

## Introduction and Summary

Low-frequency transient electromagnetic (EM) signals are commonly used in geophysical exploration of the Earth's shallow subsurface. Strong sensitivity to current conductivity of geologic formations implies they are particularly useful for inferring fluid content of saturated porous media. However, low-frequency EM wavefields are diffusive, and have significantly larger wavelengths compared to seismic signals of equivalent frequency. Seismic and (low-frequency) EM wavelengths are given by

$$\lambda(f)|_{\text{SEIS}} = \frac{c}{f}, \quad \lambda(f)|_{\text{EM}} = \frac{c(f)}{f} = \sqrt{\frac{4\pi}{\sigma\mu f}}$$

where  $c$  is phase speed, and  $\sigma$  and  $\mu$  are current conductivity and magnetic permeability. For example, the wavelength of a 30 Hz sinusoid propagating with seismic velocity of 3000 m/s in an elastic medium is 100 m, whereas the analogous EM signal diffusing through a conductive body of 0.1 S/m (clayey shale) has wavelength 1826 m. This larger wavelength has implications for the resolution capabilities of the EM prospecting method.

We are investigating the detection and resolving power of the EM method via theoretical and numerical experiments. The **thin bed reflection and transmission** problem, well known in seismology (e.g., Widess, 1973), is readily adapted to electromagnetics. Normal incidence plane wave responses for a simple three-medium earth model are amenable to an analytic solution which includes all intrabed multiples. Frequency-domain electric vector (Ex) and magnetic vector (By) responses are calculated for thin conductive beds with widths that are several orders of magnitude smaller than the dominant wavelength of an incident EM wave. Although detectable signals are calculated, the effect of bed thickness and conductivity appears to be encoded in the amplitudes, rather than arrival times, of observed time-domain signals.

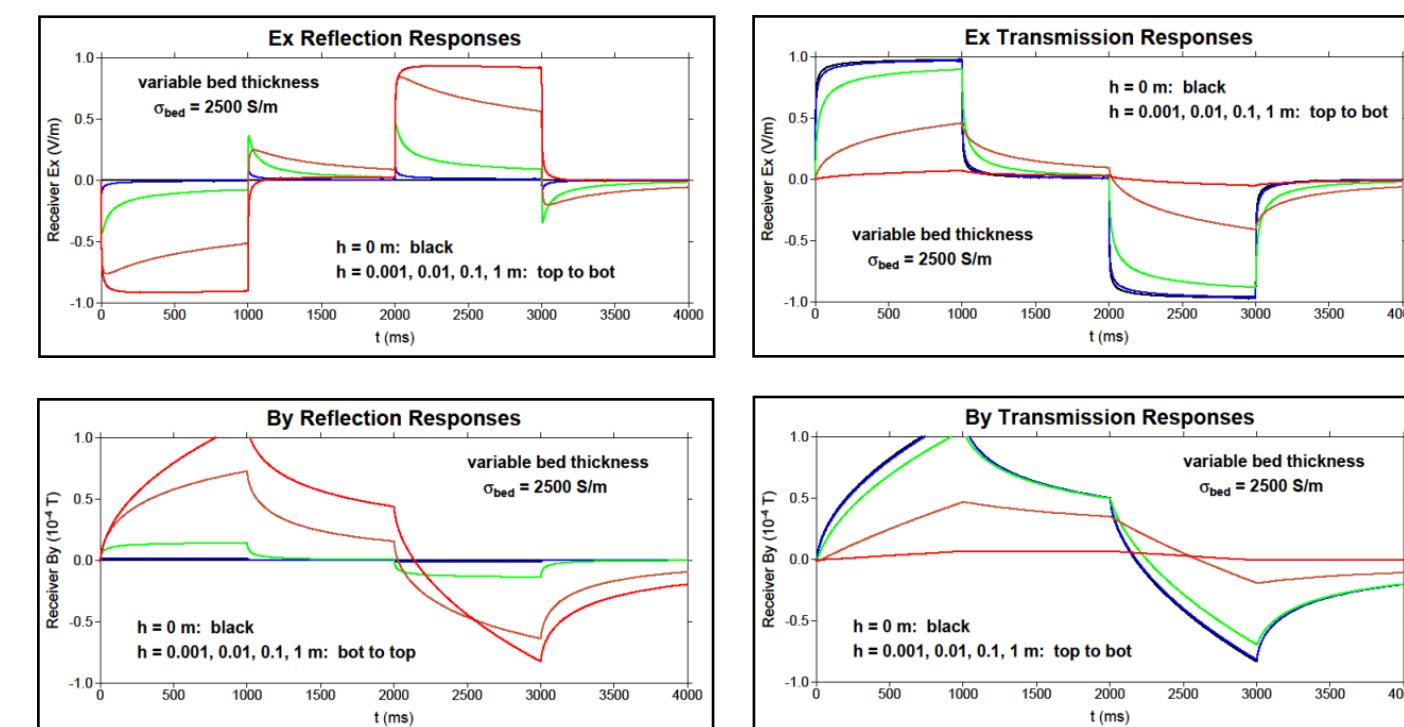
Finite-element numerical solution of Poisson's equation for the scalar electric potential function, on a three-dimensional (3D) unstructured mesh, enables an investigation of **resolution of two buried bodies** by **dc frequency** methods. Excellent agreement is obtained with an analytic solution (Aldridge and Oldenburg, 1989) for the secondary potential due to two conductive spheres, where extrema on the observed voltage curve indicates two separate bodies. However, as depth to two shallow resistive spheres increases, the secondary potential contours merge and mimic the shape of a *single* buried body.

