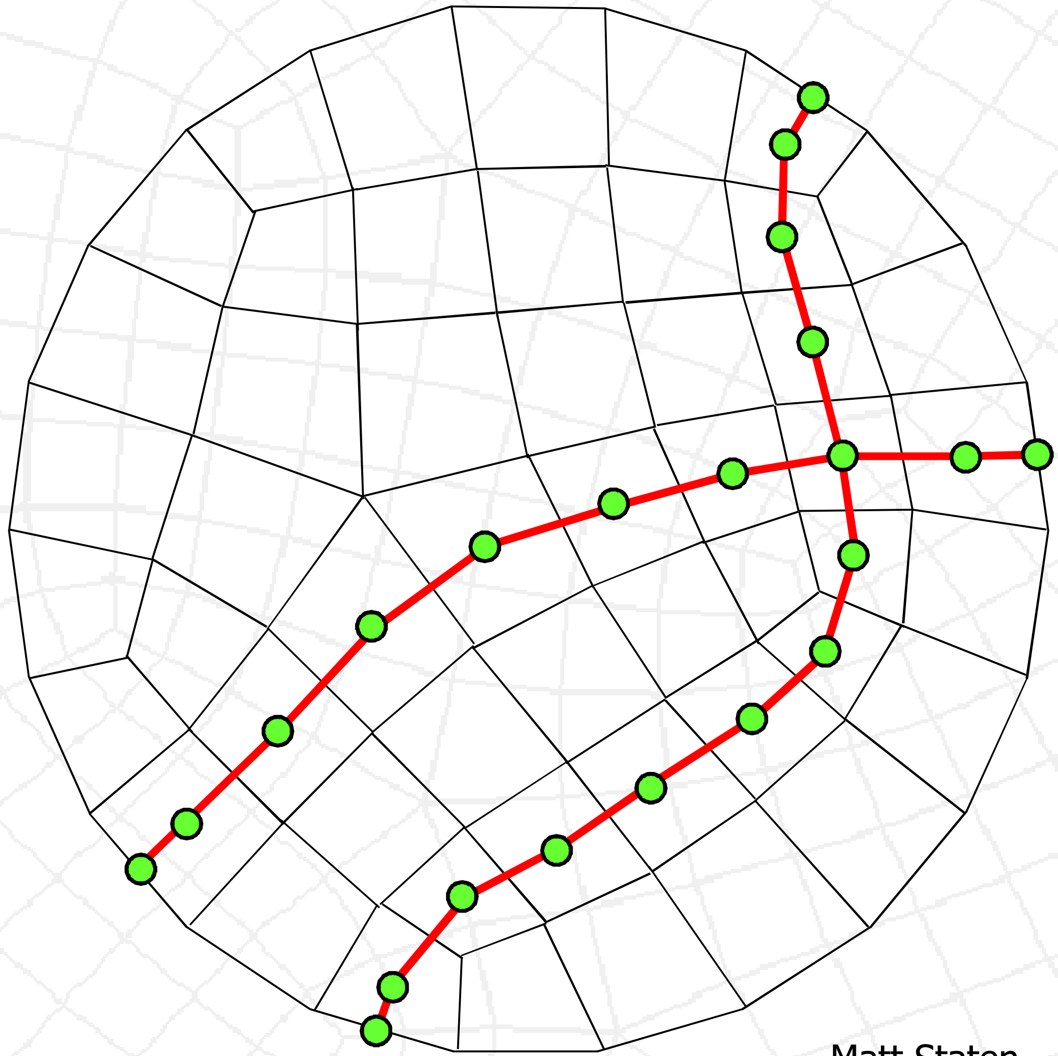
Quadrilateral Dual Representation

Each dual edge is part of exactly one dual chord.

The vertex at the centroid of a quad is the intersection of 2 dual chords.



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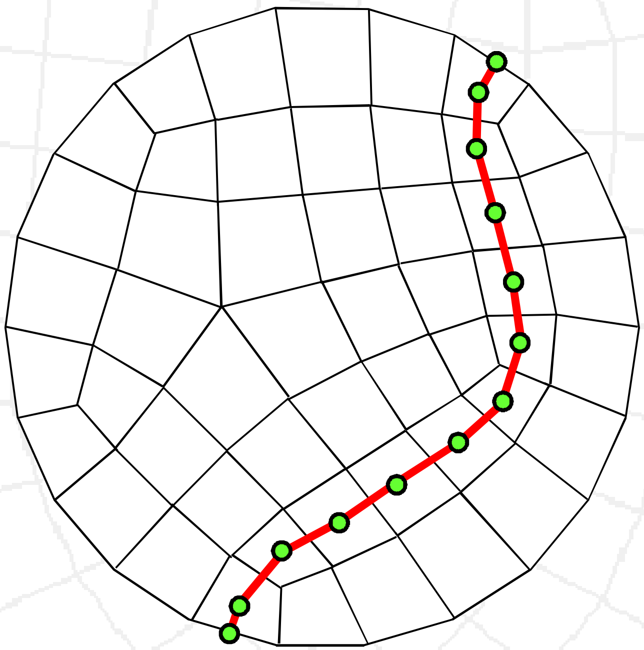
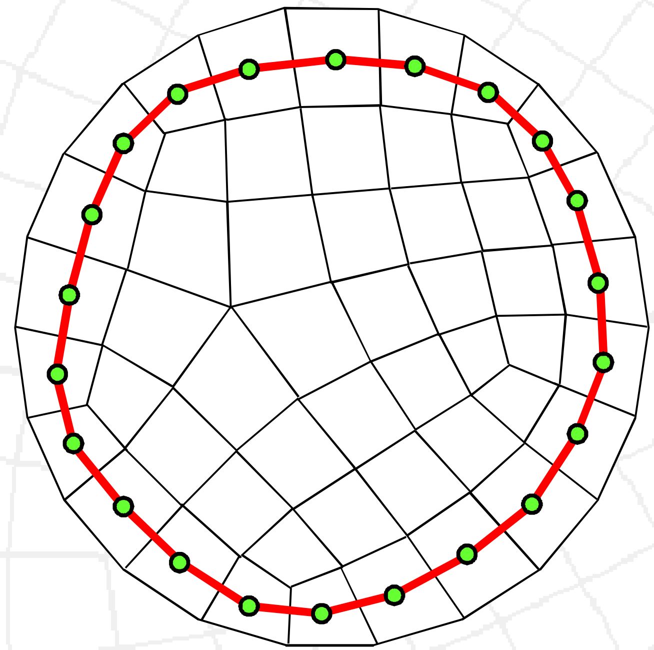
Quadrilateral Dual Representation

Dual chords must be either circular, or connect two boundaries.

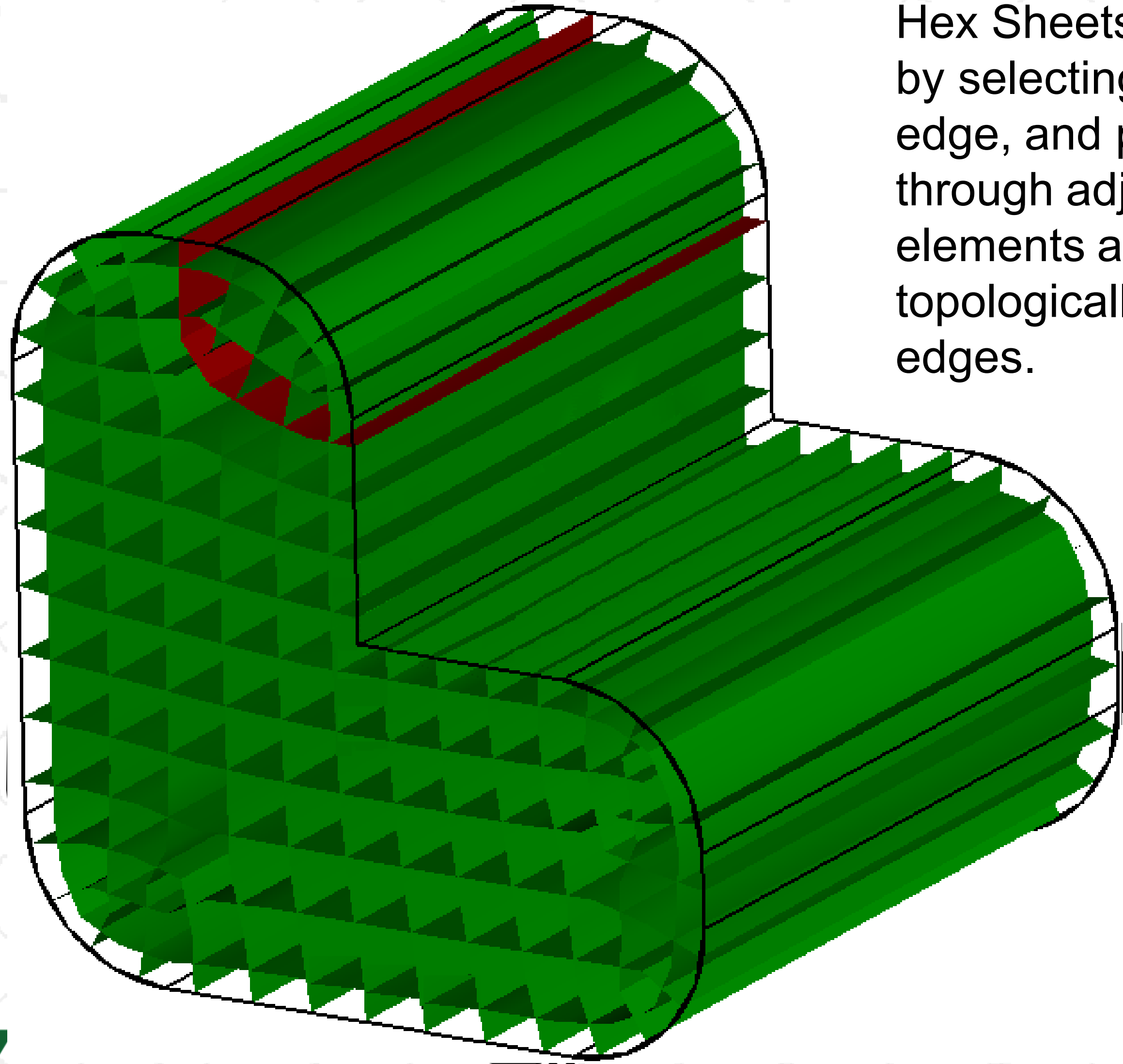


Connects two boundaries



Circular

Basic Definitions
Hex Sheets

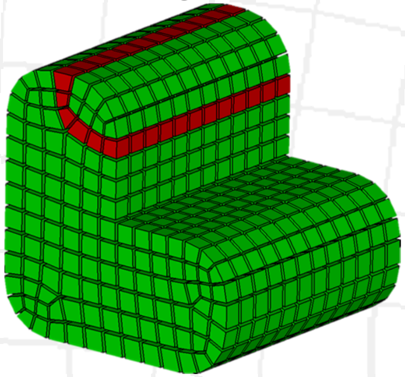


Hex Sheets are defined by selecting a single edge, and propagating through adjacent hex elements and through topologically opposite edges.

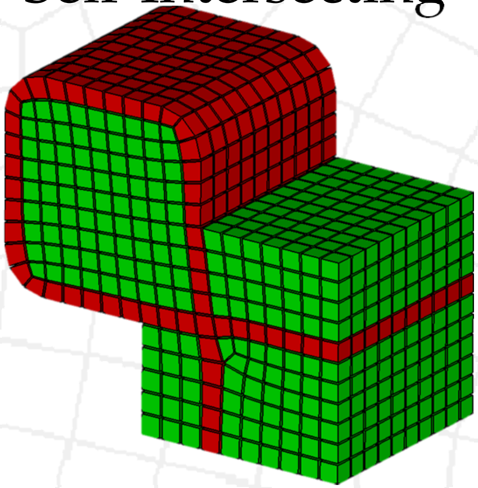
Basic Definitions
Hex Sheets

- **Hex Sheets can be classified into three types:**

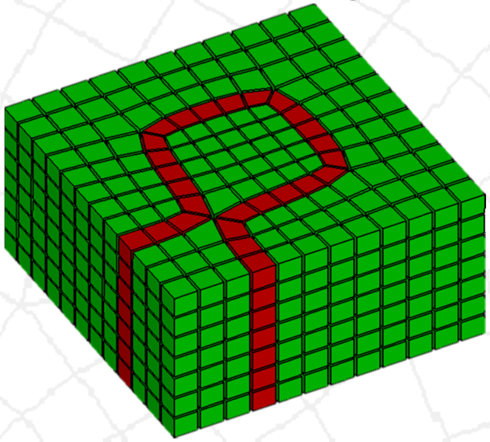
Regular



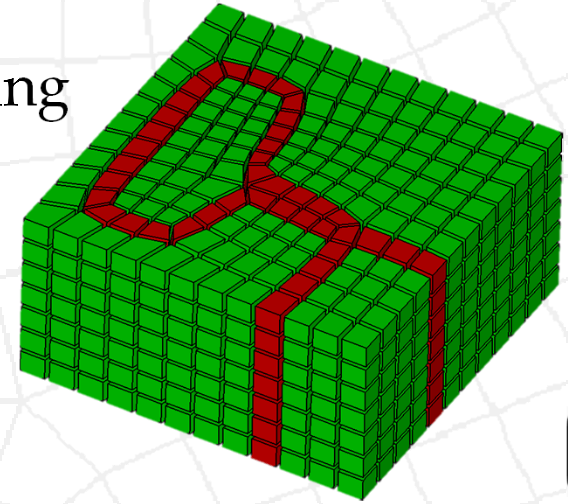
Self-Intersecting



Self-Touching



- **Types can be combined**
  – Self-Intersecting & Self-Touching



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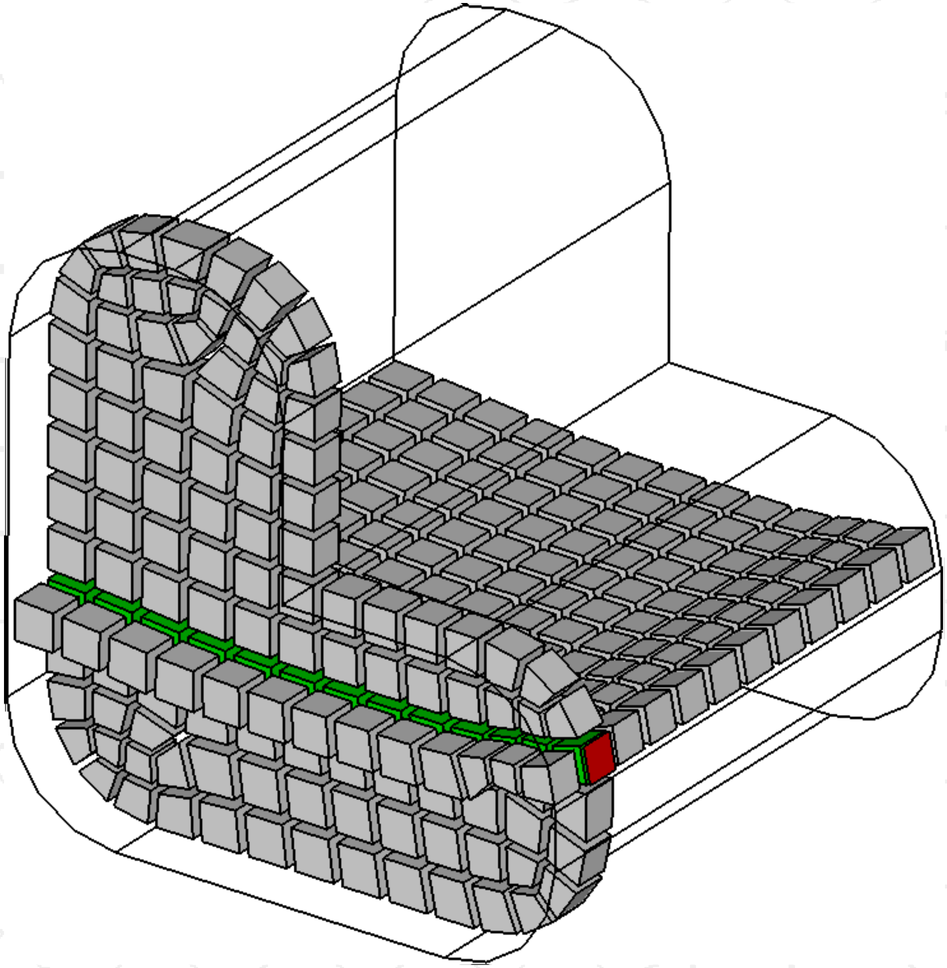
Basic Definitions
Hex Columns



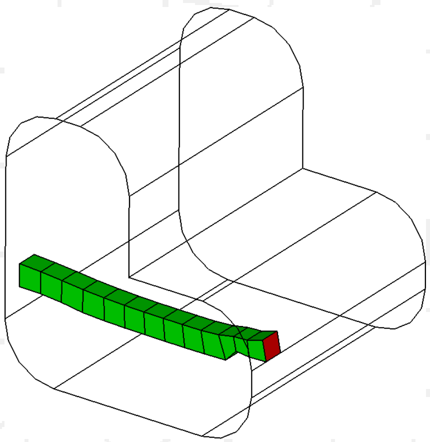
Hex columns are defined by selecting a single face. We iteratively propagate through adjacent hexahedra and opposite faces until we return back to the starting face, or terminate on the boundary.

Hex Columns define the intersection of two hex sheets.

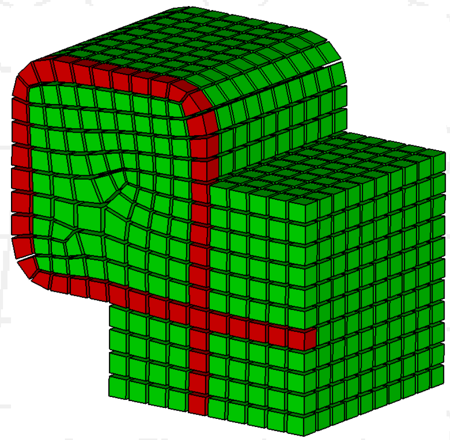
Basic Definitions
Hex Columns

- **Hex Columns can be classified into three types:**

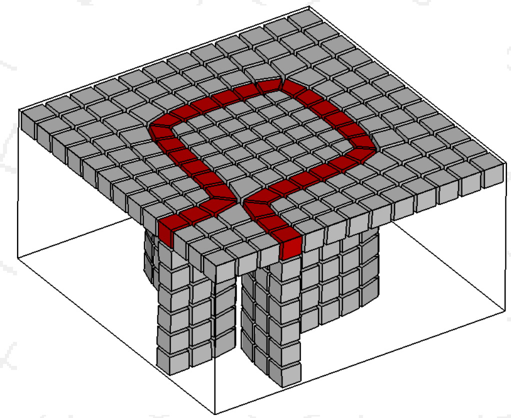
Regular



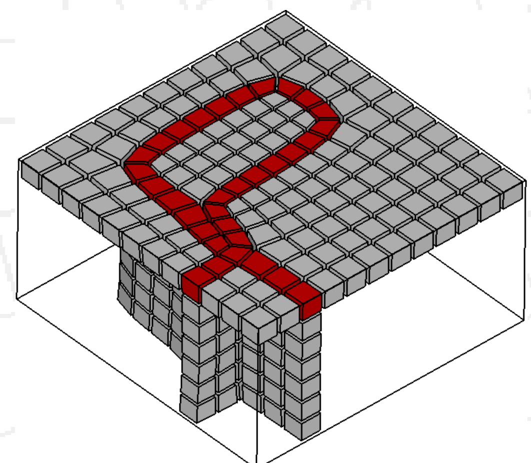
Self-Intersecting



Self-Touching



- **Types can be combined**
  - Self-Intersecting & Self-Touching



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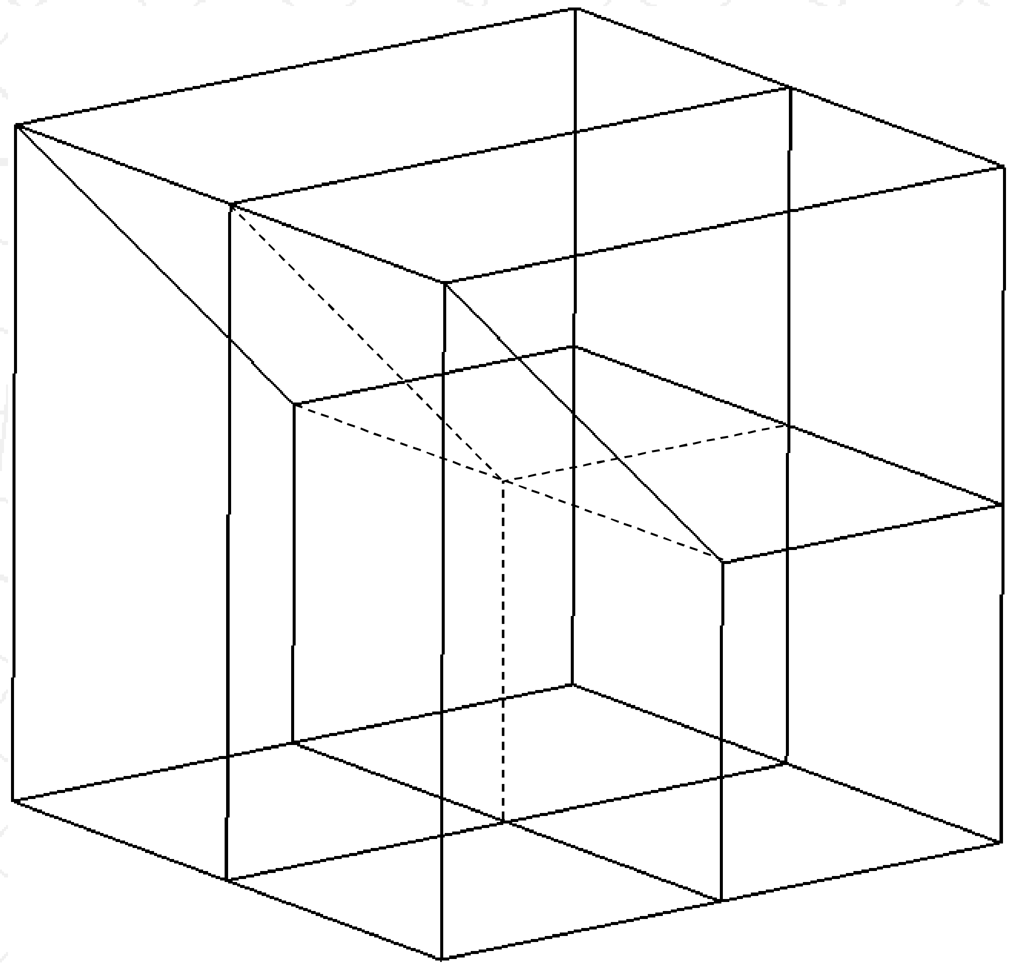
Hexahedral Dual Representation

The elemental representation of a hexahedral mesh, composed of hexahedra, faces, edges, and nodes, is known as the *primal*.

Hexahedral meshes also have a *dual* representation, similar to the voroni skeleton of a triangular delaunay mesh.



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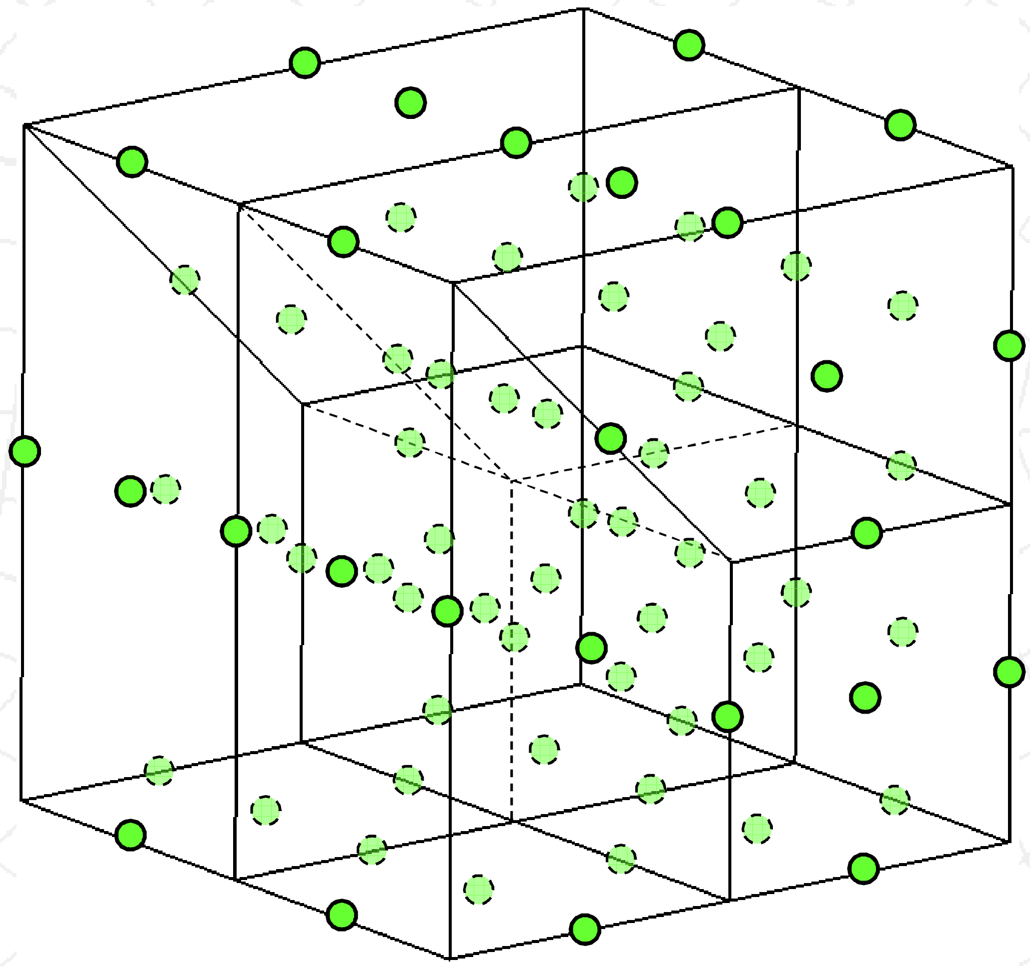
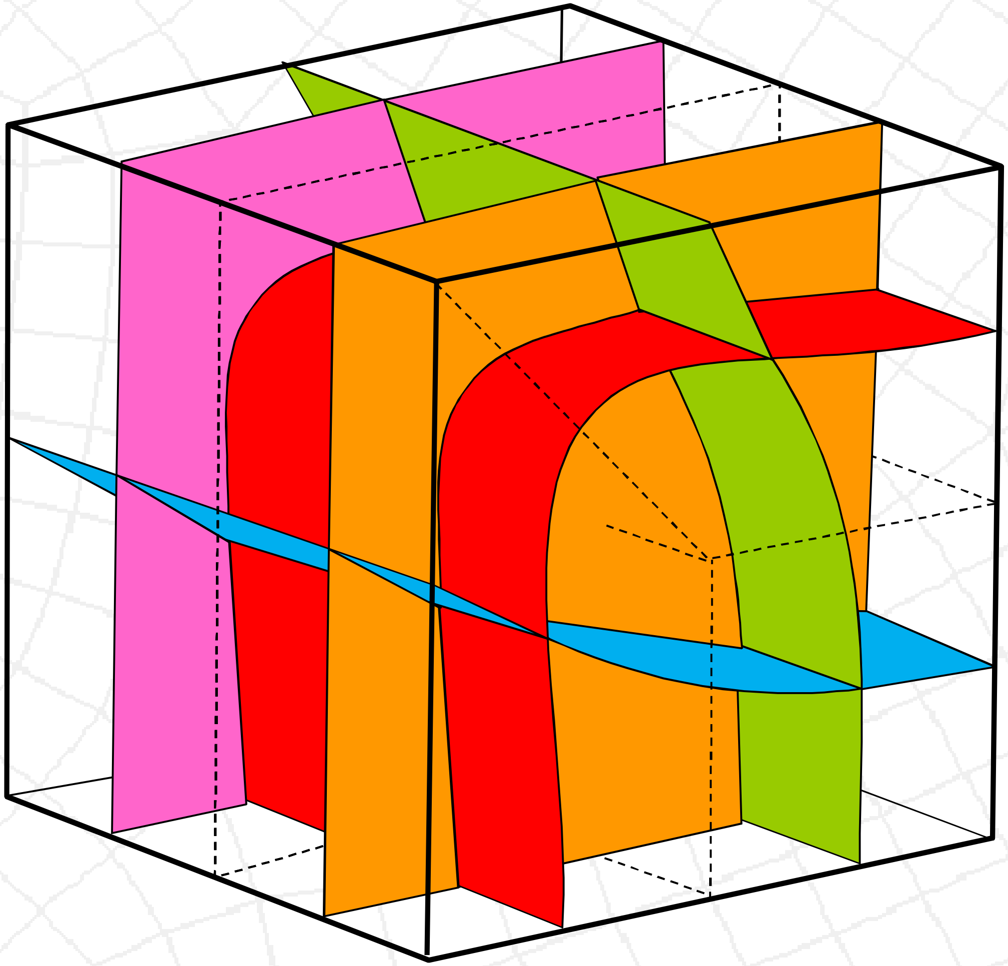
Hexahedral Dual Representation

A dual vertex, v_i, is defined at the centroid of each hexahedral element, boundary quad face, and boundary edge.



