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Vibration-Induced Rectified Motion of a Piston in a Liquid-Filled Cylinder with Bellows to Mimic Gas Regions

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Vibration-Induced Rectified Motion

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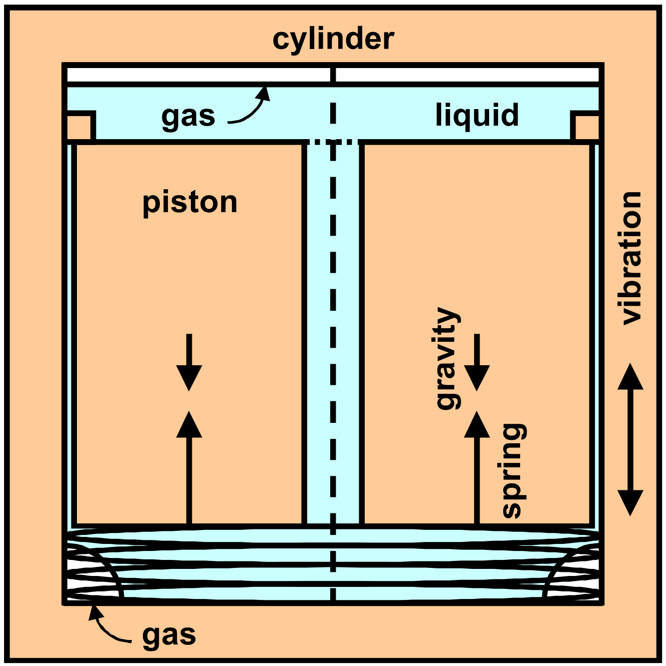


Simple fluid-solid dynamical system

- Piston moves vertically in cylinder
- Viscous liquid & gas fill open space
- Spring is stronger than gravity
- Cylinder is vertically vibrated

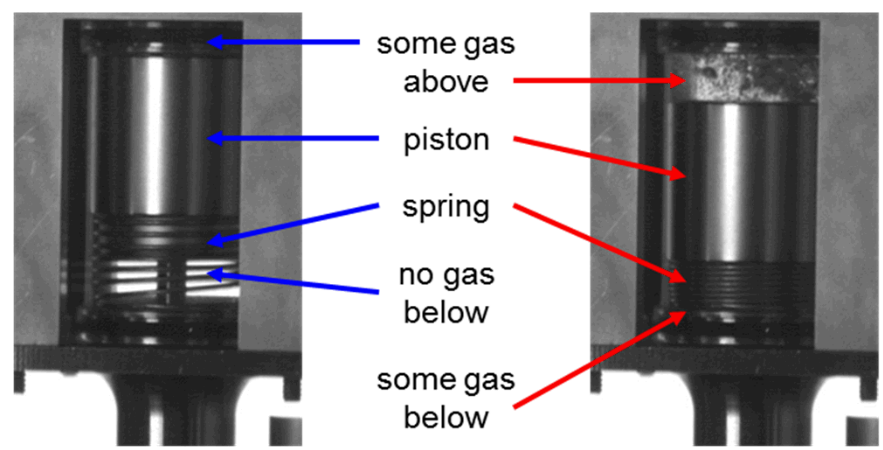
Vibration makes piston move down

- Identify rectification mechanisms
- Upper & lower gas regions are key

Dynamical model of piston and bubble motion

$$M_P \frac{dU_P}{dt} = \cdots$$

$$M_B \frac{dU_B}{dt} = \cdots$$



Vibration off
Spring supports piston

Vibration on
Piston compresses spring

Steps for Rectified Piston Motion

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Upper & lower gas regions → **Strong system resonance** → **Large force on piston**

