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Theory of using magnetic deflections to combine charged particle beams into the I^3TEM through a single port

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Theory of using magnetic deflections to combine charged particle beams into the I³TEM through a single port

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Abstract

Several radiation effects projects in the Ion Beam Laboratory (IBL) have recently required two disparate charged particle beams to simultaneously strike a single sample through a single port of the target chamber. Because these beams have vastly different mass-energy products (MEP), the low MEP beam requires a large angle of deflection toward the sample by a Colutron bending electromagnet. A second steering electromagnet, preceding the first along the beamline, provides a means to compensate for the small angle of deflection experienced by the high MEP beam as it travels through the bending magnet. This paper derives the equations used to select the magnetic fields required by these two magnets to successfully unite both beams at the target sample. A simple result was obtained when the separation between the two electromagnets was equivalent to the distance between the bending magnet and the TEM sample . This result is summarized by the equation: $B_s = \frac{1}{2}\left(\frac{r_c}{r_s}\right)B_c$, where B_s and B_c are the magnetic fields in the steering and bending magnet and $\frac{r_c}{r_s}$ is the ratio of the radii of the bending magnet to that of the steering magnet. This result is not dependent upon the parameters of the high MEP beam, i.e. energy, mass, charge state. Therefore, once the field of the bending magnet is set for the low MEP beam, and the field in the steering magnet is set as indicted in the above equation, the trajectory path of any high MEP beam will be directed into the TEM sample.

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CONTENTS

FIGURES

NOMENCLATURE

IBL	Ion Beam Laboratory at Sandia National Laboratories
QASPR	Qualification Alternatives to the Sandia Pulsed Reactor
I^3TEM	*In Situ* Ion Irradiation TEM
Colutron	an accelerator which provides gas ions up to 10 keV
Tandem	a two-stage accelerator providing any ion at energies of several MeV
TEM	Transmission Electron Microscope, a JOEL-2100
m_c, E_c, q_c	mass (amu), energy (MeV), charge state (unitless) of Colutron ion
$MEP_c = \frac{m_c E_c}{q_c^2}$	mass-energy product of Colutron ion
B_c, r_c	magnetic field (kG) and radius (.038*m*) of the Colutron bending magnet,
R_c , θ_c	radius of curvature (*m*), and deflection angle of Colutron ion
$R_c = \frac{\sqrt{2m_c E_c}}{B_c q_c}$	key equation for the Colutron ion trajectory in Colutron magnet
m_t, E_t, q_t	mass, energy and charge of the Tandem ion
$MEP_t = \frac{m_t E_t}{q_t^2}$	mass-energy product of Tandem ion
B_s, r_s	magnetic field and radius (.029*m*) of the steering magnet
R_{ts}, θ_{ts}	radius of curvature (*m*), and deflection angle of Tandem ion in steering magnet
R_{tc}, θ_{tc}	radius of curvature (*m*), and deflection angle of Tandem ion in Colutron magnet
$R_{tc} = \frac{\sqrt{2m_t E_t}}{B_c q_t}$	key equation for the Tandem ion trajectory in Colutron magnet
$R_{ts} = \frac{\sqrt{2m_s E_t}}{B_s q_t}$	key equation for the Tandem ion trajectory in steering magnet
r_T, B_T	radius (*m*) and magnetic field (kG) in TEM

1. INTRODUCTION

Two radiation effects projects in the Ion Beam Laboratory (IBL) at Sandia have recently required two disparate charged particle beams to simultaneously strike a sample through a single port of the end-station. The first project combines multi MeV-energy ions from the IBL Tandem Van de Graaff Accelerator with 100 keV electrons to simulate neutron and gamma hostile radiation effects on semiconductor devices as part of the Qualification Alternatives to the Sandia Pulsed Reactor (QASPR) program [1]. The second project is the new *In Situ* Ion Irradiation Transmission Electron Microscope (I^3TEM) system [2,3] which combines multi keV-energy $^4He^+$ and $^2D_2^+$ beams accelerated by a Colutron, with the multi MeV-energy heavy ion beams from the IBL Tandem accelerator. The research performed on this second system is directed at understanding the underlying defect physics, metallurgy, and materials science of the aging of metals exposed to high neutron fluences in future advanced fission and fusion nuclear reactors.

For this report, we concern ourselves with the I^3TEM only and derive the ion-optics theory and equations for how to set the magnetic fields in the two electromagnets used to bend and steer the Colutron and Tandem ions to the TEM target sample. Using similar principals, we derive where to aim the Colutron beam in order to compensate for the steering effect of the TEM magnetic field on the Colutron ions and ultimately direct the beam onto the sample. This approach, and the equations derived herein, will apply to any situation in which high and low MEP beams must be combined.

2. THE I^3TEM SYSTEM

2.1 The magnets

The I^3TEM System is shown schematically in Figure 1. Ion beams from the Colutron are accelerated to a maximum energy of $10q_c$ keV, where q_c is the ion charge state, and bent 20^o toward the JEOL-2100 TEM with an electromagnet. The ion beam from the Tandem enters the system along the TEM axis. However, if the bending electromagnet of the Colutron is energized, the Tandem beam is slightly deflected and misses the TEM sample. The steering magnet shown in the schematic is therefore used to offset the trajectory of the Tandem beam from the axis of the Colutron magnet and ultimately deflect the ions in the direction of the TEM target. In addition to these two magnets, there is a strong, constant magnetic field around the sample produced by the objective lens in the TEM. This magnetic field affects the trajectory of the Colutron beam. Therefore, the Colutron beam must be offset from the axis of the TEM such that the circular trajectory of the ions in this magnetic field directs them to the center of the sample.

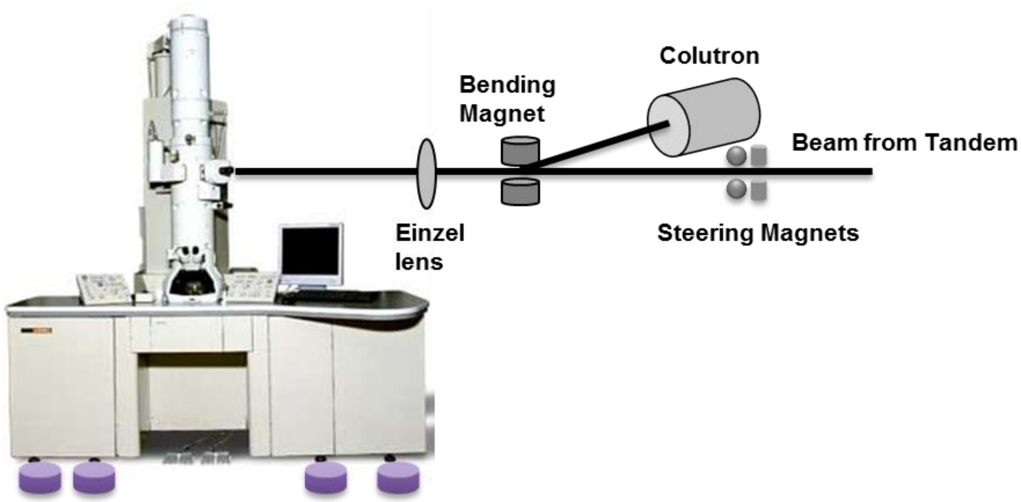

Figure 1 The I³TEM joins a multi-MeV-energy heavy ion beam from the IBL Tandem with a multi-keV-energy ion beam from the Colutron.