Hexahedral Dual Representation

Connecting the dual vertices through adjacent elements creates the edges and faces of the dual.



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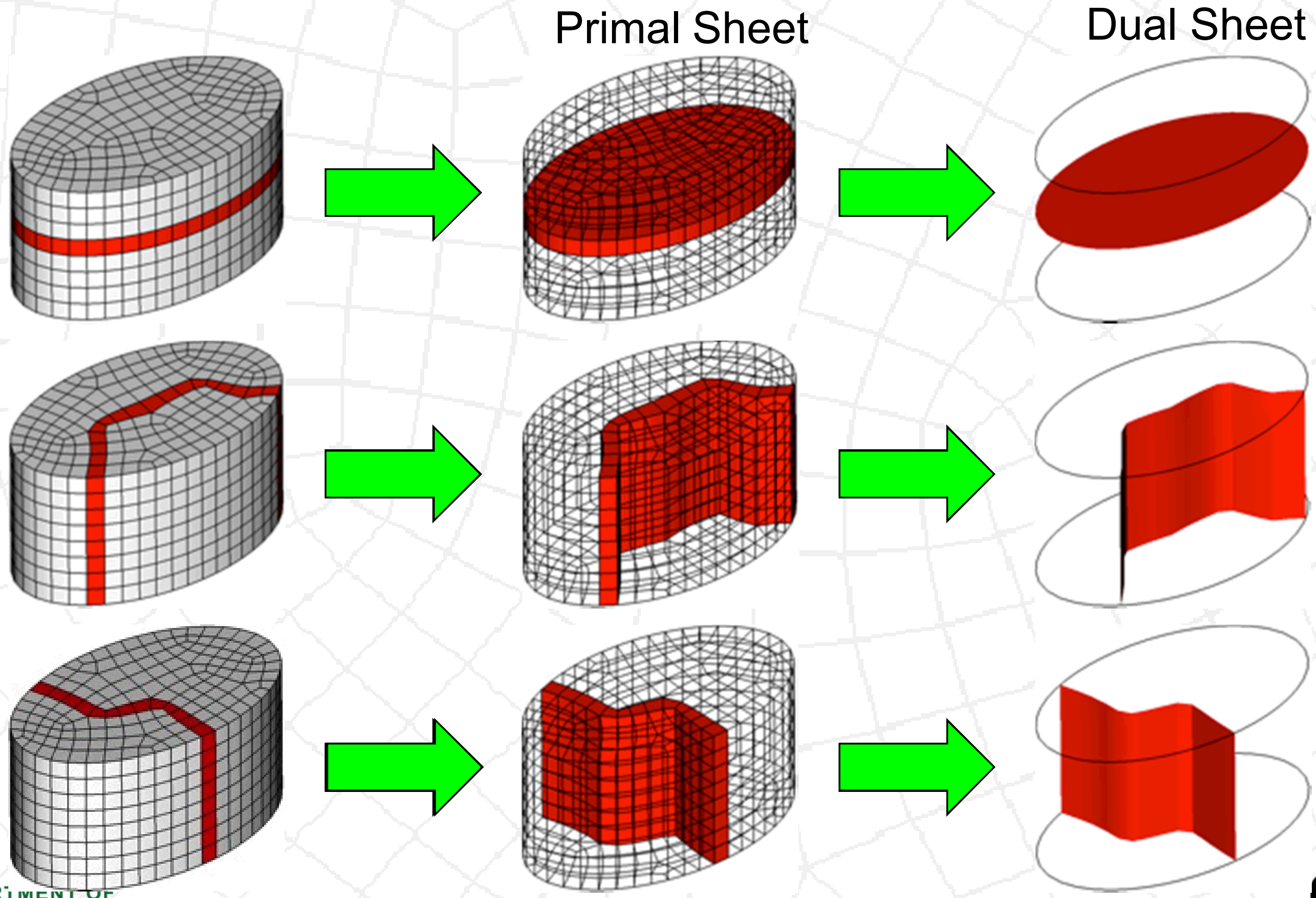
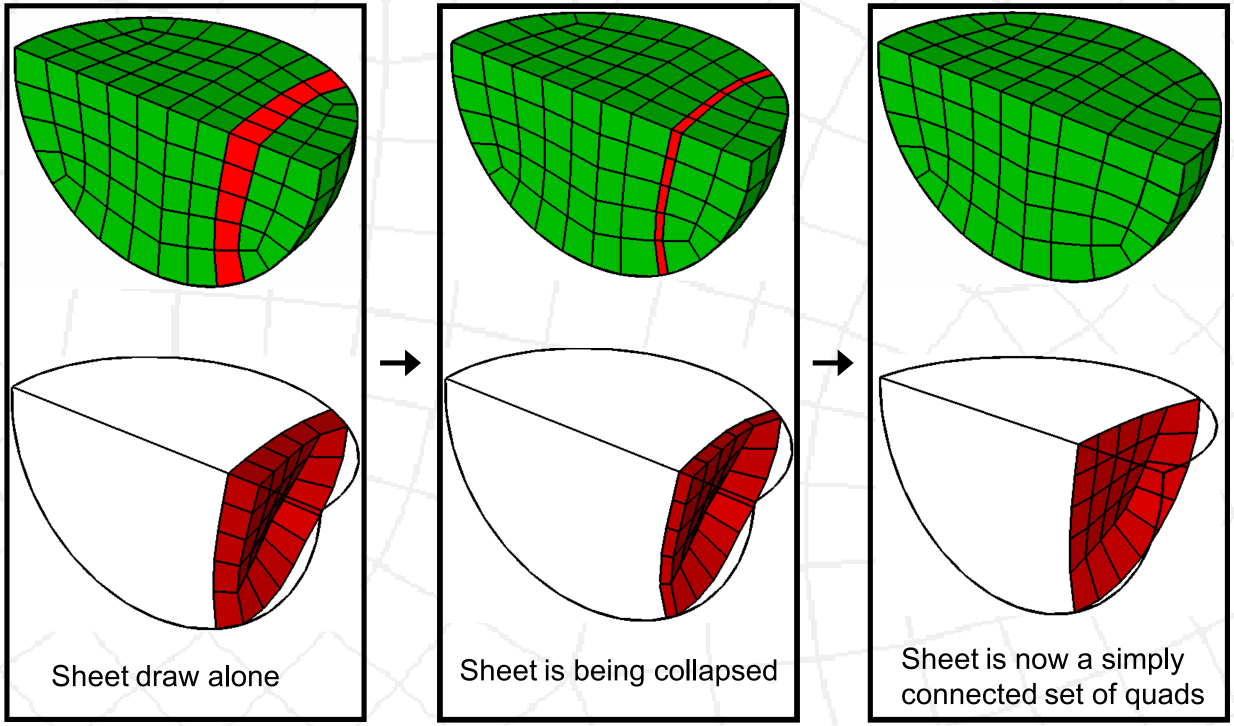
Quad/Hex Mesh Dual

Hexahedral meshes have an inherent layer structure. Each layer forms a sheet, having both a primal and a dual representation.



Hex Mesh Editing Techniques

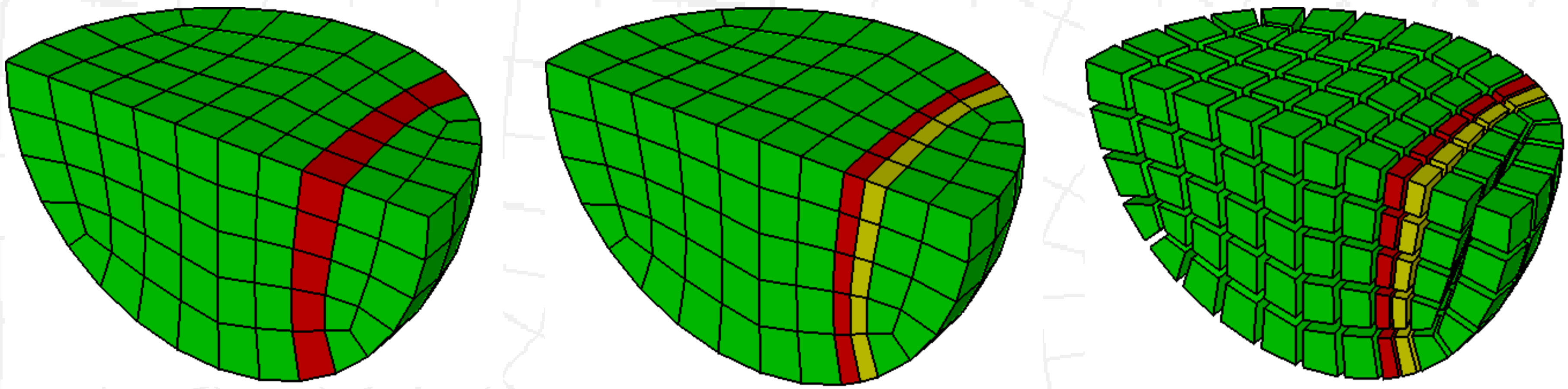
- **<u>Sheet Extract</u> – Borden et. al. 11th IMR, 2002**



- **Any sheet can be extracted, including self-intersecting, and self-touching**
- **Extract can lead to geometric node- associativity problems.**

Hex Mesh Editing Techniques: Methods of Inserting Sheets

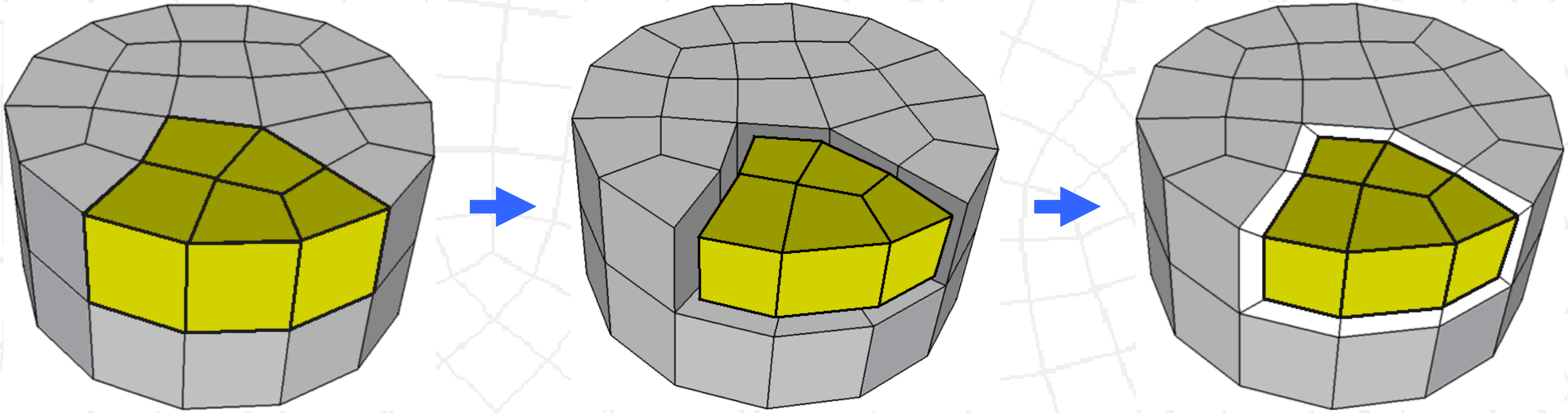
- **Sheet Dicing** – Melander et. al., 1997
  - Duplicates existing sheets by splitting each edge in the sheet



  - Any sheet can be diced, including self-intersecting, and self-touching

Hex Mesh Editing Techniques: Methods of Inserting Sheets

- **<u>Pillowing</u> – Mitchell et. al., 4th IMR 1995**
  - Inserts arbitrary "Regular" sheets only, by defining a shrink set, creating a gap, and filling with new sheet.



  - Pillowing cannot insert self-intersecting and self-touching sheets.

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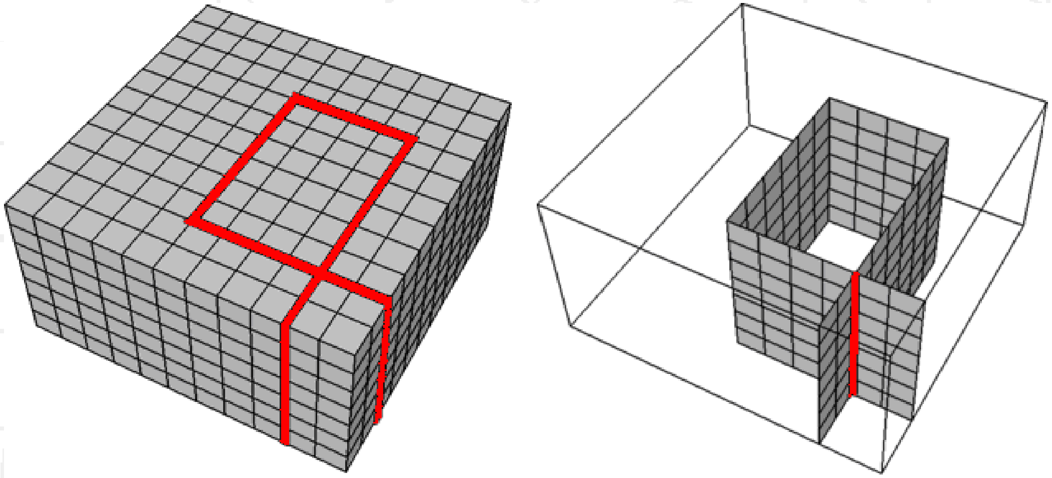
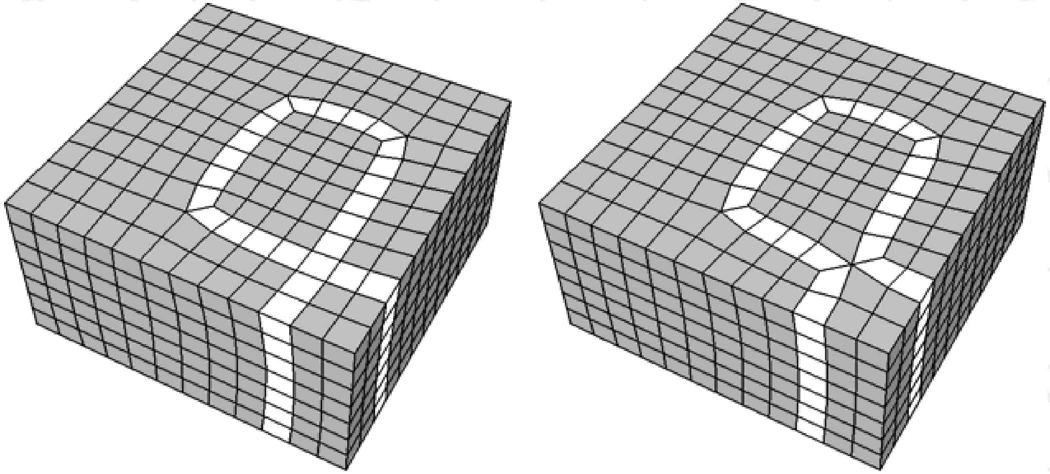
Sheet Inflation Example (4NMEsets)

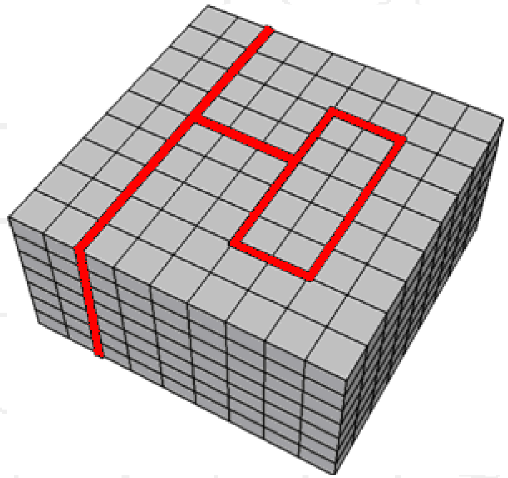


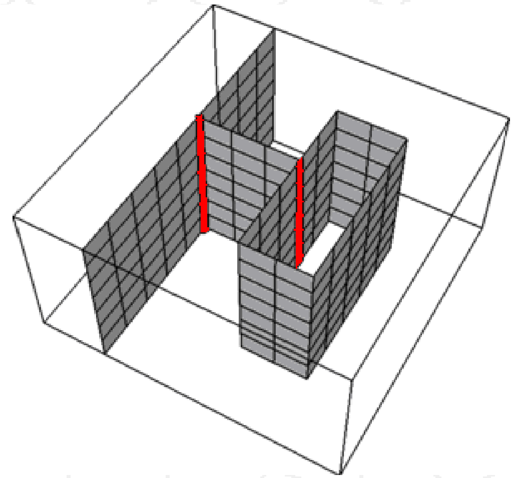
This example has one 4NMEset. When inflated, each 4NMEset can be processed as either self-intersecting, or self-touching.



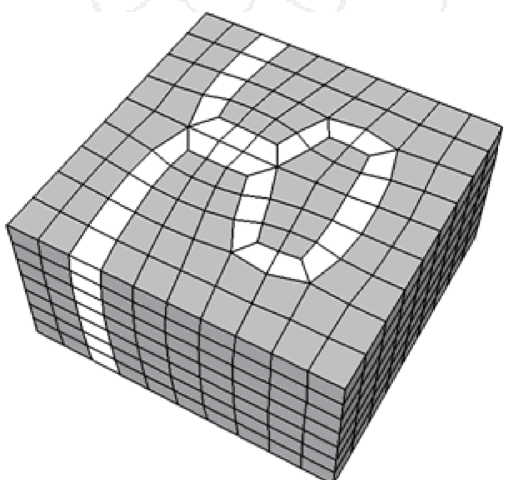
Matt

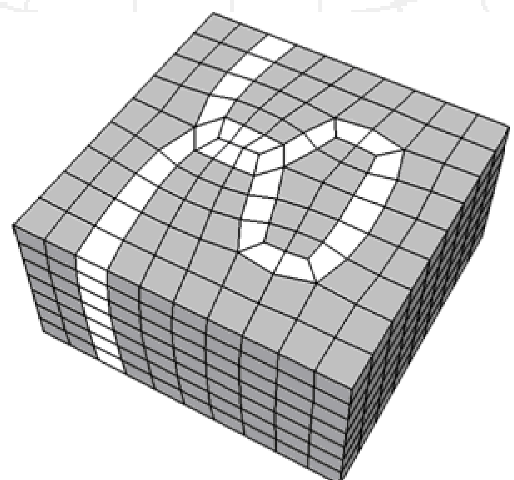
Sheet Inflation Example (3NMEsets)





This example has two 3NMEsets. When inflated, each 3NMEset can be processed as either self-intersecting, or self-touching. The quadrilaterals between the 3NMEsets are inflated into a self-touching section.





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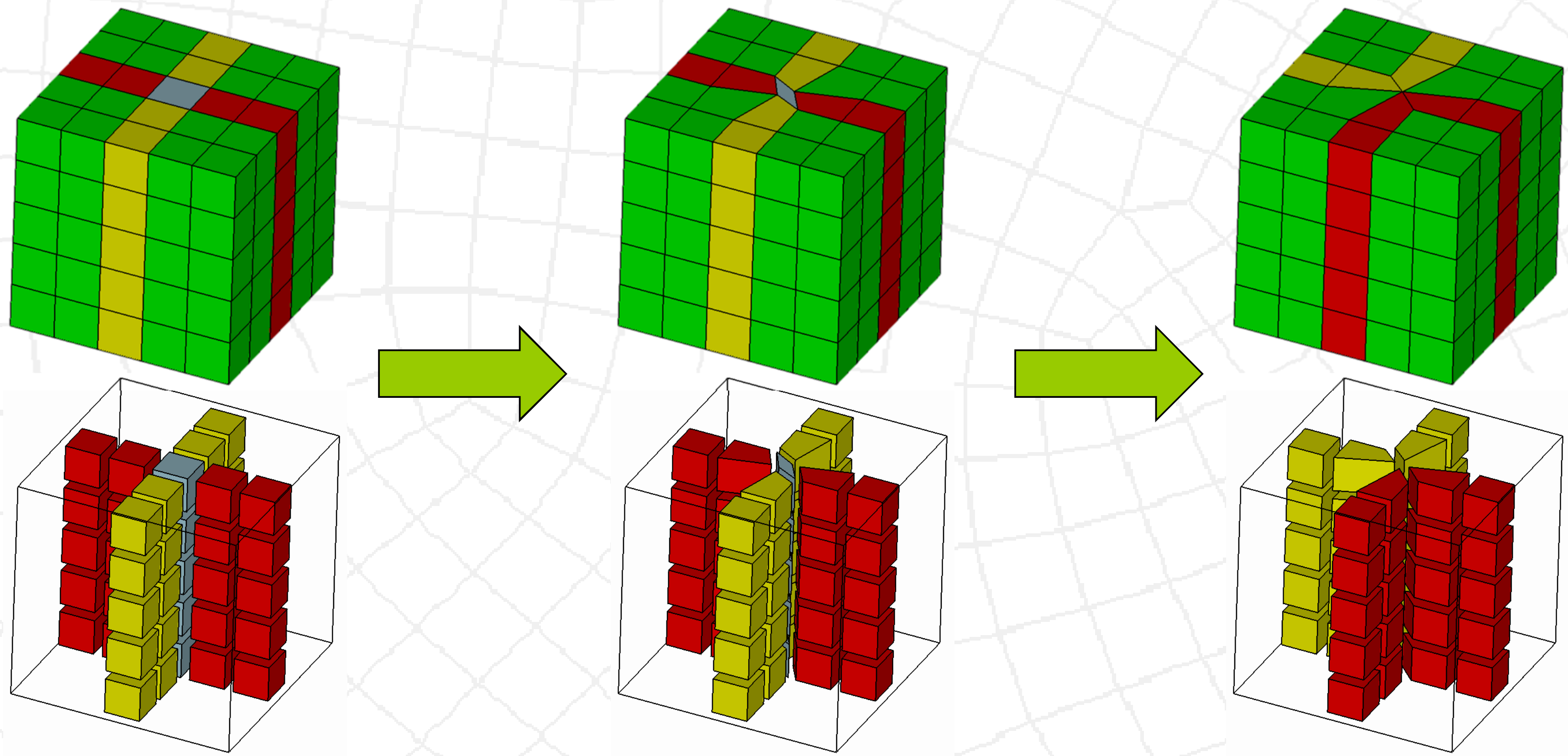
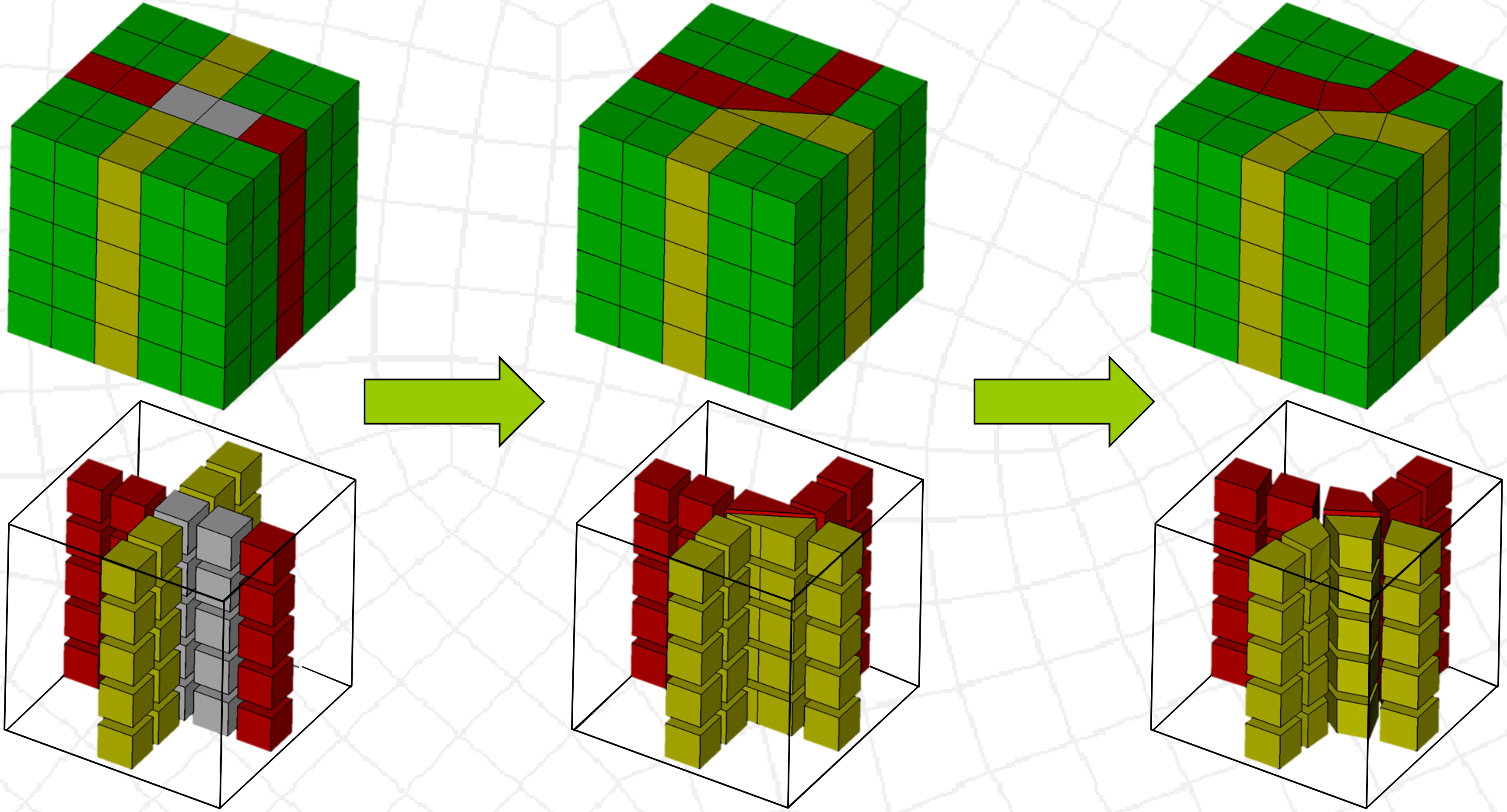
Hex Mesh Editing Techniques
Column Collapse

A column can be collapsed … which removes the intersection of the sheets … and re-directs the path of the intersecting sheets.



Column Swapping

The faces between 2 columns can be swapped… which removes the intersection of 2 sheets … and re-directs the path of the intersecting sheets.



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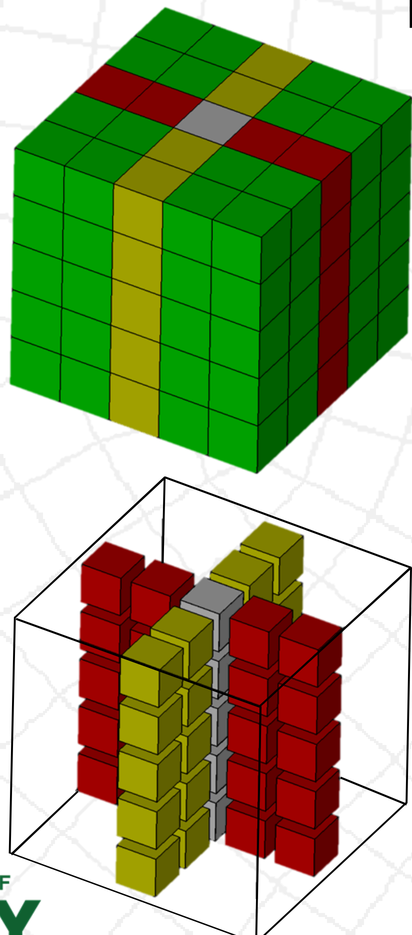
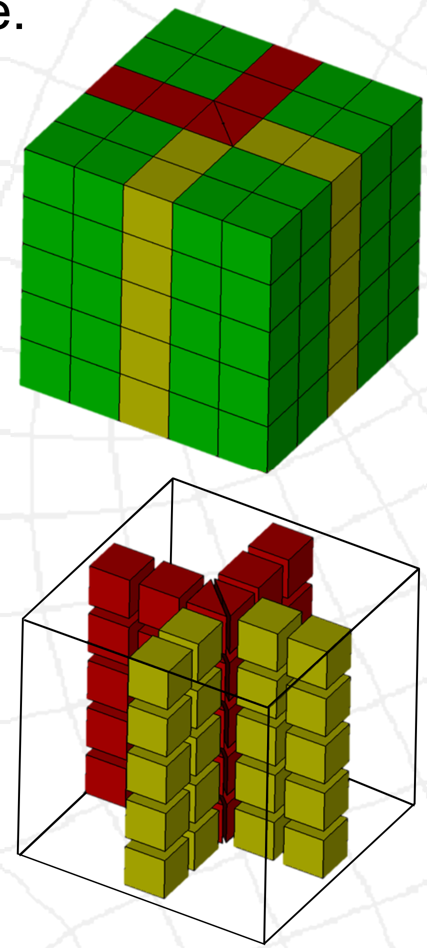
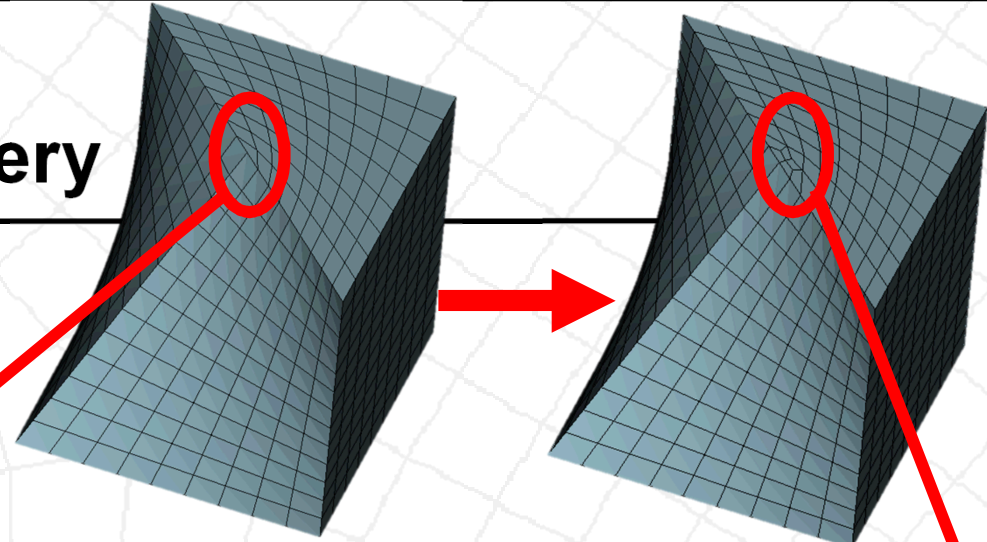
Column Split (aka Doublet Insertion)

The column at the intersection of 2 sheets can be split into 2 columns of poor quality, but redirecting the path of the intersecting sheets. Doublet insertions are always followed by a subsequent operations, such as a swap to achieve a higher objective.





