Robust distribution network reconfiguration

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***Abstract*—We propose a two-stage robust optimization model for the distribution network reconfiguration problem with load uncertainty. The first-stage decision is to configure the radial distribution network; the second-stage decision is to find the optimal A/C power flow of the reconfigured network for given demand realization. We solve the two-stage robust model by using a column-and-constraint generation algorithm, where the master problem and subproblem are formulated as mixed-integer second-order cone programs. Computational results for 16, 33, 70 and 94-bus test cases are reported. We find that the configuration from the robust model does not compromise much the power loss under the nominal load scenario compared to the configuration from the deterministic model, yet it provides the reliability of the distribution system for all scenarios in the uncertainty set.**

***Index Terms*—distribution network, reconfiguration, minimum loss, robust optimization, mixed-integer second-order cone program (MISOCP)**

NOMENCLATURE

A. Sets and parameters

N	Set of buses.
N_s	Set of substations; a subset of N.
$N(i)$	Set of buses connected to bus i.
L	Set of lines.
A_l	Indicator parameter for the initial network configuration; equals 1, if line l was initially connected, and 0, if line l was initially disconnected.
$C_l^{\text{disconnect}}$	Cost to disconnect line l.
C_l^{connect}	Cost to connect line l.
C	Parameter to convert power loss to cost.
$SW_{\max}$	Maximum number of lines to reconfigure.
P_i^D	Forecast real power demand at bus i.
Q_i^D	Forecast reactive power demand at bus i.
G_l	Conductance of line l.
B_l	Susceptance of line l.
$P_{\max,i}$	Maximum real power allowed at substation bus i.
$Q_{\max,i}$	Maximum reactive power allowed at substation bus i.
$V_{\max,i}$	Maximum voltage allowed at bus i.
$V_{\min,i}$	Minimum voltage allowed at bus i.
$I_{\max,l}$	Maximum current flow allowed on line l.

B. Variables

α_l	Binary variable for network configuration; equals 1, if line l is connected, and 0, if line l is disconnected.
β_{ij}	Binary variable; equals 1, if bus j is the parent of bus i, and 0, otherwise.
p_i	Real power injection at bus i, forcibly set to 0, if bus i is not a substation bus.
q_i	Reactive power injection at bus i, forcibly set to 0, if bus i is not a substation bus.
p_{ij}	Real power flow from bus i to bus j.
q_{ij}	Reactive power flow from bus i to bus j.
r_l	Variable introduced in the convex relaxation of A/C power flow equations; corresponding to $v_i v_j \cos(\theta_i - \theta_j)$, where v_i, v_j are voltages and θ_i, θ_j are voltage angles of the two terminal buses i, j of line l.
t_l	Variable introduced in the convex relaxation of A/C power flow equations; corresponding to $v_i v_j \sin(\theta_i - \theta_j)$.
u_i	Variable introduced in the convex relaxation of A/C power flow equations; corresponding to $v_i^2/\sqrt{2}$.
u_i^l, u_j^l	Auxiliary variables introduced in the convex relaxation of A/C power flow equations; equals u_i, u_j, respectively, if line l is connected, and 0, otherwise.

I. INTRODUCTION

The distribution network reconfiguration problem is to configure the distribution network topology in order to improve the efficiency and stability of the network by changing the status of lines. The distribution network is composed of buses, where the power is injected (substation buses) or consumed; and lines or switches, which connect the buses. The distribution network has a meshed structure but is normally operated as radial (i.e. with no loop) so as to make the protection coordination easier (upstream to downstream) and to make the distribution design easier.

We consider the distribution network reconfiguration problem, for which we decide the radial configuration on the given meshed distribution network to minimize the switching cost and power losses while satisfying operational and physical constraints of the distribution system. In the perspective of mathematical programming, such a problem is a mixed-binary nonlinear optimization problem, as the optimal power flow problem is essentially nonconvex.

Most of the studies on distribution network reconfiguration focus on finding good feasible solutions using heuristics [1]–[9], as the problem remains difficult due to its discreteness and

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nonlinearity. Besides heuristic algorithms to generate feasible solutions, many global-optimization strategies (simulation annealing, genetic algorithms, particle swarms, etc.) are also employed for the distribution network reconfiguration problem [10]–[23]. Recently, several authors have applied a mathematical programming approach to the distribution network reconfiguration problem [19], [24]–[27]. Especially reference [27] formulate the problem as a mixed-integer second-order cone program (MISOCP), for which the global optimal solution up to the desired accuracy can be found by using available commercial solvers. The convexity of the optimal power flow of the radial network is important because it is guaranteed to find the global optimal solution, and also we can further develop models that are more advanced.

In this paper, we consider the distribution network reconfiguration problem with uncertain demands, where the uncertainty of the demand arises from the daily fluctuations of the loads. We propose a two-stage robust optimization model for the distribution network reconfiguration problem with uncertain loads. In our two-stage robust optimization model, the network reconfiguration is the first-stage decision; and the optimal power flow becomes the second-stage decision which is made after the realization of the uncertain demand. The uncertainty set of the loads can be constructed as a set of possible load scenarios for a given planning horizon. We use the column-and-constraint generation algorithm [28] to solve the proposed two-stage robust problem. The solution of the robust distribution network reconfiguration problem, if exists, can be used for the planning horizon instead of changing the configuration frequently for the time-varying loads.

