The Shear Modulus of Particle-Laden Foam

SAND2006-6197C

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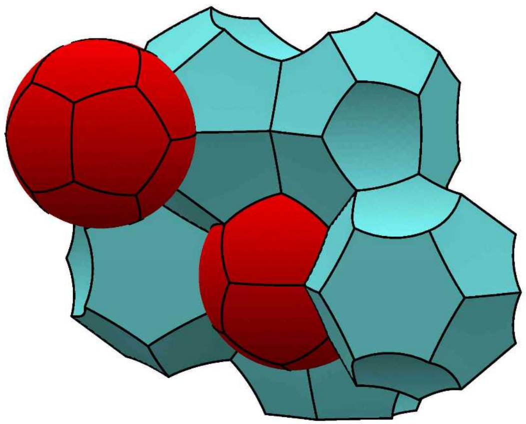

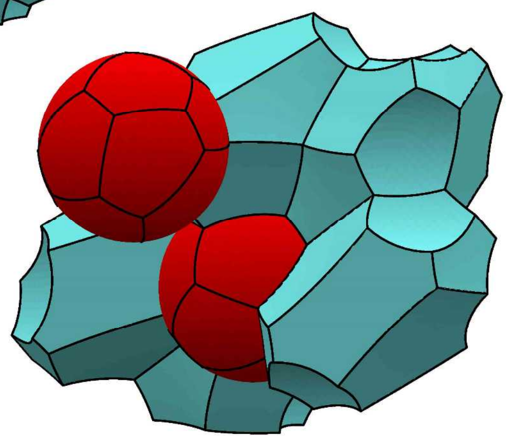

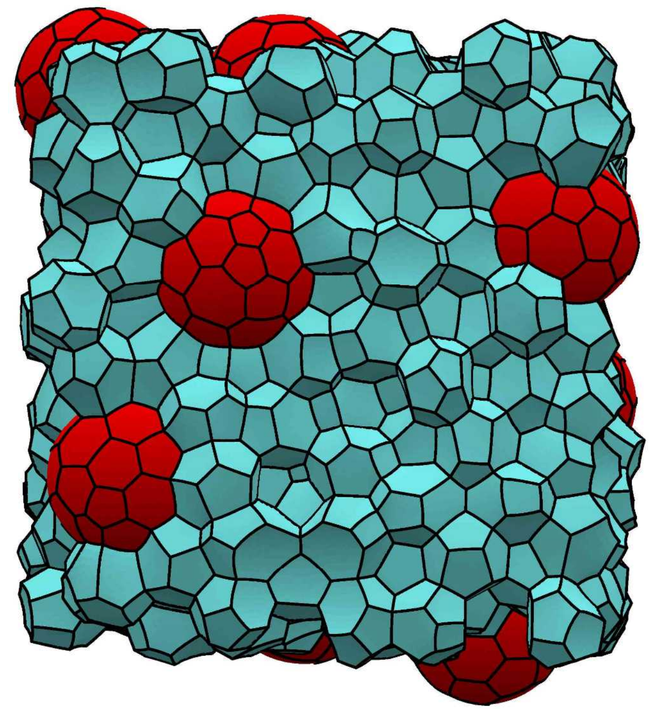

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Overview

Consider rigid spheres suspended in an “incompressible” elastic solid.

$$G/G_0 = 1 + 5/2\ \phi$$

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Investigate scale effects when the elastic medium is a dry soap froth and $R_P \approx R_B$