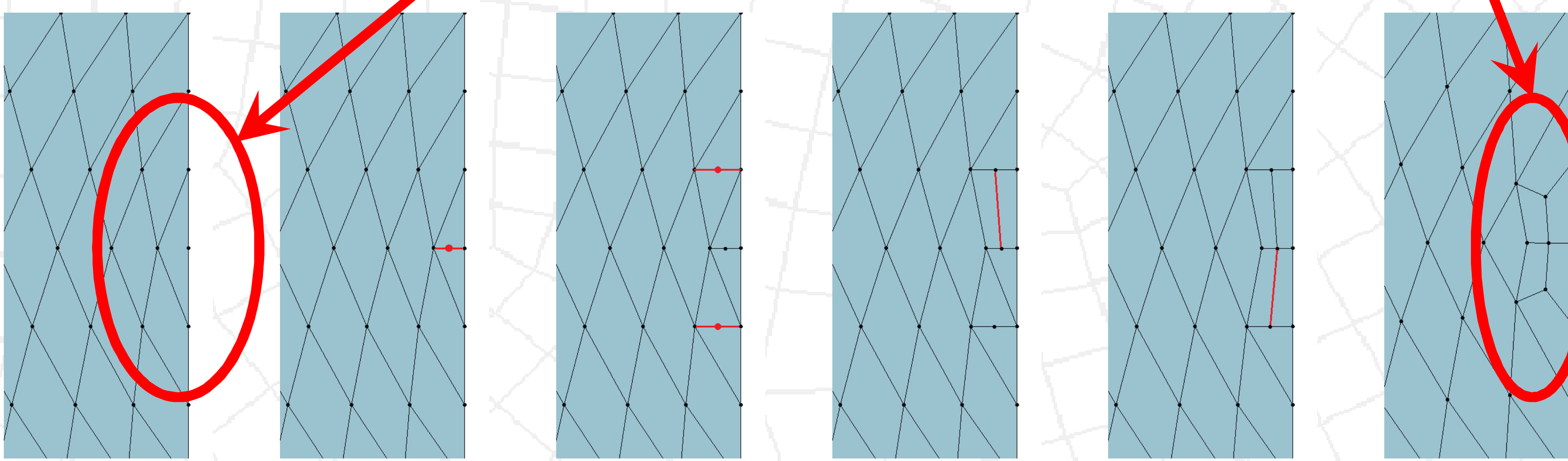
Mesh Surgery



- Several Column operations are normally done in sequence to effect a change.

Min Scaled Jacobian: 0.0

Min Scaled Jacobian: 0.47



Initial Mesh

Step 1: split column

Step 2: Split two more columns

Step 3: Swap between two columns

Step 4: Another swap between two columns

Step 5: Smooth to improve element quality.

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When to use Sheet/Column Operations?

- **Hex Mesh Quality Improvement**
  - Hex mesh quality may be poor from either initial mesh generation, shape deformation, or tweaking of CAD parameters.
  - Combinations of pillowing, dicing, and column operations can improve element quality.
  - Research Topic: Automatic application of sheet/column operations to improve element quality.

Zhu, et. Al, "Direct Editing on Hexahedral Mesh Through Dual Operations," 23rd International Meshing Roundtable, 2014.

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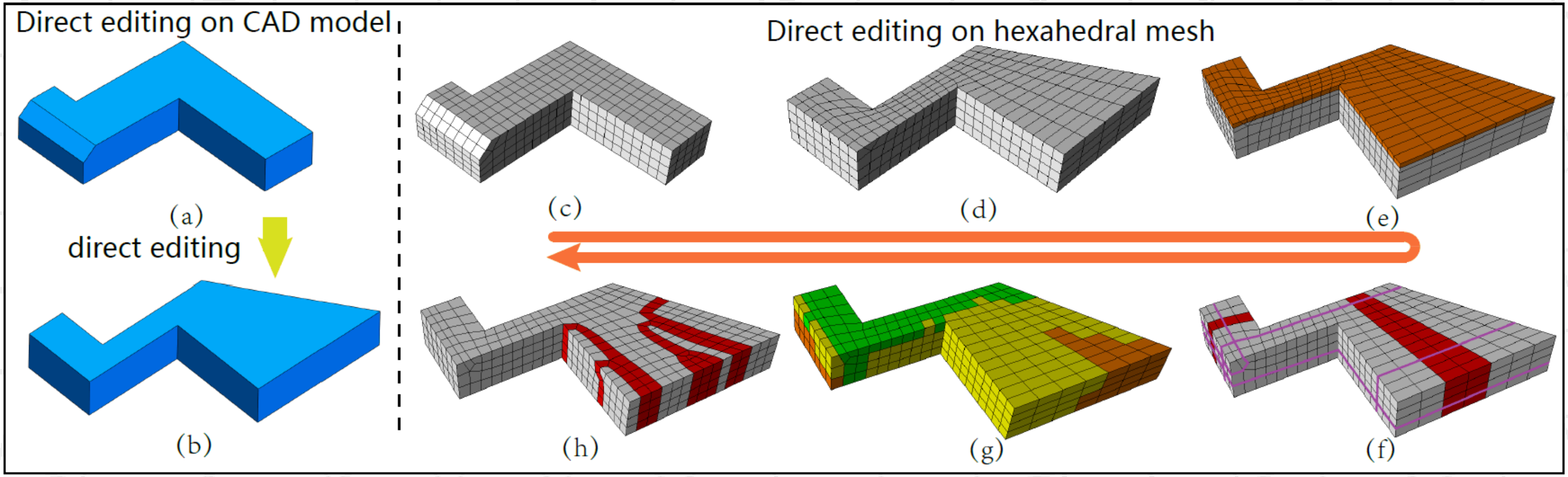
When to use Sheet/Column Operations?

Zhu, et. Al, “Direct Editing on Hexahedral Mesh Through Dual Operations,” 23rd International Meshing Roundtable, 2014.

A meshed CAD Model undergoes a parametric change. Mesh must morph with the geometry. Sheet operations performed to improve quality after morph.

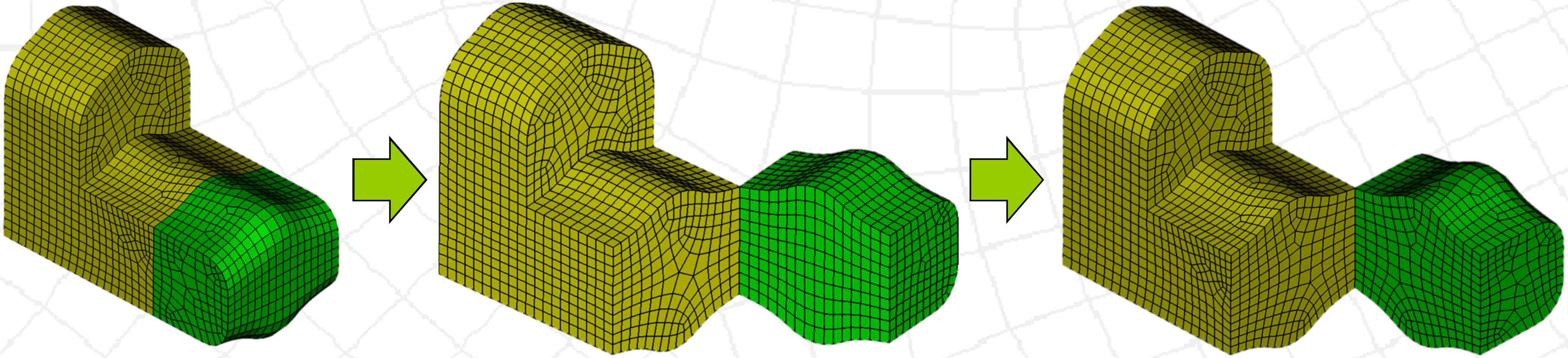


Research Topic: Can this be made robust enough to work on industrial models?

Creating Conforming Mesh Interfaces

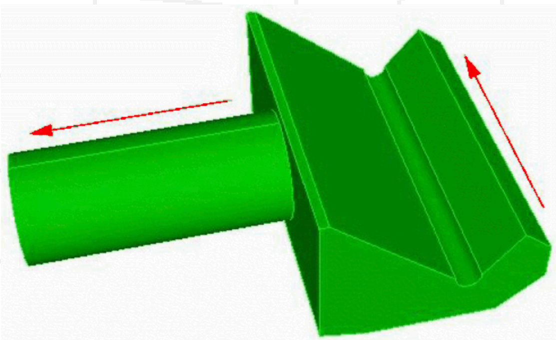
- **Hexahedral Mesh Matching**
  - Separate assembly components meshed separately can be modified later at interfaces to have conforming mesh topology.

Staten ML, Shepherd JF, Ledoux F, Shimada K. Hexahedral Mesh Matching: Converting Non-Conforming Hexahedral-to-Hexahedral Interfaces into Conforming Interfaces. *International Journal for Numerical Methods in Engineering*, Early View, 3 Dec. 2009, DOI: 10.1002/nme.2800.

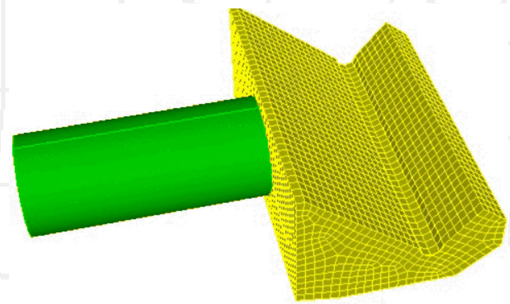


Hex Meshing Objects With More Than One Sweep Direction

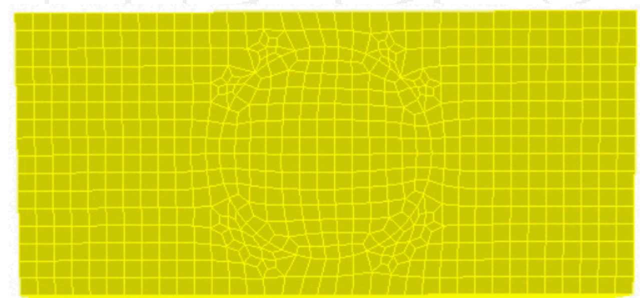
Jankovich, et al., "The Graft Tool: An All-Hexahedral Transition Algorithm for Creating a Multi-Directional Swept Volume Mesh", *8th International Meshing Roundtable*, 1999



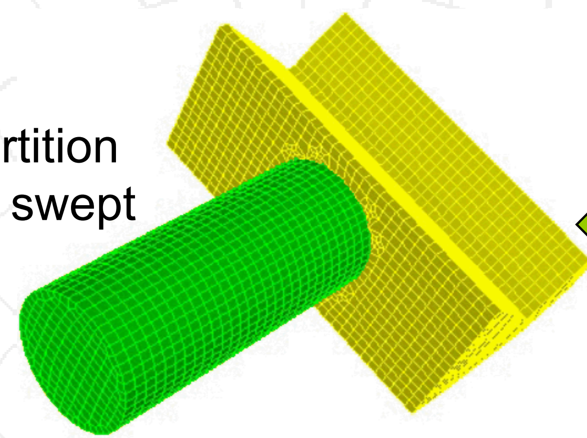
Input CAD Model



First partition (trunk) is swept



Pillowing and column operations performed to imprint source of 2nd sweep.



The 2nd Partition (branch) is swept

Identifying the Natural Internal Flow of volumes

A New Area of Hex Mesh Research

FRAME FIELDS

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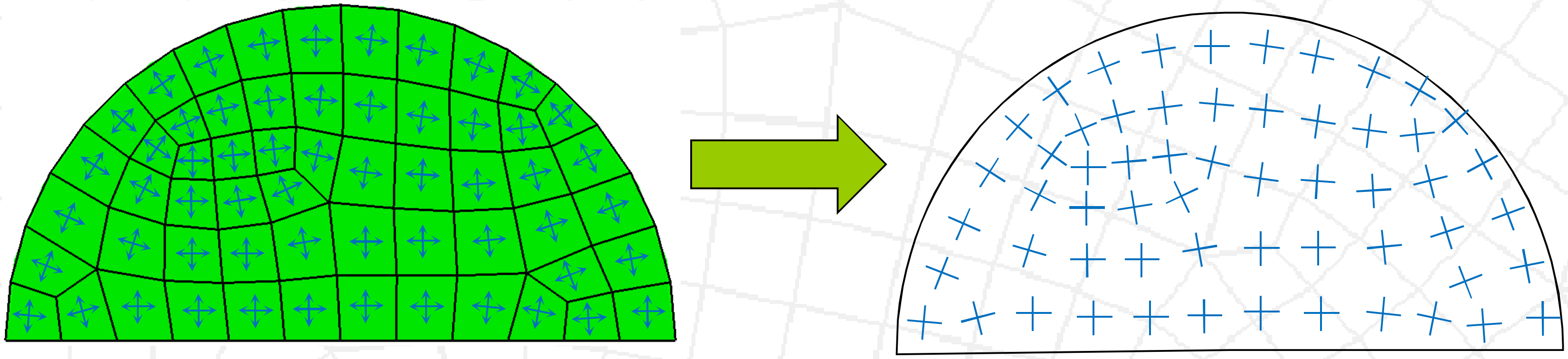
Frame Fields

Each quad in a mesh has 2 inherent directions. We can extract a field of local direction frames from any quad mesh.

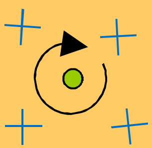


A mesh generated with Paving

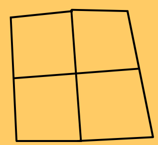
Properties of Frame Fields

Property 1:
There are singularities defined at the zeros of the piecewise linear interpolation of the frame fields.

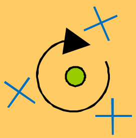
Case 1: Full 360° turn



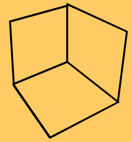
No singularity



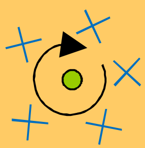
Case 2: 270° turn



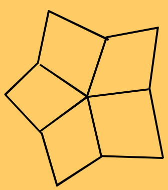
Singularity 3



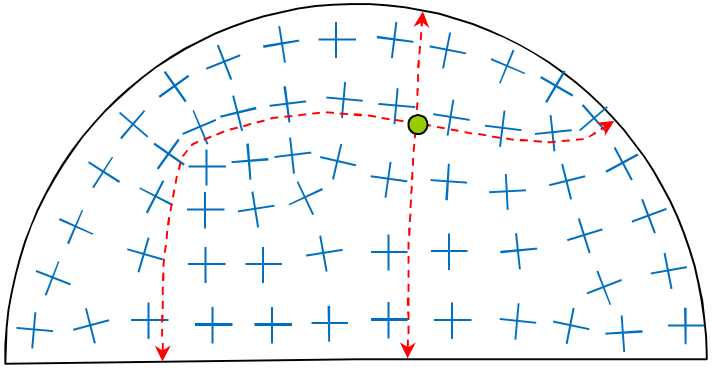
Case 3: 450° turn



Singularity 5



Property 2:
You can trace streamlines by interpolating the frames. In general Streamlines proceed in 2 directions from any point.



Steamlines that end at singularities are called separatrices.

Reference: "Kowalski, Ledoux, Frey, "A PDE based approach to multidomain partitioning and quadrilateral meshing," IMR21, 2012.

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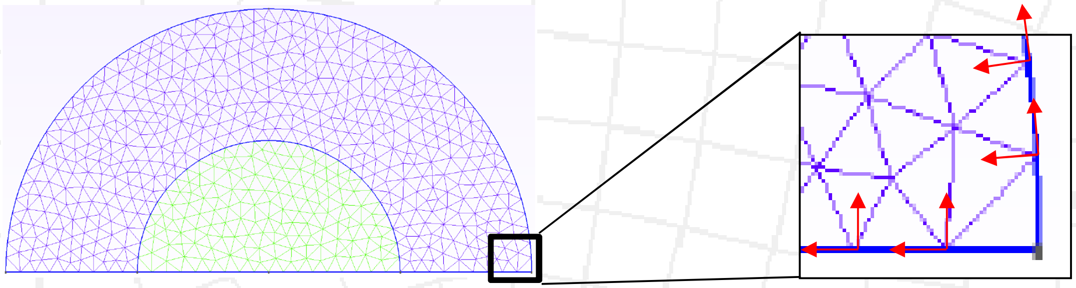
Can we first generate a frame field and then generate a quad mesh from it?

Yes, Kowalski (IMR21) builds a frame field by solving a PDE (Laplace) with Dirichlet BCs over a triangle mesh.

For BCs, normals and tangents are computed for each boundary node.

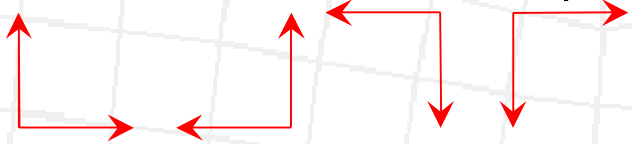


At 90 degree turns, the normal BC should pair with the adjacent tangent BC. But when to make the cut-off? We want the Laplace solution to decide. We convert the 2 vector (tangent, normal) into a single vector, N-Rosy (N-rotational symmetry direction).

N-Rosy Definition

Formal definition from Kowalski

90° rotations of a frame are equivalent



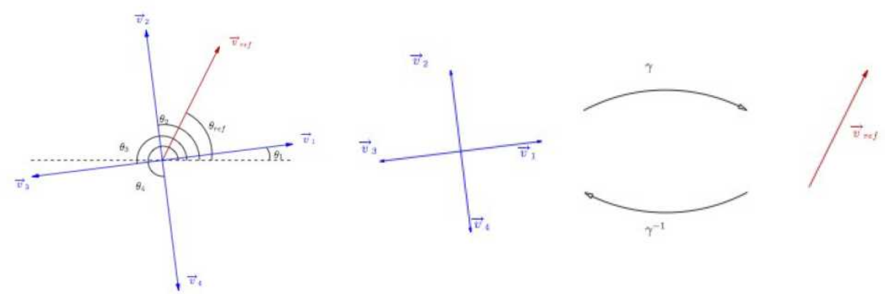
Direction field definition

Definition 2.1 *Let $N \in \mathbb{N}^+$ be an strictly positive integer, a* **N-rotational symmetry direction** $\overset{N\rightarrow}{d}$ *on Ω is a set of N unit tangent vectors on Ω invariant by rotation of $2\pi/N$ around the normal vector $\vec{n}$.*

Each of these vectors is defined as a *member vector* of $\overset{N\rightarrow}{d}$. For brevity,we call N-Rosy a N-rotational symmetry direction. A N-RoSy $\overset{N\rightarrow}{d}$ can be build from a vector $\vec{u_0}$ of Ω according to formulation

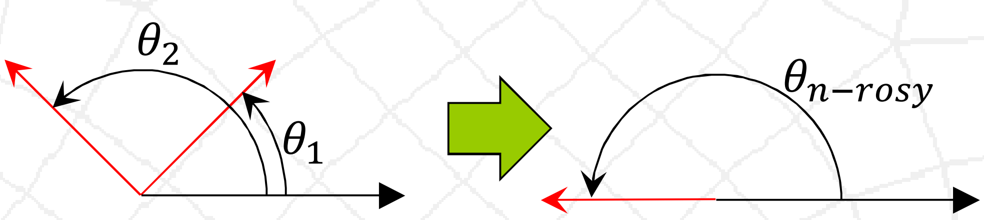
$$\overset{N\rightarrow}{d} = \{\vec{u_k} = R_{\vec{n}}(\vec{u_0}, 2k\pi/N) | k \in \mathbb{Z}\} \quad (4)$$

where $R_{\vec{n}}(., \alpha) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is the rotation of angle $\alpha \in \mathbb{R}$ around $\vec{n}$.



Convert any planar coordinate frame into a single planar vector.

$Given: (\theta_1, \theta_2)$



$$\theta_{n-rosy} = 4 * \theta_1 \,\%\, 360$$
$$= 4 * \theta_2 \,\%\, 360$$

$$\theta_1 = \frac{\theta_{n-rosy}}{4}$$

$$\theta_2 = \theta_1 + 90°$$

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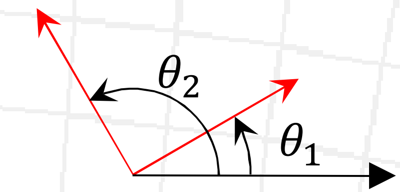
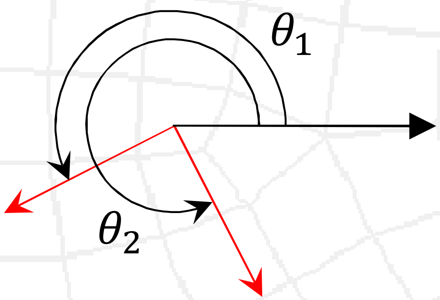
N-Rosy Examples

(30°, 120°)



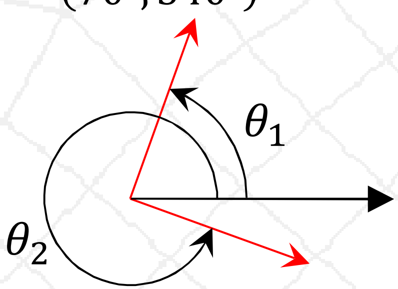
$$\theta_{n-rosy} = 4 * 30° \,\%\, 360 = 120°$$
$$\theta_{n-rosy} = 4 * 120° \,\%\, 360 = 120°$$
$$\theta_1 = \frac{120°}{4} = 30°$$
$$\theta_2 = 30° + 90° = 120°$$
$$(30°, 120°)$$

(207°, 297°)



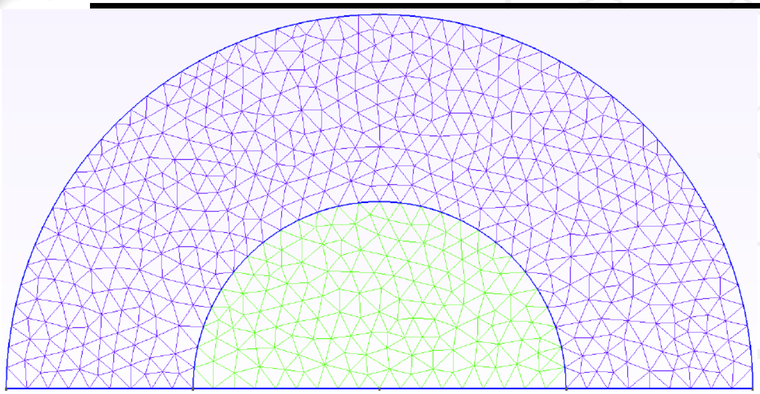
$$\theta_{n-rosy} = 4 * 207° \,\%\, 360 = 108°$$
$$\theta_{n-rosy} = 4 * 297° \,\%\, 360 = 108°$$
$$\theta_1 = \frac{108°}{4} = 27° \rightarrow 27° + 180° = 207°$$
$$\theta_2 = 27° + 90° = 117° \rightarrow 117° + 180° = 297°$$
$$(207°, 297°)$$

(70°, 340°)

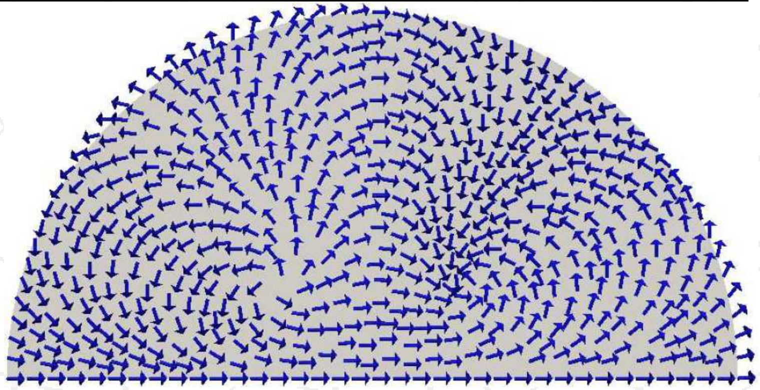


$$\theta_{n-rosy} = 4 * 70° \,\%\, 360 = 280°$$
$$\theta_{n-rosy} = 4 * 340° \,\%\, 360 = 280°$$
$$\theta_1 = \frac{280°}{4} = 70°$$
$$\theta_2 = 70° + 90° = 160° \rightarrow 160° + 180° = 340°$$
$$(70°, 340°)$$

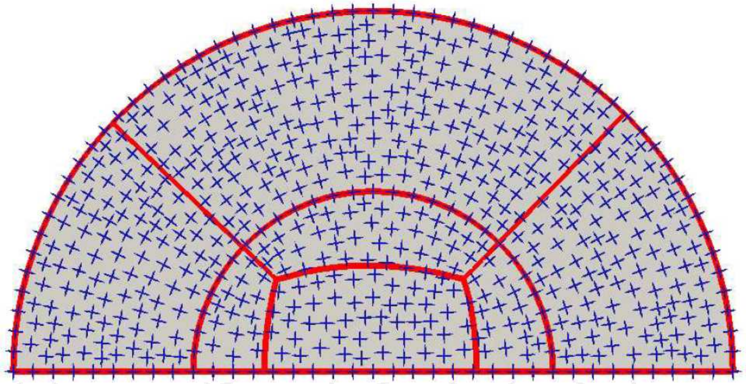
Planar Quad Mesh Generation via Frame Fields



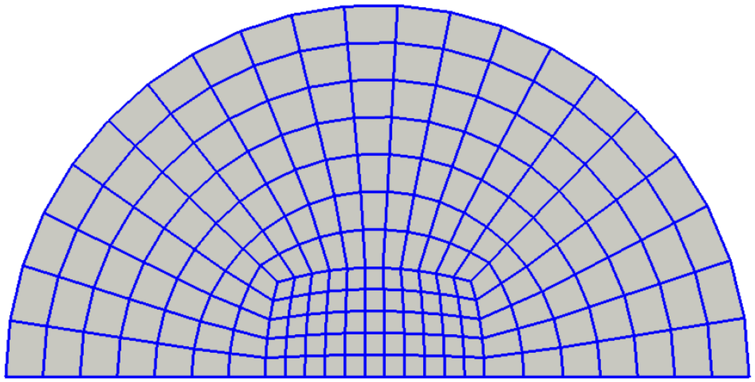
STEP 1: Build a planar tri mesh on the surface. Compute BCS: n-rosy from tangent/normal at each boundary node.



STEP 2: Solve Laplace equation.



STEP 3: Convert back to frames, compute singular points, trace separatrices, yielding a block decomposition.