This chapter is organized as follows: in Section II, we formulate the deterministic and robust distribution network reconfiguration models. Section III describes the algorithm to solve the robust distribution network reconfiguration models. Section IV shows our computational results for a few test distribution networks. Section V concludes with a discussion.

II. Robust distribution network reconfiguration model

In this section, we describe deterministic and robust distribution network reconfiguration models. First, we review a deterministic distribution network reconfiguration model with point forecast loads.

A. Deterministic distribution network reconfiguration model

Suppose we have a distribution network with $|N|$ buses and $|L|$ lines. The distribution network reconfiguration problem is to find a radial network configuration that minimizes the switching cost and power losses. Reference [27] shows that the A/C optimal power flow can be recovered from its convex relaxation when there is no loop in the distribution network configuration and formulates the distribution network reconfiguration problem as the following MISOCP:

$$\min_{\substack{\boldsymbol{\alpha},\boldsymbol{\beta},\\ \boldsymbol{p},\boldsymbol{q},\boldsymbol{r},\boldsymbol{t},\boldsymbol{u}}} \sum_{l\in L}\left[C_l^{\text{disconnect}}A_l(1-\alpha_l)+C_l^{\text{connect}}(1-A_l)\alpha_l\right] + C\sum_{i\in N}\left(p_i-P_i^D\right) \tag{1a}$$

$$\text{s.t. } \alpha_l=\beta_{ij}+\beta_{ji},\quad \forall l\in L \tag{1b}$$

$$\sum_{j\in N(i)}\beta_{ij}=1,\quad \forall i\in N_s \tag{1c}$$

$$\sum_{j\in N(i)}\beta_{ij}=0,\quad \forall i\in N\backslash N_s \tag{1d}$$

$$\sum_{l\in L}\left[A_l(1-\alpha_l)+(1-A_l)\alpha_l\right]\le 2\cdot SW_{\max} \tag{1e}$$

$$0\le\alpha_l\le 1,\quad \beta_{ij},\beta_{ji}\in\{0,1\},\quad \forall l\in L \tag{1f}$$

$$p_i-\sum_{j\in N(i)}p_{ij}=P_i^D,\quad \forall i\in N \tag{1g}$$

$$q_i-\sum_{j\in N(i)}q_{ij}=Q_i^D,\quad \forall i\in N \tag{1h}$$

$$p_i\le P_{\max,i},\quad \forall i\in N_s \tag{1i}$$

$$q_i\le Q_{\max,i},\quad \forall i\in N_s \tag{1j}$$

$$p_{ij}=\sqrt{2}G_lu_i^l-G_lr_l-B_lt_l,\quad \forall l\in L \tag{1k}$$

$$p_{ji}=\sqrt{2}G_lu_j^l-G_lr_l+B_lt_l,\quad \forall l\in L \tag{1l}$$

$$q_{ij}=-\sqrt{2}B_lu_i^l+B_lr_l-G_lt_l,\quad \forall l\in L \tag{1m}$$

$$q_{ji}=-\sqrt{2}B_lu_j^l+B_lr_l+G_lt_l,\quad \forall l\in L \tag{1n}$$

$$0\le u_i^l\le\frac{V_{\max,i}^2}{\sqrt{2}}\alpha_l,\quad \forall l\in L \tag{1o}$$

$$0\le u_j^l\le\frac{V_{\max,j}^2}{\sqrt{2}}\alpha_l,\quad \forall l\in L \tag{1p}$$

$$0\le u_i-u_i^l\le\frac{V_{\max,i}^2}{\sqrt{2}}(1-\alpha_l),\quad \forall l\in L \tag{1q}$$

$$0\le u_j-u_j^l\le\frac{V_{\max,j}^2}{\sqrt{2}}(1-\alpha_l),\quad \forall l\in L \tag{1r}$$

$$u_i^l+u_j^l-\sqrt{2}r_l\le\frac{I_{\max,l}^2}{\sqrt{2}(G_l^2+B_l^2)},\quad \forall l\in L \tag{1s}$$

$$r_l^2+t_l^2\le 2u_i^lu_j^l,\quad \forall l\in L \tag{1t}$$

$$p_i\ge 0,\quad \forall i\in N \tag{1u}$$

$$0\le r_l\le V_{\max,i}V_{\max,j},\quad \forall l\in L \tag{1v}$$

$$-V_{\max,i}V_{\max,j}\le t_l\le V_{\max,i}V_{\max,j},\quad \forall l\in L \tag{1w}$$

$$\frac{V_{\min,i}^2}{\sqrt{2}}\le u_i\le\frac{V_{\max,i}^2}{\sqrt{2}},\quad \forall i\in N \tag{1x}$$

$$u_i^l,u_j^l\ge 0,\quad \forall l\in L \tag{1y}$$

There are two types of decision variables: $\boldsymbol{\alpha},\boldsymbol{\beta}$ that are related to the network configuration, and $\boldsymbol{p},\boldsymbol{q},\boldsymbol{r},\boldsymbol{t},\boldsymbol{u}$ that are related to the optimal power flow of the distribution system. Let us describe the constraints in the above optimization problem: (1b–1d) ensure the configured network is radial, and (1e) sets the maximum number of switches to change their status from the initial network configuration $\boldsymbol{A}$. Note that index ij or ji refers to the line l of which the two terminal buses are bus i and bus j. (1g–1h) stands for the power balance of each bus. (1i–1j) enforces the capacity limits of the substations. (1k–1n) and (1t) come from the power flow equations and their convex relaxation. (1o–1r) links the network configuration variable α_l to the auxiliary variables u_i^l, u_j^l so that $p_{ij}, p_{ji}, q_{ij}, q_{ji}$ can be set to 0 when α_l is 0. (1s) sets the current flow limit on each line, and (1x) sets the voltage limit at each bus.

