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METHODOLOGY FOR DEFINING MULTI-AXIS VIBRATION SPECIFICATIONS

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The vibration excitation mechanisms for structures in service are typically multi-directional. However, sequential single axis testing has been established as the standard practice for decades. As a result, most test specifications are written for single axis tests. Developing multi-axis test specifications is challenging, because in addition to specifying the auto-spectra required for a single axis test, the cross-spectra relating the vibrations between axes must be defined as well. Often, when using legacy data, or standard specifications (e.g. MIL-STD 810G), there is no information about the cross-spectra. When collecting new field data for test specification development, there are often channel limitations that make it difficult to define all the translations, rotations, and cross-spectra. Depending on the availability of information, different techniques may be used for defining the cross-spectra. Provided there are time histories from a field test with sufficient data to define the full 6DOF motion, the cross-spectra can be defined using the collected data. In the absence of that information, different assumptions can be made for the cross-spectra including zero coherance, or defining the coherence such that the drive energy required to run the test is minimized. These different methodologies are presented and compared.

INTRODUCTION

Vibration environments in the field are inherently multi-dimensional, and can often be described in six degrees of freedom (three translations and three rotations). Deriving multi-axis test specifications can be challenging due to lack of sufficient field data. Therefore, the current common practice is to perform development and qualification testing as a series of three single-axis tests. The rotational axes are typically unknown and ignored with the assumption that the rotations are constrained to being small when excitation is provided by a single-axis shaker. Additionally, cross-axis motions are not controlled and the assumption, which may or may not be valid, is that they are small. The common practice approach assumes that if a component survives the three single-axis tests, then it would survive multi-axis excitation in a field environment.

The response of a structure, including the resulting stress states, is dependent upon the number, amplitude and phase of the participating modes and on the input excitation. It has been shown [1, 2, 3, 4, 5, 6] that the response of structures are different if they are excited in multiple axes simultaneously versus one axis at a time. Additionally, multiple researchers [5, 6, 7, 8, 9] have reported evidence of different failure modes and fatigue life estimates for multiple-axis loading versus single-axis loading, challenging the assumption that sequential testing in three single-axes is a conservative method of testing.

While it is understood that multi-axis testing has benefits, it also has challenges. One of the largest challenges is defining the test specifications. When specifying a multi-axis test, a full 6x6 spectral density (SD) matrix must be defined. A full matrix cannot usually be calculated from field data. The off-diagonal terms in the matrix are the cross spectral density (CSD) terms, and with the typical instrumentation for field tests, they cannot be calculated. However, both the rotations and CSDs, which are the aspects of the SD matrix that are difficult to get from field tests, can be calculated from high fidelity finite element models. The CSDs can also be determined from laboratory tests.

In this paper, different methods for defining the CSDs will be presented and compared. Additionally, a typical scenario will be presented, and the proposed methodology will be explained in the context of the scenario.

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COMPARISON OF METHODOLOGIES FOR DEFINING CROSS SPECTRAL DENSITIES

One of the significant questions faced when having to define multi-axis test specifications is how to write them when lacking sufficient measured field data. There are several different methods that can be used to fill in the missing gaps of information. Each of the methods that will be discussed has advantages and disadvantages. The methods include: 1) specifying the entire spectral density matrix and defining the cross spectra to meet certain objectives, 2) specifying only the diagonals of the SD matrix, and 3) forcing all of the CSDs to zero. Each of these methods to define the CSD will be discussed further.

The first method for defining CSDs is to specify the entire SD matrix and define the CSDs in such a way that they: 1) minimize the total drive energy to the system, 2) match the PSD response for certain key locations, 3) maximize the response at select locations for margin assessment, or 4) consists of some weighted combination of the other three criteria. Using the criteria of defining the CSDs so as to minimize the total drive energy of the system or matching the PSD response at key locations is advantageous because the test specifications are then based on structural response, so it is likely the best estimate of what would have occurred in the field. This then makes the tests easier for the shakers to perform and it is possible to get the most structural response for the least amount of input. The challenging aspect of this method is that the dynamics of the part must be well characterized either through a high fidelity model or through laboratory testing, which can be expensive and challenging to attain.

The second method for defining the SD matrix is to specify only the diagonal terms, allowing the CSDs to be undefined. This method is easy to define and requires no additional testing or modeling and can thus be done quickly and inexpensively. However, this method does not account for the structural response interacting with the input, can require high levels of drive, and therefore result in an unrealistic test.

The third method for defining the CSDs is to force all of the cross spectra to zero. Like the previous method, this method is easy to define and requires no additional testing or modeling and can be done quickly and inexpensively. It is commonly done in industry as an attempt to simply combine the three single-axis tests as a single three-axis test. What this doesn't account for is that single-axis tests are not truly single-axis, so the cross terms would not truly be zero. Because the system is being forced to behave in a way it would not ordinarily behave, higher stresses and earlier failures can occur in test items than would realistically occur in actual field environments. This unrealistic behavior may cause, shaker drive requirements to be quite high and perhaps unobtainable as well

The first method discussed for defining CSDs is the best choice, because it accounts for the dynamics of the system, thus giving the best estimate for what the CSDs in the field would be. Accordingly, the following section will outline a scenario for which this method can be utilized to define the environments, followed by an outline of the methodology given that scenario.

SCENARIO DEFINITION

This scenario, although fictitious, represents a realistic scenario in a laboratory that tests components residing within large systems containing many parts. These types of large systems generally have limitations on channel counts when performing field tests.