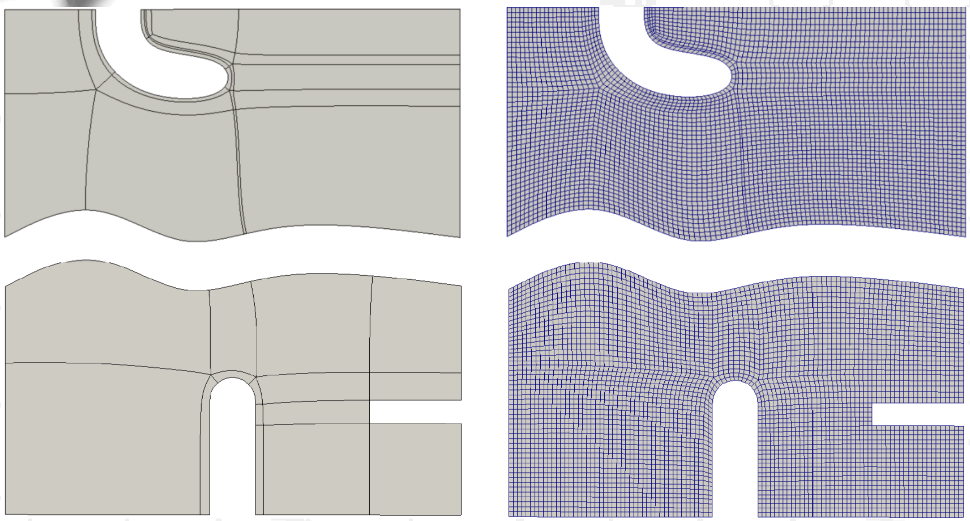


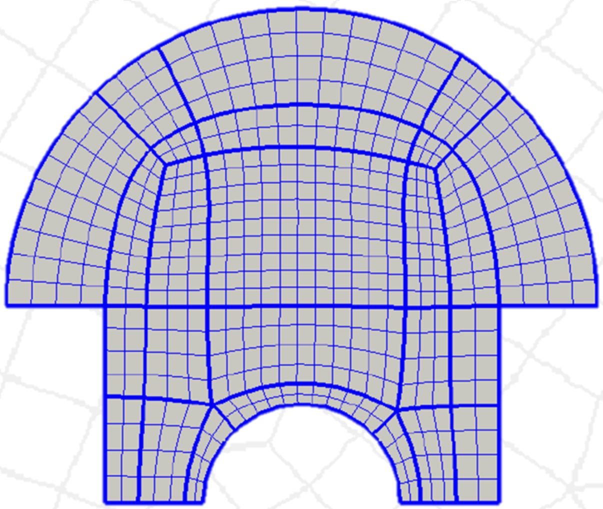
STEP 4: Map each block decomposition.

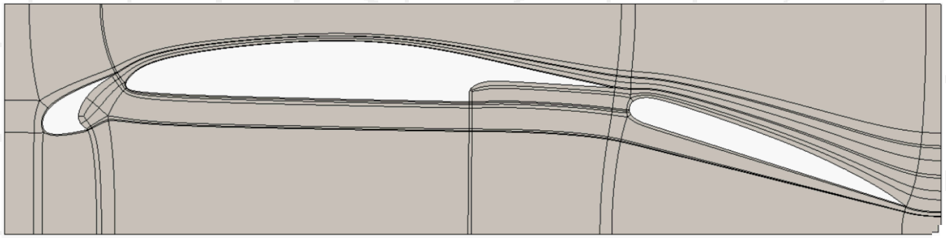
Reference: "Kowalski, Ledoux, Frey, "A PDE based approach to multi domain partitioning and quadrilateral meshing," IMR21, 2012.

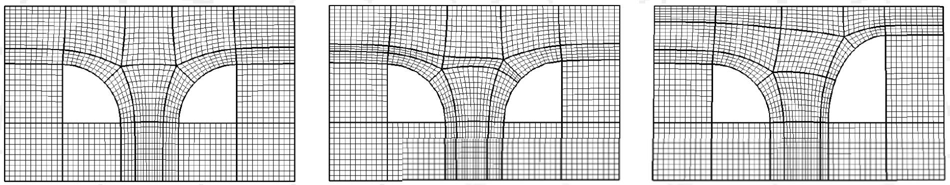
Can also be done for 3d surfaces, in parametric space.

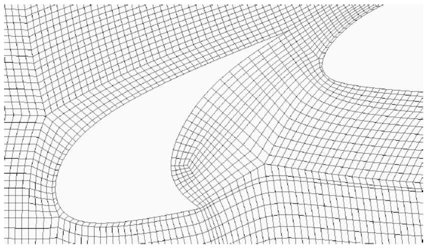
Frame Field Examples

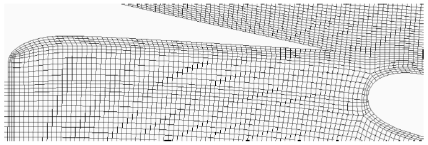


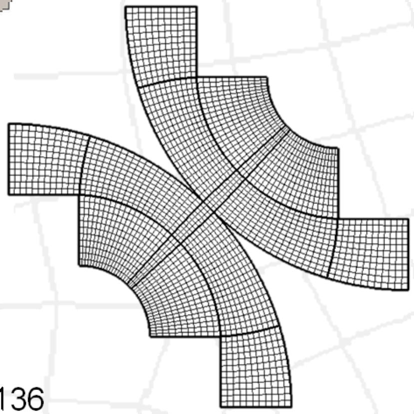


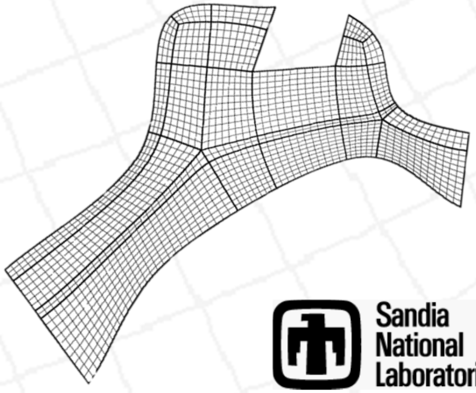






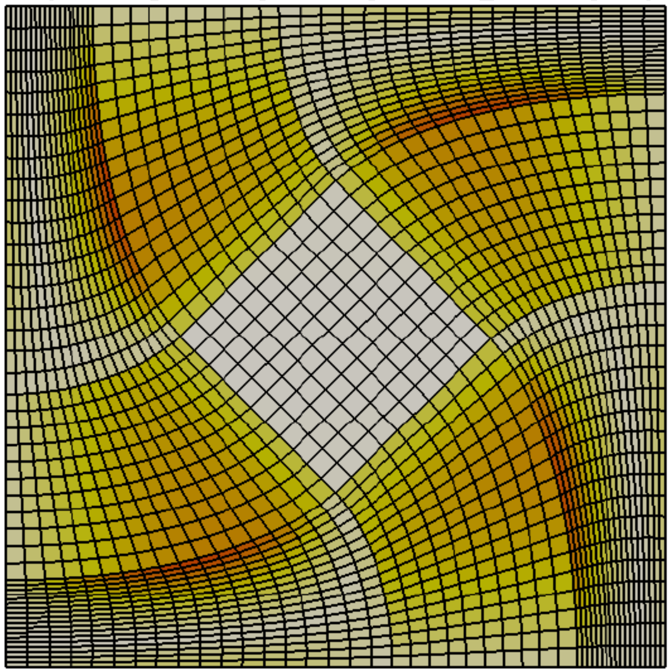






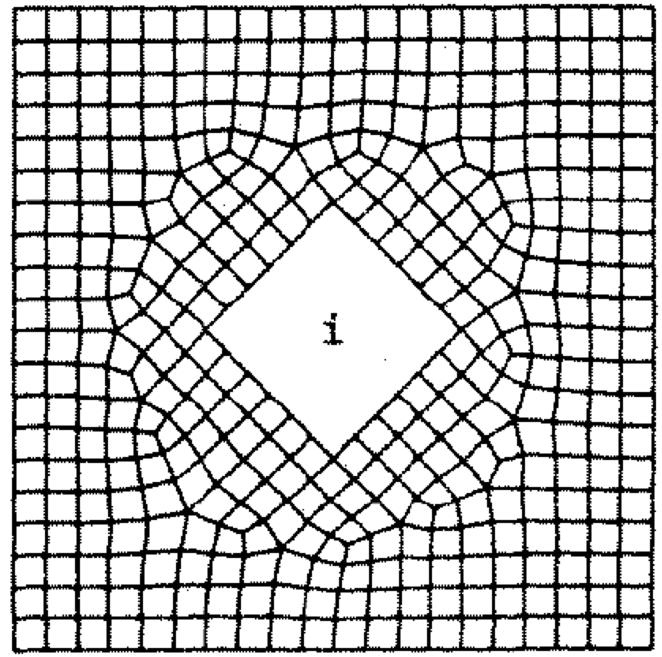
Frame Fields vs. Paving

Frame Fields, Kowalski



Sometimes generates high aspect ratio elements

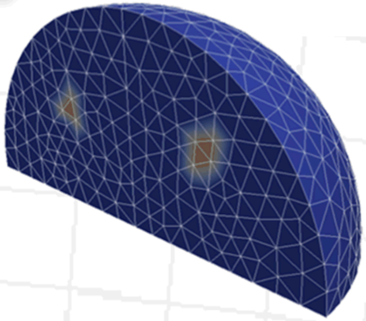
Paving, Blacker



Attempts to generate roughly isotropic elements everywhere.

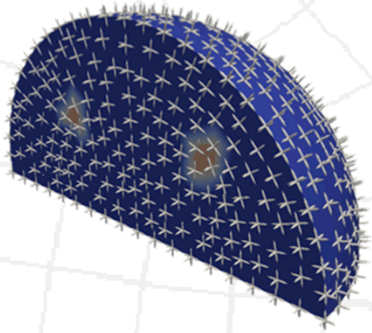
Which mesh is “Better” depends on the physics being solved.

Volumetric Frame Fields



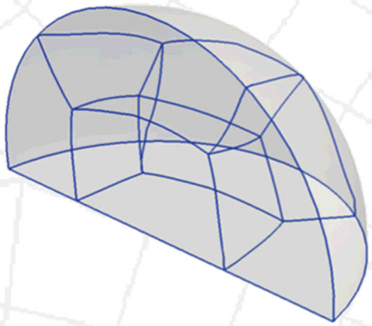
(a)

Initial Tet Mesh



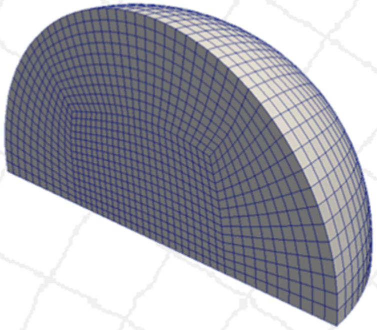
(b)

Frame field generated over Tet Mesh, via optimization approach



(c)

Singularity graph extracted defining block structure

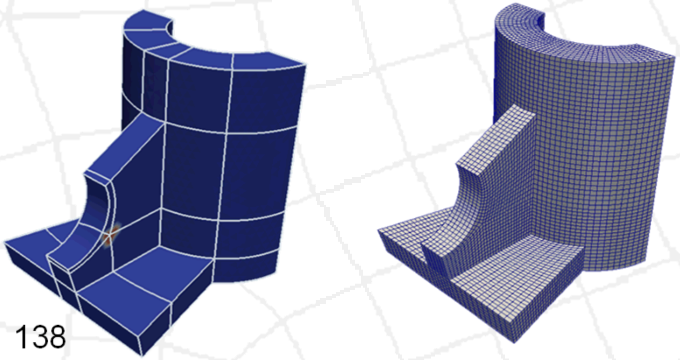


(d)

Mesh generated on blocks

Major difference, is there is no equivalent to n-rosy in 3D, thus can't solve Laplace! You must define frame field with optimization approach instead. Also, 3D block decomposition is not guaranteed to be valid.

N. Kowalski, F. Ledoux, P. Frey, "Block Structured Hexahedral Meshes for CAD Models using 3D Frame Fields", International Meshing Roundtable 2014



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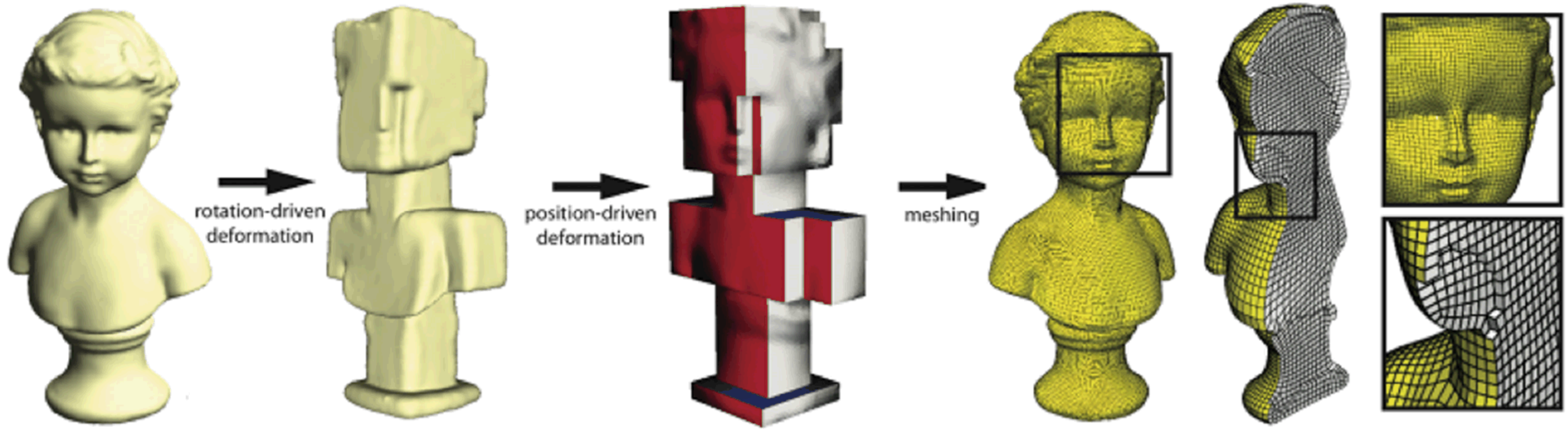
Volumetric Parameterizations

PolyCube

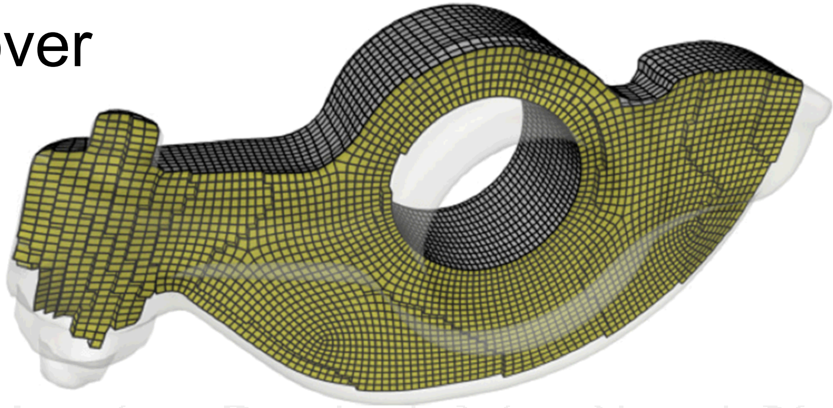


- J. Gregson, A. Sheffer, E. Zhang, “All-hex Mesh Generation via Volumetric PolyCube Deformation,” Eurographics Symposium on Geometry Processing, July 2011

Volumetric Parameterizations

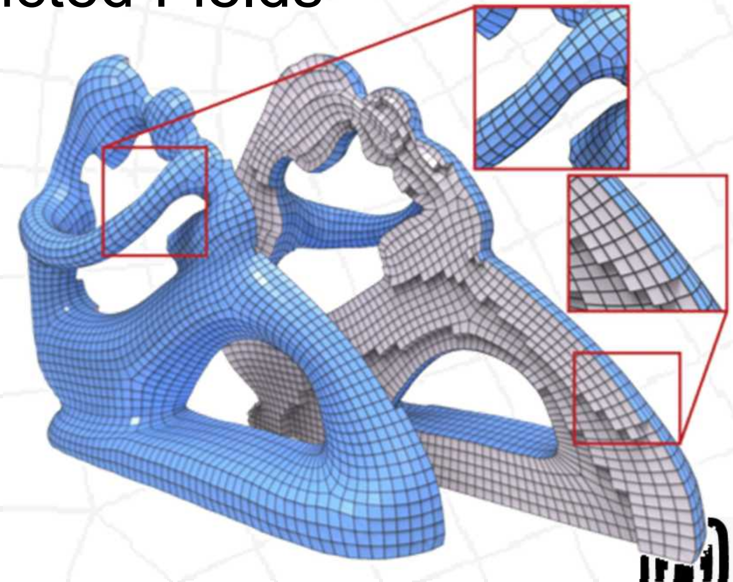
CubitCover

- M. Nieser, U. Reitebuch, K. Polthier, “CubeCover – Parameterizations of 3D Volumes”, Eurographics Symposium on Geometry Processing 2011



Singularity Restricted Fields

- Y. Li, Y. Liu, W. Xu, W. Wang, B. Guo, “All-Hex Meshing using Singularity-Restricted Field,” Microsoft Research Asia, The University of Hong Kong



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Slide from Franck Ledoux, USNCCM, 2015

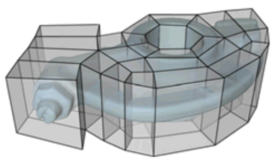
Current "implementations" of this process

Input volume (tetrahedral mesh)
↓
3D frame field (inside the volume)
↓
Singularity Graph Extraction
↓
Block Structure building (guided by 3D frame field)
↓
All-hex mesh

[Nieser et al. 11]

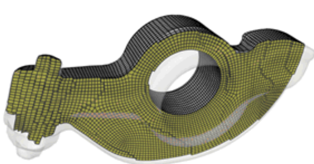
CubeCover

Frames assigned to tets and singularity on edges



Meta mesh manually designed

Volume parameterization

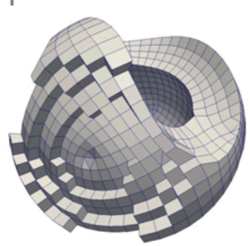


[Huang et al. 11]

Idem cube cover

Minimisation pb. using spherical harmonics

Volume parameterization

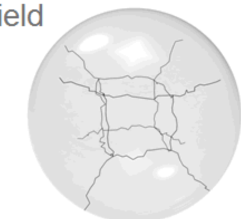


[Li et al. 12]

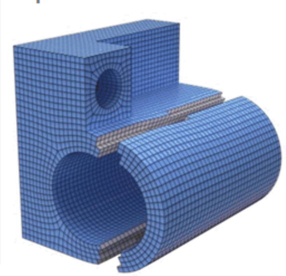
Idem cube cover

Minimisation pb. using Euler angles

generation of a singularity restricted field



Volume parameterization

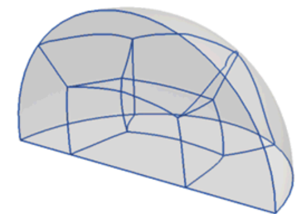


[Kowalski et al. 14]

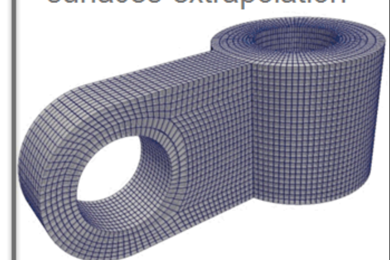
Frames assigned to vertices and singularity in tets

Minimisation pb. using Euler angles

generation of a singularity restricted field



Singularity line and surfaces extrapolation



[Nieser et al. 11] M. Nieser, U. Reitebuch, K. Polthier, CubeCover – Parameterization of 3D Volumes, Computer Graphics Forum, 30(5), pp. 1397-1406, 2011

[Huang et al. 11] J. Huang, Y. Tong, Y. Wang and H. Bao, Boundary-aligned smooth 3D cross-frame field, ACM Trans. Graph., 30(6), pp. 1-8, 2011.

[Li et al. 12] Y. Li, Y. Liu, Y. Xu, W. Wang and B. Guo, All-hex meshing using singularity-restricted field, ACM Trans. Graph., 31(6), pp. 1-11, 2012.

[Kowalski et al. 14] N. Kowalski, F. Ledoux and P. Frey, Block-Structured Hexahedral Meshes for CAD Models using 3D Frame Fields , proc. of the 23th IMR, oct. 2014.

Frame Fields
Open Research Problem

- None of the current solutions guarantee that every decomposition will be 6-sided. You can also have singularity lines that vanish inside the volume, leading to no blocks.
- This is because the constraint problem used to trace the separatrices only tries to minimize a linear alignment constraint, not a surface alignment constraint. In other words, a hex mesh is composed of chords and sheets. The constraint problems currently only optimize the chords, not the sheets.

2-Refinement & 3-Refinement

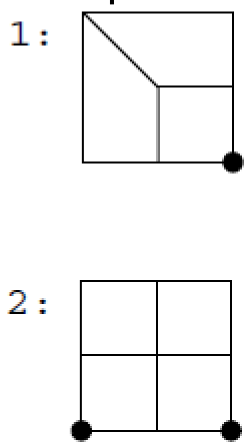
QUAD/HEX REFINEMENT

Robert Schneiders, "Algorithms for Quadrilateral and Hexahedral Mesh Generation," Proceedings of the VKI Lecture Series on Computational Fluid Dynamic, 20-24 March 2000. VKI-LS 2000-4. http://www.robertschneiders.de/papers/vki.pdf

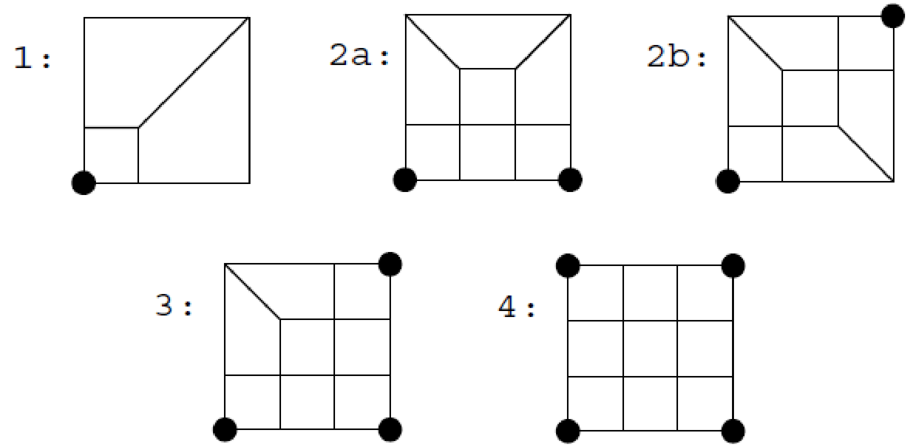
Quad Refinement With Templates

With quad meshes, refinement is done by locally replacing each element to refine with a "Template" or pattern of quads. There are 2 sets of templates, call the 2-refinement templates and the 3-refinement templates:

2-refinement Templates

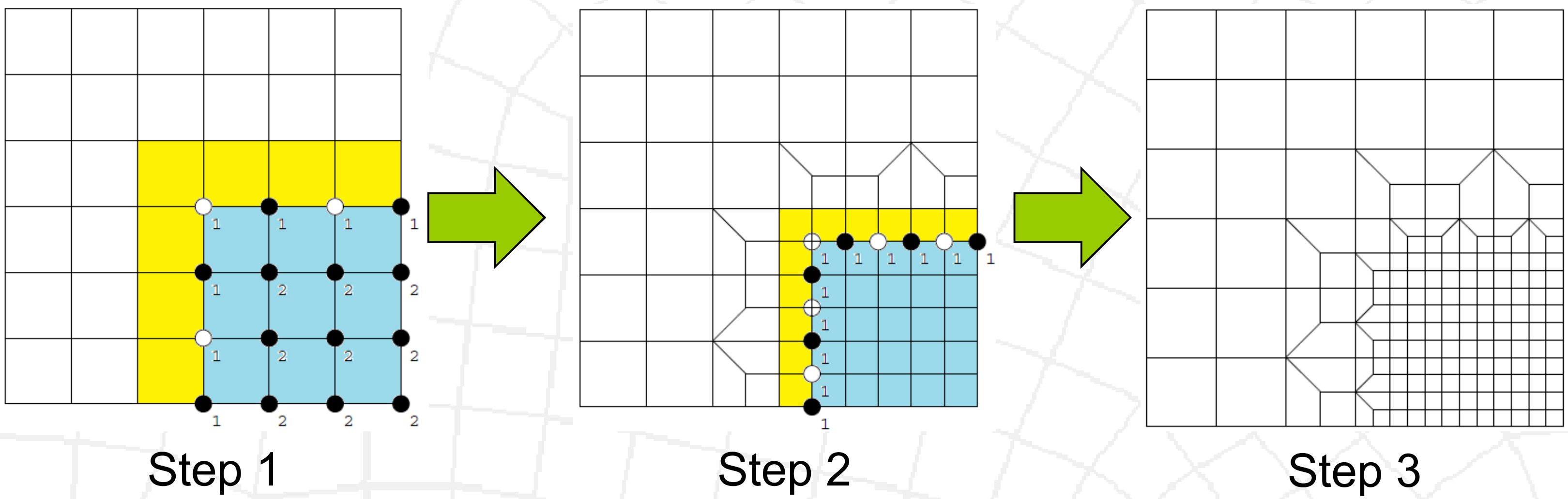


3-refinement Templates



Robert Schneiders, "Algorithms for Quadrilateral and Hexahedral Mesh Generation," Proceedings of the VKI Lecture Series on Computational Fluid Dynamic, 20-24 March 2000. VKI-LS 2000-4. http://www.robertschneiders.de/papers/vki.pdf

Quad Refinement With 2-refinement Templates

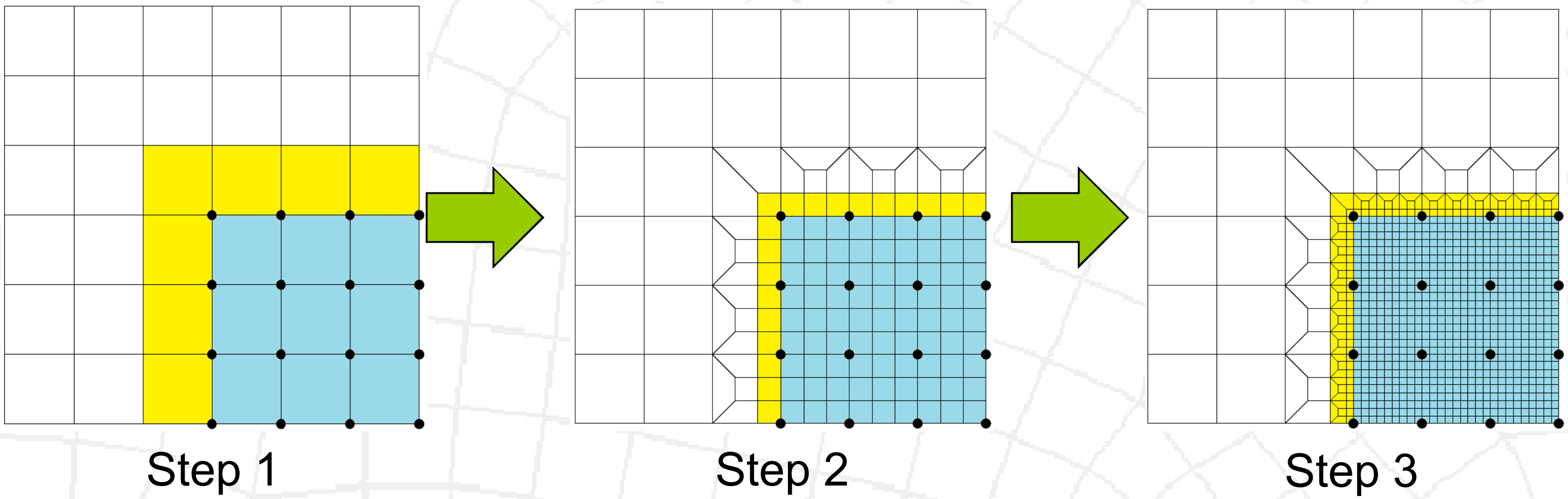


Step 1: Quads to be refined identified (blue). Transition elements identified (yellow). Nodes on blue elements are marked. Nodes shared between blue and yellow elements are marked in alternating pattern.

Step 2: Each element replaced with a pattern depending on which nodes are marked. Nodes are re-marked for a 2nd iteration.

Step 3: Templates inserted for 2nd iteration.

Quad Refinement With 3-Refinement Templates



Step 1: Quads to be refined identified (blue). Transition elements identified (yellow). Nodes on blue elements are marked. No need to alternate node markings.

Step 2: Each element replaced with a pattern depending on which nodes are marked. Nodes are re-marked for a 2nd iteration.

Step 3: Templates inserted for 2nd iteration.