Experiments at the University of Marne-la-Vallee

Model rigid spheres as drops with large surface tension

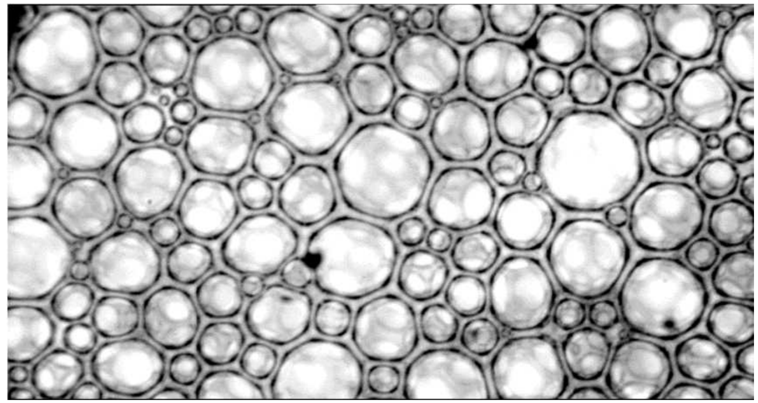
Surface Evolver simulations

- Weaire-Phelan and random monodisperse foams
- $R_P = R_B$, $2R_B$

Model rigid spheres

Foam and particles

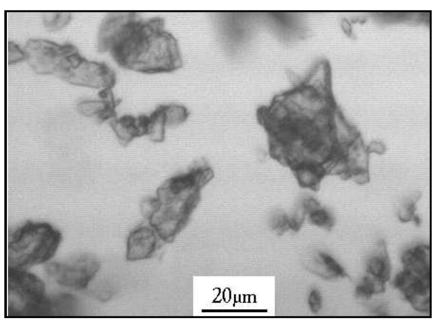
Gillette shaving cream



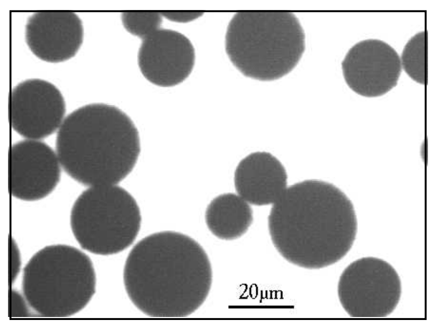
Gas volume fraction 92%

Bubble size 20 μm

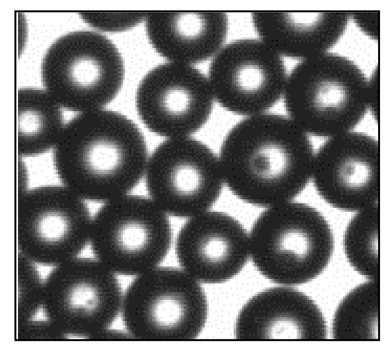
Talc platelets 10 μm



Carbon 25 μm



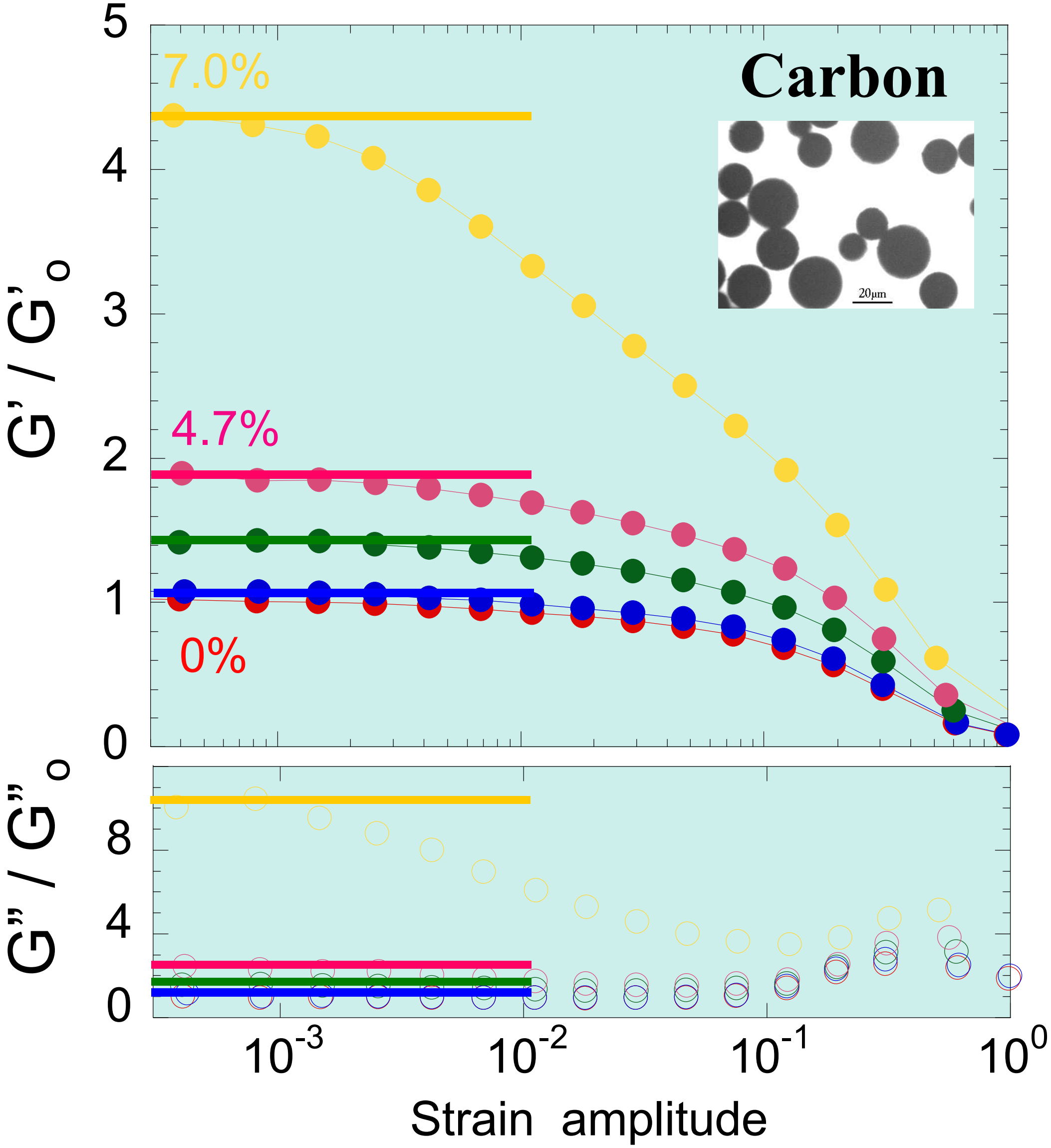
Glass



Size
30 μm
40 μm
60 μm
75 μm

Foam and hydrophilic particles mixed by whipping

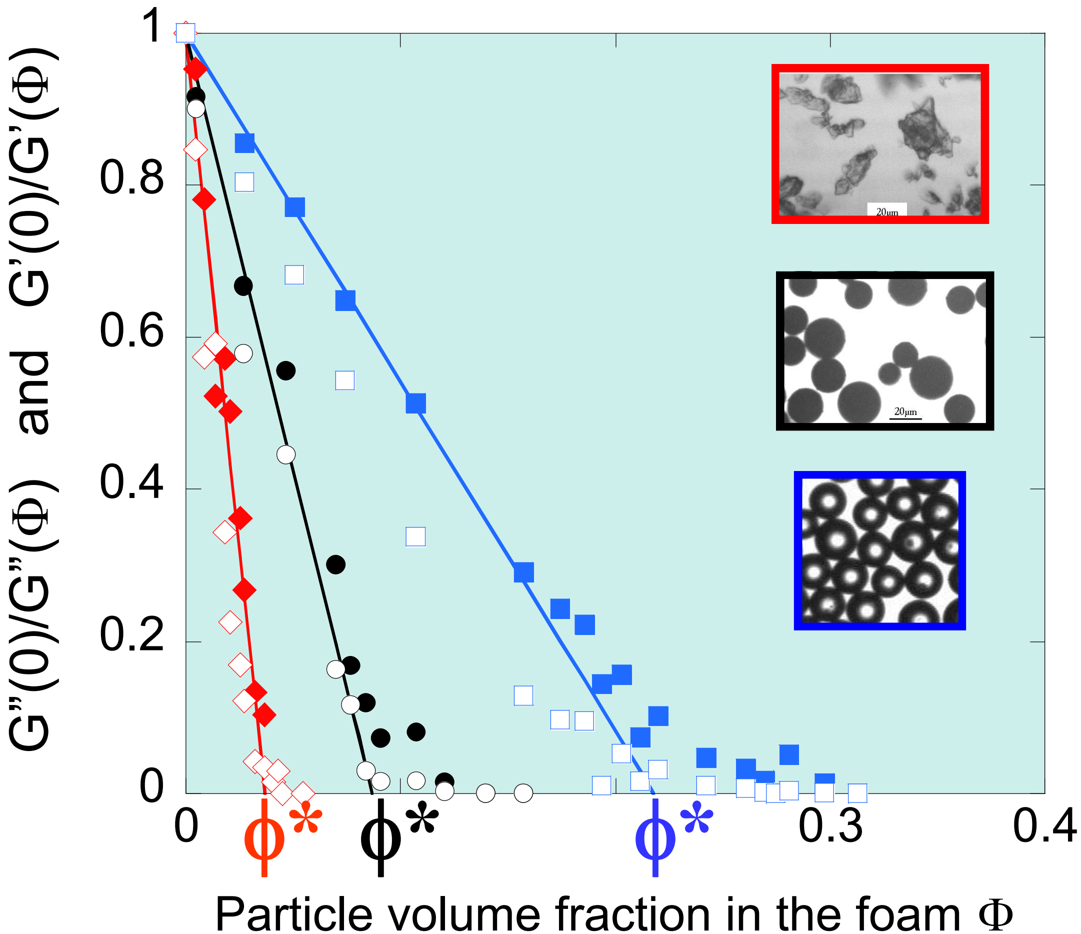
Particles strongly increase the complex shear modulus



