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# LayTracks3D: Hex Meshing using MAT

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# Goal of LayTracks3D

- Handle General Solids & Assemblies
- Orientation Insensitive
- Boundary Sensitive
- Preserves Imprints and Sharpe Features
- Geometry Adaptive
- Fast Remeshing
- Morphable Meshes
- Parallel Friendly
- Potential All-Hex

# Advantages of MAT

- Orientation Insensitive:
  - MA of a solid does not change with orientation of the solid
- Boundary Sensitive:
  - AFM fronts meet at MA, i.e., structured mesh along the boundary
- Robust Geometry Decomposition:
  - Medial branch points represent critical singularity regions
- Dimension Reduction:
  - Hex meshing can be reduced to surface meshing
- Geometry Adaptive:
  - Medial radius function provides local feature size to control mesh size.
- Symmetric Skeleton:
  - Simplifies interval assignment and restricts irregular nodes at MA
- Homotopy Equivalence:
  - Preserves sharp boundary features

# Medial Axis Transform

## Two-Way Mapping

$$f: p \rightarrow Mp$$

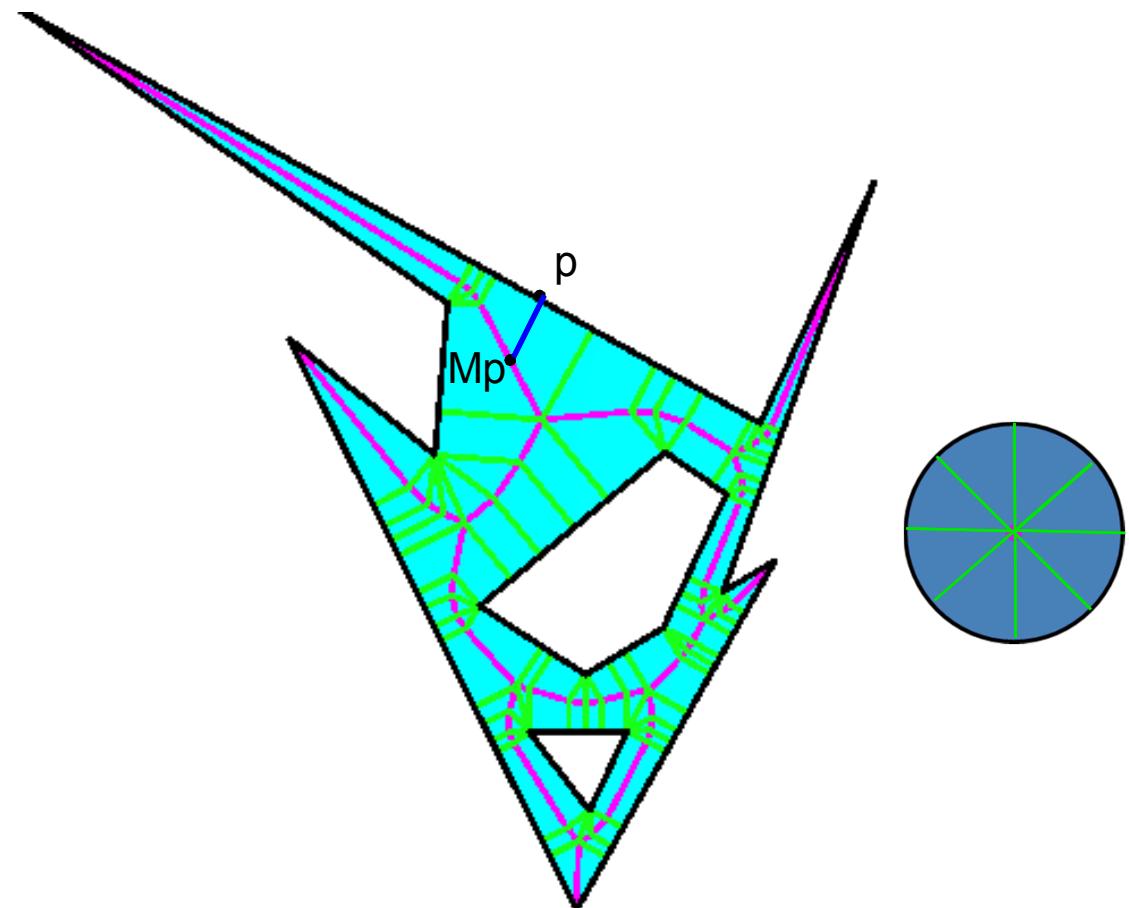
$$g: Mp \rightarrow p$$

## Types of Mapping

*1-to-1 Map*

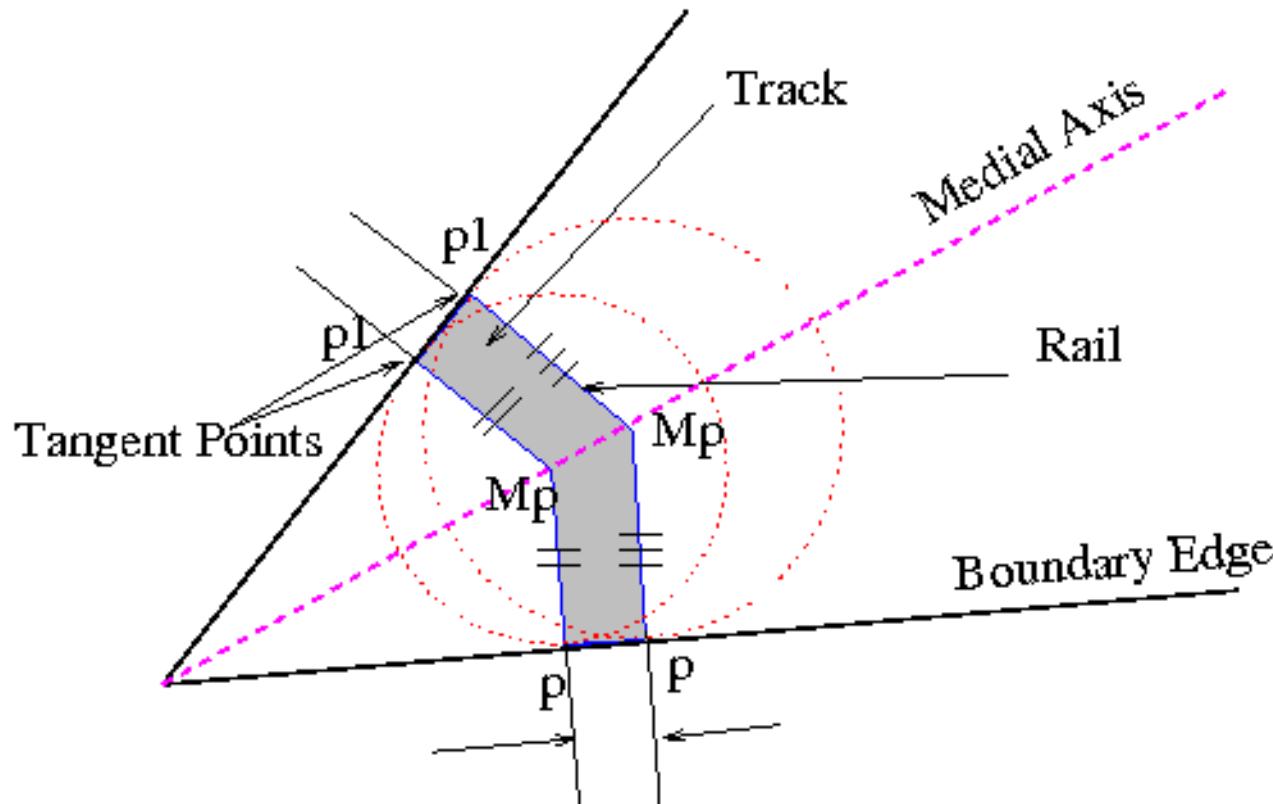
*1-to-N Map*

*N-to-1 Map*



# Projection Operator

- Uses map  $f$  to connect  $p \rightarrow Mp$
- Uses map  $g$  to connect  $Mp \rightarrow p_1$



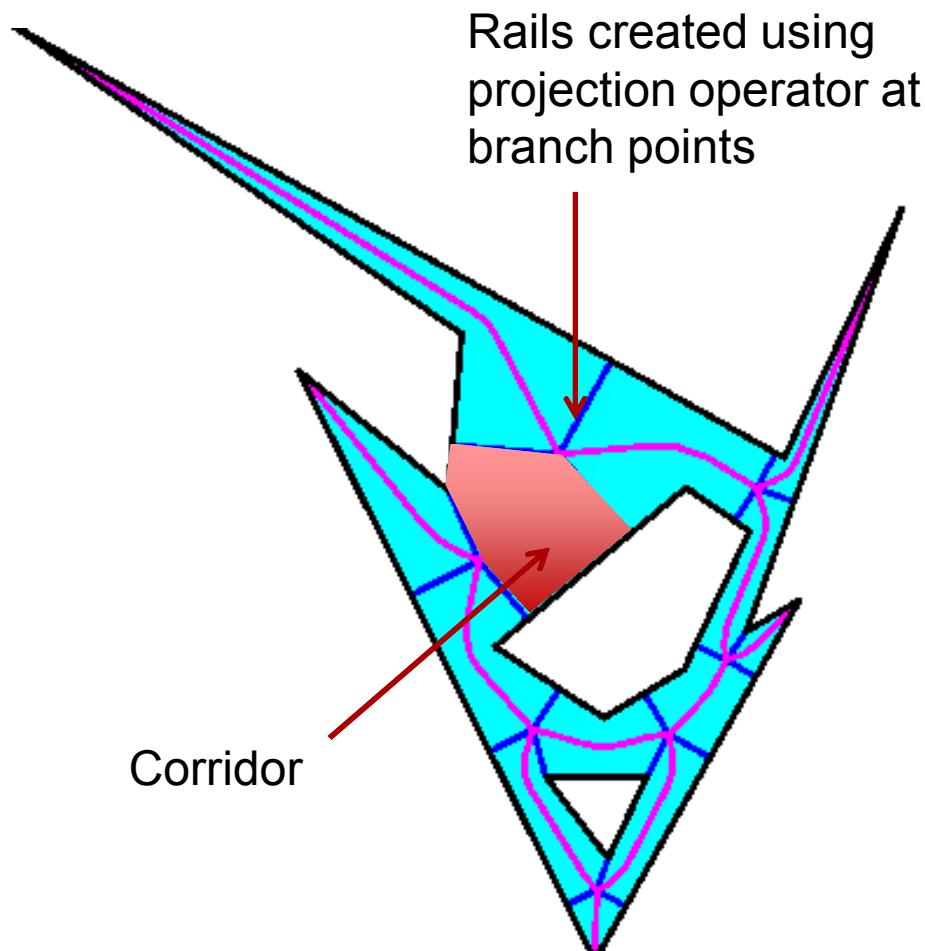
# LayTracks: All-Quad Meshing, 9<sup>th</sup> IMR, 2000

Combines merits of decomposition and advancing front methods

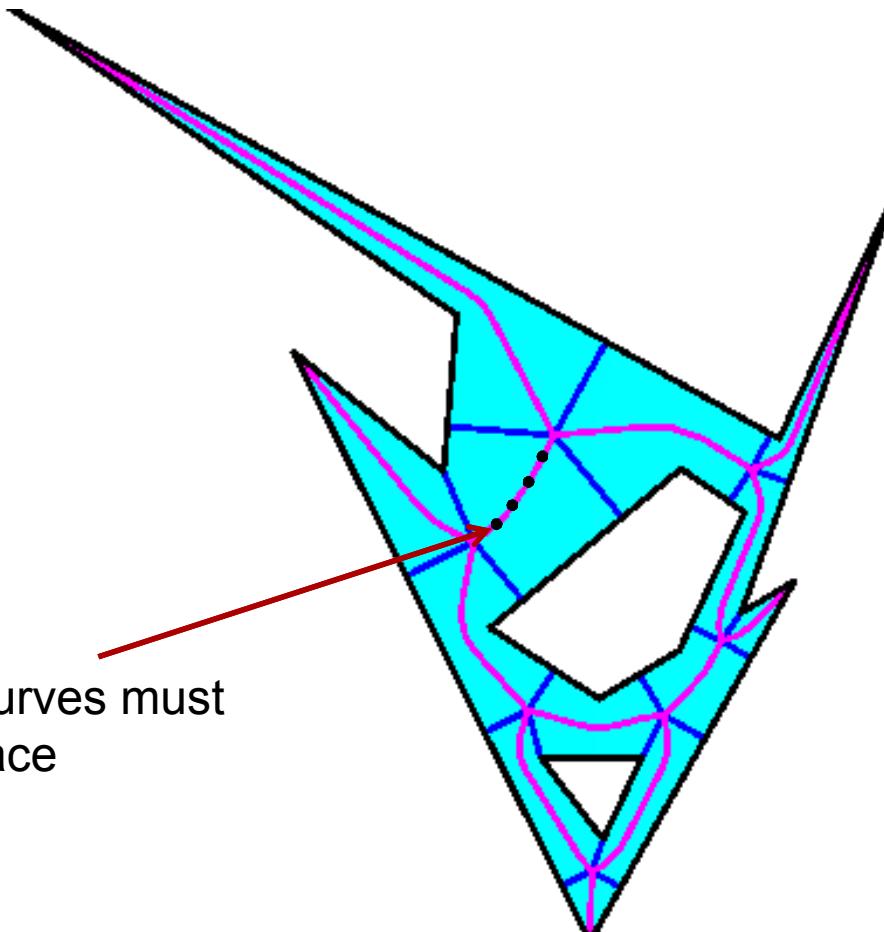
- Generate MAT: establish two-way map between domain boundary and medial
- Generate Corridors: decompose domain using medial branch points and projection operator
- Mesh Medial: surface meshing is reduced to curve meshing. Medial radius function can be used to control mesh size.
- Generate Tracks: further decompose the domain/Corridors into simpler/meshable Tracks using projection operator
- Mesh Tracks: generate all-quad in each track using symmetric property of the medial

# Generate Corridors:

## Domain Decomposition



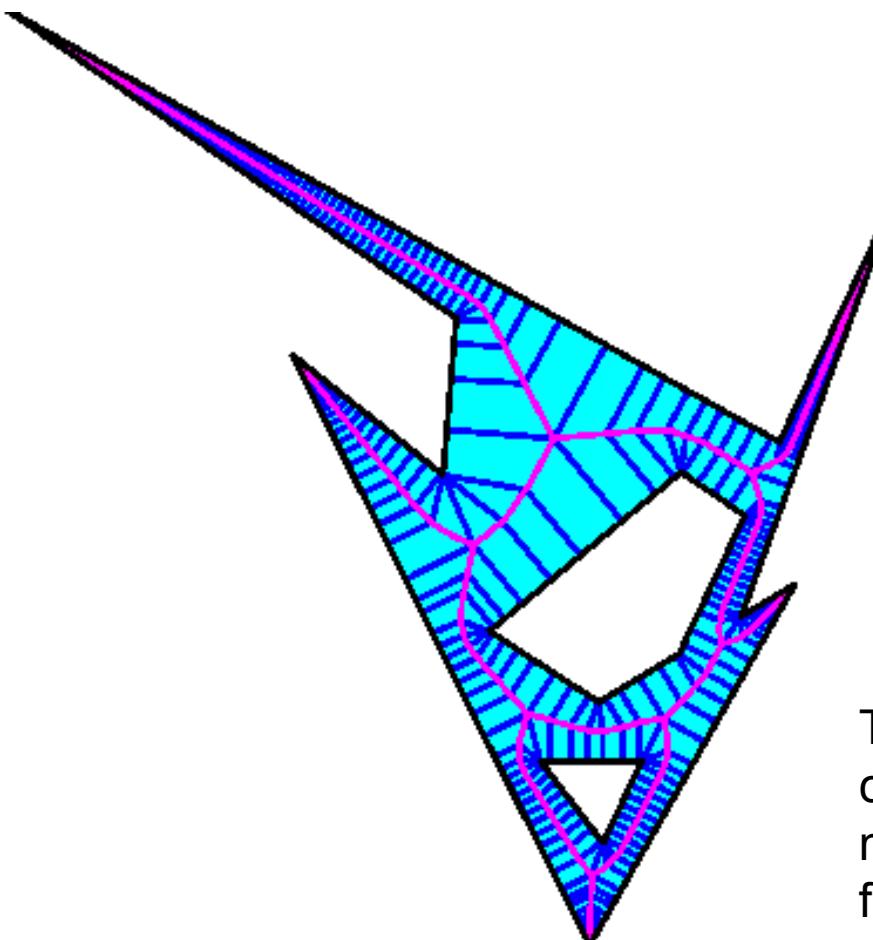
# Mesh Medial: Surface meshing is Reduced to Curve Meshing



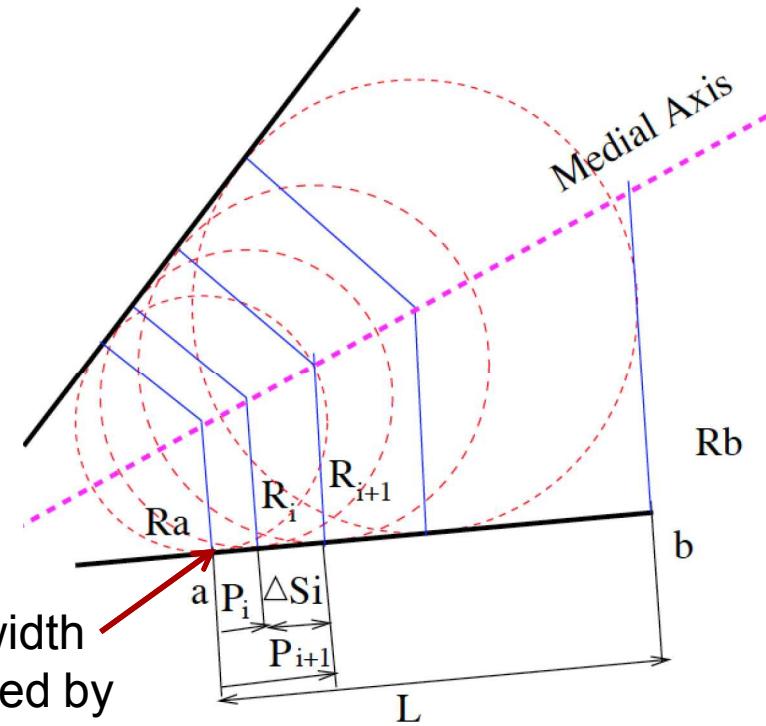
Meshing all medial curves must cover the entire surface

# Generate Tracks: Projection Operator

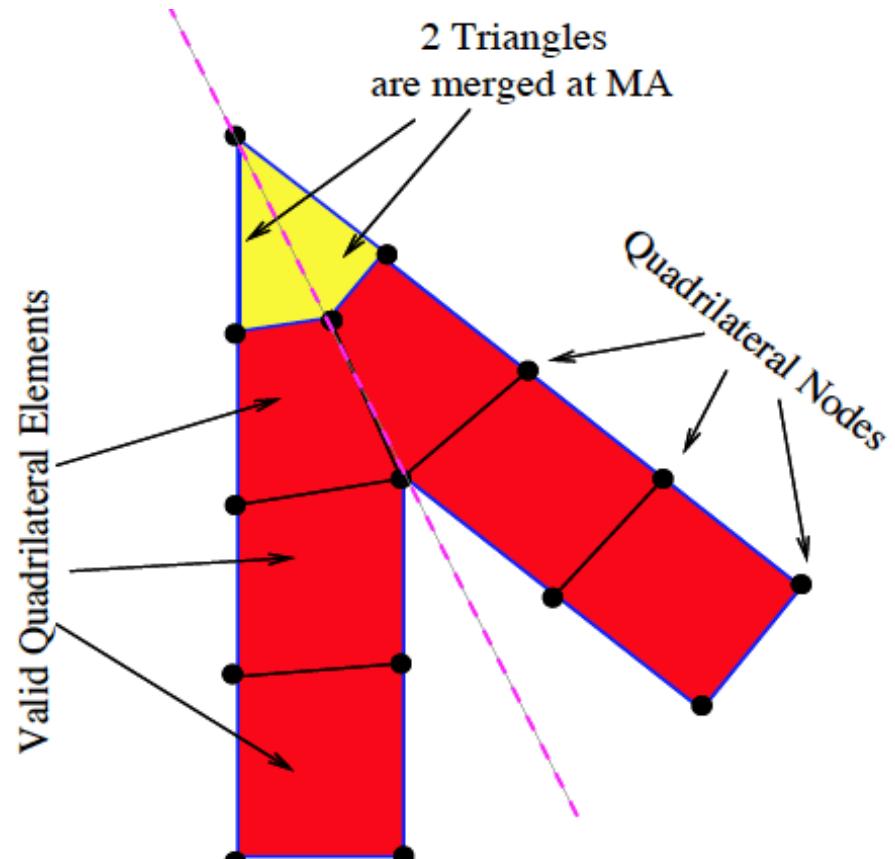
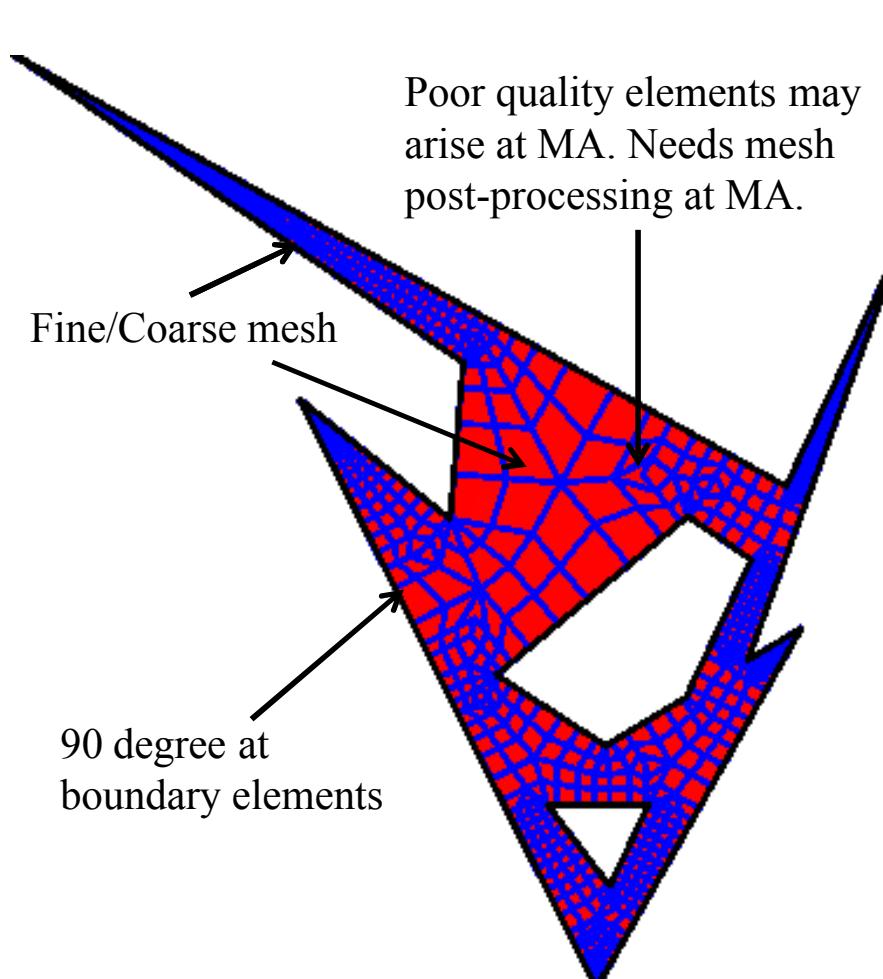
## Connects MA Nodes to Boundary



