

*Exceptional service in the national interest*



# LayTracks3D: Hex Meshing using MAT

William Roshan Quadros



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. SAND NO. 2011-XXXXP

# Goal of LayTracks3D

- Handle General Solids & Assemblies
- Orientation Insensitive
- Boundary Sensitive
- Preserves Imprints and Sharpe Features
- Geometry Adaptive
- Fast Remeshing
- Morphable Meshes
- Parallel Friendly
- Potential All-Hex

# Advantages of MAT

- Orientation Insensitive:
  - MA of a solid does not change with orientation of the solid
- Boundary Sensitive:
  - AFM fronts meet at MA, i.e., structured mesh along the boundary
- Robust Geometry Decomposition:
  - Medial branch points represent critical singularity regions
- Dimension Reduction:
  - Hex meshing can be reduced to surface meshing
- Geometry Adaptive:
  - Medial radius function provides local feature size to control mesh size.
- Symmetric Skeleton:
  - Simplifies interval assignment and restricts irregular nodes at MA
- Homotopy Equivalence:
  - Preserves sharp boundary features

# Medial Axis Transform

## Two-Way Mapping

$$f: p \rightarrow Mp$$

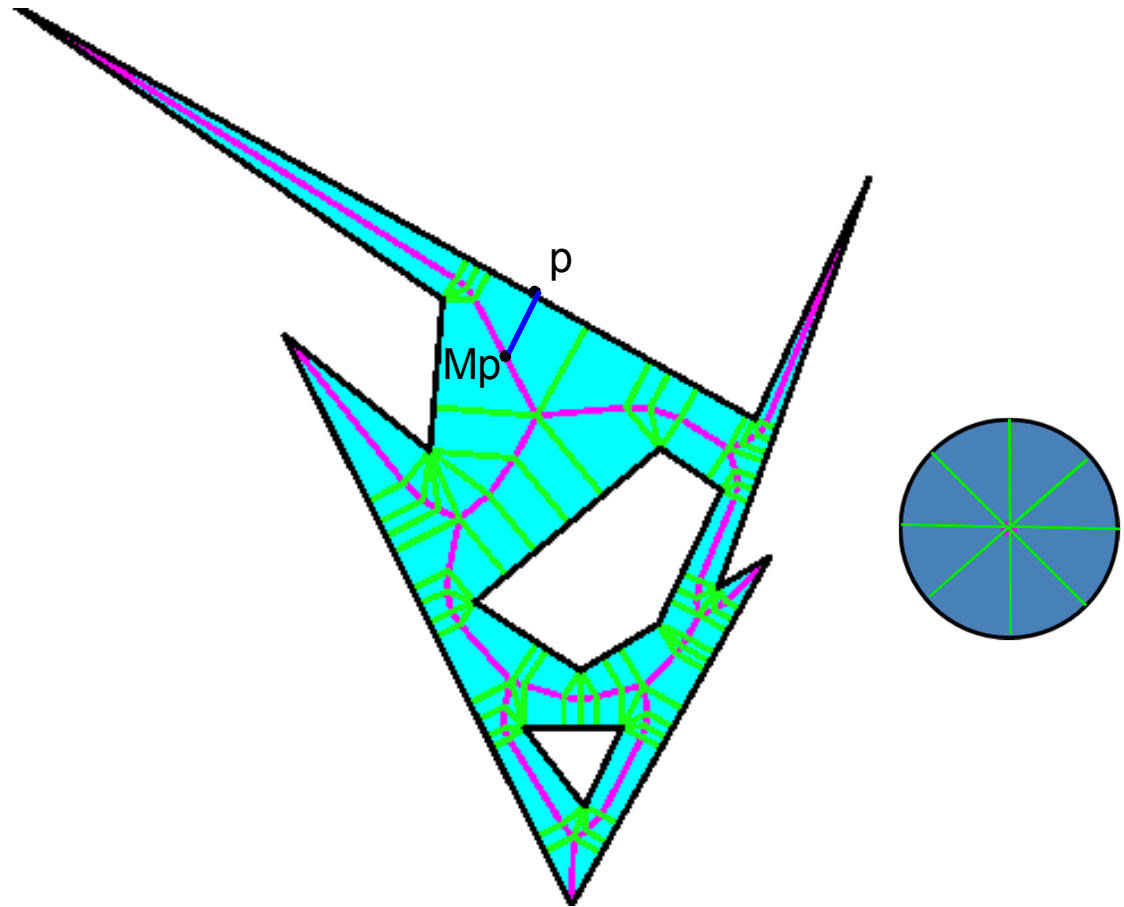
$$g: Mp \rightarrow p$$

## Types of Mapping

1-to-1 Map

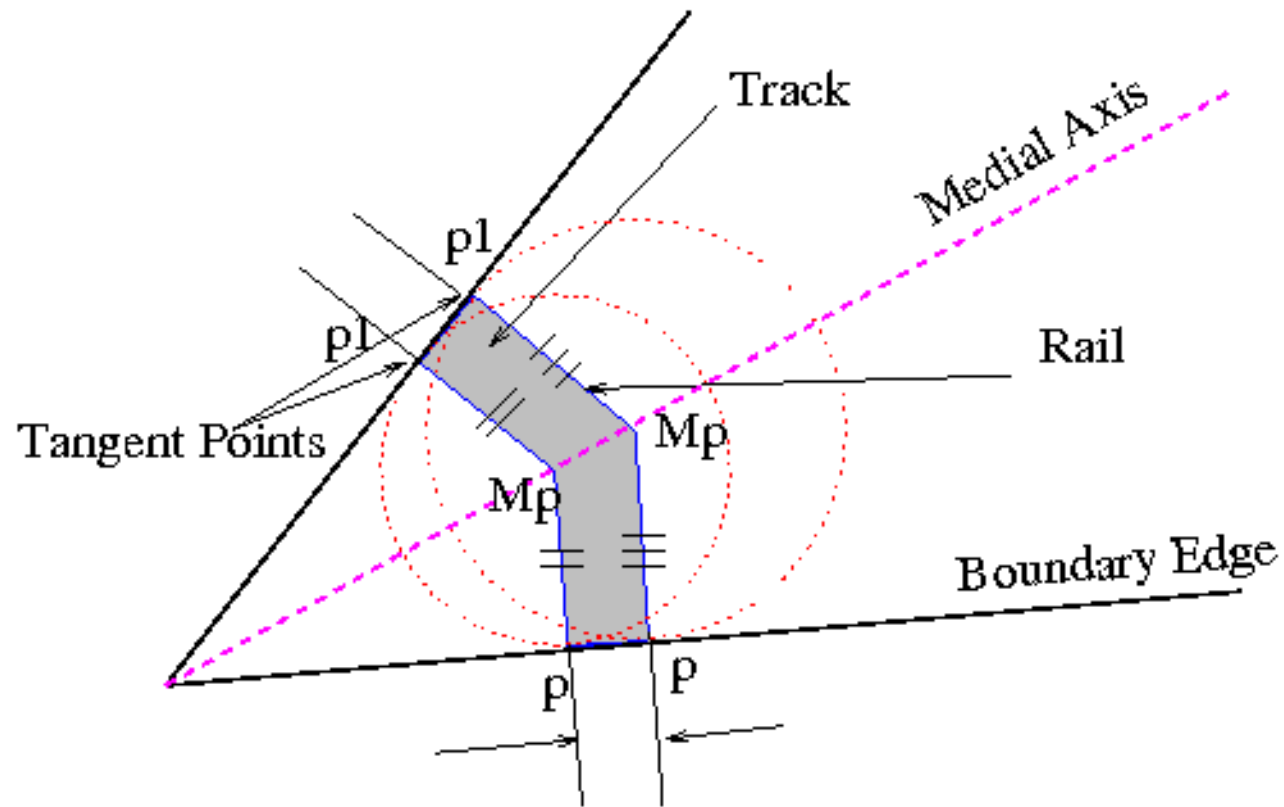
1-to-N Map

N-to-1 Map



# Projection Operator

- Uses map  $f$  to connect  $p \rightarrow M_p$
- Uses map  $g$  to connect  $M_p \rightarrow p_1$

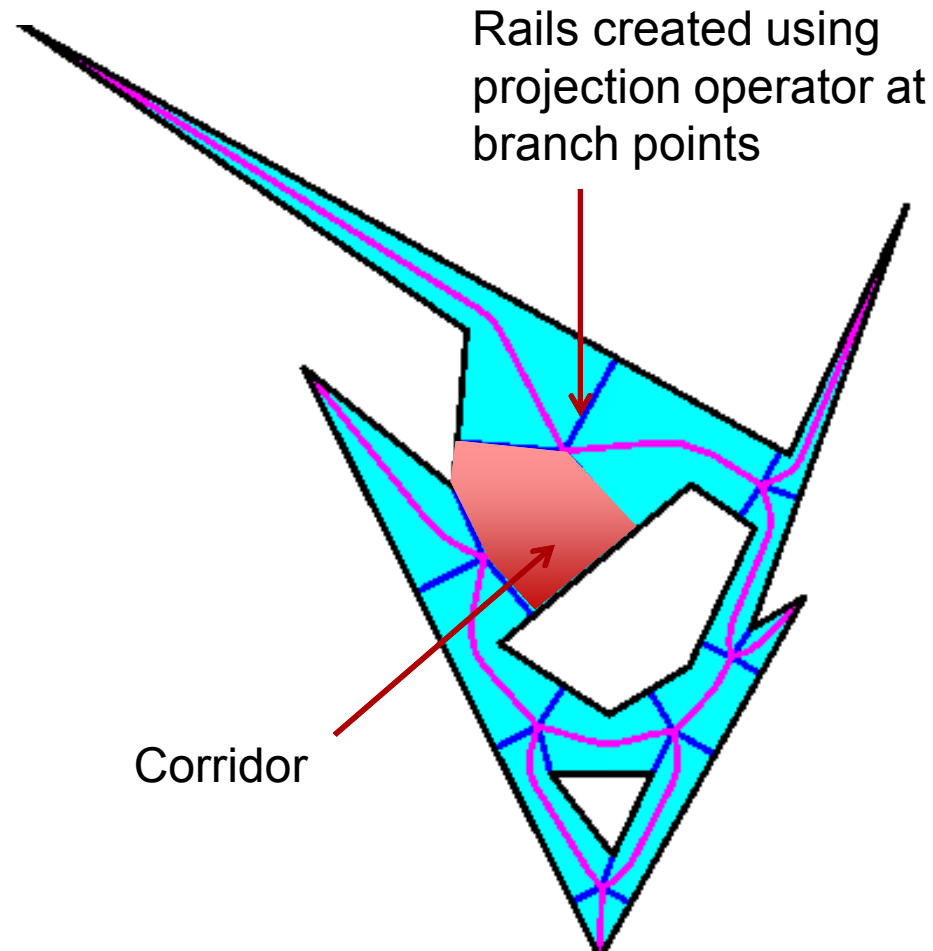


# LayTracks: All-Quad Meshing, 9<sup>th</sup> IMR, 2000

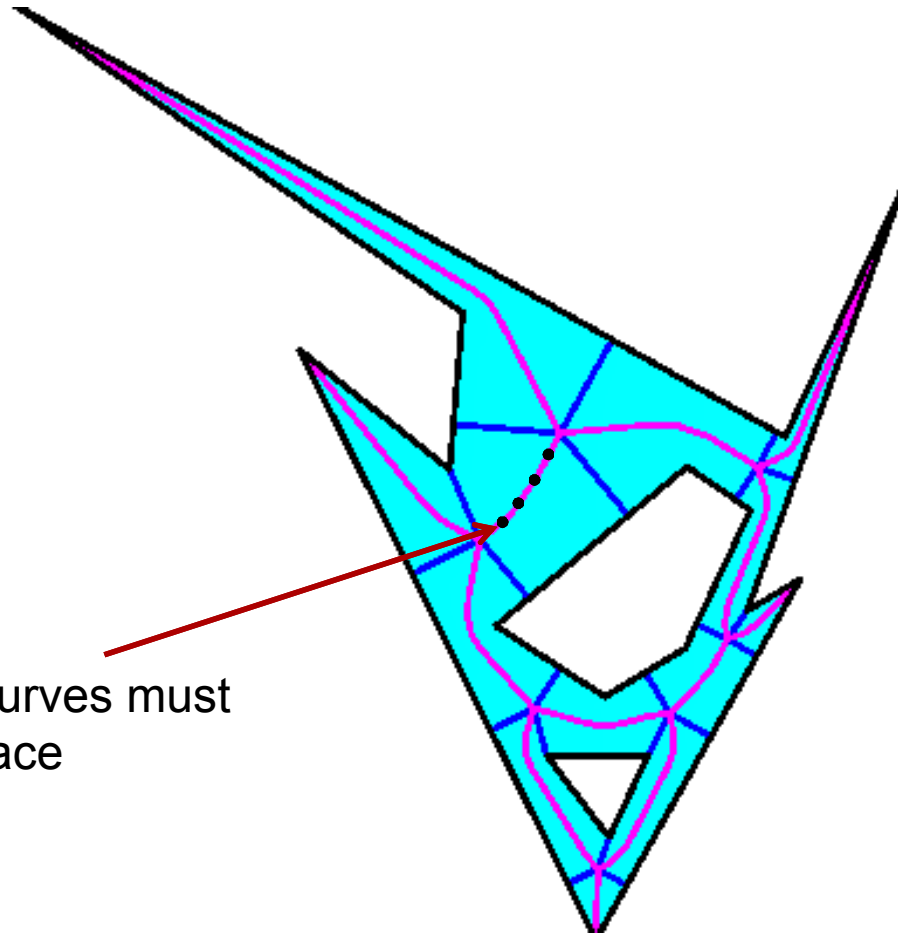
Combines merits of decomposition and advancing front methods