Finally, the ability of EM observations to **resolve two buried point current density sources** (which can either be physical body sources, or interpreted as two scattering loci via the First Born Approximation) is examined with a 3D Green function algorithm. Three-component (3C) time-domain EM traces recorded along a surface profile exhibit subtle amplitude and character changes as separation distance between two deeply-buried current sources increases. However, the changes are appreciable only at large separations, which implies resolution of closely-spaced scatterers will be difficult with low-frequency EM methods.

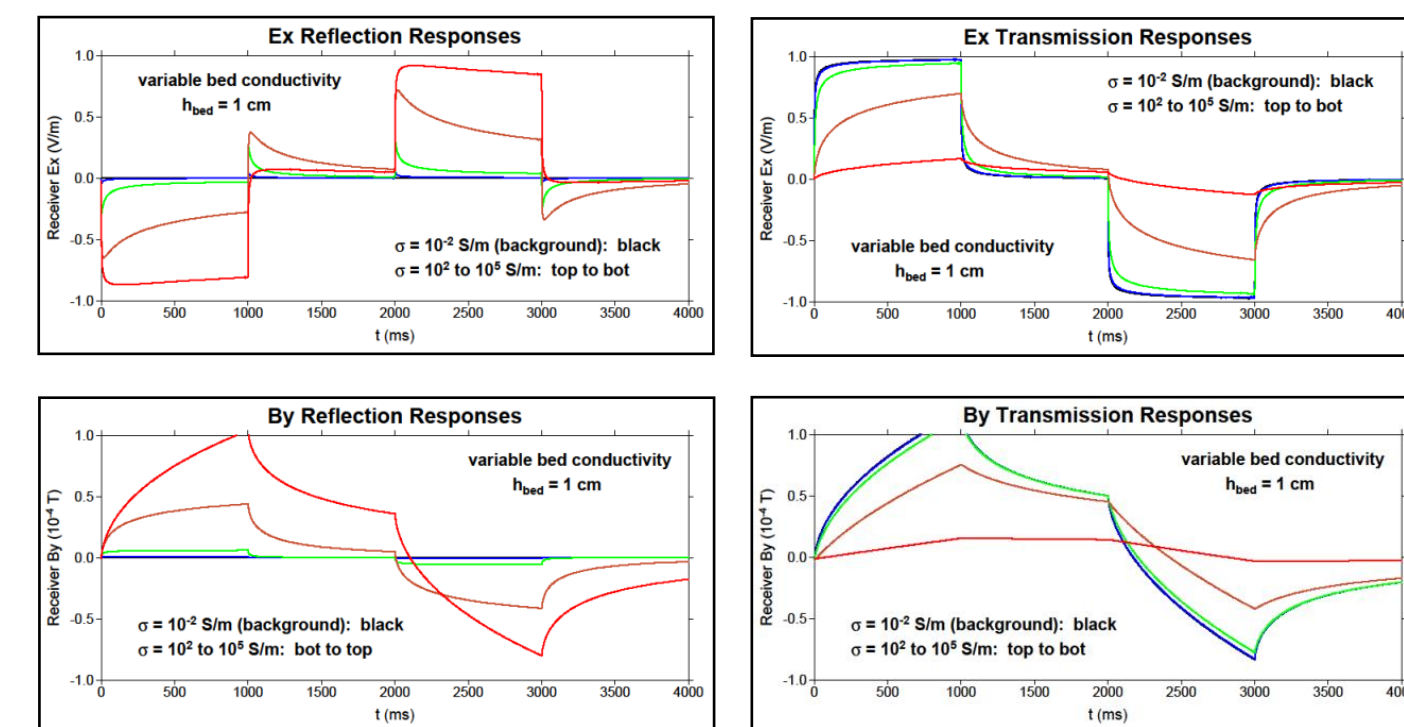
## Time-Domain EM Responses

Obtained by inverse discrete Fourier transforming frequency-domain spectra. Source waveform is alternating polarity square pulse sequence (1 s on+, 1 s off, 1 s on-).

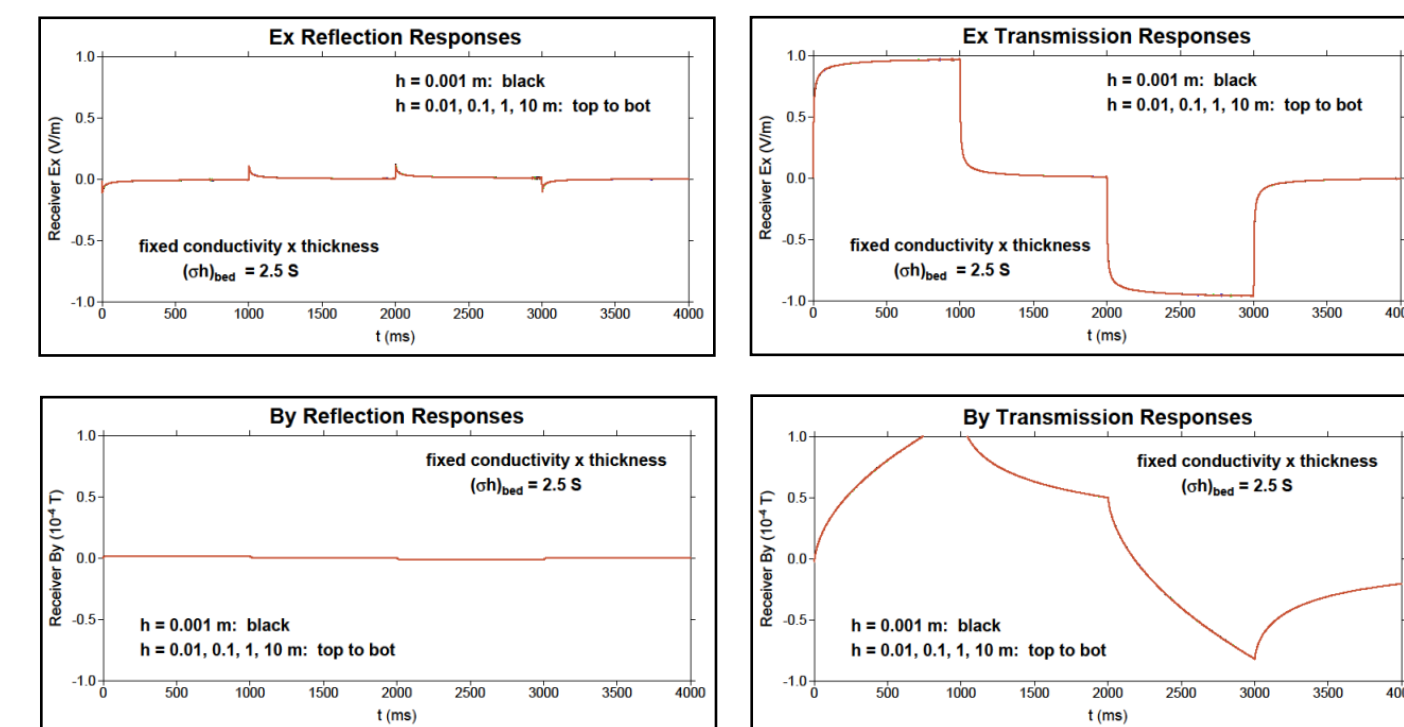
### Variable Bed Thickness



### Variable Bed Conductivity



### Fixed Conductivity x Thickness



No sensitivity to conductivity x thickness product!

