

## **Transport and Dynamics in Toroidal Fusion Systems**

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## 1. Executive Summary

The study entitled, "Transport and Dynamics in Toroidal Fusion Systems," (TDTFS) applied analytical theory and numerical computation to investigate topics of importance to confining plasma, the fourth state of matter, with magnetic fields. A central focus of the work is how non-thermal components of the ion particle distribution affect the "sawtooth" collective oscillation in the core of the tokamak magnetic configuration. Previous experimental and analytical research had shown and described how the oscillation frequency decreases and amplitude increases, leading to "monster" or "giant" sawteeth, when the non-thermal component is increased by injecting particle beams or by exciting ions with imposed electromagnetic waves. The TDTFS study applied numerical computation to self-consistently simulate the interaction between macroscopic collective plasma dynamics and the non-thermal particles. The modeling used the NIMROD code [Sovinec, Glasser, Gianakon, *et al.*, J. Comput. Phys. **195**, 355 (2004)] with the energetic component represented by simulation particles [Kim, Parker, Sovinec, and the NIMROD Team, Comput. Phys. Commun. **164**, 448 (2004)]. The computations found decreasing growth rates for the instability that drives the oscillations, but they were ultimately limited from achieving experimentally relevant parameters due to computational practicalities. Nonetheless, this effort provided valuable lessons for integrated simulation of macroscopic plasma dynamics. It also motivated an investigation of the applicability of fluid-based modeling to the ion temperature gradient instability, leading to the journal publication [Schnack, Cheng, Barnes, and Parker, Phys. Plasmas **20**, 062106 (2013)].

Apart from the tokamak-specific topics, the TDTFS study also addressed topics in the basic physics of magnetized plasma and in the dynamics of the reversed-field pinch (RFP) configuration. The basic physics work contributed to a study of two-fluid effects on interchange dynamics, where "two-fluid" refers to modeling independent dynamics of electron and ion species without full kinetic effects. In collaboration with scientist Ping Zhu, who received separate support, it was found that the rule-of-thumb criteria on stabilizing interchange has caveats that depend on the plasma density and temperature profiles. This work was published in [Zhu, Schnack, Ebrahimi, *et al.*, Phys. Rev. Lett. **101**, 085005 (2008)]. An investigation of general nonlinear relaxation with fluid models was partially supported by the TDTFS study and led to the publication [Khalzov, Ebrahimi, Schnack, and Mirnov, Phys. Plasmas **19**, 012111 (2012)]. Work specific to the RFP included an investigation of interchange at large plasma pressure and support for applications [for example, Scheffel, Schnack, and Mirza, Nucl. Fusion **53**, 113007 (2013)] of the DEBS code [Schnack, Barnes, Mikic, Harned, and Caramana, J. Comput. Phys. **70**, 330 (1987)]. Finally, the principal investigator over most of the award period, Dalton Schnack, supervised a numerical study of modeling magnetic island suppression [Jenkins, Kruger, Hegna, Schnack, and Sovinec, Phys. Plasmas **17**, 12502 (2010)].

## 2. Objectives and Accomplishments

The central theme of Transport and Dynamics in Toroidal Fusion Systems (TDTFS) was the application of numerical computation and analytical theory to plasma macroscopic dynamics in magnetic confinement. Funds provided by the Department of Energy for TDTFS through grant DE-FG02-06ER54868 were primarily used to support salary and travel for the original Principal Investigator (PI), Dalton Schnack. Smaller fractions of the funds were also used to support former graduate student Mio Suzuki for 6 months, a fraction of postdoctoral research associate Ivan Khalzov over 10 months, and a month of postdoctoral research associate Eric Howell. The PI since January 2014, Carl Sovinec, also drew partial academic-year support (over 6 months) and 0.63 months of summer support from the grant. This section summarizes the objectives and accomplishments for the technical topics that were addressed in this study. The set of objectives is taken from the final proposal that Schnack wrote for this study, and they focus on sawtooth modeling.