- Generate MAT: establish two-way map between domain boundary and medial
- Generate Corridors: decompose domain using medial branch points and projection operator
- Mesh Medial: surface meshing is reduced to curve meshing. Medial radius function can be used to control mesh size.
- Generate Tracks: further decompose the domain/Corridors into simpler/meshable Tracks using projection operator
- Mesh Tracks: generate all-quad in each track using symmetric property of the medial

# Generate Corridors: Domain Decomposition



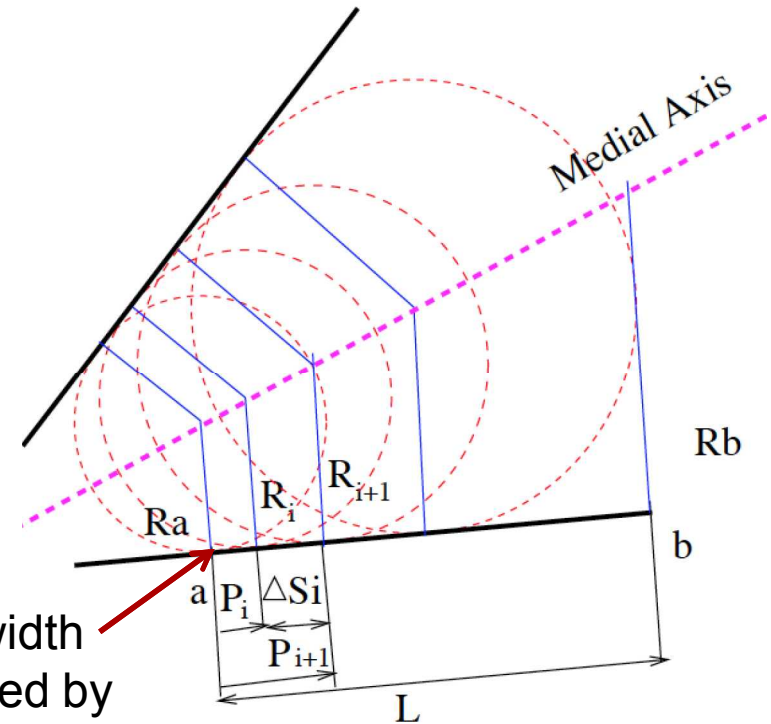
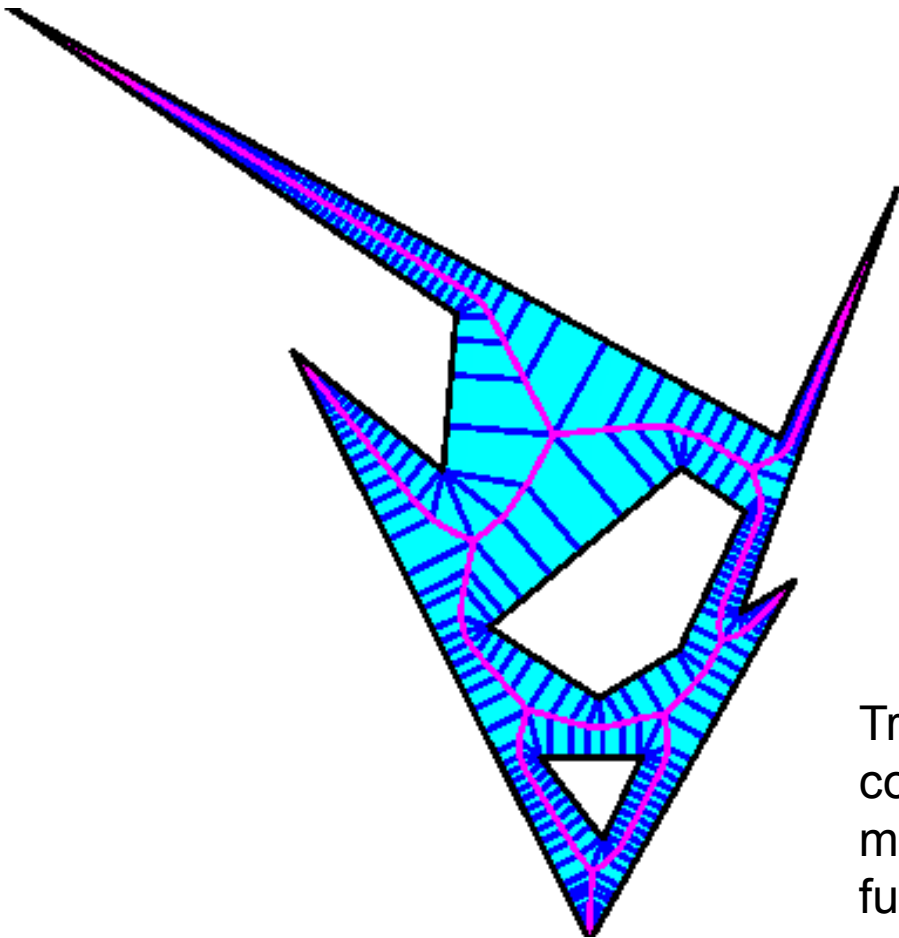
# Mesh Medial: Surface meshing is Reduced to Curve Meshing



Meshing all medial curves must cover the entire surface

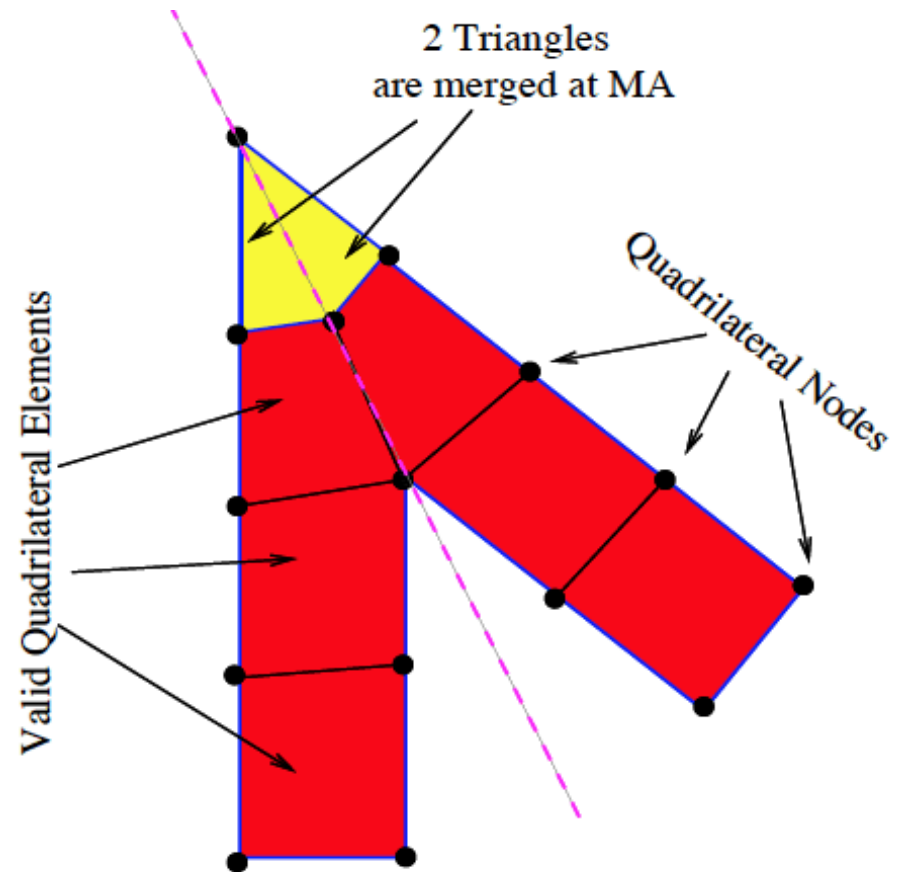
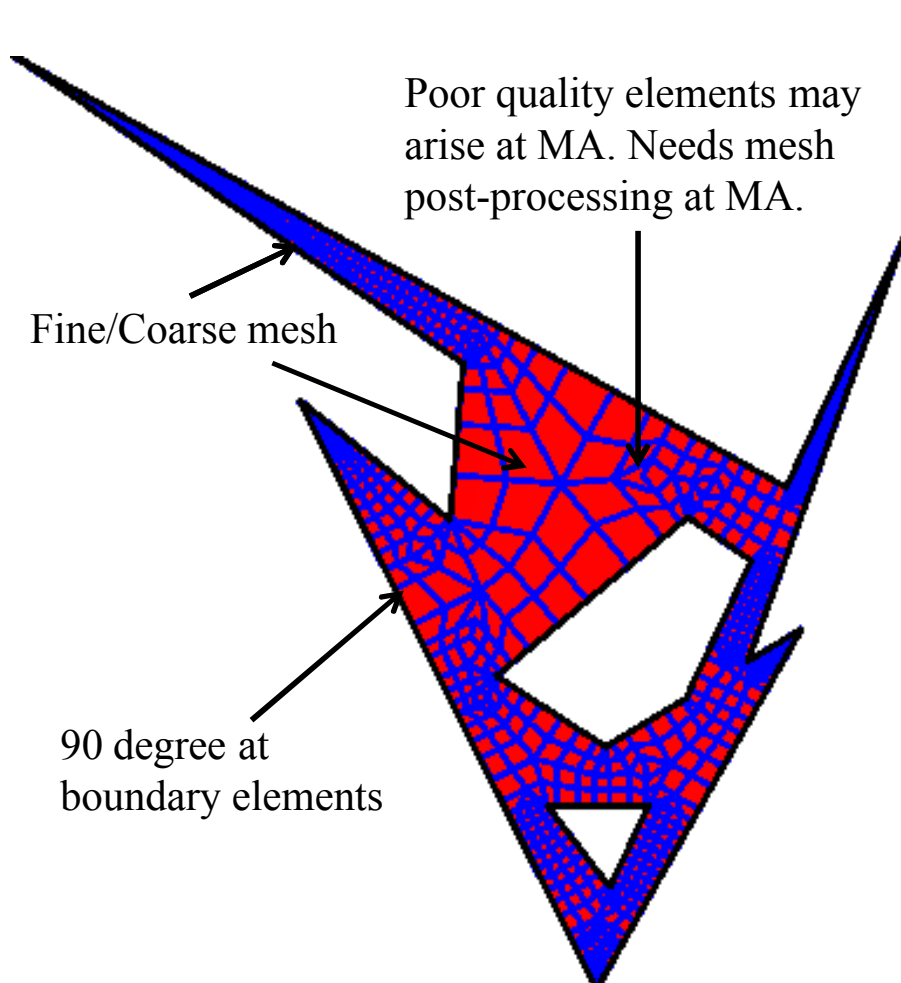


