SAND2015-7741C

Geometric Hitting Set for Segments of Few Orientations

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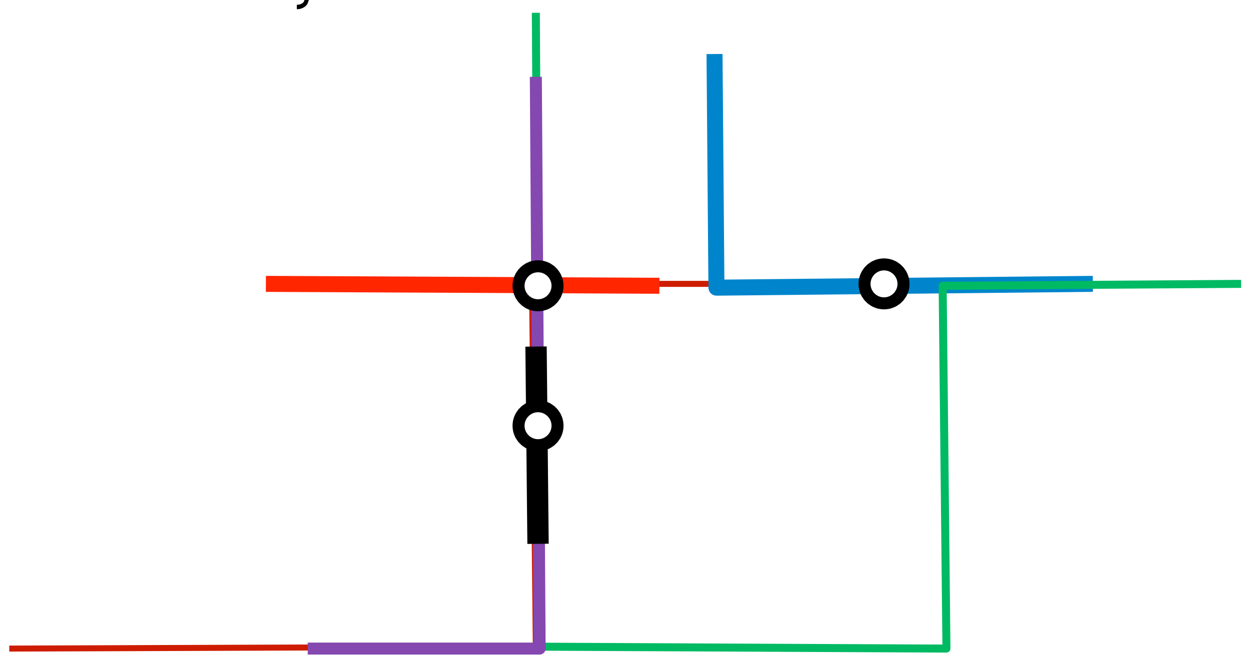
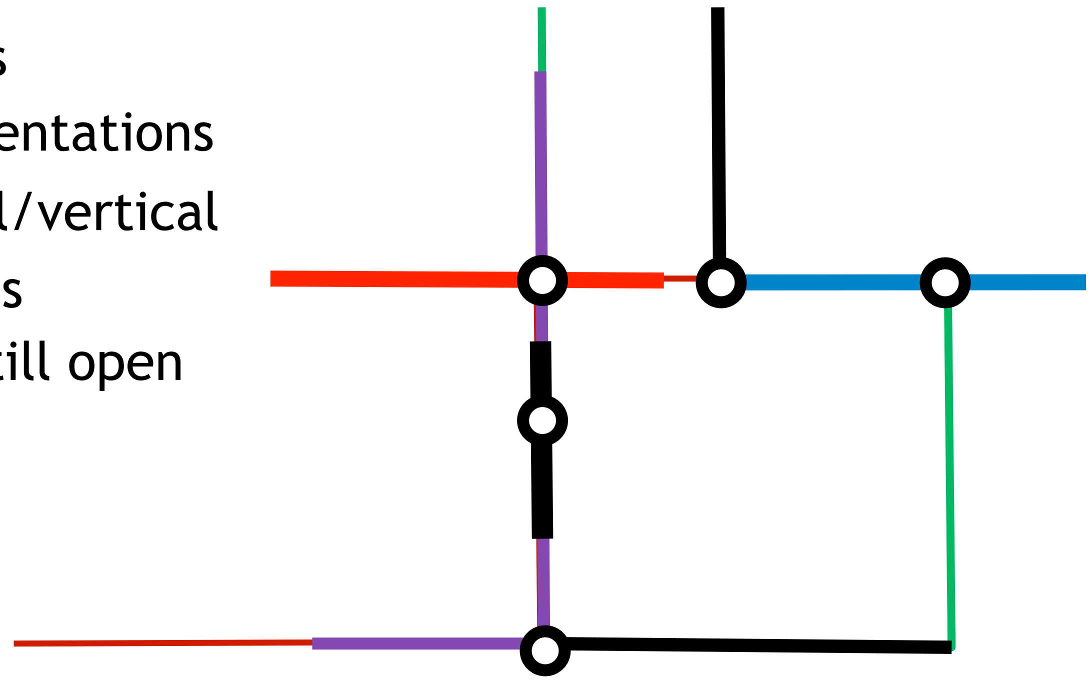
Path Monitoring: Ultimate Goal

- Given: Trajectories in the plane. Segments connecting waypoints.
- Observe (hit) each trajectory at least once
  - "Sensor placement"
  - Service placement in a city