### 1. Test the efficacy of the integrated coupling of particle-kinetic and fluid models

Schnack's computations with the NIMROD code sought to model conditions in the DIII-D experiment that Choi and colleagues used to test [1] Porcelli's analytical model [2] for giant sawtooth triggering and saturation. The input data included equilibrium profiles that were fit to relevant instances during the DIII-D discharge, but the modeled energetic particle distribution only represents the distribution resulting from beam sources. The experiment used beams to inject 80 keV ions, and the radio frequency (RF) heating excited ions to many hundreds of keV, possibly close to 1 MeV. Both the maximum beam-particle energy and the fraction of energetic particles were varied in NIMROD computations that incorporate minority-species kinetics with a single-fluid representation of the majority species. As shown in Fig. 1, the linear growth rate of the internal kink mode is reduced to an increasing extent as the maximum energy of the particles is increased. However, even with numerical sub-stepping of the minority distribution information, computational turnaround is reduced with increasing particle energy, and it was not practical to attempt the energies that result from the RF excitation.

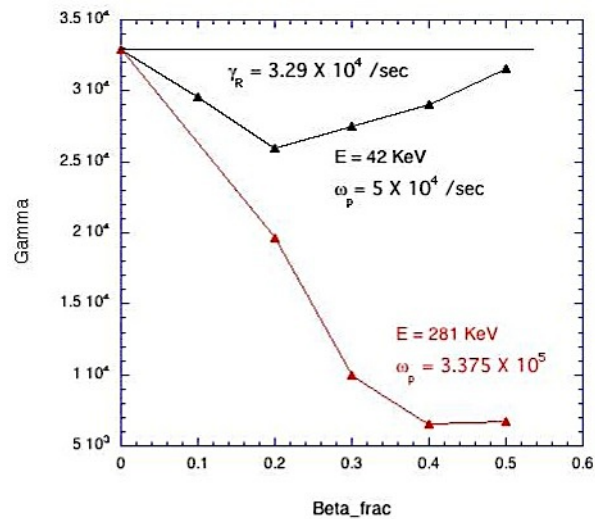


Figure 1. Results for the internal-kink growth rate as a function of the pressure-fraction of hot particles, as computed with the maximum hot particle energies of 42 keV and 281 keV.

2. Provide direction for improving the algorithms underlying these models

With the simulation-particle based approach to modeling kinetics, the drift orbits of the energetic particles need to be resolved, numerically. The computational cost increases with particle energy, because the number of computational steps increases as the particles move more quickly through the spatial mesh used for the fields. Schnack collaborated with other members of the NIMROD Team to find improvements and alternative strategies. This led to corrections in the particle module, but the cost issue for high-energy distributions in multi-scale problems seems inherent for simulation-particle based approaches. One alternative is modeling the energetic particles through drift kinetic computation in the Eulerian reference frame, where implicit methods are more tractable. Team member Eric Held had been developing Eulerian drift kinetics, and Schnack's findings prompted Held to add a variant for the energetic particles. Results on this sawtooth application, reported in Held's August 2015 summer Team meeting [presentation](#),<sup>1</sup> are encouraging.

3. Provide insight into the linear stability of the internal kink mode in the presence of energetic ions

The importance of particles excited by RF to very high energies is corroborated by results from calculations with energetic-particle kinetics and single-fluid modeling of majority species. See the following item for context.

4. Provide verification and validation of the NIMROD code by comparison with experiment and previous theoretical/computational studies

Validation of NIMROD's energetic particle modeling with respect to the giant sawtooth study in DIII-D was the motivation of the TDTFS study. The linear results showing residual growth of the internal kink instability at realistic energies for the beam particles is consistent with Choi's analysis, which showed that the RF-driven particles were critical for the DIII-D giant sawtooth observations [1]. However, the computations were not able to consider fully realistic conditions with the RF-driven component, nor were they able to address nonlinear evolution. Further work on validating giant sawtooth behavior awaits the more comprehensive and practical modeling that is being developed through Eulerian drift kinetics.