# Generate Tracks: Projection Operator Connects MA Nodes to Boundary



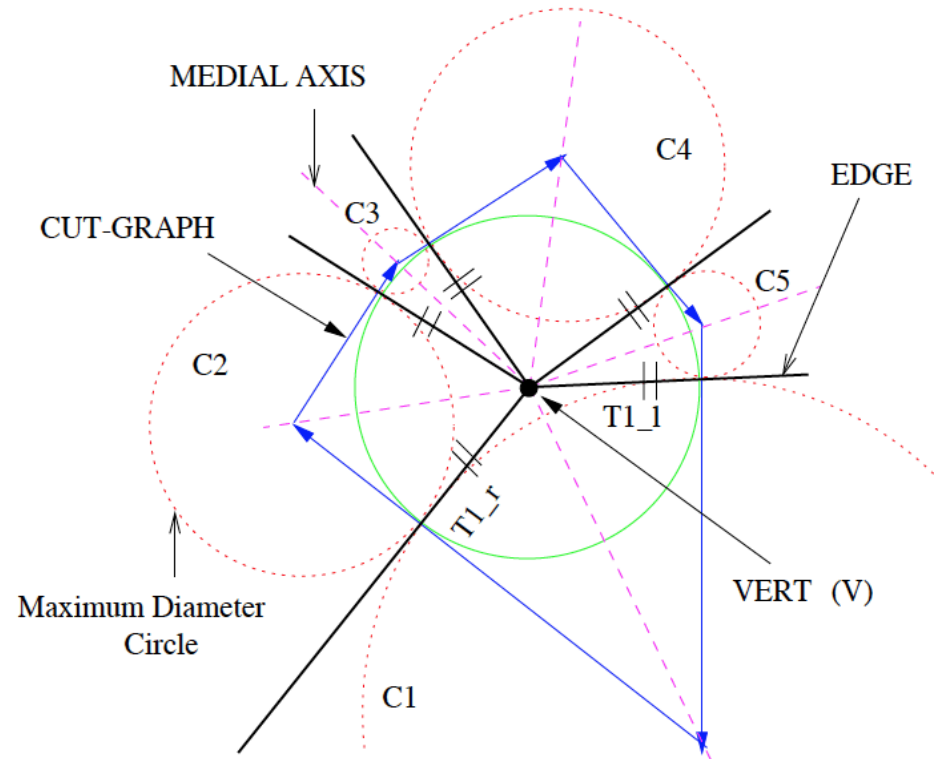
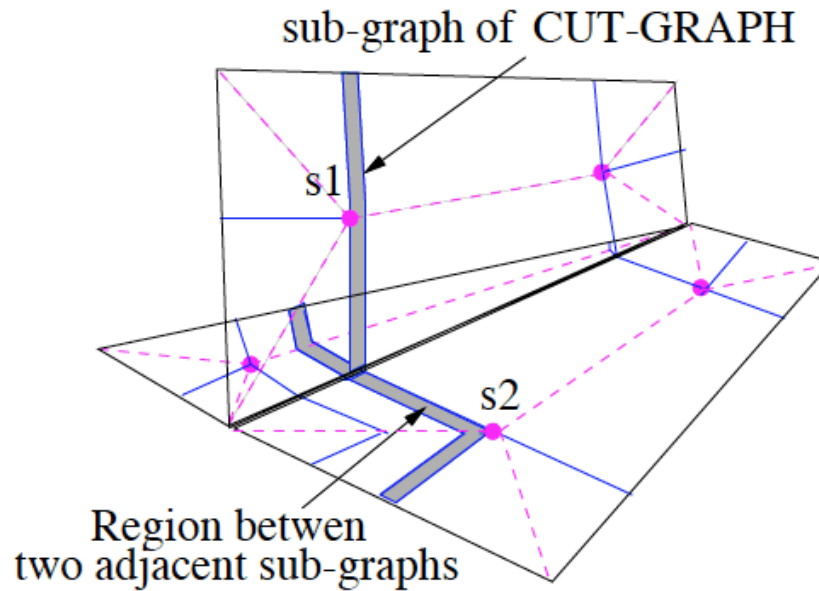
Track width  
controlled by  
medial radius  
function

