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A Modal Craig-Bampton Substructure for Experiments, Analysis, Control and Specifications

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Nomenclature

CB	Craig-Bampton method of substructuring
CMIF	complex mode indicator function
FE	finite element model
MCB	Modal Craig-Bampton model form
TS	transmission simulator - the fixture attached to the experimental substructure of interest
dof	degree of freedom
sdof	single degree of freedom
mdof	multiple degree of freedom
p	modal dof of the experimental substructure with fixed boundary
q	modal dof of free modes extracted from experimental substructure with TS attached
s	free modal dof of the transmission simulator
x	physical displacement dof
ω	frequency in radians per second
ζ	modal damping ratio
K	stiffness matrix
L_{fix}	reduction matrix applying fixed boundary constraint to experimental equations of motion
M	mass matrix
T	transformation matrix to convert free modal model to modal CB model
Φ	free mode shape matrix extracted for experimental substructure with TS attached
Ψ	free mode shape matrix of the TS
Γ	eigenvectors resulting from fixed boundary constraint of experimental equations of motion
b	subscript for the fixture or boundary
fix	subscript for the fixed boundary modes of the experimental substructure with TS as the boundary
$free$	subscript for the free modes obtained in the modal test of the experimental substructure with TS
+	superscript indicating the Moore-Penrose pseudo-inverse of a matrix

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1) Abstract

This work was motivated by a desire to transform an experimental dynamic substructure derived using the transmission simulator method into the Craig-Bampton substructure form which could easily be coupled with a finite element code with the Craig-Bampton option. Near the middle of that derivation, a modal Craig-Bampton form emerges. The modal Craig-Bampton (MCB) form was found to have several useful properties. The MCB matrices separate the response into convenient partitions related to 1) the fixed boundary modes of the substructure (a diagonal partition), 2) the modes of the fixture it is mounted upon, 3) the coupling terms between the two sets of modes. Advantages of the MCB are addressed. 1) The impedance of the boundary condition for component testing, which is usually unknown, is quantified with simple terms. 2) The model is useful for shaker control in both single degree of freedom and multiple degree of freedom shaker control systems. 3) MCB provides an energy based framework for component specifications to reduce over-testing but still guarantee conservatism.

Keywords – Experimental Dynamic Substructures, Substructuring, Craig Bampton, Shaker Control, Six DOF Shaker Control, Environmental Specifications, Energy Methods

2) Introduction

The main value of this work comes as an accidental discovery in an investigation focused on experimental dynamic substructuring using the transmission simulator (TS) method[1]. The motivation of the work was to take the standard form of the TS method and convert it to a form that could be used as a standard Craig-Bampton substructure in FE codes. This is described more completely in another work[2]. After the associated theory was developed, the utility of the intermediate modal CB form was realized. In the modal CB form, the boundary degrees of freedom (dof) are characterized with generalized dof instead of the classic physical dof. In the method provided here, this can potentially provide a drastic reduction in the number of boundary dof with a mild cost in modal truncation error. The theory here utilizes a modal basis of free modes of the TS to quantify the boundary motion, but this is not a limitation, and it may be that other bases may be found that are more accurate.

For those who are not familiar with TS method, the TS is a fixture that is attached to the substructure of interest in exactly the same way as the complement of the real system will be attached, which might be modeled with FE or another experiment. The TS is instrumented with enough sensors to capture the motion adequately with a truncated set of modes to the desired frequency band. The TS instrumentation does not have to be located at every connection dof, and rotational dof are not required. However, the rotational and connection dof are inherently carried along in the modal dof of the TS. This method captures the stiffness and damping of the joint between the connected structures as well as the characteristics of the substructure of interest. All the measured dof on the TS are considered as part of the CB boundary dof, which leads to the utility to be discussed later.

In the following theory, it will be demonstrated that the experimental free modes model of the TS connected to the substructure of interest can be transformed to a matrix form called a modal CB. One partition of the stiffness, mass and damping matrices is diagonal. It is exactly the same as the standard CB form and accounts for the fixed boundary modes, where the TS fixture is considered the boundary. The other modal dof are associated with the motion of the TS. Coupling terms connect the two in some cases. The value here is that the motion of the substructure has been separated into the dof that strain the substructure, and the dof describing the motion of the base. This separation provides the capability to gain tremendous insight that is not possible when the motion of the boundary is described in standard physical coordinates. With this approach one acquires the following power directly from the analysis. 1. The effects of the impedance of the boundary are directly quantified, mode by mode. 2. The motion input to the boundary dof can be directly calculated to produce a desired substructure modal response (for example in sdof or mdof shaker control). 3. One can utilize the fixed base modal dof to specify energy based qualification testing for the substructure. This method drastically reduces the classic but unnecessary over-conservatism at many frequencies, but still theoretically guarantees conservatism at all frequencies of interest, which is not the case with current standard methodologies for dynamic testing.