Track width  
controlled by  
medial radius  
function

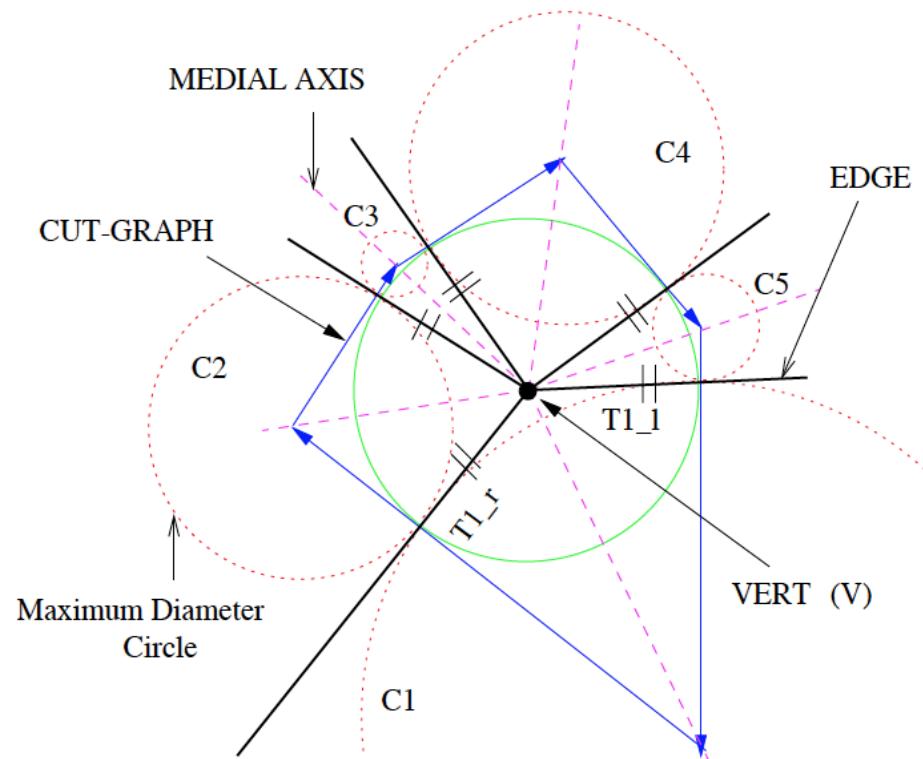
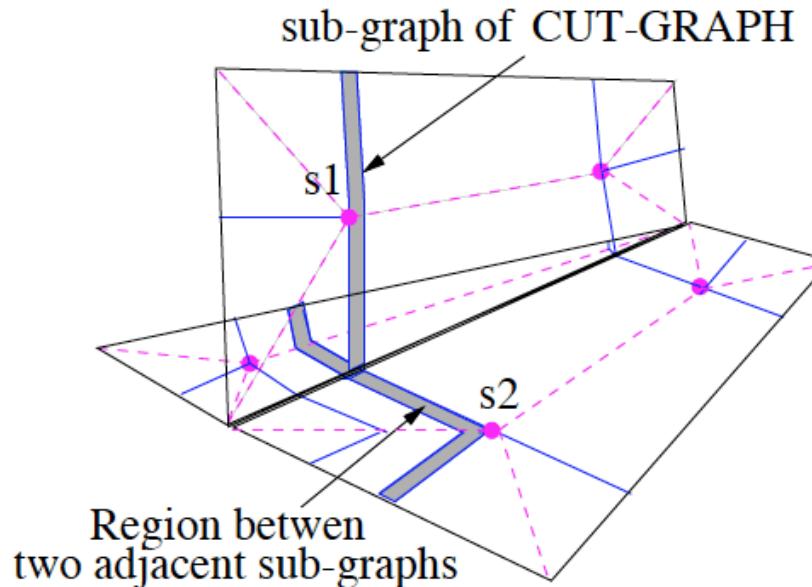