# Mesh Tracks: All-Quad Mesh can be Generated in 2D

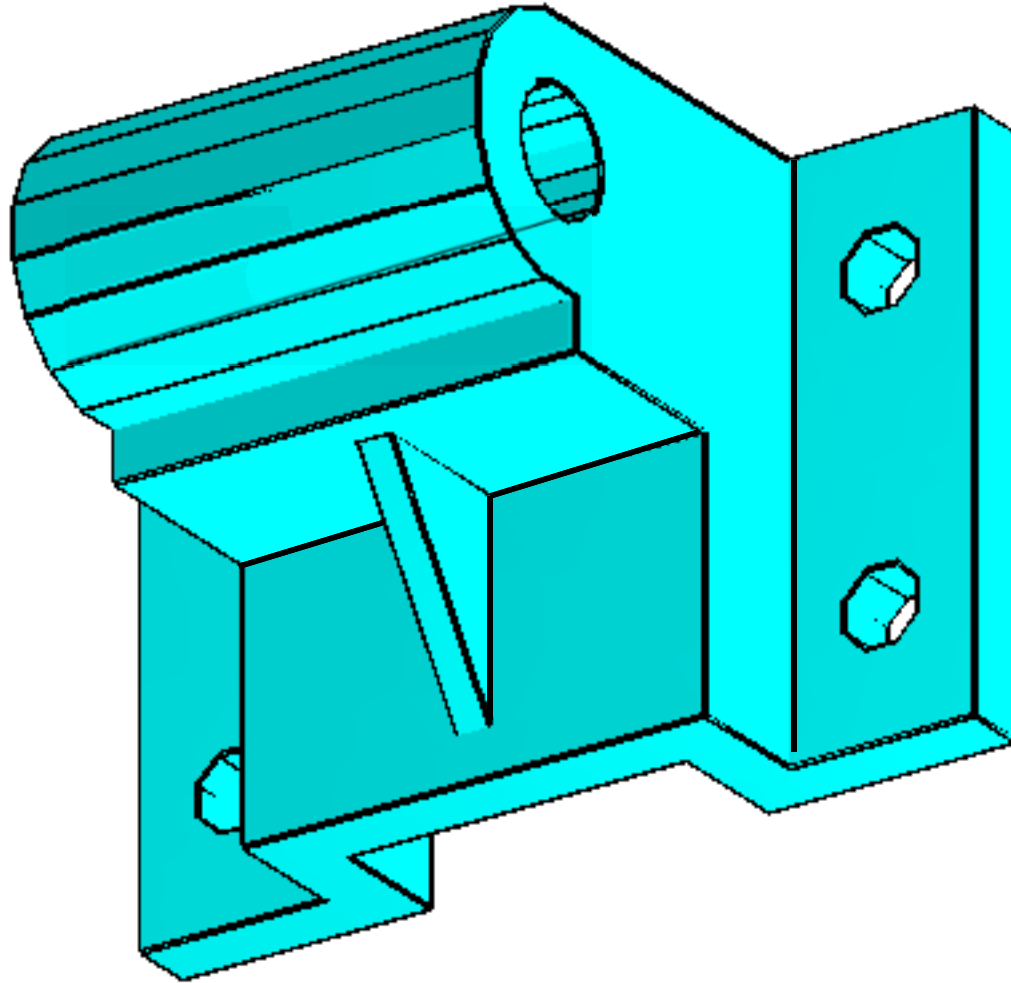


All-Quad: A track is bounded by even number of edges via symmetric property

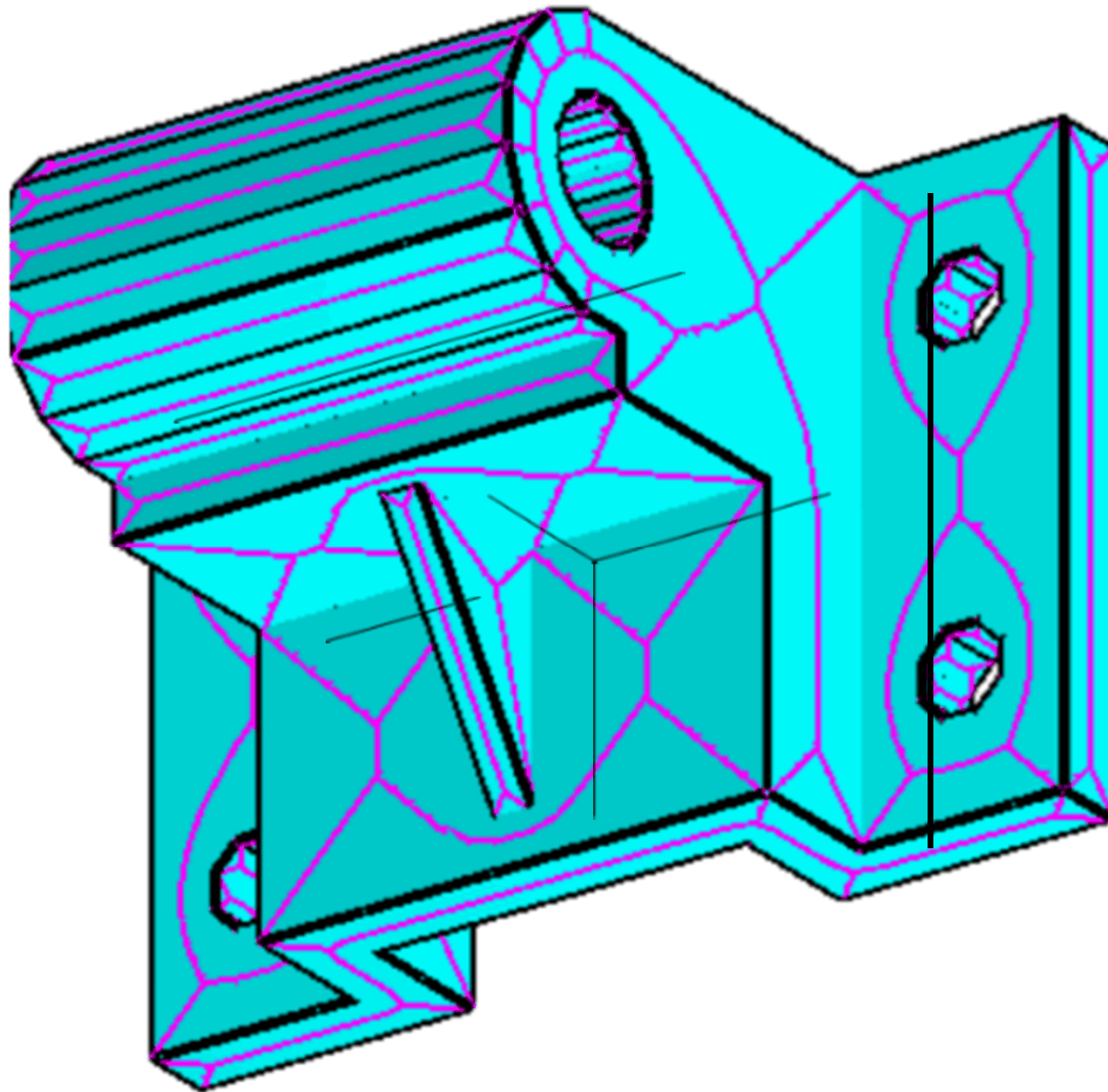
# LayTracks on Multiple Surfaces



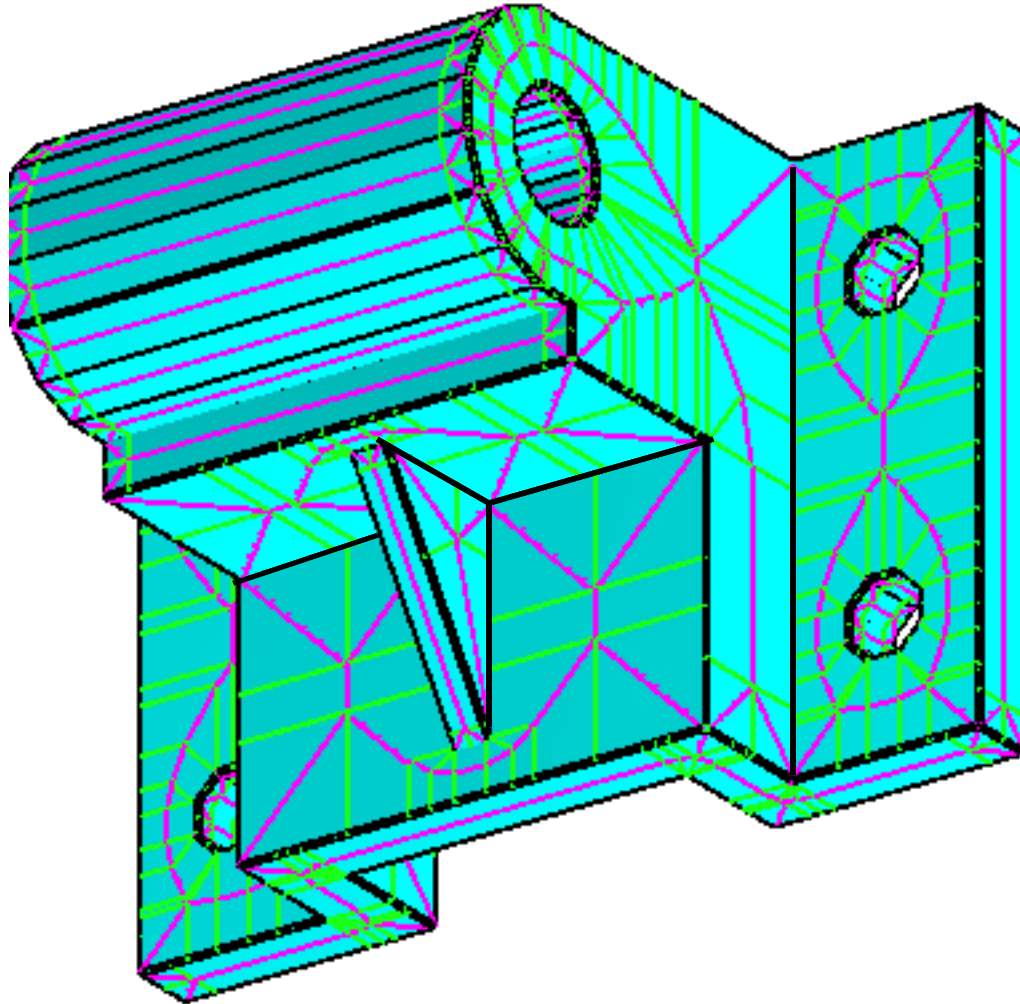
# Assembly of Surfaces



# Medial of Multiple Surfaces



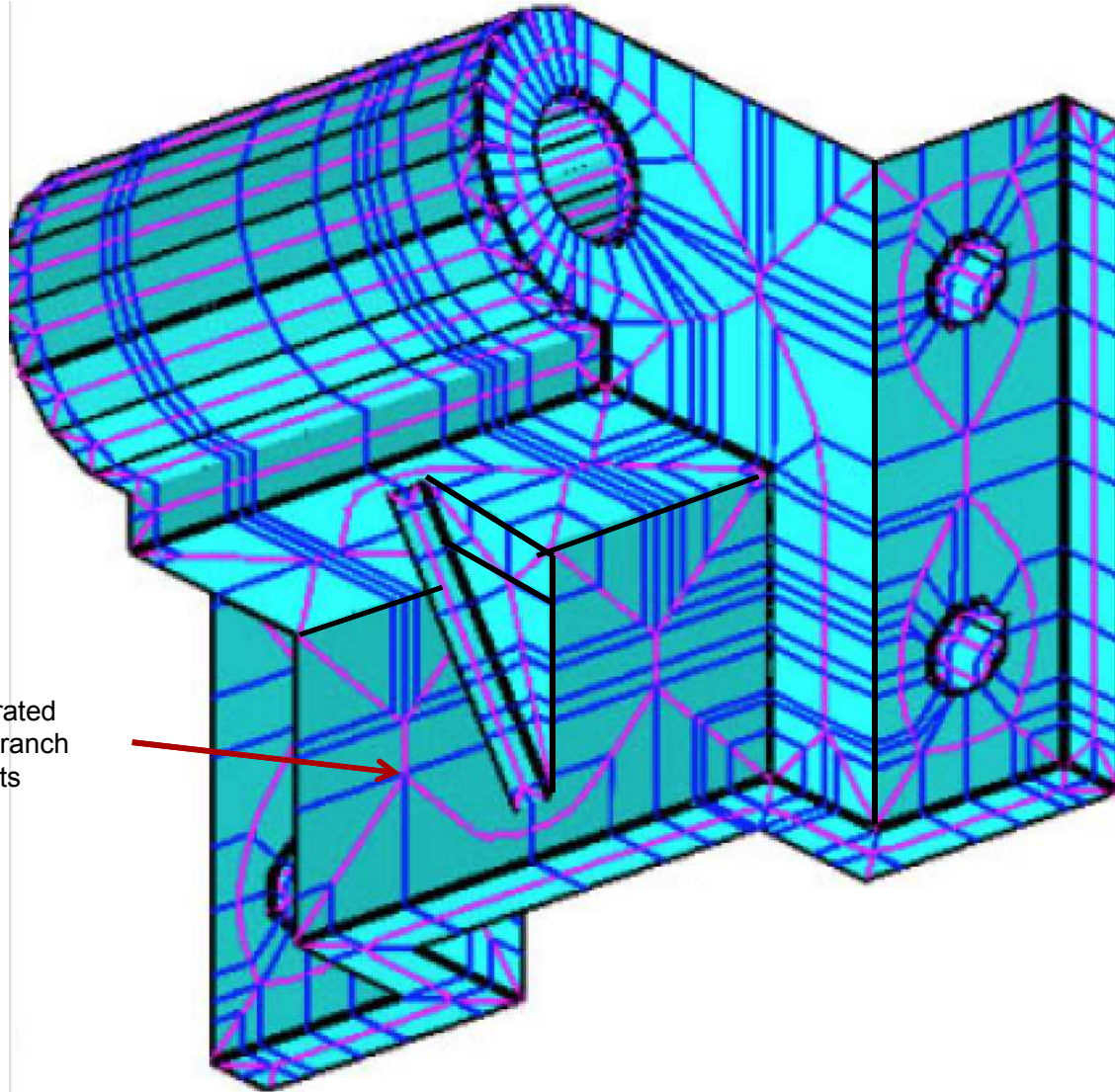
# Two-Way Map on Multiple Surfaces



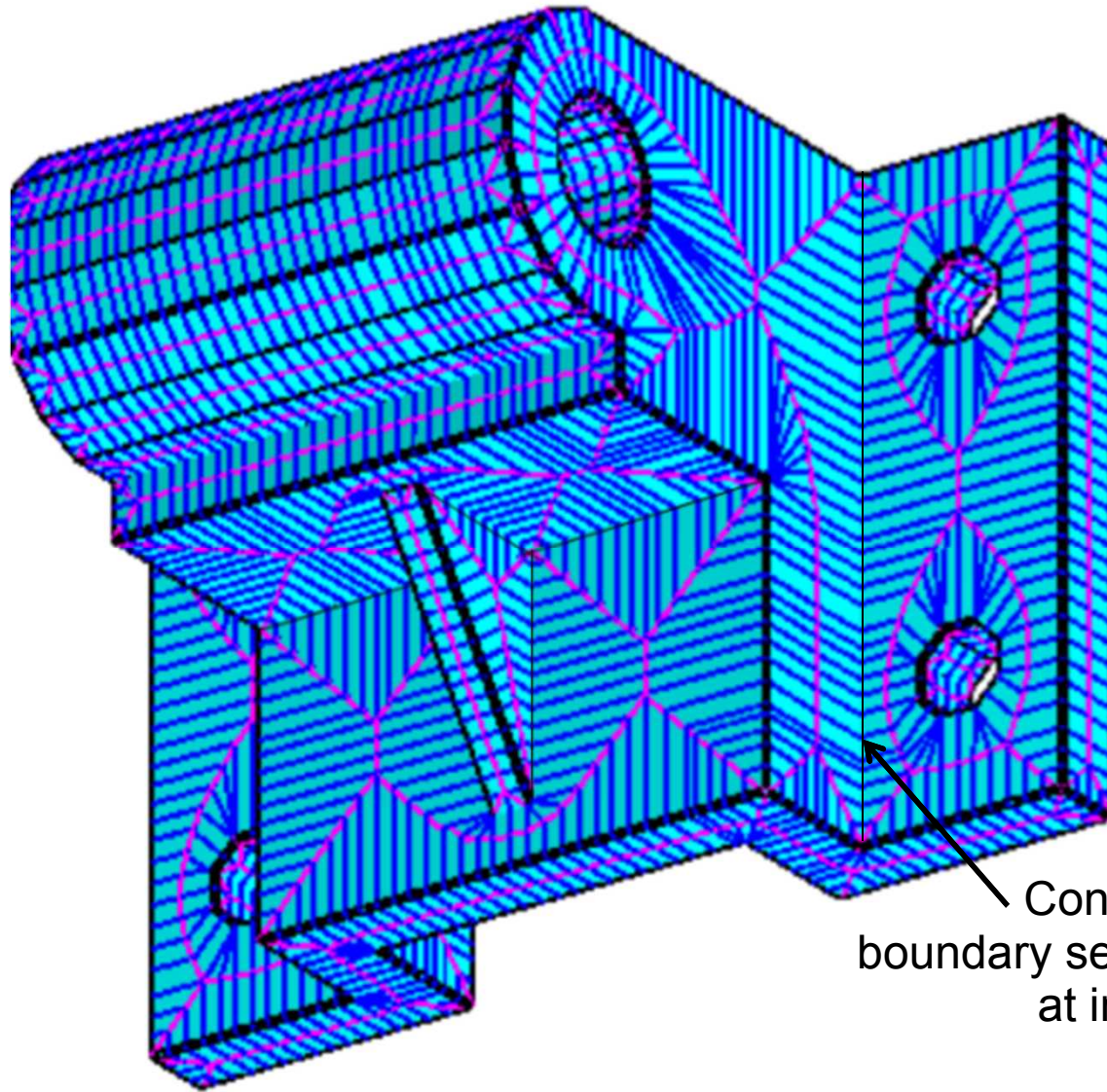


