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Mixed-Integer Formulations for Constellation Scheduling

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Abstract

Remote sensing systems have expanded the set of capabilities available for and critical to national security. Cooperating, high-fidelity sensing systems and growing mission applications have exponentially increased the set of potential schedules. A definitive lack of advanced tools places an increased burden on operators, as planning and scheduling remain largely manual tasks. This is particularly true in time-critical planning activities where operators aim to accomplish a large number of missions through optimal utilization of single or multiple sensor systems. Automated scheduling through identification and comparison of alternative schedules remains a challenging problem applicable across all remote sensing systems. Previous approaches focused on a subset of sensor missions and do not consider ad-hoc tasking.

We have begun development of a robust framework that leverages the Pyomo optimization modeling language [1, 2] for the design of a tool to assist sensor operators planning under the constraints of multiple concurrent missions and uncertainty. Our scheduling models have been formulated to address the stochastic nature of ad-hoc tasks inserted under a variety of scenarios. Operator experience is being leveraged to select appropriate model objectives. Successful development of the framework will include iterative development of high-fidelity mission models that consider and expose various schedule performance metrics. Creating this tool will aid time-critical scheduling by increasing planning efficiency, clarifying the value of alternative modalities uniquely provided by multi-sensor systems, and by presenting both sets of organized information to operators. Such a tool will help operators more quickly and fully utilize sensing systems, a high interest objective within the current remote sensing operations community.

Preliminary results for mixed-integer programming formulations of a sensor-scheduling problem will be presented. Assumptions regarding sensor geometry and sensing activity time constraints, durations, priorities, etc. will be outlined. Finally, solver speed and stochastic programming details for uncertain activities and scheduling impediments will be discussed.

1 INTRODUCTION

We describe mathematical programming formulations for managing a constellation of remote sensors scheduled to monitor physical locations in space and time. The focus herein will be on remote sensing systems where available sensor time is less than the time required to observe all requested activities [3]. The models herein have are particularly relevant for earth observing satellites, such as NASA's fleet that now includes the joint U.S. Geological Survey's LANDSAT 8, and space observing satellites, such as the U.S. Air Force's Geosynchronous Space Situational Awareness Program (GSSAP) satellites. It should be noted that the models developed herein are also aimed at describing a variety of terrestrial sensor networks including those with stationary video cameras or others that are comprised of drones or other airborne sensing systems.

Management of mobile sensors is challenging because they provide very flexible capabilities for simultaneously monitoring a diverse range of locations and events. In the following, we assume that the performance of mobile sensors will be evaluated with respect to a fixed set of *activities* that are defined by:

- **Start time**: when the activity starts.
- **Duration**: fixed and known before building a constellation schedule. Some activities may be divisible.
- **Physical location**: the location that needs to be *observed*; the precise requirements for *observing* a location will depend on the nature of the sensor technology.
- **Configuration**: the operational configuration required to *observe* a location; dependent on the nature of the sensor technology and the *observable*.
- **Quality**: a minimum quality for the *observation*, which may be impacted by the physical location of the sensor, the time of day, and other factors. Quality is measured relative to the observable.
- **Priority**: the importance of this activity relative to other activities. Priority is normalized between 0 and 1, inclusive.
- **Category**: a hierarchal system establishing the *necessity* of an activity for a given sensor.

Managing sensors involves determining which activity is to be scheduled on each sensor, at each moment in time, and doing so to optimize the resultant set of observations. In general, we assume that time is suitably discretized. It is further assumed that the position of each sensor is known accurately enough at each time-step so that activities can be scheduled on an appropriate sensor at a feasible time-step. Additionally, the time-windows of certain activities may shrink and move with time. Hence, there is a general need to schedule activities to enable resilient rescheduling of sensors.

Finally, there are several general observations that need to be considered:

- Although we use the term *scheduling* here, there are no precedence constraints amongst most activities. Hence, a schedule of sensors simply reflects the quality, number, and selection of activities that they cover.
- There may be times where sensors cannot be employed (e.g. it is too dark to use an optical sensor). This could be captured with a low quality assessment for the sensor scheduled for that activity at those times, or through an explicit modeling constraint.
- Mobile sensors may need to regularly schedule activities where they are off-line (e.g. to refuel or recalibrate). This may be accomplished with constraints for the interval allowed between the activities or by assigning suitable priorities and time windows for those activities.
- Depending on a variety of criteria, multiple sensors may be able to cover an activity simultaneously (though perhaps with different quality scores).