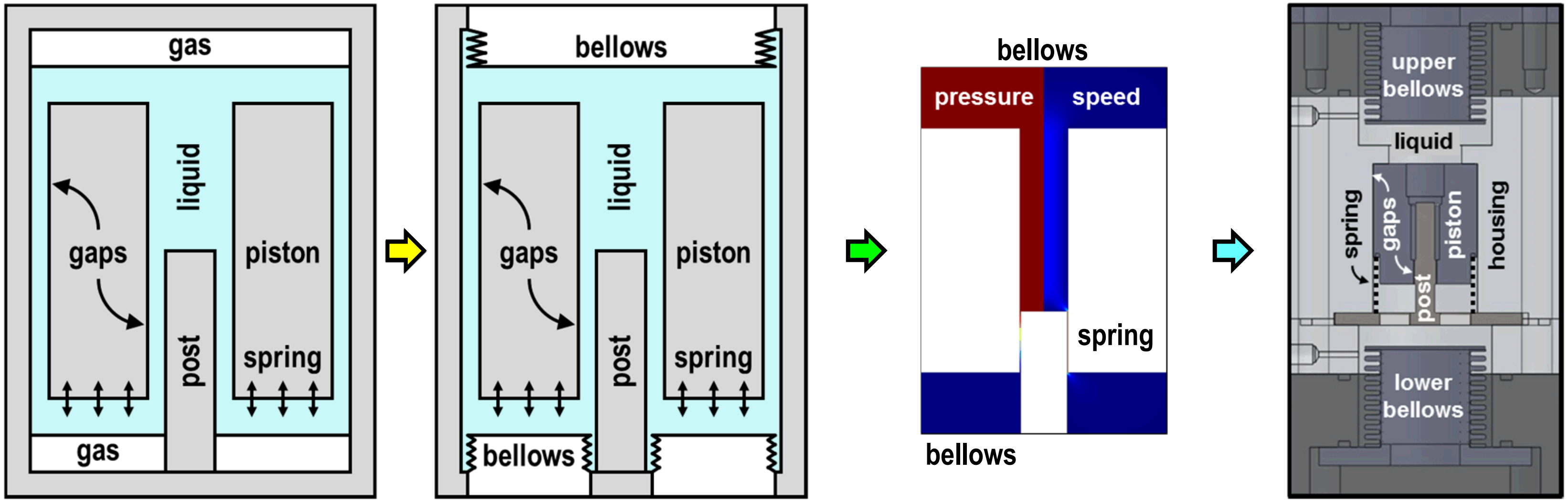
Three steps are needed to produce rectified piston motion

1. Upper and lower gas regions must both be present
   - Lower region is from trapped gas or rectified bubble motion
   - Regions make a gas spring, giving a new degree of freedom
2. Large resonance must be excited for piston-gas system
   - Seems counterintuitive since damping is very large
   - Viscous liquid must be forced through narrow gaps
3. Nonlinear effects produce large net force on piston
   - Quadratic terms become large at resonance

Investigate Step 2: large resonance despite large damping

- First step, rectified bubble motion, already investigated

Analogous Piston-Bellows System



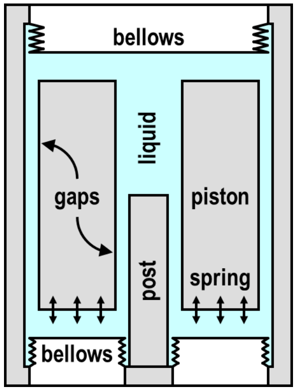
Replace piston-gas system with piston-bellows system

- Upper & lower bellows mimic upper & lower gas regions
  - Pressure-volume dependence like gas but well defined
  - Dynamics like gas but without complicated free surfaces
- Upper and lower bellows oscillate vertically like piston

Understand behavior of this simplified dynamical system

- Ultimately extend analysis to corresponding experiment

Dynamics of Liquid, Piston, Bellows



Liquid

$$\frac{\partial}{\partial \mathbf{x}}\cdot\mathbf{u}=0$$

$$\rho\left(\frac{\partial}{\partial t}+\mathbf{u}\cdot\frac{\partial}{\partial \mathbf{x}}\right)\mathbf{u}=\frac{\partial}{\partial \mathbf{x}}\cdot\boldsymbol{\sigma}$$

$$\boldsymbol{\sigma}=-p\mathbf{I}+\mu\left(\frac{\partial\mathbf{u}}{\partial\mathbf{x}}+\frac{\partial\mathbf{u}^{T}}{\partial\mathbf{x}}\right)$$