# Mesh Tracks: All-Quad Mesh can be Generated in 2D

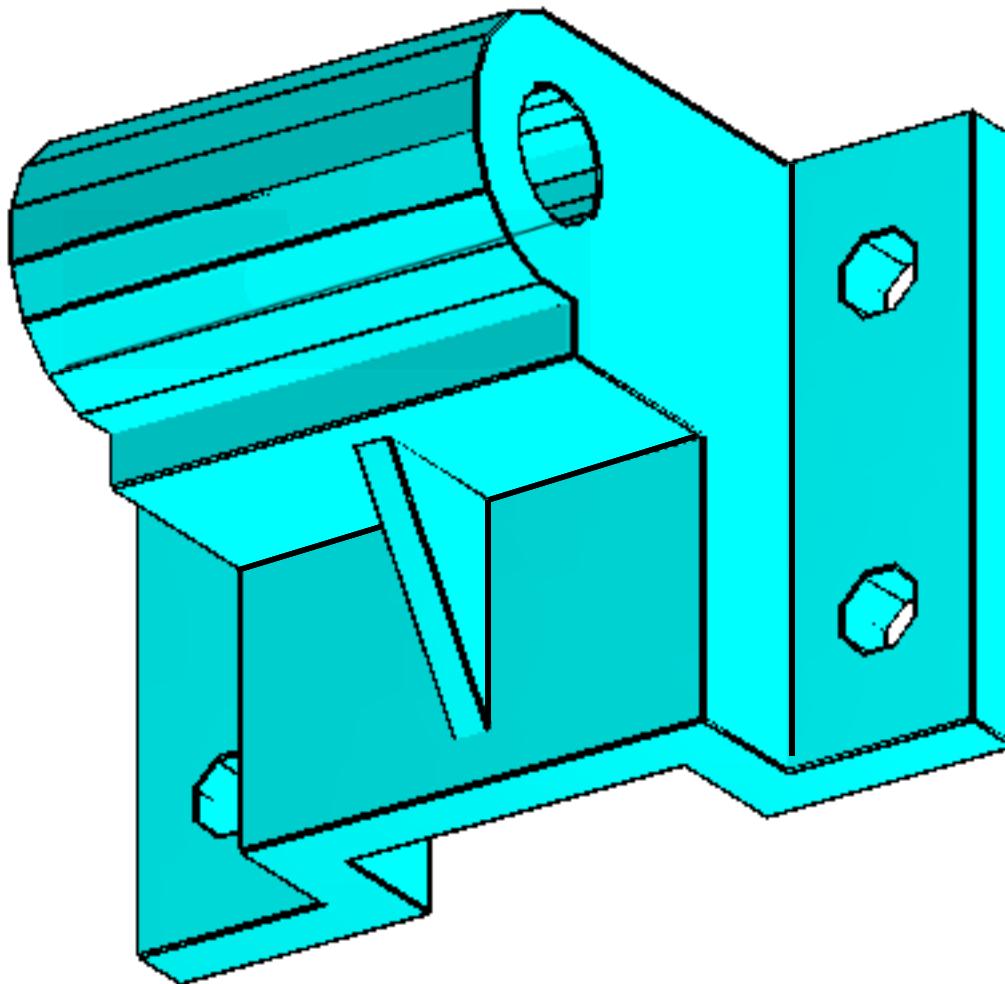


All-Quad: A track is bounded by even number of edges via symmetric property

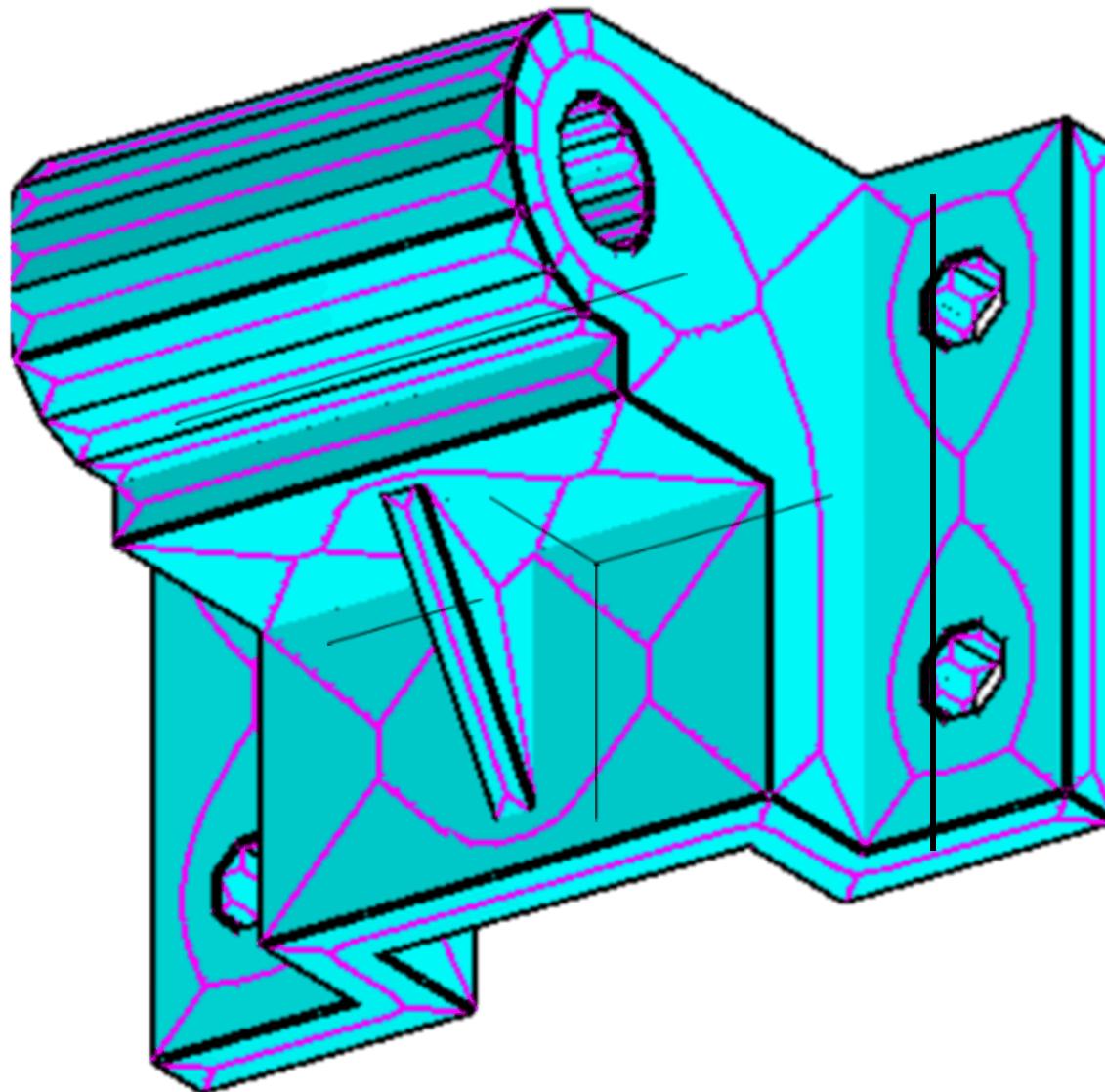
# LayTracks on Multiple Surfaces



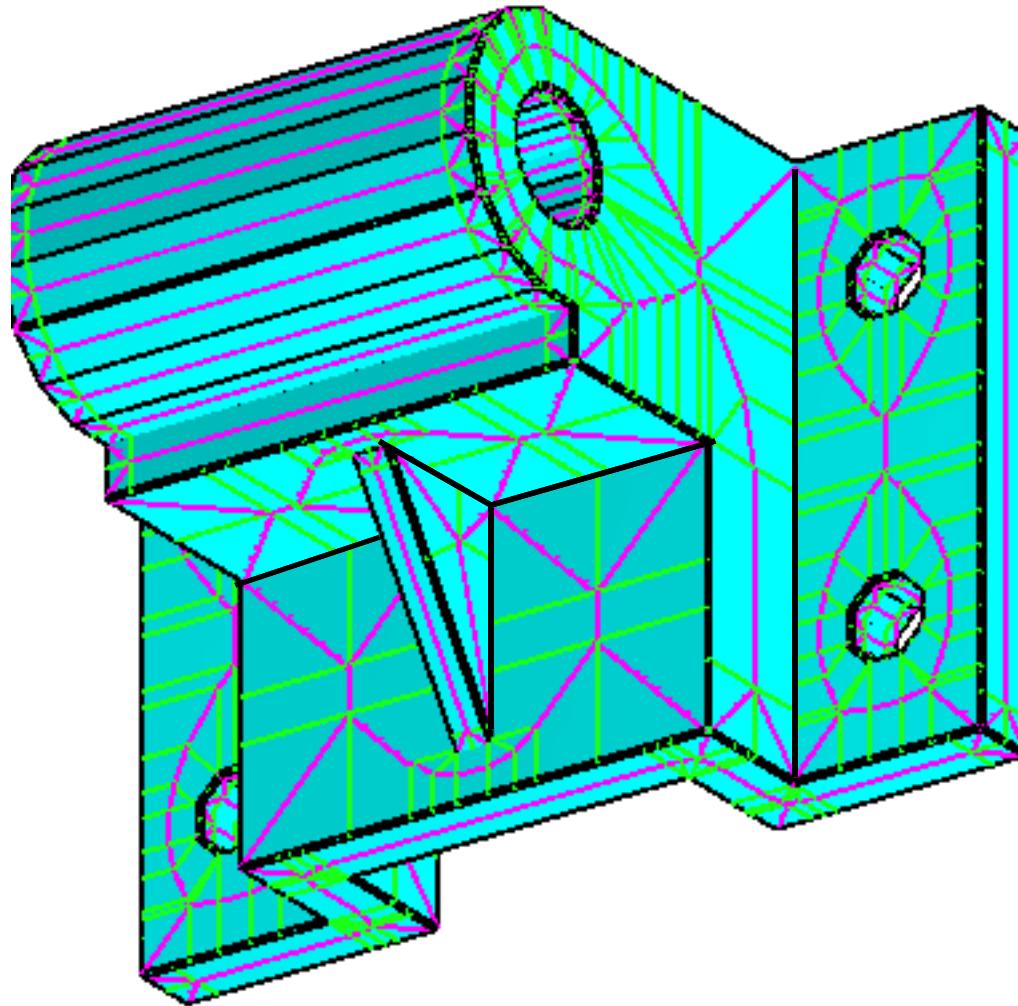
# Assembly of Surfaces



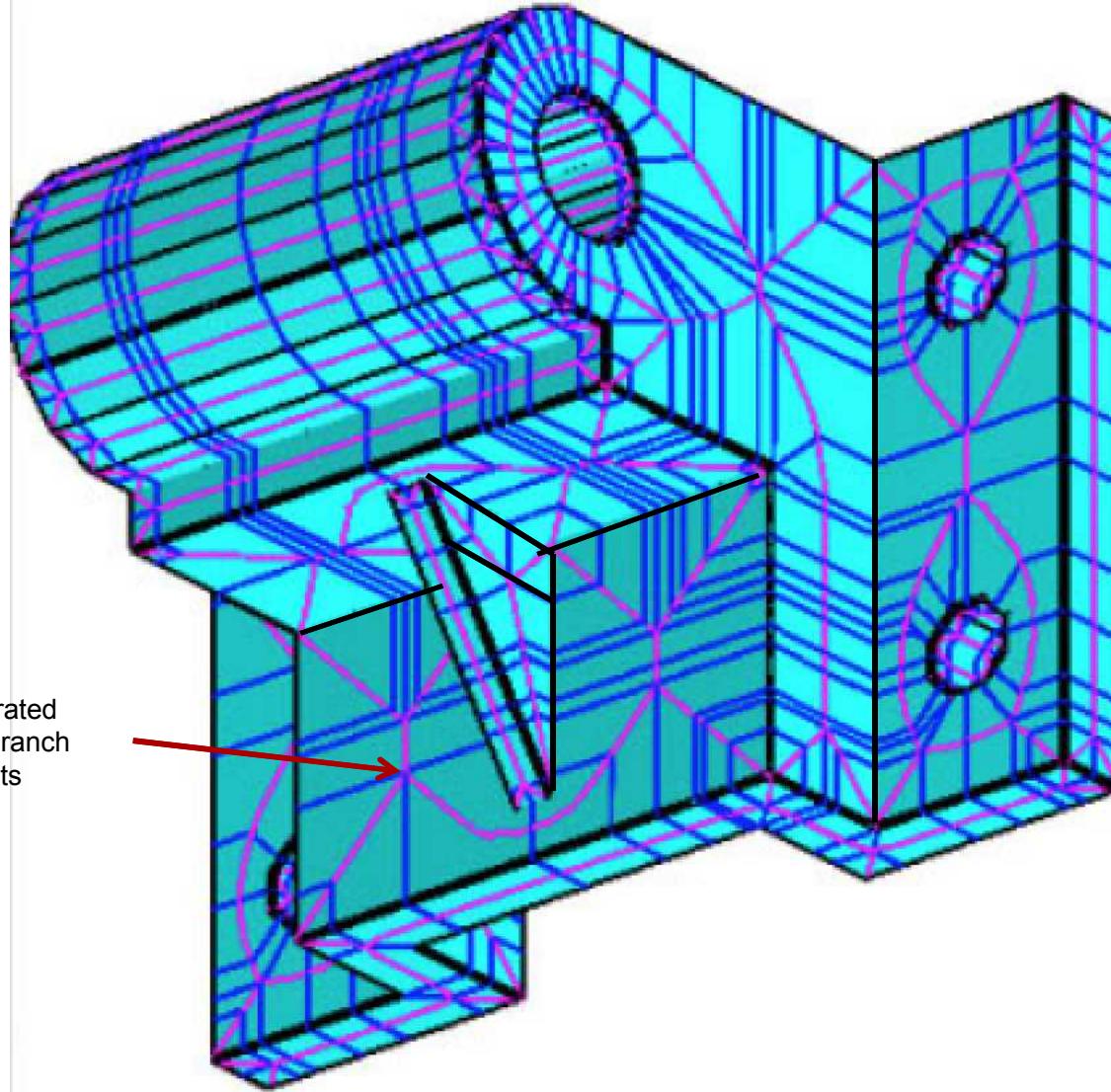
# Medial of Multiple Surfaces



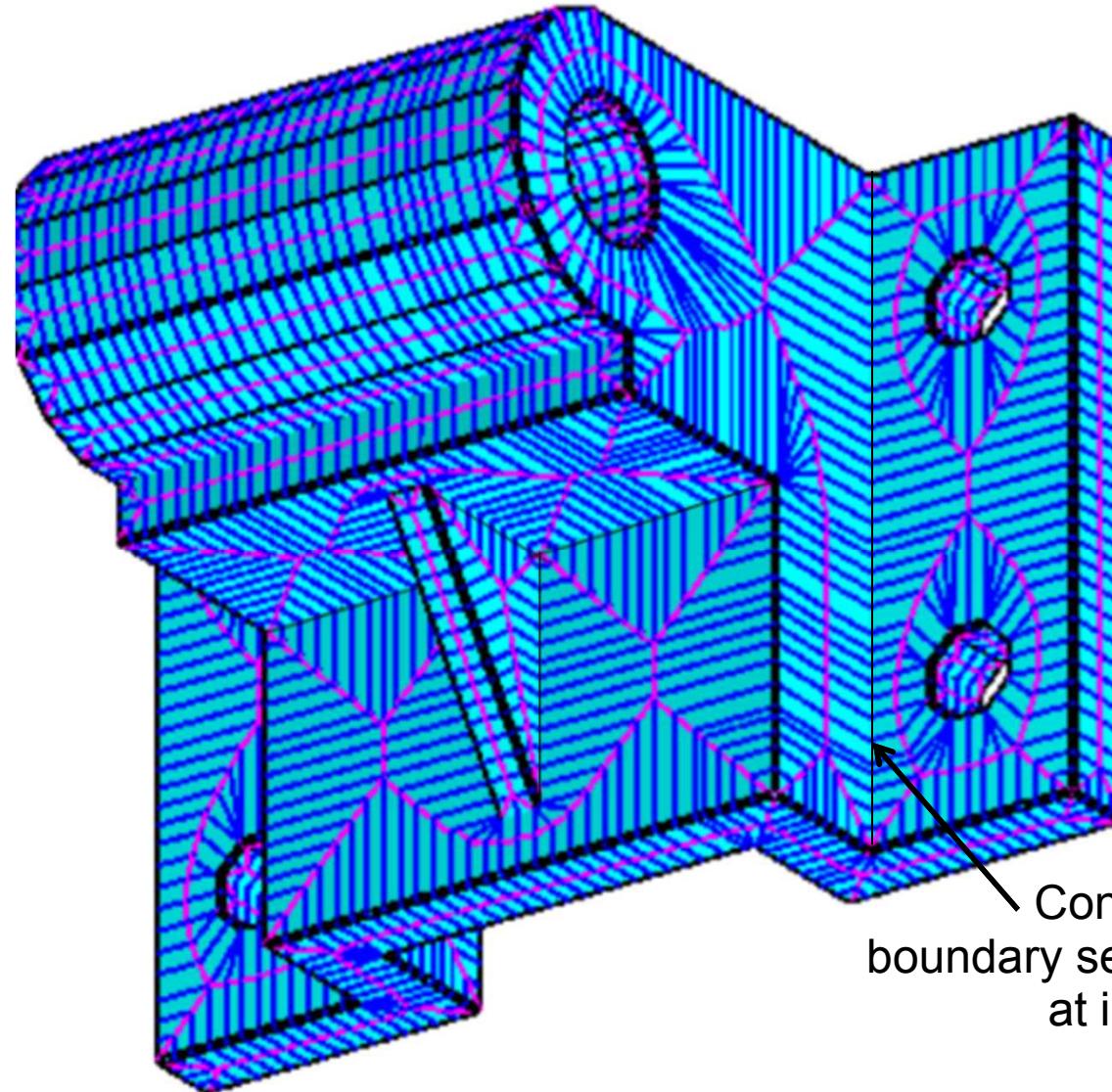
# Two-Way Map on Multiple Surfaces



# Automatic Geometry Decomposition

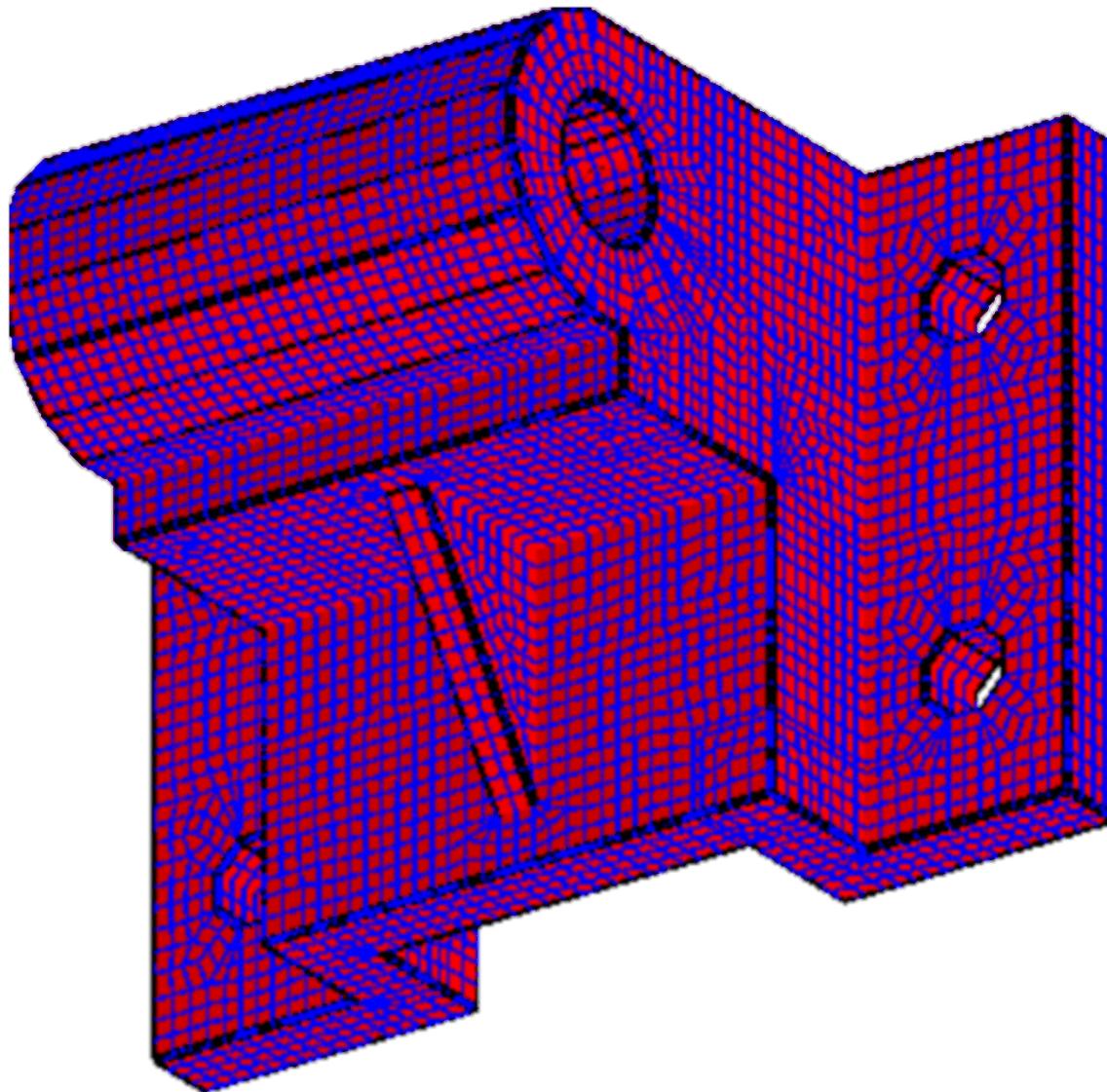


# Tracks on Multiple Surfaces

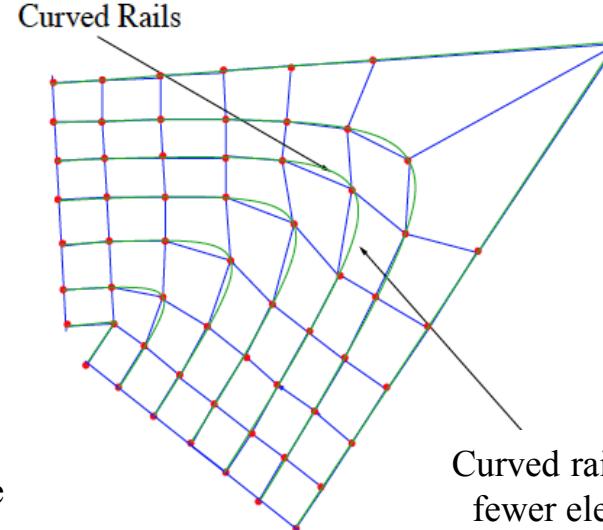
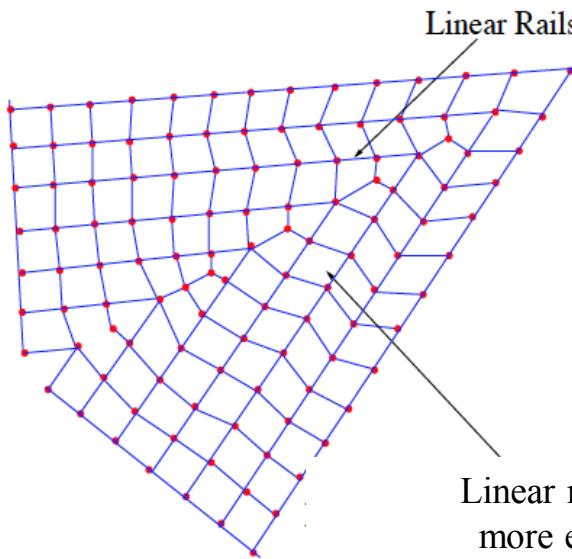
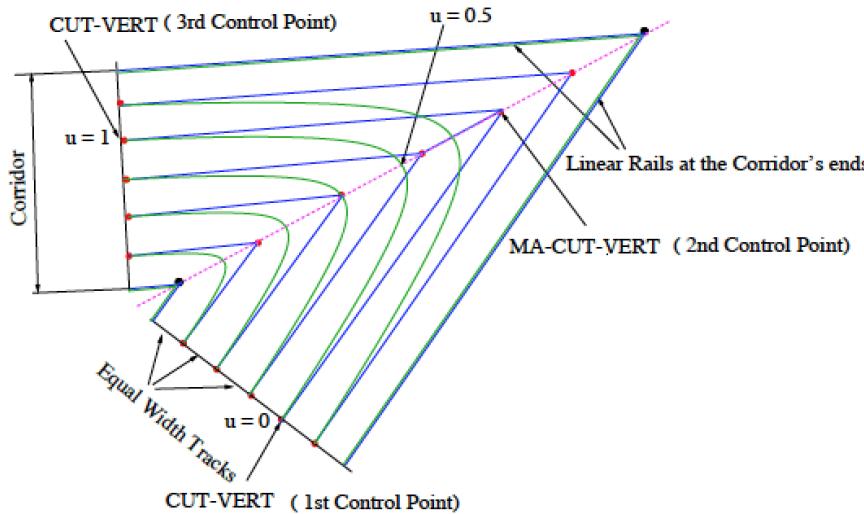


Conformal &  
boundary sensitive elements  
at interface

# Quad Mesh on Multiple Surfaces (with no post-meshing operations)

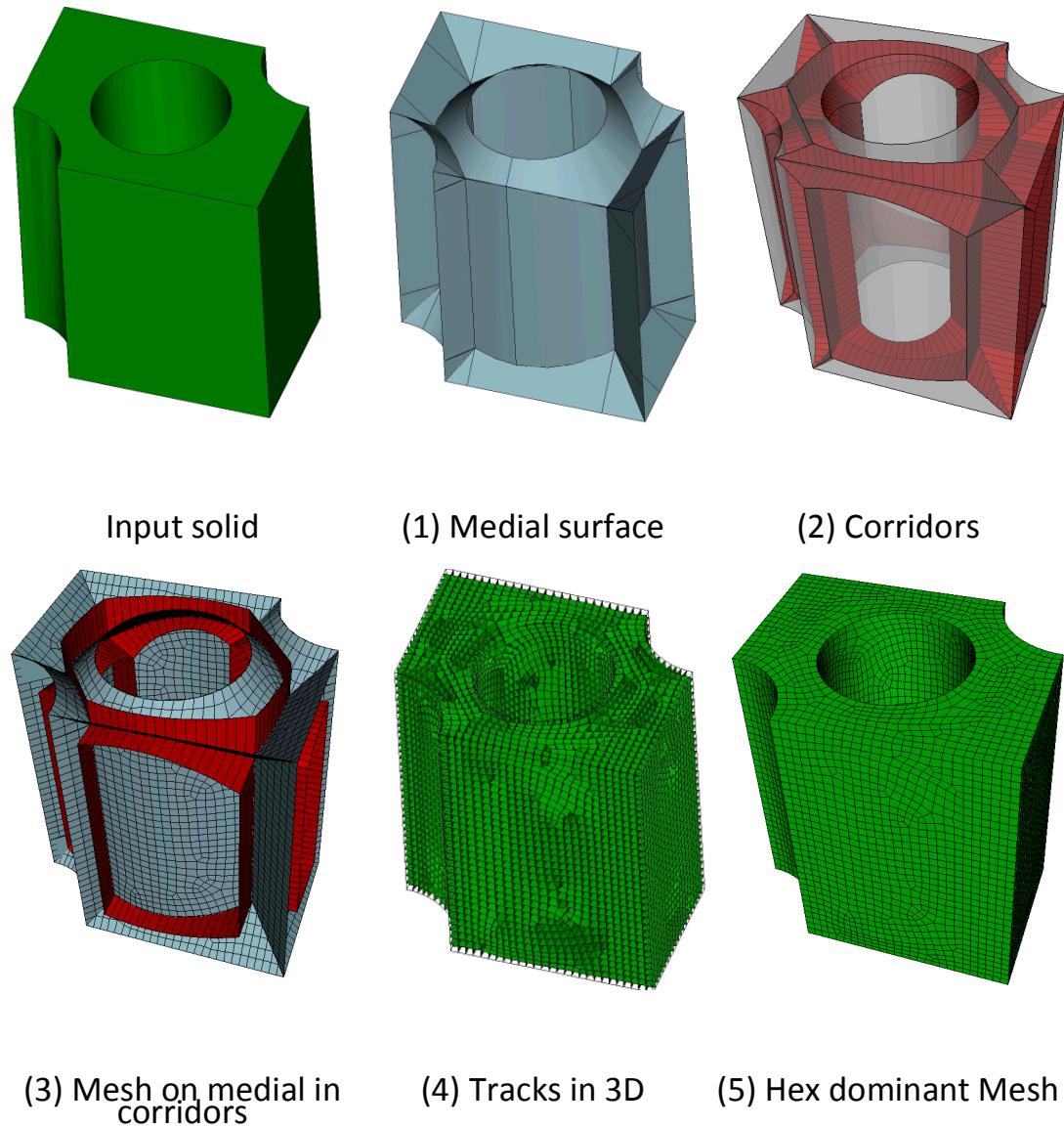
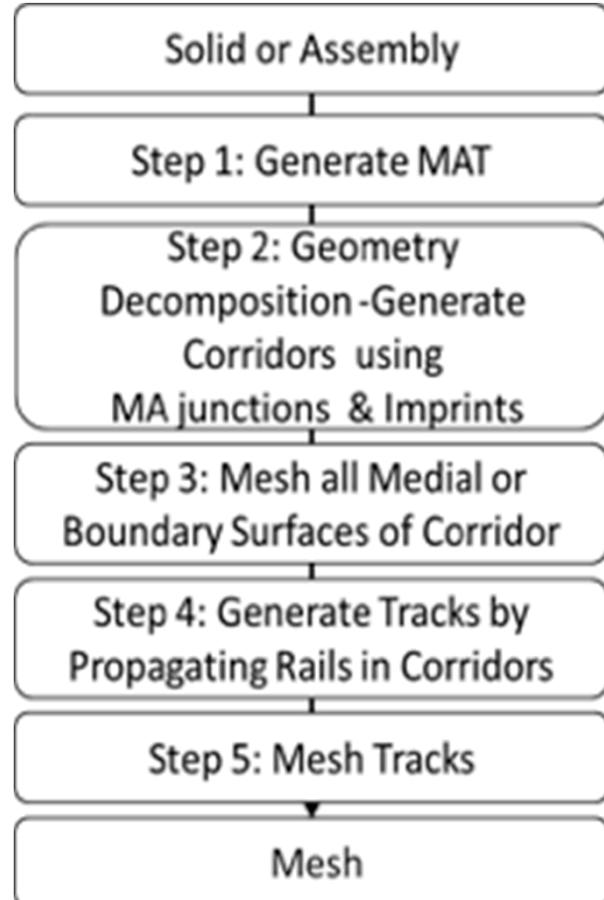


# LayTracks with Curved Rails & Tracks

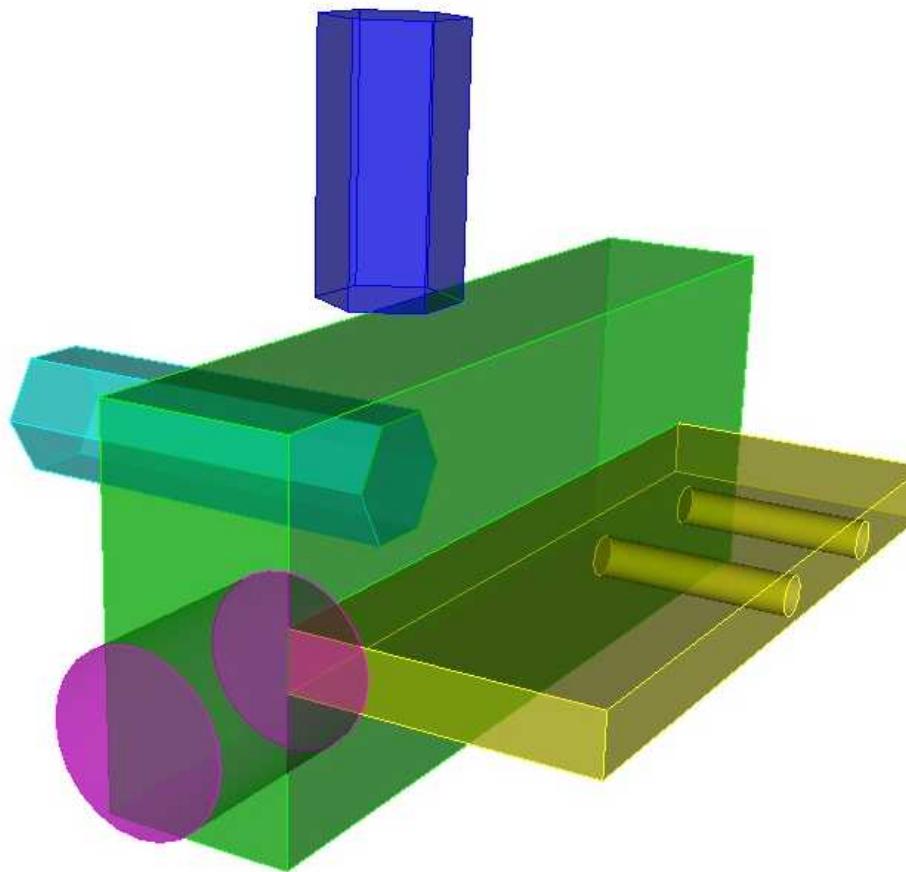


# LayTracks3D: Extension of LayTracks to 3D

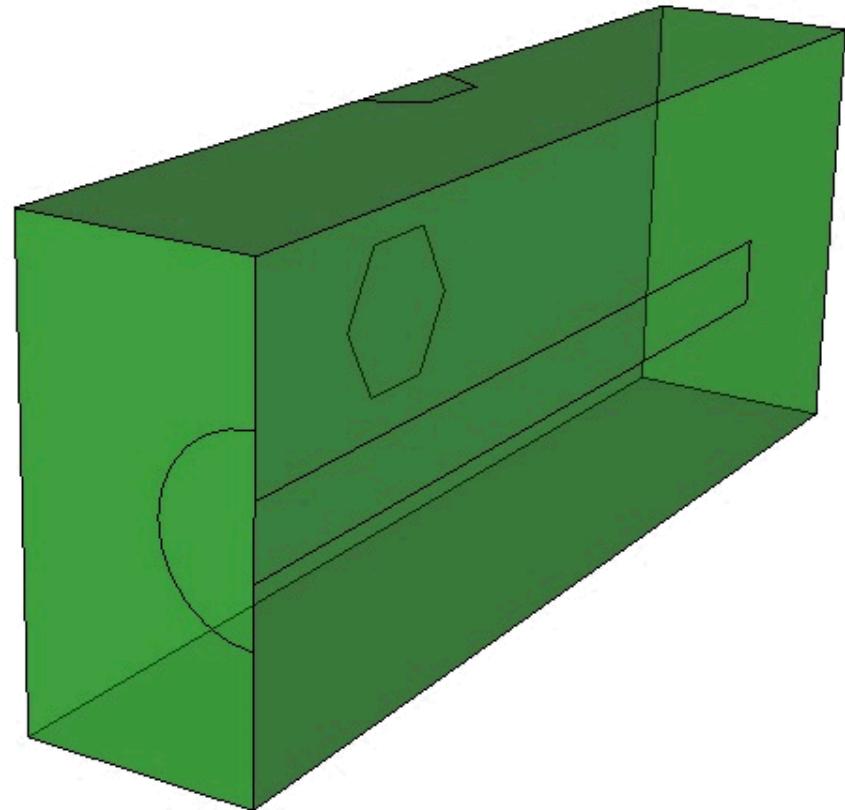
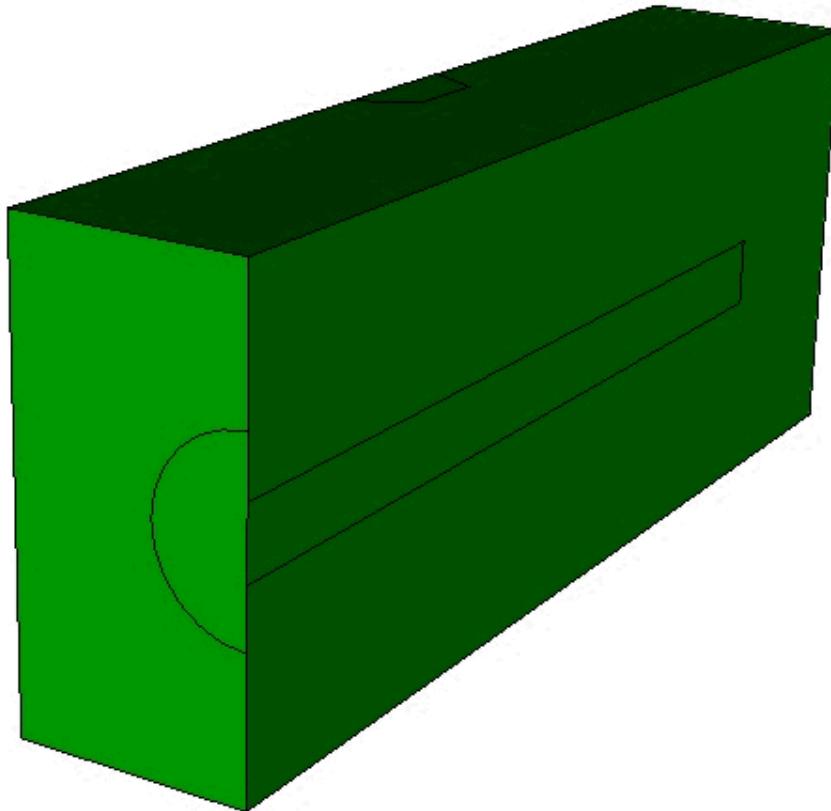
# Overview of LayTracks3D