2 A COVERAGE MODEL

We begin with a simple coverage model for sensor management. Let $\mathcal{I} = \{1, \ldots, I\}$ be the set of sensors that are being managed and let $\mathcal{T} = \{1, \ldots, T\}$ be the scheduling horizon, or number of time steps that are to be scheduled. Each sensor shares the same scheduling horizon. In other words, $\mathcal{T}_i = \{1, \ldots, T\}$, $\forall i \in \mathcal{I}$. For simplicity, we assume that all sensors are distinct. In the case of a satellite or platform with multiple sensors, each individual sensor is modeled separately. In general, the following objectives might be considered when scheduling a sensor network:

- minimize the number of sensors.
- minimize the planning horizon (e.g. minimize the "make span").
- maximize the average priority of scheduled activities.
- maximize the quality of scheduled activities.
- maximize the number of scheduled activities.

2.1 ACTIVITY SPECIFICATION

Let $\mathcal{K} = \{1, \ldots, K\}$ be the set of activities to be scheduled. For each activity k we have

- (e_k, l_k) is the time window over which this activity can be *started*.
- d_k is the duration of the activity.
- q_{ikt} is the quality of observing activity k with sensor i at time t. It is assumed that the quality of sensor observation can be precomputed in advance.

- $q_k^{\min}$ is the minimal allowed quality for activity k when it is scheduled; by convention $q_k^{\min} > 0$.
- p^k is the priority associated with a given activity where $p^k \in [0, 1]$.

2.2 ACTIVITY CATEGORIES

In addition to the above specifications, activities are organized into one of four *categories* to denote the necessity of being scheduled on a given sensor or a potential set of sensors.

- **Category 1** activities are unique to a given sensor. They are constrained to be scheduled and are therefore not assigned priority. Examples of this category of activity include solar outage or other sensor safety activities.
- **Category 2** activities are also unique to a given sensor. These activities cannot be scheduled during category 1 activities. Category 2 acivities are limited to be within the priority range $p_2^k \in [p_2, 1]$, with $0 < p_2 < 1$ and p_2 generally much larger than 0. Higher priority category 3 and category 4 activities can preempt category 2 activities. Examples of this type of activity include periodic calibration or maintenance activities.
- **Category 3** activities cannot be scheduled during category 1 activities. category 3 activities represent the largest portion of activities to be scheduled. They are capable of being scheduled on multiple sensors and span the range of activity priorities. Category 3 activities consist of any type of *observation* activity and are limited to be within the priority range $p_3^k \in [0, p_3)$, with $0 < p_3 < 1$.
- **Category 4**: activities have the priority $p_4^k = 1$ but cannot preempt category 1 activities. These activities have start time windows that are time-varying. It is assumed that the time windows are monotonically decreasing in span. Examples include uncertain or evolving activities where uncertainty decreases with time such as observing wildfires or flooding near critical infrastructure.

2.3 DECISION VARIABLES AND OBJECTIVE FUNCTIONS

Let δ_{ikt} be a binary decision variable where $\delta_{ikt} = 1$ if sensor i is scheduled to observe activity k starting at time period t. We assume that the set of activities are organized into the categories given above. Category 1 activities must be scheduled, but category 2 and 3 activities may be deferred due to scheduling limitations; let $\bar{\mathcal{K}} \subseteq \mathcal{K}$ be the indices of the categroy 1 activities. A natural objective may be to maximize the number of category 2 and 3 activities that are scheduled:

$$\max \sum_{k \in \mathcal{K} \setminus \bar{K}} \omega_k \tag{1}$$

where

$$\omega_k = \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \delta_{ikt}, \forall k \in \mathcal{K}. \tag{2}$$