Foam with particles
$G^* = G' + i\,G''$

Foam without particles
at low strain amplitude
$G^*_o = G'_o + i\,G''_o$

Smaller particles magnify the effect

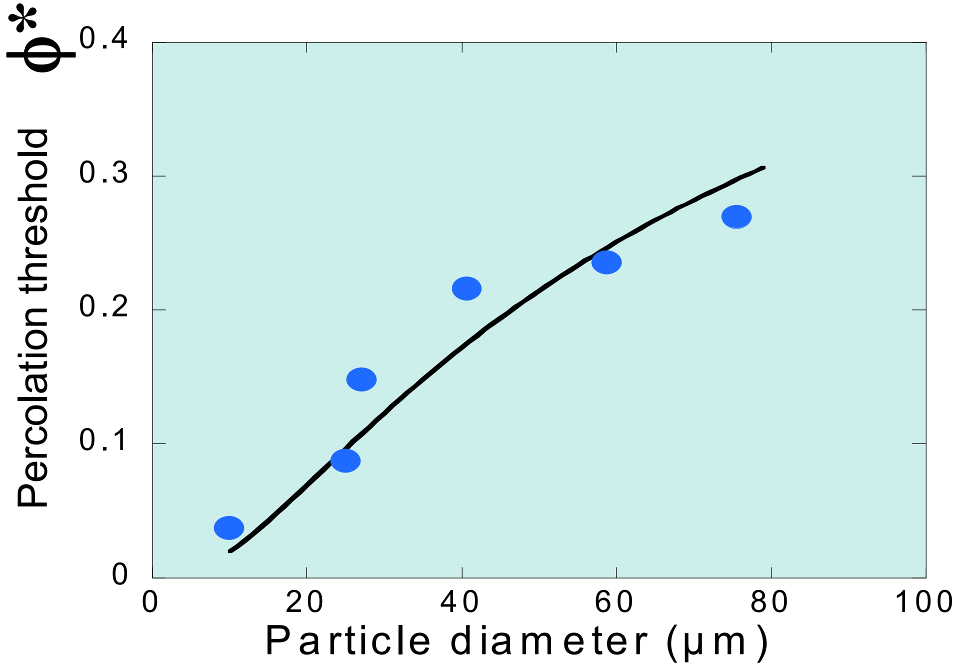


$$\frac{G^*(0)}{G^*(\Phi)} \cong 1 - \frac{\Phi}{\Phi^*}$$

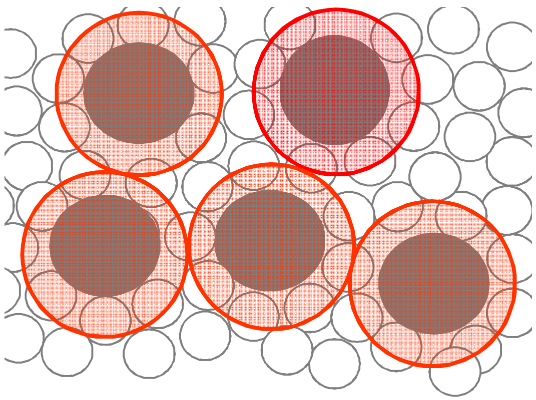
Effective Medium Theory:
Garboczi & Thorpe 1986;
Torquato 2002

G* scales with particle volume fraction in agreement with the rigidity percolation model in the superelastic limit.

Rigidity percolation threshold



At ϕ^* the average distance between particles is of the order of one bubble diameter.



$$\Phi^* = \Phi^*_{eff}\left(\frac{D_{particle}}{D_{particle}+D_{bubble}}\right)^3$$

with $\Phi_{\mathrm{eff}}{}^* \cong 0.64$

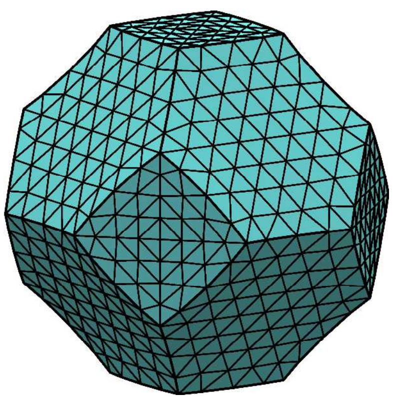
Surface Evolver

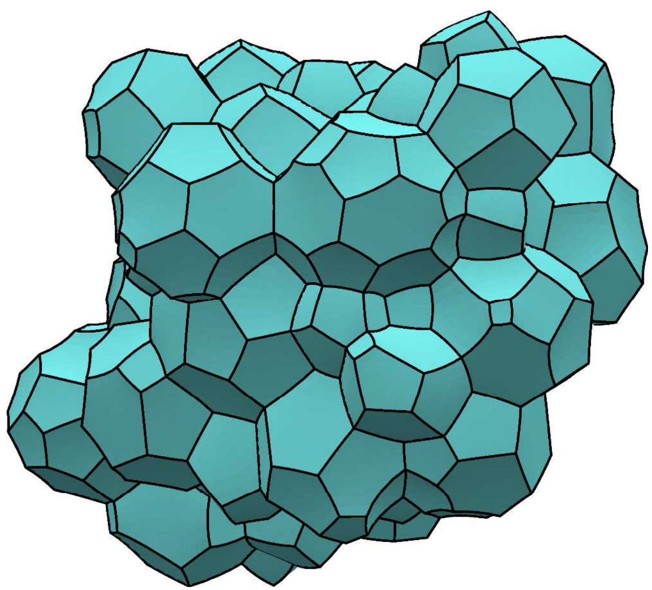
An interactive program for modeling liquid surfaces shaped by various forces (surface tension) and constraints (spatial periodicity and cell volumes). The surface evolves toward minimal energy by simulating the process of evolution by mean curvature.

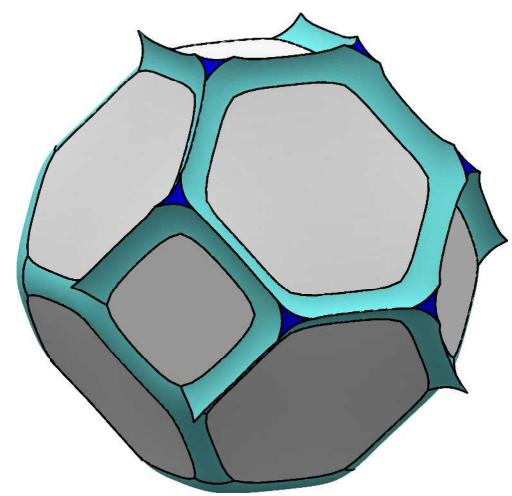
Developed by Ken Brakke, Mathematics Department, Susquehanna University

Free download

K.A. Brakke (1992) *Exp Math* **1**, 141.



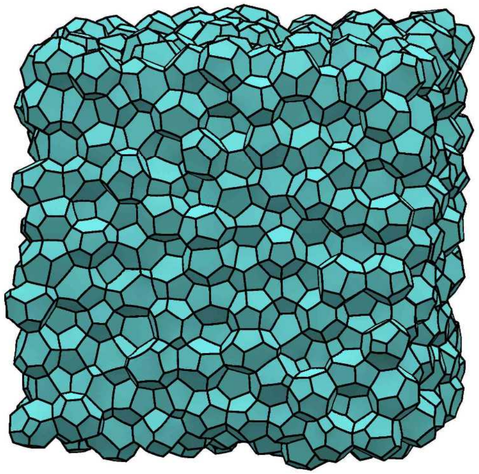




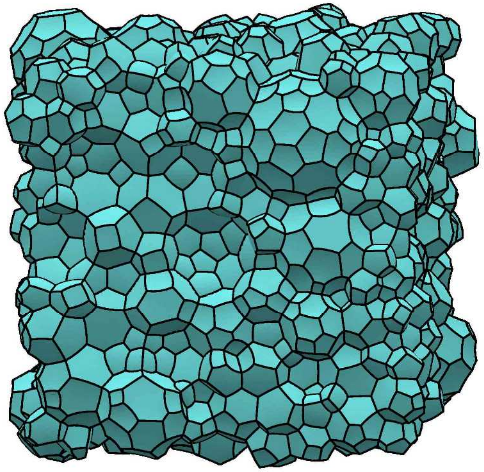
Random Foams

Kraynik, Reinelt & van Swol (2003) *Phys Rev E* **67**, 031403;

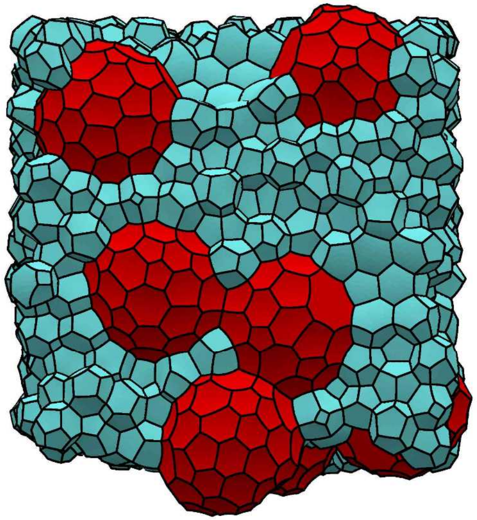
(2004) *Phys Rev Lett* **93**, 208301; (2005) *Colloids Surfaces A* **263** 11-17.