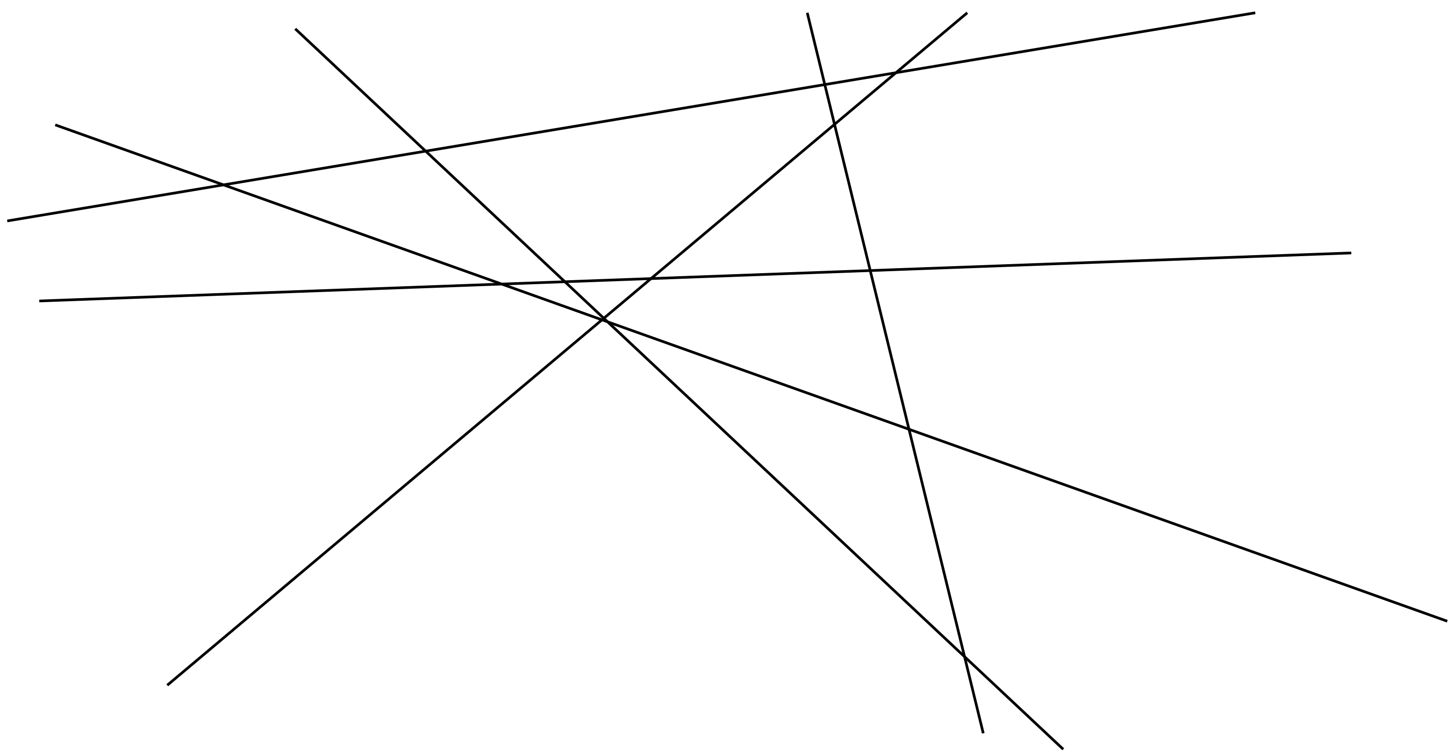
Path Monitoring: Simplified Goal

- Straight lines
  - Lines, rays, segments
  - Arbitrary overlaps
- Limited number of orientations
  - Largely horizontal/vertical
  - Also 3 orientations
- Considerable theory still open
- Guide heuristics



Hitting lines in the Plane (previous results)

- Arbitrary orientations
- Max SNP-hard (Kumar, Arya, Ramesh 2000); APX-hard (Brodén, Hammar, Nilsson, 2001)
- Greedy is log(OPT) approximation (Kovaleva, Spieksma 2006)
- Greedy is Ω(log n) approximation (Dumitrescu, Jiang 2013)



Results

Objects to hit	Complexity	Approximation
Lines, 3 slopes	NP-hard	1.4
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Vertical lines/ Horizontal rays	O(nM), where M = Bipartite matching	1
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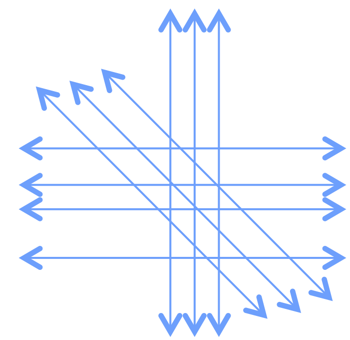
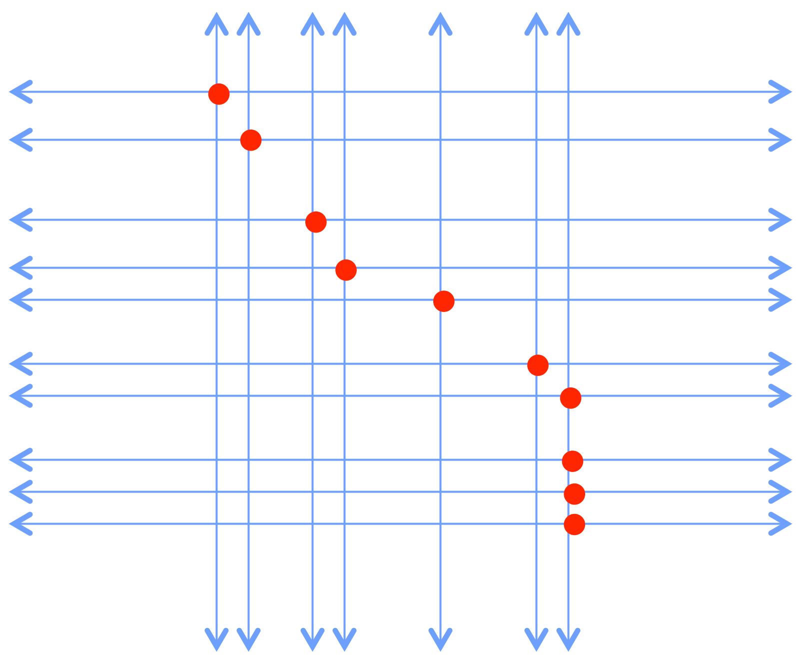
Triangle-free segments	O(n) not LP-based	3

- n total objects to hit
- All objects in the plane unless otherwise indicated.