The following constraint ensures that all category 1 activities are scheduled:

$$\omega_k = 1, \forall k \in \bar{\mathcal{K}}. \tag{3}$$

To enforce that each category 2 or category 3 activity can be scheduled at most once, the following constraint is used:

$$\omega_k \leq 1, \forall k \in \mathcal{K} \setminus \bar{\mathcal{K}}. \tag{4}$$

Optionally, to enforce a minimum quality of sensor observation, we have

$$\sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} q_{ikt} \delta_{ikt} \geq q_k^{\min}, \forall k \in \bar{\mathcal{K}} \tag{5}$$

and

$$\sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} q_{ikt} \delta_{ikt} \geq q_k^{\min} \omega_k, \forall k \in \mathcal{K} \setminus \bar{\mathcal{K}}. \tag{6}$$

Now if a sensor is scheduled to begin observing an activity at time $\bar{t}$, then it will continue to observe the activity for the next $d_k - 1$ time steps. Let $\mathcal{C}(k, \bar{t})$ be the set of feasible time steps where activity k could have been started prior to time step $\bar{t}$ and for which the sensor observation would conflict with starting a new sensor observation beginning at time step $\bar{t}$. Thus

$$\mathcal{C}(k, \bar{t}) = \max(e_k, \bar{t} - d_k + 1), \ldots, \min(l_k, \bar{t} - 1). \tag{7}$$

The constraint to prevent concurrent observations over sensor i's scheduling horizon is

$$\sum_{k \in \mathcal{K}} \delta_{ik\bar{t}} \leq 1 - \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{C}(k, \bar{t})} \delta_{ik\bar{t}}, \forall \bar{t} \in \mathcal{T}, i \in \mathcal{I}. \tag{8}$$

This constraint prohibits scheduling an activity k at time $\bar{t}$ if another activity was scheduled "recently", where that refers to some time $t \in \mathcal{C}(k, \bar{t})$.

The right hand side of equation (8) is closely related to the definition of a gap in a sensor schedule. Notably, if we define the following

$$g_{i\bar{t}} := 1 - \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{C}(k, \bar{t})} \delta_{ik\bar{t}}, \bar{t} \in \mathcal{T}, i \in \mathcal{I} \tag{9}$$

then on sensor i whenever no activities are scheduled for a given time $t \in \mathcal{C}(k, \bar{t})$ or whenever an activity starts at time $\bar{t}$, $g_{i\bar{t}} = 1$. In light of this, we can define a schedule gap for sensor i at time step $\bar{t}$ as:

$$g_{i\bar{t}} - \sum_{k \in \mathcal{K}} \delta_{ik\bar{t}}, \bar{t} \in \mathcal{T}, i \in \mathcal{I}. \tag{10}$$

Depending on the scheduling strategy, gaps can be incorporated into the scheduling model in one of two ways. Gaps can be prohibited with constraints. Alternatively, gaps can be penalized with a time-weighted term in the objective function to discourage gaps at certain times. There may be strategies where one or the other is preferred over the other. By making use of equation (10), we can propose an alternative objective function that penalizes gaps in any sensor's schedule, penalizes lower quality activities, and penalizes activities that don't make it on to any of the sensor schedules:

$$\min \quad \begin{array}{l} \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} \mathcal{Z}_{it}[g_{it} - \sum_{k\in\mathcal{K}} \delta_{ikt}] \\ + \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}[q^k_{\max} - q_{ikt}]\delta_{ikt} \\ + \sum_{k\in\mathcal{K}} q^k_{\max}[1-\omega_k]. \end{array} \tag{11}$$

Above, $\mathcal{Z}_{it}$ is the time-dependent penalty for sensor i and where $q^k_{\max}$ is the maximum possible quality for activity k, determined *a priori*.

2.4 PRIORITY AND QUALITY OBJECTIVE FUNCTION

The purpose of this section is to define an alternative to the objective functions given in Equations (1) and (11). The following objective function has been the focus of recent model analysis. Sensor operators and planners have worked extensively with priority-based scheduling of activities. A scheduling model that also includes measures of sensor performance that are activity and sensor dependent provides several potential scheduling enhancements. Depending on the nature of the activity and the customer requesting the activity, different combinations of measures of sensor performance can be used to define quality and their relative weightings can be adjusted by sensor experts and/or customers. By using quality thresholds, activities with large time windows, $span(e_k, l_k)$, can be scheduled according to time and sensor dependent quality. Subsequently, if a certain level of quality cannot be achieved, it represents an opportunity to prevent sensor resources from be wasted by producing observations that are unsatisfactory and instead scheduling an alternative activity that meets its respective quality threshold.