Monodisperse



Polydisperse



Bidisperse

Spatially periodic structure

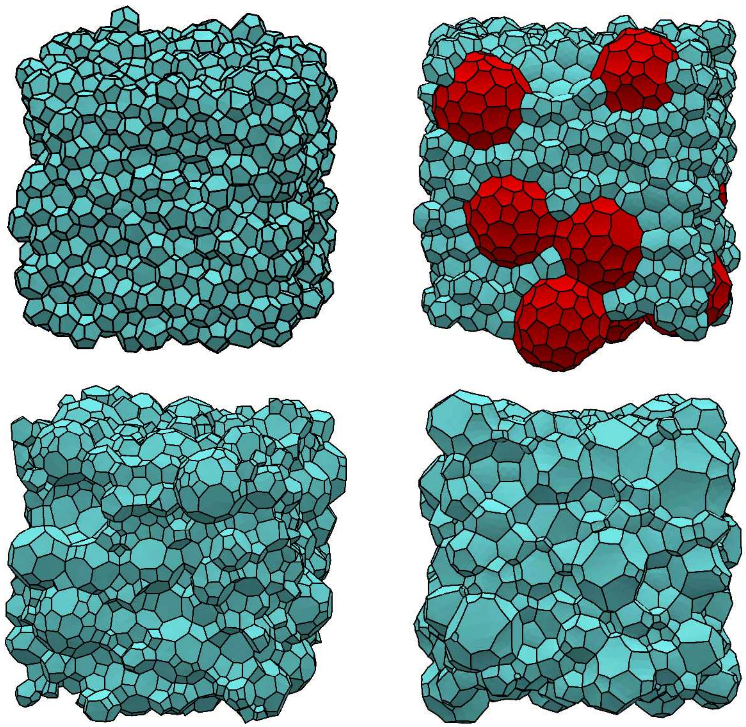
1728 cells

Cell volumes vary by three orders of magnitude

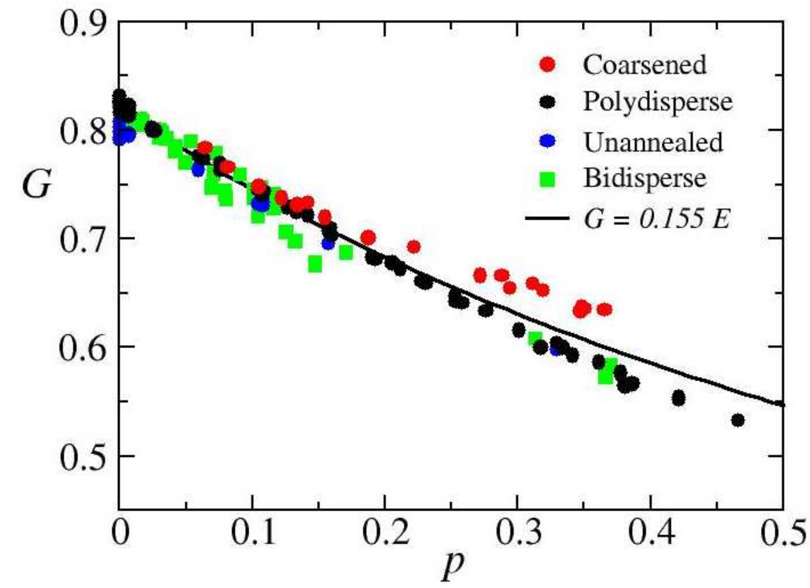
Shear Modulus of Random Foam

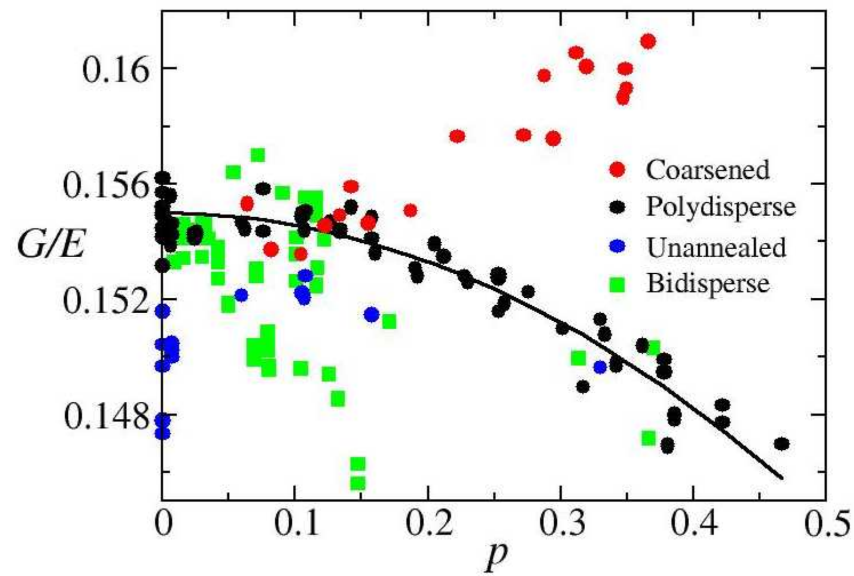
$$G \approx 0.155\,E = 0.512\,\frac{\sigma}{R_{32}}$$

$$E = 3.30\,\frac{\sigma}{R_{32}} = \frac{5.32}{(1+p)}\,\frac{\sigma}{\langle V\rangle^{\frac{1}{3}}}$$



G depends on polydispersity and the shape of the cell-size distribution

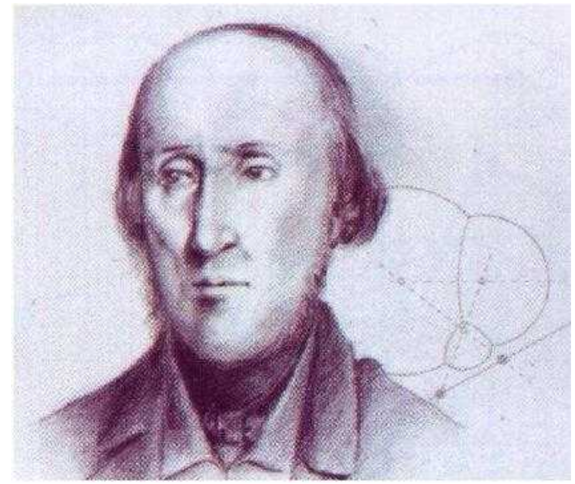




Plateau's Laws for Equilibrium Structure

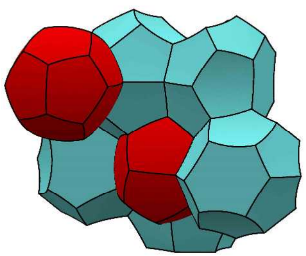
Ideal soap froth: 2D surfaces (with uniform surface tension σ) define trivalent polyhedral cells.