In addition to the energetic-particle effects, the relative drifting of electrons and ions and fast magnetic reconnection, which are described in two-fluid models, are important for internal kink dynamics. The former are described phenomenologically in Porcelli's sawtooth model, and two-fluid effects are within the scope of NIMROD's fluid algorithm. Two-fluid computations with the DIII-D profiles found the internal kink and high-wavenumber instabilities. The high-wavenumber instabilities were identified as fluid ion temperature gradient (ITG) modes. This finding prompted Schnack to coordinate a fluid/kinetic comparison of ITG in slab geometry with Jianhua Cheng, Scott Parker, and Daniel Barnes. They identified limits in parameter space on the applicability of two-fluid modeling to ITG, and their results are published in the 2013 Physics of Plasmas paper [3]. A related effort is Eric Howell's verification of cylindrical internal two-fluid kink with the analytical results of [4]. A small fraction of Howell's work and supervision from Sovinec were supported by this grant.

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<sup>1</sup> The link is to [https://nimrodteam.org/meetings/team\\_mgt\\_8\\_15/held\\_nimrod\\_summer.pdf](https://nimrodteam.org/meetings/team_mgt_8_15/held_nimrod_summer.pdf).

### 5. Provide insight into the non-linear evolution and final state of a giant sawtooth crash

Given the computational challenges in computing linear evolution of the DIII-D giant sawtooth with simulation-particle kinetics, nonlinear computations were not attempted during this study. This topic awaits further development with Eulerian drift kinetics.

### 6. Contribute to an improved overall integrated modeling capability for fusion plasmas

At the time that this study was proposed, it was understood that modeling giant sawteeth in conditions representative of DIII-D would be computationally challenging. Complete modeling was to include energetic-ion kinetics, two-fluid majority species dynamics, and a large separation of transport and Alfvén-wave timescales. While the results on giant sawteeth physics are limited, valuable lessons for integrated modeling were learned. The computational practicalities of modeling energetic ions with simulation particles over the timescales of macroscopic dynamics are not tractable, and implicit (or semi-implicit) Eulerian kinetic computation has emerged as the preferred approach for integrated simulation. While fluid modeling alone was never thought to be sufficient, we've learned that there are classes of two-fluid instabilities at large wavenumber that are not stabilized by drift effects. Depending on profiles, this may lead to unphysical dynamics if large-orbit effects would stabilize the same modes in a full kinetic model, an example for ITG and its implications are discussed in [3]. Work on two-fluid interchange reached similar conclusions that are reported in [5].

Another aspect of integrated modeling that was partially supported by this effort is modeling RF suppression of magnetic islands. Schnack supervised a study based on single-fluid modeling with a simple representation of RF-driven current. (Then-postdoctoral associate Thomas Jenkins was supported by the Center for Simulation of RF Wave Interaction with Magnetohydrodynamics, and Schnack was an unfunded participant in CSWIM.) The study investigated requirements for suppression through current-profile, hence stability-parameter  $\Delta'$ , effects in nonlinear single-fluid simulations. This work represented a first step in comprehensive modeling of RF-MHD dynamics.

Finally, Sovinec used a small amount of the remaining funds to support preparation of an invited tutorial presentation on simulating macroscopic dynamics in magnetic confinement systems. The [presentation](#)<sup>2</sup> is the final item in the products list and is available through the NIMROD Team website.