The goal of the objective function given below is to promote a schedule in which long duration, high priority activities are scheduled at times that result in high quality observations. Higher priority, higher quality, and longer duration activities are preferred over other scheduled activity arrangements.

$$\max \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} \frac{(\delta_{ikt})(p^k)(d_k)(q_{ikt})}{\sigma_n} \tag{12}$$

In Equation (12), the variable σ_n denotes a normalization constant. Let $\sigma_n := 100/(\sum_{k\in\mathcal{K}} p^k d_k q^*_k)$ so that an objective function value of 100 will denote a set of sensor schedules in which all activities are scheduled. As defined above, this normalization constant will depend on the set of *a priori* activities. Using the constraints defined above, together with the objective function in (12), we get the following mixed-integer program:

$$
\begin{array}{lll}
\max & \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\frac{(\delta_{ikt})(p^k)(d_k)(q_{ikt})}{\sigma_n} & \\
\text{s.t.} & \omega_k = \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\delta_{ikt} & \forall k\in\mathcal{K} \\
& \omega_k = 1 & \forall k\in\bar{\mathcal{K}} \\
& \omega_k \leq 1 & \forall k\in\mathcal{K}\setminus\bar{\mathcal{K}} \\
& \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} q_{ikt}\delta_{ikt} \geq q_k^{\min} & \forall k\in\bar{\mathcal{K}} \\
& \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} q_{ikt}\delta_{ikt} \geq q_k^{\min}\omega_k & \forall k\in\mathcal{K}\setminus\bar{\mathcal{K}} \\
& \sum_{k\in\mathcal{K}}\delta_{ik\bar{t}} \leq 1 - \sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{C}(k,\bar{t})}\delta_{ik\bar{t}}, & \forall\bar{t}\in\mathcal{T}, i\in\mathcal{I}. \\
& \delta_{ikt} = 0 & \forall i\in\mathcal{I}, k\in\mathcal{K}, \text{and } t < e_k \text{ or } l_k < t \\
& \delta_{ikt}\in\{0,1\} & \forall i\in\mathcal{I}, k\in\mathcal{K}, t\in\mathcal{T}
\end{array}
\tag{13}
$$

3 PRELIMINARY RESULTS

In this section, we provide preliminary results from realistic examples using the mixed-integer program of Equation 13. Models were coded in Python and made use of the Pyomo optimization modeling language. In general, the solver used to produce sensor schedules was the IBM CPLEX mixed-integer program solver. The solver ran on a shared Linux machine with 1TB of RAM and 64 cores. No special tuning of solver parameters was employed. Several experiments were run and are summarized in the table below. Each row represents preliminary results of a set of experiments described by:

- the number of sensors $\mathcal{I}$ to produce schedules for,
- the number of category 2 and category 3, $\mathcal{K}_2$ and $\mathcal{K}_3$ respectively, activities to be scheduled,
- the periodicity ω, periodicity variation, and duration d_k of the category 2 activities in time steps,
- the number of time steps to produce a schedule over $\mathcal{T}$,
- and the approximate wall clock time taken to compute a 95% optimal solution, or better, and the optimal solution. By a 95% optimal solution, we are referring to a gap between the solution and the optimal linear relaxation.

For reference, 1440 time steps is a 24-hour schedule discretized at minute resolution. For the experiments, the time durations of the category 2 activities were determined from a histogram of common activity durations for a set of earth observing sensors. A seeded random number generator was used to generate activity durations for each set of *a priori* activities so that solution times could be compared across different sets. It should be noted that for these preliminary

Table 1: Preliminary results for priority and quality based objective function.

$\mathcal{I}$	$\mathcal{K}_3$	$\mathcal{K}_2$	ω_2	$\pm$	d_k	$\mathcal{T}$	95% solution time	time to optimal
2	40	20	20	2	1	200	10-30s	30-120s
2	353	72	20	2	1	1440	50-180s	$>$ 12 hours
4	706	144	20	2	1	1440	180s	...