1) Each film has constant mean curvature: $(R_1^{-1} + R_2^{-1})$
2) Three films meet at 120° angles at cell edges
3) Four edges meet at tetrahedral angles: *acos*(-1/3) = 109.47°

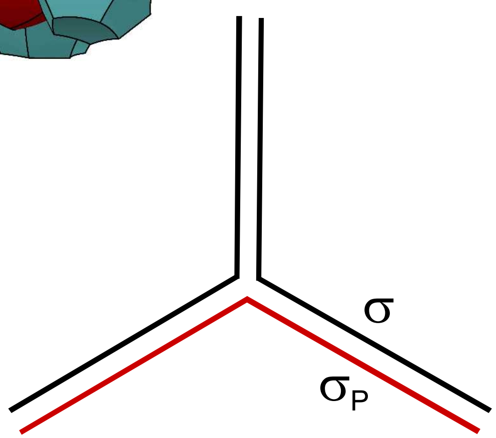


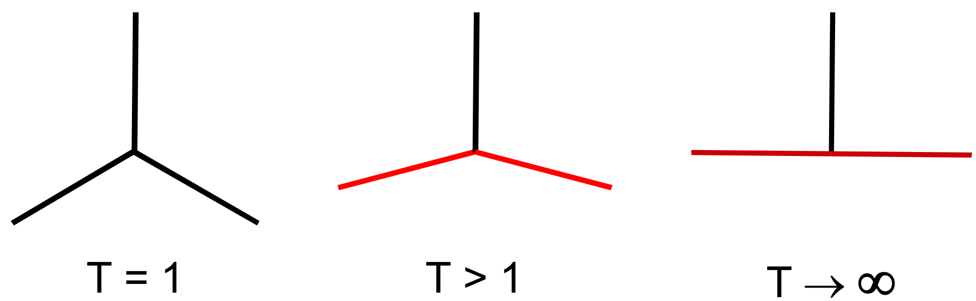
Courtesy of Weaire and Hutzler

J.A.F. Plateau (1873) "Statique Experimentale et Theorique des Liquides Soumis aux Seules Forces Moleculaires," Gauthier-Villard, Paris.



Excess film tension T: $2T\sigma = \sigma + \sigma_P$

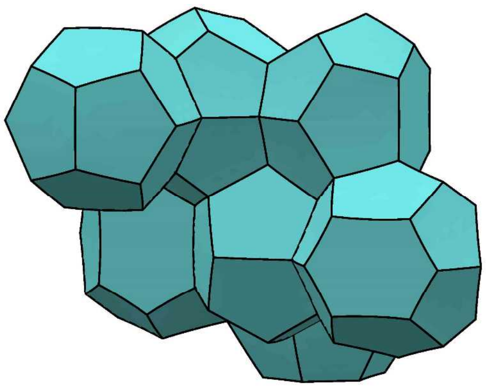




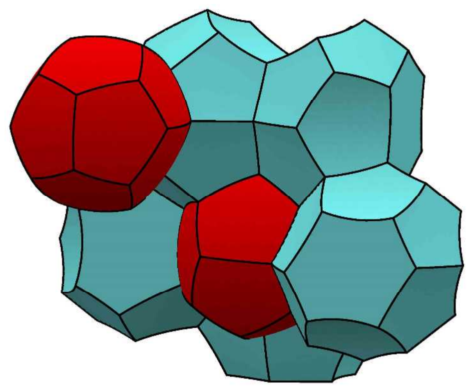
The foam films "slip" along the particle surface.

Weaire-Phelan foam with high-tension drops: $V_P = V_B$

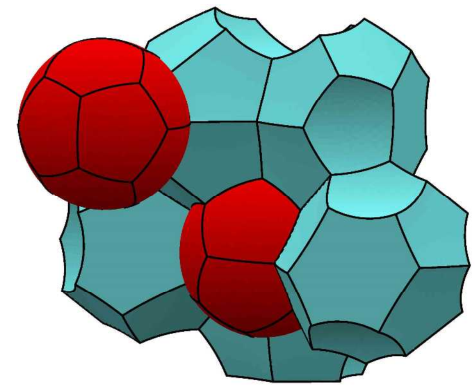
Film tension: $2T\sigma = \sigma + \sigma_P$