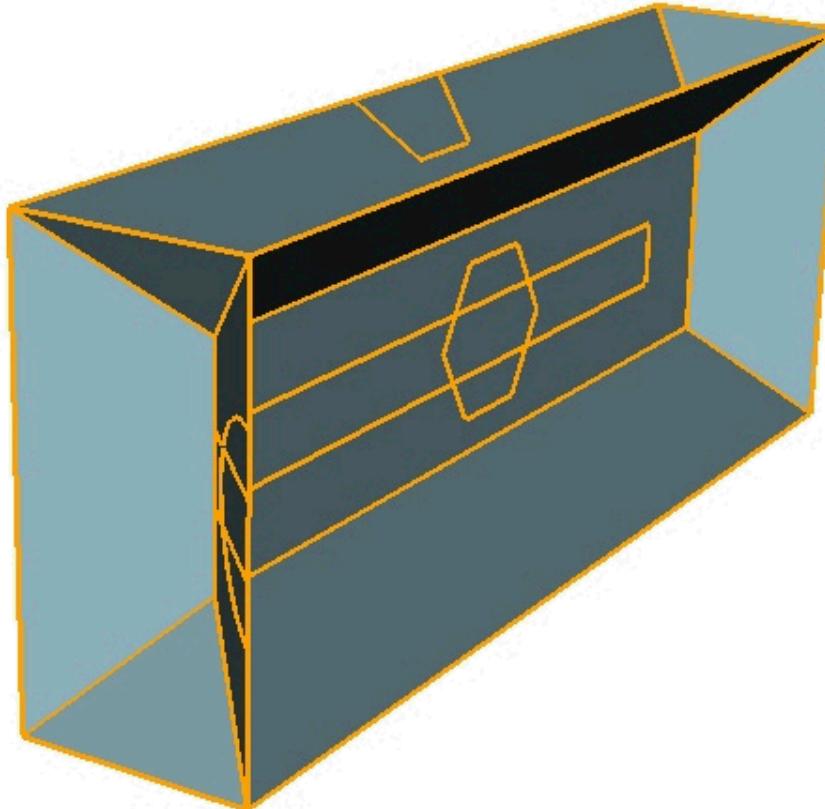
# Extension to Assembly Model



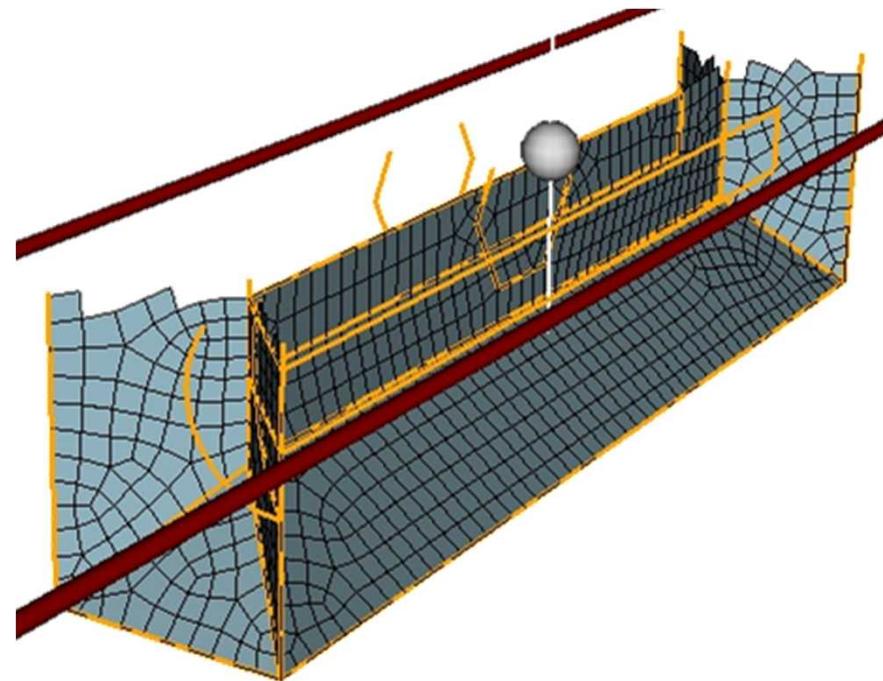
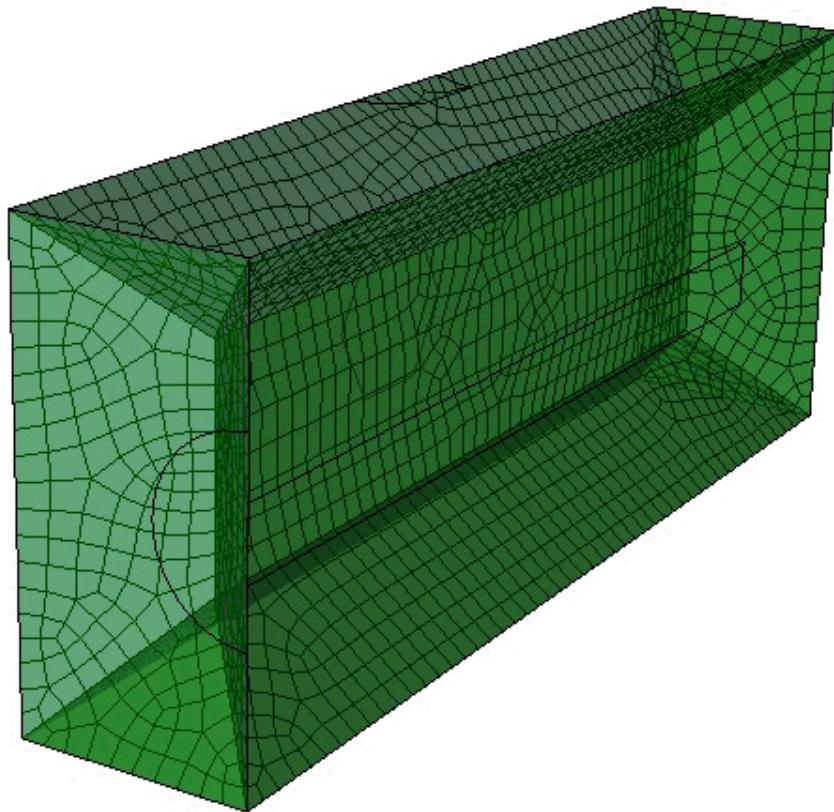
# Imprints on Top, Bottom, and Sides



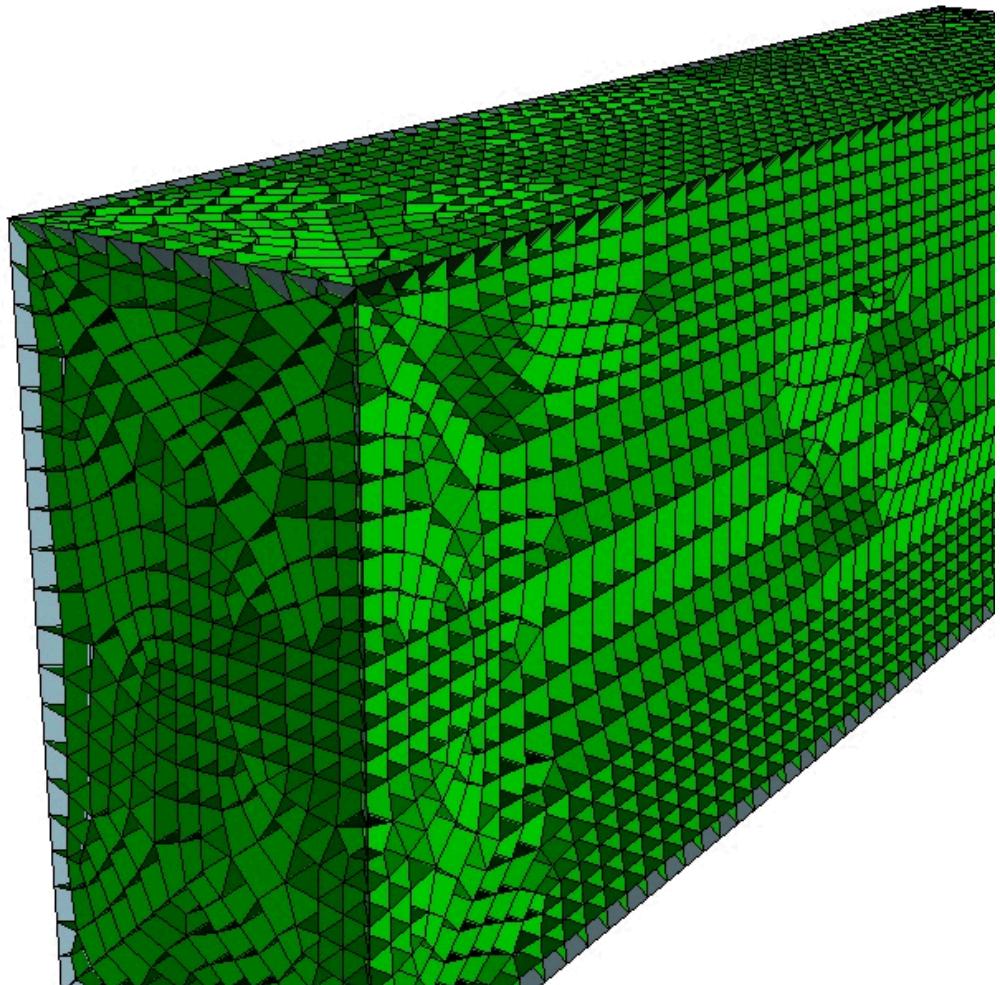
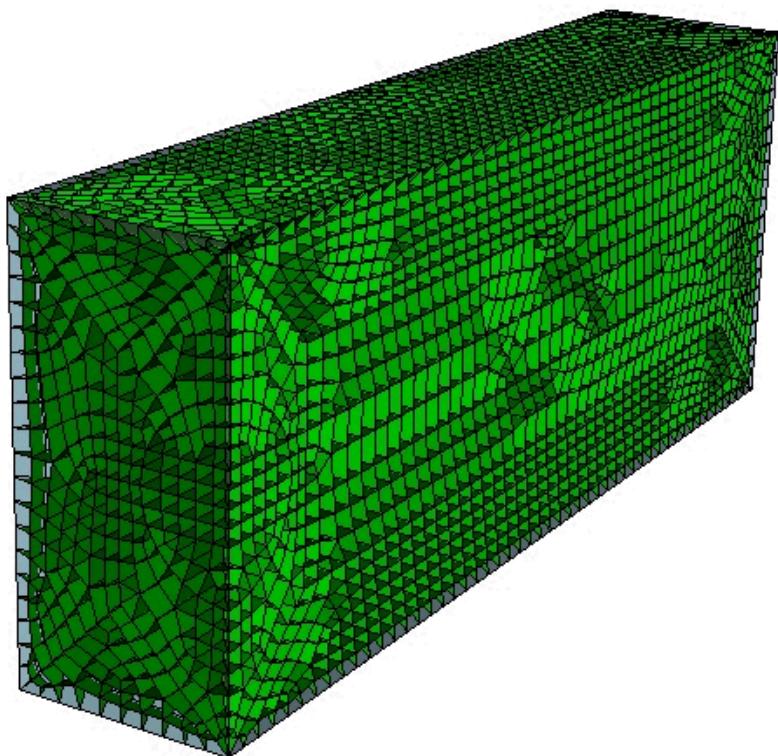
# All Imprints Resolved at MA using Projection Operator



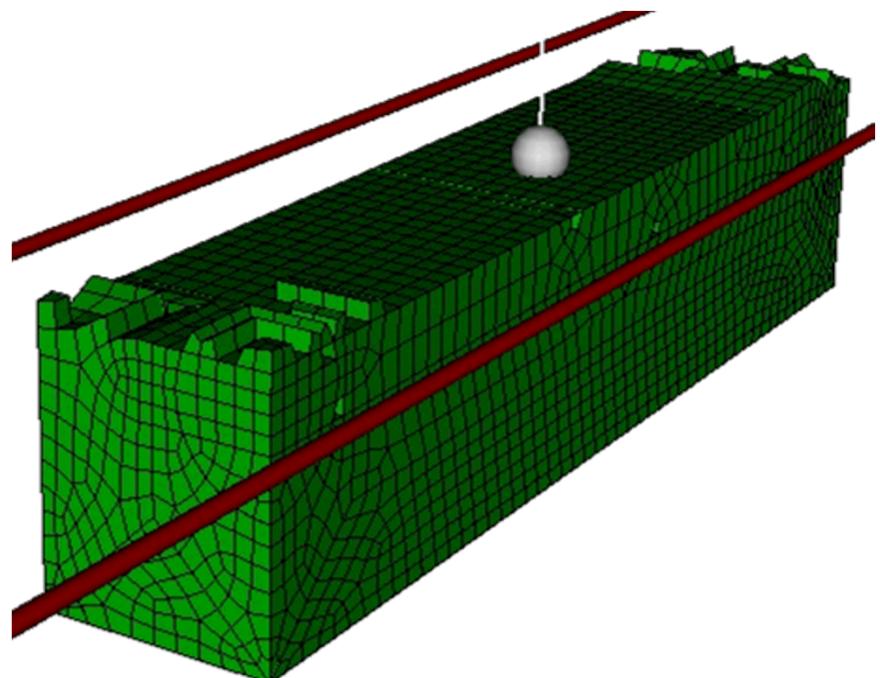
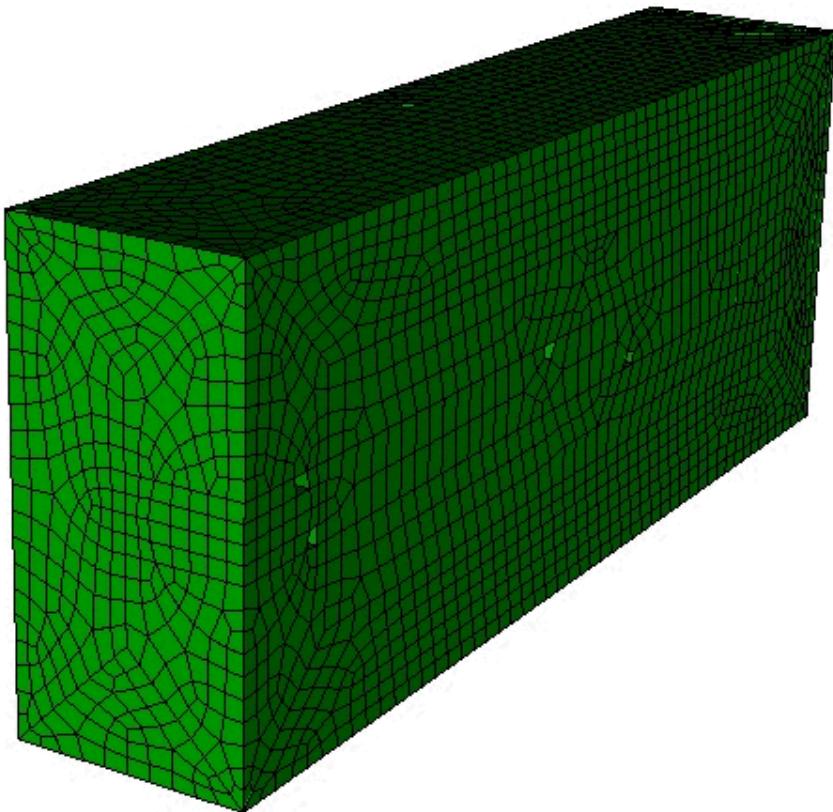
# Quad Mesh on Imprinted Medial