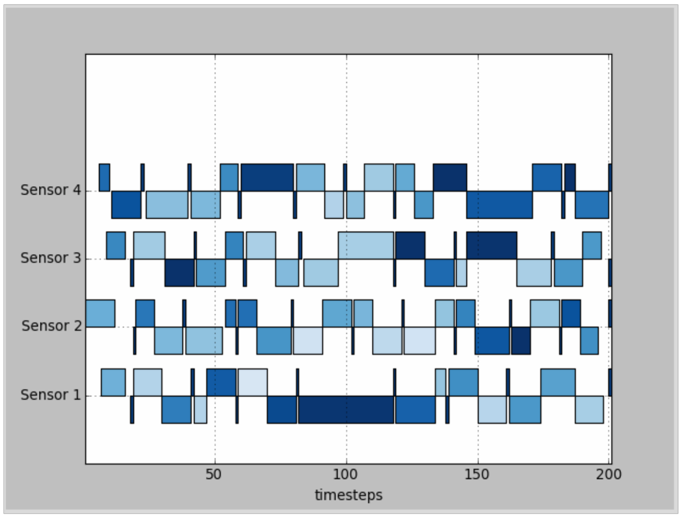

Figure 1: Example sensor schedule.

results, it was assumed that observation quality across the activities' time windows did not change. This may have introduced additional symmetry to the set of feasible solutions, possibly resulting in slower average solution times. An example schedule is depicted in Figure 1. Priority is denoted by the shade of blue in each of the activities. Darker blue denotes higher priority. Category 2 activities are colored black. Activities alternate above and below the sensor timeline to clearly show where one activity ends and the next begins. Schedule gaps are shown as space between activities.

4 STOCHASTIC MIXED-INTEGER LINEAR PROGRAM

Current research includes a focus on constructing a scenario-based stochastic mixed-integer program (SMIP) to address common areas of remote sensing uncertainty. Observation quality can be heavily impacted by weather at the location to be observed. Historical and future forecast data will be aggregated and studied, from a service such as [4], to build scenario distributions over areas of interest and across the globe. An additional use for a near-real time weather forecast service is to build quality values nearer to the time the sensor schedules are built.

Another source of uncertainty that that the scenario-based SMIP aims to address is caused by the effects of category 4 activities on remote sensor schedules. The planned approach will build scenarios for category 4 activities that have time windows that span longer than the scheduling horizon. The scenarios will assume that if a category 4 or set of category 4 activities need to be scheduled, they will displace any activity that was previously scheduled. It will be assumed that the duration of the category 4 activities will be known *a priori*. The goal will be to minimize the disruption in the overall schedule by placing activities according to the category 4 probability distributions.

5 FUTURE WORK

There are many opportunities for continued research of these remote sensor scheduling models. A thorough sensitivity and scalability analysis of solution time with respect to activity time window, time window overlap, priority, duration, and quality will help establish the utility of the approach. Further interaction with remote sensor operators and planners as well as continued use of activity distributions from a variety of remote sensors will guide research into additional types of constraints, quality functions for different observation activities, and development of the stochastic program discussed in Section 4. Distributions for weather and *ad hoc* activity scenarios will need to be derived. Finally, a large amount of work will be focused on studying optimal time horizons over which a schedule should be constructed and the effects on schedule optimality stemming from the associated period of rescheduling.

References

[1] W. E. Hart, C. Laird, J.-P. Watson, and D. L. Woodruff, *Pyomo–optimizaiton modeling in python*, vol. 67. Springer Science & Business Media, 2012.

[2] W. E. Hart, J.-P. Watson, and D. L. Woodruff, "Pyomo: modeling and solving mathematical programs in python," *Mathematical Programming Computation*, vol. 3, no. 3, pp. 219–260, 2011.

[3] A. Globus, J. Crawford, J. Lohn, and A. Pryor, "A comparison of techniques for scheduling earth observing satellites," in *AAAI*, pp. 836–843, 2004.

[4] Forecast.io, "Forecast.io." Online; accessed 4-September-2015, 2015.