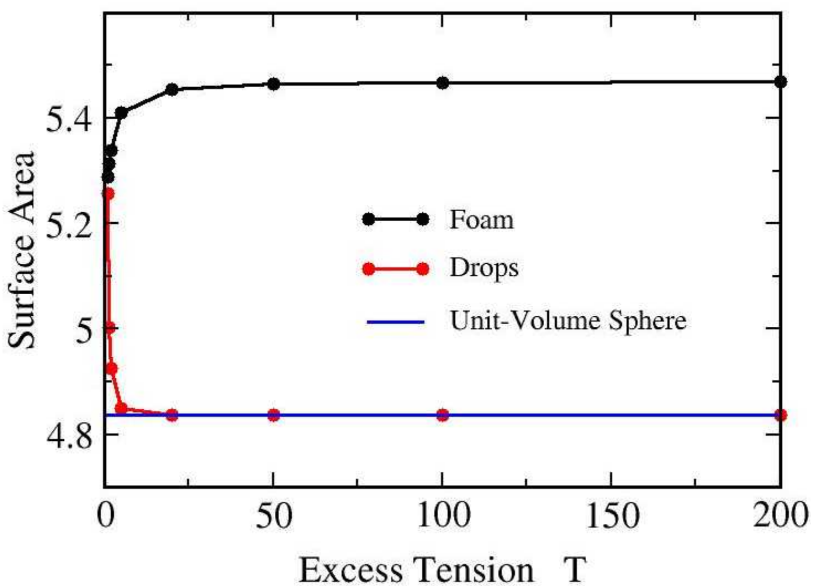
T = 1

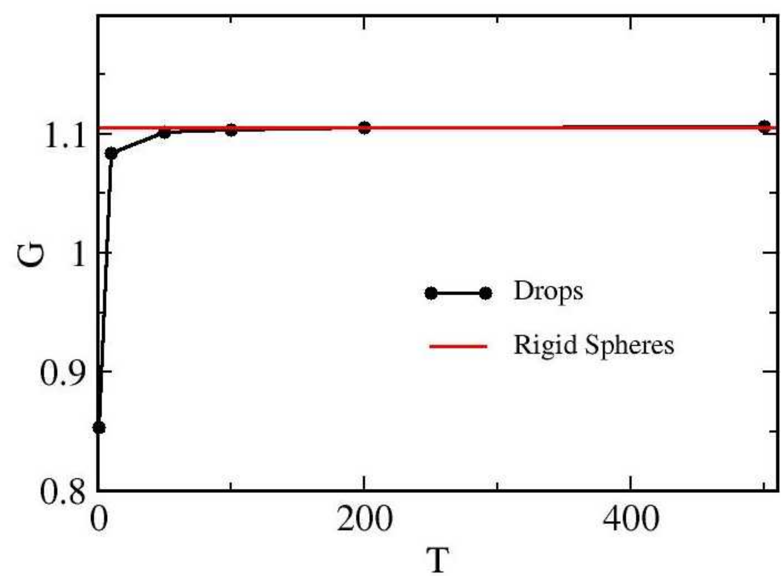


T = 2

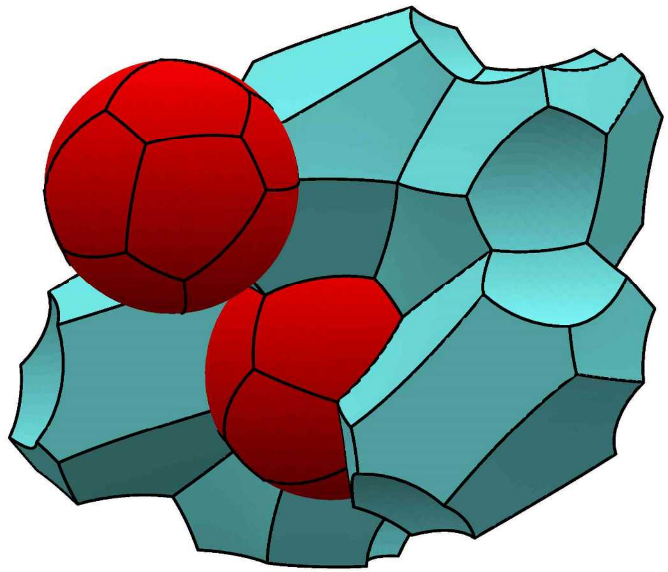


T = 20



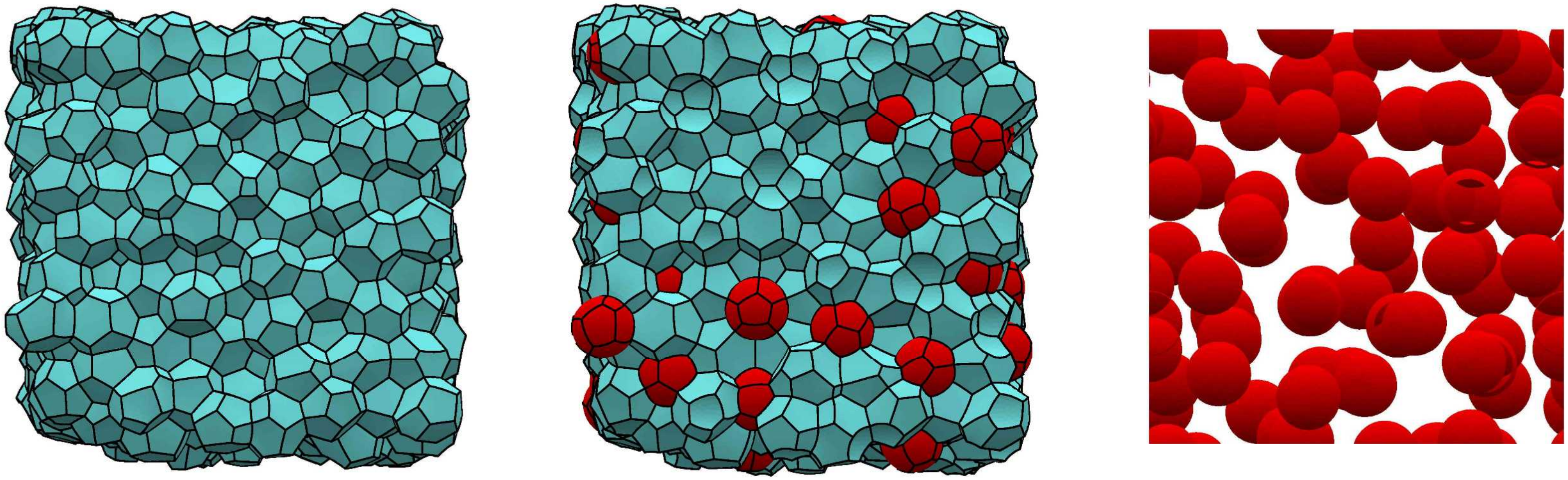


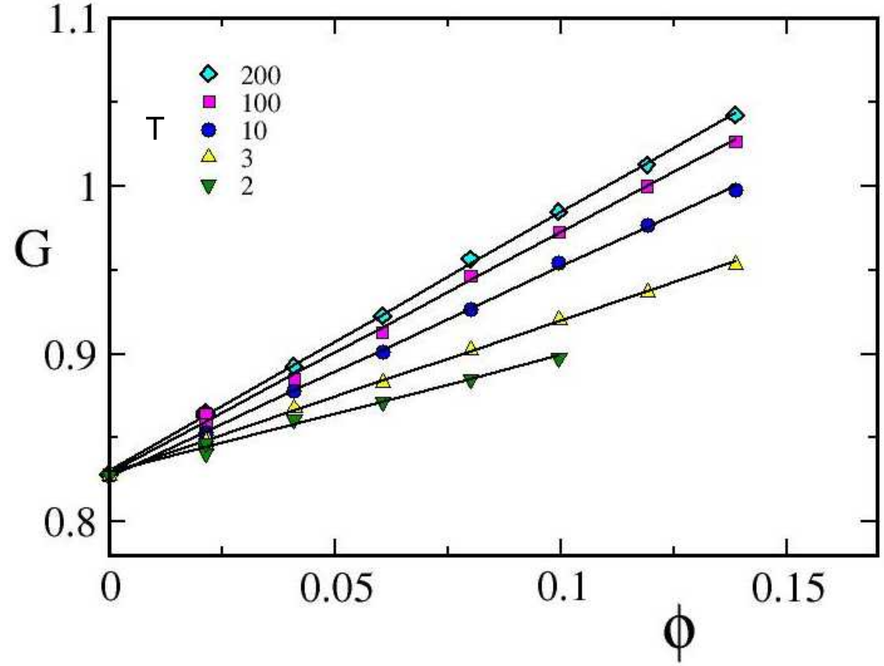
Evaluating the Foam Stress



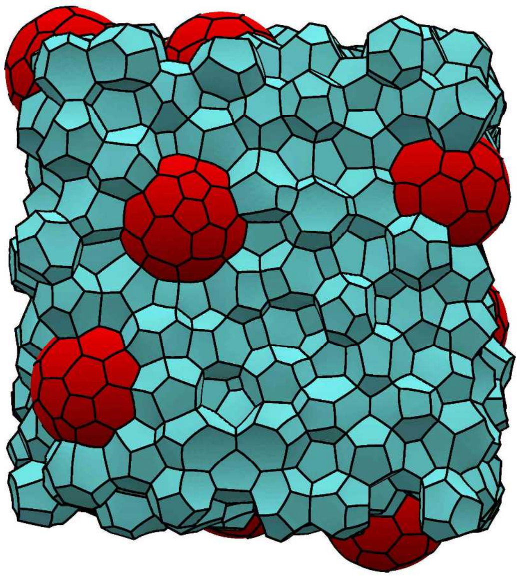
$$\Sigma_{ij} = -\frac{1}{V} \sum_k p_k V_k \, \delta_{ij} + \frac{1}{V} \sum_k \iint_{\mathcal{S}_k} \sigma \, (\delta_{ij} - n_i n_j) \, ds$$

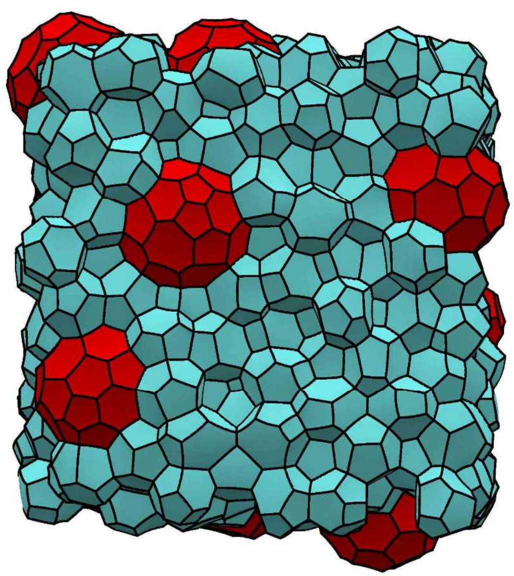
Random monodisperse foam with high-tension drops: $V_P = V_B$