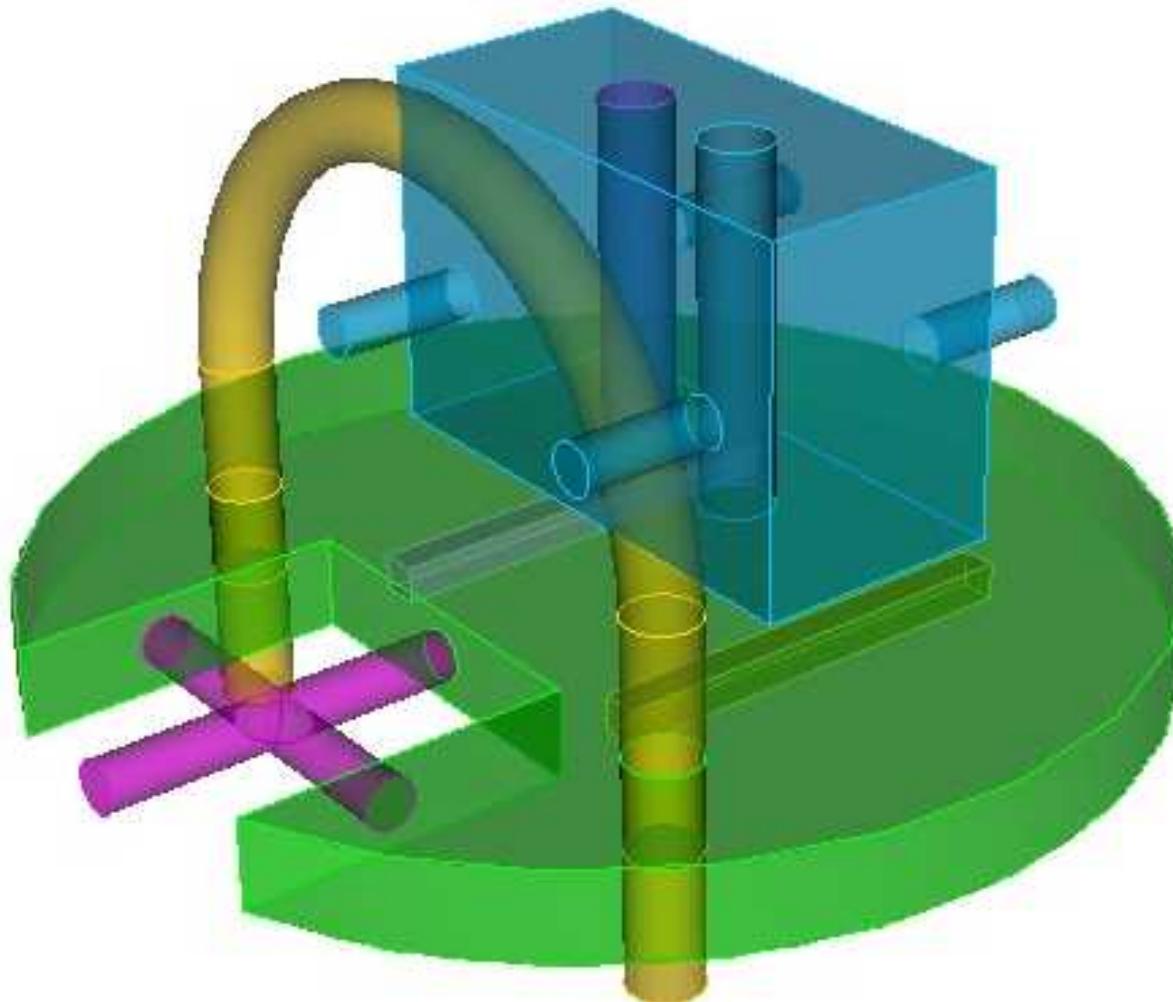
# Tracks in 3D



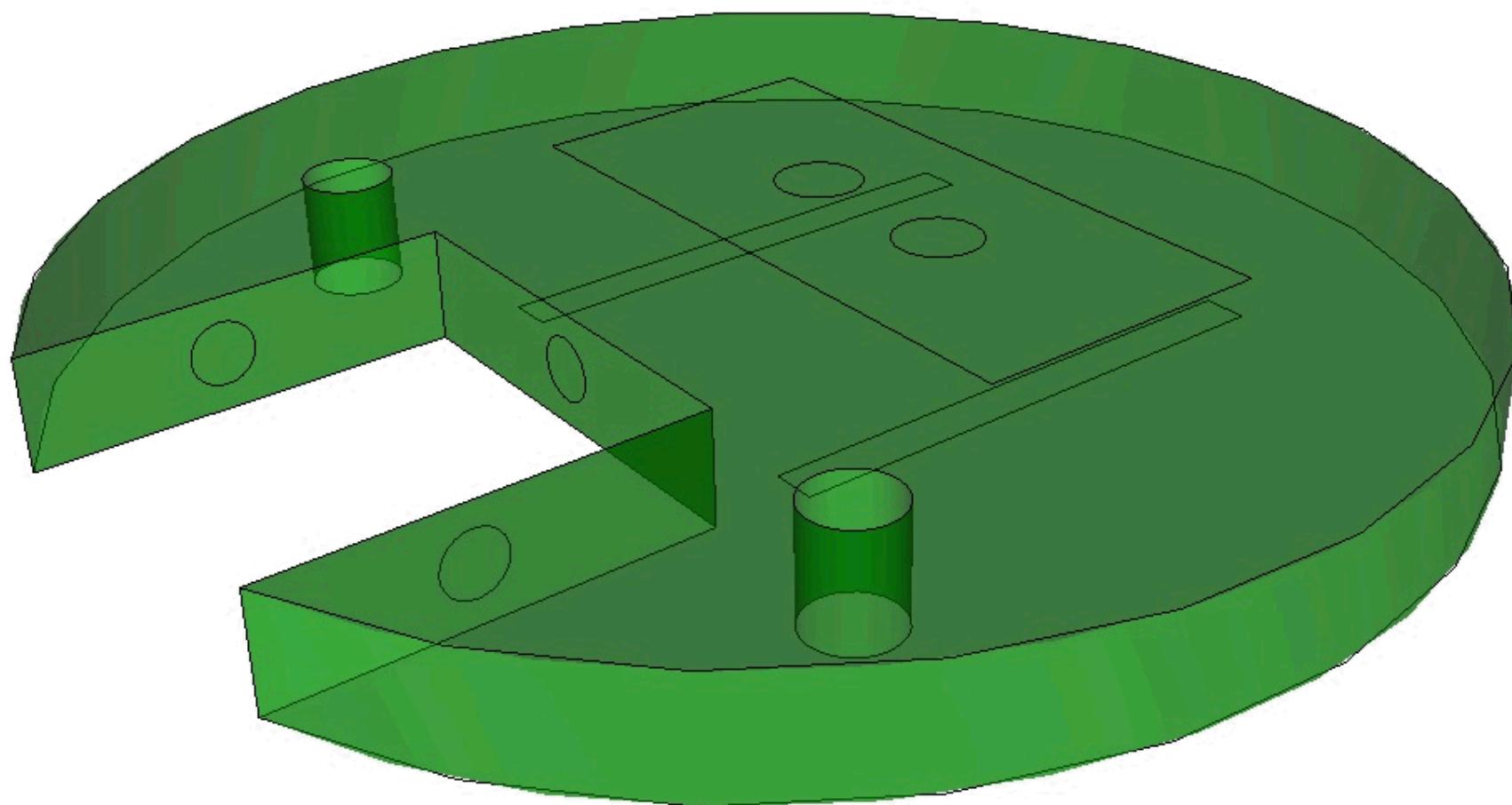
# Hex-dominant Mesh



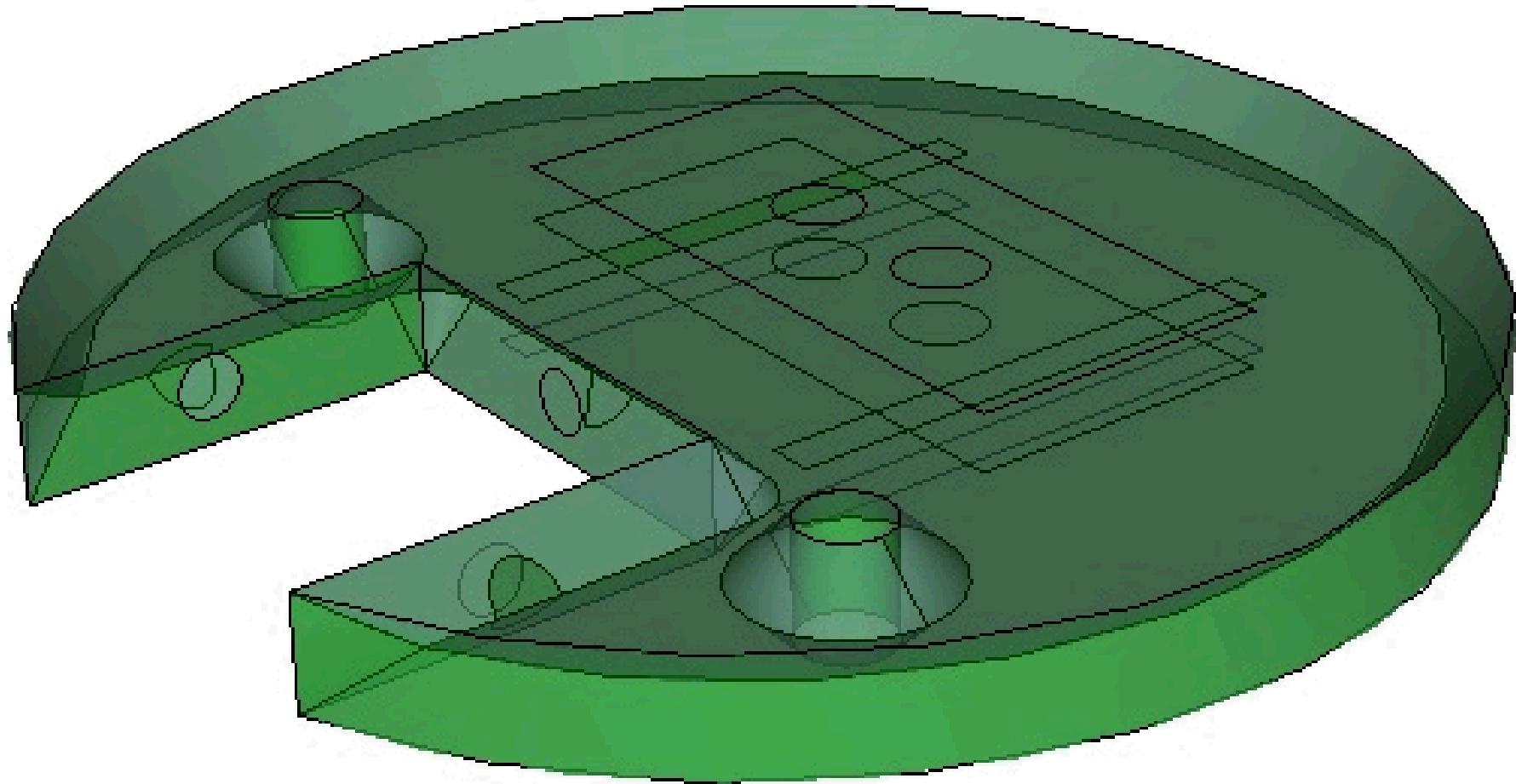
# Extension to Assembly Model



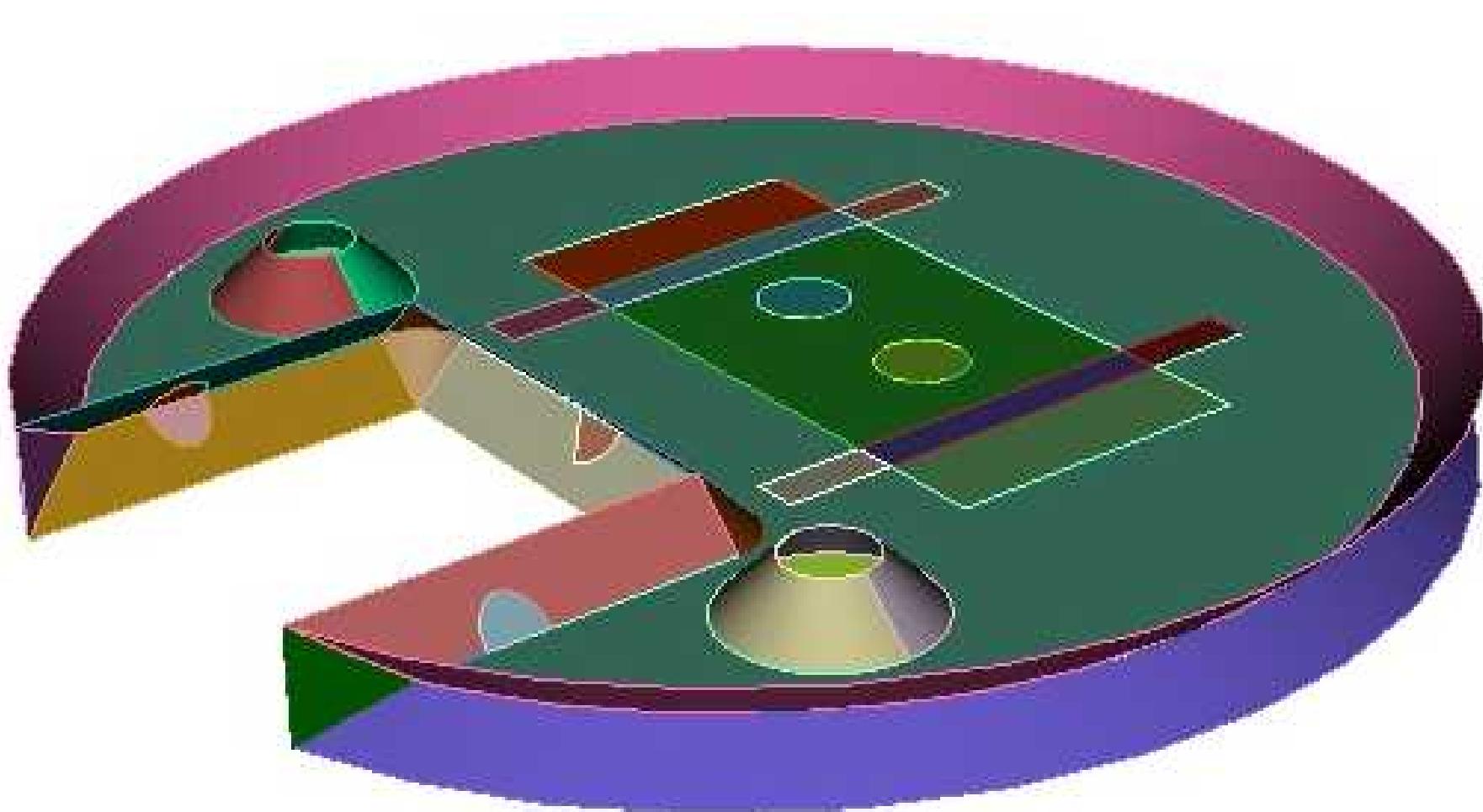
# Imprints on Top, Bottom, and Sides



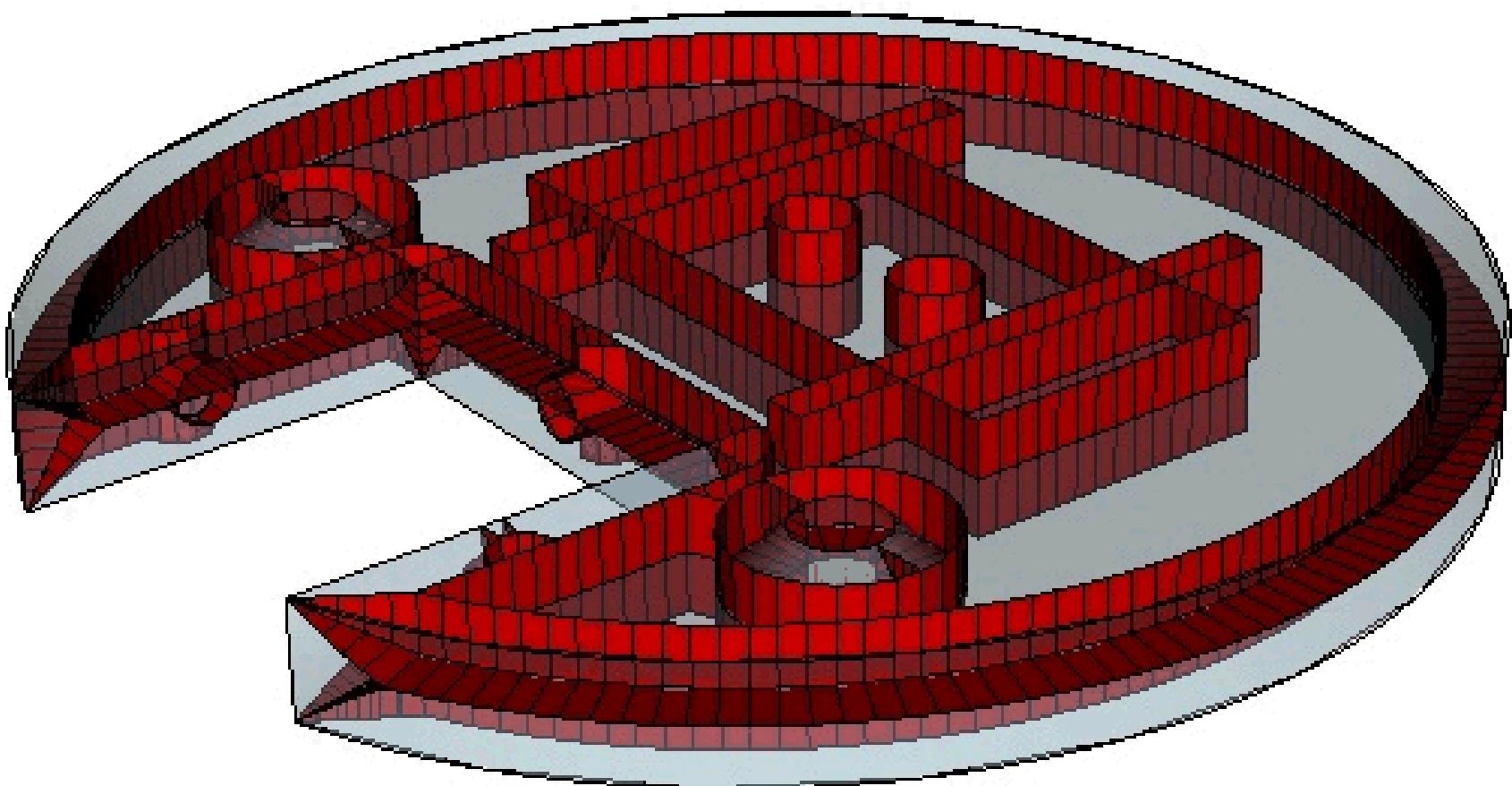
# Resolve Boundary Imprints on Medial using Projection Operator