# Automatic Geometry Decomposition

Corridors generated  
using Medial Branch  
Points & Imprints



# Tracks on Multiple Surfaces

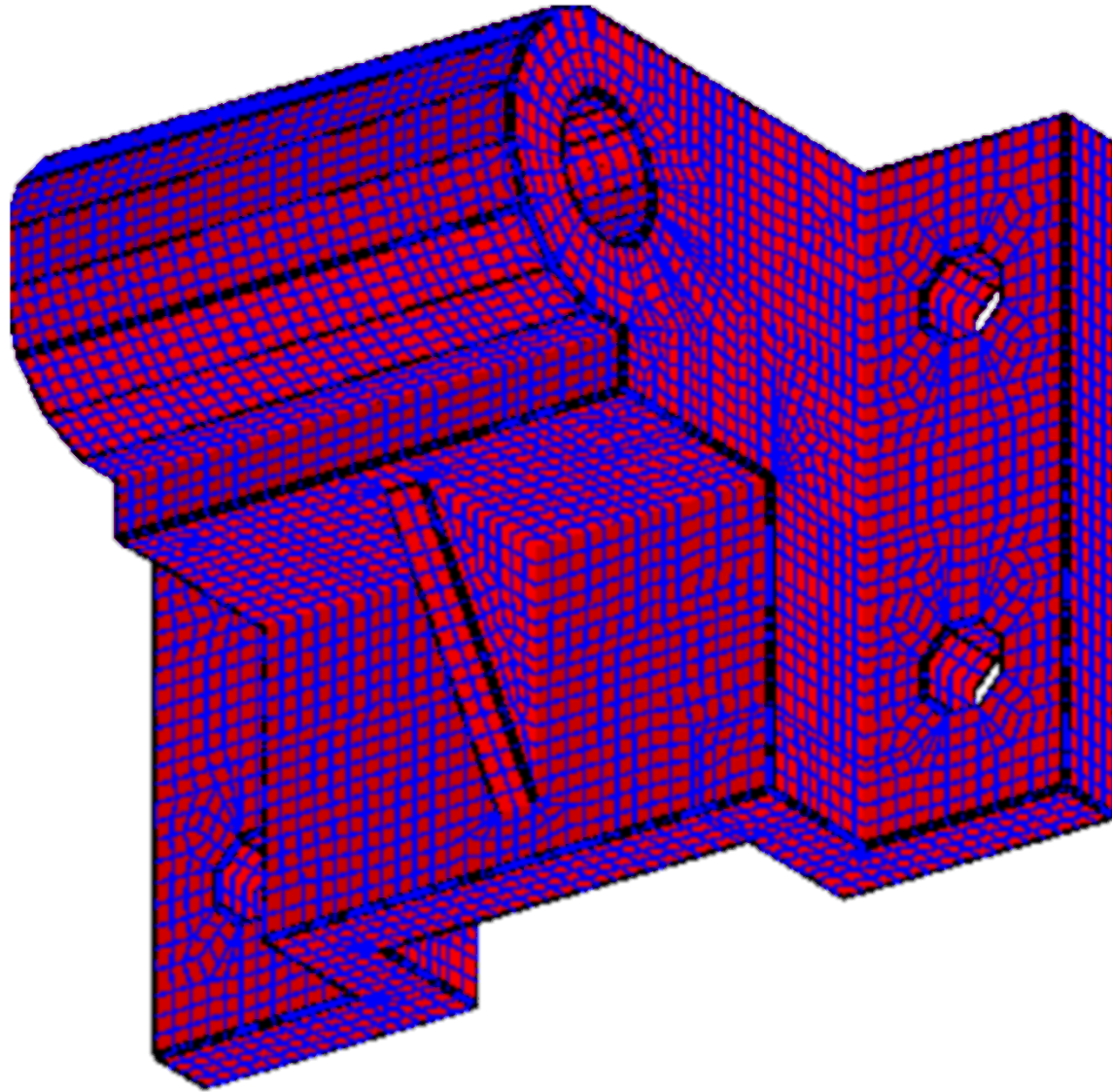


Conformal &  
boundary sensitive elements  
at interface

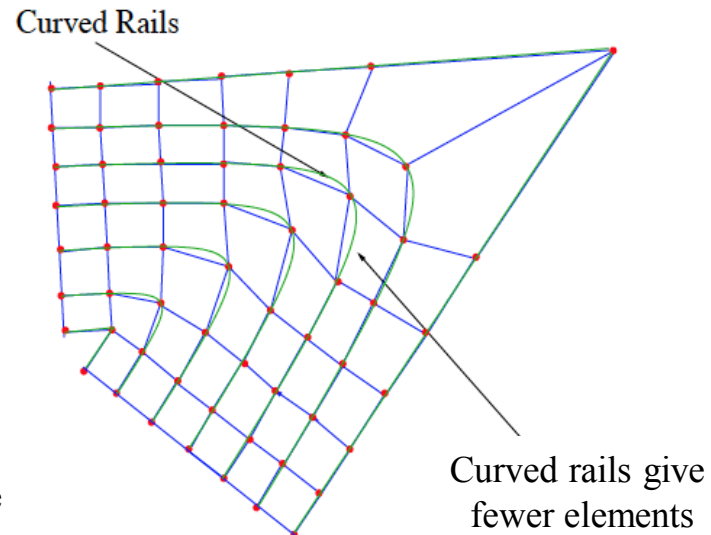
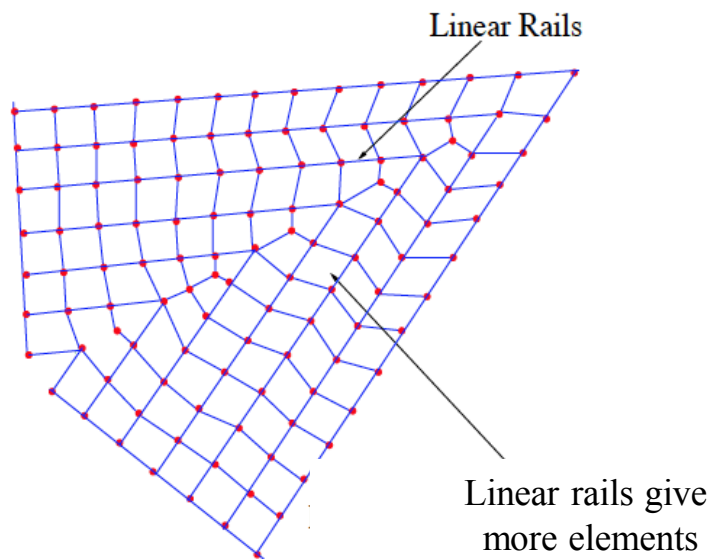
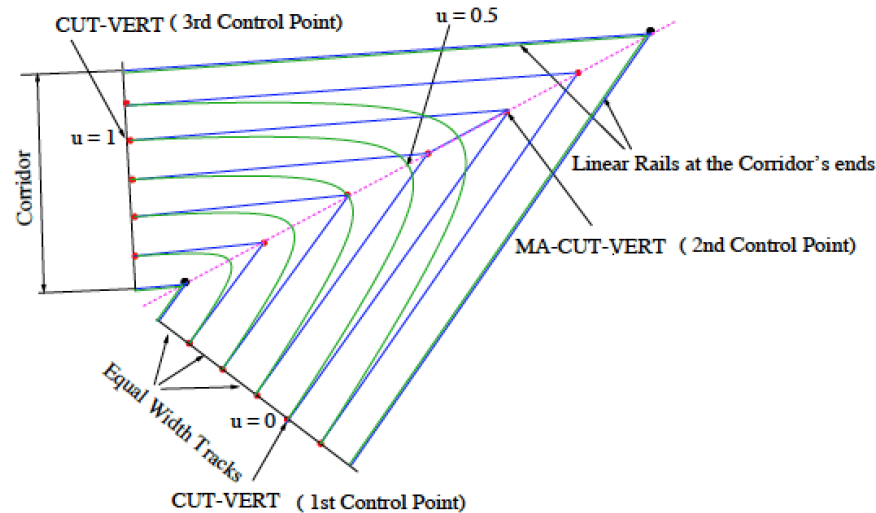


# Quad Mesh on Multiple Surfaces

(with no post-meshing operations)

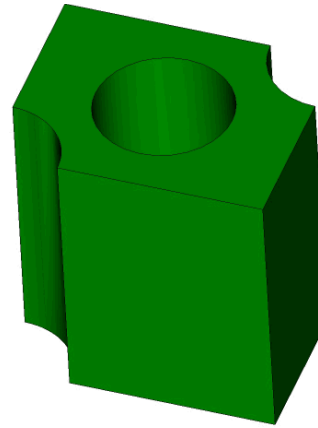
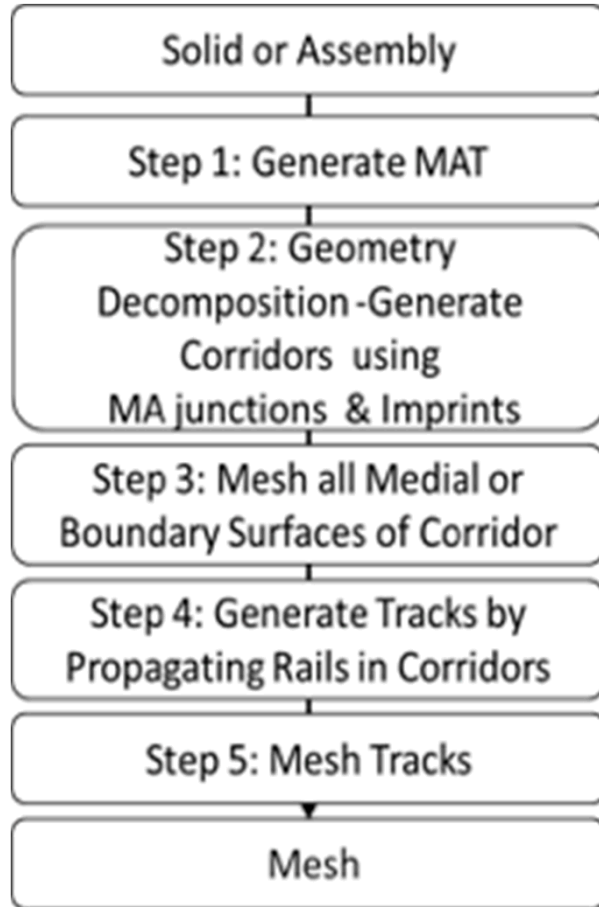