For a clear presentation in the subsequent discussion, we recast the above distribution network reconfiguration problem in the following compact form:

$$\min_{\boldsymbol{x},\boldsymbol{y}} \quad \boldsymbol{c}^\top \boldsymbol{x} + \boldsymbol{b}^\top \boldsymbol{y} - \boldsymbol{e}^\top \boldsymbol{d}_0 \tag{2a}$$

$$\text{s.t.} \quad \boldsymbol{x} \in \mathcal{X} \tag{2b}$$

$$\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y} \geq \boldsymbol{f} \tag{2c}$$

$$\boldsymbol{F}\boldsymbol{y} = \boldsymbol{d}_0 \tag{2d}$$

$$\|\boldsymbol{G}_l \boldsymbol{y}\| \leq \boldsymbol{g}_l^\top \boldsymbol{y}, \qquad \forall l = 1, \cdots, m \tag{2e}$$

The vector $\boldsymbol{x}$ denotes the variables $\boldsymbol{\alpha}, \boldsymbol{\beta}$ regarding the distribution network configuration. The vector $\boldsymbol{y}$ represents the rest of the continuous variables related to optimal power flow of the distribution network. The vector $\boldsymbol{d}_0$ is the forecast loads. The term $\boldsymbol{c}^\top \boldsymbol{x}$ represents the switching cost and $\boldsymbol{b}^\top \boldsymbol{y} - \boldsymbol{e}^\top \boldsymbol{d}_0$ represents the cost associated with the power losses. $\mathcal{X}$ is the feasible set of radial distribution network configurations described by (1b–1f). (2d) summarizes the power balance equations (1g–1h), (2e) represents second-order-cone constraints (1t), and (2c) denotes all the other optimal power flow constraints that are linear.

B. Two-stage robust distribution network reconfiguration model

We assume that the network configuration decision is made prior to the realization of loads. For each realization of uncertain loads, we can compute the power losses for the given radial network configuration by solving the optimal power flow problem. We make the first-stage network configuration which minimizes the worst-case second-stage power loss. This can be formulated as the following two-stage robust model:

$$\min_{\boldsymbol{x}} \; \boldsymbol{c}^\top \boldsymbol{x} + \max_{\boldsymbol{d} \in \mathcal{D}} L(\boldsymbol{x}, \boldsymbol{d}) \tag{3a}$$

$$\text{s.t.} \; \boldsymbol{x} \in \mathcal{X} \tag{3b}$$

$$\mathcal{Y}(\boldsymbol{x}, \boldsymbol{d}) \neq \emptyset, \qquad \forall \boldsymbol{d} \in \mathcal{D} \tag{3c}$$

where

$$L(\boldsymbol{x}, \boldsymbol{d}) := \min_{\boldsymbol{y} \in \mathcal{Y}(\boldsymbol{x}, \boldsymbol{d})} \boldsymbol{b}^\top \boldsymbol{y} - \boldsymbol{e}^\top \boldsymbol{d} \tag{4}$$

and

$$\mathcal{Y}(\boldsymbol{x}, \boldsymbol{d}) := \left\{ \boldsymbol{y} \;\middle|\; \begin{array}{l} \boldsymbol{B}\boldsymbol{y} \geq \boldsymbol{f} - \boldsymbol{A}\boldsymbol{x} \\ \boldsymbol{F}\boldsymbol{y} = \boldsymbol{d} \\ \|\boldsymbol{G}_l \boldsymbol{y}\| \leq \boldsymbol{g}_l^\top \boldsymbol{y}, \qquad \forall l = 1, \cdots, m \end{array} \right\} \tag{5}$$

Here, $\mathcal{D}$ is the uncertainty set of the loads. We assume that the uncertainty set of the loads is a polyhedron. The polyhedral assumption guarantees the finite convergence of the column-and-constraint generation algorithm that solves the proposed model [28], since the polyhedron has a finite number of extreme points. We further assume that we can characterize all the extreme points of the polyhedral uncertainty set using binary variables and linear constraints as in [29]. This assumption allows us to convert the bilinear subproblem into a mixed-integer linear program (MILP) as shown in the next section.

The innermost problem solves the optimal power flow of a given radial network configuration $\boldsymbol{x}$ with a given load scenario $\boldsymbol{d}$, which replaces the forecast load $\boldsymbol{d}_0$ to obtain the cost of power losses $L(\boldsymbol{x}, \boldsymbol{d})$. In the midlevel, the load scenario that maximizes the minimum cost of power losses $L(\boldsymbol{x}, \boldsymbol{d})$ for the given network configuration is obtained. In the outermost level, the robust program finds a network configuration that minimizes the sum of switching cost and worst-case cost of power losses.