Easy problem: Lines, 2 Orientations

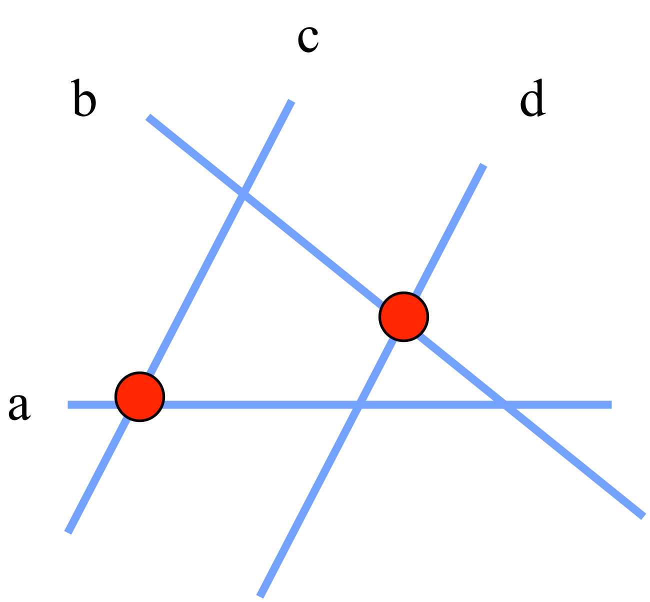
- Simple greedy
- m vertical, n horizontal
- WLOG m≤n then OPT = n
- But NP hard for 3 orientations in the plane

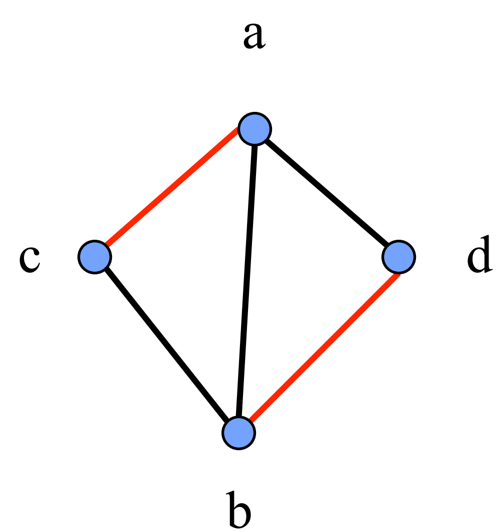




Easy problem: At Most Two Objects per Point

- Edge cover in the intersection graph. Matching + greedy
- Arbitrary orientations

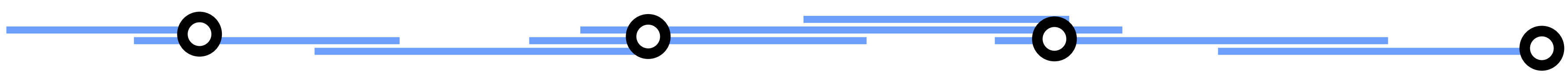




Intersection graph

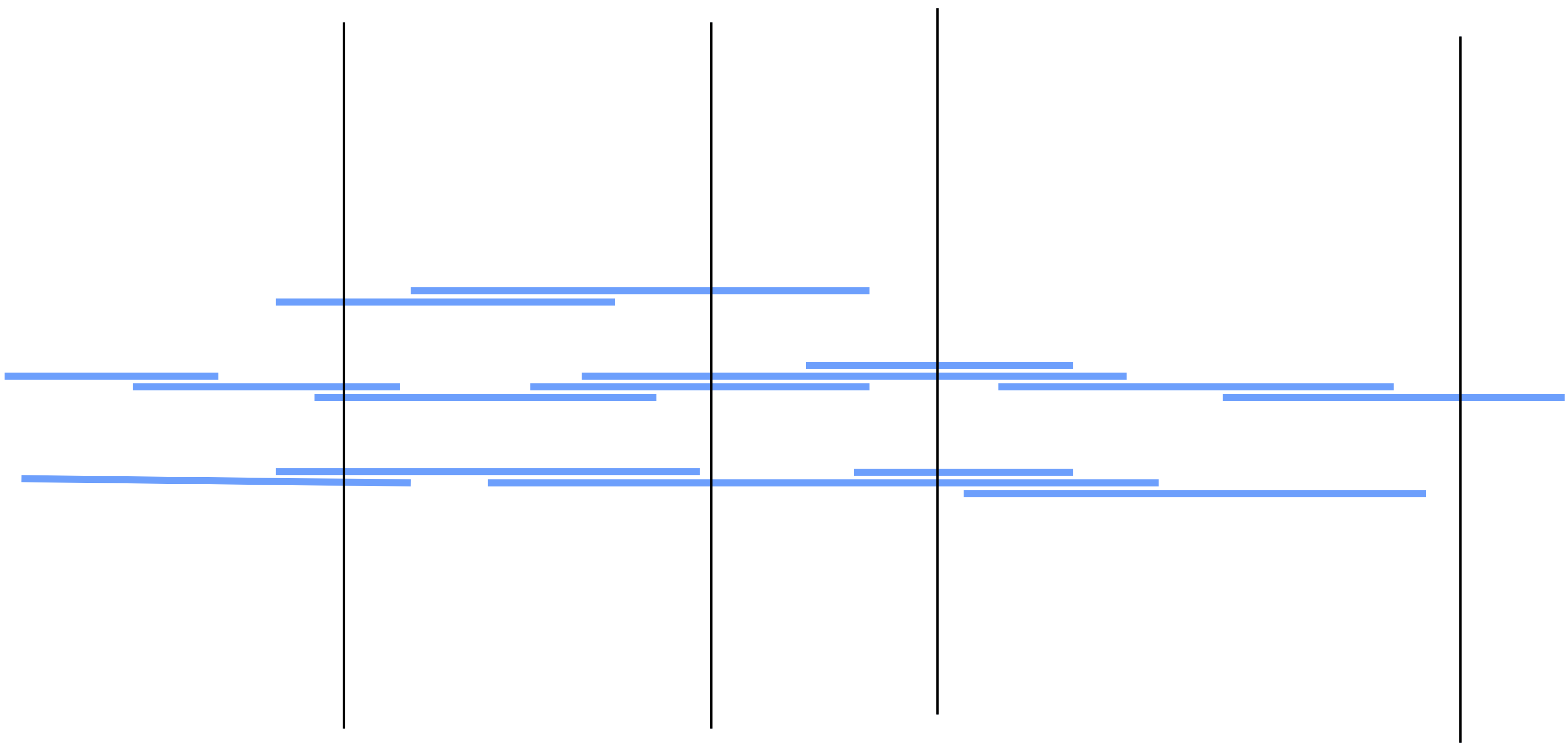
Easy Problem: Segments on a Line

- Interval stabbing on a line
- Optimal Algorithm:
  - Scan left to right (events = segment endpoints)
  - Place a point when a segment ends and remove all segments that are hit
- Linear time after sorting