2.2 The beams

The most typical beams from the Colutron are keV-energy $^{4}He^{+}$ and $^{2}D_{2}^{+}$. These beams simulate the effects of neutron-induced transmutation through (n,α) and (n,p) nuclear reactions in metals used in fission and fusion nuclear reactors exposed to enormous fluxes of high energy neutrons during use. The energy of these ions is selected so the ions range out in the electron transparent TEM sample, which is typically only 100*nm*. Energies <10 keV are therefore commonly used. Deuterium ions were selected instead of protium (i.e. ^{1}H) to match the MEP of the $^{4}He^{+}$. Because of this matching beam rigidity, both beams are steered identically by the Colutron bending magnet and consequently act identically in the TEM magnet field.

The Tandem beam has a much higher MEP than that of the Colutron because the Tandem ions are typically several MeV and are much heavier, up to ^{197}Au. These ions pass directly through the TEM sample and produce atomic displacements along their trajectory. This simulates neutron-induced defect formation in metals of interest for fission and fusion reactors. After all, it is the neutron-induced recoil atoms, called the primary knock-on atoms, in the metal that cause the collision cascades resulting in virtually all of the displaced atoms in the metal.

By adjusting the intensities and energies of each of the three beams (two from the Colutron and one from the Tandem) to have all three concurrently intersect the target sample, one can observe nanoscopic changes in the sample morphology both in real time and *in situ* with the TEM. The I^{3}TEM facility therefore creates in macrocosm the harsh radiation environments experienced by materials used in nuclear reactors, and the TEM enables the observation and documentation of the effects of this radiation.

3. THE MAGNET PROBLEMS

The problems we will solve in the theory section of this report are: What are the magnetic fields required in 1) the Colutron bending magnet and 2) the Tandem steering magnet to set all beams on trajectories to hit the center of the TEM target sample? How do these fields account for the ion bend effect of the magnetic field in the TEM?

3.1 The Colutron magnet

As indicated above, the Colutron bending magnet is required to bend Colutron beams by 20^{o} to direct them toward the TEM sample. Beams must be aimed slightly away from the sample center to compensate for the change in trajectory within the TEM due to the strong objective lens magnetic field. This circular pole magnet was manufactured at Sandia, and has a radius of 0.038*m*. In the remainder of this report, we will refer to this magnet as the Colutron magnet, and use the subscript "c" to denote accelerated Colutron beams and Colutron magnet parameters such as magnetic field and radius.

3.2 The steering magnet

Once the Colutron magnetic field is established, we must determine the field in the steering magnet (NEC model 2EA038100, effective radius=0.029*m*) [4] required to accurately offset the Tandem beam from the axis of the Colutron magnet. This offset will allow the Tandem beam to be deflected by the Colutron magnet into the TEM sample. There is a restriction placed on the size of this offset by the 0.0127*m* inside radius of the Einzel lens located directly after the Colutron magnet along the beamline. This restriction sets a lower limit on the MEP of Tandem beams that are steerable to the sample. Before entering the magnetic field of the Colutron magnet, the Tandem beam is first bent 45^{o} by a switching magnet. This switching magnet places an upper limit on the MEP of the Tandem beam because the large angle of deflection is only effective on beams with MEP<35 MeVamu/q^2. For the remainder of this paper, we refer to the magnet preceding the Colutron magnet along the Tandem beamline as the steering magnet, and use the subscript "s" to denote the parameters of this magnet. A subscript "t" is used for the parameters of the Tandem beam.

3.2 The TEM magnet

The Colutron beam cannot be aimed directly at the TEM sample because the TEM objective lens emits a magnetic field. Because the Colutron ions have a small mass and energy, this field deflects the Colutron beam away from the target sample. Therefore, in order to hit the sample, the Colutron beam must be offset from the axis of the TEM to account for this internal deflection. For the purposes of this paper, we use a magnetic field of 10 kG and an effective magnet radius of 0.02*m*. We anticipate that the magnetic field in the TEM, while quite high, will have little effect on the Tandem ions due to their high MEP, and this is borne out below. In the remainder of this report, the subscript "T" will indicate parameters of the TEM magnet.

4. THEORY

The first part of this section derives a theory for the equations of motion of both the Colutron and Tandem beams as they pass through the Colutron and steering magnets. The second part derives equations for the offset of these beams from the axis of the TEM to account for the change in motion of ions caused by the TEM magnetic field.

4.1 Assumptions

This section documents the assumptions and simplifications employed in this paper. We assume that all three magnets have circular poles, and that the magnetic fields have hard edges, i.e. these fields are constant within the radii of these magnets and drop to zero outside these radii. The algebra is significantly simplified by 1) assuming the separation between the steering magnet and Colutron magnet is the equal to the separation between the Colutron magnet and the TEM, and 2) that the deflection of the Tandem beam, while very important, is quite small.

4.2 Colutron ions through Colutron magnet

The equation for the radius of curvature of a Colutron ion in the Colutron magnetic field results from equating the Lorentz force on the ion moving in a perpendicular magnetic field with the centripetal force acting on the ion. This is given by:

$$R_c = \frac{\sqrt{2m_c E_c}}{B_c q_c} \tag{4.1}$$

where R_c is the radius of curvature in meters, B_c is the magnetic field in kG, r_c is the radius of the Colutron magnet (0.038*m*) and $m_c, E_c,$ and q_c are equal to the mass (amu), energy (MeV), and charge state (unitless) of the Colutron ion. A diagram of the trajectory of the Colutron ion from the source, through the magnet, and into the TEM is shown in Figure 2.