If a distribution network configuration $\boldsymbol{x} \in \mathcal{X}$ satisfies (3c) (that is, given such a configuration, if the distribution network is feasible for any possible demand scenario $\boldsymbol{d} \in \mathcal{D}$), then we say the network configuration $\boldsymbol{x}$ is *robustly feasible*. If no *robustly feasible* configuration $\boldsymbol{x}$ exists (in other words, there is no configuration under which the distribution network is feasible for all possible demand scenario $\boldsymbol{d} \in \mathcal{D}$), then the robust program is infeasible. It is possible for the robust program to be infeasible, if the uncertainty set is too large and so there cannot be a configuration that is feasible for all possible scenarios in the uncertainty set. For such case, one can make the uncertainty set smaller, for example, by considering a shorter planning horizon. When there is no solution for the robust distribution network model, the distribution system should change the configuration more frequently to adapt to the rapidly changing load.

III. Solution Methods for Two-Stage Robust Distribution Network Reconfiguration Model

In order to solve the two-stage robust distribution network reconfiguration model, we use the column-and-constraint generation algorithm that is introduced in reference [28]. The distinctive feature of the proposed two-stage robust distribution network reconfiguration model is that the innermost problem is a second-order cone program. However, the column-and-constraint generation algorithm presented in [28] can be naturally extended for the robust distribution network reconfiguration model. We outline the application of the algorithm as follows. The column-and-constraint generation algorithm attempts to solve the following extensive formulation of (3):

$$\min_{\boldsymbol{x}, \boldsymbol{y}(\boldsymbol{d}), L} \quad \boldsymbol{c}^\top \boldsymbol{x} + L \tag{6a}$$

$$\text{s.t.} \; \boldsymbol{x} \in \mathcal{X} \tag{6b}$$

$$L \geq \boldsymbol{b}^\top \boldsymbol{y}(\boldsymbol{d}) - \boldsymbol{e}^\top \boldsymbol{d}, \qquad \forall \boldsymbol{d} \in \mathcal{D} \tag{6c}$$

$$\boldsymbol{y}(\boldsymbol{d}) \in \mathcal{Y}(\boldsymbol{x}, \boldsymbol{d}), \qquad \forall \boldsymbol{d} \in \mathcal{D} \tag{6d}$$

$$L \geq 0 \tag{6e}$$

The above problem has an infinite number of variables and constraints corresponding to each demand scenario $\boldsymbol{d}$.

1) Master problem: Uncountably many number of constraints (6c) and (6d), indexed by the uncertainty set $\mathcal{D}$, make it impossible to solve (6) directly. The following reduced problem, in which $\mathcal{D}$ is replaced by its finite subset, provides a lower bound for (3):

$$\min_{\boldsymbol{x},\boldsymbol{y}_{(i)},L} \boldsymbol{c}^\top \boldsymbol{x} + L \tag{7a}$$
$$\text{s.t. } \boldsymbol{x} \in \mathcal{X} \tag{7b}$$
$$L \geq \boldsymbol{b}^\top \boldsymbol{y}_{(i)} - \boldsymbol{e}^\top \boldsymbol{d}_{(i)}, \qquad \forall i = 1, \cdots, k \tag{7c}$$
$$\boldsymbol{y}_{(i)} \in \mathcal{Y}\left(\boldsymbol{x}, \boldsymbol{d}_{(i)}\right), \qquad \forall i = 1, \cdots, k \tag{7d}$$
$$L \geq 0 \tag{7e}$$

where $\boldsymbol{d}_{(i)} \in \mathcal{D},\ i = 0, \cdots, k$. We call (7) the master problem of the robust distribution network reconfiguration model. The master problem is an MISOCP with binary variables in $\boldsymbol{x}$ and $|L| \cdot k$ number of second-order cone constraints in (7d).

2) Subproblem: The next key step in the column-and-constraint generation algorithm is to generate the worst-case scenario for the master problem and to obtain an upper bound for (3). We achieve these goals by solving the following separation problem for the given distribution network configuration $\boldsymbol{x}^*$:

$$L(\boldsymbol{x}^*) := \max_{\boldsymbol{d}\in\mathcal{D}} \min_{\boldsymbol{y}} \boldsymbol{b}^\top \boldsymbol{y} - \boldsymbol{e}^\top \boldsymbol{d} \tag{8a}$$
$$\text{s.t. } \boldsymbol{B}\boldsymbol{y} \geq \boldsymbol{f} - \boldsymbol{A}\boldsymbol{x}^* \qquad (\boldsymbol{\pi}) \tag{8b}$$
$$\boldsymbol{F}\boldsymbol{y} = \boldsymbol{d} \qquad (\boldsymbol{\lambda}) \tag{8c}$$
$$\|\boldsymbol{G}_l \boldsymbol{y}\| \leq \boldsymbol{g}_l^\top \boldsymbol{y}, \quad \forall l \qquad (\boldsymbol{\sigma}_l, \mu_l) \tag{8d}$$

Given the radial network configuration $\boldsymbol{x}^*$, the above separation problem finds the worst-case load scenario $\boldsymbol{d}$ and corresponding optimal power flow $\boldsymbol{y}$. The optimal objective value of the separation problem $L(\boldsymbol{x}^*)$ can be used to compute an upper bound for (3) because it is the worst-case power losses for a given network configuration $\boldsymbol{x}^*$.