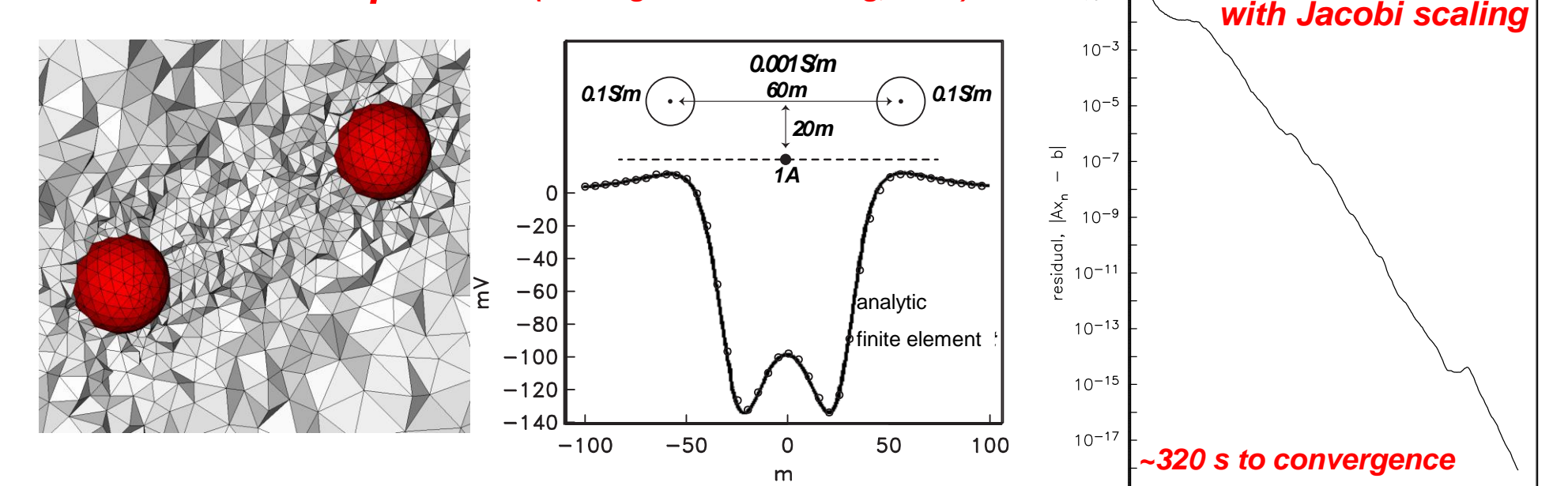
## Resolution Analysis of Buried Spherical Targets in the DC Frequency Limit

### Problem Statement

$$-\nabla \cdot (\sigma(\mathbf{x}) \nabla \Phi(\mathbf{x})) = \nabla \cdot \mathbf{J}_s(\mathbf{x})$$

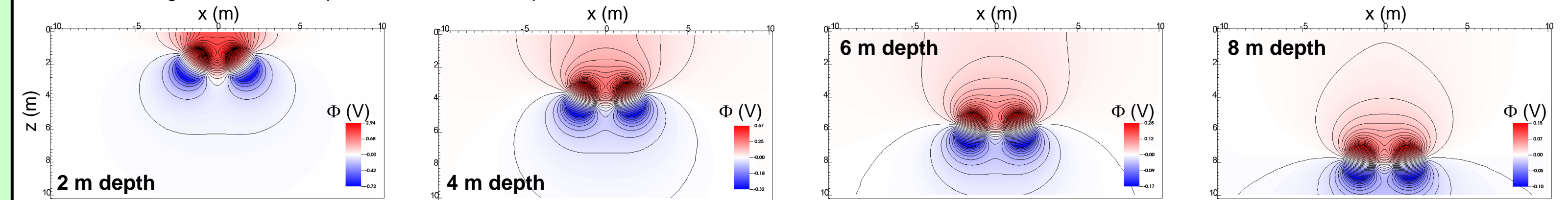
$$\partial_n \Phi|_{\Gamma_N} = 0 \quad \Phi|_{\Gamma_D} = 0$$