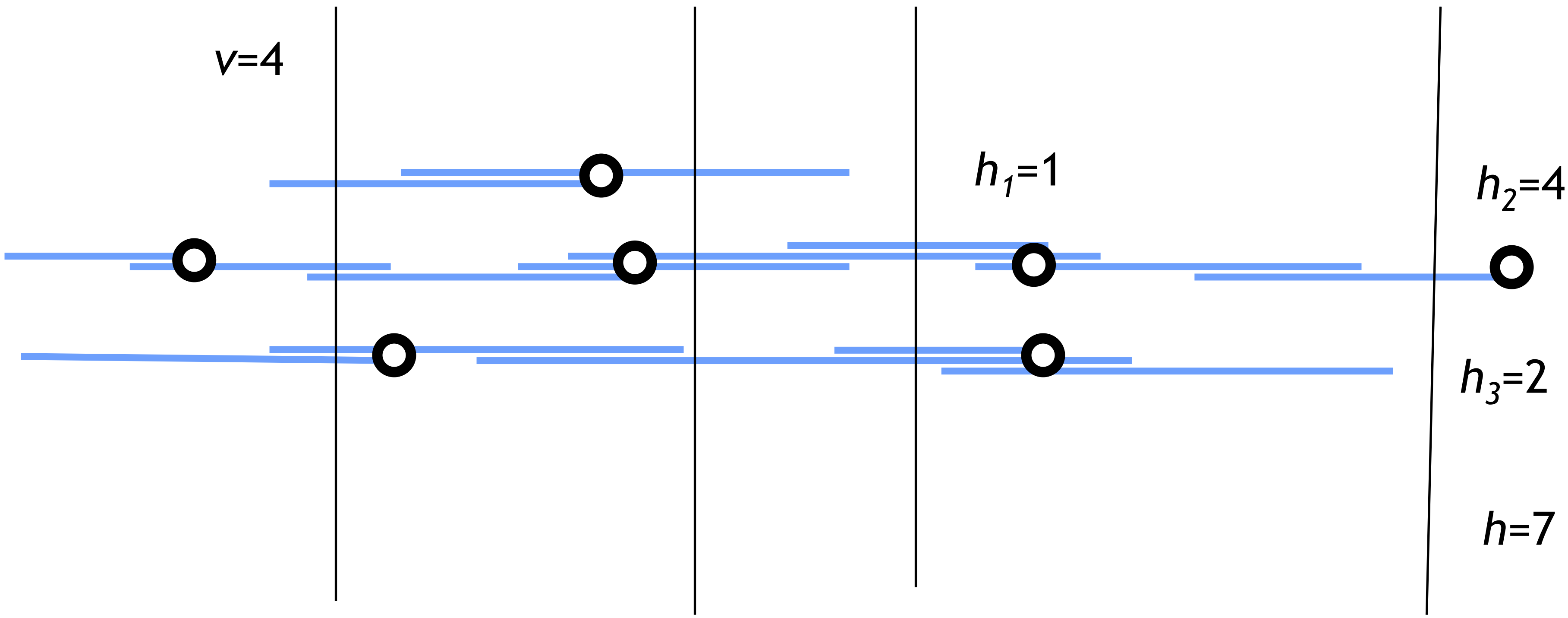
Vertical Lines, Horizontal Segments Approx

- 5/3-approximation (recall NP-hard)



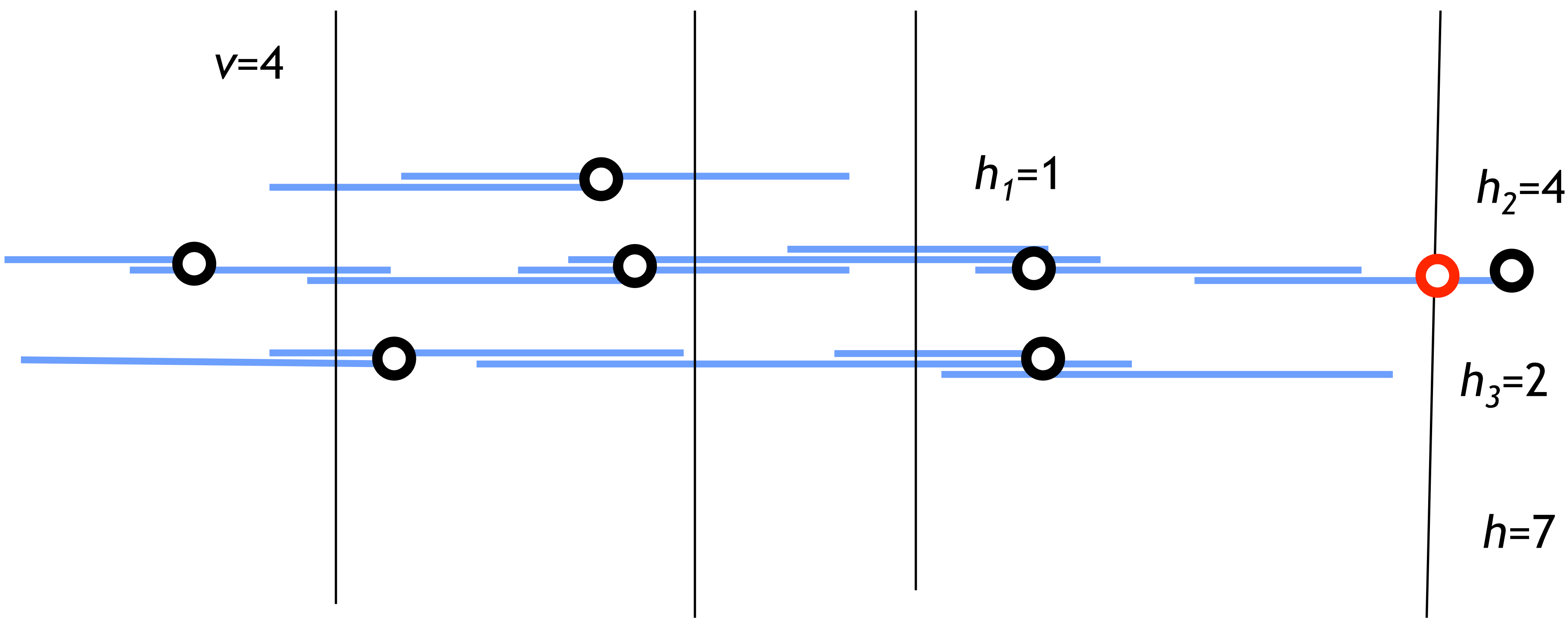
Vertical Lines, Horizontal Segments Approx