We dualize the inner conic program to convert the subproblem into a monolithic form.

$$\max_{\boldsymbol{d},\boldsymbol{\pi},\boldsymbol{\lambda},\boldsymbol{\sigma},\boldsymbol{\mu}} \left(\boldsymbol{f} - \boldsymbol{A}\boldsymbol{x}^*\right)^\top \boldsymbol{\pi} + \boldsymbol{d}^\top \boldsymbol{\lambda} - \boldsymbol{e}^\top \boldsymbol{d} \tag{9a}$$
$$\text{s.t. } \boldsymbol{B}^\top \boldsymbol{\pi} + \boldsymbol{F}^\top \boldsymbol{\lambda} + \sum_l \left(\boldsymbol{G}^\top \boldsymbol{\sigma}_l + \boldsymbol{g}\mu_l\right) = \boldsymbol{b} \tag{9b}$$
$$\|\boldsymbol{\sigma}_l\| \leq \mu_l, \quad \forall l \tag{9c}$$
$$\boldsymbol{\pi}, \boldsymbol{\mu} \geq \boldsymbol{0}, \quad \boldsymbol{\lambda}, \boldsymbol{\sigma} : \text{free}, \quad \boldsymbol{d} \in \mathcal{D} \tag{9d}$$

The objective function is linear except for the bilinear term $\boldsymbol{d}^\top \boldsymbol{\lambda}$, and the constraints are linear and second-order cone constraints. Assuming that the uncertainty set $\mathcal{D}$ is polyhedral and that its extreme points can be described by a set of binary variables and linear constraints, we can transform the bilinear term $\boldsymbol{d}^\top \boldsymbol{\lambda}$ with a bilinear term between the continuous variable $\boldsymbol{\lambda}$ and binary variables. This will allow us to linearize the bilinear term by introducing big-M constraints. Then, the subproblem is equivalent to an MISOCP.

3) Feasibility subproblem: For a given distribution network configuration $\boldsymbol{x}^*$, the subproblem (9) would not be well-defined if there exists a scenario $\boldsymbol{d}$ such that the innermost optimal power flow problem (4) is infeasible. For such a case, the subproblem (9) is unbounded. Hence, before solving the subproblem (9), we must check whether the given configuration $\boldsymbol{x}^*$ is *robustly feasible*. We can check the robust feasibility of the given configuration $\boldsymbol{x}^*$ by solving the following feasibility subproblem:

$$e\left(\boldsymbol{x}^*\right) := \max_{\boldsymbol{d}\in\mathcal{D}} \min_{\boldsymbol{y},s,t} s + t \tag{10a}$$
$$\text{subject to } \boldsymbol{B}\boldsymbol{y} \geq \boldsymbol{f} - \boldsymbol{A}\boldsymbol{x}^* \qquad (\boldsymbol{\pi}) \tag{10b}$$
$$\boldsymbol{F}\boldsymbol{y} = \boldsymbol{d} + (s - t)\boldsymbol{1} \qquad (\boldsymbol{\lambda}) \tag{10c}$$
$$\|\boldsymbol{G}_l \boldsymbol{y}\| \leq \boldsymbol{g}_l^\top \boldsymbol{y}, \quad \forall l \qquad (\boldsymbol{\sigma}_l, \mu_l) \tag{10d}$$
$$s, t \geq 0 \tag{10e}$$

The feasibility subproblem adds non-negative slack and surplus variable s and t to the power balance equation (10c), and these variables are adjusted according to the infeasibility of the optimal power flow problem. If $e\left(\boldsymbol{x}^*\right) > 0$, there exists a load scenario $\boldsymbol{d}^* \in \mathcal{D}$ where there is no feasible distribution. On the other hand, if $e\left(\boldsymbol{x}^*\right) = 0$, then under the given network configuration $\boldsymbol{x}^*$, there is a feasible distribution for each load scenario in the uncertainty set.

4) Column-and-constraint generation algorithm: Overall procedures to solve the two-stage robust distribution network reconfiguration problem are summarized in Figure 1.

- **initialization** $LB \leftarrow 0, UB \leftarrow \infty, k \leftarrow 0$. Set tolerance level ϵ.
- Solve the master problem to get optimal solution $\boldsymbol{x}^*$ and L^*. $LB \leftarrow \max\{LB, \boldsymbol{c}^\top \boldsymbol{x} + L^*\}$.
- **While** $UB - LB < \epsilon$
  - Given $\boldsymbol{x}^*$, solve the feasibility subproblem to get optimal objective $e\left(\boldsymbol{x}^*\right)$, corresponding to worst-case load scenario $\boldsymbol{d}^*$.
  - **If** $e\left(\boldsymbol{x}^*\right) > 0$
    - $\boldsymbol{d}_{(k+1)} \leftarrow \boldsymbol{d}^*$, create variables $\boldsymbol{y}_{(k+1)}$ and add corresponding constraints (6d) to the reduced master problem.
  - **Else**
    - Given $\boldsymbol{x}^*$, solve the subproblem to get optimal objective $L\left(\boldsymbol{x}^*\right) > 0$, corresponding to worst-case load scenario $\boldsymbol{d}^*$. $UB \leftarrow \min\{UB, \boldsymbol{c}^\top \boldsymbol{x} + L\left(\boldsymbol{x}^*\right)\}$.
    - $\boldsymbol{d}_{(k+1)} \leftarrow \boldsymbol{d}^*$, create variables $\boldsymbol{y}_{(k+1)}$ and add corresponding constraints (6c) and (6d) to the reduced master problem.
  - $k \leftarrow k + 1$ and solve the master problem to get optimal solution $\boldsymbol{x}^*$ and L^*. $LB \leftarrow \max\{LB, \boldsymbol{c}^\top \boldsymbol{x} + L^*\}$.