The test item, Part A, contains 4 internal components, 1-4 as shown in Figure 1. Due to channel count limitations during field testing, internal components 1-4 were not instrumented, so no field test data is available. However, there is a single channel of field test data from two locations on Part A. A series of 3 single-axis system level ground tests were performed on the system. During the ground tests, all of the internal components were instrumented to collect 3 mutually perpendicular channels of data. In parallel to the testing, a high fidelity model was developed and validated using test data from the laboratory. The equipment for doing the multi-axis testing is a shaker system with a large table, for which six degrees of freedom (6DoF) can be defined, so given the constraints on available measured data, it is desirable to come up with 6DoF inputs into internal component 2.

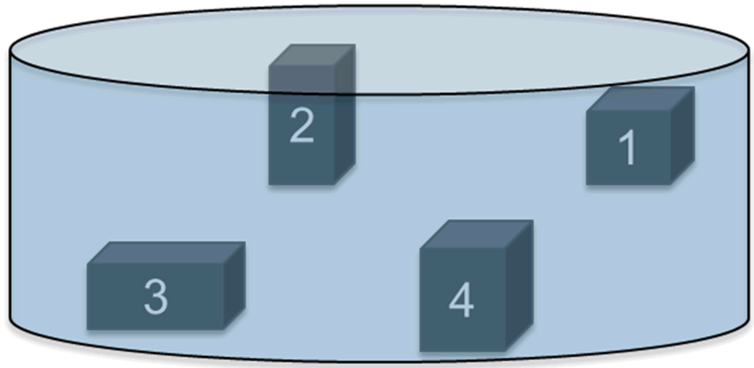

Figure 1: Schematic of Part A with Internal Components 1-4

EXAMPLE SOLUTION

The first step in defining a complete SD matrix is to determine the dynamic response of the part. There are two methods for achieving this: using a model or using test data. Given that a high fidelity model exists, it is possible to put in uncorrelated white noise in all six degrees of freedom, and calculate the transfer functions for the responses for each of the internal components relative to that input. An alternate method for determining the dynamic response is to put uncorrelated white noise into the physical part in the laboratory and measure the responses. An advantage to characterizing the dynamics using a laboratory test is that the actual part is going to be used, so there is no modeling uncertainty to influence the calculations. The disadvantage to getting the dynamics from the laboratory tests is that it can require multiple tests when trying to find the CSDs for internal components.

The second step in defining the SD matrix is to calculate the CSDs for all of the internal parts; in this case it will be a 12 x 12 matrix. There are two pieces of information that are fed into algorithms for calculating the CSDs: the power spectral densities (PSDs) from the ground level tests and the transfer functions from either a high fidelity model or tests in the laboratory. There are a number of algorithms that can be used to calculate the CSDs and can be designed to fit various metrics, such as minimizing drive energy. In this case the algorithms used are described in [10, 11].

The third step is to ensure that the resulting SD matrix is positive definite. There are multiple calculations that can be done, such as the Choleski decomposition or an Eigenvalue decomposition that can give an indication of the positive definiteness of the matrix. If the matrix is not positive definite, the terms that are causing the spectral density matrix to not be positive definite need to be varied until the matrix is positive definite. It is critical that the spectral density matrix is positive definite because the control algorithms invert the spectral density matrix, and this cannot be done on a matrix that is not positive definite. Additionally, a matrix that is not positive definite implies that negative drive is required to run the test, which is not physically possible.

Step four is to do an inverse problem on the high fidelity model to determine the 6DoF inputs into Part A so that the responses for internal components 1-4 are matched. It is likely that it will not be possible to match all of the responses of all of the internal components. Therefore, a least squares fit would be required calculate a 6DoF input that yields responses that most closely match the responses from the ground test.

The fifth step is to determine the 6DoF inputs into component 2. Response points from the model on Component 2, which would be representative of how the component would be attached to the multi-axis shaker system, can be used to determine what the 6DoF inputs to Component 2 should be in order to make it respond as it does while contained in Part A. Depending on the orientation of Component 2 within Part A, coordinate transformations might be necessary to align the required inputs to that of the table.

SUMMARY OF METHODOLOGY

The methodology for determining 6DoF inputs to a part that cannot be sufficiently instrumented in field tests described in this paper can be summarized by the following steps:

1. Determine the transfer functions to describe the dynamic response of the part in which the component of interest resides.
2. Use the transfer functions from a high-fidelity model or laboratory tests, as well as data from ground tests, to calculate a spectral density matrix in such a way as to minimize the drive energy.
3. Ensure that the spectral density matrix is positive definite; if not iterate on the spectral density matrix terms until it is.
4. Perform an inverse problem to determine the input into the part that would yield the response in the spectral density matrix; perform a least squares fit if it is not possible to match all the responses.
5. Use the input calculated in Step 4 to calculate the response of the internal component of interest and the corresponding 6DoF inputs to the component required to make the response occur.

CONCLUSIONS

Multi-axis testing has many benefits. With multi-axis testing, it is possible to get more realistic stress states in the parts and more accurately simulate field environments, which allows for more realistic screening of parts. Additionally, the inputs are more controllable with multi-axis testing than with single-axis testing, for which it is not possible to control the off-axis excitation. Therefore, multi-axis testing will allow for more accurate data with which to calibrate models, leading to more predictive models and improved designs.

It is challenging to define test specifications when there is limited data available from field tests. The most common pieces of information that are missing are the cross-spectral terms. Several methods have been proposed for how to define spectral density matrices in the absence of the information about the CSDs. The methodology in this paper proposes using a combination of high-fidelity models and known test data to determine reasonable CSDs based on the dynamic characteristics of the test item for a multi-axis test.

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