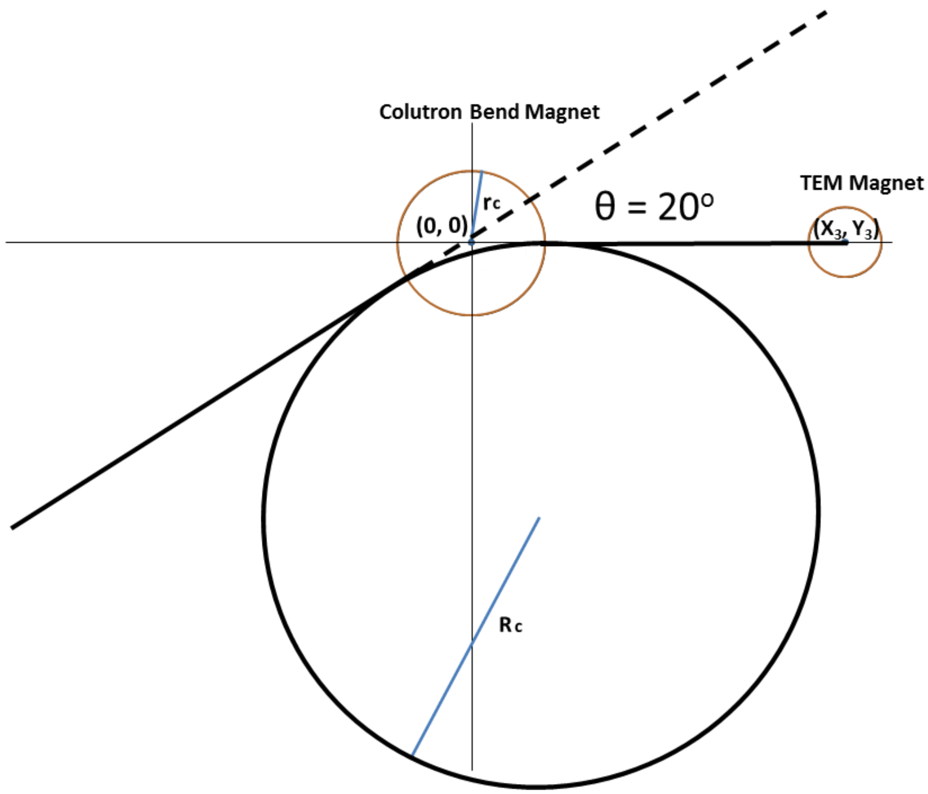

Figure 2 Trajectory of Colutron ion through Colutron magnet

For this, and future calculations, the Colutron magnet is placed at (0, 0) in the x,y coordinate system, and the TEM axis and sample position is at (x_3, y_3). The radius of the circular magnet is r_c. It is straightforward to show that the angle of ion deflection is given by:

$$\tan\frac{\theta_c}{2} = \frac{r_c}{R_c} \tag{4.2}$$

where, in this context, θ_c is 20^o. Rearranging and solving for the field in the Colutron magnet gives:

$$B_c = \frac{\tan\frac{\theta_c}{2}\sqrt{2m_c E_c}}{r_c q_c}. \tag{4.3}$$

For example, the magnetic field required to bend a 10 keV $^4He^+$ ion twenty degrees is 1.3 kG.

4.3 Tandem ions through steering and Colutron magnets

The trajectory of the Tandem ion as it passes through the steering magnet and the Colutron magnet to hit the TEM sample is shown in Figure 3.

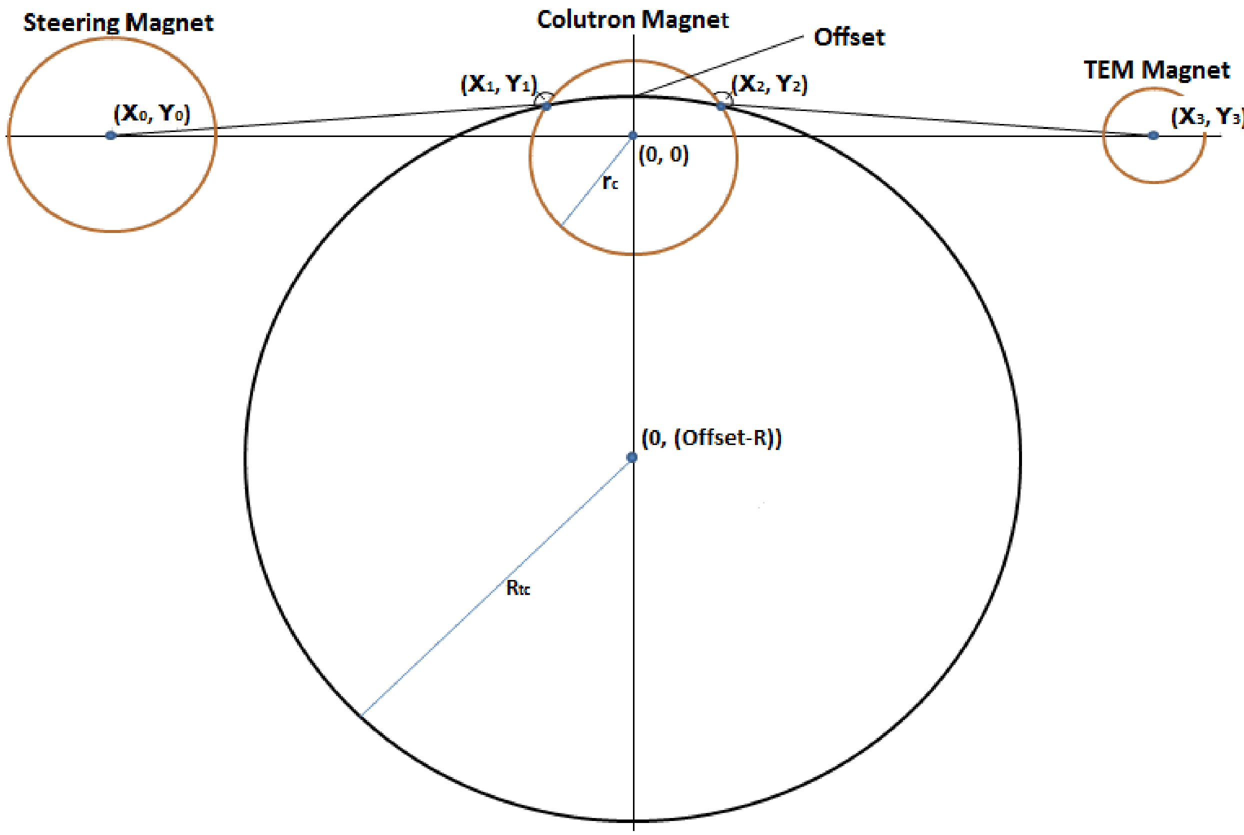

Figure 3 The geometry of Tandem ion trajectory to the TEM target at (x_3, y_3). The steering magnet is at (x_0, y_0). The beam enters the Colutron magnet at (x_1, y_1) and exits at (x_2, y_2), continuing on to the TEM. It has a radius of curvature in the magnet of R_{tc}.

The distance from the steering magnet to the Colutron magnet is 1.2m, and the distance from the Colutron magnet to the TEM is 1.5m. Approximating symmetry for this system, we average these two separations where $x_0 = -1.35m$ and $x_3 = 1.35m$.

4.3.1 Steering of the Tandem beam by the Colutron magnet

In this context, we assume the following:

- Symmetry where $x_0 = -x_3$
- $R \gg r$, and therefore the Offset $\cong y_1$
- Time reversal symmetry results in $y_1 = y_2$ and the entry and exit angles of the ion in the Colutron magnet are identical. Therefore, because $x_0 = -x_3$, the ions exiting the Colutron magnet are set on a trajectory path to hit the center of the TEM sample.

4.3.1.1 Equations of the straight line parts of the trajectory

The equation of the linear trajectory of the Tandem ion from the steering magnet to the Colutron magnet entry point is given by:

$$y = y_1 \frac{(x - x_0)}{(x_1 - x_0)} \quad . \tag{4.4}$$

4.3.1.2 Equations of the circular part of the trajectory through the Colutron magnet

The equation of motion (an arc of a circle) of the Tandem beam through the Colutron magnet, where subscript "tc" denotes Tandem beam in the Colutron magnet, is given by:

$$x^2 + (y - (Offset - R_{tc}))^2 = R_{tc}^{\ 2} \quad . \tag{4.5}$$

Applying the approximation: Offset = y_1:

$$x^2 + (y - (y_1 - R_{tc}))^2 \approx R_{tc}^{\ 2} \tag{4.6}$$

where R_{tc} is the radius of curvature of a Tandem ion as it travels through the Colutron magnetic field, and is equal to:

$$R_{tc} = \frac{\sqrt{2m_t E_t}}{q_t B_c}. \tag{4.7}$$

The path of an ion as it just enters and travels through the Colutron magnetic field must be smooth with respect to its linear trajectory before it enters the magnet. Therefore, the slope of the line given in Eq. 4.4 must be equal to the first derivative, dy/dx, of the equation of motion of the Tandem ion in the magnet given by Eq. 4.6, both evaluated at $\left(x_1, y_1\right)$.