$$\mathbf{u}=\mathbf{u}_{\text{surface}}\ \text{on walls}$$

Piston $M_P\ddot{Z}_P=-K_PZ_P-M_{PG}g_V+F_P,\quad M_{PG}=M_P-M_{LP}$

Bellows $M_B\ddot{Z}_B=-K_BZ_B-M_{BG}g_V+F_B-\rho g(1-\kappa)A_BZ_B$

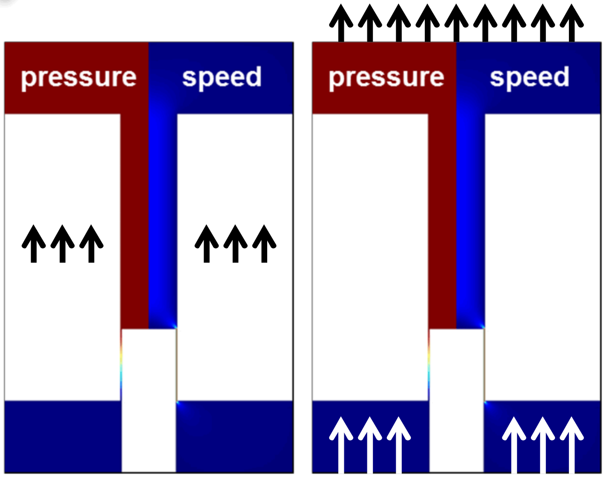
$$Z_{BU}=Z_B,\quad Z_{BL}=\kappa Z_B,\quad \kappa=A_{BU}/A_{BL},\quad A_B=A_{BU},\quad g_V=g_1\cos\omega t$$

$$M_B=M_{BU}+\kappa^2M_{BL},\quad K_B=K_{BU}+\kappa^2K_{BL},\quad M_{BG}=M_{BU}+\kappa M_{BL}+\rho L_BA_B$$

Liquid Forces $F_P=\hat{\mathbf{e}}_z\cdot\int_{S_P}\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\,dS,\quad F_B=\hat{\mathbf{e}}_z\cdot\int_{S_{BU}}\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\,dS+\kappa\hat{\mathbf{e}}_z\cdot\int_{S_{BL}}\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\,dS$

- All motions are analyzed in housing frame of reference
  - Housing motion gives oscillating gravity in housing frame
- Liquid obeys incompressible Navier-Stokes equations
  - Hydrostatic pressure is removed but gives buoyancy forces
- Piston and bellows obey Newton's 2^{nd} Law (F = ma)
  - Piston has spring force, bellows are springs themselves
  - Liquid forces are integrals of stress tensor over bodies
- Incompressibility constrains two bellows to move as one
  - Constraint eliminates unknown time-varying pressure offset

System Has Quasi-Steady Behavior



i = 1:
piston moves
bellows fixed

i = 2:
piston fixed
bellows move

liquid added masses — liquid damping coefficients — gravity-buoyancy

$$\left(\tilde{\mathbf{M}} + \mathbf{M}\right)\ddot{\mathbf{Z}} + \left(\tilde{\mathbf{B}} + \mathbf{B}\right)\dot{\mathbf{Z}} + \tilde{\mathbf{K}}\mathbf{Z} = \mathbf{f}$$

object masses — object damping coefficients — object spring constants

$$\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u}_i = 0, \quad \frac{\partial}{\partial \mathbf{x}} \cdot \boldsymbol{\sigma}_i = 0$$

$$\mathbf{Z} = \begin{pmatrix} Z_P \\ Z_B \end{pmatrix} \qquad \mathbf{f} = -\begin{pmatrix} M_{PG} \\ M_{BG} \end{pmatrix} g_V \cos \omega t$$

$$\tilde{\mathbf{K}} = \begin{pmatrix} K_P & 0 \\ 0 & K_B \end{pmatrix}$$

$$\mathbf{u}_i = \begin{cases} U\hat{\mathbf{e}}_z \text{ on } S_i \\ 0 \text{ on other walls} \end{cases}$$

$$\mathbf{B} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \qquad \beta_{ij} = \frac{2\mu}{U^2} \int_V \mathbf{S}_i : \mathbf{S}_j \, dV$$

$$\tilde{\mathbf{B}} = \begin{pmatrix} B_P & 0 \\ 0 & B_B \end{pmatrix}$$

$$\mathbf{S}_i = \frac{1}{2}\left(\frac{\partial \mathbf{u}_i}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}_i}{\partial \mathbf{x}}^T\right)$$

$$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \qquad m_{ij} = \frac{\rho}{U^2} \int_V \mathbf{u}_i \cdot \mathbf{u}_j \, dV$$

$$\tilde{\mathbf{M}} = \begin{pmatrix} M_P & 0 \\ 0 & M_B \end{pmatrix}$$

In quasi-steady regime, liquid forces have simple forms

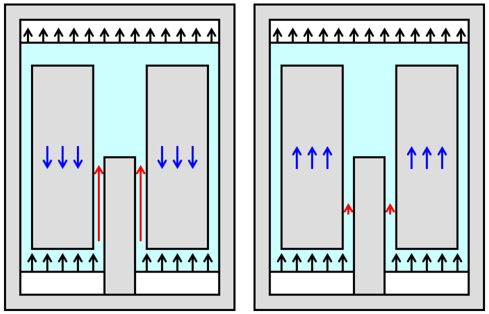
- Damping forces: proportional to object velocities
- Added-mass forces: proportional to object accelerations