# LayTracks with Curved Rails & Tracks

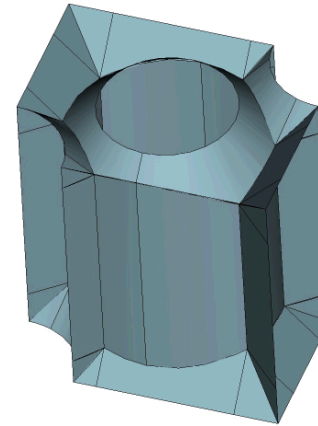


# LayTracks3D: Extension of LayTracks to 3D

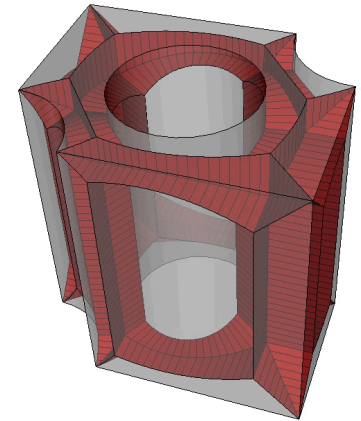
# Overview of LayTracks3D



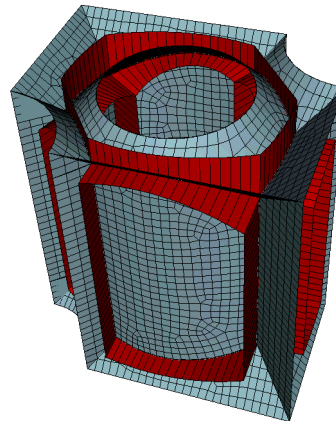
Input solid



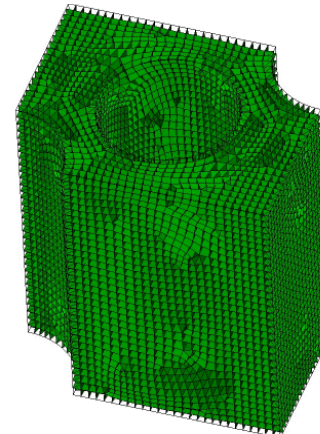
(1) Medial surface



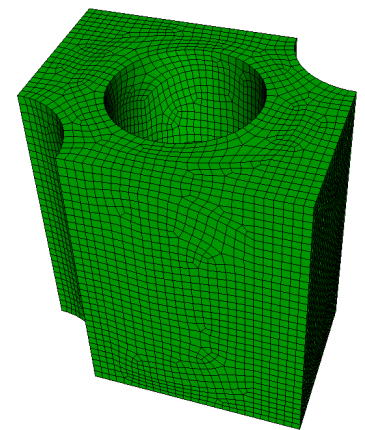
(2) Corridors



(3) Mesh on medial in corridors

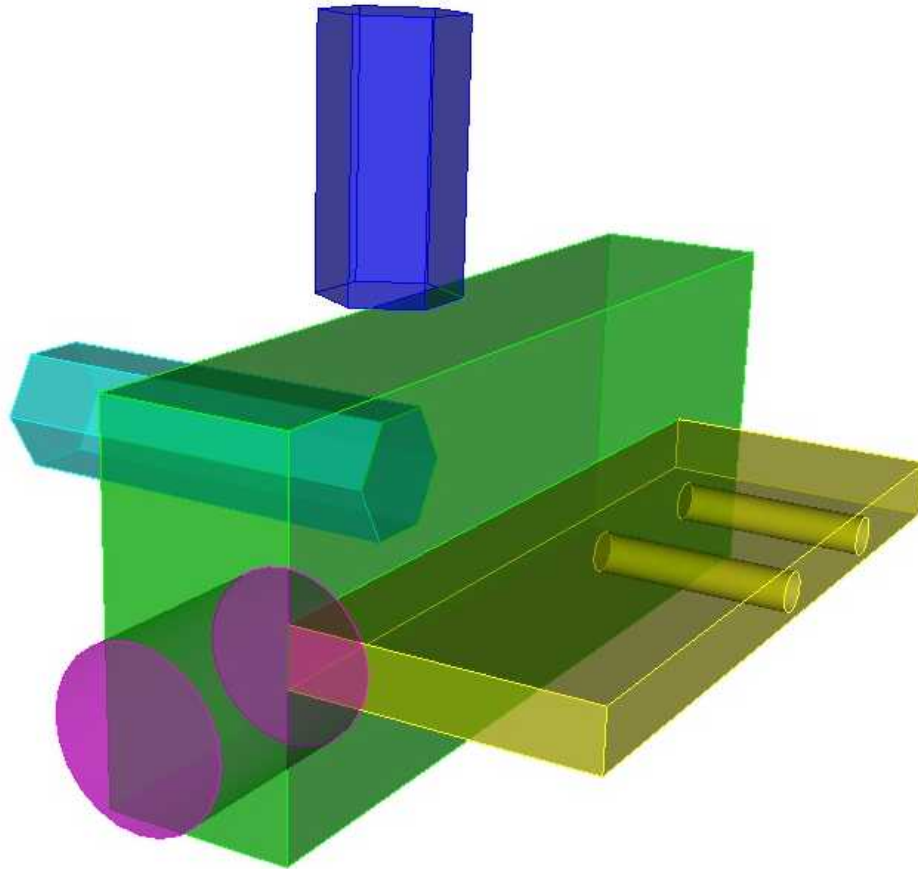


(4) Tracks in 3D

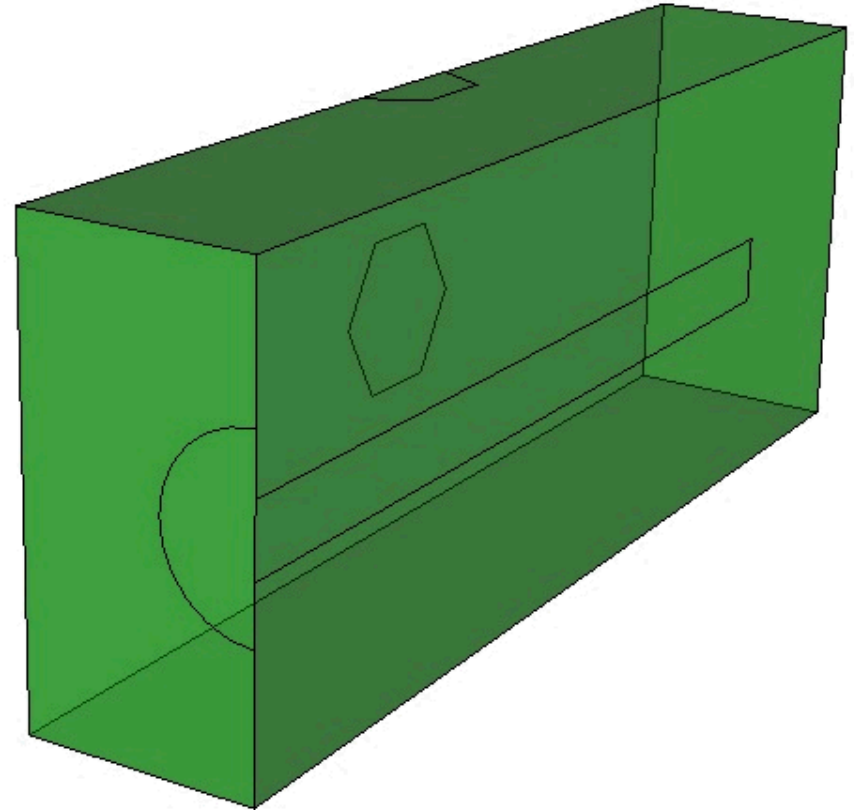
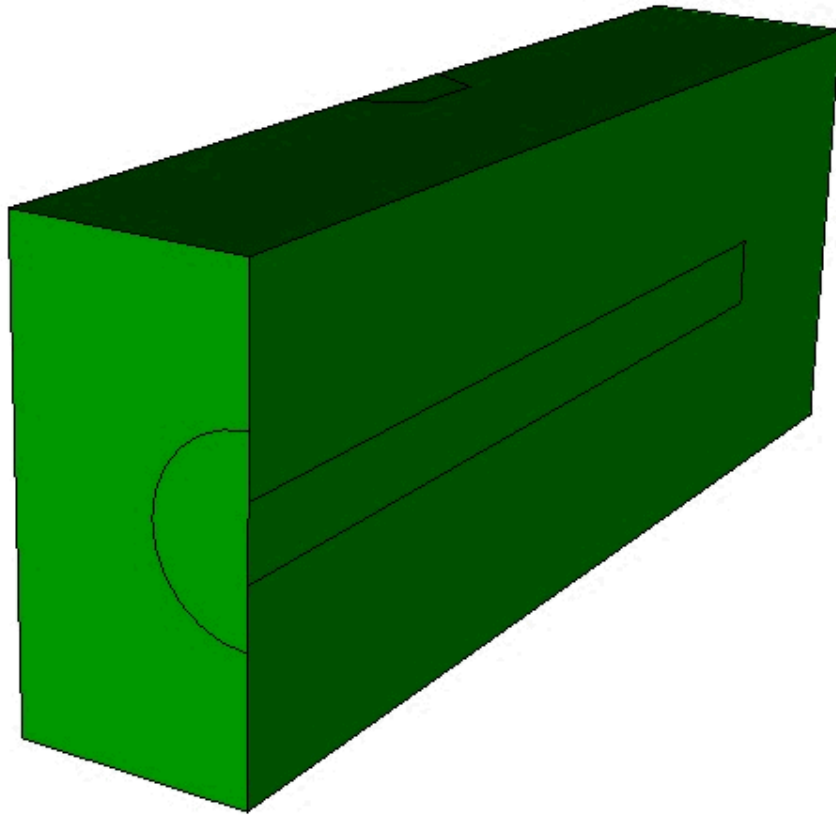


(5) Hex dominant Mesh

# Extension to Assembly Model

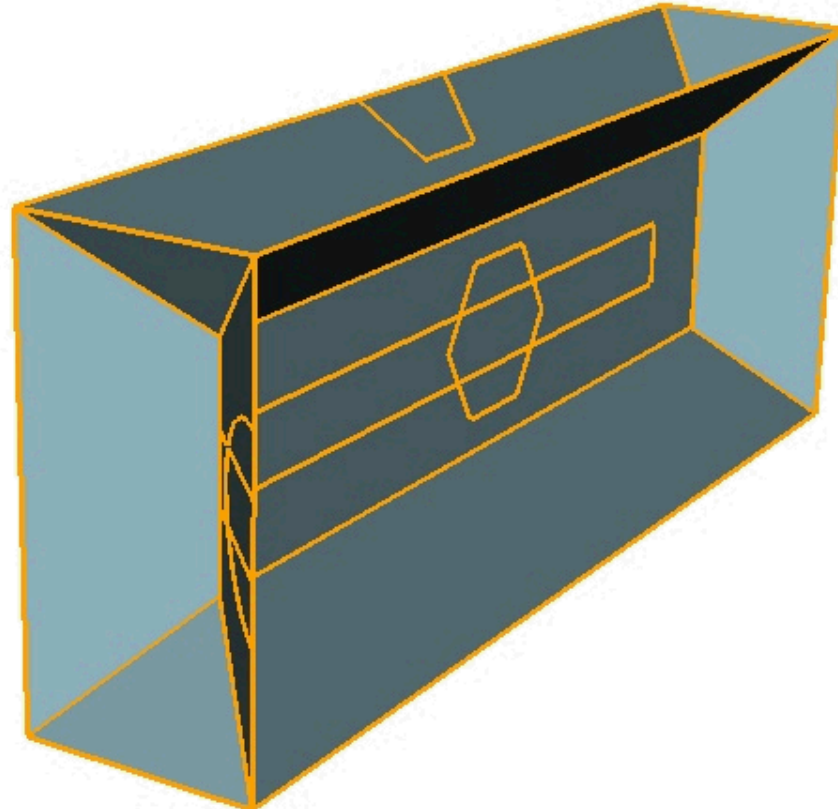


# Imprints on Top, Bottom, and Sides

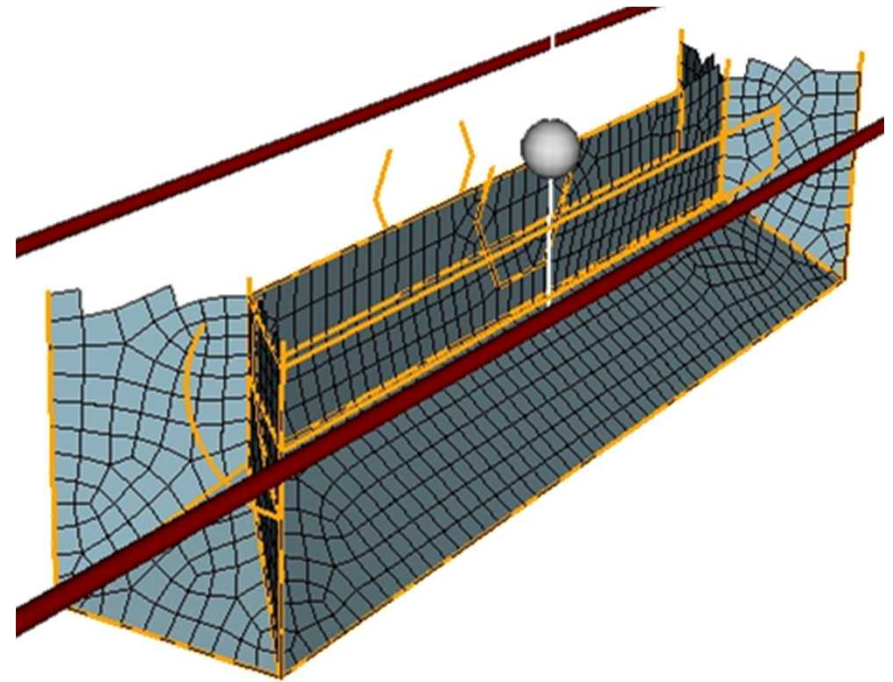
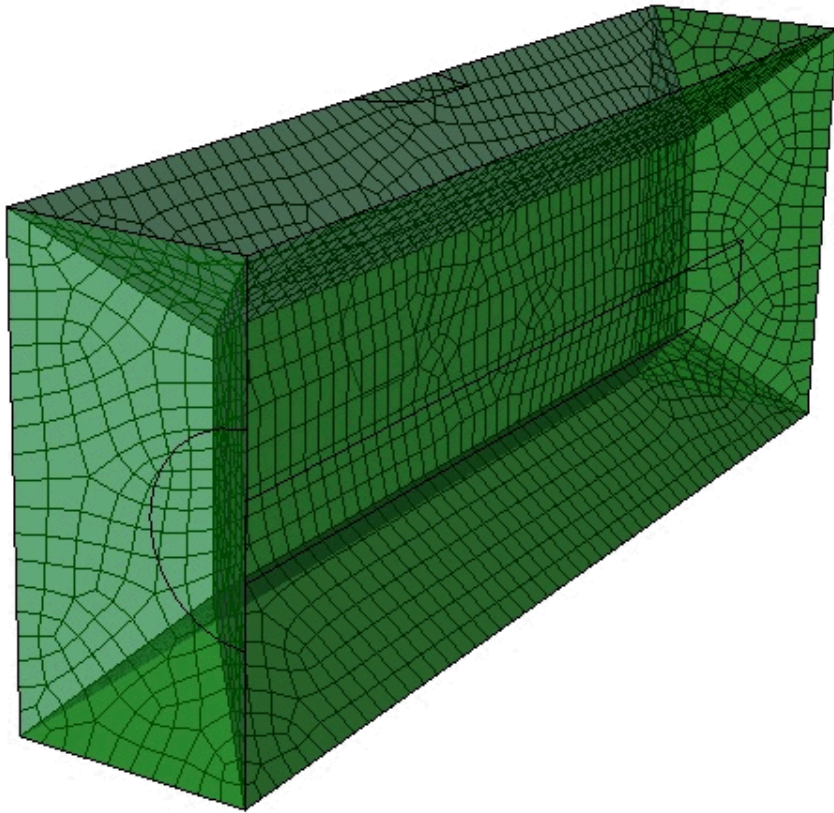