Fig. 1. Column-and-constraint generation algorithm for robust distribution network reconfiguration

IV. Computational Experiments

We consider four test cases from the literature: 16-bus [2], 33-bus [3], 70-bus [30], and 94-bus [15] distribution network for our computational experiments (see Figure 2 for 94-bus system). A basic information of each the distribution network is given in Table I. In our case study, the switching costs are ignored.

TABLE I
SUMMARY OF TEST CASES

	16-bus	33-bus	70-bus	94-bus
No. of substations	3	1	2	11
No. of lines	16	37	79	96
No. of lines to disconnect	3	5	11	13

All the algorithms are implemented in AMPL and solved with CPLEX 12.5. The test environment is a laptop with a 2.53-GHz Duo CPU and 8 GB of RAM.

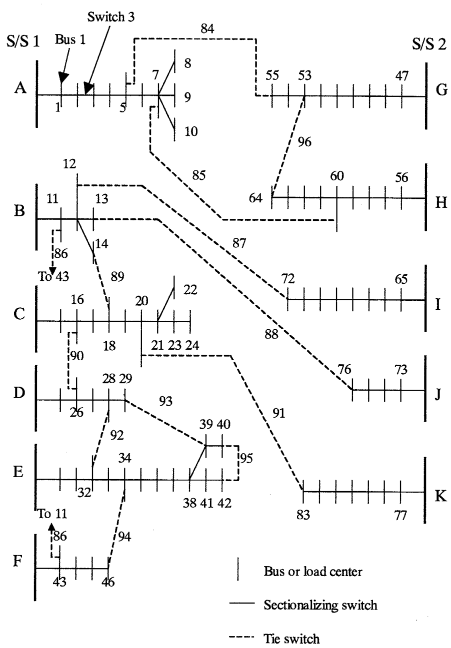

Fig. 2. 94-bus distribution network [15]

A. Characteristics of robust distribution network reconfiguration

We consider the following budget uncertainty set for the loads:

$$\mathcal{D} = \left\{ \boldsymbol{P} \left| \begin{array}{l} P_i^{\min} \le P_i \le P_i^{\max}, \forall i \in N \\ \displaystyle\sum_{i \in N} \left[\left(\frac{P_i^D - P_i}{P_i^D - P_i^{\min}} \right)^+ \right] \\ \displaystyle\quad + \sum_{i \in N} \left[\left(\frac{P_i - P_i^D}{P_i^{\max} - P_i^D} \right)^+ \right] \le B \end{array} \right. \right\}. \quad (11)$$

We assume the real loads can vary between the lower and upper limit $P_i^{\min}$ and $P_i^{\max}$. We further assume that the overall variation is controlled by a user-defined budget parameter B. For simplicity, we assume that the uncertain reactive loads will be proportional to the uncertain real loads where the ratio is presumed to be fixed. However, one can assume an independent uncertainty set for the reactive loads as well. To make the problem more tractable, we aggregate the load buses to make a zone. We assume that the load variation (in percentage) of each bus within the same zone is identical. This approximation reduces the number of binary variables in the subproblem and the solution time. We assume the lower and upper limit parameters of the uncertainty set to be $\pm$ 5% of the nominal value. For all cases, we aggregate load buses into 9 zones and assume the budget parameter B to be 8.

The solution of the robust distribution network reconfiguration problem is compared with the solution of the deterministic model in Table II. For 16-bus test case, the topology of the network is rather simple, and the configuration decision from the robust model is identical to the one from the deterministic model. But, for the other test cases, we observe that the the two configurations are similar in terms of the total power losses under the forecast scenario; namely, nominal power losses in Table II. However, the solution from the deterministic model is not robust, as there exists at least one demand scenario within the uncertainty set under which the delivery of power is not possible without violating the physical and operational constraints of the network. On the other hand, the robust model finds the configuration that is *robustly feasible*.

In all of our test cases, the algorithm terminates in two iterations: we solve the deterministic reconfiguration problem with the forecast load to generate an initial configuration decision; we then solve the feasibility and optimality subproblems to generate the worst-case scenarios; and the algorithm terminates after we solve the master problem again with the added worst-case scenarios.