# Medial Resolves Imprints from All Directions

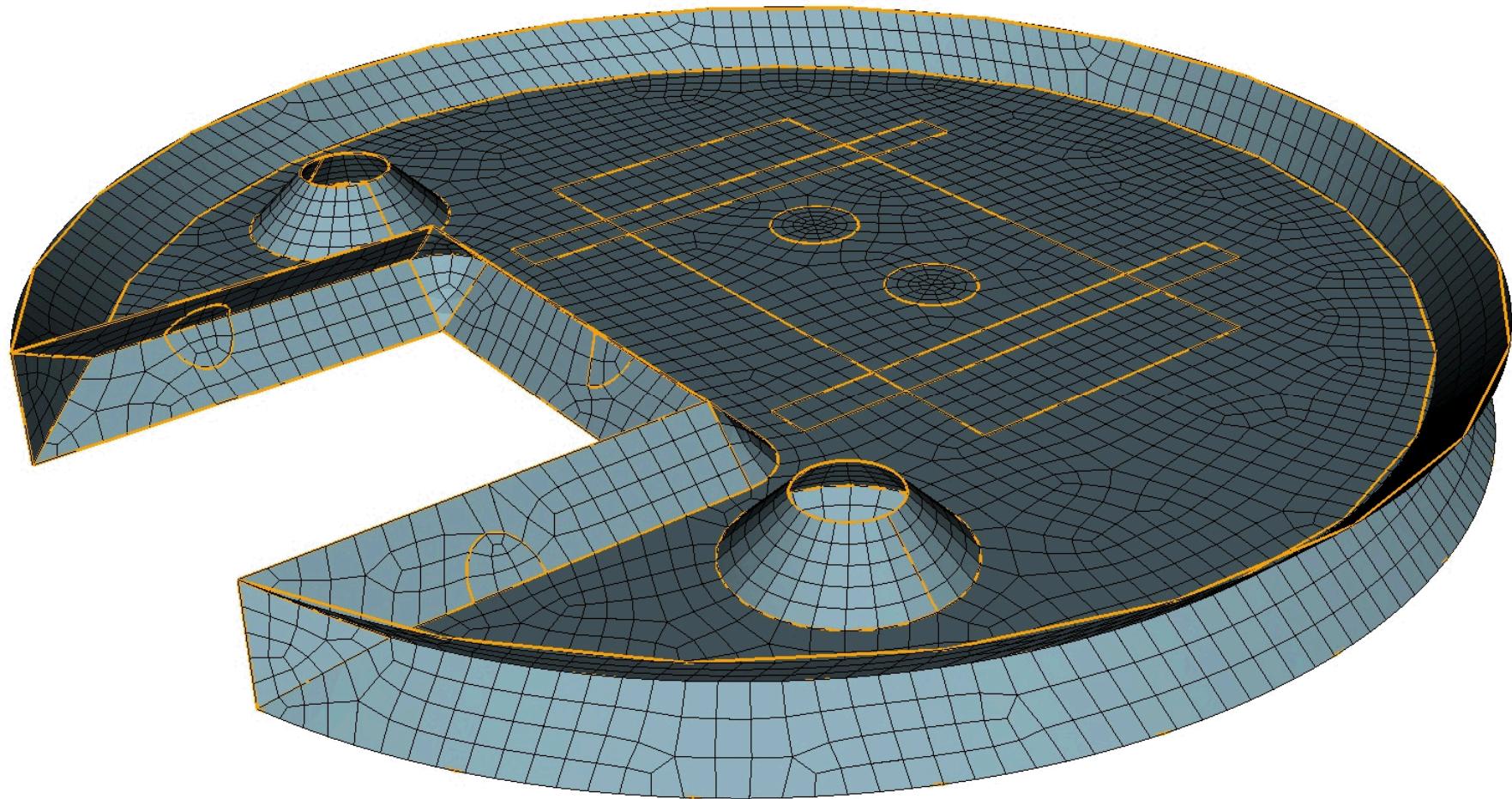


# Automatic Geometry Decomposition using Imprints and Medial Junctions

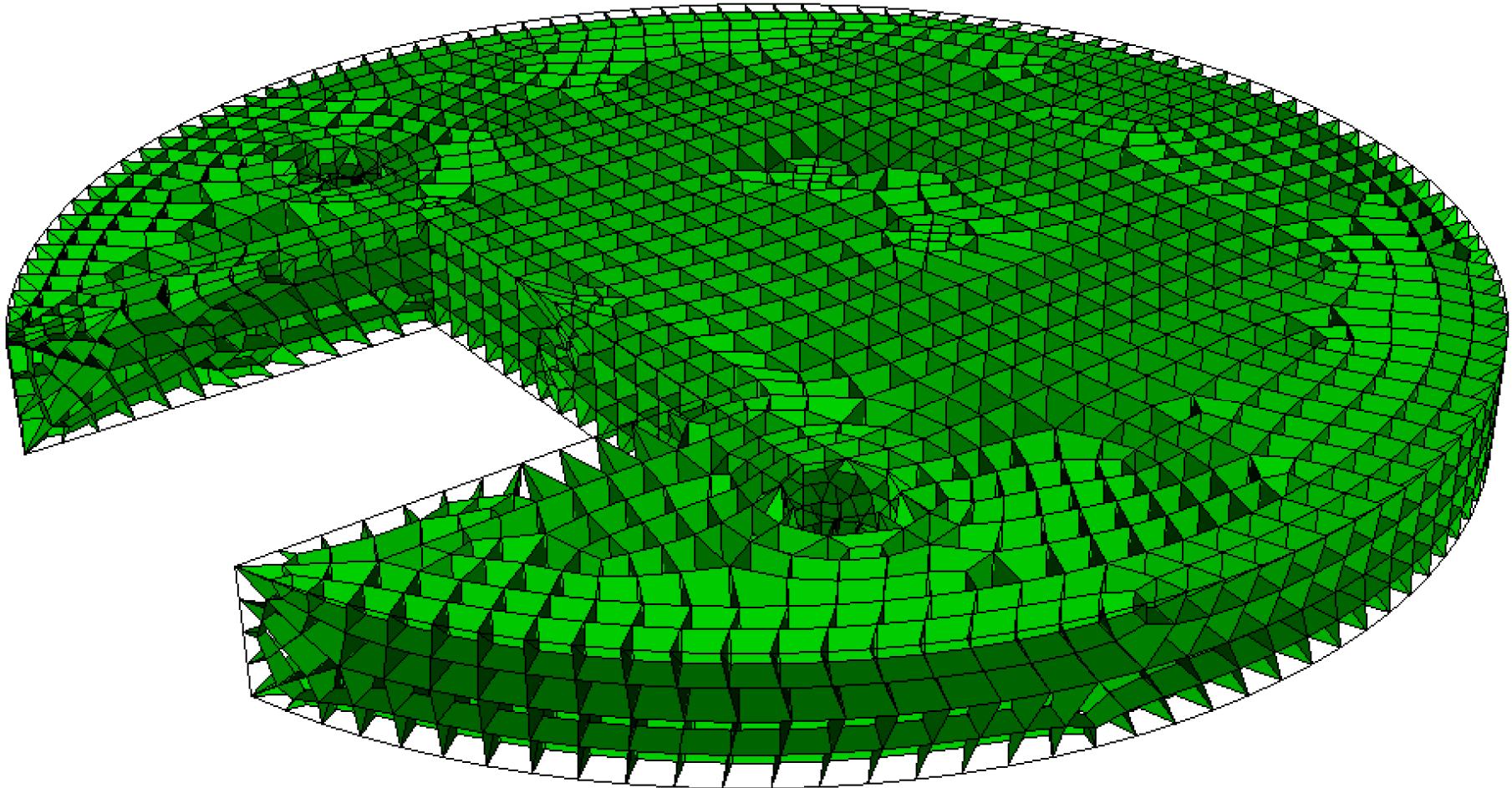


Corridors

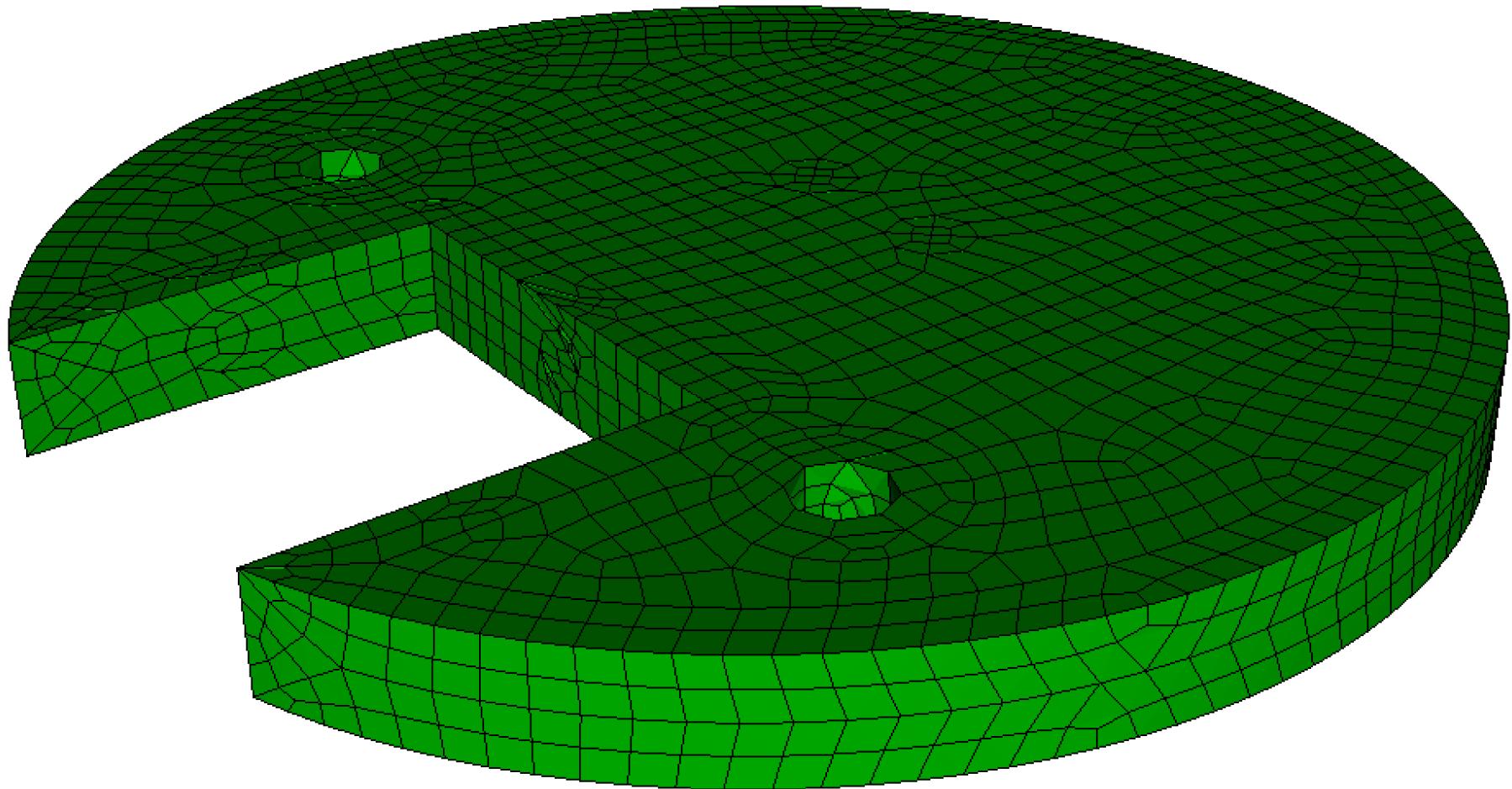
# Quad Mesh on Imprinted Medial



# Tracks in 3D

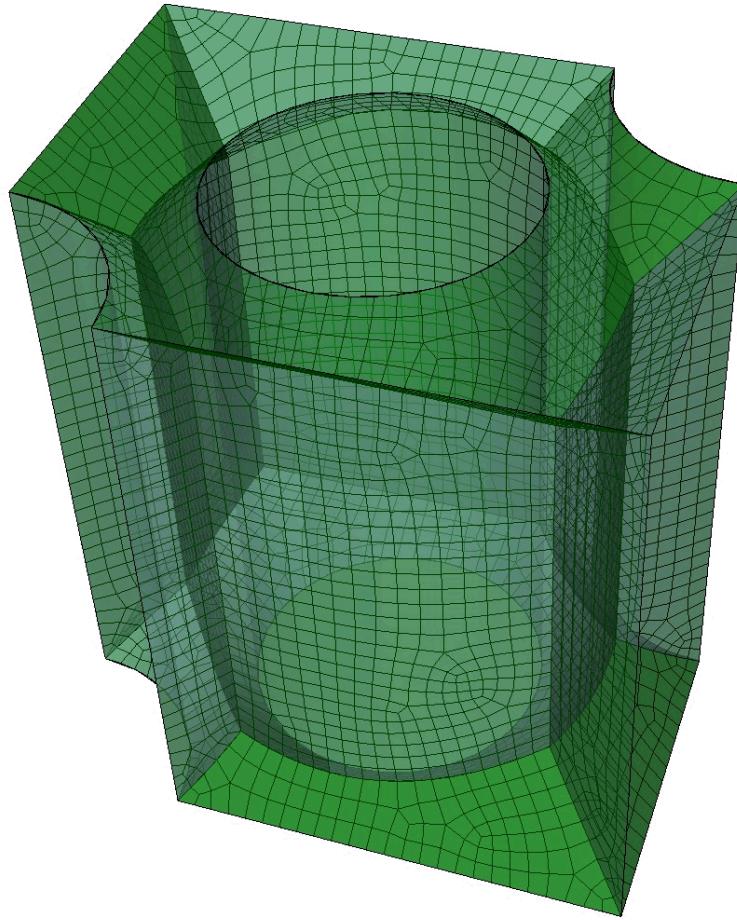


# Hex-Dominant Mesh

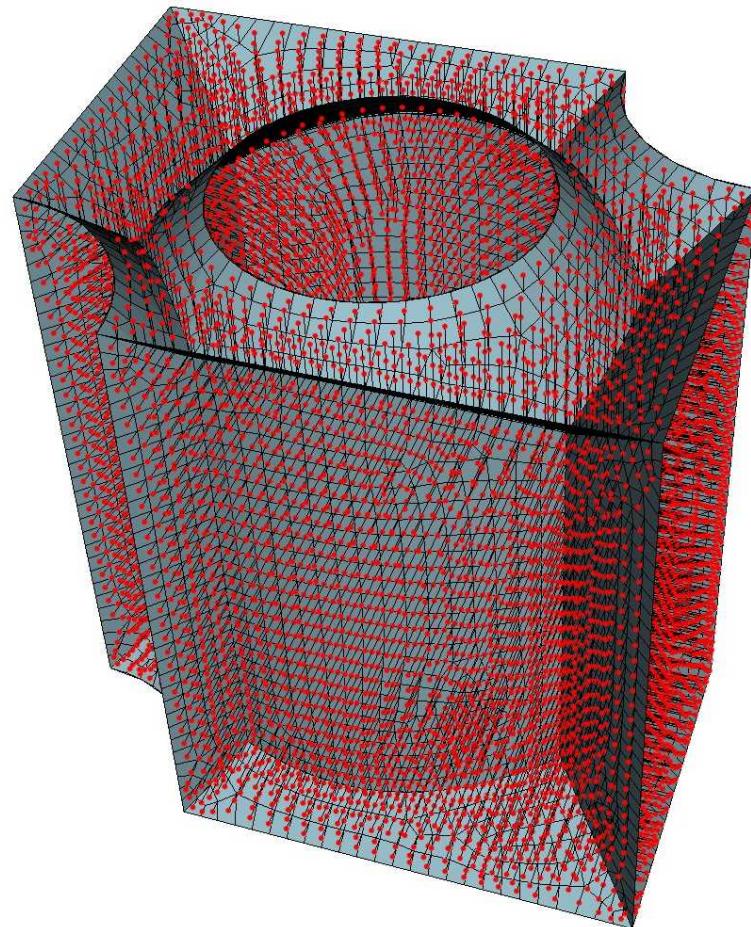


# Improve Quality at Common 3-manifold MA Curve

# All-Quad Mesh on Medial is not Ideal!

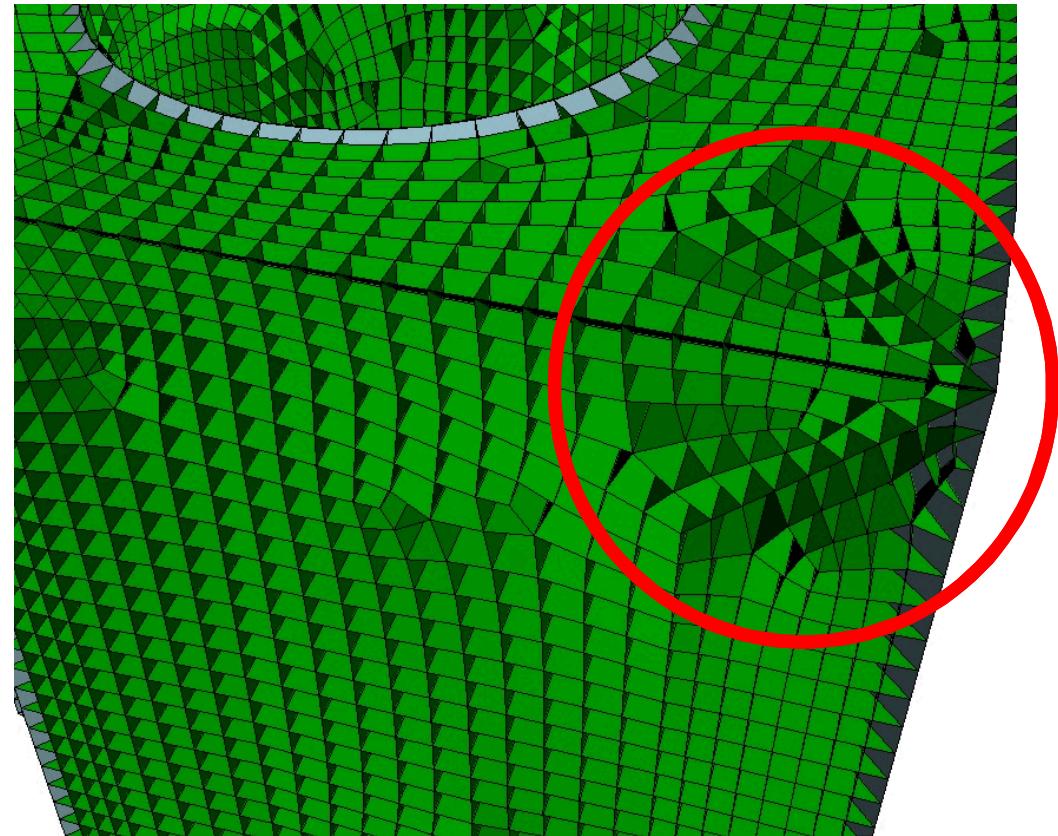
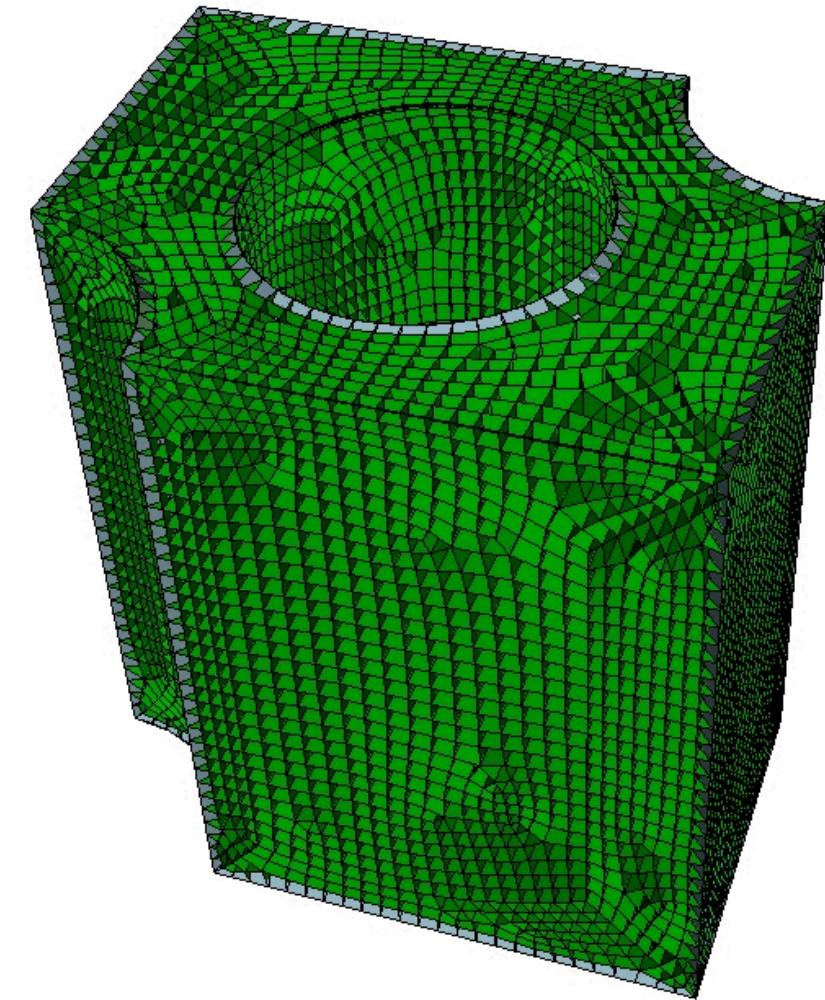