### Benchmark Comparison (Aldridge and Oldenburg, 1989)



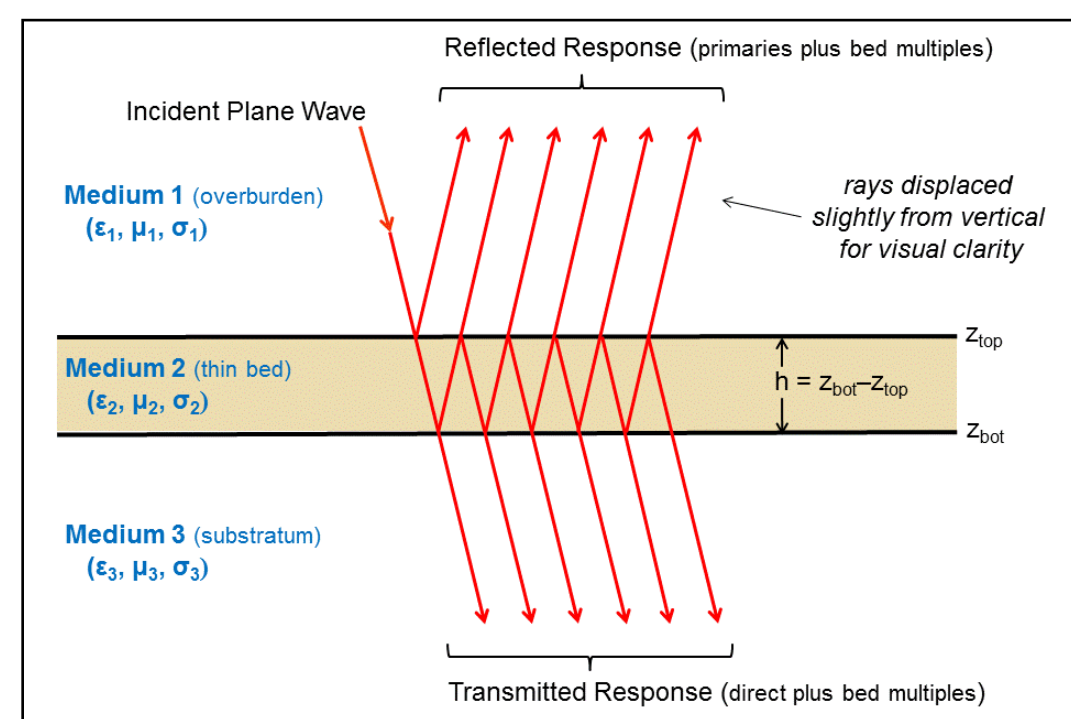
### Scattered Potentials: total - background

Finite element mesh: 280,431 nodes; 1,595,186 elements on 60x60x30 m grid  
0.01 S/m background, 1e-8 S/m spheres, 1m radius, 3m separation



## Thin Bed Reflection and Transmission

### The Geophysical Basis



### The Mathematical Basis

Sum the infinite series of individual arrivals (with appropriate phase delays and amplitude attenuations) to obtain