Taking the derivative of Eq. 4.4 to find the slope of the Tandem ion trajectory line at $\left(x_1, y_1\right)$ we find:

$$\frac{dy}{dx} line \Rightarrow \frac{dy}{dx}\Big| x_1, y_1 = \frac{y_1}{(x_1 - x_0)} \quad . \tag{4.8}$$

Then taking the derivative of Eq. 4.6 to find the slope of the circular trajectory and evaluating at $\left(x_1, y_1\right)$ we get:

$$\frac{dy}{dx} circle \Rightarrow 2x + 2(y - y_1 + R_{tc}) \frac{dy}{dx} = 0 \tag{4.9}$$

$$\frac{dy}{dx}\Big| x_1, y_1 = -\frac{x_1}{R_{tc}} \quad . \tag{4.10}$$

Setting the slopes (Eq. 4.8 and Eq. 4.10) equal to determine a smooth Tandem ion trajectory path as it enters and travels through the Colutron magnet, and solving in terms of x_1 and y_1, i.e., the Offset, yields:

$$\frac{y_1}{(x_1 - x_0)} = -\frac{x_1}{R_{tc}} \quad . \tag{4.11}$$

Rearranging:

$$R_{tc} y_1 + x_1^{\,2} - x_0 x_1 = 0 \tag{4.12}$$

and dividing by $R_{tc}^{\,2}$:

$$\frac{y_1}{R_{tc}} + \frac{x_1^{\,2}}{R_{tc}^{\,2}} - \frac{x_0 x_1}{R_{tc}^{\,2}} = 0. \tag{4.13}$$

We can omit the term that is second order in $\frac{x_1}{R_{tc}}$ because it is <<1, and find:

$$y_1 = \frac{x_0 x_1}{R_{tc}}. \tag{4.14}$$

Using the equation for the Colutron magnet perimeter, x_1 and y_1 can be expressed in the equation:

$$x_1^{\,2} + y_1^{\,2} = r_c^{\,2} \quad . \tag{4.15}$$

Therefore:

$$x_1 = \sqrt{r_c^{\,2} - y_1^{\,2}} \tag{4.16}$$

and substituting this value for x_1 into Eq. 4.14, we find:

$$y_1 = \frac{x_0}{R_{tc}} \left(\sqrt{r_c^{\,2} - y_1^{\,2}} \right) \quad . \tag{4.17}$$

Solving for y_1, i.e. the offset of the Tandem beam from the axis of Colutron magnet:

$$y_1 = \frac{r_c}{\sqrt{\left(\frac{R_{tc}}{x_0} \right)^2 + 1}} \quad . \tag{4.18}$$

We can then apply this equation to solve for the lower Tandem beam mass-energy product (MEP) limit as a function of the magnetic field in the Colutron magnet by replacing R_{tc} with its value in Eq. 4.7:

$$y_1 = \frac{r_c}{\sqrt{\frac{2m_t E_t}{x^2{}_0 B^2{}_c q^2{}_t} + 1}} \quad . \tag{4.19}$$

To check the accuracy of the assumptions and simplifications made above, an exact solution for the Offset of the Tandem beam at the Colutron magnet was obtained and the results are compared in Figure 4.

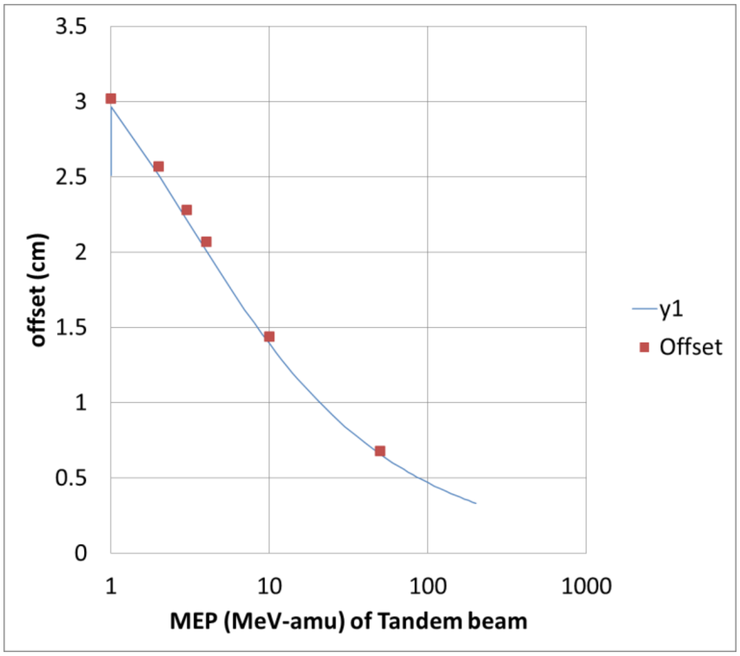

Figure 4 Exact (Offset) vs. analytical (y_1) offsets for Tandem beams thru Colutron magnet set for 10keV $^4He^+$ B = 1.3kG

Solving for the MEP of the Tandem beam, i.e. $\left(\frac{mE}{q^2}\right)_t$, in Eq. 4.19 gives:

$$\left(\frac{mE}{q^2}\right)_{tc} = \frac{B_c{}^2 x_0{}^2}{2}\left[\left(\frac{r_c}{y_1}\right)^2 - 1\right]. \tag{4.20}$$

As mentioned above, the Einzel lens presents an aperture with a 0.0127m radius along the beamline just as the Tandem ions exit the Colutron magnet. This aperture limits $y_1 < 0.0127m$. Therefore, any Tandem beam that requires an offset greater than 0.0127m to overcome the bend effect of the magnetic field in the Colutron magnet will not hit the TEM sample. The lower MEP limit on the Tandem beam can be calculated in Eq. 4.20 as a function of the magnetic field in the Colutron magnet. For example, and as derived above in Section 4.2, a 10 keV $^4He^+$ beam requires a magnetic field of 1.3 kG in order to bend 20° into the TEM. As discussed above, we are using $|x_0| = 1.35m$. When substituting these values into Eq. 4.20, and including the upper limit of 35 MEP placed on the Tandem beam by the 45° magnet, we find:

$$13 < \left(\frac{mE}{q^2}\right)_{tc} < 35 \ . \tag{4.21}$$

Therefore, any Tandem beam with a MEP below 13 will not be properly bent by the Colutron magnet when set for 10 keV $^4He^+$ and will ultimately miss the center of the TEM.