# All Imprints Resolved at MA using Projection Operator

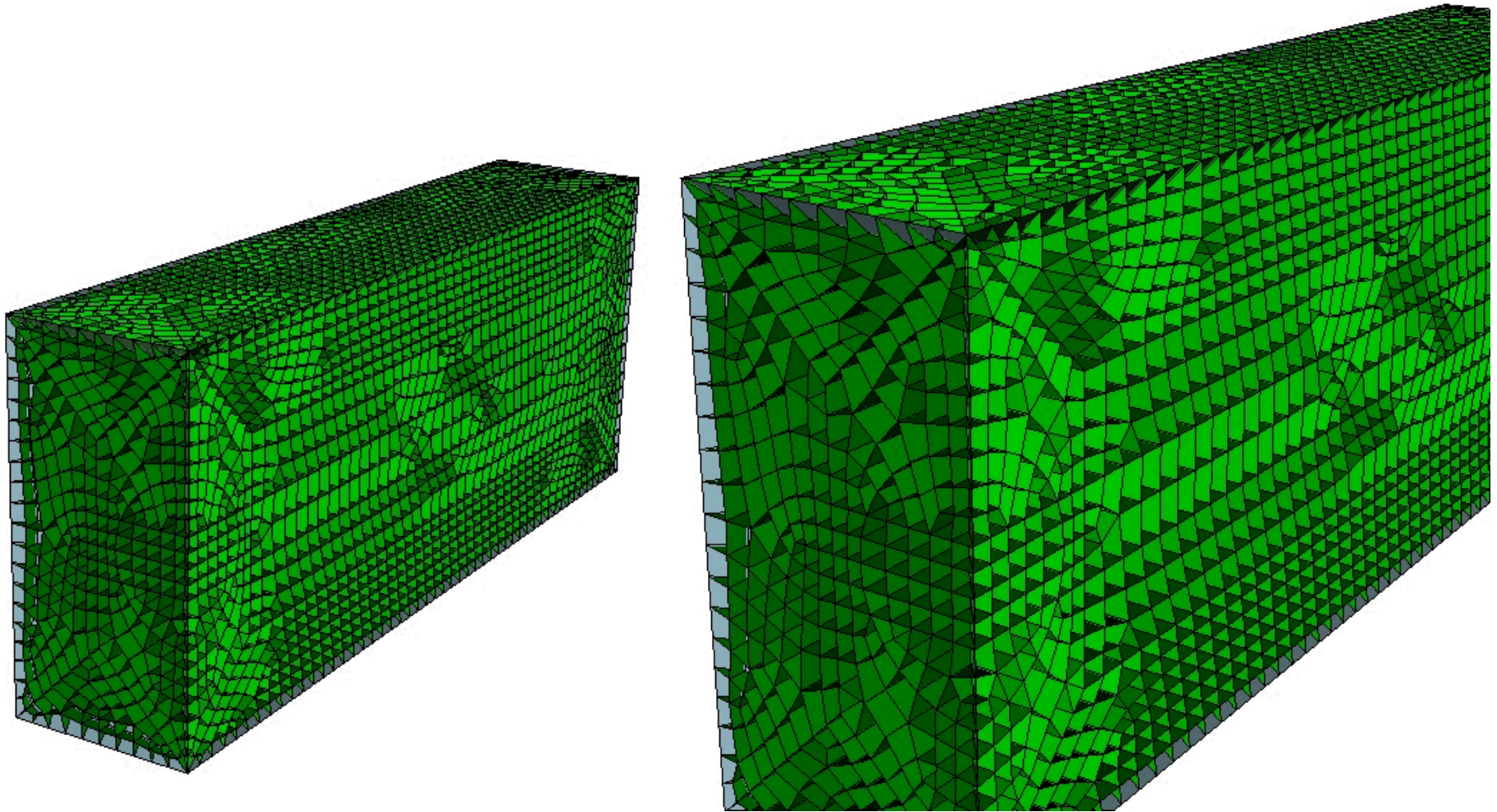


# Quad Mesh on Imprinted Medial

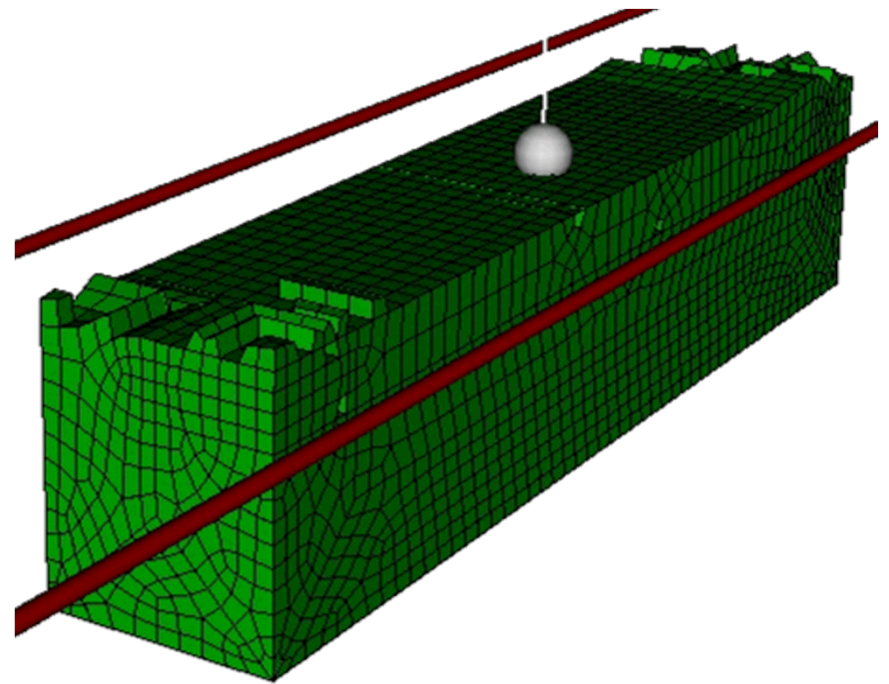
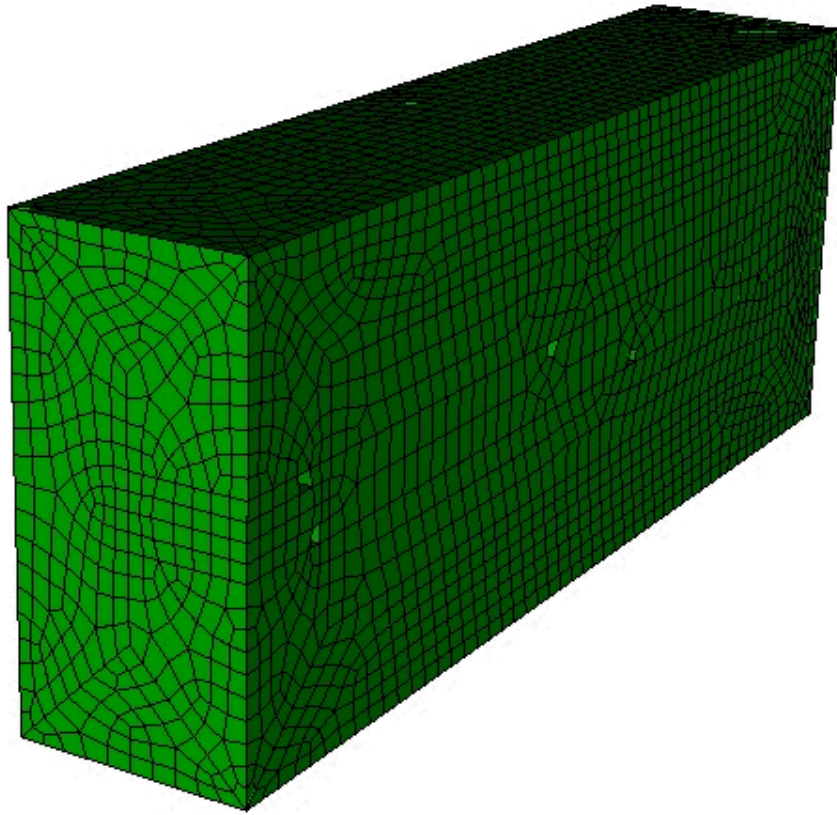