- Lower bounds:
  1. v = # vertical lines,
  2. optimal coverage of each horizontal line. $h = h_1 + h_2 + h_3$



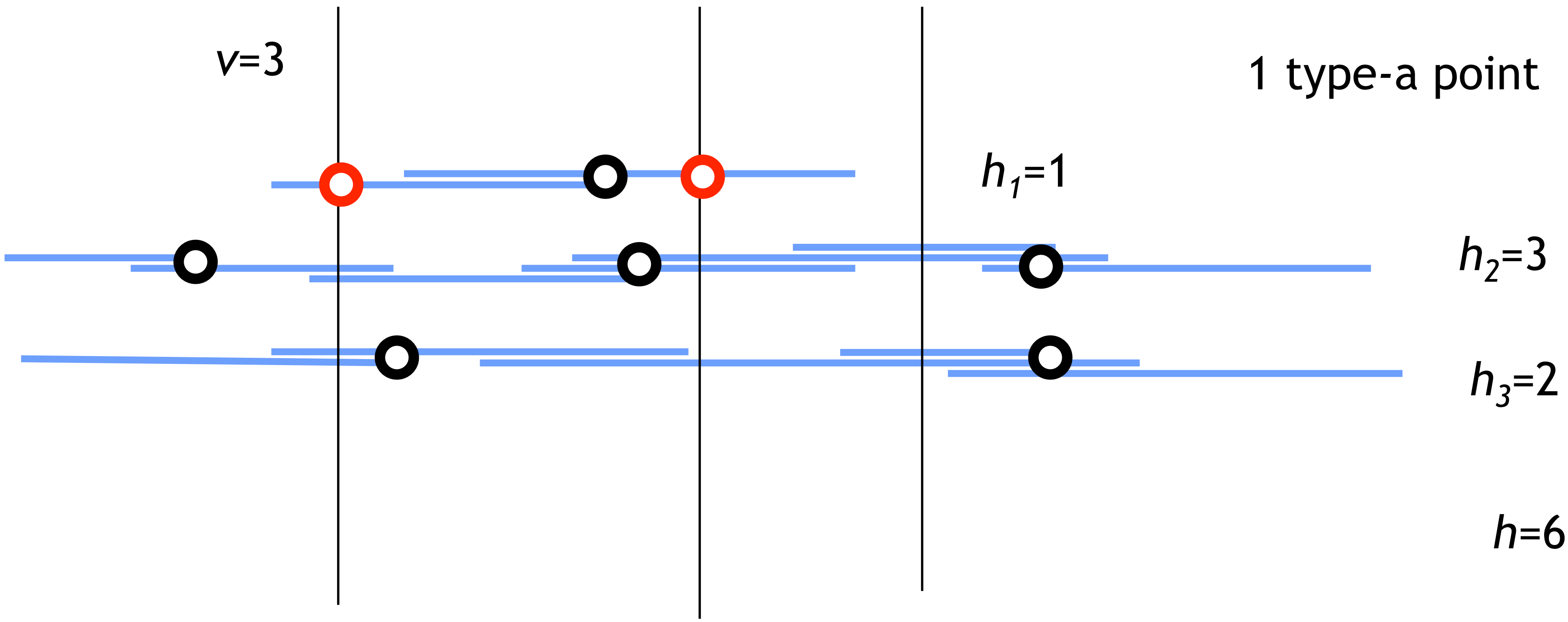
Vertical Lines, Horizontal Segments Approx

- Stage 1a: maximally productive points
  - Place points that cover a line and reduce the associated h_i by 1
  - Remove the line and all covered segments



Vertical Lines, Horizontal Segments Approx

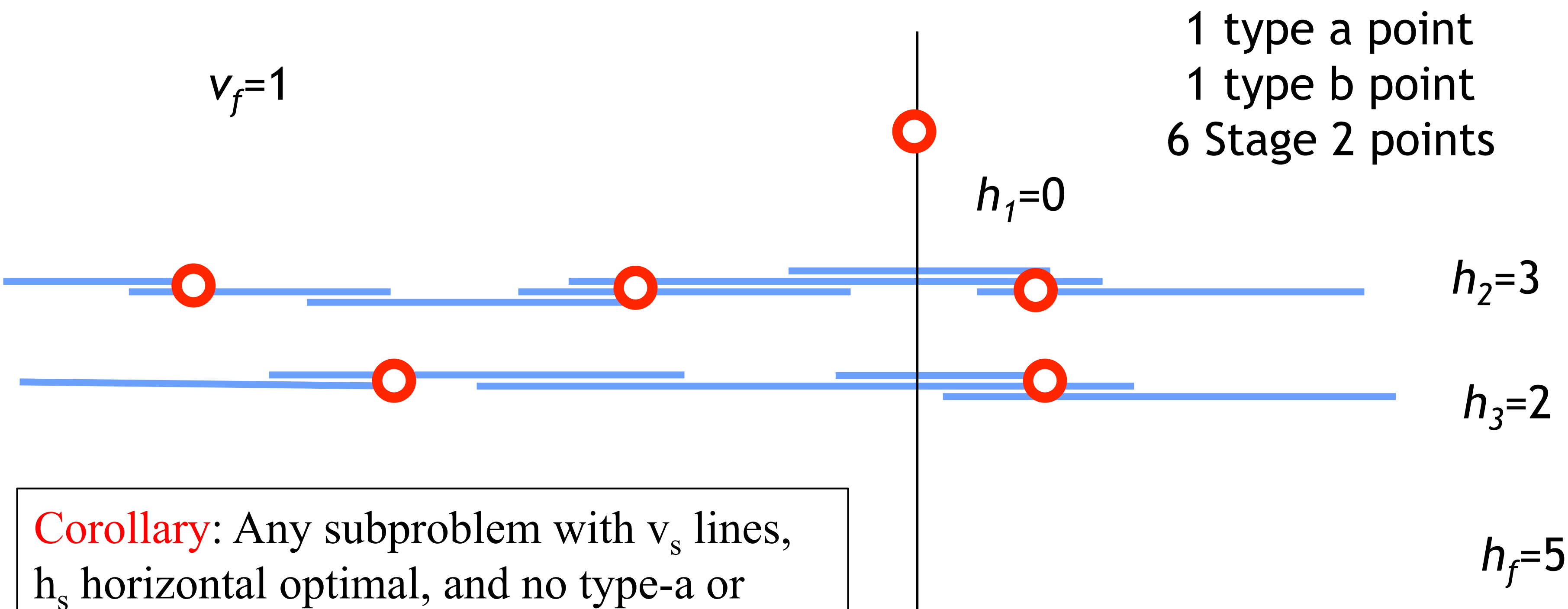
- Stage 1b:
  - Place 2 points that cover 2 lines and reduce the associated h_i by 1
  - Remove the lines and all covered segments



