

A Simple, Mixed-Mode Dependent Cohesive Zone Model

E. D. Reedy, Jr.

Sandia National Laboratories, Albuquerque, NM, USA (edreedy@sandia.gov)

Abstract

A conceptually simple, mixed-mode dependent cohesive zone model (MDG_c CZM) is described. The application of this model is illustrated by analyzing an edge-cracked elastic layer that is sandwiched between rigid adherends. Various combinations of edge-normal and edge-tangential displacements were applied to the elastic layer to generate a wide range of applied mode-mixity. The calculated effective toughness versus applied mode-mixity relationship displays a strong dependence on mode-mixity with the effective toughness increasing rapidly with the magnitude of the mode-mixity. The calculated relationship also displays a pronounced asymmetry with respect to the applied mode-mixity. This dependence is similar to that observed experimentally, and calculated results are in good agreement with published data for a glass/epoxy interface that was generated using a test specimen of the same type as analyzed here.

Introduction

The measured apparent interfacial toughness of many polymer solid interfaces increases with increasing crack-tip mode-mixity (Liechti and Chai, 1992; Swadener and Liechti, 1998; Mello and Liechti 2006). Furthermore this dependence on mode-mixity can have large impact on observed behavior (Hutchinson and Suo, 1992). A mode-mixity dependent toughness can be realized in an analysis that uses a mode-mixity independent CZM if the analysis includes plastic dissipation within the bulk materials that are adjacent to the interface (Tvergaard and Hutchinson, 1993). However, this requires a detailed analysis that models nonlinear material behavior at a very fine scale. An alternate approach is to include mode-mixity effects directly in the CZM. In this case the bulk materials could be modeled as linear elastic. Unfortunately, attempts to directly include a mode-mixity dependent toughness in the CZM have proved difficult. For example, in recent work a polynomial-based potential formulation that is defined in terms of four fracture parameters in each fracture mode does replicate a mode-dependent toughness; however, determining all eight fracture parameters is a challenging task (Park and Paulino 2011). In an alternate approach, a nonpotential-based method that defines Mode I and Mode II response independently has been used to model the mode-mixity dependent failure of adhesive joints (Yang and Thouless 2001). This approach uses a mixed-mode failure criterion to link the two independently defined traction-separation relationships.

The present study describes a simple model that generates

mode-mixity dependent interfacial toughness. This model has been implemented within a cohesive surface element framework for implicit quasistatic finite element calculations and this code was used to generate the results presented below.

Mode-mixity Dependent Interfacial Toughness Cohesive Zone Model

The MDG_c CZM has two elements. Mode I energy dissipation is determined by a traction-separation T - U relationship that depends only on normal separation. The two key parameters defining this relationship are the interfacial strength σ^* and the intrinsic mode I work of separation/unit area of interface Γ . This study uses a simple trapezoidal T - U relationship. Mode II (III) dissipation is generated by perfectly plastic shear yielding and slip in the cohesive surface elements that lie in front of the region where Mode I separation (softening) occurs. The yield strength is τ^* and plastic slip is associated with the tangential displacement jump.

Crack Growth along the Interface of a Thin Elastic Layer Held between Rigid Grips

The plane strain problem of crack growth along the interface of a thin elastic layer held between rigid grips was analyzed (Fig. 1). The elastic layer is loaded by applying edge-normal and edge-tangential displacements to the upper rigid grip while the bottom rigid grip is fixed. The interface's effective toughness Γ_e is defined as the value of the energy release rate when the interfacial crack begins to propagate. This is calculated using the well-known analytical energy release rate calibration for an edge-crack along the interface of a thin elastic layer held between rigid grips

$$\Gamma_e = \frac{h}{2E_u} (\bar{\sigma}_{yy}^c)^2 + \frac{h}{2G} (\bar{\sigma}_{xy}^c)^2 \quad (1)$$

where $\bar{\sigma}_{yy}^c$ and $\bar{\sigma}_{xy}^c$ are the calculated critical values of the normal and shear stress in the uniformly stressed ligament when the crack begins to propagate, h is the layer thickness, $E_u = (1-\nu) E / ((1+\nu)(1-2\nu))$ is the uniaxial strain modulus, and G is the shear modulus. In order to provide a formal connection to the crack-tip mode-mixity as defined in linear elastic fracture mechanics solution for the same problem geometry, an applied mode-mixity ψ_a is defined as

$$\psi_a \equiv \tan^{-1}(2\bar{\sigma}_{xy}^c / \bar{\sigma}_{yy}^c) \quad (2)$$