Hereinafter is presented the theory of the transformation in section 3, the value of the modal CB form for quantifying the uncertainty in the boundary condition (impedance) in section 4, the value in sdof or mdof control in section 5, and the value for superior qualification specifications in section 6. Afterward are the conclusions in section 7.

3) Theory

Consider an experimental substructure tested with the TS fixture attached. The test captures modal parameters for the free modes of the substructure and the attached TS fixture. It is desired to transform the experimental model to a modal CB form which contains fixed boundary modes for the substructure of interest, free modes of the fixture and coupling terms to connect the generalized dof. As an example, consider component A, the substructure of interest attached to a fixture, the TS, in Figure 1. The test article would be instrumented according to the traditional TS method. The goal is to transform the free mode test results to a modal CB form, which has useful properties that will then be examined. The free modal test will produce modal parameters associated with the *q* dof. After a transformation, the TS has free modal parameters associated with the *s* dof, and the motion of component A will be described with the fixed boundary modal dof *p*.

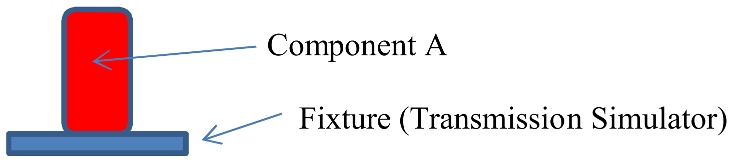

Figure 1 - Example Component on Test Fixture

Generally, there is a FE model of the TS. The FE model is used in test planning to define measurement locations that will achieve independent mode shape measurements for all free modes of the TS slightly beyond the frequency band of interest. The TS fixture is thus instrumented. The transmission simulator hardware is attached to the experimental substructure and the free TS mode shapes are assumed to span the space of the fixture motion when connected to the experimental substructure. How well it spans the actual connection motion space affects the fidelity of the substructure model. The modal parameters from a free modal test of the experimental substructure with the TS attached can be used to produce the following equations of motion as

$$\left[\omega^2{}_{free} + j2\omega\omega_{free}\zeta_{free} - \omega^2 I\right]\bar{q} = 0 \tag{1}$$

where the subscript *free* represents the set of modes obtained from the experimental modal test. The subscript *free* is used because the structure is typically suspended by bungee cords or some very soft suspension whose mass, stiffness and damping are considered negligible. The mass-normalized mode shapes derived from the test will be contained in the measured mode shape matrix, Φ. For convenience, the rest of this derivation will drop the damping matrices, but they may easily be included. The goal is to derive a square matrix transformation, *T*, that will convert eqn. (1) to a modal CB form. Define the generalized coordinates, *p,* as the fixed-boundary modal coordinates and the generalized coordinates, *s,* as the coordinates that account for the motion of the TS, which is considered to be on the boundary of the experimental substructure as

$$\bar{q} = T\begin{Bmatrix}\bar{p}\\ \bar{s}\end{Bmatrix} \quad . \tag{2}$$

First consider a constraint that ties the free TS to the tested structure. Use the modal approximations to set the motion of the experiment on the boundary (TS dof) to match the free modal motion of the TS as

$$\Phi_b\bar{q} \approx \Psi_b\bar{s} \tag{3}$$

where the subscript *b* dof will actually be a subset of the boundary dof where the measurements are made, Φ is the experimental mode shape and Ψ is the chosen truncated set of free modes of the TS. Ψ usually comes from a TS FE model, but could also be measured. Then the relation between *q* and *s* is

$$\bar{q} = \Phi_b^+\Psi_b\bar{s} \tag{4}$$

where the + sign represents the Moore-Penrose pseudo inverse. This provides the right hand partition of the transformation, *T,* associated with the *s* dof.

To obtain the fixed boundary modal dof, *p*, describing the elastic motion of component A, fix the boundary dof with

$$\bar{x}_b = \Phi_b \bar{q} = 0 \quad . \tag{5}$$

Previous work[3] has shown that a practical way to accomplish eqn. (5) is to fix the TS dof with

$$\Psi_b^+ \Phi_b \bar{q} = s = 0 \quad . \tag{6}$$

Using Rixen's primal assembly[4], the modal dof are replaced with

$$\bar{q} = L_{fix} \bar{\eta} \tag{7}$$

which is substituted back into eqn. (6) to obtain

$$\Psi_b^+ \Phi_b L_{fix} \bar{\eta} = 0 \quad . \tag{8}$$

Since $\bar{\eta}$ can be many vectors, depending on the forcing motion, L_{fix} is chosen to guarantee satisfaction of the constraint as

$$L_{fix} = null(\Psi_b^+ \Phi_b) \quad . \tag{9}$$

Pre and post-multiply eqn. (1) using the transformation L_{fix} appropriately to give

$$L_{fix}^T \left[\omega^2{}_{free} - \omega^2 I\right] L_{fix} \bar{\eta} = 0 \quad . \tag{10}$$