TABLE II
COMPARISON OF DETERMINISTIC AND ROBUST MODELS

		Deterministic	Robust model
16-bus	Configuration	7, 8, 16	7, 8, 16
	Nominal power losses (kW)	466	466
	Maximum power losses (kW)	517.8	517.8
	CPU Time (s)	0.8	17.8
	Number of iteration	-	1
33-bus	Configuration	7, 9, 14, 29, 32	7, 11, 14, 29, 32
	Nominal power losses (kW)	129.9	131.6
	Maximum power losses (kW)	infeasible	145.3
	CPU Time (s)	3.9	209.3
	Number of iteration	-	2
70-bus	Configuration	14, 30, 39, 46, 51, 66, 71, 75, 76, 77, 79	14, 30, 39, 46, 51, 66, 70, 71, 76, 77, 78
	Nominal power losses (kW)	204.1	207.7
	Maximum power losses (kW)	infeasible	224.5
	CPU Time (s)	13.3	123.1
	Number of iteration	-	2
94-bus	Configuration	7, 13, 33, 37, 40, 63, 72, 82, 84, 86, 89, 90 92	7, 13, 34, 39, 42, 61, 72, 82, 84, 86, 89, 90 92
	Nominal power losses (kW)	471.9	472.7
	Maximum power losses (kW)	infeasible	521.1
	CPU Time (s)	3	160.1
	Number of iteration	-	2

B. Sensitivity with respect to the uncertainty set

The users can adjust the uncertainty set (11) by changing the value of the demand variation limit $P^i_{\max}, P^i_{\min}$ and the budget of the uncertainty parameter B. Table III shows the sensitivity of the minimum worst-case power losses of the robust distribution network reconfiguration problem with respect to the different variation limits of the uncertainty set of the 94-bus test case with $B = 8$. We can see that the worst-case power losses increases as the size of the uncertainty set gets bigger. When the uncertainty set allows a variation of more than 7% from the forecast demand, the robust program is infeasible and there is no radial configuration that is feasible in all possible scenarios. This implies that with the current design of the distribution network and operational requirements, it is impossible to meet all possible demand scenarios within the prescribed uncertainty set. Table IV shows the worst-case power losses of the robust distribution network reconfiguration with the uncertainty sets with different values of the budget parameter B while the maximum demand fluctuation levels are set at $\pm$ 5%, where we observe the similar pattern as in the previous table.

TABLE III
COMPARISON OF ROBUST MODELS WITH DIFFERENT VARIATION LIMITS OF THE UNCERTAINTY SETS WITH $B = 8$

Variation limits	1%	2%	3%	4%	5%	6%
Maximum power loss (kW)	480	490	502	507	523	534
Number of iteration	1	1	1	1	2	2

TABLE IV
COMPARISON OF ROBUST MODELS WITH DIFFERENT BUDGET OF UNCERTAINTY SET PARAMETER WITH $\pm$ 5% LIMIT

Budget parameter B	2	4	6	8
Maximum power loss (kW)	481	493	510	523
Number of iteration	2	2	2	2

V. CONCLUSION

We propose the use of the two-stage robust optimization for the distribution network reconfiguration problem with the uncertain loads. The robust model is solved by using the column-and-constraint generation algorithm, in which the master problem and subproblem are transformed into an equivalent MISOCP and can be solved by using available commercial software. In our case study, we found that the solution from the deterministic distribution network reconfiguration model is not necessarily *robustly feasible*. We also found that the robust distribution network model can generate the network configuration solution that is robust with regard to all possible scenarios considered in the uncertainty set and yet does not compromise much in terms of the power losses under the forecast demand scenario.

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REFERENCES

[1] A. Merlin and H. Back, "Search for a minimal-loss operating spanning tree configuration in an urban power distribution system," in *Proc. 5th Power System Computation Conference (PSCC)*, Cambridge, U.K., Sep. 1975.

[2] S. Civanlar, J. J. Grainger, H. Yin, and S. S. H. Lee, "Distribution feeder reconfiguration for loss reduction," *Power Delivery, IEEE Transactions on*, vol. 3, no. 3, pp. 1217–1223, 1988.

[3] M. Baran and F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing," *Power Delivery, IEEE Transactions on*, vol. 4, no. 2, pp. 1401–1407, 1989.

[4] S. Goswami and S. Basu, "A new algorithm for the reconfiguration of distribution feeders for loss minimization," *Power Delivery, IEEE Transactions on*, vol. 7, no. 3, pp. 1484–1491, 1992.

[5] R. Taleski and D. Rajicic, "Distribution network reconfiguration for energy loss reduction," *Power Systems, IEEE Transactions on*, vol. 12, no. 1, pp. 398–406, 1997.

[6] A. Morton and I. M. Y. Mareels, "An efficient brute-force solution to the network reconfiguration problem," *Power Delivery, IEEE Transactions on*, vol. 15, no. 3, pp. 996–1000, 2000.

[7] H. Schmidt, N. Ida, N. Kagan, and J. Guaraldo, "Fast reconfiguration of distribution systems considering loss minimization," *Power Systems, IEEE Transactions on*, vol. 20, no. 3, pp. 1311–1319, 2005.

[8] F. Gomes, J. Carneiro, S., J. L. R. Pereira, M. Vinagre, P. Garcia, and L. R. de Araujo, "A new distribution system reconfiguration approach using optimum power flow and sensitivity analysis for loss reduction," *Power Systems, IEEE Transactions on*, vol. 21, no. 4, pp. 1616–1623, 2006.

[9] G. Raju and P. R. Bijwe, "An efficient algorithm for minimum loss reconfiguration of distribution system based on sensitivity and heuristics," *Power Systems, IEEE Transactions on*, vol. 23, no. 3, pp. 1280–1287, 2008.