Results

The interfacial edge-cracked adhesive bond geometry analyzed in this study is similar to that which has been used by Swadener and Liechti (1998) to measure the mixed-mode dependent interfacial toughness of a glass/epoxy interface. In the Swadener and Liechti study (referred to as the SL study) an epoxy layer is sandwiched between relatively stiff aluminum and glass adherends that are subjected to bond-normal and bond-tangential edge displacements. The epoxy layer's E is reported to be two GPa, while the layer thickness h falls within the range of 0.13 to 0.4 mm, and the intrinsic toughness Γ was determined to be in the range of 1 to 2 J/m². Various combinations of applied edge displacements propagated a long interfacial edge crack along the glass/epoxy interface. Figure 2 compares SL experimental results with the calculated Γ_e vs. ψ_a relationship. The nondimensional MDG_c CZM parameters used in these calculations are based on the reported SL test configuration (i.e., $E = 2$ GPa, $h=0.25$ mm, and $\Gamma=1.5$ J/m²). Specifically, $\sigma^*/E=0.02$, $\Gamma/(h\sigma^*)=1.5e-4$, and τ^*/σ^* equals 0.5. Note that the SL results are reported in terms of crack-tip mode-mixity and the epoxy layer is considered to be "material one" in the definition of the bimaterial constant ε . To enable a direct comparison with the calculated Γ_e/Γ vs. ψ_a relationships, the SL crack-tip mode mixity is converted to applied mode mixity ψ_a and the SL toughness data was also normalized by the apparent intrinsic toughness (i.e., by the minimum of the measured Γ_e vs. ψ_a relationship). The calculated Γ_e/Γ vs. ψ_a relationship is in good agreement with the experimental results. In particular the asymmetry in the Γ_e/Γ vs. ψ_a relationship is remarkably similar.

Acknowledgements

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Figures

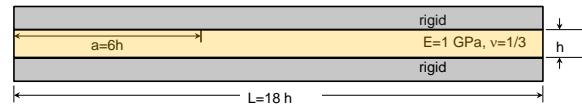


Figure 1. Analyzed the growth of an edge-crack along the interface of a thin elastic layer sandwiched between rigid grips where edge-normal and edge-tangential displacements are applied to the upper grip while the bottom grip is fixed.

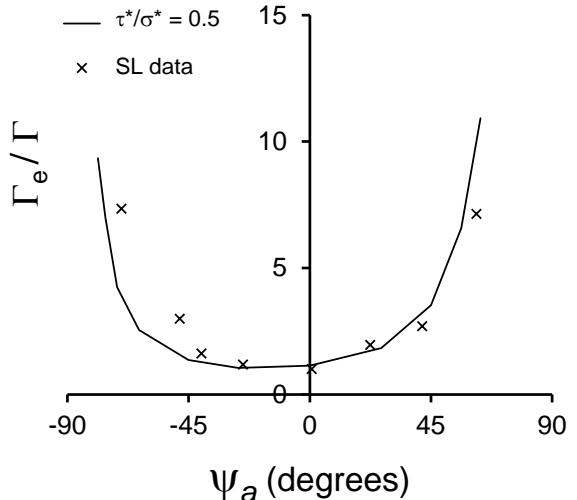


Figure 2. Comparison of finite element analysis predictions that use the MDG_c CZM with experimental interfacial toughness data published by Swadener and Liechti (SL).