All-Quad Mesh on  
Medial Surface

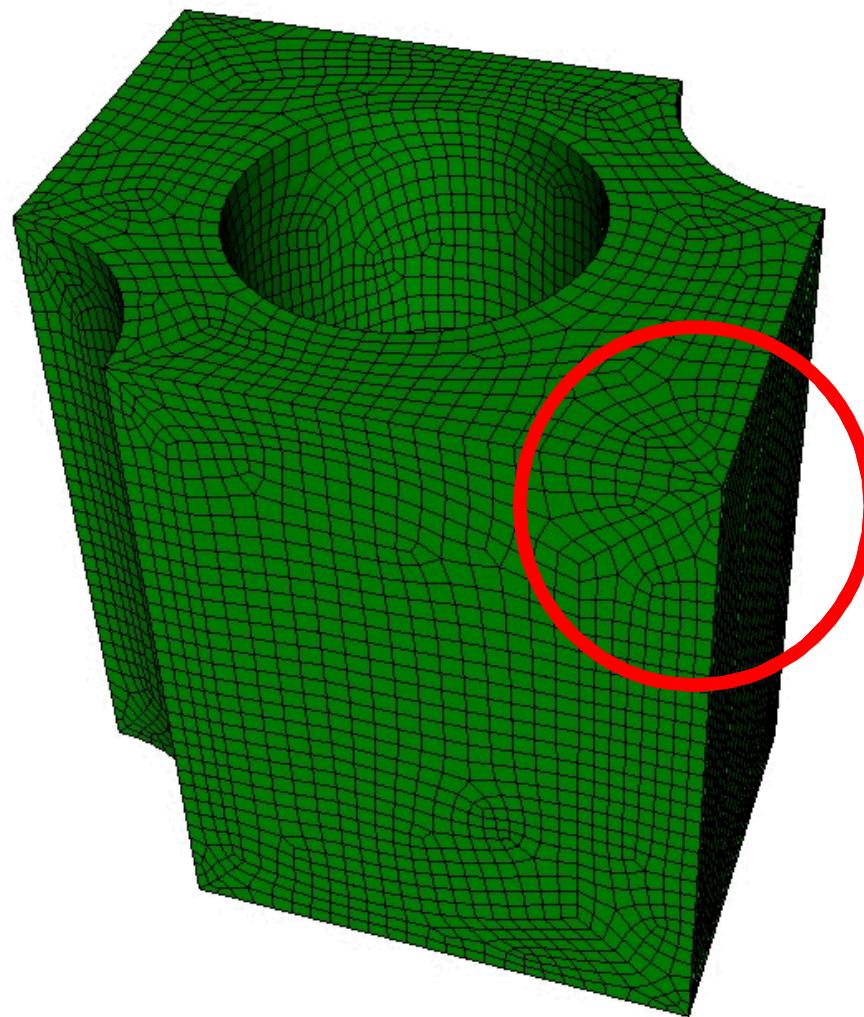


Rails Generated using  
Projection Operator

# Complex Tracks at a Convex Vertex

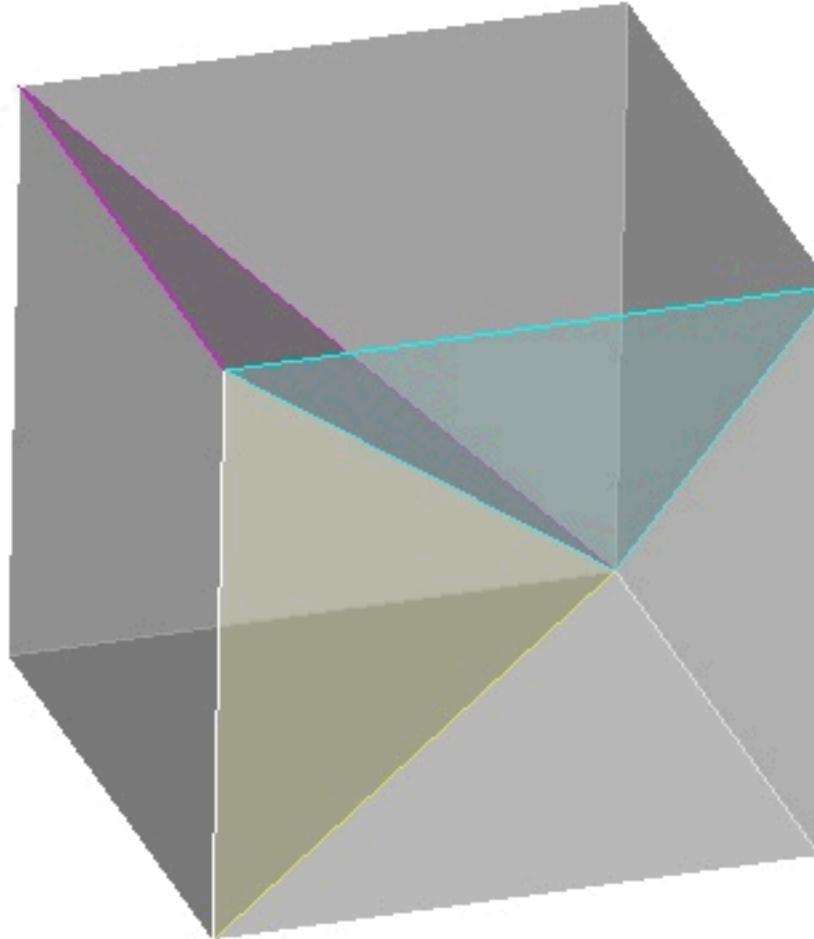


# Hex-dominant Mesh at Convex Vertex

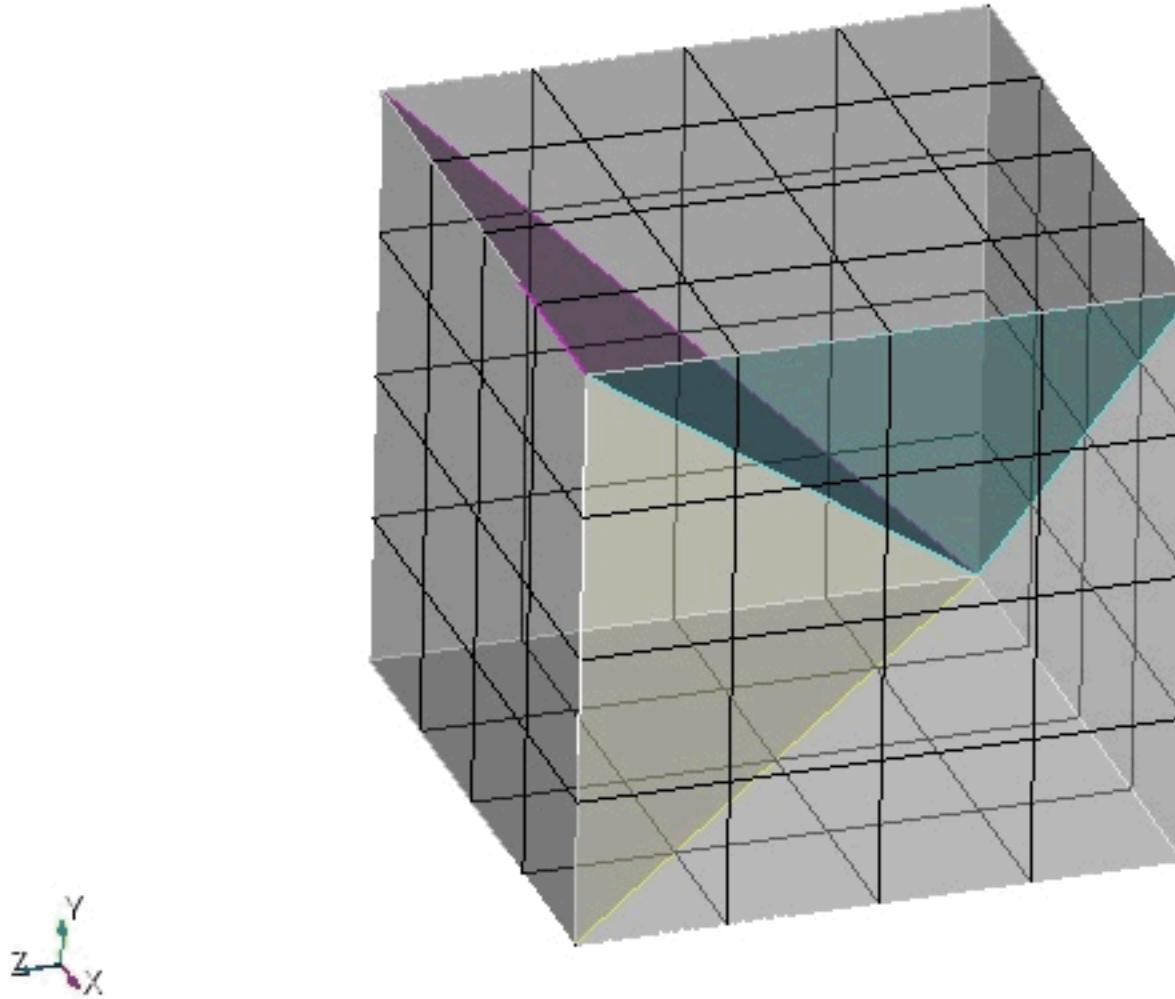


# How to Improve Hex Quality at Convex Vertex?

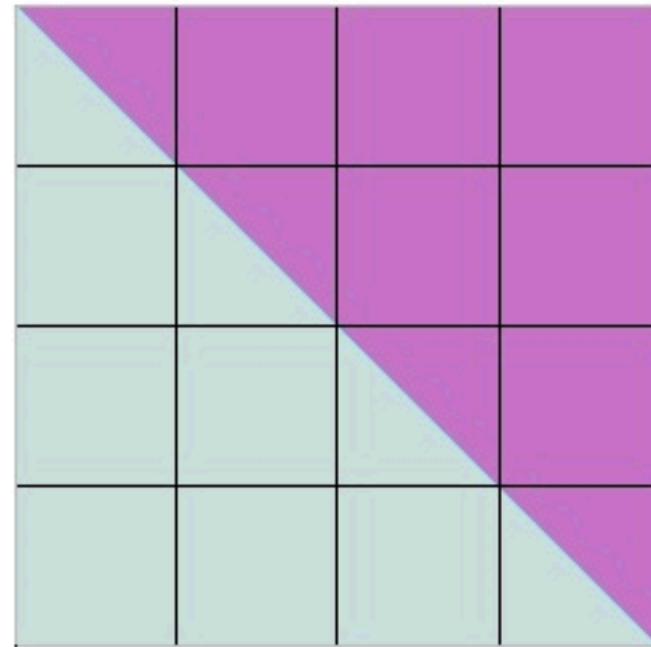
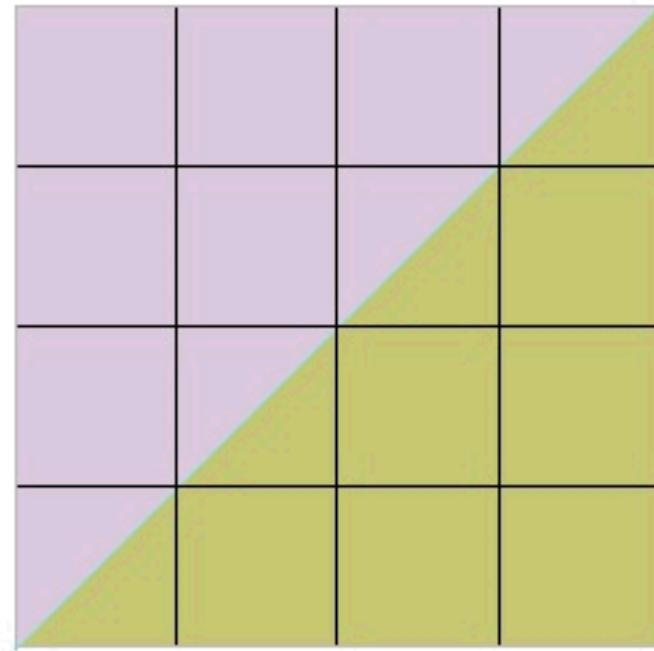
# Typical 3-manifold Medial at a Convex Vertex



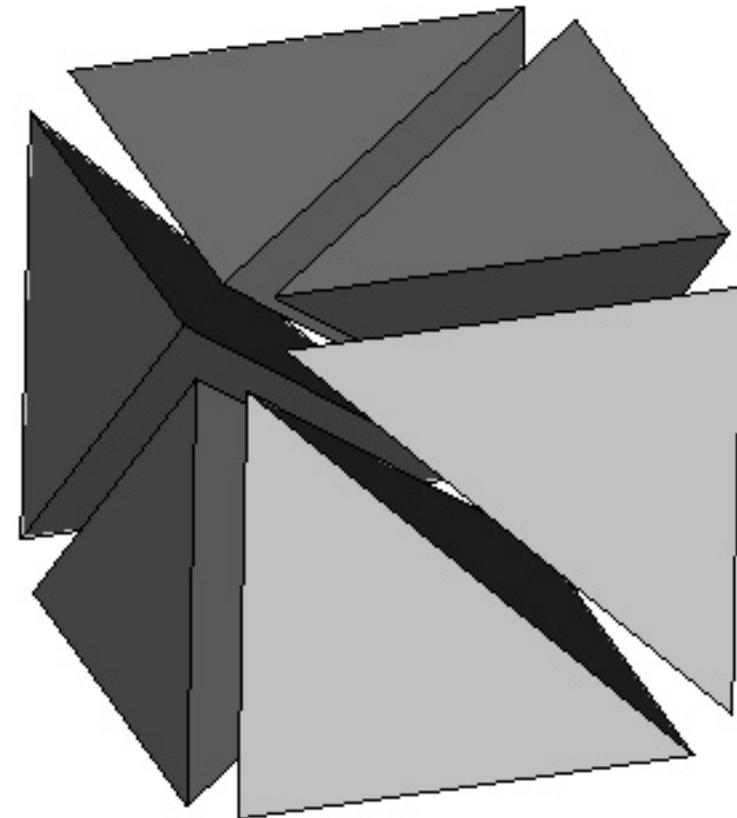
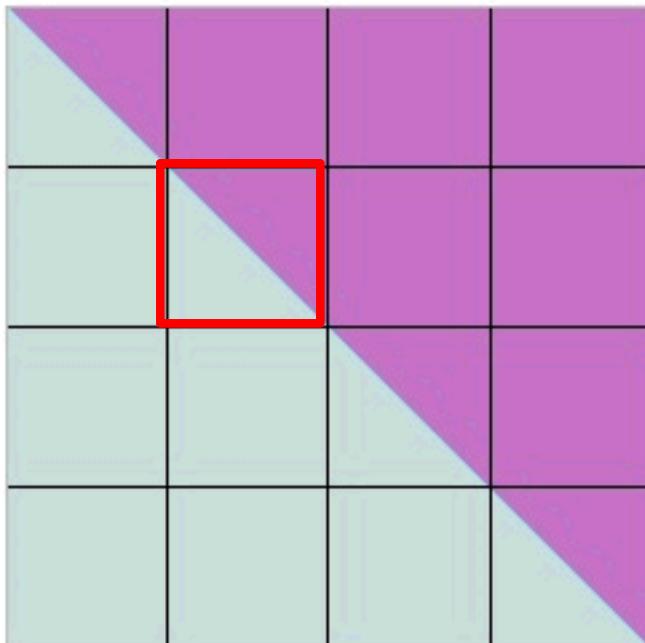
# Ideal Hex Mesh at a Convex Vertex



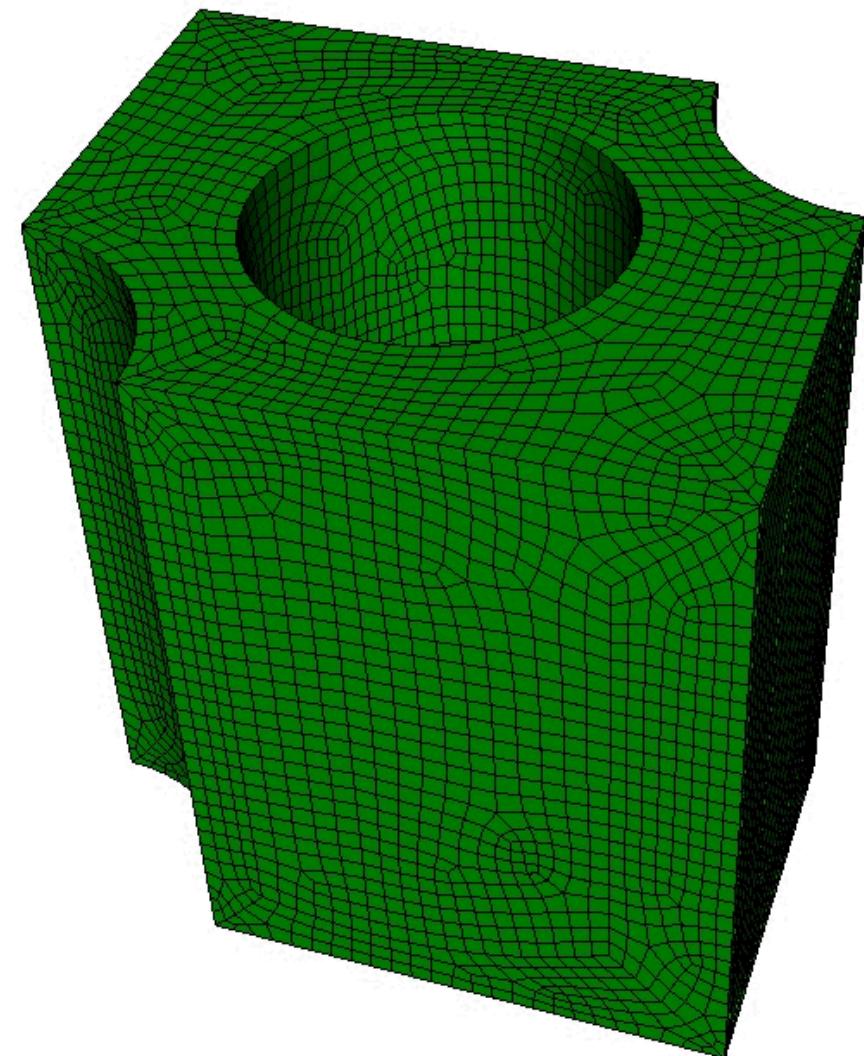
# Ideal Mesh Should Contain Tri Along 3-manifold Medial Curve!!!



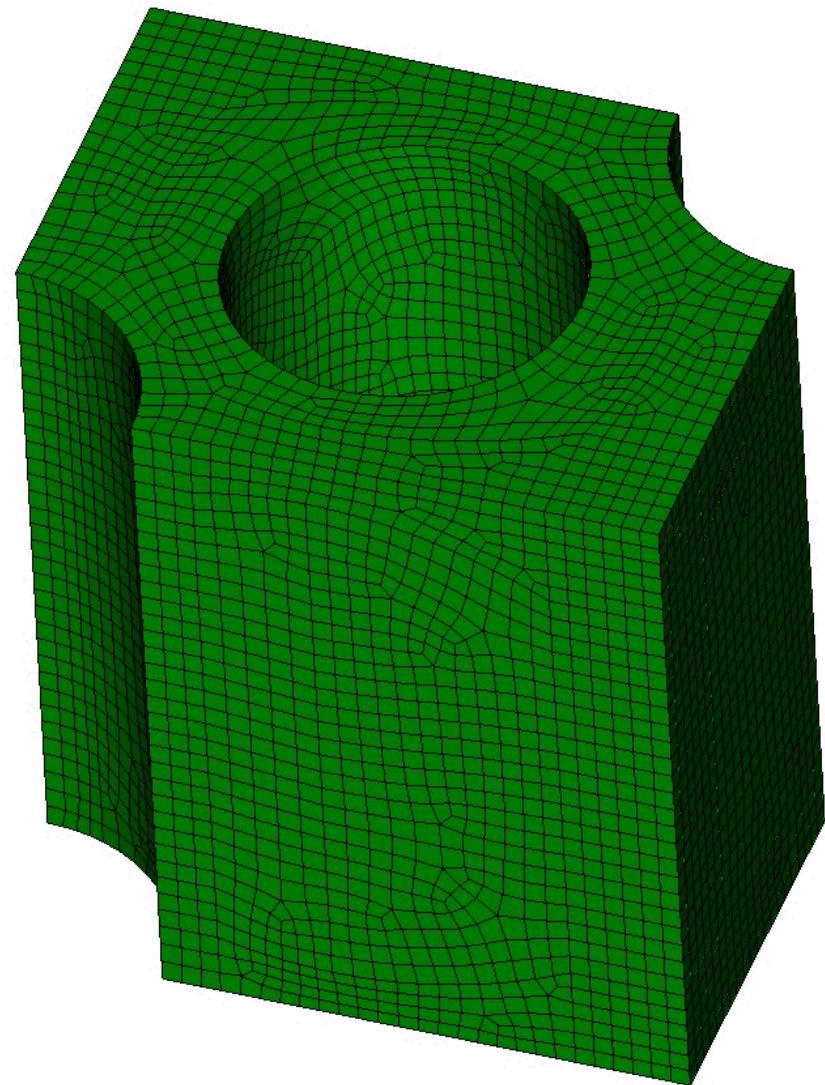
# Merge Six Tets to form a Hex along 3-Manifold Medial Edge!!!