Eq. 4.20 can be further modified to solve for the lower Tandem MEP limit as a function of Colutron ion energy. Inserting the value of B_c in Eq. 4.3 into Eq. 4.20, we find:

$$\left(\frac{mE}{q^2}\right)_{tc} = \tan^2\frac{\theta_c}{2}\frac{x_0^{\ 2}}{r_c^{\ 2}}\left[\left(\frac{r_c}{y_1}\right)^2 - 1\right]\frac{m_c E_c}{q_c^{\ 2}} \ . \tag{4.22}$$

This proves that the lower MEP limit of the Tandem beam scales linearly with the energy and MEP of the Colutron beam directed into the TEM. As the MEP of the Colutron beam decreases, so does the strength of the magnetic field required to bend the Colutron ions into the TEM. This, in turn, proportionally lowers the MEP lower limit of the Tandem beam. For example, if the energy of the $^4He^+$ beam is reduced to 5 keV, the Tandem beam MEP $\left(\frac{mE}{q^2}\right)_{tc}$ can then be > 6.5, half the value calculated in Eq. 4.20.

4.3.1 Steering of the Tandem beam by the steering magnet

We can use the setting of the Colutron magnet to determine the magnetic field required in the steering magnet. The form of Eq. 4.3 can be adapted in terms of the Tandem ion in the steering magnet:

$$B_s = \frac{\tan\left(\frac{\theta_s}{2}\right)\sqrt{2m_t E_t}}{q_t r_s} \ . \tag{4.23}$$

Where r_s is the radius of the steering magnet (0.029*m*). The slope of the ion trajectory path as it exits the steering magnet can be written as:

$$\frac{dy}{dx} = \tan\left(\theta_s\right) \ . \tag{4.24}$$

When we set this value equal to the calculated slope in Eq. 4.8 we find:

$$\tan(\theta_s) \cong \frac{y_1}{(x_0 - x_1)} \cong \frac{y_1}{x_0} \ . \tag{4.25}$$

This approximation is viable because the steering magnet is small compared to its separation from the Colutron magnet, i.e. $x_1 << x_o$. Then using the expression in Eq. 4.18 for y_1, and $\tan\left(\frac{\theta_s}{2}\right) \cong \frac{1}{2}\tan(\theta_s)$ because the deflection of the steering magnet is always very small, we find:

$$\tan\left(\frac{\theta_s}{2}\right) \cong \frac{r_c}{\sqrt{\left(\frac{R_{tc}}{x_0}\right)^2 + 1}} \left(\frac{1}{2x_0}\right) . \tag{4.26}$$

Which we then substitute this expression for $\tan\left(\frac{\theta_s}{2}\right)$ into Eq. 4.23:

$$B_s = \frac{r_c}{\sqrt{\left(\frac{R_{tc}}{x_0}\right)^2 + 1}} \left(\frac{1}{2x_0}\right) \left(\frac{2m_t E_t}{r_s^2 q_t^2}\right)^{1/2} . \tag{4.27}$$

Squaring both sides:

$$B_s^2 = \frac{r_c^2}{\left(\frac{R_{tc}}{x_0}\right)^2 + 1} \left(\frac{1}{4x_0^2}\right) \left(\frac{2m_t E_t}{r_s^2 q_t^2}\right) . \tag{4.28}$$

In this context we can assume $\frac{R_{tc}}{x_0} \gg 1$, therefore the "1" term can be omitted from the denominator in Eq. 4.28.

Squaring and rearranging Eq. 4.7 yields:

$$\frac{2m_t E_t}{q_t^2} = R_{tc}^2 B_c^2 \tag{4.29}$$

and substituting this expression into Eq. 4.28:

$$B_s^{\,2} = \frac{r_c^{\,2}}{\left(\frac{R_{tc}}{x_0}\right)^2}\left(\frac{1}{4x_0^{\,2}}\right)\left(\frac{R_{tc}^{\,2}B_c^{\,2}}{r_s^{\,2}}\right) . \qquad (4.30)$$

Simplifying we find:

$$B_s^{\,2} = \left(\frac{r_c}{r_s}\right)^2 \left(\frac{B_c}{2}\right)^2 \qquad (4.31)$$

and finally we take the square root of Eq. 4.31 to solve for the magnetic field in the steering magnet as a function of the magnetic field in the Colutron magnet:

$$B_s = \frac{1}{2}\left(\frac{r_c}{r_s}\right)B_c \ . \qquad (4.32)$$

This simple equation indicates that the magnetic field in the steering magnet required to accelerate the Tandem beam thru the Colutron magnet and into the TEM sample depends only upon 1) the magnitude of the field in the Colutron magnet and 2) the dimensions of the steering and Colutron magnets. Of particular note, the required magnetic field in the steering magnet is not dependent upon the MEP of the Tandem beam. Therefore, once the magnetic field in the Colutron magnet is set to bend a beam into the TEM, the steering magnetic field can then be set according to Eq. 4.32 such that any beam from the Tandem will automatically be directed into the TEM.

4.4 Effect of the TEM magnet on the Colutron and Tandem beams

The trajectory path of an ion entering and inside the magnetic field of the TEM is shown in Figure 5. In this coordinate system, the TEM magnet is centered at (0, 0) with a radius of $r_{T.}$ We assume here that the linear trajectory of the ion is parallel to the x-axis of the coordinate system, and is offset by y_1. This assumption considerably simplifies the determination of the equation of motion of an ion in the TEM magnetic field. The center of this circular trajectory is then determined to be at (0, y_1-R_T), where R_T is the radius of curvature of the ion in the magnetic field of the TEM.