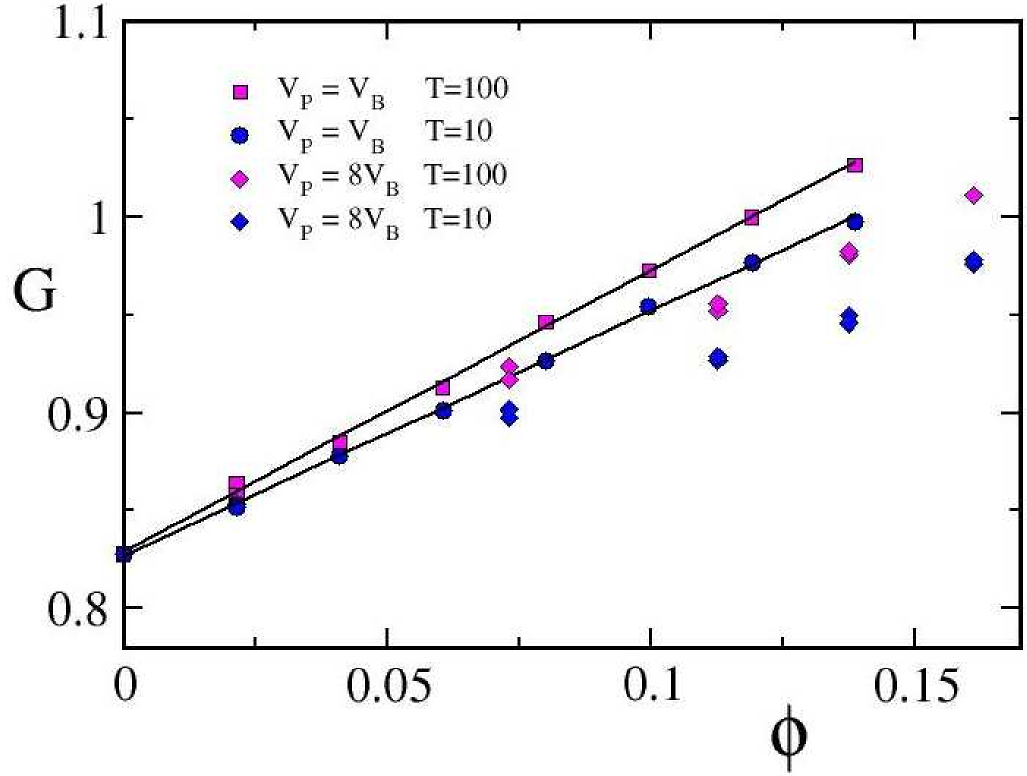




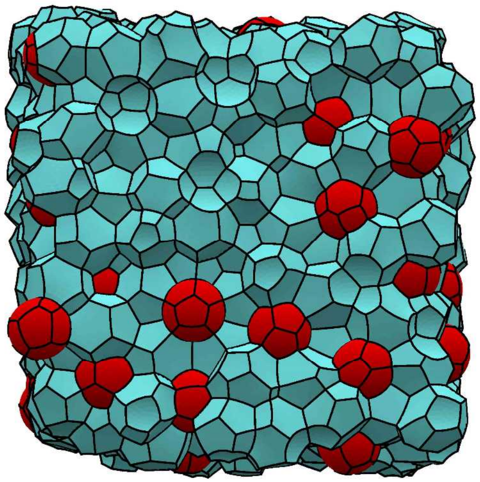
Random monodisperse foam with high-tension drops: $V_P = 8\ V_B$



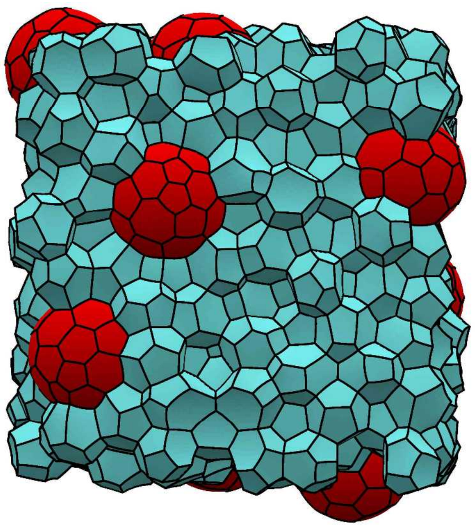




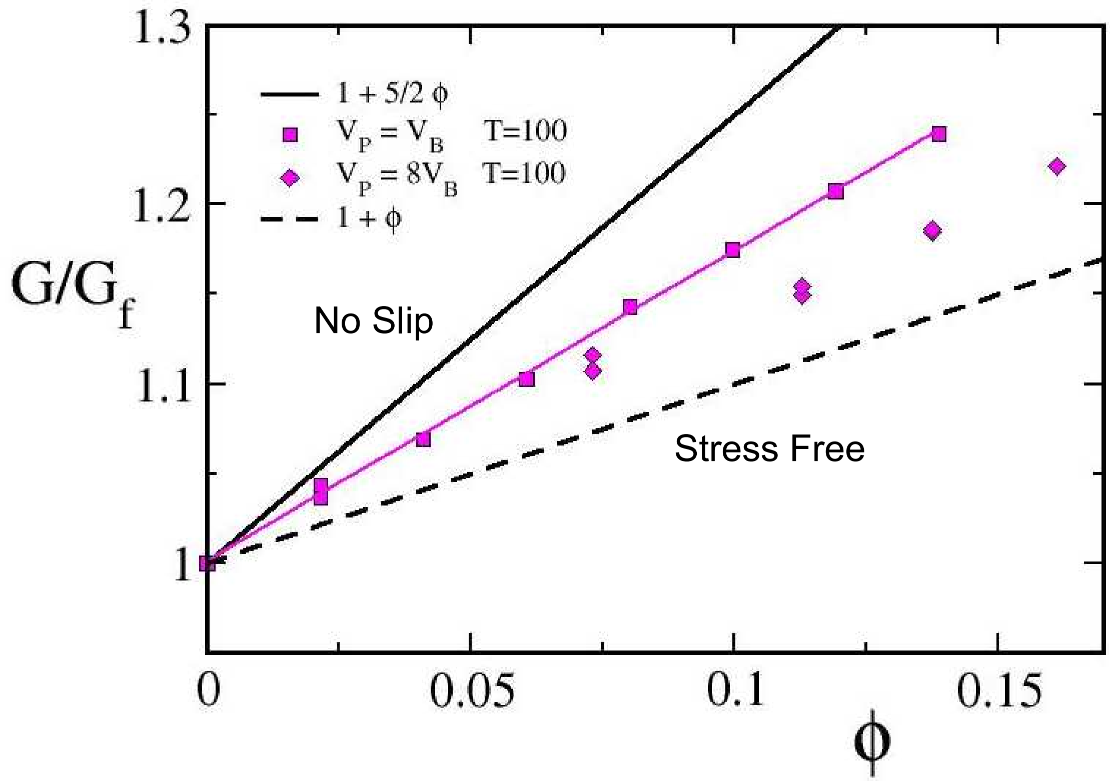
Random monodisperse foam with high-tension drops