### References

- [1] M. Choi, A. D. Turnbull, V. S. Chan, *et al.*, Phys. Plasmas **14**, 112517 (2007).
- [2] F. Porcelli, D. Boucher, and M. N. Rosenbluth, Plasma Phys. Control. Fusion **38**, 2163 (1996).
- [3] D. D. Schnack, J. Cheng, D. C. Barnes, and S. E. Parker, Phys. Plasmas **20**, 062106 (2013).
- [4] L. Zakharov and B. Rogers, Phys. Fluids B **4**, 3285 (1992).
- [5] P. Zhu, D. D. Schnack, F. Ebrahimi, *et al.*, Phys. Rev. Lett. **101**, 085005 (2008).

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<sup>2</sup> The link is to [https://nimrodteam.org/presentations/sovinec\\_IEEE16\\_PSminicourse.pdf](https://nimrodteam.org/presentations/sovinec_IEEE16_PSminicourse.pdf).

### 3. Products

#### Journal Publications

1. Batchelor, DB, D'Azevedo, E, Bateman, G, Bernholdt, DE, Berry, LA, Bonoli, PT, Bramley, R, Breslau, J, Chance, M, Chen, J, Choi, M, Elwasif, W, Fu, GY, Harvey, RW, Houlberg, WA, Jaeger, EF, Jardin, SC, Keyes, D, Klasky, S, Kruger, S, Ku, LP, McCune, D, Ramos, J, Schissel, DP, Schnack, D, Wright, JC, "[Integrated physics advances in simulation of wave interactions with extended MHD phenomena](#)," Journal of Physics Conference Series **78**, 012003 (2007).
2. Batchelor, DB, Alba, G, Bateman, G, Bernholdt, DE, Berry, L, Bonoli, PT, Bramley, R, Breslau, J, Chance, M, Chen, J, Choi, M, Elwasif, W, Fu, G, Harvey, RW, Jaeger, E, Jardin, Jenkins, T, SC, Keyes, D, Klasky, S, Kruger, S, Ku, L, Lynch, V, McCune, D, Ramos, J, Schissel, D, Schnack, D, Wright, J, "[Simulation of wave interactions with MHD](#)," Journal of Physics Conference Series **125**, 12039 (2008).
3. P. Zhu, D. D. Schnack, F. Ebrahimi, E. G. Zweibel, M. Suzuki, C. C. Hegna, and C. R. Sovinec, "[Absence of Complete Finite-Larmor-Radius Stabilization in Extended MHD](#)," Physical Review Letters **101**, 085005 (2008).
4. T. G. Jenkins, S. E. Kruger, C. C. Hegna, D. D. Schnack, and C. R. Sovinec, "[Calculating Electron Cyclotron Current Drive Stabilization of Resistive Tearing Modes in a Nonlinear MHD Model](#)," Physics of Plasmas **17**, 012502 (2010).
5. I. V. Khalzov, F. Ebrahimi, D. D. Schnack, and V. V. Mirnov, "[Minimum energy states of the cylindrical plasma pinch in single-fluid and Hall magnetohydrodynamics](#)," Physics of Plasmas **19**, 012111 (2012).
6. D. D. Schnack, J. Cheng, D. C. Barnes, and S. E. Parker, "[Comparison of kinetic and extended magnetohydrodynamics computational models for the linear ion temperature gradient instability in slab geometry](#)," Physics of Plasmas **20**, 062106 (2013).
7. J. Scheffel, D. D. Schnack, and A. A. Mirza, "[Static current profile control and RFP confinement](#)," Nuclear Fusion **53**, 113007 (2013).

#### Conference Presentations

1. D.D. Schnack, D.C. Barnes, D.P. Brennan, A.Y. Pankin, and C.R. Sovinec, "Validation of 2-Fluid and Gyro-Viscous Terms in Nimrod," BAPS.2006.DPP.UP1.40, presented at the 48th Annual Meeting of the Division of Plasma Physics, American Physical Society, Philadelphia, PA, Oct. 30-Nov. 3, 2006.
2. D. D. Schnack, P. Zhu, F. Ebrahimi, E. G. Zweibel, M. Suzuki, C. C. Hegna, and C. R. Sovinec, "The Absence of Complete FLR Stabilization in Extended MHD," Poster 1B.10, presented at the 2008 International Sherwood Fusion Theory Conference, Boulder, CO, Mar. 30-Apr. 2, 2008.
3. M. Suzuki, F. Ebrahimi, and D. D. Schnack, "Linear stability of resistive interchange modes in a cylindrical reversed-field pinch plasma," BAPS.2008.DPP.NP6.28, presented at the 50th Annual Meeting of the Division of Plasma Physics, American Physical Society, Dallas, TX, Nov. 17-21, 2008.