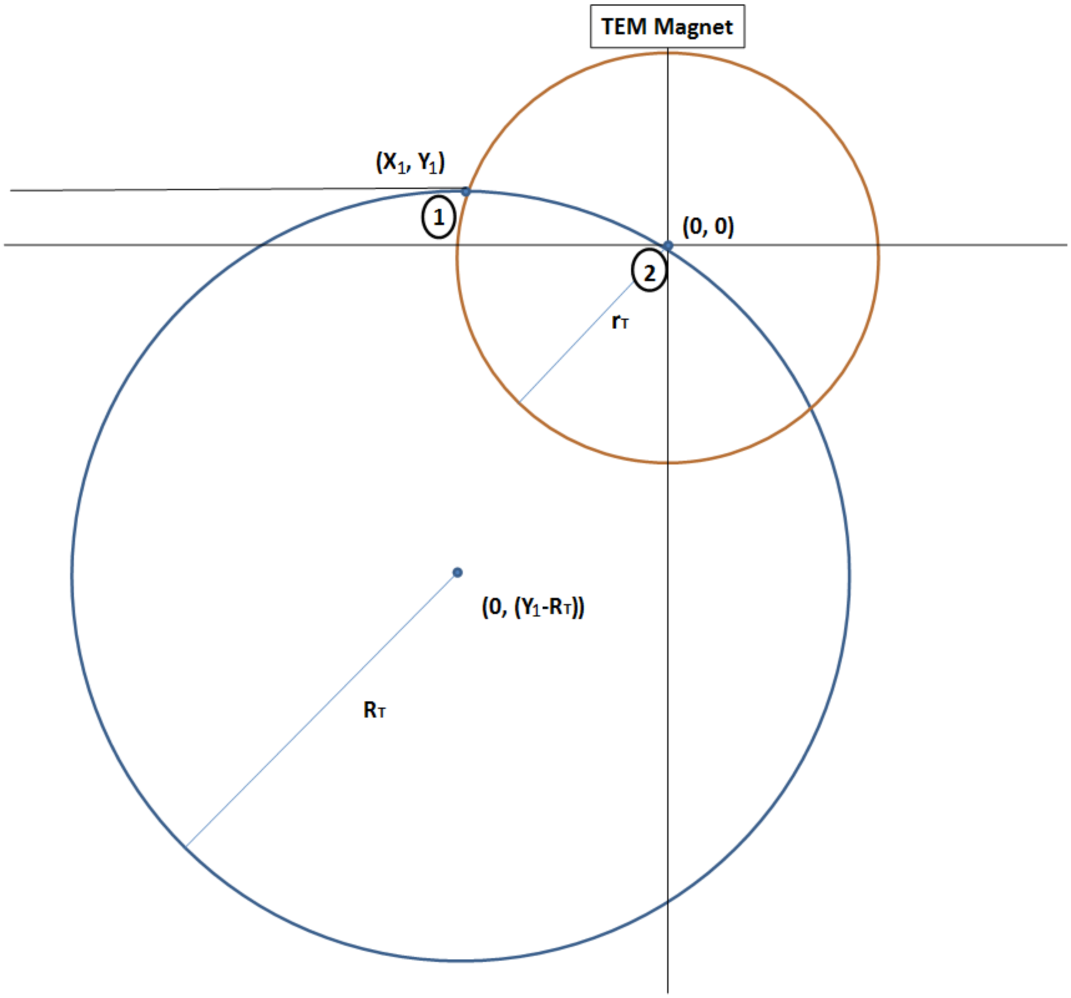

Figure 5 Trajectory of an ion in the magnetic field of the TEM.

The position of the ion beam entry into the magnet (Point 1) can be related to the equation of the circular perimeter of this magnet, and is given by:

$$x_1^{\,2} = r_T^{\,2} - y_1^{\,2} \ . \tag{4.33}$$

The equation of motion (the arc of a circle) of an ion beam through the TEM is given by:

$$(x - x_1)^2 + (y - (y_1 - R_T))^2 = R_T^{\,2} \ . \tag{4.34}$$

Solving for x_1 and y_1 at the position of the sample at Point 2 (0, 0) in the diagram above we find:

$$x_1^{\,2} + (y_1 - R_T)^2 = R_T^{\,2} \ . \tag{4.35}$$

We can then solve for y_1 (Offset) by substituting the expression for $x_1^{\,2}$ in Eq. 4.33 into Eq. 4.35:

$$y_1 = \frac{r_T^{\ 2}}{2R_T} \ . \tag{4.36}$$

The radius of curvature of an ion in the TEM magnet is:

$$R_T = \frac{\sqrt{2mE}}{B_T q} \ . \tag{4.37}$$

Therefore the Offset, y_1, by which the ions must be displaced upon entering the magnetic field of the TEM in order to strike the sample at (0, 0) is:

$$y_1 = \frac{r_T^{\ 2}}{2} \frac{B_T q}{\sqrt{2mE}} \ . \tag{4.38}$$

4.4.1 MEP limits to Colutron beam

As mentioned above, the magnetic field in the TEM is ~10 kG and has an effective radius of 0.02*m*. Therefore, the y_1 offset for 10 keV $^4He^+$ is 0.007*m*, or 7*mm*. In contrast, the offset for the highest MEP Tandem beam that can be steered toward the TEM from the 35 MeV-amu switching magnet is only 0.0002*m*, or 0.2*mm*. For light, lower energy Tandem beams with a MEP of 1 MeV-amu, the offset is 0.0014*m*, or 1.4*mm*. Both of these deflections are less than the standard 3*mm* diameter of TEM targets, therefore, Tandem ions will still strike the sample regardless of the TEM magnetic field. This validates our initial assumption, and observations by the I^3TEM users, that the TEM magnet steering effect is important for the Colutron beams, but has little effect on the Tandem beams.

The question then becomes: what is the lower Colutron MEP beam limit such that the Colutron ions can still be steered to the TEM sample? The trajectory of such an ion would arrive tangential to the sample. Therefore, we must solve for the limit where the ion has an unbounded slope tangential to the top surface of the TEM sample. Taking the derivative of Eq. 4.34 and setting it equal to infinity we find:

$$\frac{dy}{dx} circle \Rightarrow 2(x - x_1) - 2(y - y_1 + R_T)\frac{dy}{dx} = \infty \ . \tag{4.39}$$

Solving for dy/dx at the sample position, (0, 0):

$$\frac{dy}{dx} = \frac{x_1}{y_1 - R_T} \rightarrow \infty \ . \tag{4.40}$$

Or upon inverting this equation:

$$\frac{dx}{dy} = \frac{y_1 - R_T}{x_1} \rightarrow 0 \ . \tag{4.41}$$

Therefore, an ion trajectory tangential to the target will result when:

$$y_1 = R_T = \frac{\sqrt{2m_c E_c}}{B_T q_c} \quad . \tag{4.42}$$

Substituting this value of y_1 into Eq. 4.38 and solving for the MEP of the Colutron beam:

$$MEP_{\text{minimum}} = \frac{m_c E_c}{q_c^{\ 2}} \geq \frac{B_T^{\ 2} r_T^2}{4} \quad . \tag{4.43}$$

Beams must obey this limit or the ions will not hit the TEM sample. For the Sandia I^3TEM system, B = 10 kG and r_T = 0.02*m*. Therefore, the minimum MEP beam that can be steered by the TEM magnetic field to the sample is $1\text{x}10^{-2}$ MeV-amu, or 10 keV-amu. This limits Colutron beams that can be accelerated to 2.5 keV D_2^+ and ${}^4He^+$, 5 keV D^+ and 10 keV H^+. One should therefore follow these limits when designing experiments for the I^3TEM system, and never plan to use H^+ with the TEM magnet at full strength.

4.4.2 Angle of incidence of Colutron beam on tilted TEM sample

It is important to know the angle of incidence of the Colutron beam with respect to the surface normal of the TEM target in order to determine the concentration depth profile of the Colutron ions implanted into the target. Figure 6 shows the trajectory of these ions as they enter the magnetic field of the TEM where they take a circular trajectory as shown in Figure 5 with a radius of curvature R_T given in Equation 4.37 and striking the sample in the xy plane with an angle of incidence of α. The TEM samples must be rotated about the y axis so that the ions can enter the target and at the same time perform microscopy with the TEM electrons. This angle is defined as β in Figure 6, and can be varied by the operator for most samples up to 30^o. The combined effect of the beam's incident angle α in the xy plane and this target rotation β which is in the xz plane complicates the determination of the effective angle of incidence δ that should be used in the determination of implantation fluences and resulting depth concentration profiles.