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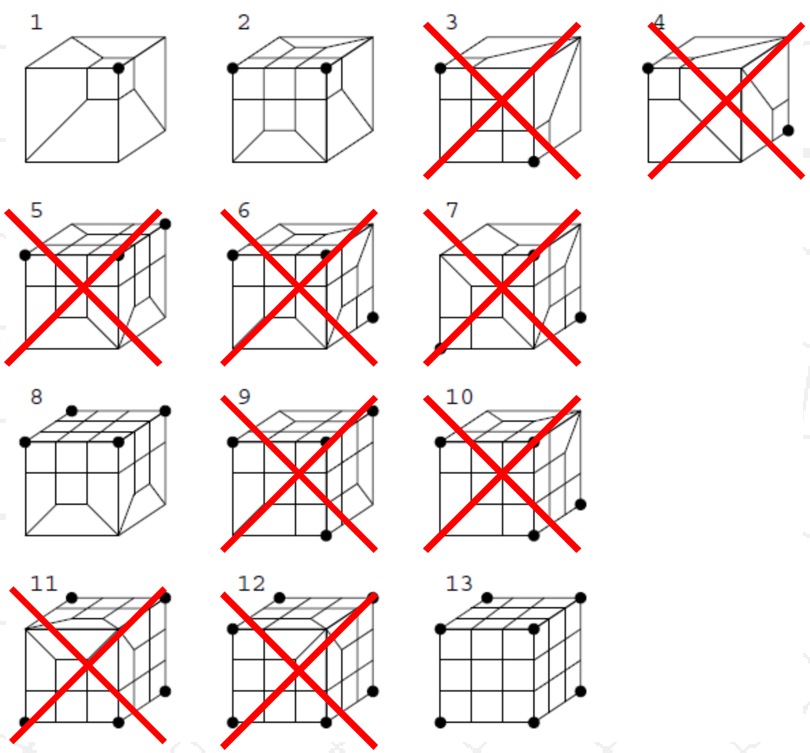
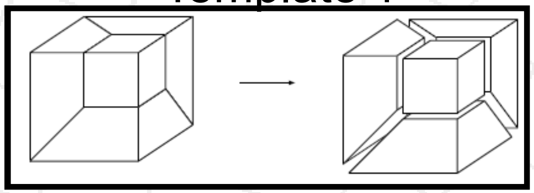
Hex Refinement With 3-Refinement Templates

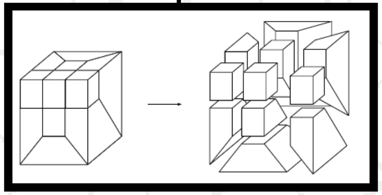
There are 13 potential node markings for hex refinement, requiring 13 transition templates. However, only 4 of them are defined (1, 2, 8 & 13 below)



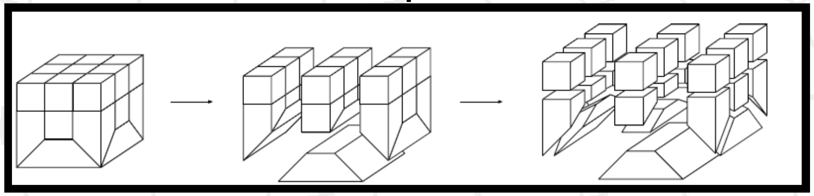
Template 1



Template 2

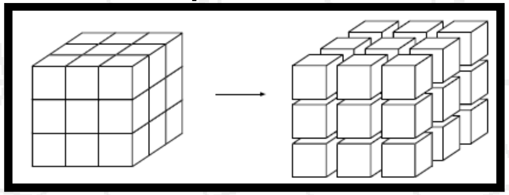


Template 8



The missing template mean that only concave refinement zones can be handled.

Template 13



Hex Refinement With 2-Refinement Templates

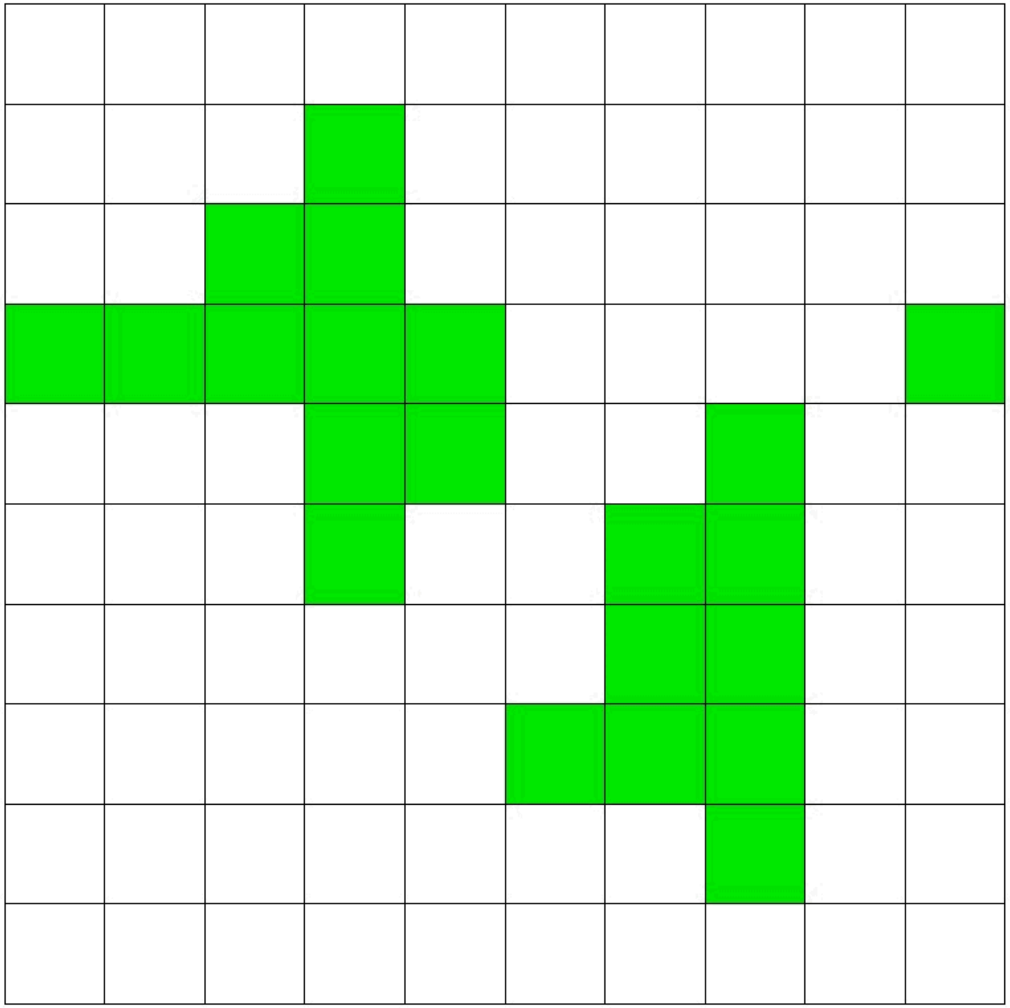
Because of the non-symmetric templates, hex Refinement with 2-Refinement templates is significantly more complicated than hex 3-Refinement, or quad 2-Refinement.

For structured meshes (i.e. no singular edges), a directional approach is taken to apply the 2-Refinement Templates. The next few slides demonstrate direction 2-Refinement for quad refinement. The 3D extension to directional refinement will be discussed on Monday Morning's talk by Steve Owen: *A Template-Based Approach for Parallel Hexahedral Two-Refinement, IMR24.*

For a general unstructured hex mesh, this direction approach is difficult to apply because directions are not always obvious, general unstructured hex 2-refinement remains an open problem.

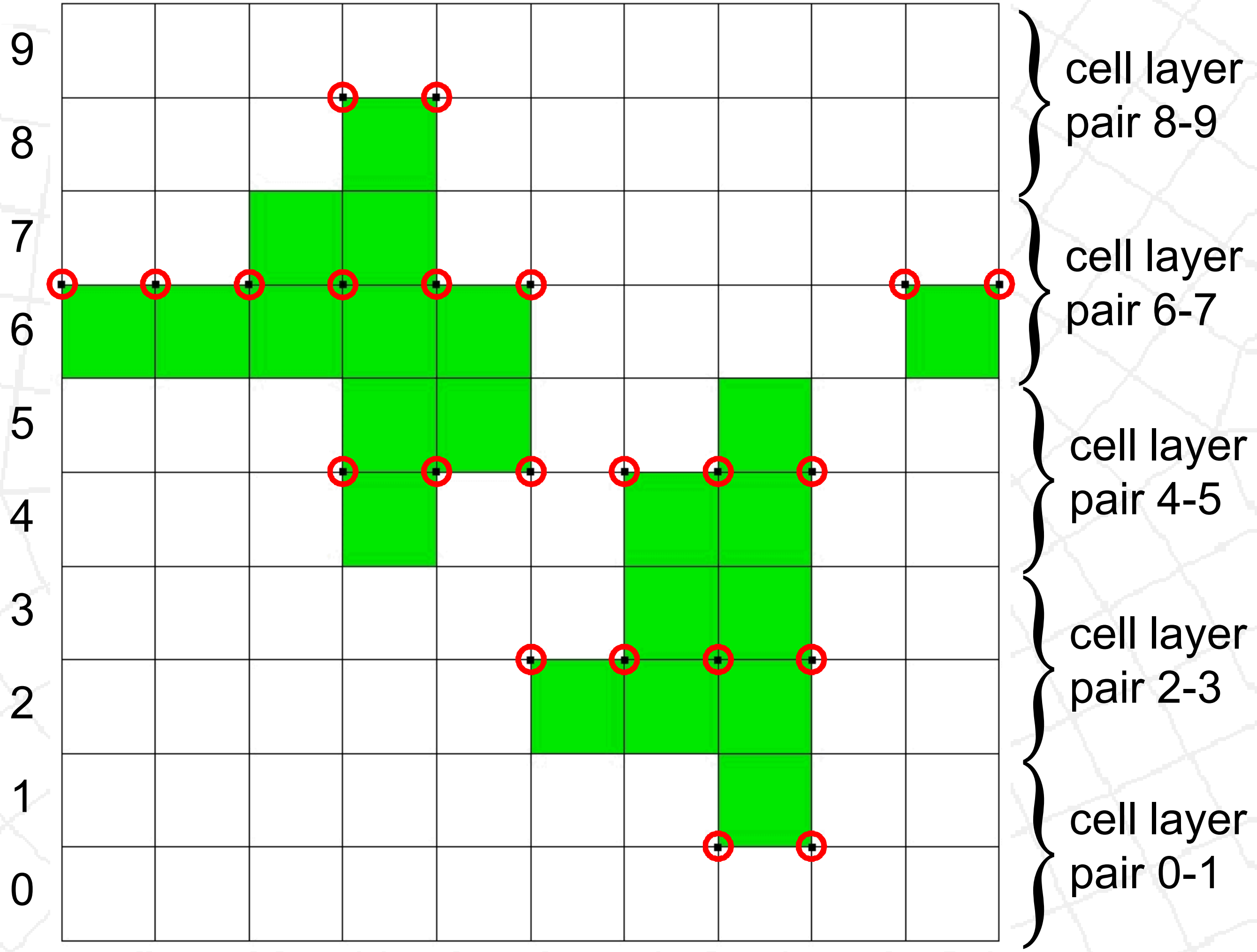
Directional 2-Refinement

Green quads are marked for refinement



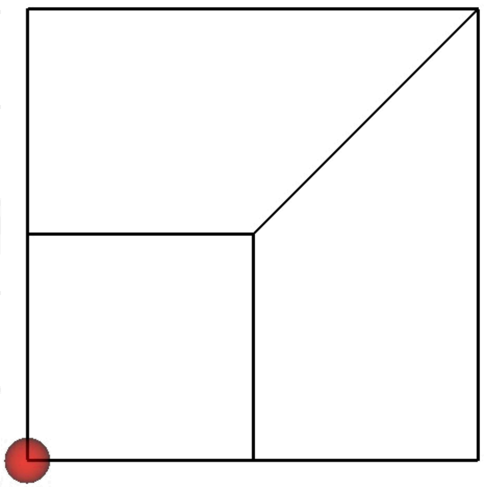
Directional 2-Refinement

The layers in the horizontal direction are numbered 0-9, and put into pairs

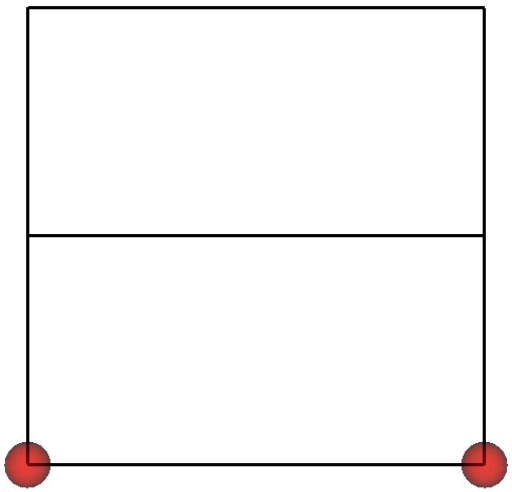


Directional 2-Refinement

With directional refinement in 2D, there are only 2 templates to consider.



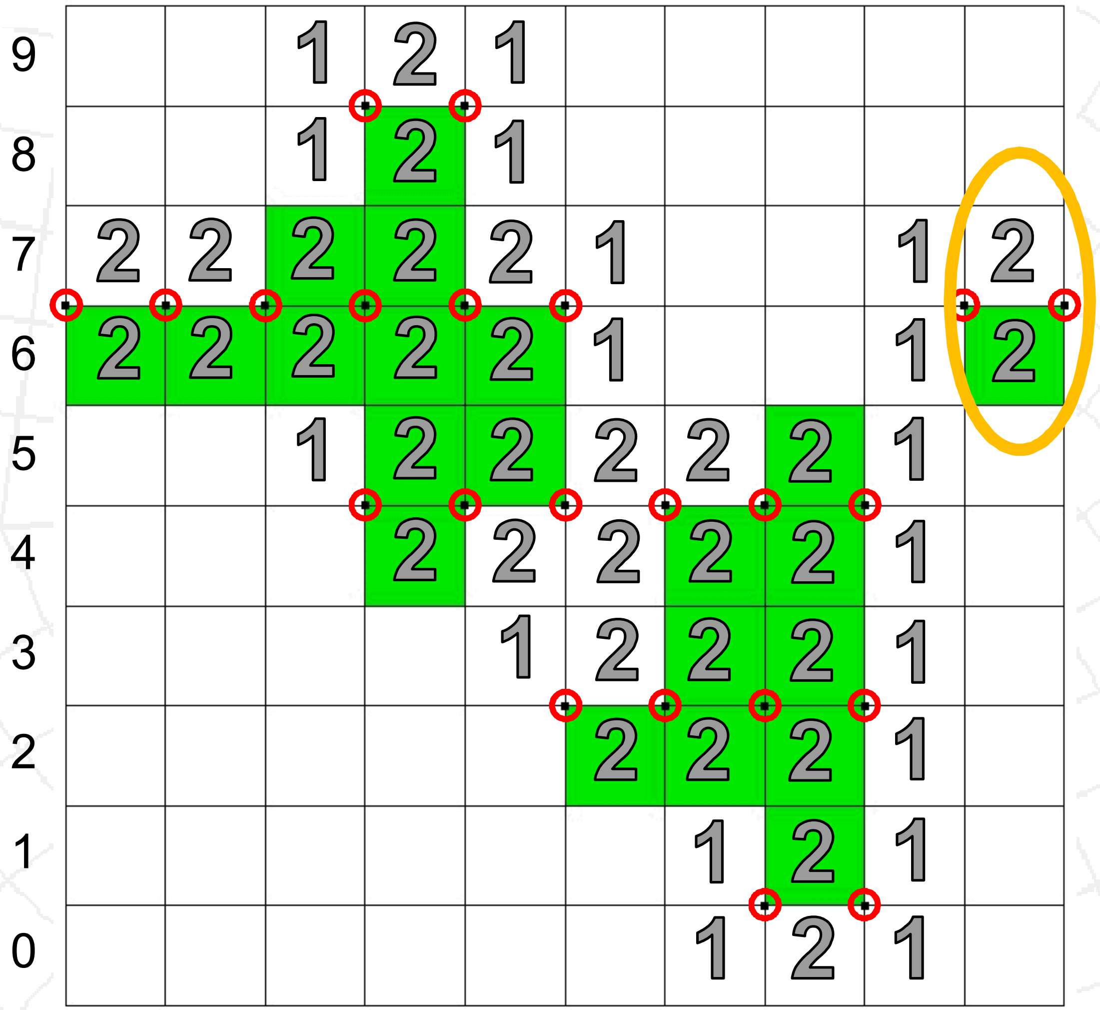
1



2

Directional 2-Refinement

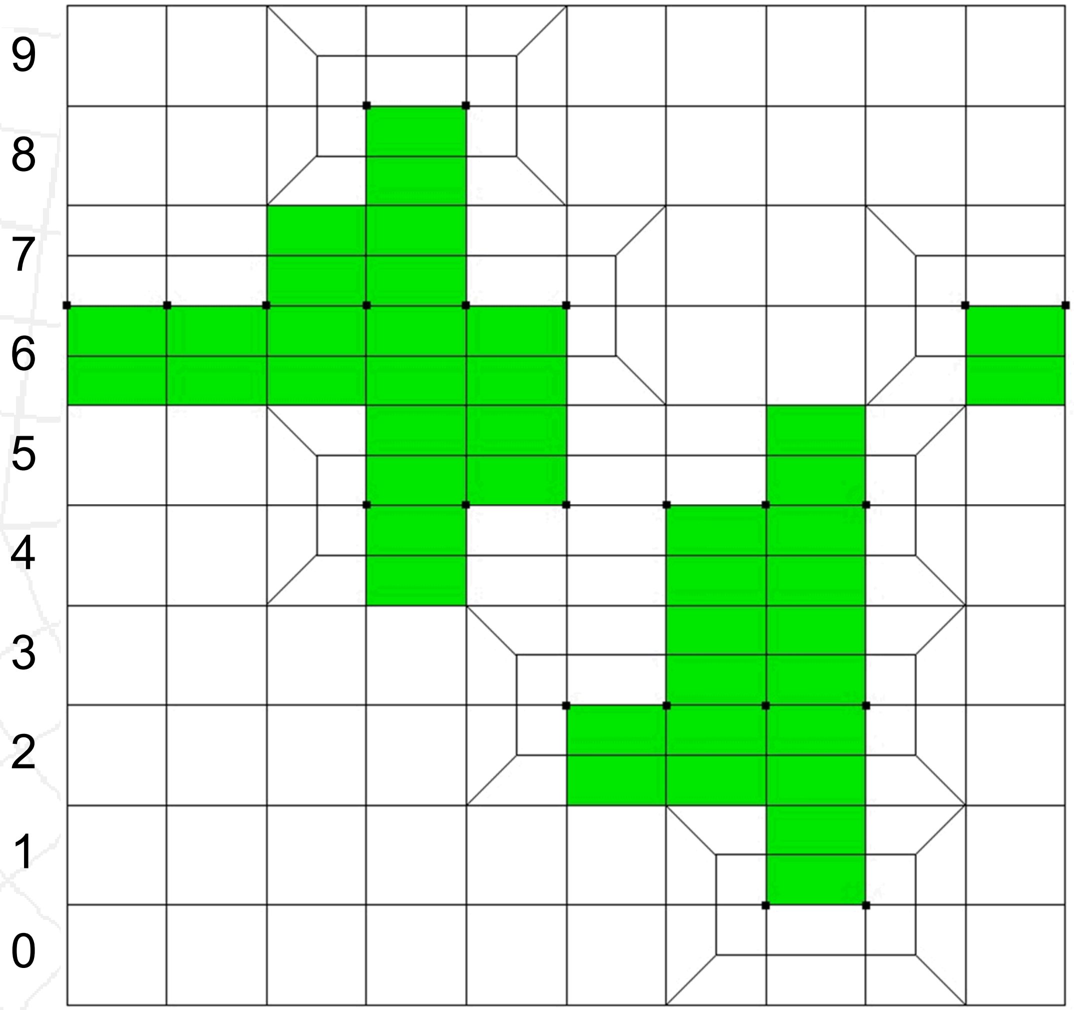
Templates are assigned to the elements. Note that the same template is applied to pairs of elements in each pair of rows, even if they are marked for refinement differently.



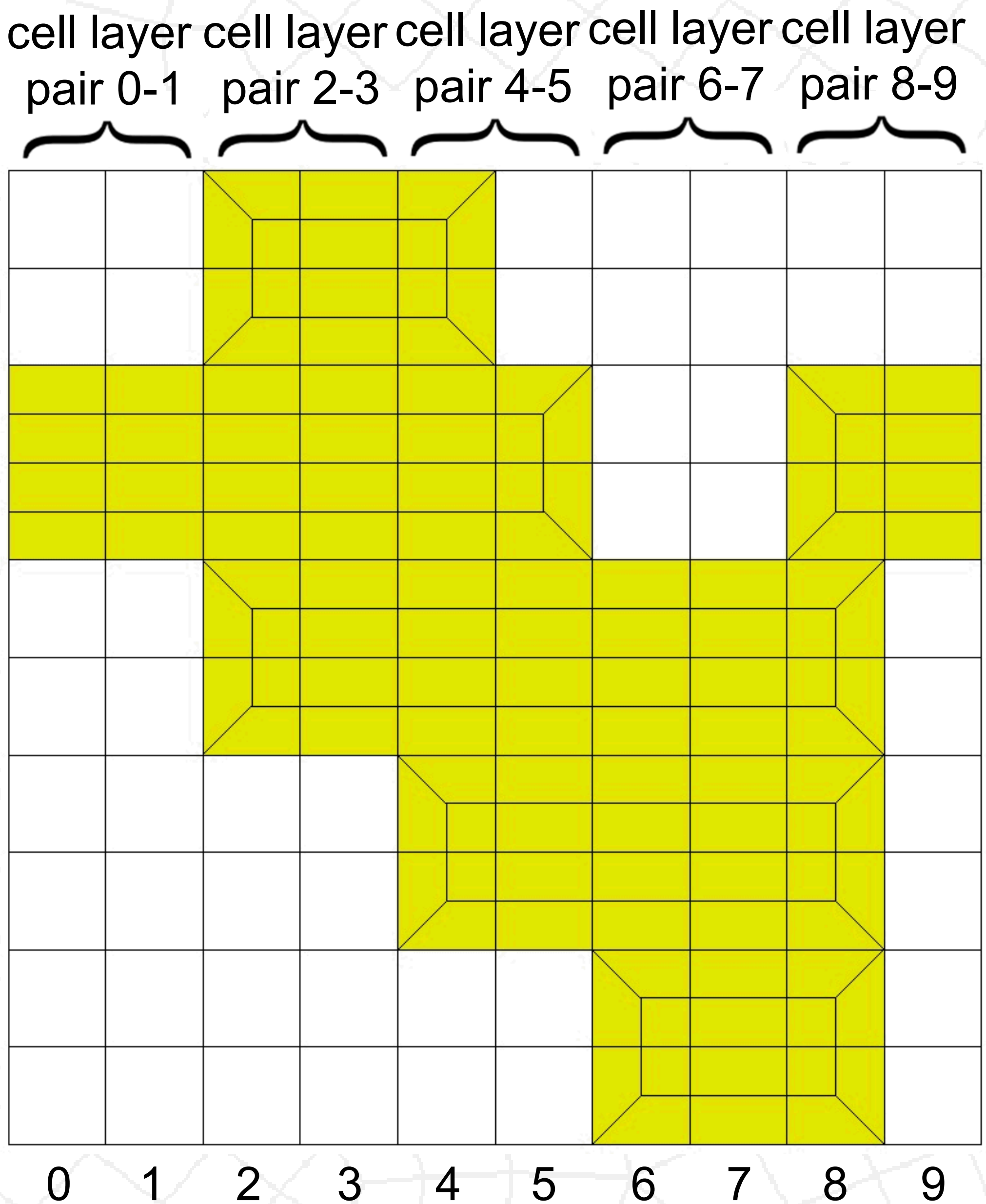
Both of these 2 elements will be replaced with template 2, even though only 1 of them is in the marked refinement zone.

Directional 2-Refinement

This slide shows the result after the first direction of refinement is complete.

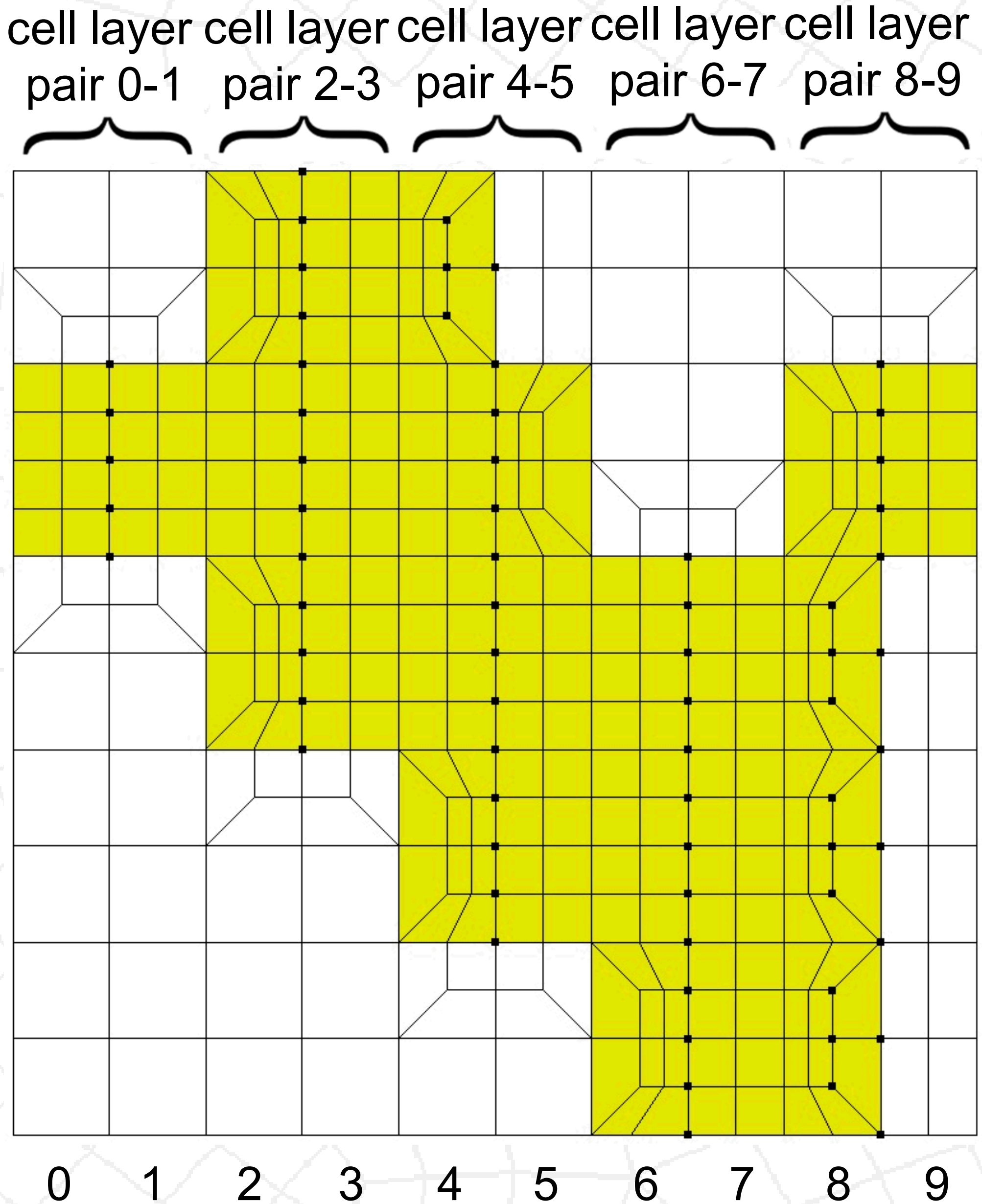


Directional 2-Refinement



The vertical rows are now numbered and paired.

Directional 2-Refinement



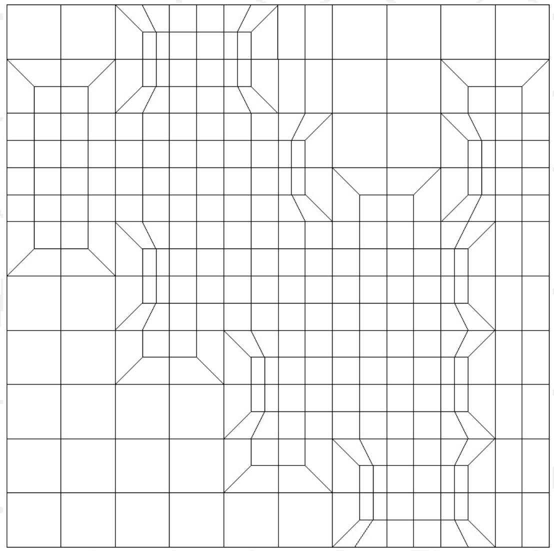
Templates are inserted in vertical direction.