# Improved Hex-dominant Mesh

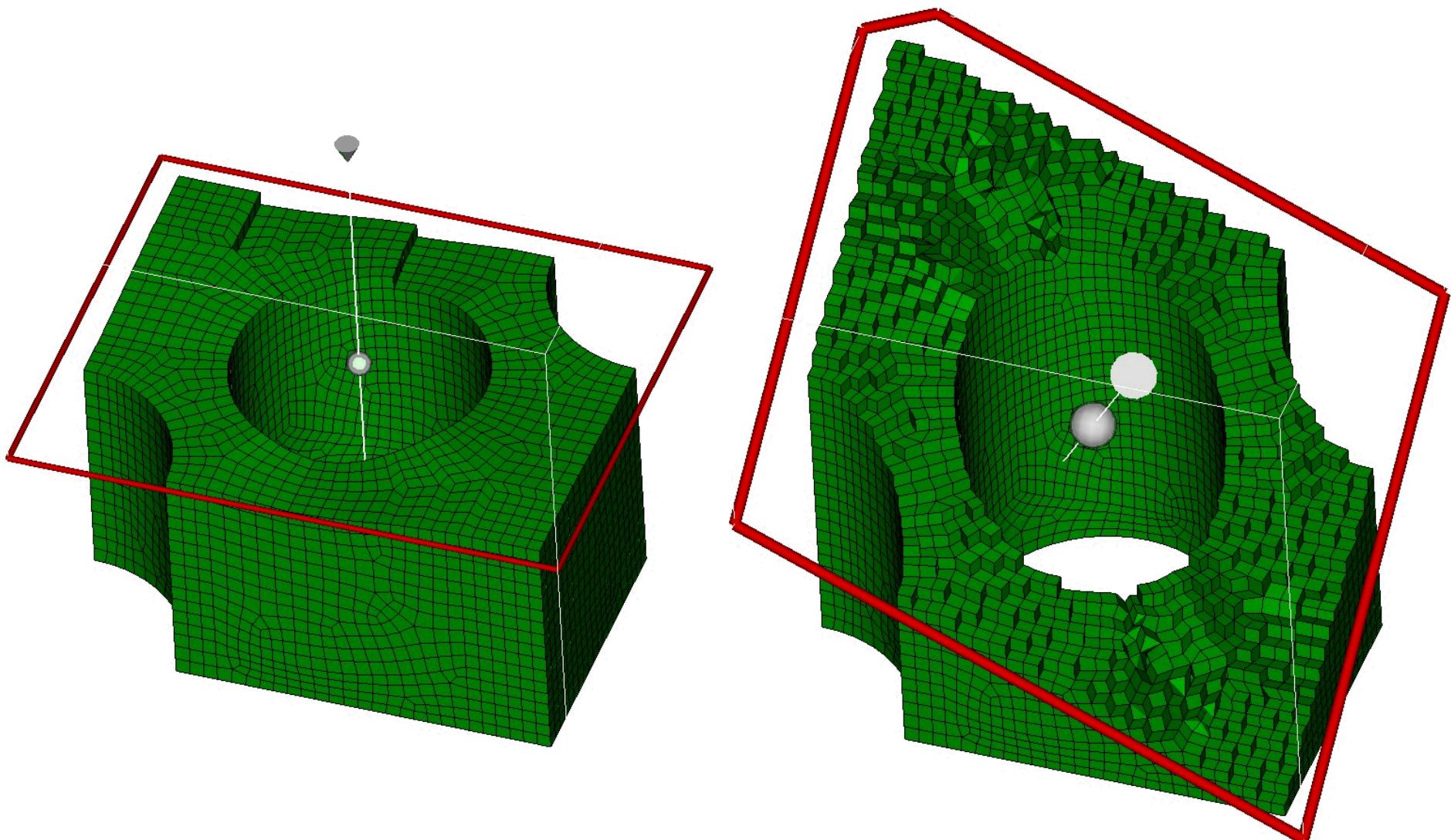


All-Quad Mesh on Medial

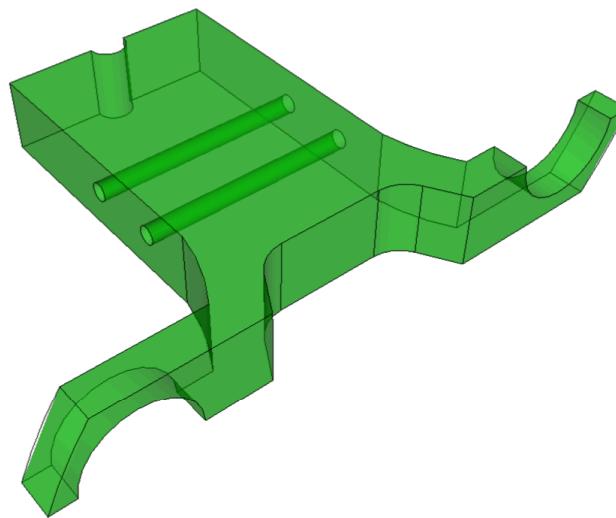


Tri at 3-Manifold Medial Curve

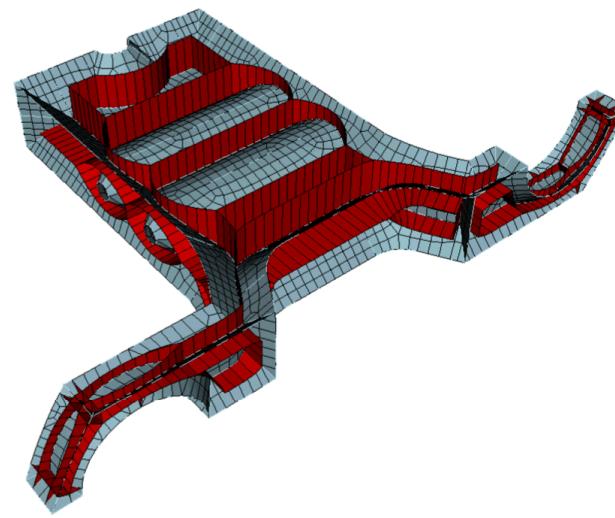
# Sectional View



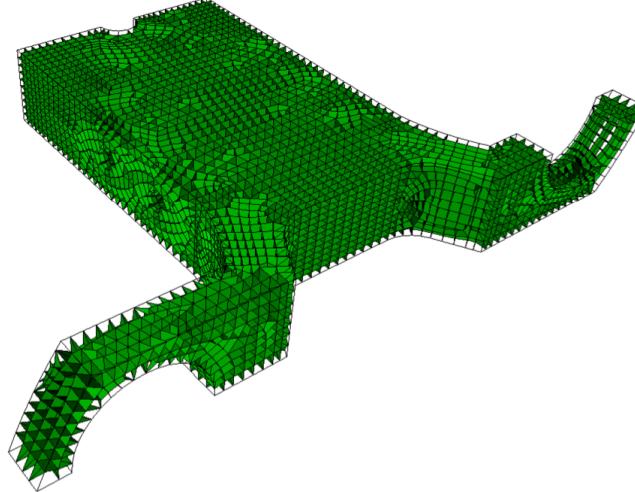
# Demo



(a) Solid with holes



(b) Mesh on MA inside corridors

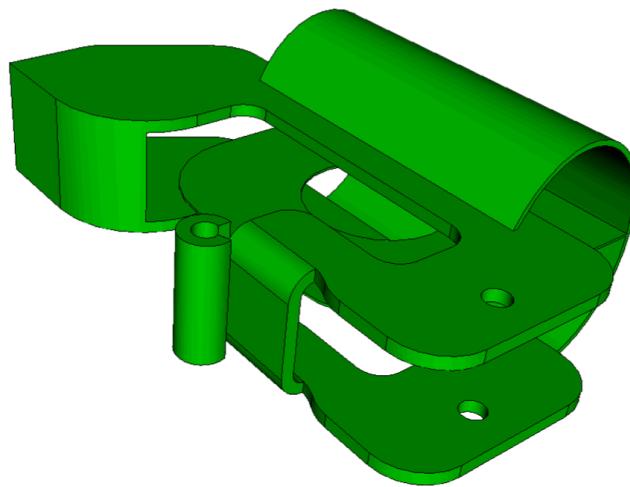


(c) Tracks in 3D

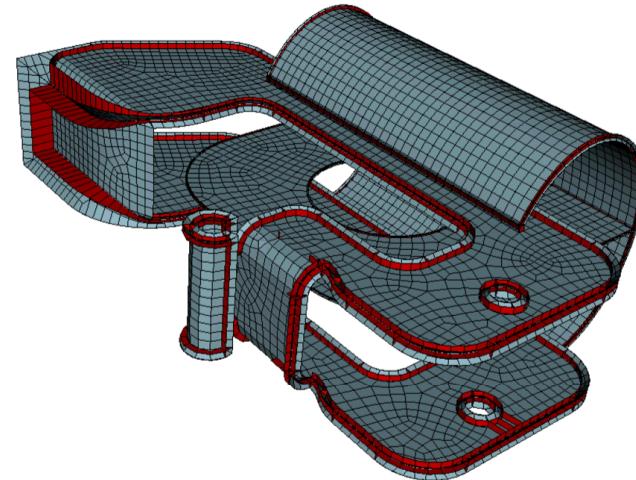


(d) Mesh cross section

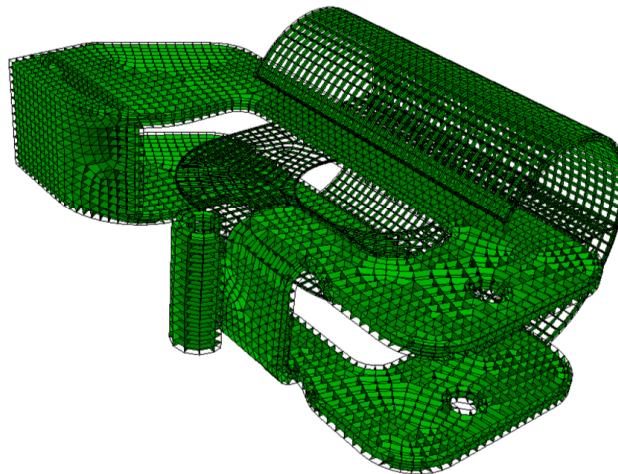
# Demo



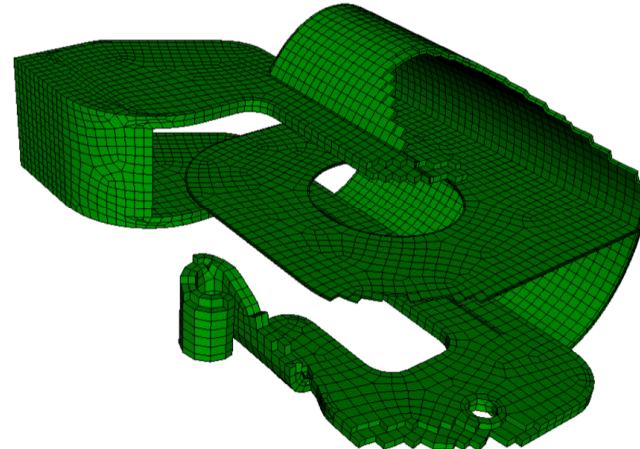
(a) Thin-wall solid



(b) Mesh on MA inside corridors



(c) Tracks in 3D



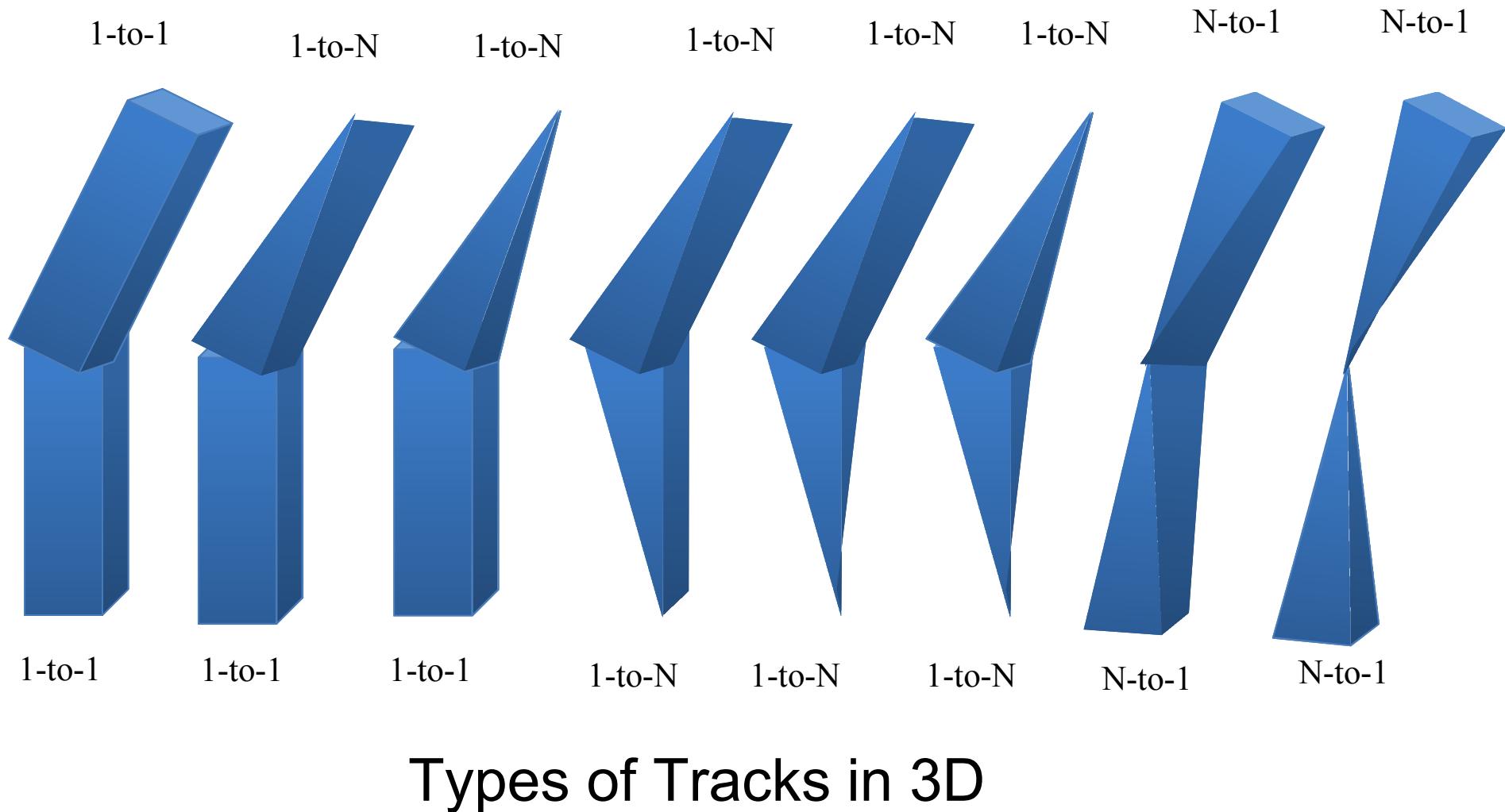
(d) Mesh cross section

# Extension to All-Hex Meshing

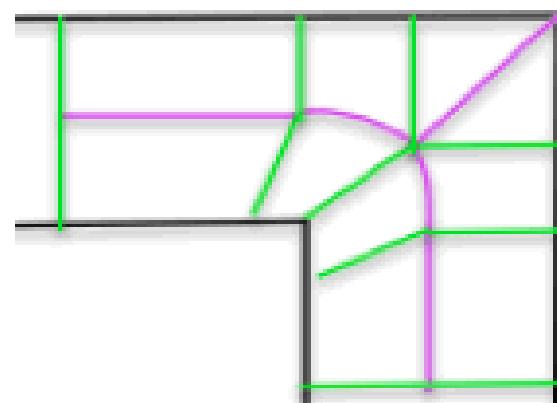
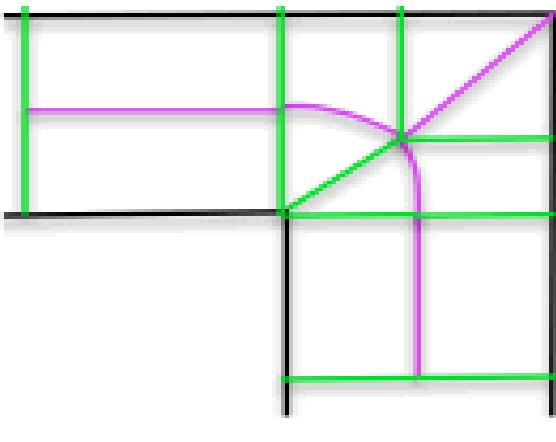
***Claim 1: A general solid can be decomposed into a set of connected quad cross-section 3D tracks***

***Claim 2: A quad cross-section 3D track can be meshed with all-hex elements***

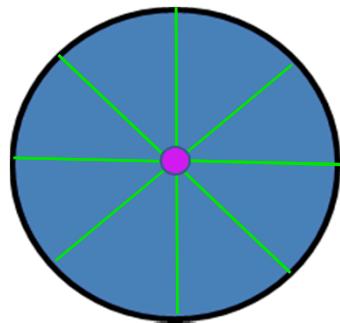
# Claim 1: A general solid can be decomposed into a set of connected quad cross-section 3D tracks



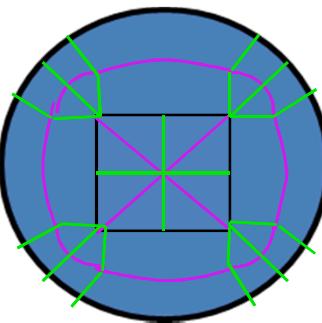
# 1-to-N Track Transformed to Quad Cross-Section 1-to-1 Track



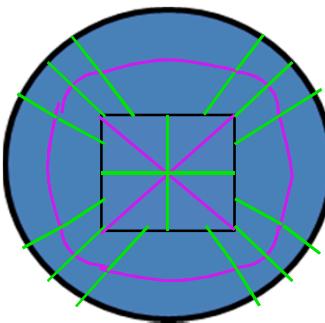
# N-to-1 Track Transformed into Quad Cross-Section 1-to-1 Track



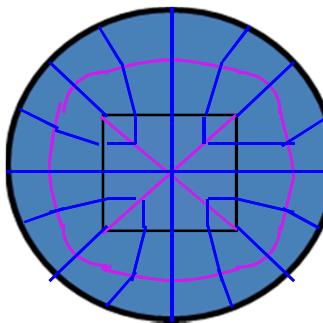
(a)N-to-1



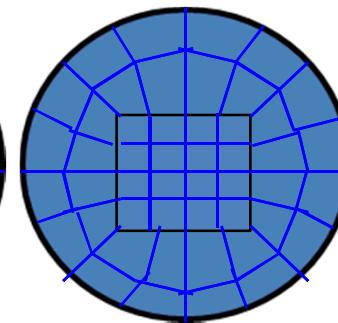
(b)1-to-N



(c)1-to-1



(d)Tracks



(e)Mesh

# Claim 2: A quad cross-section 3D track can be meshed with all-hex elements

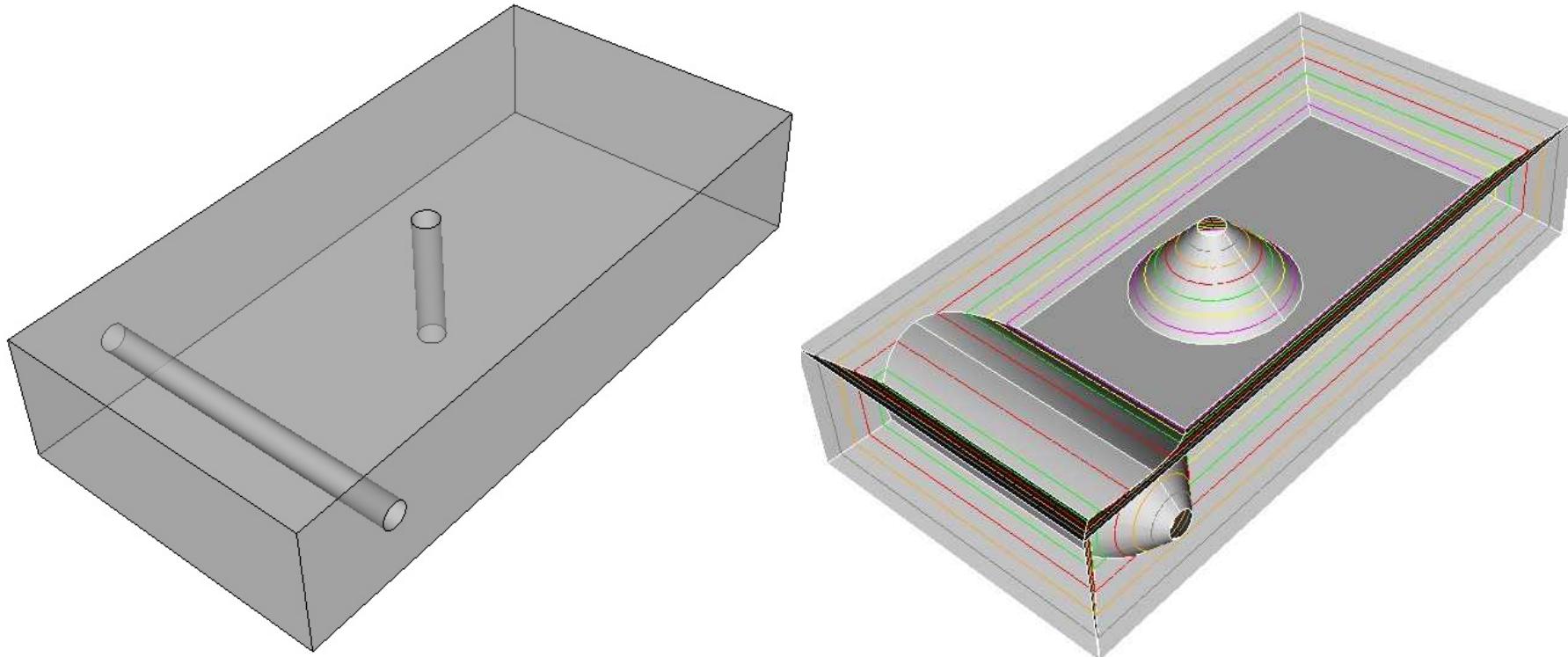
In order to obtain an all-hex mesh inside a quad cross-section track,

Case 1: all four rails must have the same intervals or Case 2: two rails must have  $2N$  intervals and the other two rails must have  $2(N+1)$  intervals.

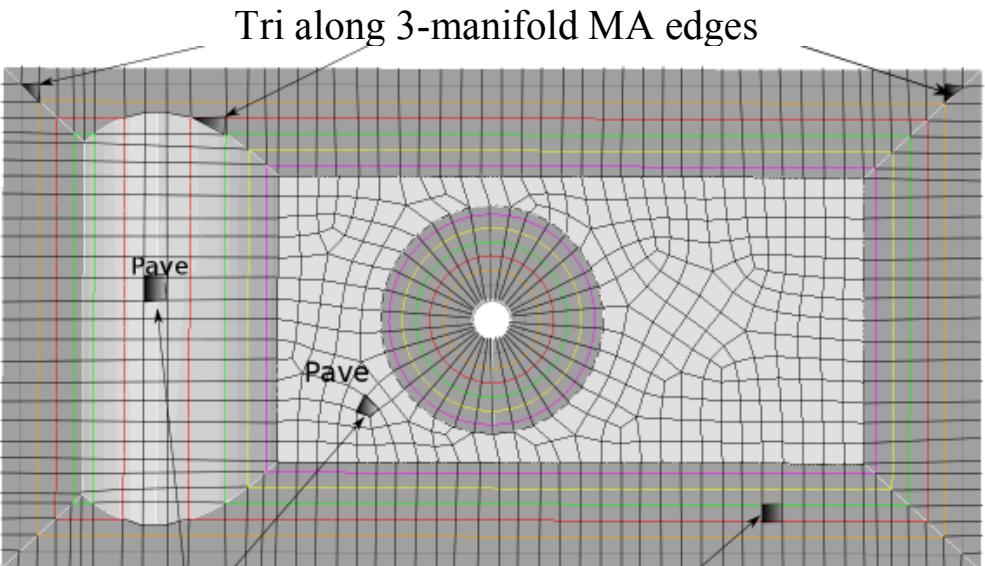
With Case 1, equal number of hex elements are generated on both sides of MA and

with Case 2, two wedges at the MA are merged to form a hex.

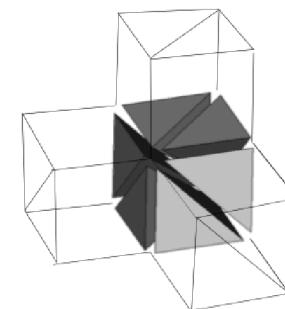
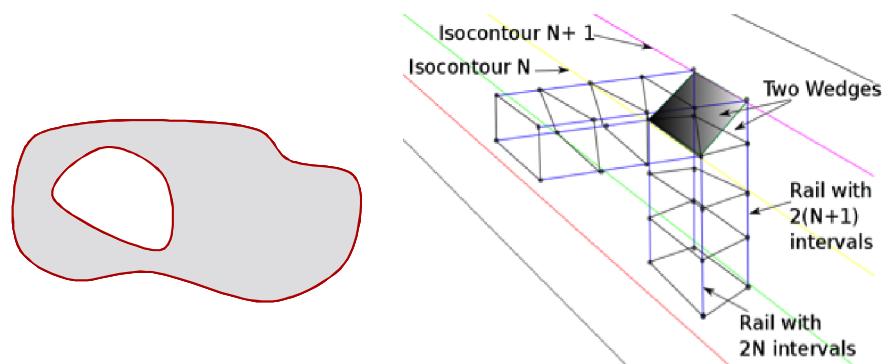
# Isocontours of Medial Radius Function Controls Mesh Intervals on Rails



# All-Hex Mesh Topology Visualization on Isocontours of MA



(c) Meshing MA using isocontours guarantees correct intervals on rails to satisfy Case 1 or Case 2 in each track



(d) Six tets form a hex at 3-manifold MA

# Acknowledgements

*Thanks to Henry Bucklow, Robin Fairey, and Mark Gammon at ITI TranscenData Europe Ltd for providing the CADfix medial object library.*

# Thank You

## Questions?

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