4. F. Ebrahimi, and D. D. Schnack, "Finite pressure effects in the Reversed Field Pinch," BAPS.2008.DPP.CP6.70, presented at the 50th Annual Meeting of the Division of Plasma Physics, American Physical Society, Dallas, TX, Nov. 17-21, 2008.
5. D. D. Schnack, S. E. Kruger, C. C. Kim, and A. D. Turnbull, "Numerical Simulation of Giant Sawteeth in Tokamaks Using the NIMROD Code," BAPS.2009.DPP.NP8.114, presented at the 51st Annual Meeting of the Division of Plasma Physics, American Physical Society, Atlanta, GA, Nov. 2-6, 2009.
6. D. D. Schnack, E. D. Held, C. C. Kim, S. E. Kruger and A. D. Turnbull, "Numerical Simulation of the Interaction of the Internal Kink Mode with an Energetic Ion Species using the NIMROD Code," presented at the 2010 International Sherwood Fusion Theory Conference, Seattle, WA, Apr. 19-21, 2010.
7. D. D. Schnack, S. E. Kruger, C. C. Kim, and A. D. Turnbull, "MHD and 2-Fluid Stability of DIII-D Shot #96043 using the NIMROD Code," BAPS.2010.DPP.BP9.107, presented at the 52nd Annual Meeting of the Division of Plasma Physics, American Physical Society, Chicago, IL, Nov. 8-12, 2010.
8. F. Ebrahimi, D. D. Schnack, B. E. Chapman, and K. Caspary, "Two-fluid and gyroviscosity effects on pressure-driven instabilities," BAPS.2010.DPP.PP9.45, presented at the 52nd Annual Meeting of the Division of Plasma Physics, American Physical Society, Chicago, IL, Nov. 8-12, 2010.
9. D. Schnack, P. Zhu, C. Sovinec, and C. Hegna, "The Linear Stability of the ITG Mode using the NIMROD Code," BAPS.2011.DPP.GP9.13, presented at the 53rd Annual Meeting of the Division of Plasma Physics, American Physical Society, Salt Lake City, UT, Nov. 14-18, 2011.
10. D. D. Schnack, J. Cheng, D. C. Barnes, and S. E. Parker, "Comparison of Kinetic and Extended MHD Computational Models for the Ion Temperature Gradient Instability in Slab Geometry," Poster III.12, presented at the 2013 International Sherwood Fusion Theory Conference, Santa Fe, NM, Apr. 15-17, 2013.
11. K. J. McCollam, D. J. Den Hartog, J. A. Reusch, J. S. Sarff, J. P. Sauppe, D. D. Schnack, C. R. Sovinec, and S. Masamune, "MHD validation studies for RFPs," BAPS.2013.DPP.CP8.68, presented at the 55th Annual Meeting of the Division of Plasma Physics, American Physical Society, Denver, CO, Nov. 11-15, 2013.
12. E. C. Howell and C. R. Sovinec, "Two-fluid benchmarking of the 1/1 internal kink," Poster P3.025, presented at the 2016 International Sherwood Fusion Theory Conference, Madison, WI, Apr. 4-6, 2016.
13. C. R. Sovinec, "Simulation of macroscopic dynamics in magnetically confined plasma" (invited tutorial), Plasma Simulation Minicourse, IEEE International Conference on Plasma Science, Banff, Canada, June 23-24, 2016.