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Parallel solution of nonlinear contingency-constrained network problems

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Abstract

This paper presents a nonlinear stochastic programming formulation for a large-scale contingency-constrained optimal power flow problem. Using a rectangular IV formulation to model AC power flow in the transmission network, we construct a nonlinear, multi-scenario optimization formulation where each scenario considers failure an individual transmission element. Given the number of potential failures in the network, these problems are very large; yet need to be solved rapidly. In this paper, we demonstrate that this multi-scenario problem can be solved quickly using a parallel decomposition approach based on nonlinear interior-point methods. Parallel and serial timing results are shown using a test example from Matpower, a MATLAB-based framework for power flow.

Keywords: ACOPF, contingency-constrained, parallel computing, interior-point methods.

1. Introduction

In the traditional alternating current optimal power flow (ACOPF) problem we seek to find the optimal generator setpoints that minimize operational costs while satisfying load demands across the electrical transmission network. The ACOPF is an important problem that has been studied for five decades, but solution of this optimization problem is still challenging because of the large size of the system and the nonlinearities of the model. The Federal Energy Regulatory Commission (FERC) website lists several papers (Cain et al., 2012; Campaigne et al., 2013; Castillo and O'Neill, 2013a,b; Lipka et al., 2013; O'Neill et al., 2012a,b; Pirnia et al., 2013; Schecter and O'Neill, 2013) about the history, formulation, approximation, and solution of the ACOPF problem. In their studies, they conclude that the rectangular IV formulation performs better than the polar formulation for larger problems.

The electrical grid is a critical infrastructure, and in addition to reducing operational costs, one would like to guarantee that the system is resilient to failure. In this paper, we consider an extension to the traditional ACOPF problem, called the contingency-constrained ACOPF problem. Using a rectangular IV formulation to model AC power flow in the transmission network, we construct a nonlinear, multi-scenario optimization formulation that minimizes the operating cost for the nominal case while including a large number of contingency scenarios, where each scenario considers failure of an individual transmission element. We solve the corrective form of the problem, with terms added to the objective function to penalize deviations in the control variables between the nominal case and the contingency case. Given the number of potential failures in the network, these problems can become very large; yet we need to solve them efficiently to rapidly determine new optimal operating conditions over changing network demands.

For a realistic power network, with numerous contingencies considered, the overall problem size will increase dramatically, quickly exhausting the capabilities of a single workstation. Fortunately, the structure of these multi-scenario problems can be exploited to allow solution in parallel (Borges and Alves, 2007; Phan and Kalagnanam, 2012). This problem is nonlinear (and non-convex), and interior-point methods provide a powerful tool for local solution of these formulations (Capitanescu et al., 2011; M.Bhaskar et al., 2011). The dominant computational expense in an interior-point method is the solution a linear system at each iteration arising from a modified Newton's method, and a number of researchers have investigated structure-exploiting approaches to allow parallel solution of this structured linear system (Chiang and Grothey, 2012; Kang et al., 2013; Lubin et al., 2011; Qiu et al., 2005). In this paper, we show that fast solution of the contingency-constrained ACOPF problem is possible with our parallel nonlinear interior-point code, Schur-IPOPT. This code uses a Schur-complement decomposition strategy to allow parallel solution of the KKT system in a nonlinear interior-point method based on the popular algorithm, IPOPT (Wächter and Biegler, 2006).

The remainder of this paper is organized as follows. Section 2 describes the contingency-constrained ACOPF formulation. Numerical solution and timing results are shown and discussed in Section 3. In Section 4 we conclude this paper and propose some future work.

2. Problem Formulation

When modelling the transmission network, we consider a rectangular IV formulation. This model, shown below in equations (1a-1q), is based on the traditional π model for transmission lines as derived in the Matpower User's Manual (Zimmerman and Murillo-Sánchez, 2011). All sets, parameters, and variables used in the model are given in Table 1. The model includes relationships between voltages and current as defined by the transmission network, bus current balances, constraints on required power for all loads, and bounds that constrain operation within reasonable limits.

$$\begin{bmatrix} i^l_{fr} \\ i^l_{fj} \\ i^l_{tr} \\ i^l_{tj} \end{bmatrix} = Y^l_{br} \begin{bmatrix} v^l_{fr} \\ v^l_{fj} \\ v^l_{tr} \\ v^l_{tj} \end{bmatrix} \qquad \forall\, l \in \mathcal{L} \qquad (1a)$$

$$\begin{bmatrix} i^b_{Sr} \\ i^b_{Sj} \end{bmatrix} = Y^b_{sh} \begin{bmatrix} v^b_r \\ v^b_j \end{bmatrix} \qquad \forall\, b \in \mathcal{B} \qquad (1b)$$

$$P^l_t = (v^{bt(l)}_r \cdot i^l_{tr} + v^{bt(l)}_j \cdot i^l_{tj}) \qquad \forall\, l \in \mathcal{L} \qquad (1c)$$