Solve eqn. (10) to get the eigenvectors, Γ, and the eigenvalues to uncouple the dof, p. Then the relationship between q and the fixed boundary dof, p, is

$$\bar{q} = L_{fix} \Gamma \bar{p} \tag{11}$$

which provides the rest of the transformation. T is written from eqn.(4) and (11) as

$$T = \left[L_{fix}\Gamma \quad \Phi_b^+ \Psi_b\right] \quad . \tag{12}$$

Pre multiplying eqn. (1) by the transpose of T and substituting eqn. (2) into eqn. (1) for q yields the following transformed equations of motion for free vibration

$$\left[\begin{bmatrix} \backslash\omega_{fix\backslash}^2 & K_{ps} \\ K_{ps}^T & K_{ss} \end{bmatrix} - \omega^2 \begin{bmatrix} I & M_{ps} \\ M_{ps}^T & M_{ss} \end{bmatrix}\right] \begin{Bmatrix} \bar{p} \\ \bar{s} \end{Bmatrix} = 0 \tag{13}$$

for which the eigenvalue and eigenvector solution have not changed from eqn. (1). It has exactly as many dof as eqn. (1), but now they have been transformed to the fixed base modes associated with p and the free TS modes which were on the boundary as modal dof s. The upper left portion of the matrices is diagonal. Now there are coupling terms between the fixed base modes and the free TS motion. Considering the upper partition of eqn. (13) and moving the boundary TS dof, s, to the right hand side develops equations of motion from enforced boundary motion as

$$\left[\left[\backslash\omega_{fix\backslash}^2\right] - \omega^2[I]\right]\{\bar{p}\} = \left[\left[0 \quad -K_{ps_e}\right] + \omega^2\left[M_{ps_{rb}} \quad M_{ps_e}\right]\right] \begin{Bmatrix} \bar{s}_{rb} \\ \bar{s}_e \end{Bmatrix} \tag{14}$$

where the e subscript is associated with the free elastic modes of the TS structure and the rb subscript is associated with the free rigid body modes of the TS structure. Notice that there is no coupling of the p dof with the s_{rb} dof through stiffness but there is coupling through mass terms. The p dof are coupled with the elastic s_e dof through both stiffness and mass terms. In general there are many p, six s_{rb}, and many s_e dof. For discussion purposes let us assume there is only one s_{rb} dof and one s_e dof. Since the left hand side is uncoupled, we can consider the scalar equation of motion for the very first p dof as

$$(\omega_{fixed\_1}^2 - \omega^2) p_1 = \omega^2 m_{rb\_1} s_{rb\_1} + (-k_{e\_1} + \omega^2 m_{e\_1}) s_{e\_1} \quad . \tag{15}$$

Eqn. (15) determines how much elastic modal dof p_1 is excited by enforce rigid body motion and elastic motion of the TS.

4) Value of the Modal CB Form for Removing Uncertainty in the Boundary Condition

Suppose one performs a free modal test of the structure in Figure 1, has a FE model of the fixture, and transforms the results of eqn. (1) into the modal CB form as in eqn. (14). One might also achieve the results of eqn. (14) with a modal model derived from a FE model of component A and the fixture. Such a model conveniently quantifies the effect of the boundary condition (the fixture) on the elastic motion of a substructure such as component A. Consider one mode of component A in eqn. (15). The elastic motion of component A characterized by generalized dof p_1 is influenced by each TS rigid body modal dof times its mass coupling term, which is classically called the modal participation factor[3]. In addition, mode p_1 is influenced by the elastic motion of each mode of the TS multiplied by both a stiffness and a mass coupling term. This immediately describes the impedance effects of the TS on component A on a mode by mode basis. All of these mass, stiffness and damping coupling terms come directly from the transformation that is applied to the free modal model of component A attached to the fixture (TS). This quantification removes the uncertainty associated with the boundary condition that has clouded virtually all component qualification testing.

It was observed by Savoie[4] that if a similar transformation were applied not only to component A with the TS as its vibration/shock testing fixture, but also to component A with the TS as the FIELD SYSTEM, the impedance effects could be directly compared between field environment and ground test to see how different they are. Many times there is a FE of the rest of the system that could be used as the TS. This difference between the field and test boundary conditions has always been a massive uncertainty using traditional methods of qualification specification and testing. In addition, environments engineers have related stories of overtesting of components at certain resonant frequencies associated with elastic modes of the fixture, s_{e_k}, which were not experienced in the actual field environment.