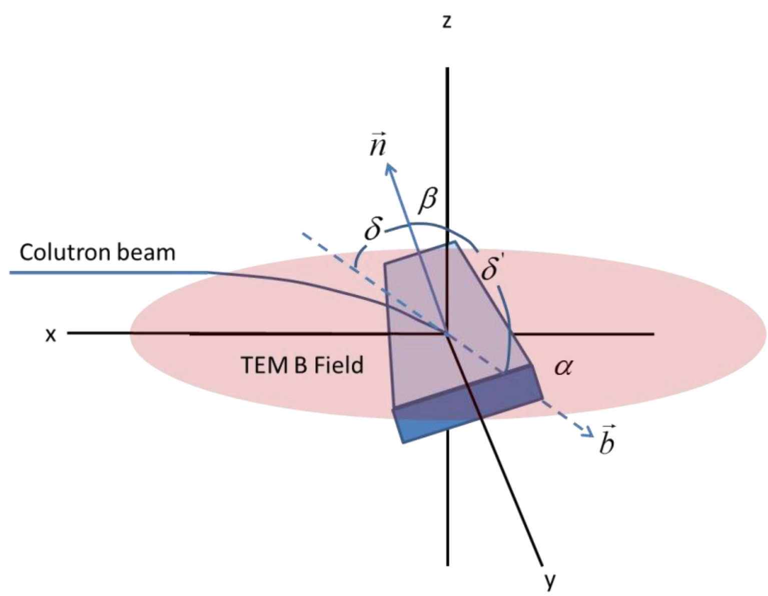

Figure 6 Trajectory of Colutron ion striking TEM sample tilted at an angle

As derived above, the slope of the Colutron beam's trajectory as it enters the TEM sample is given by:

$$\frac{dy}{dx} = \frac{x_1}{y_1 - R_{cT}} \tag{4.44}$$

Where R_{cT} is the radius of curvature of the Colutron ion, and is given by:

$$R_{cT} = \frac{\sqrt{2m_c E_c}}{B_T q_c} \ . \tag{4.45}$$

Using Equations 4.33 and 4.36 above and a little algebra, the beams incident angle in the xy plane is related to this slope by:

$$\alpha = \tan^{-1}\left(\frac{dy}{dx}\right) = \tan^{-1}\left(\frac{r_T}{R_{cT}}\sqrt{\frac{1+\frac{1}{2}\left(\frac{r_T}{R_{cT}}\right)^2}{1-\frac{1}{2}\left(\frac{r_T}{R_{cT}}\right)^2}}\right) \tag{4.46}$$

Where as above, r_T is the radius of the TEM magnet.

The unit vector describing the trajectory of the Colutron beam as it enters the TEM sample is:

$$\vec{b} = -\cos(\alpha)\vec{i} + \sin(\alpha)\,\vec{j}\ . \tag{4.47}$$

The unit vector for the surface normal is:

$$\vec{n} = \sin(\beta)\vec{i} + \cos(\beta)\vec{k}\ . \tag{4.48}$$

These unit vectors and angles are indicated in Figure 6.

The angle between $\vec{b}$ and $\vec{n}$ is found by taking the dot-product of these two vectors:

$$\delta' = \cos^{-1}(\vec{b}\cdot\vec{n}) = \cos^{-1}(-\cos(\alpha)\sin(\beta)) \tag{4.49}$$

This angle is obviously obtuse, and it is more common in calculating fluence, implantation profiles and the range of ions to use the acute angle given by:

$$\delta = \pi - \delta' \tag{4.50}$$

Again using B_T = 10 kG and r_T = 0.02*m*, the true angle of incidence of the Colutron beam onto the TEM sample tilted at an angle of β can be calculated as a function of the MEP of the beam. This is shown in Figure 7.

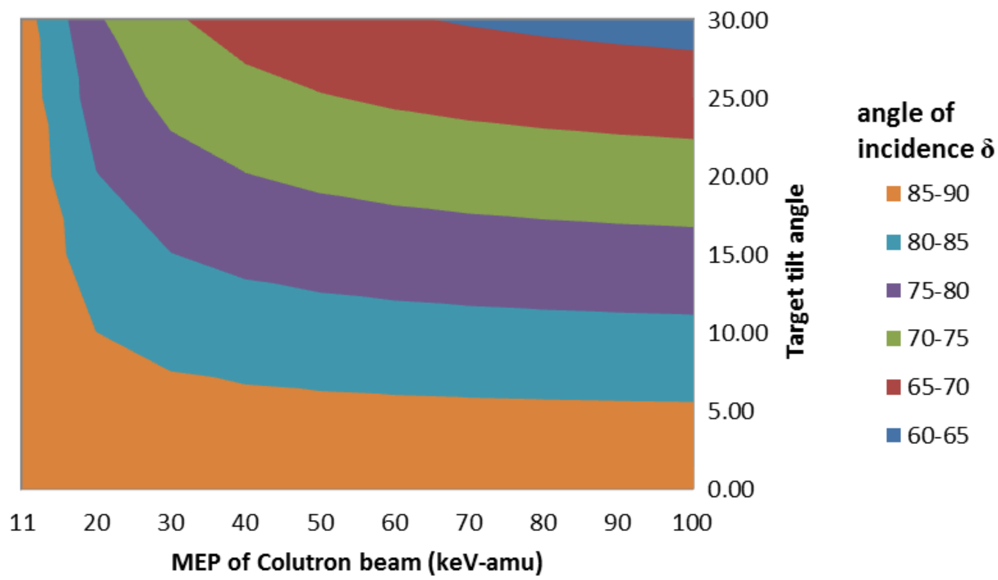

Figure 7 Angle of incidence of Colutron beam onto tilted TEM sample

For low MEP Colutron beams the angle of incidence can be quite large, which is the same as saying they will be at grazing incidence. As mentioned above, this angle needs to be taken into account when calculating fluences and the depth of implants. If the current and area of the beam is measured upstream of the TEM at normal incidence, then the fluence and range of the implants must be multiplied by $\cos(\delta)$. If on the other hand the beam area is measured in the TEM with the sample tilted, then only the range needs to be multiplied by $\cos(\delta)$.

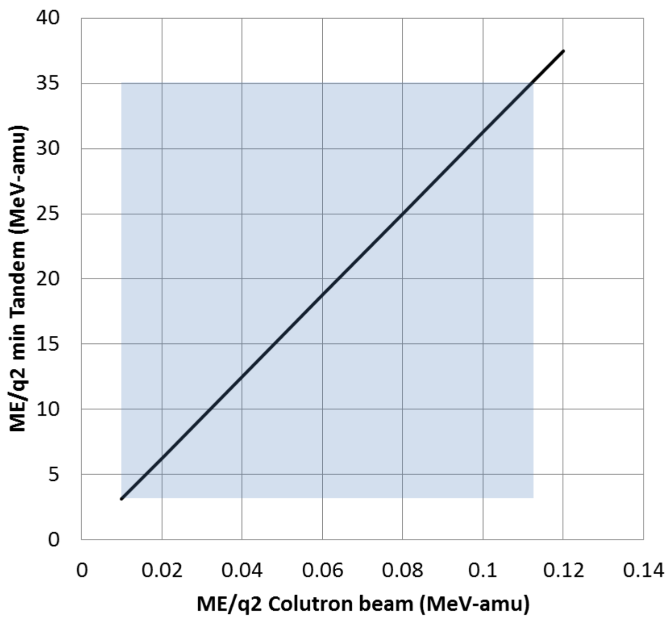

Figure 8 Minimum MEP of tandem beam as function of MEP of Colutron beam

The physical restriction placed by the ½" radius of the Einzel lens just after the Colutron magnet places a lower limit on the MEP of the tandem beam that can then be steered into the TEM, and the expression of this limit can be found in Equation 4.43. There is a maximum MEP of 35 MeV-amu for the Tandem beam to be steered 45^o into the TEM beamline. There is also a minimum MEP placed on the Colutron beam so that this beam can be steered by the TEM magnet onto the sample.