$$Q^l_t = (v^{bt(l)}_j \cdot i^l_{tr} - v^{bt(l)}_r \cdot i^l_{tj}) \qquad \forall\, l \in \mathcal{L} \qquad (1d)$$

$$P^l_f = (v^{bf(l)}_r \cdot i^l_{fr} + v^{bf(l)}_j \cdot i^l_{fj}) \qquad \forall\, l \in \mathcal{L} \qquad (1e)$$

$$Q^l_f = (v^{bf(l)}_j \cdot i^l_{fr} - v^{bf(l)}_r \cdot i^l_{fj}) \qquad \forall\, l \in \mathcal{L} \qquad (1f)$$

$$S_t^l = (P_t^l)^2 + (Q_t^l)^2 \qquad \forall\, l \in \mathcal{L} \qquad (1g)$$

$$S_f^l = (P_f^l)^2 + (Q_f^l)^2 \qquad \forall\, l \in \mathcal{L} \qquad (1h)$$

$$0 = \sum_{l\in\mathcal{B}_{in}^b} i_{tr}^l + \sum_{l\in\mathcal{B}_{out}^b} i_{fr}^l + i_{Sr}^b + i_{Lr}^d - \sum_{g\in\mathcal{G}^b} i_{Gr}^g \qquad \forall\, b \in \mathcal{B} \qquad (1i)$$

$$0 = \sum_{l\in\mathcal{B}_{in}^b} i_{tj}^l + \sum_{l\in\mathcal{B}_{out}^b} i_{fj}^l + i_{Sj}^b + i_{Lj}^d - \sum_{g\in\mathcal{G}^b} i_{Gj}^g \qquad \forall\, b \in \mathcal{B} \qquad (1j)$$

$$P_L^d = (v_r^d \cdot i_{Lr}^d + v_j^d \cdot i_{Lj}^d) \qquad \forall\, d \in \mathcal{D} \qquad (1k)$$

$$Q_L^d = (v_j^d \cdot i_{Lr}^d - v_r^d \cdot i_{Lj}^d) \qquad \forall\, d \in \mathcal{D} \qquad (1l)$$

$$P_G^g = (v_r^g \cdot i_{Gr}^g + v_j^g \cdot i_{Gj}^g) \qquad \forall\, d \in \mathcal{D} \qquad (1m)$$

$$Q_G^g = (v_j^g \cdot i_{Gr}^g - v_r^g \cdot i_{Gj}^g) \qquad \forall\, d \in \mathcal{D} \qquad (1n)$$

$$v_m^b = (v_r^b)^2 + (v_j^b)^2 \qquad \forall\, b \in \mathcal{B} \qquad (1o)$$

$$v_j^{ref} = 0 \qquad (1p)$$

$$\text{bounds on } v_m^b, P_G^g, Q_G^g, S_f^l, S_t^l \qquad (1q)$$

Table 1: Set, Parameter and Variable Description

$\mathcal{L}$	set of all branches (transmission lines)
$\mathcal{B}$	set of all bus nodes
$\mathcal{G}$	set of all generators (subset of $\mathcal{B}$)
$\mathcal{D}$	set of all buses that are loads (a subset of $\mathcal{B}$)
$\mathcal{B}_{in}^b$	set of all inlet branches to bus b
$\mathcal{B}_{out}^b$	set of all outlet branches from bus b
$\mathcal{G}^b$	set of all generators at bus b
$bt(l)$, $bf(l)$	index of bus attached to the to and from end of line l
Y_{br}^l, Y_{sh}^b	matrices of the IV relationships for branch l and shunt for bus b
P_L^d, Q_L^d	parameter values for real (P) and reactive (Q) power of each load d
v_r^b, v_j^b	real and complex voltages at each bus b
v_m^b	square of voltage magnitude (at each bus b)
P_G^g, Q_G^g	real (P) and reactive (Q) power for each generator g
P_f^l, Q_f^l	real (P) and reactive (Q) power at the $from$ end of each branch l
P_t^l, Q_t^l	real (P) and reactive (Q) power at the *to* end of each branch l
S_f^l	S at the $from$ end of each branch l
S_t^l	S at the *to* end of each branch l
i_{fr}^l, i_{fj}^l	real and complex current at the $from$ end of each branch l
i_{tr}^l, i_{tj}^l	real and complex current at the *to* end of each branch l
i_{Lr}^d, i_{Lj}^d	real and complex current for each load d
i_{Sr}^b, i_{Sj}^b	real and complex current for shunt at bus b
i_{Gr}^g, i_{Gj}^g	real and complex current for generator g

When formulating the multi-scenario contingency-constrained model, we repeat these equations for each contingency case, except we assume a single line failure (modifying the corresponding entries in the transmission matrix) for each scenario. The full multi-scenario optimization formulation is shown in equations (2) below, where x_0 and x_c represent the state variables for normal operation (i.e., no failure occurs) and the contingency cases, respectively. Vectors u_0 and u_c represent the control variables for normal operation and the contingency cases (active generator power in our studies).