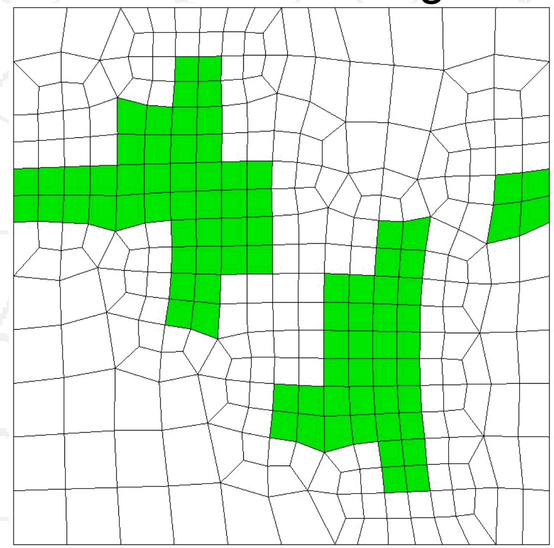
2-Refinement vs 3-Refinement

After smoothing

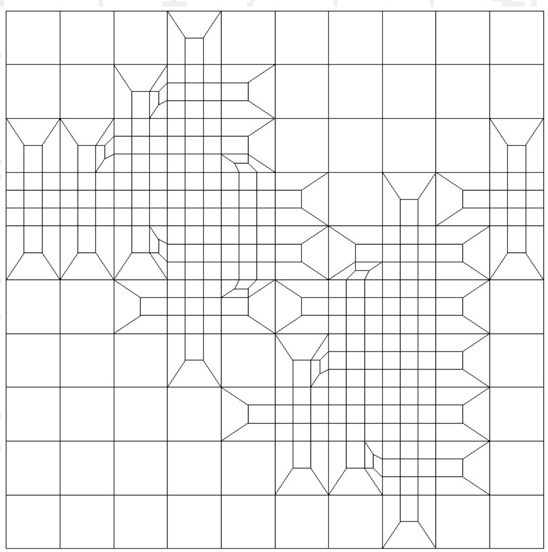
Green indicates elements marked for refinement

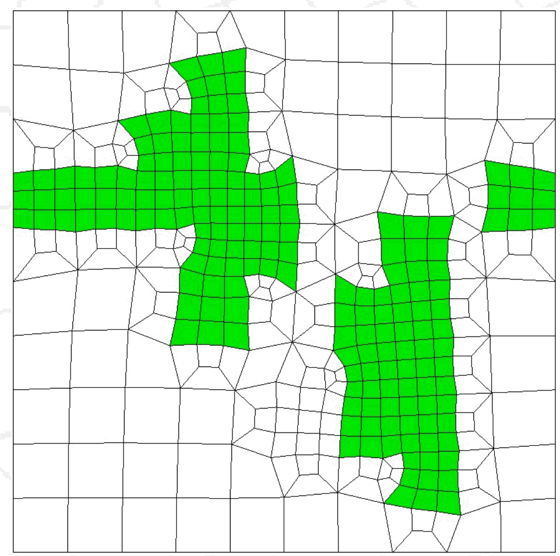
2-Refinement





3-Refinement





Quad/Hex Refinement With Templates

2-Refinement

1. Every edge gets split in halves.
2. Care must be taken to alternate template direction in transition region.
3. Cannot handle the special case of refining around a hole with an odd number of intervals.
4. Can be extended to 3D for structured refinement zones, using a directional approach.

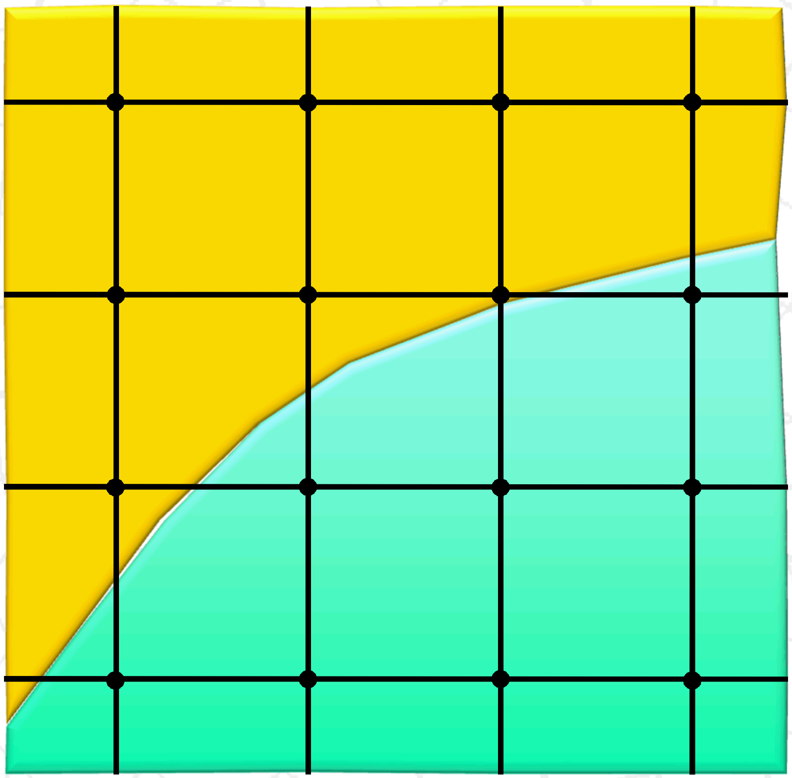
3-Refinement

1. Every edge gets split in thirds (over-refined?).
2. No need to alternate template directions in transition region.
3. Can handle refinement of all cases.
4. Extension to 3D refines only convex refinement zones.

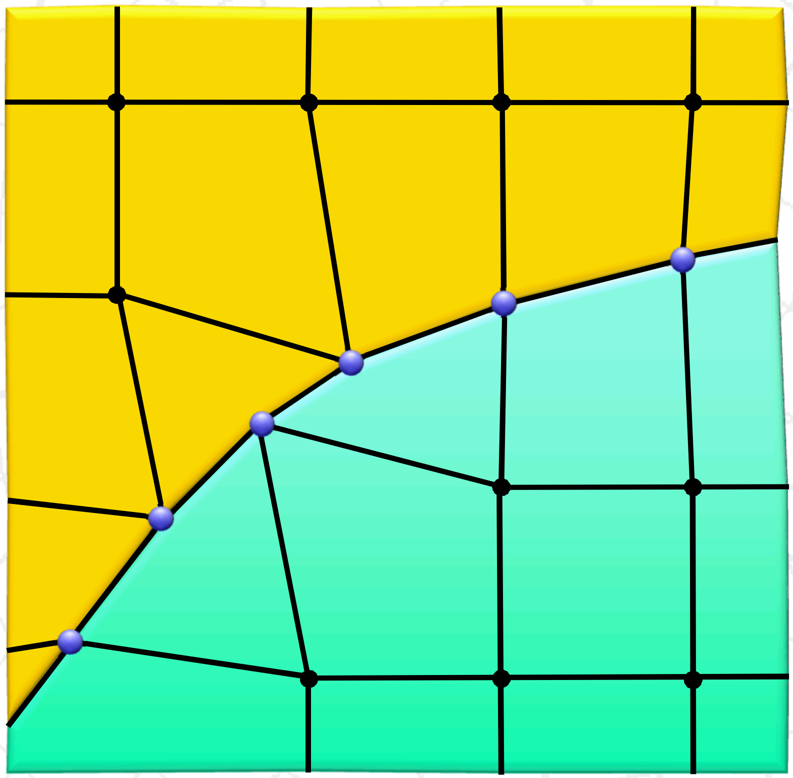
Truly Automated Hex Mesh Generation for Some Applications

OVERLAY GRID METHODS

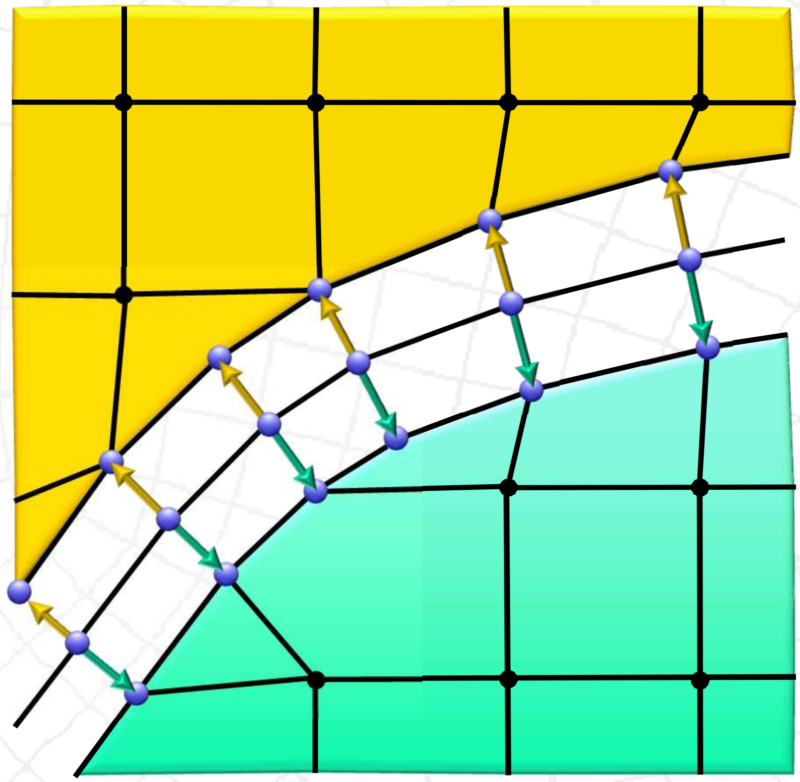
Overlay Grid Method



Overlay Grid Method



Overlay Grid Method



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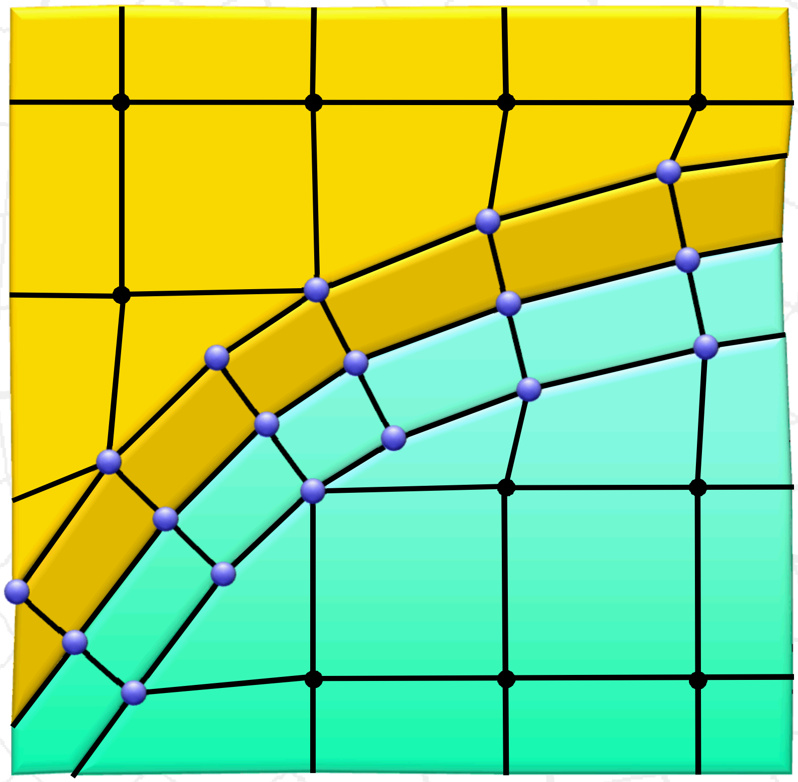
Overlay Grid Method



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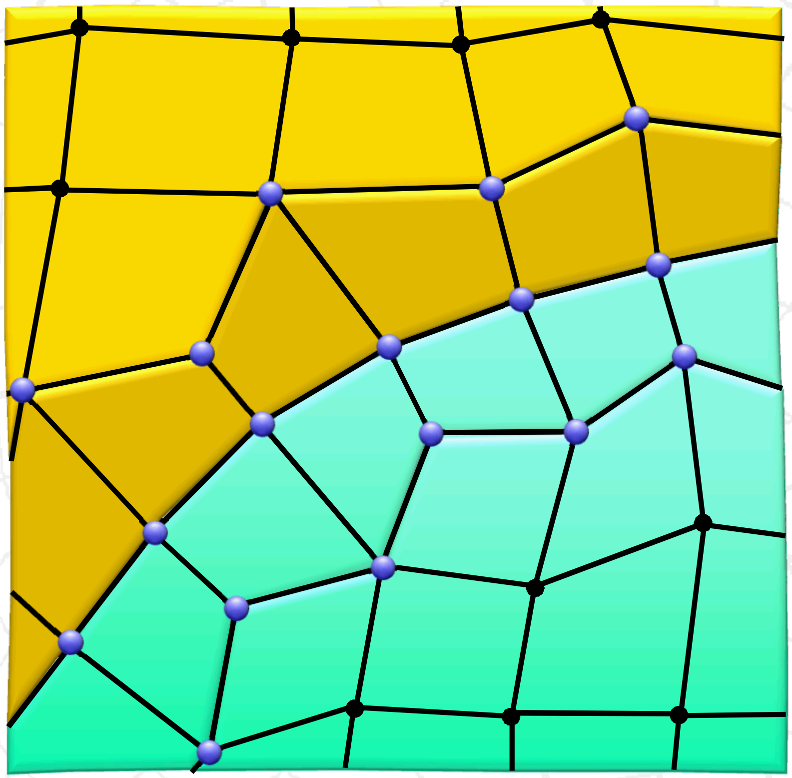
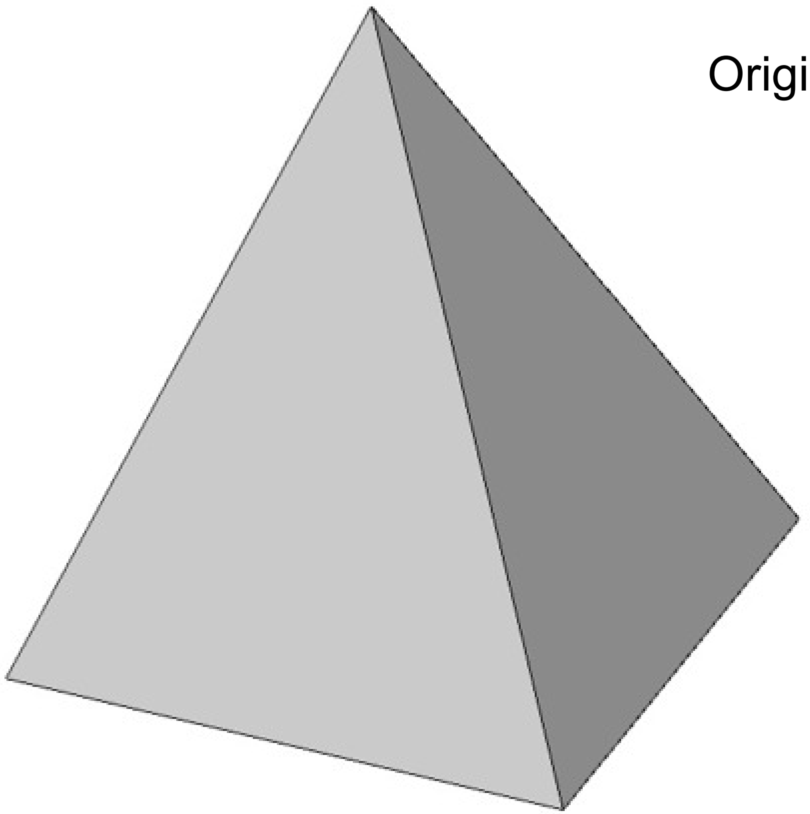
Overlay Grid Method

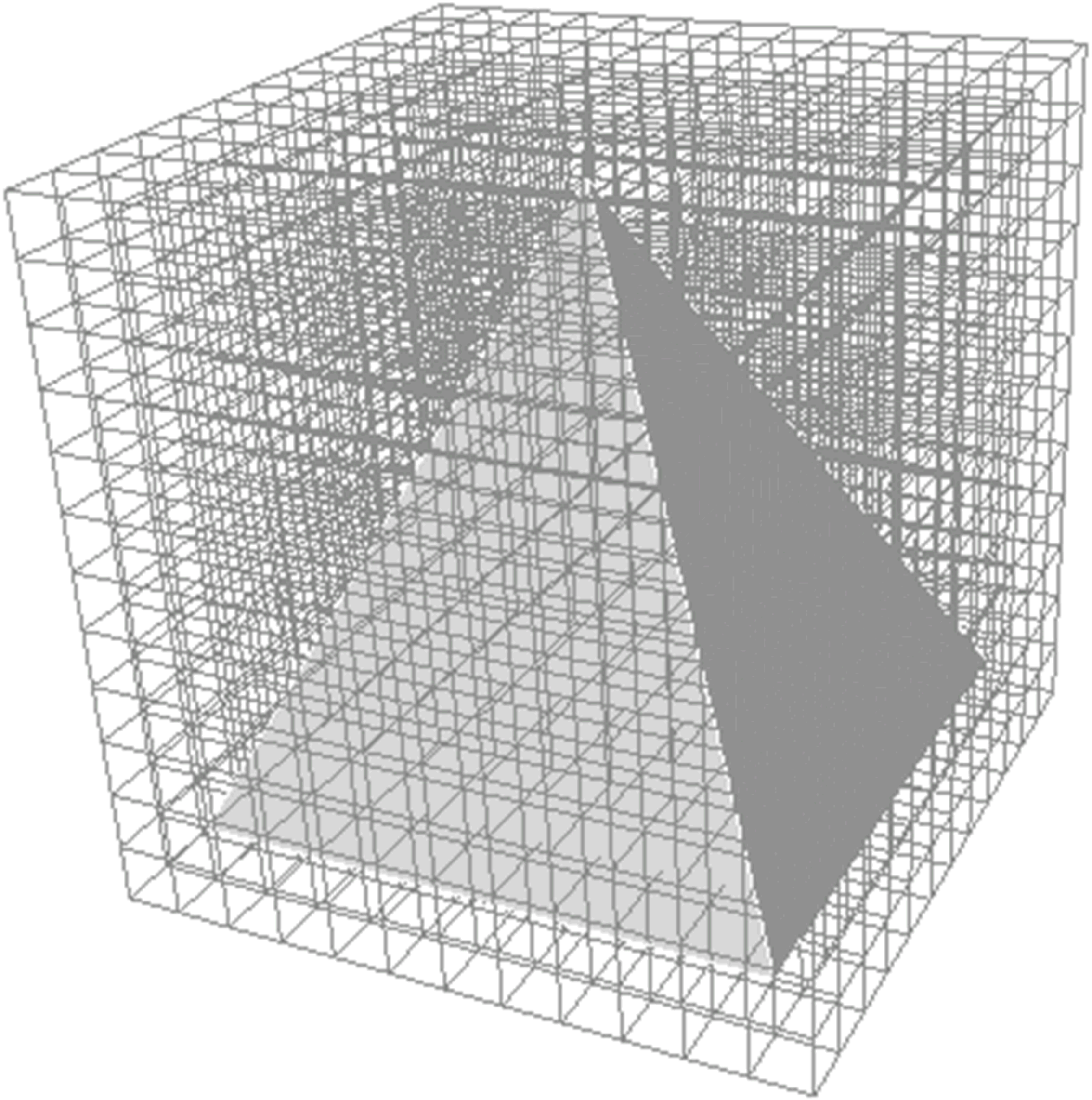


Grid-Based Methods



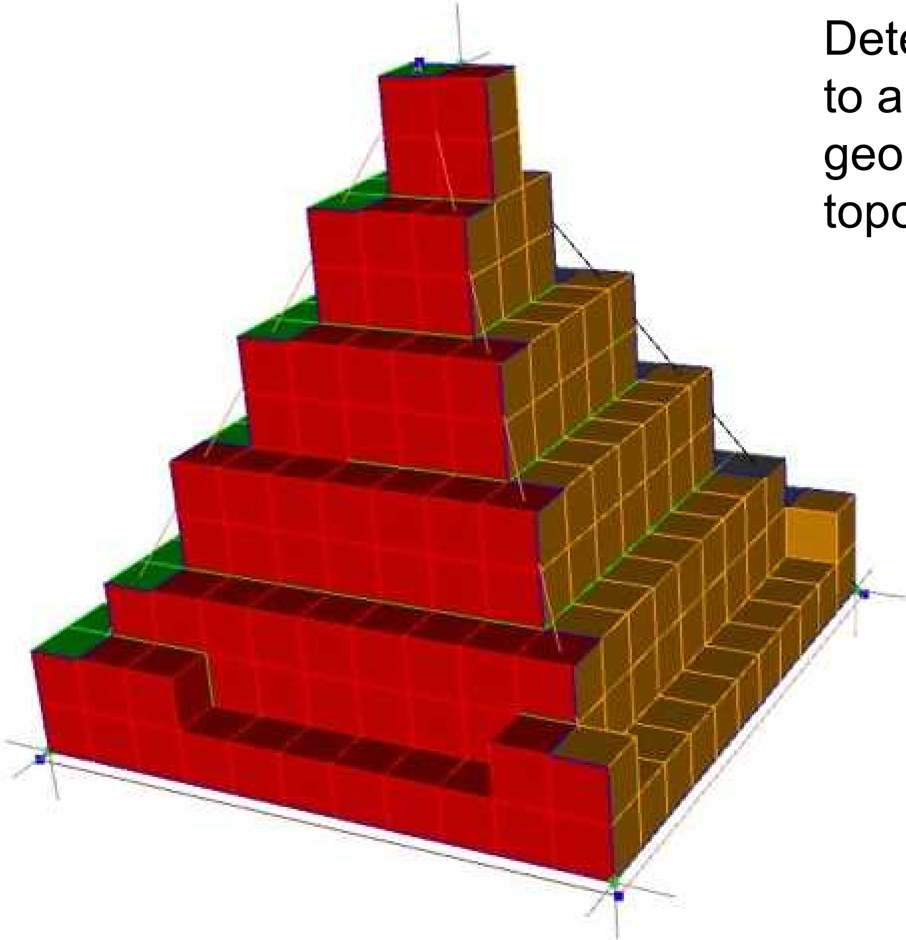
Original geometry

Grid-Based Methods



Overlay Grid

Grid-Based Methods



Determine grid cells to approximate geometry and topology

Most often accomplished by throwing out all cells whose centroid lies outside of the geometry

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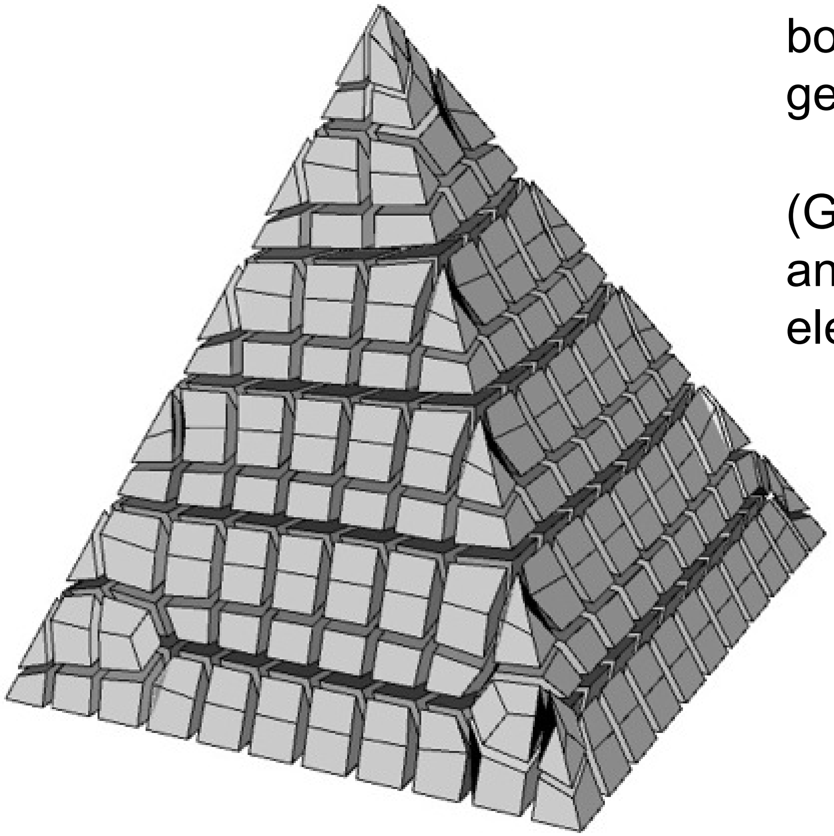
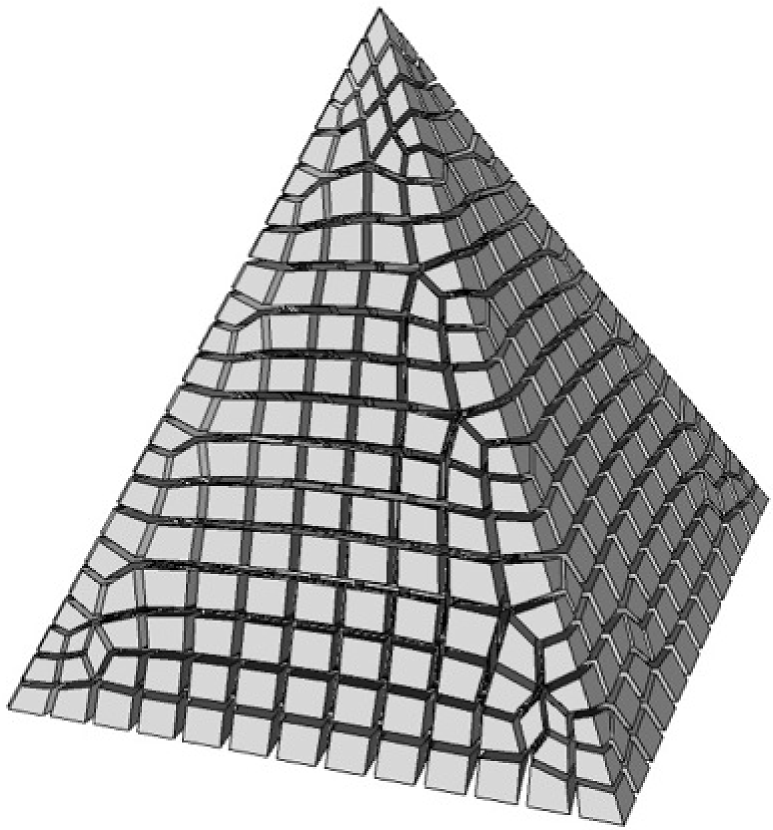
Grid-Based Methods



Snap nodes at boundary to geometry

(Generates poor and inverted elements)

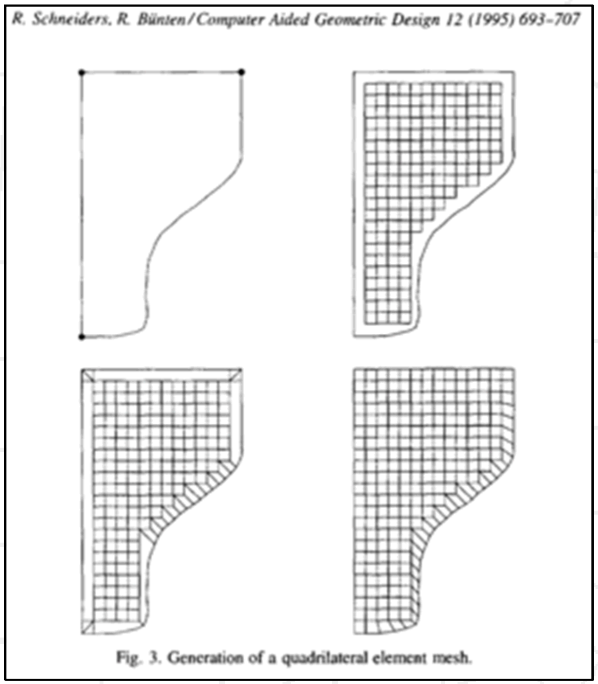
Grid-Based Methods

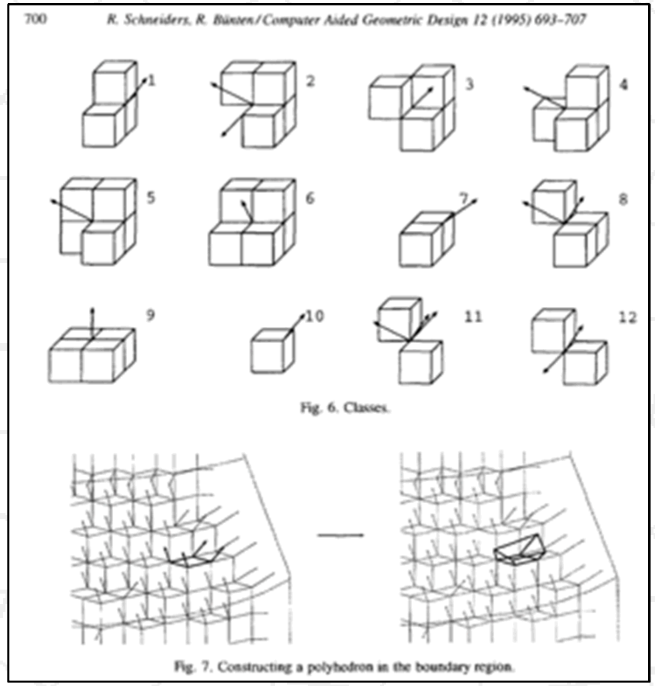


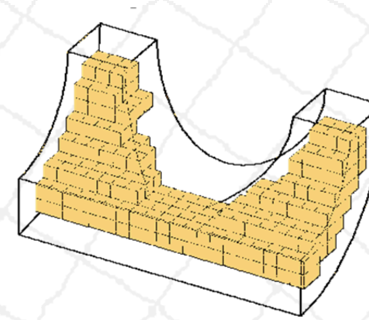
Insert fundamental sheets at boundary

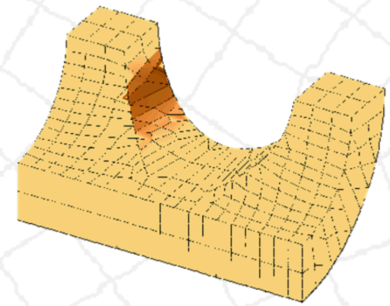
Smooth

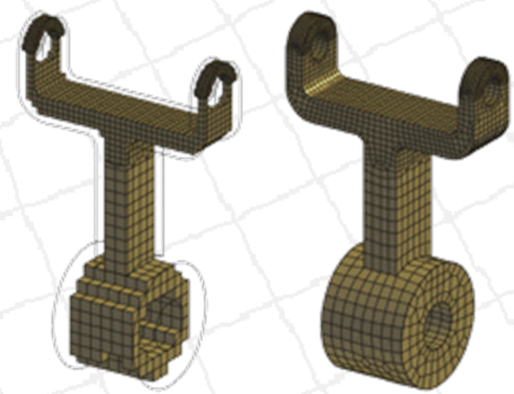
Early Overlay Grid Methods











Early papers on overlay grid method

Robert Schneiders, “A Grid-based Algorithm for the Generation of Hexahedral Element Meshes,” Engineering with Computers, Vol 12, pp. 68-77 (1996)

Reza Taghavi, “Automatic, Parallel and Fault Tolerant Mesh Generation from CAD,” Engineering with Computers, Vol. 12, pp. 178-185 (1996)

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Recent Overlay Grid References

Yongjie (Jessica) Zhang, Thomas Hughes, and Chandrijit Bajaj, "Automatic 3D Mesh Generation for a Domain with Multiple Materials", Proceedings, 16th International Meshing Roundtable, pp. 367-386, (2007).