Found from steady Stokes equations and first correction

- Damping coefficients involve integrals of dissipation
- Added masses involve integrals of kinetic energy

System described by 2×2-matrix ODE for driven oscillator

Thin Gaps Strongly Damp Motion



Poiseuille | Couette

$$\mathbf{B} \approx \beta_{11}\begin{pmatrix} 1 & -\lambda \\ -\lambda & \lambda^2 \end{pmatrix} \qquad \lambda = \frac{A_B}{A_P + (A_G/2)} \qquad M_\lambda = \mathbf{\Lambda}^T\left(\tilde{\mathbf{M}} + \mathbf{M}\right)\mathbf{\Lambda} \qquad \mathbf{Z} = \xi\mathbf{\Lambda},\ \text{so that } \dot{Z}_P = \lambda \dot{Z}_B$$

$$B_\lambda = \mathbf{\Lambda}^T\left(\tilde{\mathbf{B}} + \mathbf{B}\right)\mathbf{\Lambda} \qquad M_\lambda \ddot{\xi} + B_\lambda \dot{\xi} + K_\lambda \xi = f_\lambda$$

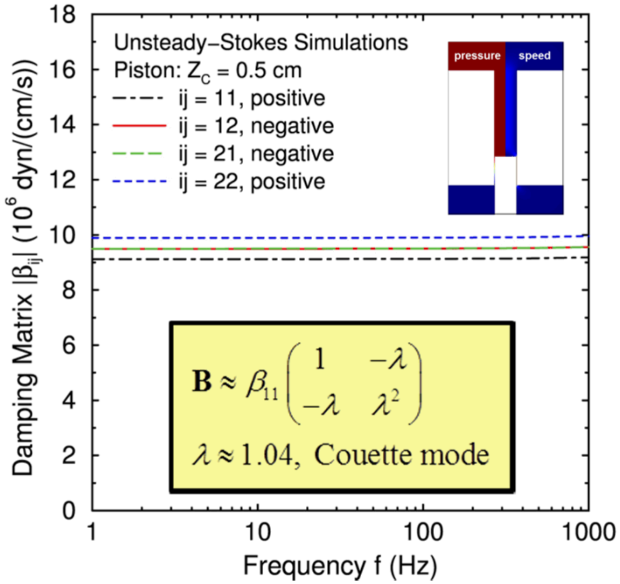
$$\mathbf{M} \approx m_{11}\begin{pmatrix} 1 & -\lambda \\ -\lambda & \lambda^2 \end{pmatrix} \qquad \mathbf{\Lambda} = \begin{pmatrix} \lambda \\ 1 \end{pmatrix} \qquad K_\lambda = \mathbf{\Lambda}^T \tilde{\mathbf{K}} \mathbf{\Lambda} \qquad \omega_C^2 = \frac{K_\lambda}{M_\lambda},\ B_\lambda^2 << K_\lambda M_\lambda$$

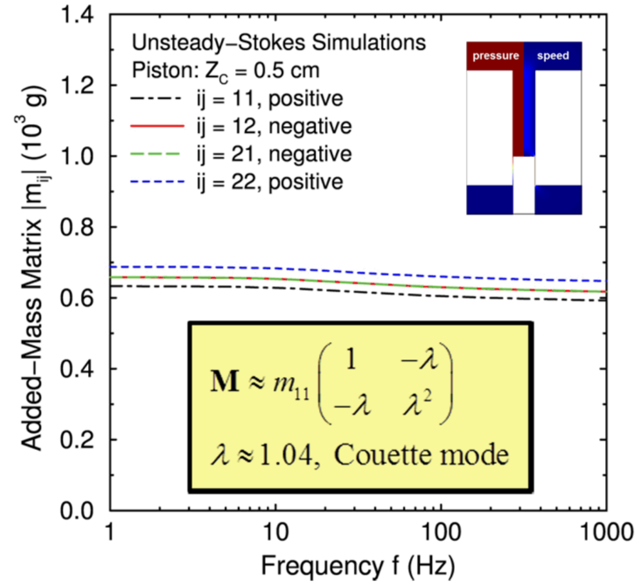
$$f_\lambda = \mathbf{\Lambda}^T \mathbf{f}$$