$$\min_{x_0,u_0,x_c,u_c} \quad f(u_0) + \rho \sum_{c \in \mathcal{C}} f_c\left(u_c, u_0\right) \tag{2a}$$

$$\text{s.t.} \quad g_0\left(x_0, u_0\right) = 0 \tag{2b}$$

$$g_c\left(x_c, u_c\right) = 0 \qquad \forall \ c \in \mathcal{C} \tag{2c}$$

$$\underline{x}_0 \le x_0 \le \bar{x}_0, \ \underline{u}_0 \le u_0 \le \bar{u}_0 \tag{2d}$$

$$\underline{x}_c \le x_c \le \bar{x}_c, \ \underline{u}_c \le u_c \le \bar{u}_c \qquad \forall \ c \in \mathcal{C} \tag{2e}$$

Equations (2b) represent the complete network model corresponding to normal operation, while equations (2c) represent the network models for each of the contingency. The function $f(u_0)$ is a polynomial describing the generator operating cost. The penalty function $f_c(u_c, u_0)$ is the sum of the square of the 2-norm of the deviation between each vector u_c and u_0. However, other measures could be considered instead, such as ramp rate constraints.

3. Solution Approach and Numerical Results

Problem (2) represents a large-scale nonlinear, non-convex optimization problem where each scenario is independent except for the coupling in the penalty term between the normal case and the contingency cases. In this paper, we solve the problem with our previously developed code Schur-IPOPT, a parallel nonlinear interior-point method. In particular, we make use of the explicit Schur-complement decomposition approach from Kang et al. (2013). This multi-scenario problem is formulated using Pyomo (Hart et al., 2012), a Python-based, mathematical programming language. Pyomo allows formulation of the problem in parallel and produces *nl* files that can be processed by Schur-IPOPT to allow model evaluation and solution in parallel.

For numerical timing, we consider the problem *case118* distributed with Matpower 4.1. This test problem has 118 buses, 54 active generators, and 186 branches. We first test our model by solving the single-scenario ACOPF problem with no contingencies using using our interior-point solver and comparing this with the optimal results produced with Matpower. Both codes can solve the single-scenario ACOPF problem quickly (less than a second on a 2.3 GHz Intel Core i5 MacBook Pro), obtaining the same objective function value (129660.69 $/hr) and generator setpoints.

Next, we report timing on the multi-scenario problem with 128 scenarios in total. We consider the normal operating scenario and 127 contingencies. All timing results are wall-clock times obtained from the Red Mesa supercomputing cluster at Sandia National Laboratories in Albuquerque, NM. This cluster is made up of computing nodes each with two, 2.93 GHz quad-core, Nehalem X5570 processors (giving 8 computing cores per node). Each node has 12 GB of DDR3 RAM. To compute parallel speedup, we first solve the problem with two serial options in our interior-point algorithm. The FS-S algorithm solves the full-space KKT system using a direct linear solver, while the ESC-S algorithm solves the KKT system with the explicit Schur-complement decomposition approach. FS-S solves the problem in 199 seconds and, as expected, the ESC-S algorithm is slower, solving the problem in 412 seconds. Next, we compare these timing results with the ESC-P algorithm, the parallel explicit Schur-complement approach. Table 4 lists the timing results for this problem as we increase the number of processors.

Table 4: Strong scaling results for 128 scenarios (127 contingencies)

# processors	ESC-P Time(s)	Speedup (based on ESC-S)	Speedup (based on FS-S)
1	412.18	1.00	0.48
2	215.35	1.91	0.92
4	110.21	3.74	1.80
8	60.99	6.76	3.26
16	31.72	13.00	6.26
32	16.05	25.68	12.37
64	8.63	47.75	23.01
128	4.55	90.54	43.63

The total wall-clock time for solving this problem can be decreased dramatically to less than 5 seconds with 128 processors. This represents an overall speedup of approximately 90 times when compared to the ESC-S approach, and over 40 times when compared with the FS-S approach.

4. Conclusions and Future Work

This paper presents a rectangular IV formulation for the contingency-constrained ACOPF problem and demonstrates that the parallel Schur-complement based, nonlinear interior-point method described in Kang et al. (2013) can dramatically reduce solution times for this problem. In future work, we will address a much larger transmission network from the Matpower test suite (with over 3000 buses). With a larger number of generators, we suspect that the implicit PCG approach we introduced in Kang et al. (2013) will be much more effective. Finally, while the explicit Schur-complement approach yields significant speedup, the parallel efficiency is much lower than we have seen in other problems. This can be explained by the difference in factorization time for individual scenarios based on which transmission element was removed. We will also seek to address this load balancing issue and increase parallel efficiency.

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