Yasushi Ito, Alan M. Shih, and Bharat K. Soni, "Octree-based reasonable-quality hexahedral mesh generation using a new set of refinement templates," International Journal for Numerical Methods in Engineering, published online DOI: 10.1002/nme.2470 (2008)

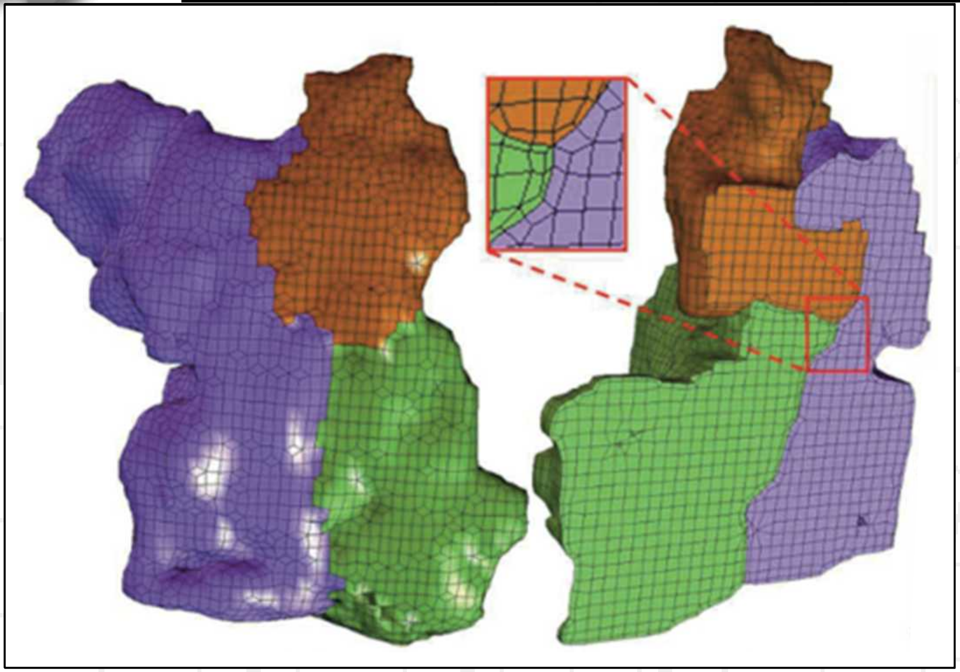
Loic Marechal, "Advances in Octree-Based All-Hexahedral Mesh Generation: Handling Sharp Features," 18th International Meshing Roundtable, 2009.

Steven J. Owen, Matthew L. Staten, Marguerite C. Sorensen, "Parallel hexahedral meshing from volume fractions," Engineering With Computers, Vol 30, Issue 3, pp. 301-313, July 2014

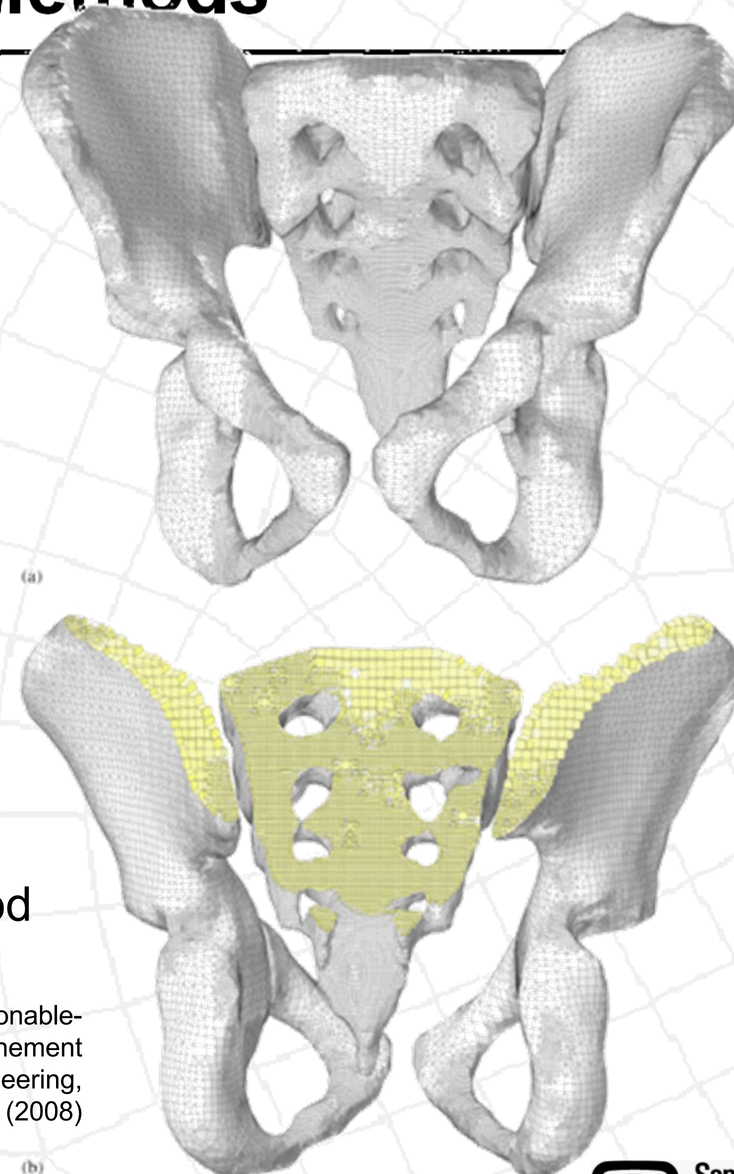
Steven J. Owen, Tim R. Shelton, "Evaluation of grid-based hex meshes for solid mechanics," Engineering With Computers, Vol 31, Issue 3, pp. 529-543, July 2015

Steven J. Owen, "A Template-Based Approach for Parallel Hexahedral Two-Refinement", Proceedings 24th International Meshing Roundtable, Oct 2015

Overlay Grid Methods



Yongjie (Jessica) Zhang, Thomas Hughes, and Chandrijit Bajaj, "Automatic 3D Mesh Generation for a Domain with Multiple Materials", Proceedings, 16th International Meshing Roundtable, pp. 367-386, (2007)

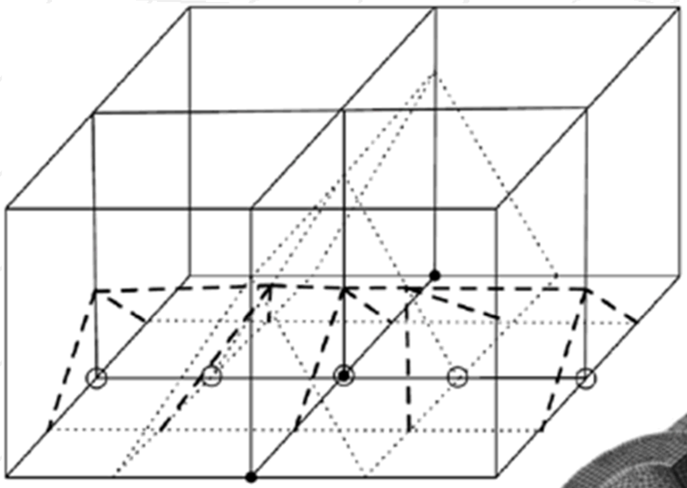


Recent publications using overlay grid method

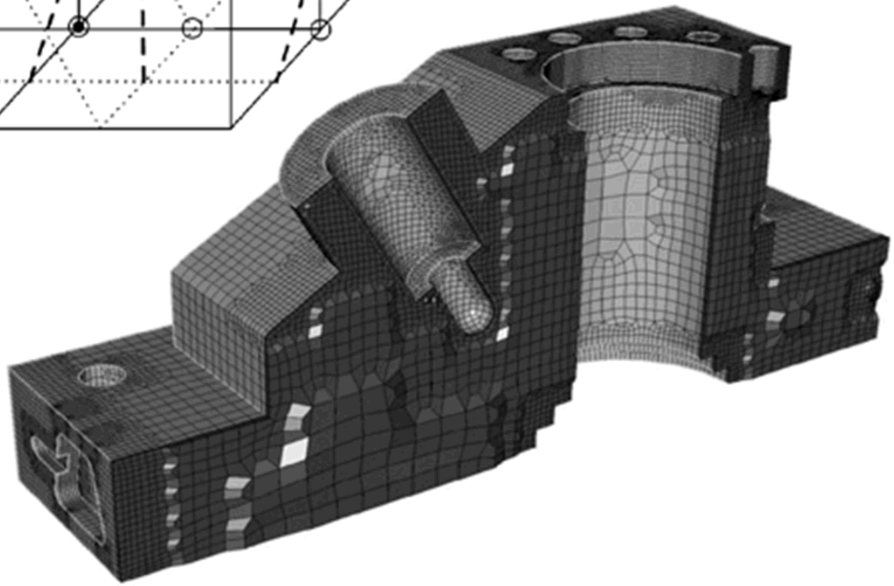
Yasushi Ito, Alan M. Shih, and Bharat K. Soni, "Octree-based reasonable-quality hexahedral mesh generation using a new set of refinement templates," International Journal for Numerical Methods in Engineering, published online DOI: 10.1002/nme.2470 (2008)

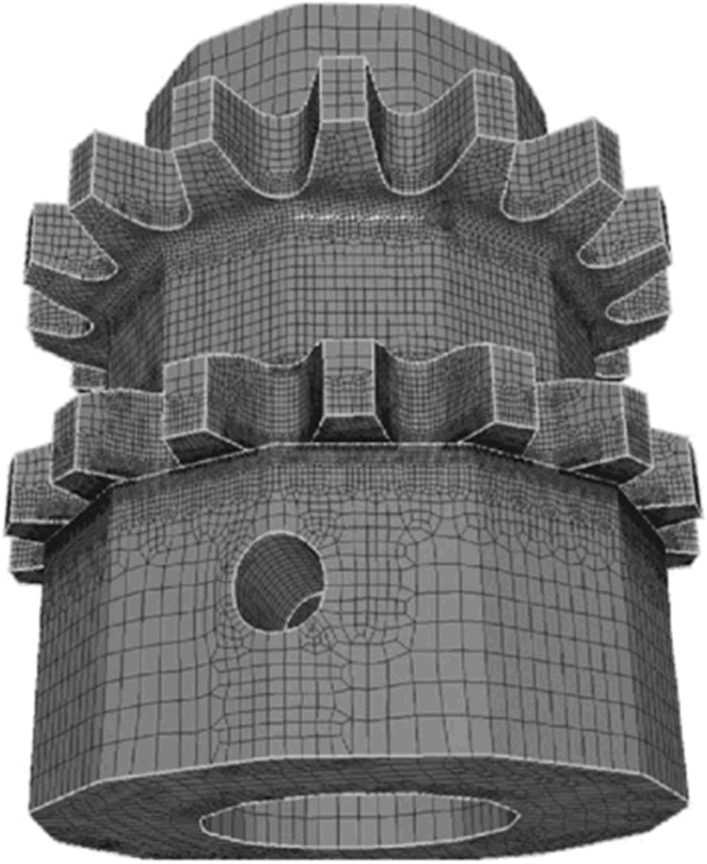
Overlay Grid Methods

Loïc Maréchal [IMR 2009]

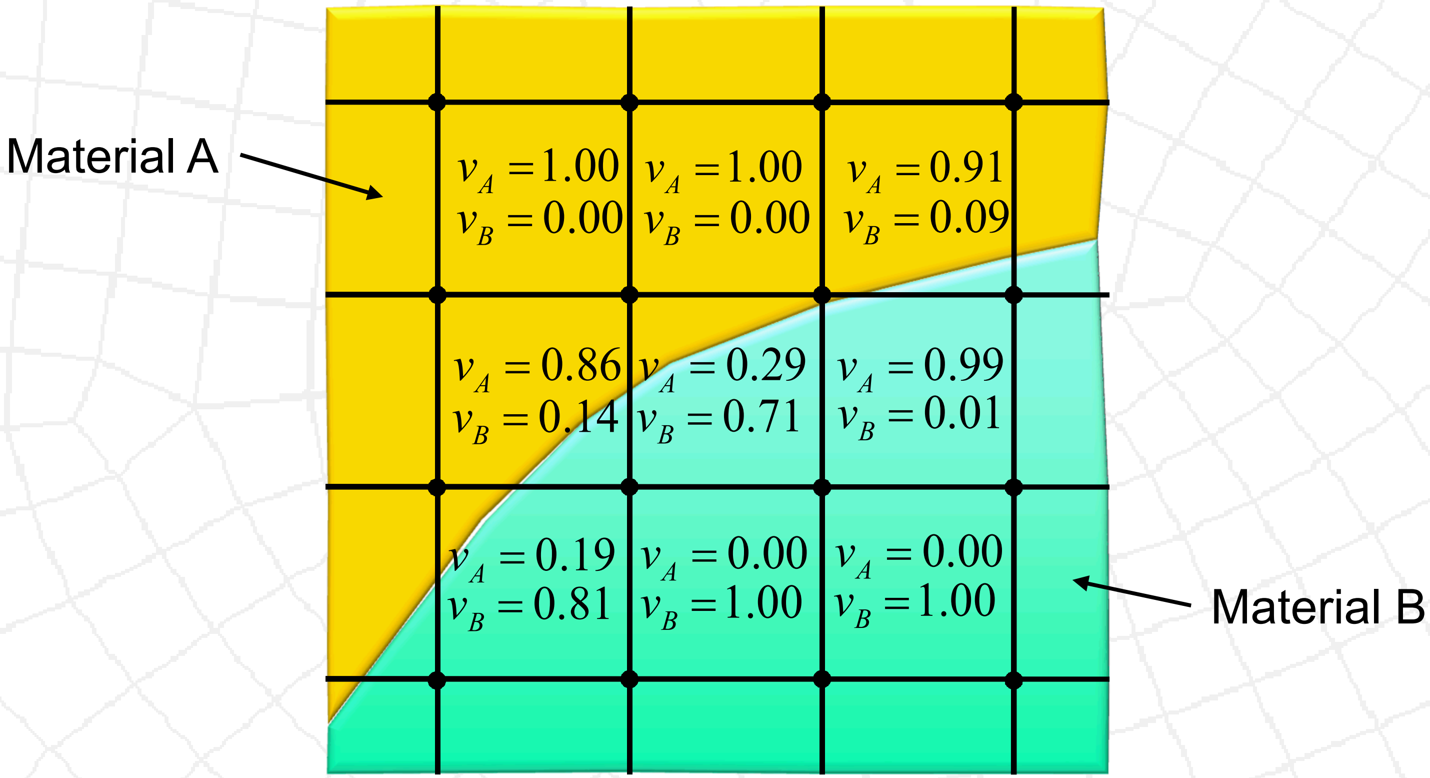


Incorporates feature capture and local refinement





Sculpt



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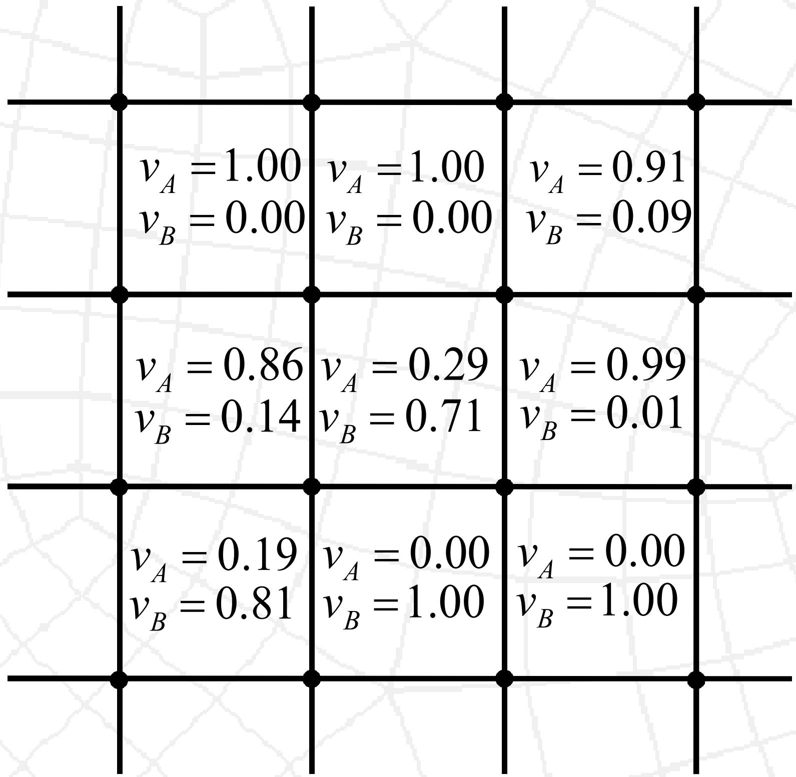
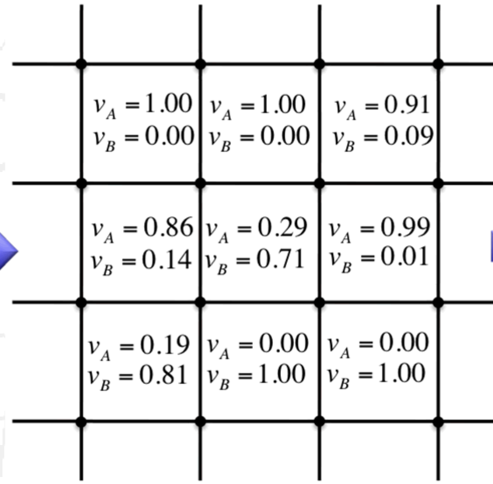
Sculpt



Volume Fractions

Sculpt

Solid Model



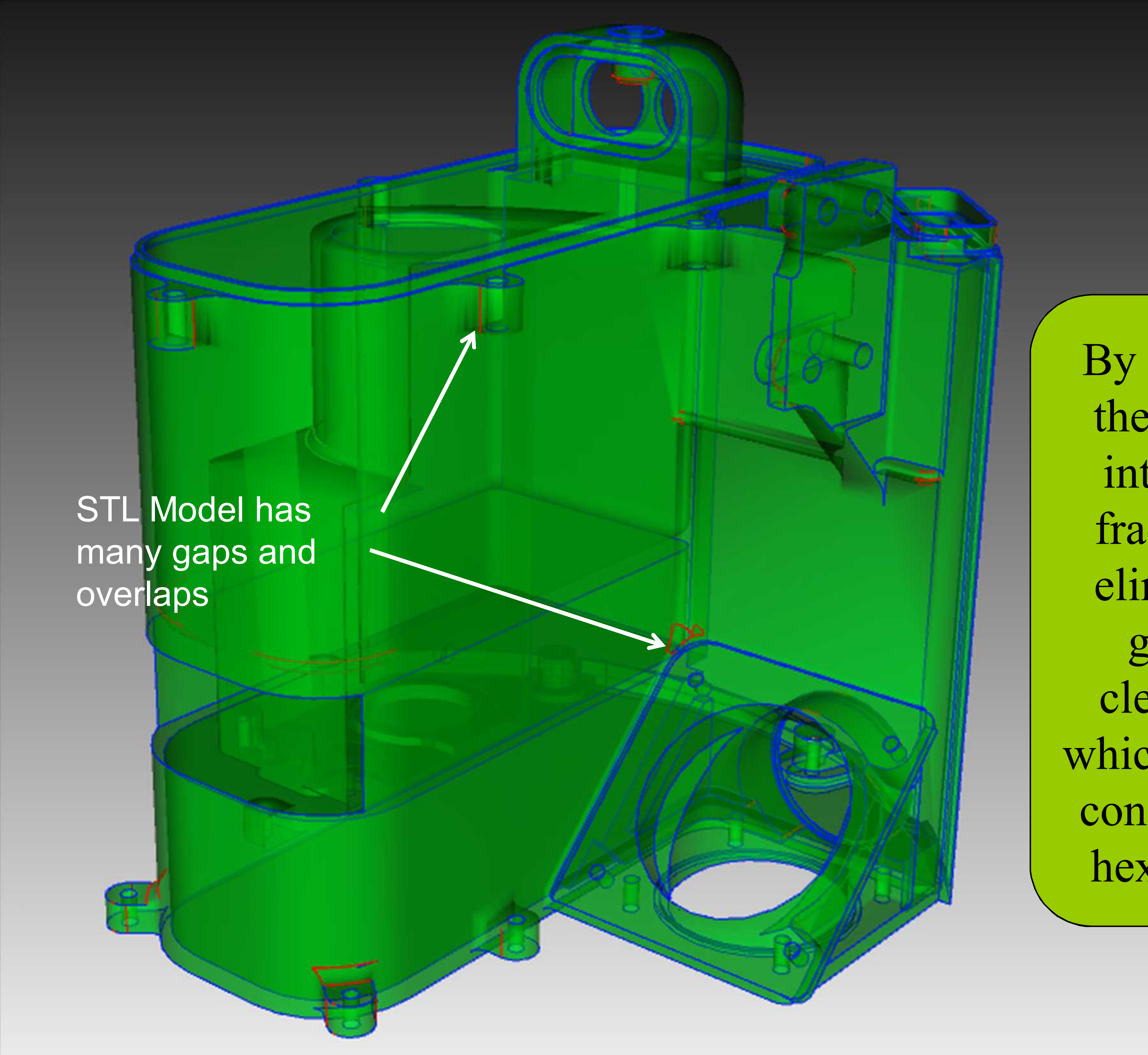
Volume Fractions

All-Hex Mesh

Steven J. Owen, Matthew L. Staten, Marguerite C. Sorensen, "Parallel hexahedral meshing from volume fractions," Engineering With Computers, Vol 30, Issue 3, pp. 301-313, July 2014

Steven J. Owen, Tim R. Shelton, "Evaluation of grid-based hex meshes for solid mechanics," Engineering With Computers, Vol 31, Issue 3, pp. 529-543, July 2015

Steven J. Owen, "A Template-Based Approach for Parallel Hexahedral Two-Refinement", Proceedings 24th International Meshing Roundtable, Oct 2015



By converting the geometry into volume fractions, we eliminate the geometry cleanup step which is so time consuming for hex meshing.

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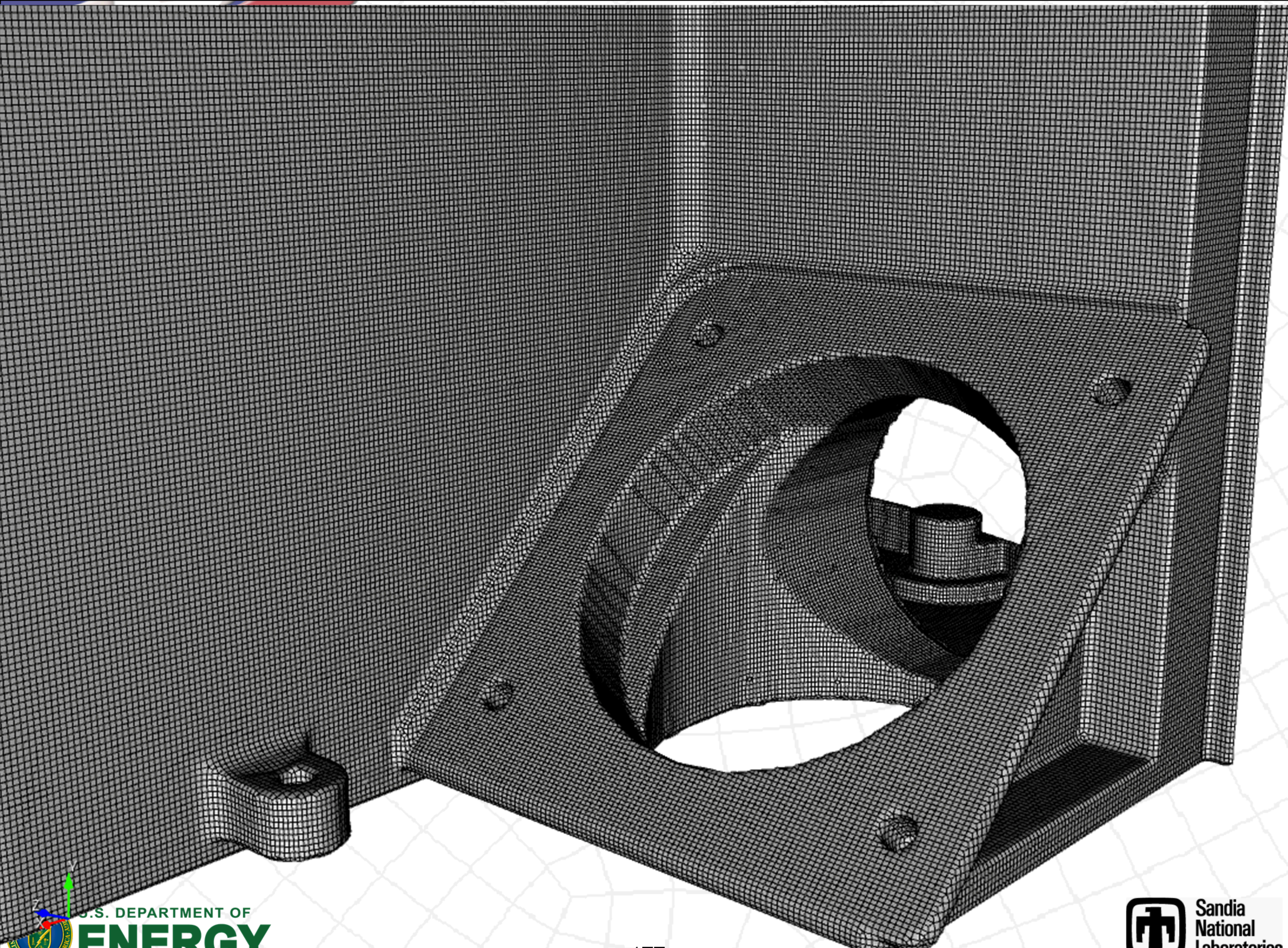