The reasoning in the paragraph above also suggests that we can quantify the quality of the test fixture in replicating the boundary conditions for the next higher level assembly. Future work could explore improving fixture design as well as test specification tailored to the different test fixtures used in qualification (e.g. different vibration fixtures for different axes, shock fixtures, etc.)[4].

5) Value of the Modal CB Form for Vibration Control in SDOF or MDOF Tests

Eqn. (14) could also be very useful from the standpoint of vibration control for testing of component A. The ideal fixture for shock or vibration control in qualification of a substructure like component A would be rigid, having only s_{rb} terms and no elastic s_e terms like the one shown in eqn. (15). Then one could excite the associated p_i terms exactly as desired by controlling the rigid body motion of the fixture and knowing the modal participation factors. This applies to the standard sdof (one translation direction) excitation as well as emerging mdof (e.g. 3 dof or 6 dof) shaker control. Unfortunately, there are usually elastic fixture modes in the bandwidth of interest. For sdof control, usually only one s_{rb} term (for one desired direction) is excited by the shaker, but there will also be elastic s_e terms excited. Using the modal CB model, one can see how much excitation of p_i comes from the desired rigid body fixture motion AND how much excitation comes from elastic motion of the fixture. Over-excitation of a particular p_i of component A due to elastic fixture modes can be remedied with this knowledge.

There is also the possibility of identifying uncontrollable input. For example, in a sdof shaker, the terms of the right hand side may add in such a way to produce a near zero value at some frequency. For a sdof shaker, all the s terms will be proportional to the input, i.e., the ratio of one s term to another is always the same. No matter what control algorithm is used, one will not have any control at this frequency. Nonlinearities in the system may produce undesired motion at this frequency. This formulation can warn the vibration engineer that there is danger of an uncontrollable frequency regime.

6) Value of the Modal CB Form as a New Paradigm for Qualification Specifications in the Energy Domain

At Sandia National Laboratories, energy methods are being used for quantification of environmental margin of components, such as component A. Edwards[5] has shown how such a method can predict the damaging energy absorbed by a test article subject to random vibration. Damage is generally induced in a component by elastic motion, not rigid body motion. Standard environmental specifications do not distinguish the rigid body motion from the elastic motion, i.e. these motions are confounded in the specification. A great advantage of the modal CB method is that it conveniently quantifies ONLY the damaging elastic motion with the generalized p dof, the motion of true interest for qualification. Based on eqns. (14,15) the amount of strain, dissipative or kinetic energy in the substructure may be determined for any specific mode as

$$SE = 1/2\omega_{fixed\_i}^2 p_i^2 \tag{6-1}$$

$$DE = \int 2\zeta_i \omega_{fixed\_i} \dot{p}_i p_i \, dp_i \tag{6-2}$$

$$KE = 1/2\dot{p}_i^2 \tag{6-3}$$

where SE is strain energy, DE is dissipative energy and KE is kinetic energy. To specify environments, one could quantify them in terms of these values. For example, if an environment causes a failure by strain, one could determine the maximum strain from an environment in each of the p modes. Uncertainty in the fixed base natural frequency could be included in Monte Carlo or other statistical analyses to determine the maximum SE that might be achieved in any subsystem for the environments being considered and a specification for the vibration or shock testing written based on each p. This would then be guaranteed to be conservative, unlike current acceleration frequency based specifications in a uniaxial direction. It has been shown for certain cases that the rate of fatigue damage is increased over uniaxial random input by a factor of two with more realistic three axis excitation[6]. However, the over-conservatism in many frequency lines of current acceleration frequency based specifications would be removed using energy methods with the fixed boundary modal dof. Eqn. (14) could then be used in the vibration test to perform the control for the qualification test.

Future work might also bring another enhancement to this model. Suppose there are some p_i dof that have no relevant contribution to the failure modes of component A. If even some of those can be identified, their effect could be minimized, or possibly even removed from the qualification specification[4].

7) Conclusions

The modal CB form conveniently separates the elastic response of the substructure of interest from the modal dof response of the structure that it is mounted upon, the TS. Simple coefficients relate the substructure response mode to the modal motion of the TS. The TS can be a test fixture, or it can be the entire system upon which the substructure is mounted. The differences in impedance between a test fixture and the field system can be quantified, which would introduce a new capability. The modal dof of the fixture can be used directly in establishing control parameters to excite the substructure in vibration or shock testing. This is of value for either traditional sdof input testing or emerging mdof testing capabilities. It is suggested that the energy methods, sometimes used in margin quantification, could be utilized with this modal CB form to specify environments in a way that guarantees conservatism, but reduces traditional over-conservatism in many frequency bands by a large margin.

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