[10] H.-D. Chiang and R. Jean-Jumeau, "Optimal network reconfigurations in distribution systems. i. a new formulation and a solution methodology," *Power Delivery, IEEE Transactions on*, vol. 5, no. 4, pp. 1902–1909, 1990.

[11] H.-C. Chang and C.-C. Kuo, "Network reconfiguration in distribution systems using simulated annealing," *Electric Power Systems Research*, vol. 29, no. 3, pp. 227–238, 1994.

[12] Y. H. Song, G. Wang, A. Johns, and P. Wang, "Distribution network reconfiguration for loss reduction using fuzzy controlled evolutionary programming," *Generation, Transmission and Distribution, IEEE Proceedings*, vol. 144, no. 4, pp. 345–350, 1997.

[13] Y.-J. Jeon, J.-C. Kim, J.-O. Kim, J.-R. Shin, and K. Lee, "An efficient simulated annealing algorithm for network reconfiguration in large-scale distribution systems," *Power Delivery, IEEE Transactions on*, vol. 17, no. 4, pp. 1070–1078, 2002.

[14] J. Zhu, "Optimal reconfiguration of electrical distribution network using the refined genetic algorithm," *Electric Power Systems Research*, vol. 62, no. 1, pp. 37–42, 2002.

[15] C.-T. Su and C.-S. Lee, "Network reconfiguration of distribution systems using improved mixed-integer hybrid differential evolution," *IEEE Transactions on Power Delivery*, vol. 18, no. 3, pp. 1022–1027, 2003.

[16] X. Jin, J. Zhao, Y. Sun, K. Li, and B. Zhang, "Distribution network reconfiguration for load balancing using binary particle swarm optimization," in *Power System Technology, International Conference on*, vol. 1, 2004, pp. 507–510.

[17] J.-P. Chiou, C.-F. Chang, and C.-T. Su, "Variable scaling hybrid differential evolution for solving network reconfiguration of distribution systems," *Power Systems, IEEE Transactions on*, vol. 20, no. 2, pp. 668–674, 2005.

[18] L. Daniel, I. Khan, and S. Ravichandran, "Distribution network reconfiguration for loss reduction using ant colony system algorithm," in *INDICON, 2005 Annual IEEE*, 2005, pp. 619–622.

[19] E. Ramos, A. Exposito, J. Santos, and F. Iborra, "Path-based distribution network modeling: application to reconfiguration for loss reduction," *Power Systems, IEEE Transactions on*, vol. 20, no. 2, pp. 556–564, 2005.

[20] C.-T. Su, C.-F. Chang, and J.-P. Chiou, "Distribution network reconfiguration for loss reduction by ant colony search algorithm," *Electric Power Systems Research*, vol. 75, no. 2–3, pp. 190–199, 2005.

[21] J. Mendoza, R. Lopez, D. Morales, E. Lopez, P. Dessante, and R. Moraga, "Minimal loss reconfiguration using genetic algorithms with restricted population and addressed operators: real application," *Power Systems, IEEE Transactions on*, vol. 21, no. 2, pp. 948–954, 2006.

[22] E. Carreno, R. Romero, and A. Padilha-Feltrin, "An efficient codification to solve distribution network reconfiguration for loss reduction problem," *Power Systems, IEEE Transactions on*, vol. 23, no. 4, pp. 1542–1551, 2008.

[23] R. Rao, S. V. L. Narasimham, M. Ramalinga Raju, and A. Srinivasa Rao, "Optimal network reconfiguration of large-scale distribution system using harmony search algorithm," *Power Systems, IEEE Transactions on*, vol. 26, no. 3, pp. 1080–1088, 2011.

[24] J. F. Franco, M. J. Rider, M. Lavorato, and R. Romero, "A mixed-integer lp model for the reconfiguration of radial electric distribution systems considering distributed generation," *Electric Power Systems Research*, vol. 97, pp. 51–60, 2013.

[25] E. Romero-Ramos, J. Riquelme-Santos, and J. Reyes, "A simpler and exact mathematical model for the computation of the minimal power losses tree," *Electric Power Systems Research*, vol. 80, no. 5, pp. 562–571, 2010.

[26] H. Khodr, J. Martinez-Crespo, M. Matos, and J. Pereira, "Distribution systems reconfiguration based on OPF using benders decomposition," *Power Delivery, IEEE Transactions on*, vol. 24, no. 4, pp. 2166–2176, 2009.

[27] R. Jabr, R. Singh, and B. Pal, "Minimum loss network reconfiguration using mixed-integer convex programming," *Power Systems, IEEE Transactions on*, vol. 27, no. 2, pp. 1106–1115, 2012.

[28] B. Zeng and L. Zhao, "Solving two-stage robust optimization problems using a column-and-constraint generation method," *Operations Research Letters*, vol. 41, no. 5, pp. 457–461, Sep. 2013.

[29] C. Lee, C. Liu, S. Mehrotra, and M. Shahidehpour, "Modeling transmission line constraints in two-stage robust unit commitment problem," *Power Systems, IEEE Transactions on*, vol. 29, no. 3, pp. 1221–1231, May 2014.

[30] D. Das, "A fuzzy multiobjective approach for network reconfiguration of distribution systems," *Power Delivery, IEEE Transactions on*, vol. 21, no. 1, pp. 202–209, Jan 2006.

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