5. CONCLUSION

The purpose of this conclusion section is to highlight and consolidate the key equations derived above. The importance of these equations is then discussed in context of the operation of the I^3TEM. The units used for all calculations are mass in amu, energy in MeV, magnetic field in kG, radii and offset in meters, and q (ion charge state) is unitless.

The magnetic field required to bend a Colutron ion 20^o, directing it toward the TEM, is:

$$B_c = \frac{\tan\frac{\theta_c}{2}\sqrt{2m_cE_c}}{r_cq_c} \tag{5.1}$$

where the subscript "c" denotes the magnetic field and radii (0.038*m*) of the Colutron magnet, θ_c is the 20^o deflection angle, and $m_c, E_c,$ and q_c are the mass, energy, and charge state of the Colutron ion. For 10 keV $^4He^+$ ions, the magnetic field, B, in the Colutron magnet is calculated to be 1.3 kG.

The magnetic field in the steering magnet required to direct the Tandem ions through the Colutron bending magnet and into the TEM is:

$$B_s = \frac{1}{2}\left(\frac{r_c}{r_s}\right)B_c \tag{5.2}$$

where the subscript "s" denotes the magnetic field and radii (0.029*m*) of the steering magnet. Since r_s = 0.029*m* and r_c = 0.038*m* for the I^3TEM system, we solve:

$$B_s = 0.655B_c\ . \tag{5.3}$$

Eq. 5.2 proves that the magnetic field setting in the steering magnet is independent of the MEP of the Tandem beam. This is significant because, once the steering magnetic field is set according Eq. 5.2, all beams from the Tandem will automatically be deflected through the Colutron magnet to hit the TEM. Therefore, this field does not have to be changed to account for differing Tandem beams. If a different Tandem beam is required later in an experiment, the magnet setting can remain constant. This means that high and low intensity Tandem beams can be accelerated into the TEM in the same experiment. One could initially select a high intensity beam, which is easy to diagnose and visualize on the beam viewer in the TEM, and then introduce the low intensity beam, which is difficult to diagnose and visualize, as it will be on exactly the right trajectory to hit the TEM sample.

Such a trajectory involves shifting the point of entry of the Tandem ions into the Colutron magnet by an offset from the axis of the magnet. This offset is given by:

$$Offset = y_1 = \frac{r_c}{\sqrt{\frac{2m_tE_t}{x_0{}^2q_t{}^2B_c^2}+1}} \tag{5.4}$$

where the subscript "t" denotes the mass, energy and charge state of the Tandem ion. x_o is the 1.35*m* separation distance between the steering and Colutron magnets. This distance is also equal to the separation between the Colutron magnet and the TEM. The calculated offset value is limited by a 0.0127*m* opening in the Einzel lens. This restriction sets the lower mass-energy product (MEP) limit of the Tandem beams that can be deflected toward the TEM. This limit is given by:

$$\left(\frac{mE}{q^2}\right)_{\text{lower limit}} = \tan^2\left(\frac{\theta}{2}\right)\frac{x_0^{\ 2}}{r_c^{\ 2}}\left[\left(\frac{r_c}{y_1}\right)^2 - 1\right]\frac{m_c E_c}{q_c^{\ 2}} \ . \qquad (5.5)$$

The lower MEP limit of the Tandem beam scales linearly with the MEP of the Colutron ions. For 10 keV $^4He^+$ ions from the Colutron, the lower MEP limit for the Tandem beam is $MEP_t > 13$ MeV-amu. There also exists an upper MEP Tandem beam limit of 35 MeV-amu set by a 45^o switching magnet preceding the Colutron magnet along the Tandem beamline. Eq. 5.5 indicates that the MEP of the Colutron beam should be selected as low as possible in order to maximize the accessible range of MEP values of the Tandem beam.

The Colutron beam also requires an offset from the axis of the TEM because the TEM objective lens field has a significant steering effect on the low-mass Colutron ions. This offset, y_1, by which the Colutron ions must be displaced upon entering the magnetic field of the TEM in order to strike the sample is:

$$y_1 = \frac{r_T^{\ 2}}{2}\frac{B_T q_c}{\sqrt{2m_c E_c}} \ . \qquad (5.6)$$

The magnetic field in the TEM is 10 kG and it has an effective radius $r_T = 0.02m$. For 10 keV $^4He^+$, the y_1 offset of most Colutron beams is $0.007m$. However, the offset for most Tandem beams is significantly less than one millimeter and can therefore be ignored.

There is also a lower MEP limit of the Colutron beam for ions to be effectively steered by the TEM magnetic field to the sample. This limit can be expressed as:

$$MEP_{\text{c minimum}} = \frac{m_c E_c}{q_c^{\ 2}} \geq \frac{B_T^{\ 2} r_T^2}{4} \ . \qquad (5.7)$$

For the situation in the Sandia I^{3}TEM, B = 10 kG and $r_T = 0.02m$. Therefore, the minimum MEP Colutron beam that can be steered by the TEM magnetic field to the sample is $1\text{x}10^{-2}$ MeV-amu, or 10 keV-amu. Therefore, only the lowest energy Colutron beams cannot be directed to the sample when the TEM is in operation. It is nearly impossible to direct H^+ ions into the TEM sample unless the magnetic field, B, exerted by the TEM magnet is significantly reduced.

The angle of incidence for the Colutron beam onto the TEM sample tilted at an angle of β is found to be:

$$\delta = \pi - \cos^{-1}(-\cos(\alpha)\sin(\beta)) \qquad (5.8)$$

Where α is the angle through which the beam is steered at the point of impact on the sample, and is given by:

$$\alpha = \tan^{-1}\left(\frac{dy}{dx}\right) = \tan^{-1}\left(\frac{r_T}{R_{cT}}\sqrt{\frac{1+\frac{1}{2}\left(\frac{r_T}{R_{cT}}\right)^2}{1-\frac{1}{2}\left(\frac{r_T}{R_{cT}}\right)^2}}\right) \tag{5.9}$$

The approach and the equations derived in this paper apply to any situation in which high and low MEP beams must be combined to enter a single end-station port. The low MEP beam must have a large angle of deflection caused by a bending magnet. The high MEP beam, in contrast, must be initially directed toward the sample and slightly steered upstream of the bending magnet so the deflection caused by the bending magnet corrects the beam trajectory to hit the sample. The algebra is significantly simplified by the assumption of beamline symmetry, where the distance of separation between the steering magnet and the Colutron bending magnet is equal to the distance between the Colutron bending magnet and the TEM target sample. This assumption is viable due to time reversal symmetry. The main advantage of such geometry is it allows a single setting of the magnetic field in the steering magnet to automatically project any high MEP beam thru the bending magnet and on to the sample.

The equations derived herein have been programmed in a MS Excel worksheet that is available on the IBA Periodic Table Website [5].

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