- **At first glance, system appears to be highly overdamped**
  - Thin gaps have large flow speeds, dissipation, kinetic energy
  - Liquid damping & added mass are large and oppose motion
- **But damping and added-mass matrices are nearly singular**
  - Singularity corresponds to pure Couette flow in each gap
  - Couette pressure difference is much smaller than Poiseuille
- **Piston and bellows move so as to produce Couette flow**
  - Piston/bellows velocity ratio is inverse of their area ratio
- **Couette mode is lightly damped and can have a resonance**
  - Couette drag is much smaller than Poiseuille drag

Damping and Added-Mass Matrices

Post radius	0.190 cm
Hole radius	0.200 cm
Piston radius	0.998 cm
Housing radius	1.000 cm
Inner gap width	0.010 cm
Outer gap width	0.002 cm
Inner gap length	0.500 cm
Outer gap length	2.000 cm
Piston density	8 g/cm^3
Liquid density	1 g/cm^3
Liquid viscosity	0.2 g/(cm·s)
Piston spring rate	6×10^4 dyn/cm
Bellows spring rate	1.4×10^7 dyn/cm
Piston mass	50 g
Bellows mass	0 g





Damping and added-mass matrices behave as expected

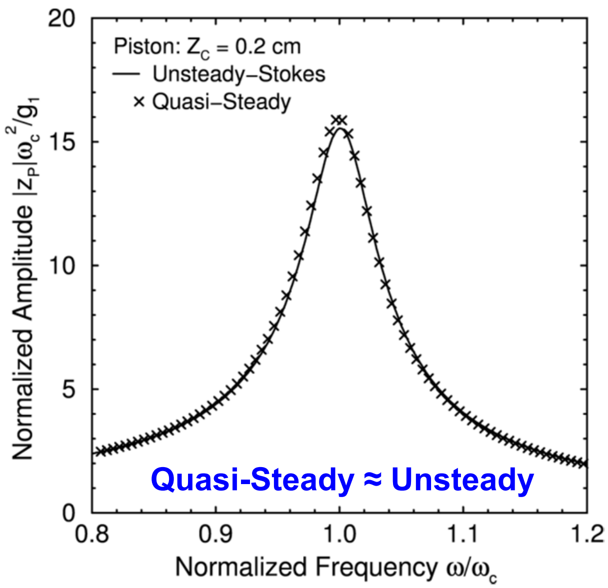
- Quasi-steady regime: almost independent of frequency
- Damping opposing Poiseuille flow is large: ~10^7 dyn/(cm/s)
- Nearly singular form with bellows-piston area ratio: 1.04

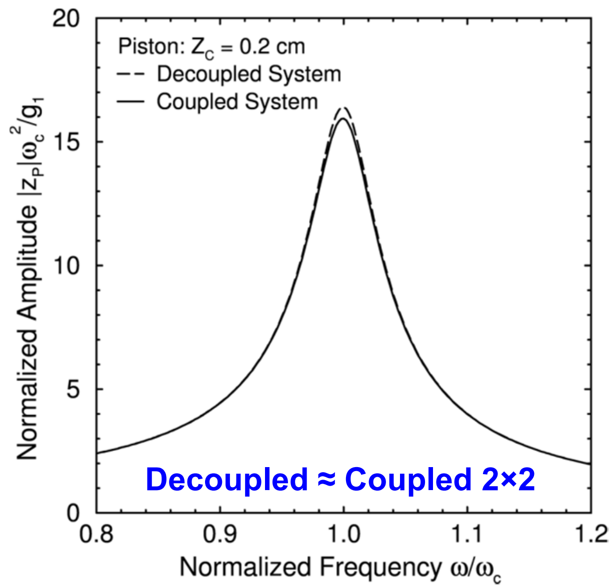
Near-singular form of matrices allows Couette mode

- Damping opposing Couette mode is small: ~10^3 dyn/(cm/s)
- Oscillates like piston mass against bellows spring: ~80 Hz