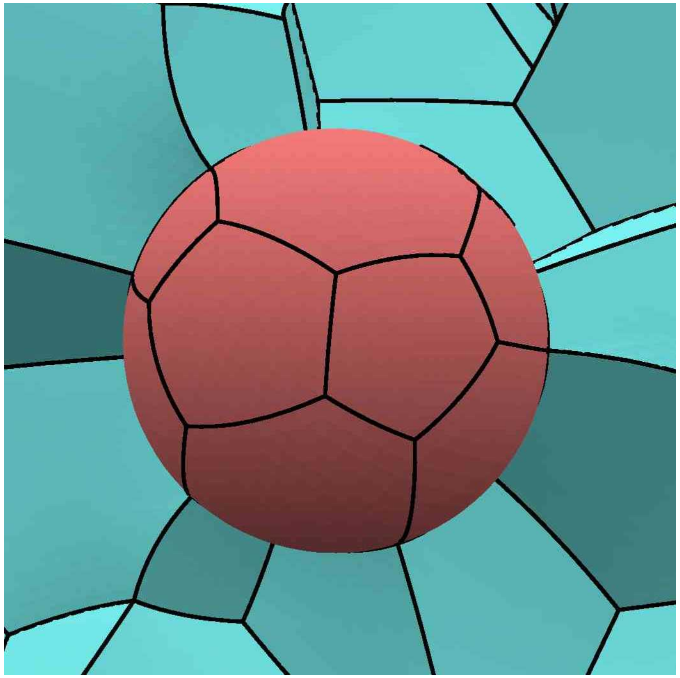
$V_P = V_B$



$V_P = 8\ V_B$

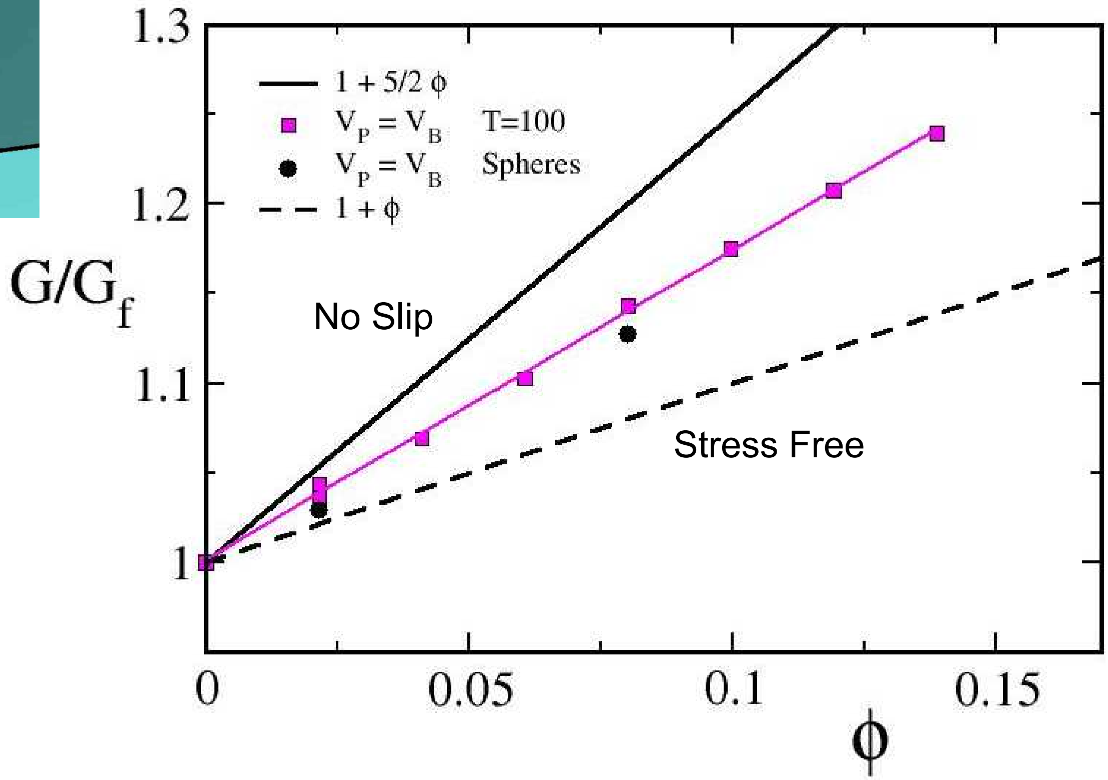


Random monodisperse foam with rigid spheres



Force-free particles

The surface tension and pressure on the spheres contributes to the foam stress.

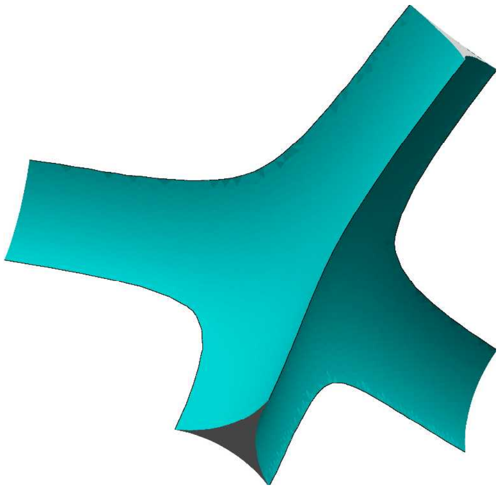


Concluding Remarks

Smaller particles increase stiffness, in agreement with the experiments.

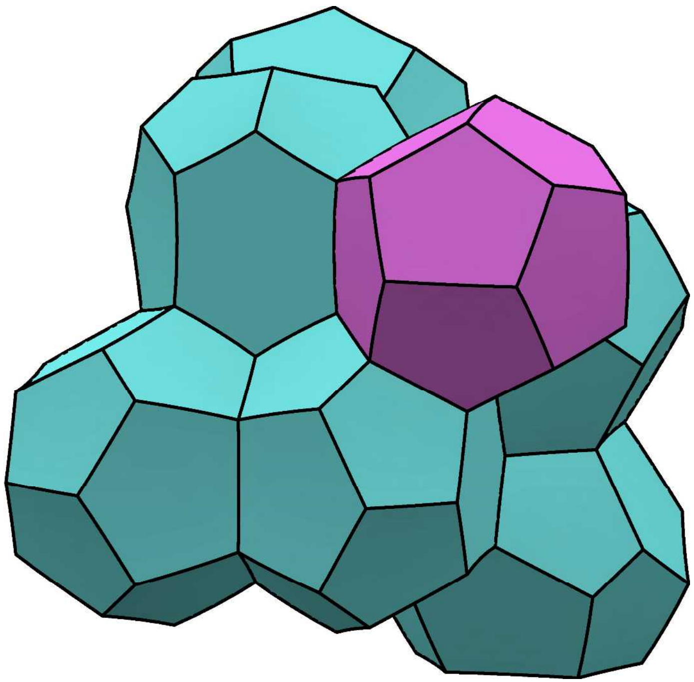
The model under predicts the experiments

- Slip ?
- Wet foam vs. dry foam ?
  - Plateau borders and liquid bridges
- Inter-particle forces ?



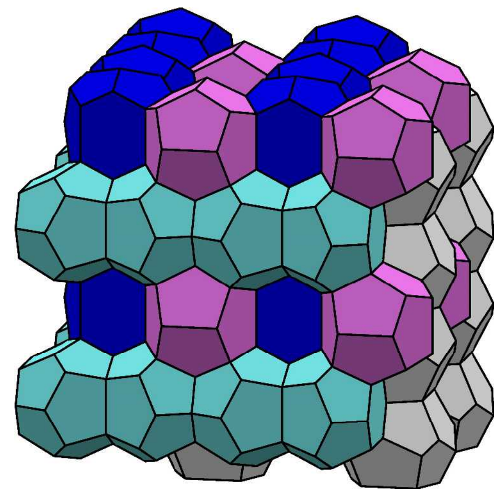
Weaire-Phelan Foam — the best monodisperse foam (lowest surface area)

D. Weaire & R. Phelan (1994) *Phil Mag Lett* **69**, 107.



A15

six 14-hedra (0-12-2)
two dodecahedra (0-12-0)



Cubic Symmetry

Tetrahedrally Closest Packed (TCP) Structure
or Frank-Kasper Structure