### Electric Vector Reflection and Transmission Response Filters:

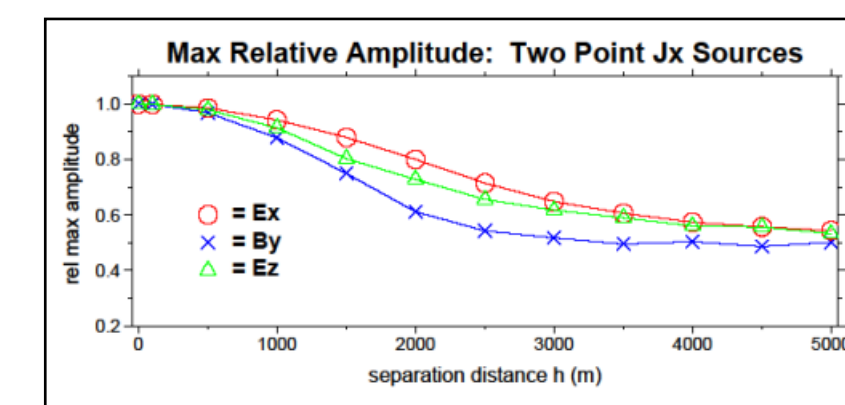
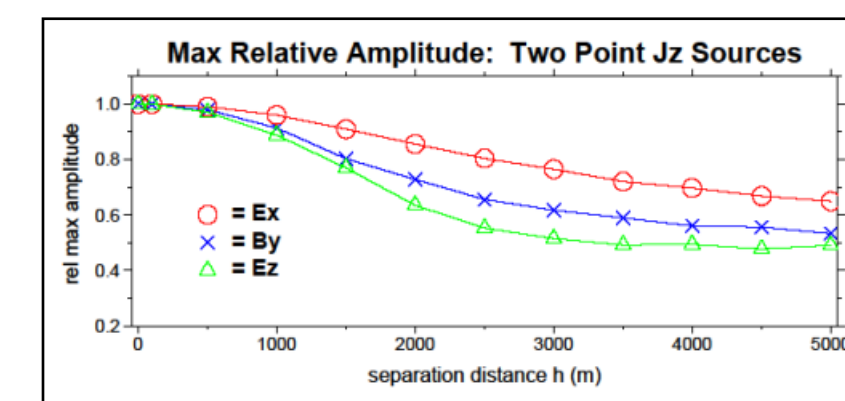
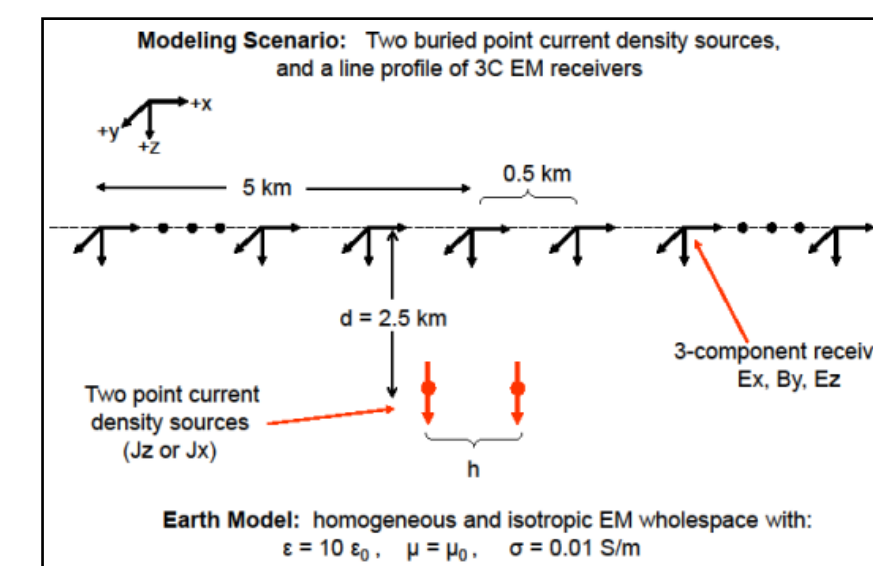
$$\text{REF}(\omega) = \frac{R_{\text{top}}(\omega) + R_{\text{bot}}(\omega)e^{+iK_2(\omega)2h}}{1 + R_{\text{top}}(\omega)R_{\text{bot}}(\omega)e^{+iK_2(\omega)2h}}, \quad \text{TRN}(\omega) = \frac{[1 + R_{\text{top}}(\omega)][1 + R_{\text{bot}}(\omega)]}{1 + R_{\text{top}}(\omega)R_{\text{bot}}(\omega)e^{+iK_2(\omega)2h}}$$

where the normal incidence **reflection coefficient** and **complex wavenumber** are:

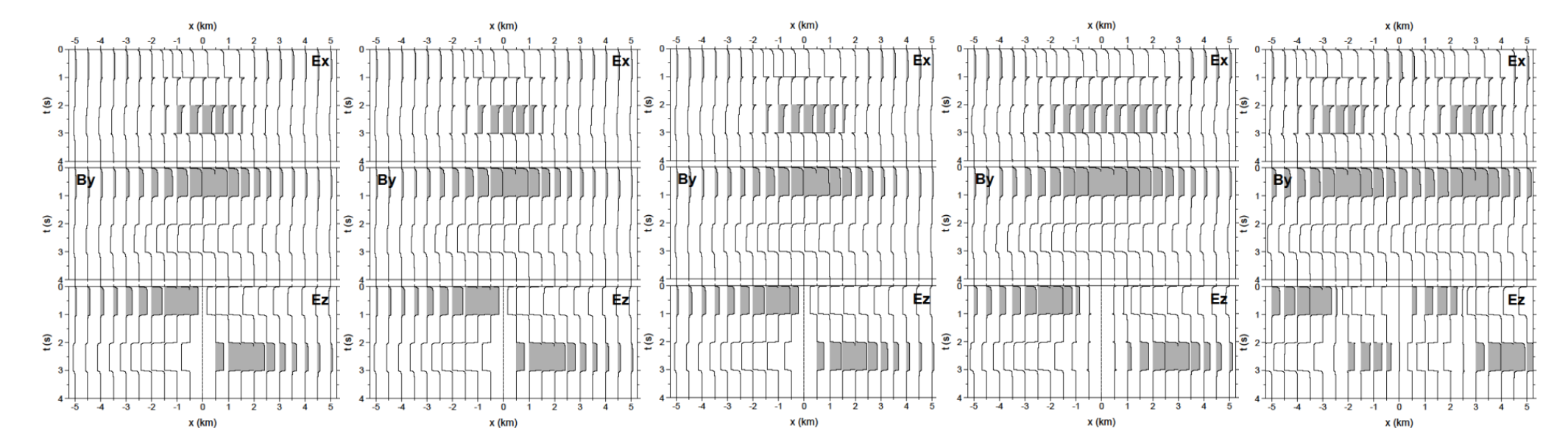
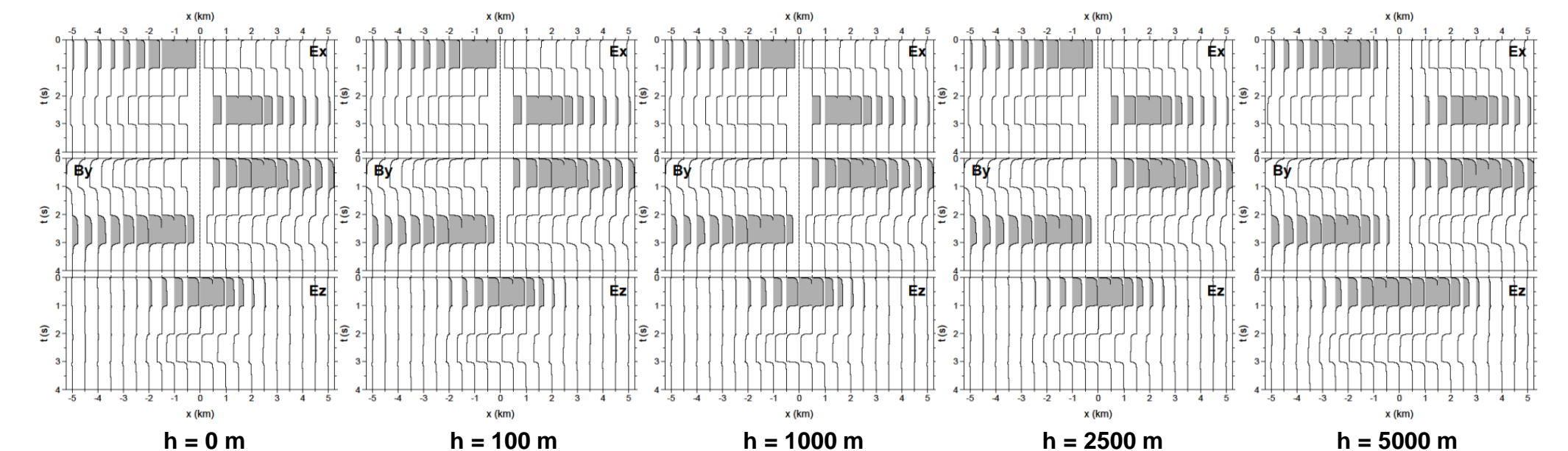
$$R_{\text{top}}(\omega) = \frac{K_1(\omega)/\mu_1 - K_2(\omega)/\mu_2}{K_1(\omega)/\mu_1 + K_2(\omega)/\mu_2}, \quad K(\omega) = \sqrt{i\omega\mu(\sigma - i\omega\epsilon)}$$

Magnetic (**B**) vector responses obtained by multiplying by  $K(\omega)/\omega$ .

## Resolving Two Buried Point Current Density Sources (or Two Point Conductivity Scatterers)



### Two Point Jz Sources



### Two Point Jx Sources

Time-domain trace character "obviously" differs only at large source separation distances.

## Acknowledgement

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## References

Aldridge, D.F., and Oldenburg, D.W., 1989, Direct current electric potential field associated with two spherical conductors in a whole-space: Geophysical Prospecting, **37**, 311-330.  
Widess, M.B., 1973, How thin is a thin bed?: Geophysics, **38**, 1176-1180.