Vertical Lines, Horizontal Segments Approx

- Stage 2: Cover horizontal and vertical pieces separately

Theorem: Instances with no type-a or type-b moves require v + h points



Corollary: Any subproblem with v_s lines, h_s horizontal optimal, and no type-a or type-b moves has OPT $\geq v_s + h_s$

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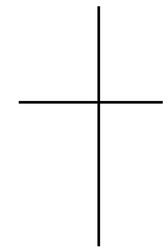
5/3 Approx: Analysis

- Select k_1 type-a points. Reduces v and h each by k_1
- Select k_2 type-b points. Reduces v by k_2 and h by $k_2/2$
- Stage 2: $v_f = (v - k_1 - k_2)$ and $h_f = (h - k_1 - k_2/2)$
  - Selects $v_f + h_f$ points (≤ OPT by previous corollary)



Type a

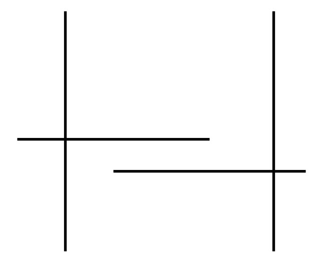
- Total selected: $v + h - k_1 - k_2/2$

Case 1: $k_1 + k_2 \leq 2OPT/3$

Stage 1 + Stage 2 ≤ 2OPT/3 + OPT = 5OPT/3

Case 2: $k_1 + k_2 > 2OPT/3$

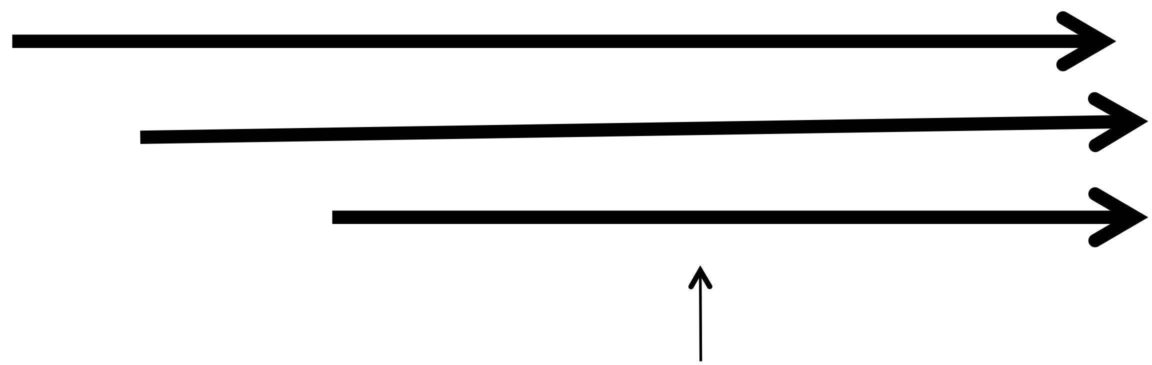
$$\begin{aligned} v + h - k_1 - \frac{k_2}{2} &\leq 2 * OPT - \frac{k_1 + k_2}{2} \\ &\leq 2 * OPT - \frac{OPT}{3} \\ &= \frac{5}{3} * OPT \end{aligned}$$



Type b

Vertical Lines and Horizontal Rays

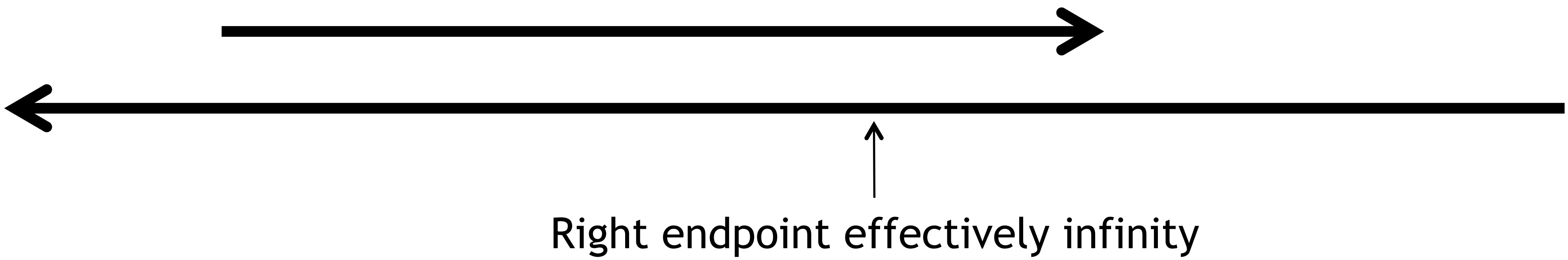
- For a given line, need to keep only one right-facing ray and one left-facing ray:



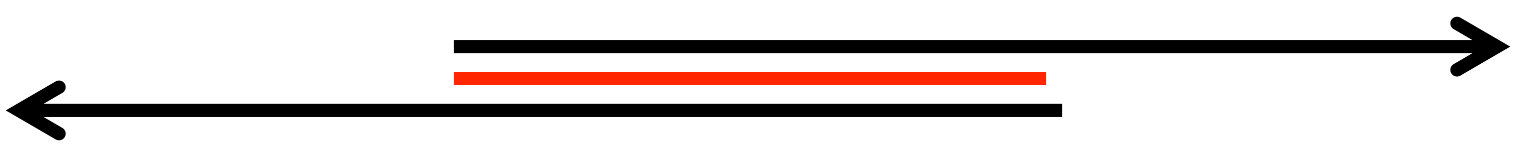
If this ray is covered, they all will be

Vertical Lines and Horizontal Rays

- WLOG, each horizontal line with rays has one in each direction

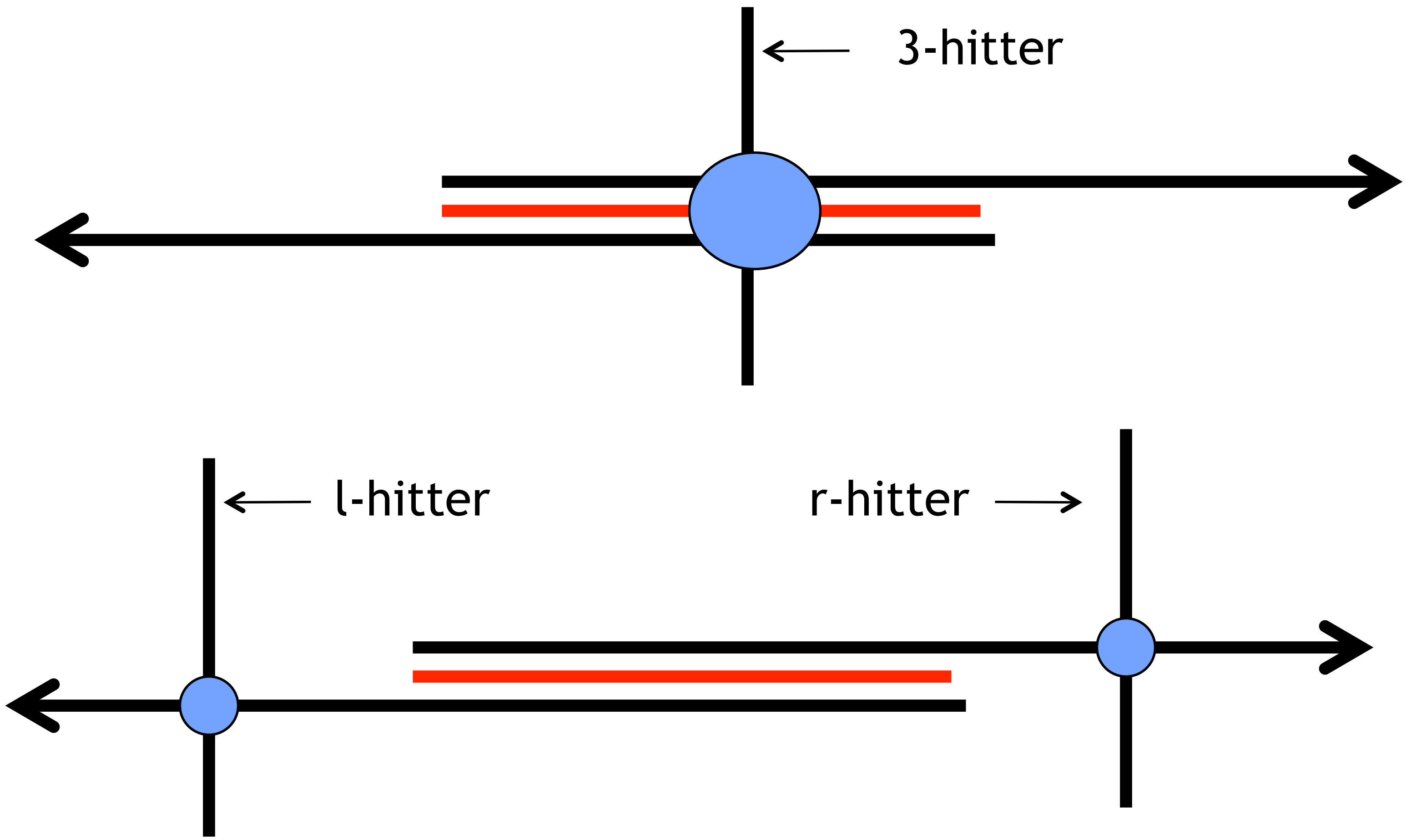


- The two rays intersect in a segment



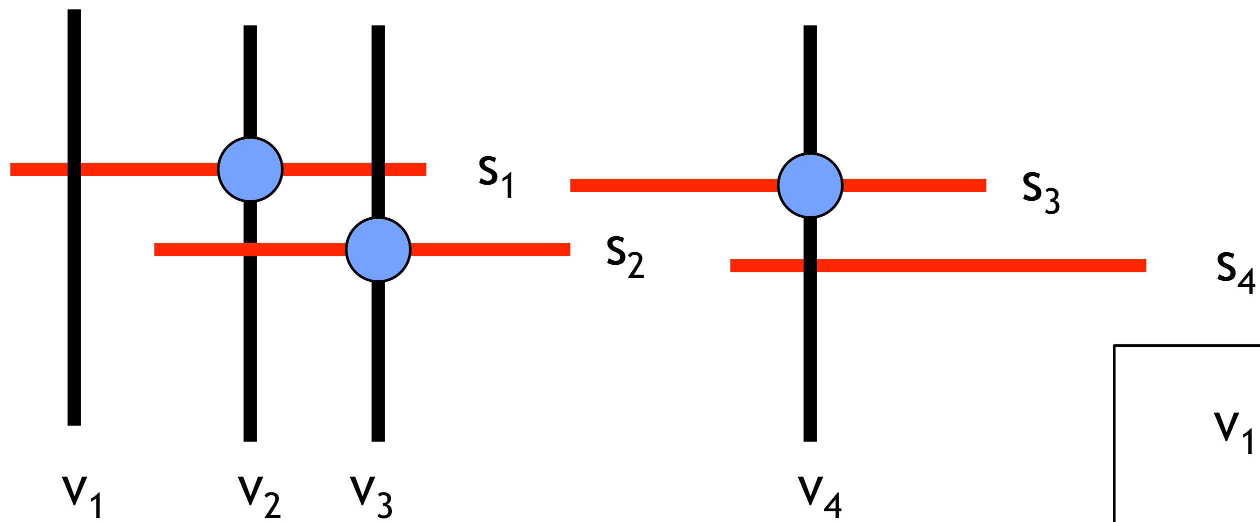
Vertical Lines and Horizontal Rays

- Best way to hit a pair of intersecting rays is in the segment
- 2 ways to hit vertical lines and horizontal rays together:

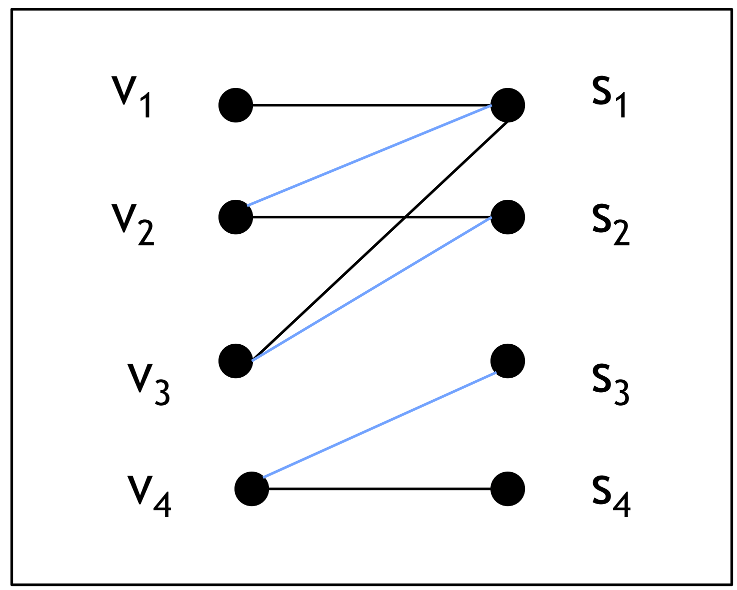


Vertical Lines and Horizontal Rays (VLHR)

- 3-hitters (line + segment) are most efficient
- Find the maximum number of 3-hitters with matching

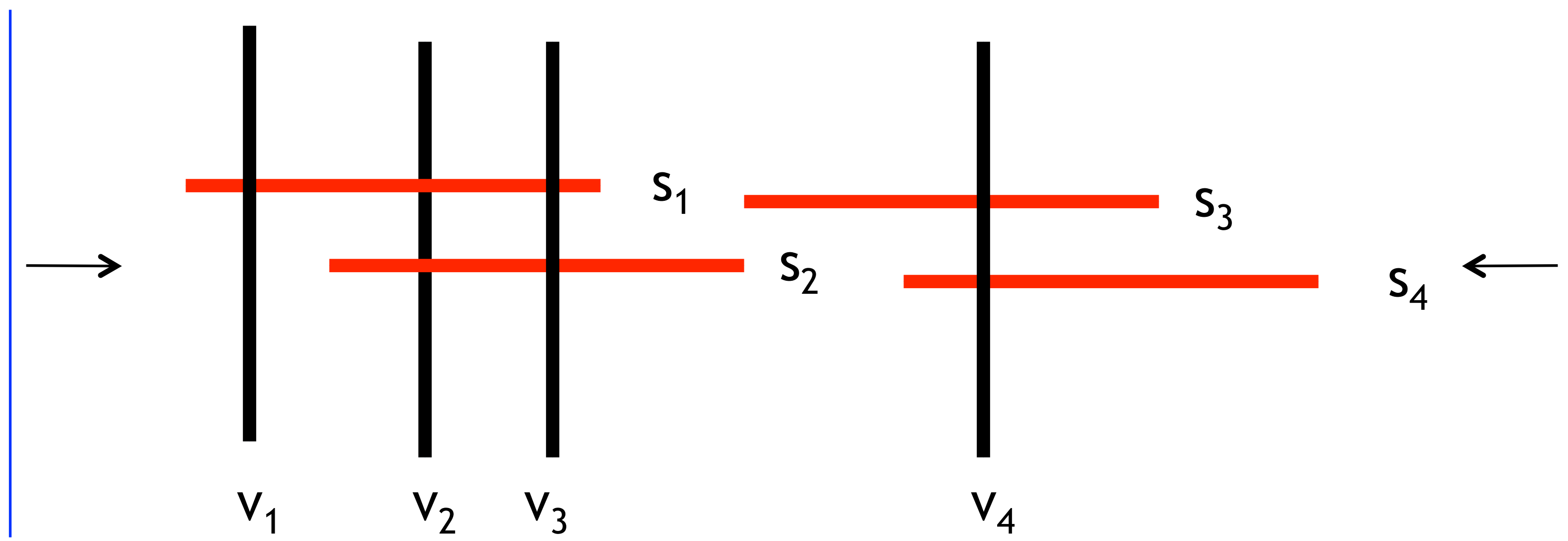


Theorem: There is a maximum matching between lines and segments that can be augmented to an optimal solution.



VLHR Algorithm sketch

- Find a suitable maximum set of 3-hitters
  - Sweep from left and right
  - Add a 3-hitter at last moment (waiting reduces number of 3-hitters, so matching is a test)
  - Balance remaining lines between left and right
    - Best chance to find a pair of lines that covers a ray pair
- Remaining objects intersect in at most pairs (easy edge cover)



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Open: Improve factors? Covering versions?

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Open Problems

- Improve approximation factors
  - Lines of 3 slopes in 2D: improve 1.4?
  - Vertical lines, horizontal segments: improve 5/3?
  - Vertical/horizontal segments: improve naïve factor 2?
- Covering problems: Cover points by fewest segments/rays/lines of few orientations