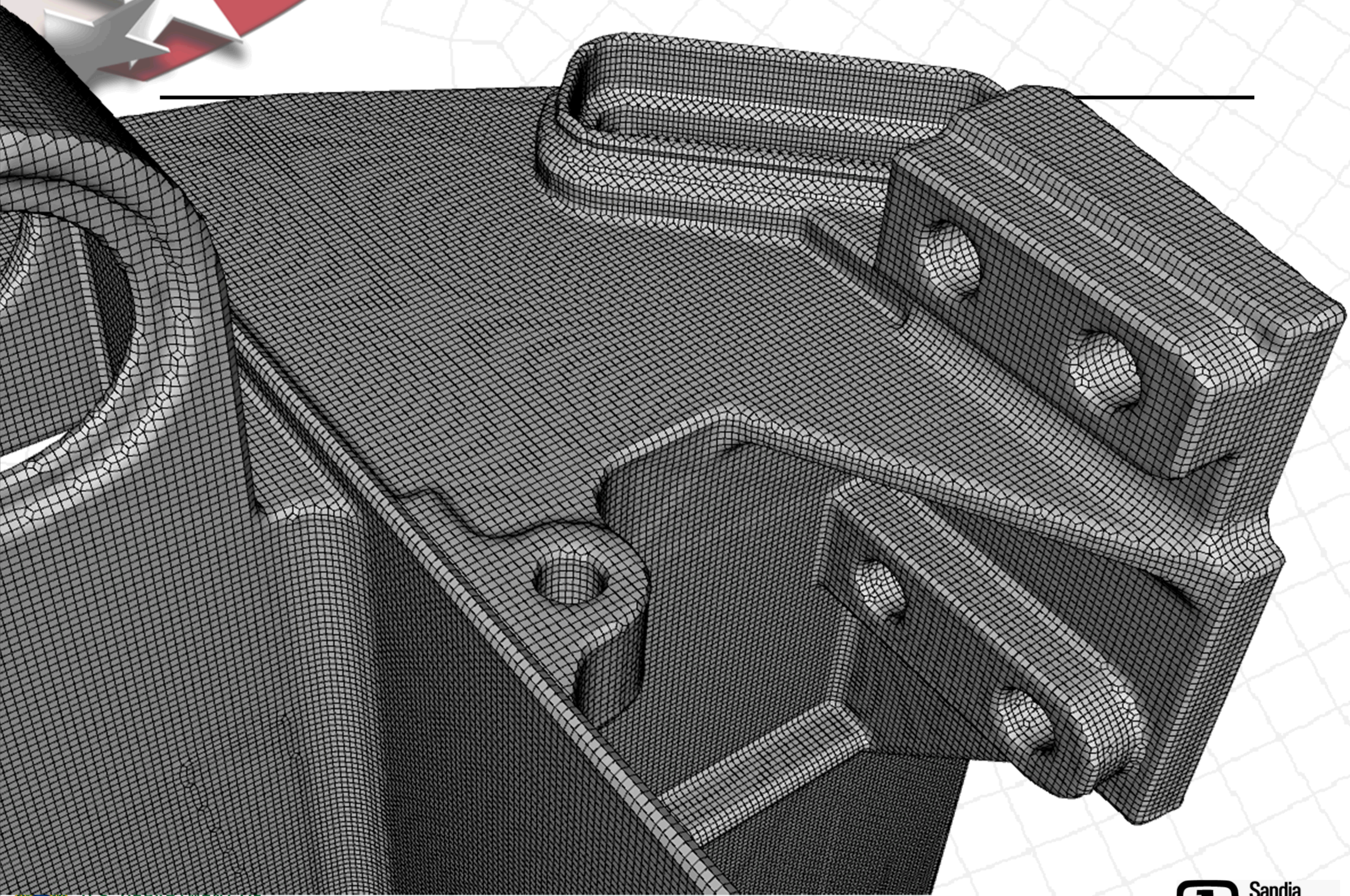

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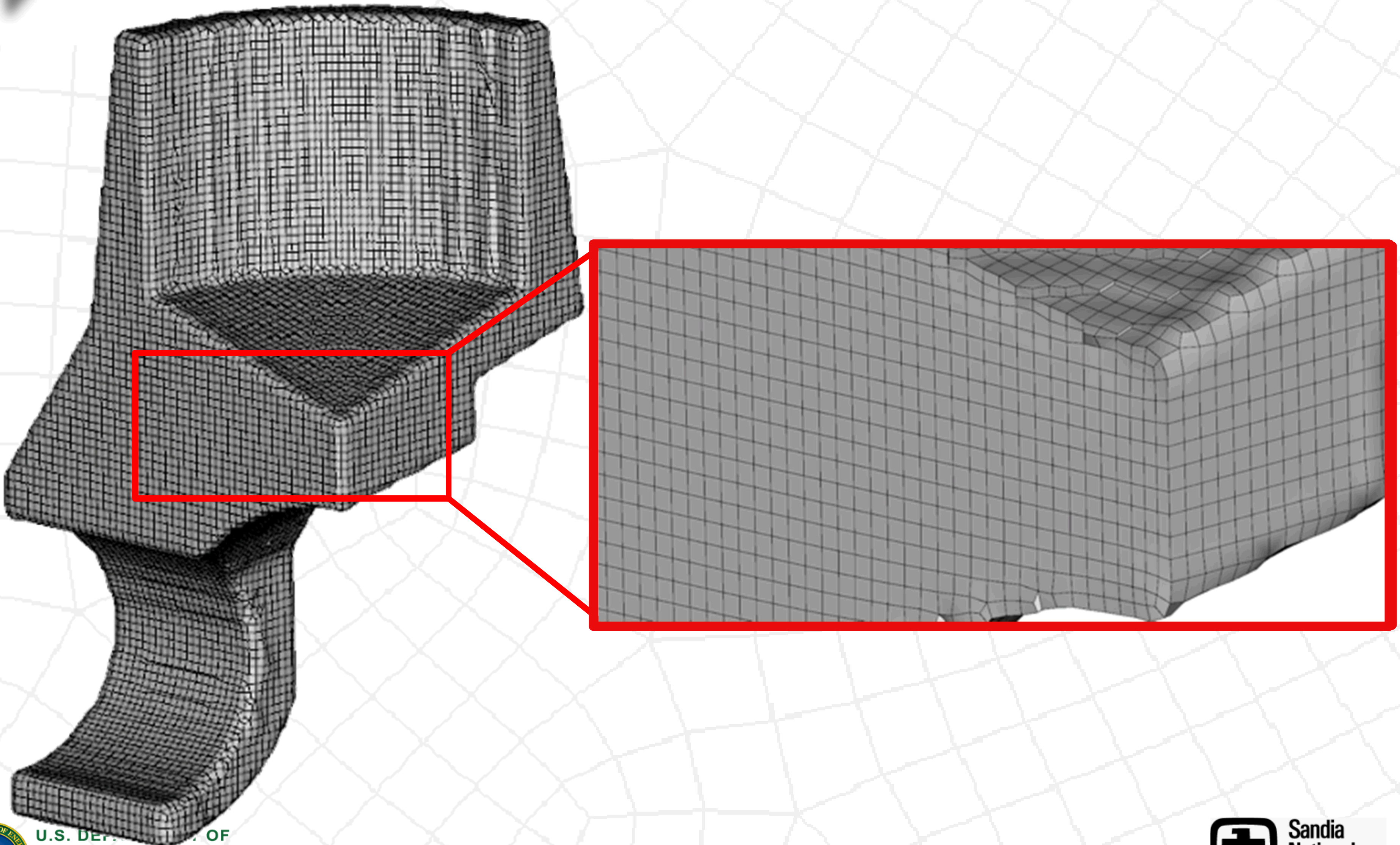

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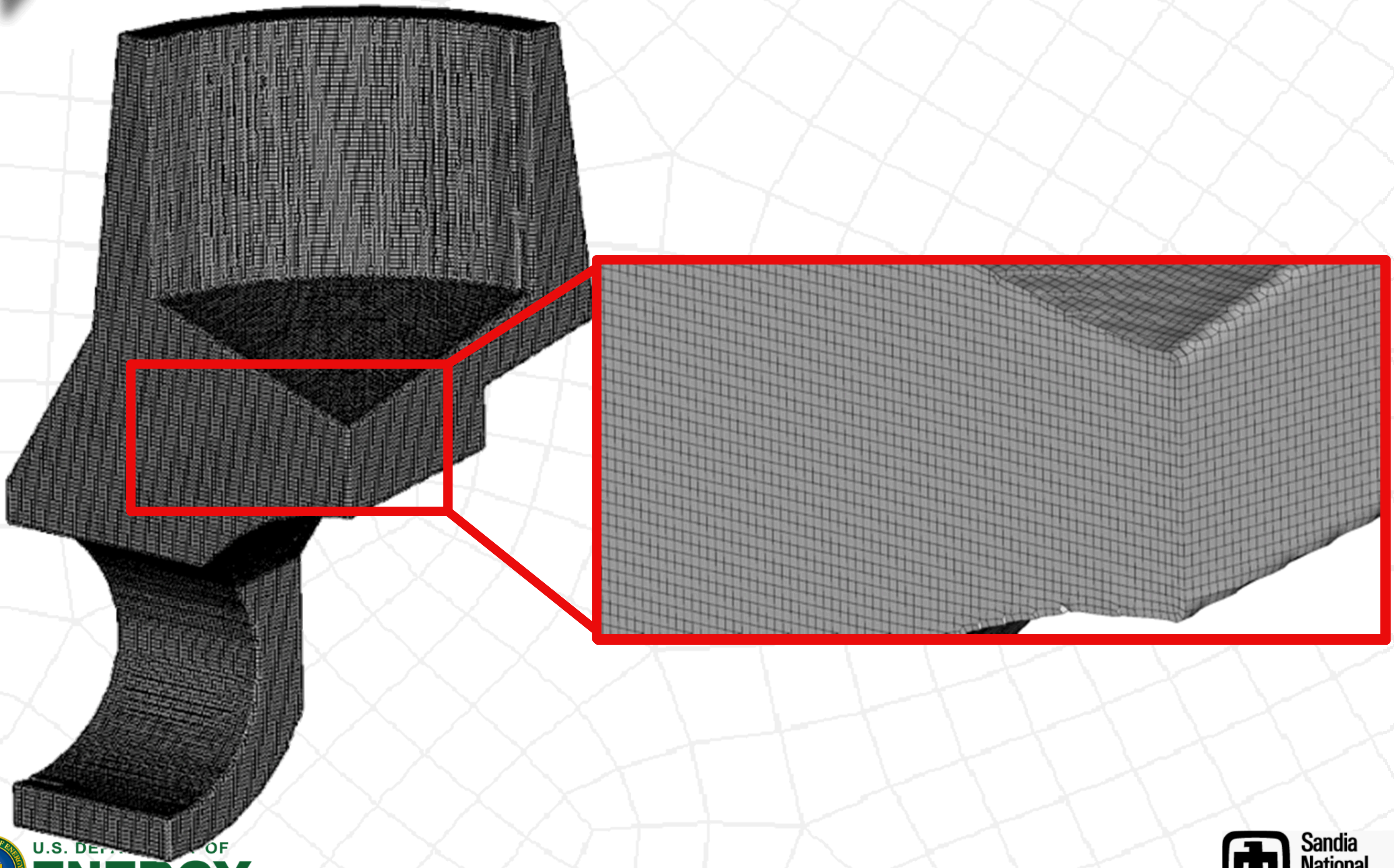
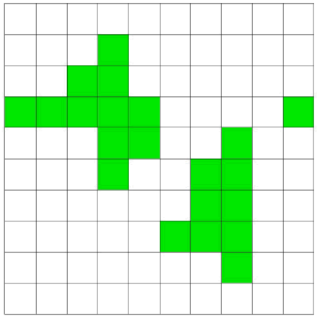
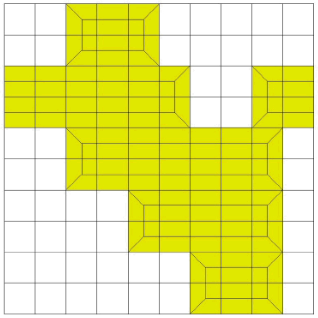


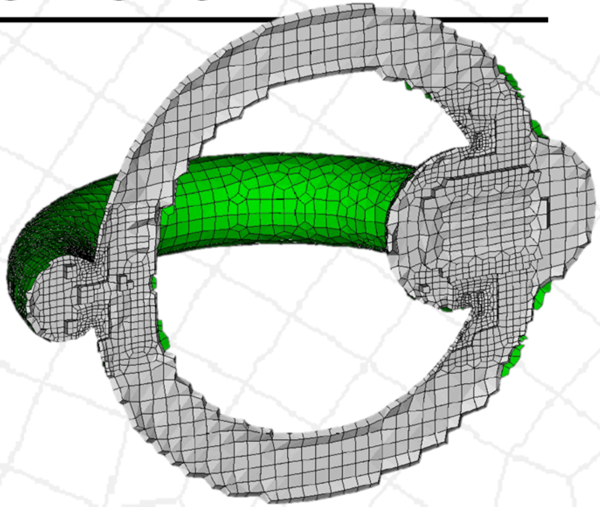
Sculpt with Geometry Adaptive Local Refinement

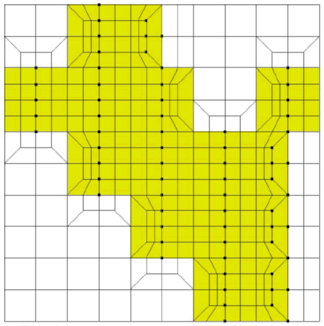


Cells flagged for refinement

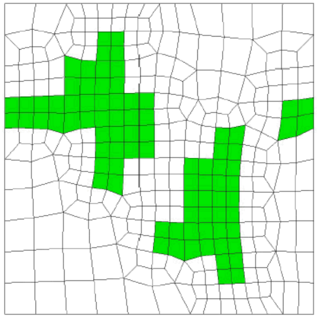


2-Refinement performed on columns in dimension 1

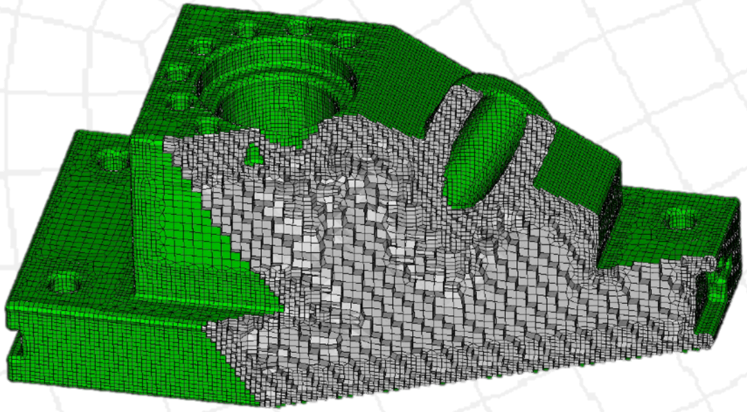




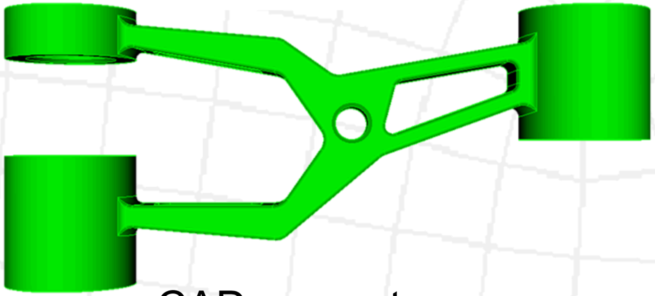
2-Refinement performed on columns in dimension 2. In 3D, dimension 3 is refined next.



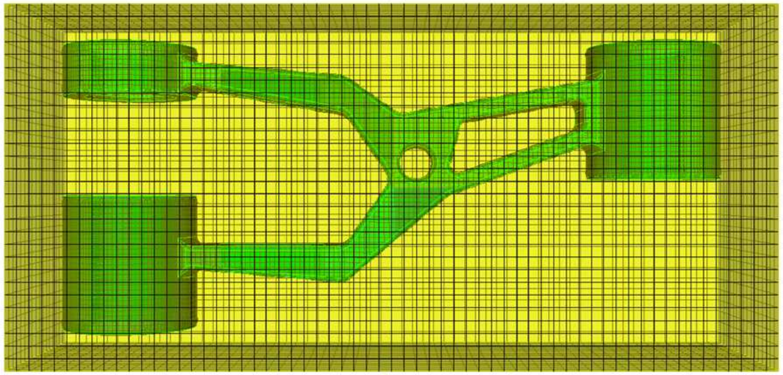
Smoothing, quality optimization



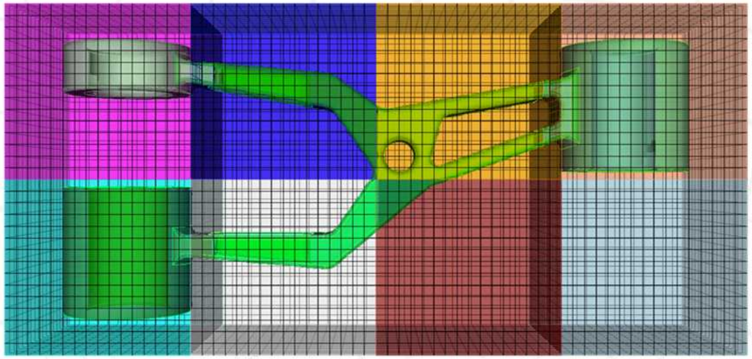
Sculpt Distributed Meshing



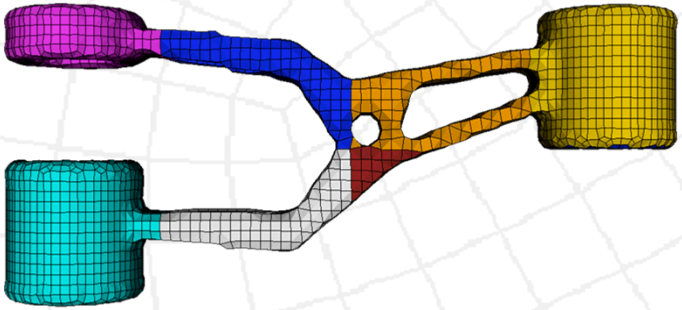
CAD geometry



Global overlay Cartesian grid



Cartesian grid decomposed and distributed amongst many processors



Each processor independently meshes its portion of Cartesian grid

Chama – 1.56B hexes from 4.10B Cartesian cells on 8192 procs in 4.7 mins.
Sequoia – 1.29B hexes from 3.28B Cartesian cells on 16,384 procs in 18.2 mins.

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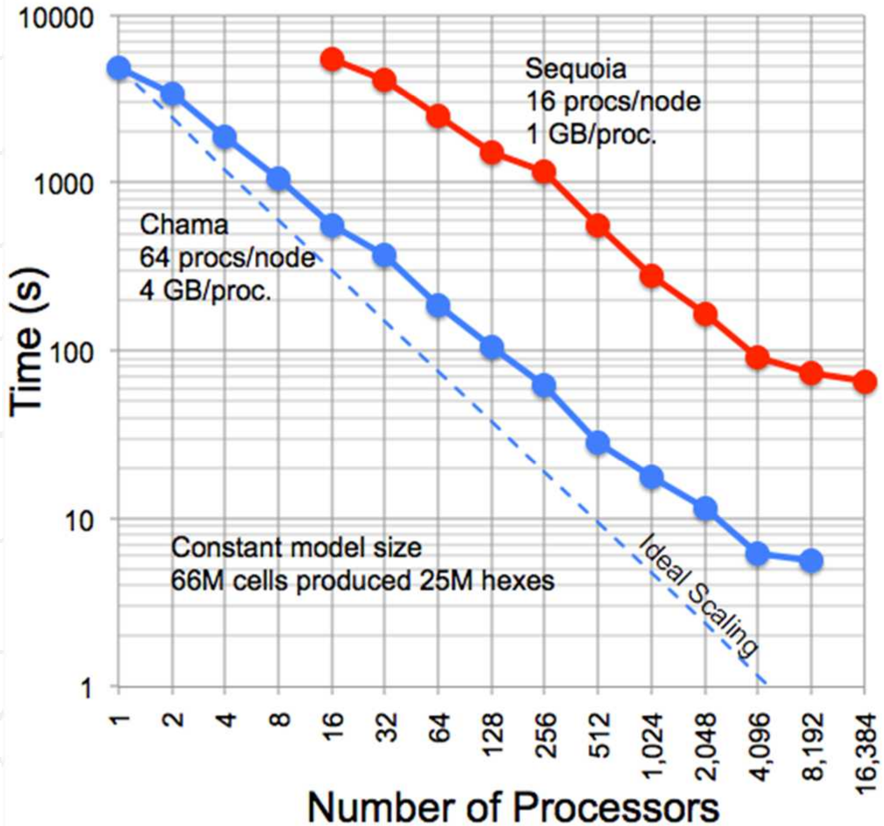
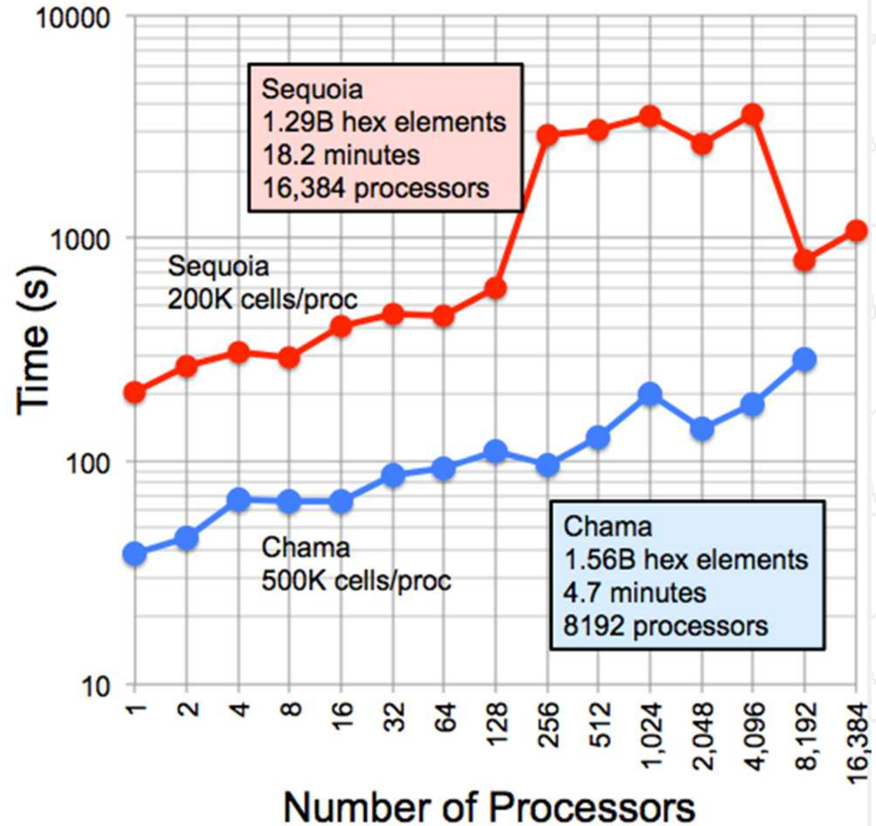
Sculpt Scaling Studies

Scaling studies performed on Sandia's Chama and LLNL's Sequoia revealed good scalability on each, but noted significant differences in performance between platforms.

Strong Scaling Study
Mesh size held constant



Weak Scaling Study
Load/processor held constant



OTHER METHODS TO CONSIDER

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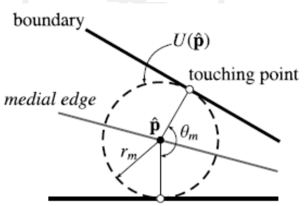
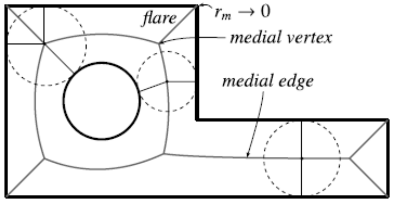
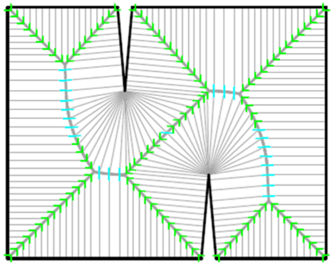
Medial Axis/Surface Methods

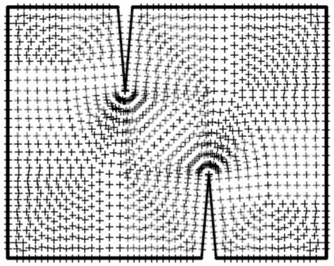
Another area of quad/hex meshing research is that of Medial Axis/Surface methods. Here are some recent references for the interested researcher:

Fogg, Harold J. and Cecil G. Armstrong, Trevor T. Robinson, "New techniques for enhanced medial axis based decompositions in 2-D," Proceedings 23rd International Meshing Roundtable, 2014.

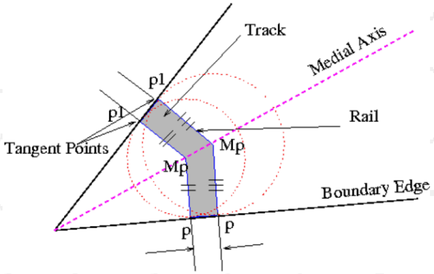


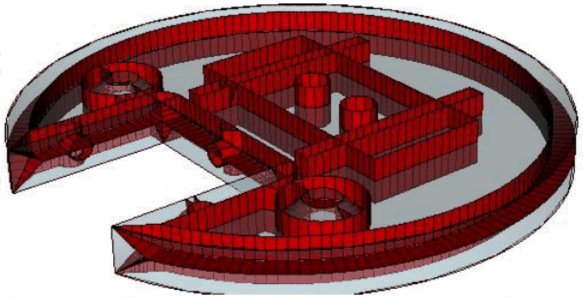


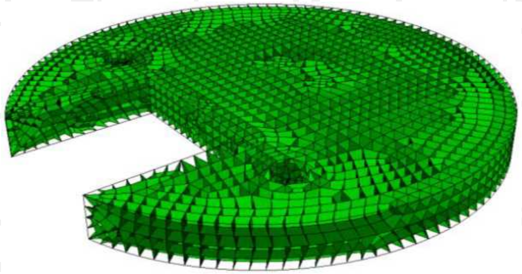




Quadros, William Roshan, "LayTracks3D: a new approach to meshing general solids using medial axis transform," Proceedings 23rd International Meshing Roundtable, 2014.

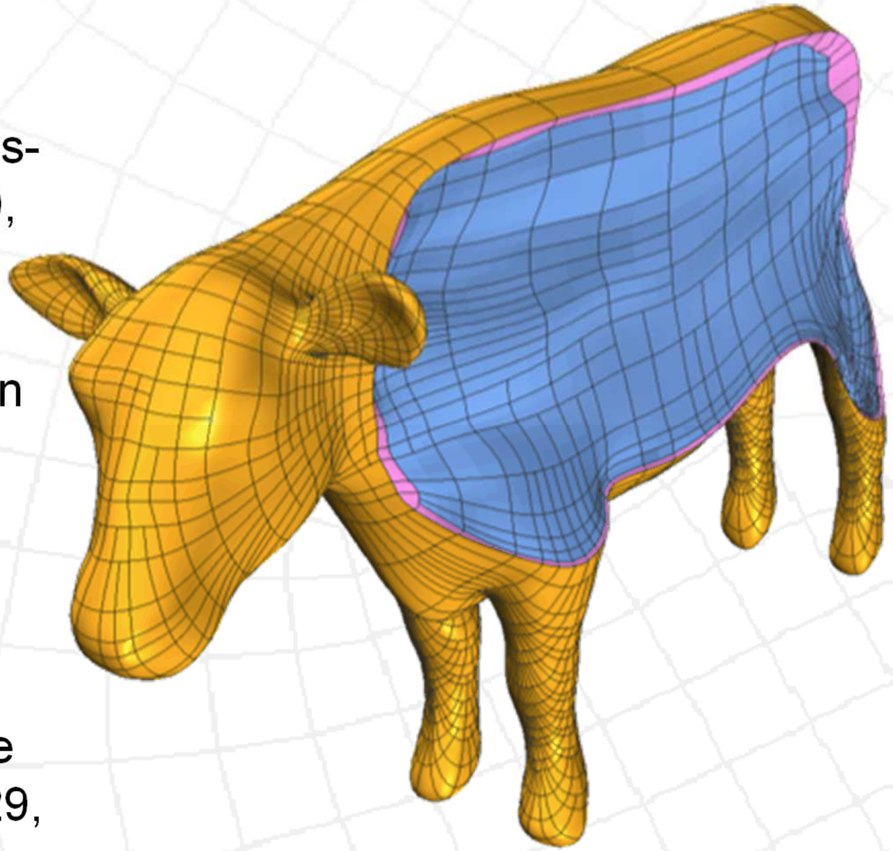






Hex Meshing and IsoGeometrics

- **Direct Generation of T-Splines**
  - Y. Zhang, W. Wang, T.J.R. Hughes, "Solid T-Spline Construction from Boundary Representations for Genus-Zero Geometry," ICES Report 11-40, November 2011
  - W. Wang, Y. Zhang, L. Liu, T.J.R. Hughes, "Solid T-Spline Construction from Boundary Triangulations with Arbitrary Genus Topology", ICES Report 12-13, April 2012
  - Y. Zhang, W. Wang, T.J.R. Hughes, "Conformal Solid T-Spline Construction from Boundary T-spline Representations," ICES Report 12-29, July 2012
  - And others…



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