Resonance of Couette Mode

Post radius	0.190 cm
Hole radius	0.200 cm
Piston radius	0.998 cm
Housing radius	1.000 cm
Inner gap width	0.010 cm
Outer gap width	0.002 cm
Inner gap length	0.800 cm
Outer gap length	2.000 cm
Piston density	8 g/cm³
Liquid density	1 g/cm³
Liquid viscosity	0.2 g/(cm·s)
Piston spring rate	6×10⁴ dyn/cm
Bellows spring rate	1.4×10⁷ dyn/cm
Piston mass	50 g
Bellows mass	0 g





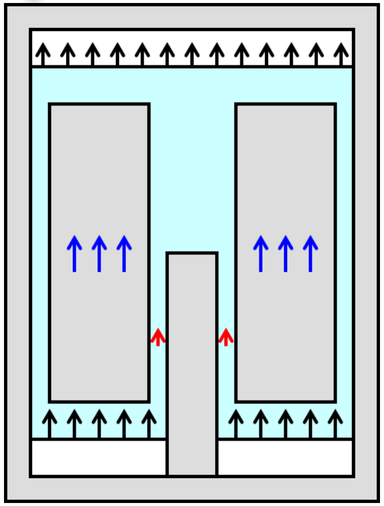
Couette mode has large resonance at expected frequency

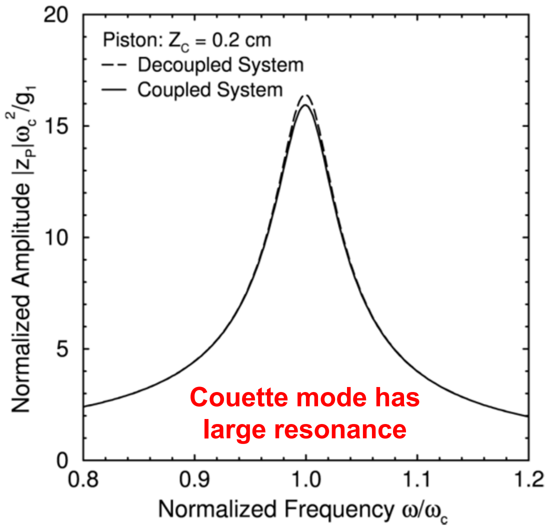
- Resonant frequency is ~80 Hz, FWHM is ~ 8 Hz
- For $g_1 = g_0 = 981$ cm/s², peak amplitude $|Z_P|$ is ~0.06 cm
- Amplitude is significant fraction of gap length even for small

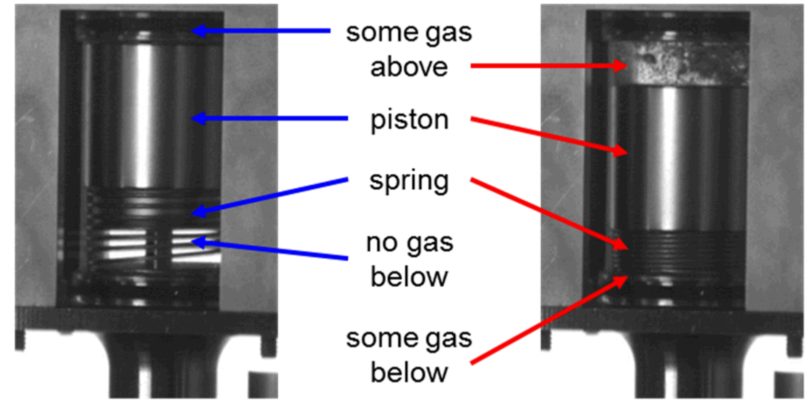
Quasi-steady Couette mode is an excellent approximation

- Frequency dependence of matrices is relatively unimportant
- Coupled (2×2) and decoupled (Couette) are almost identical

Conclusions







Vibration off
Spring supports piston

Vibration on
Piston compresses spring

Goal is to explain vibration-induced rectified piston motion

1. **Create two gas regions: downward rectified bubble motion**
2. **Large resonance: Couette mode has extremely low damping**
   - **Piston speed = bellows speed × bellows area / piston area**
3. **Rectified force: average of nonlinear terms near resonance**

Steps 1-2 have been analyzed; Step 3 is focus of future work

– **Desire ODE dynamical model informed by computation**

– **Ultimately compare model and experimental results**