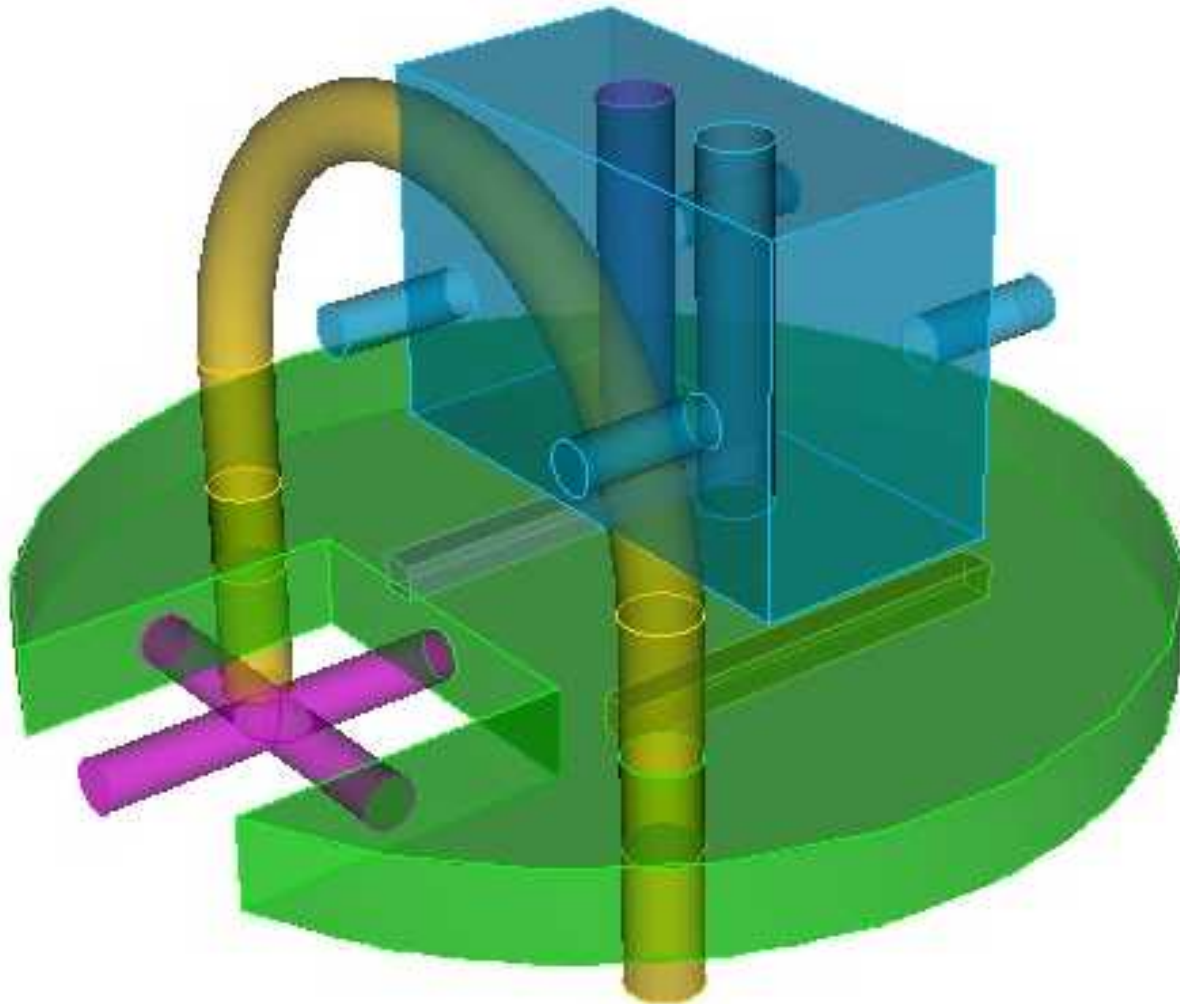
# Tracks in 3D



# Hex-dominant Mesh

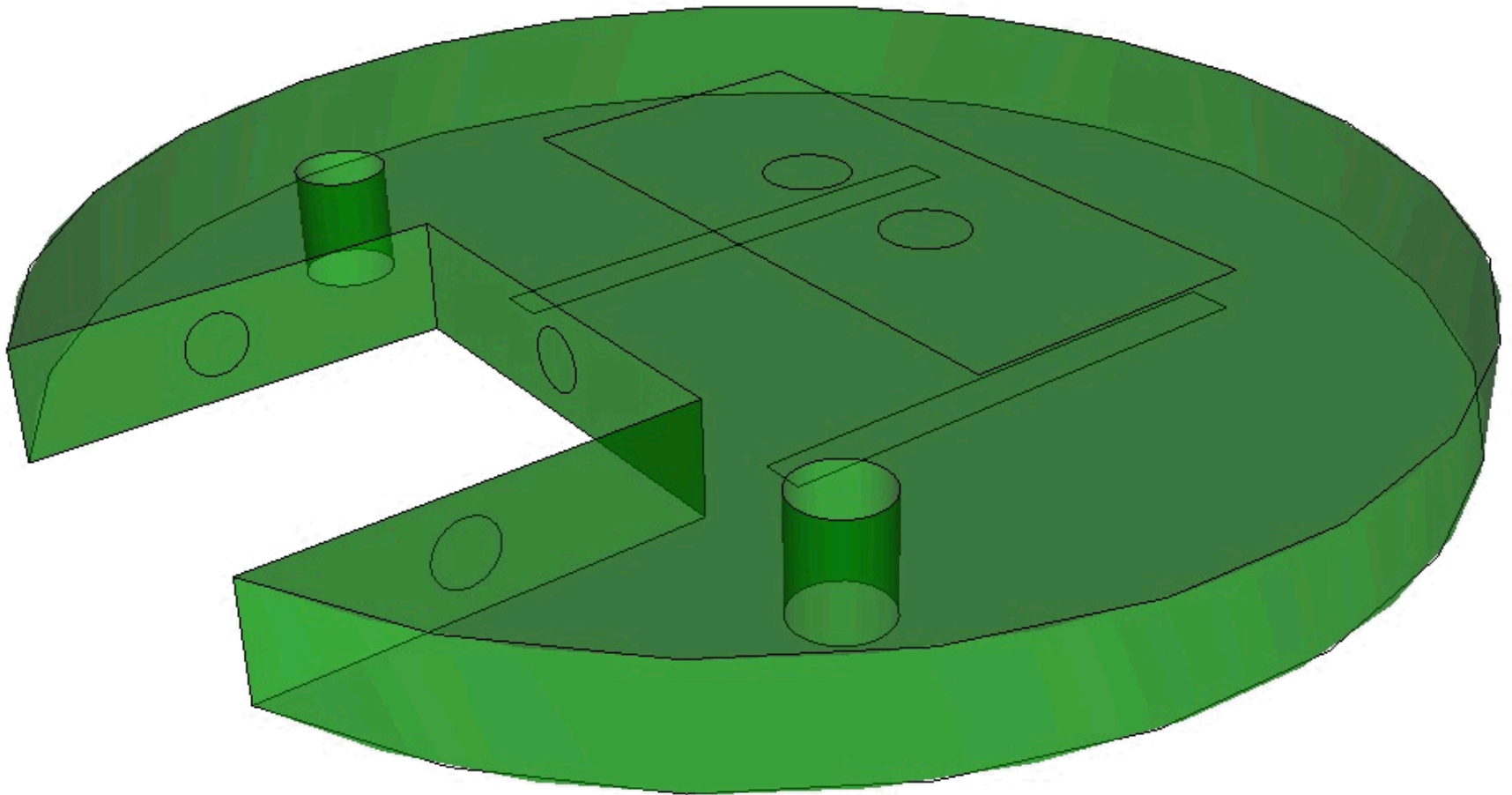


# Extension to Assembly Model

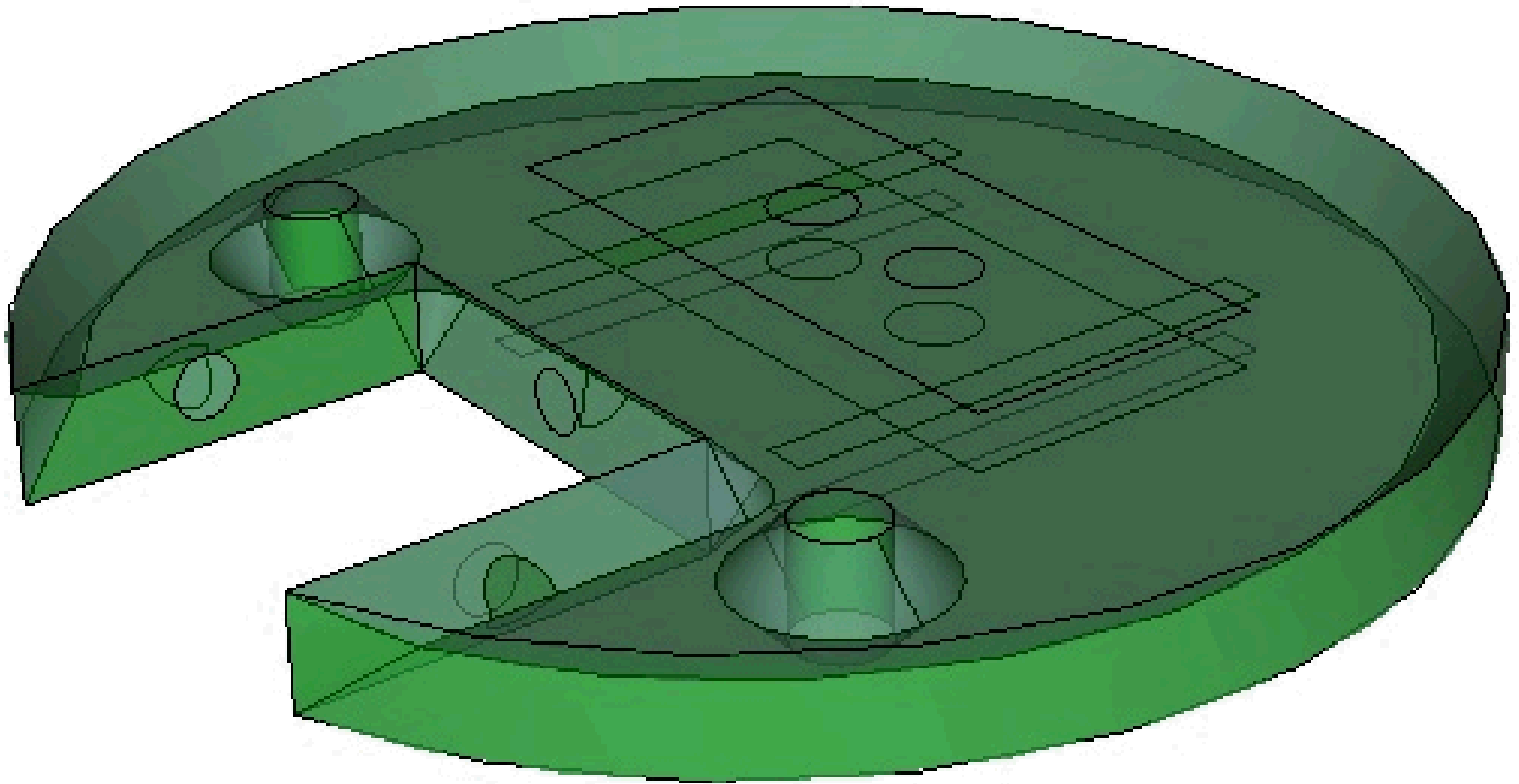




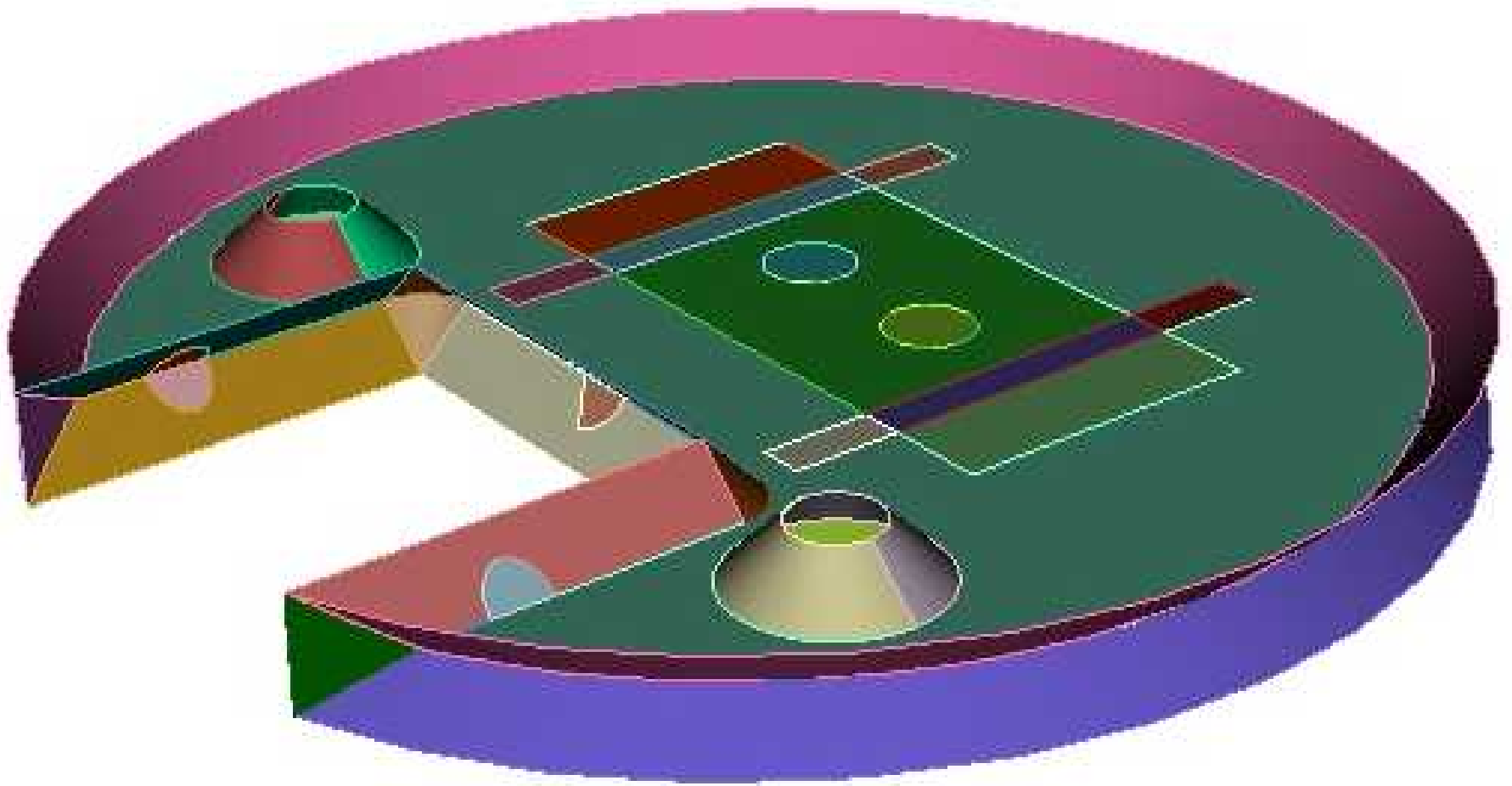
# Imprints on Top, Bottom, and Sides



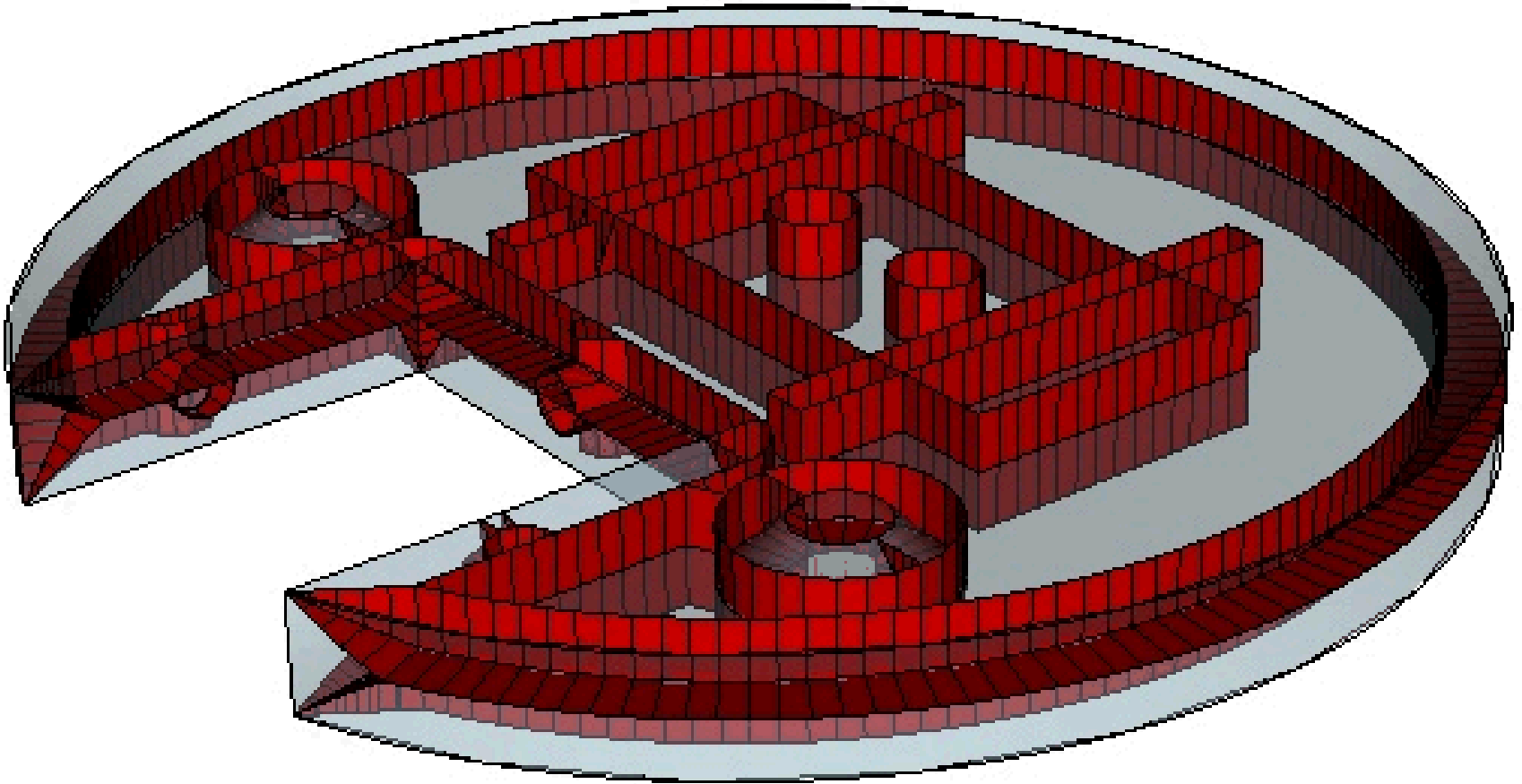
# Resolve Boundary Imprints on Medial using Projection Operator



# Medial Resolves Imprints from All Directions

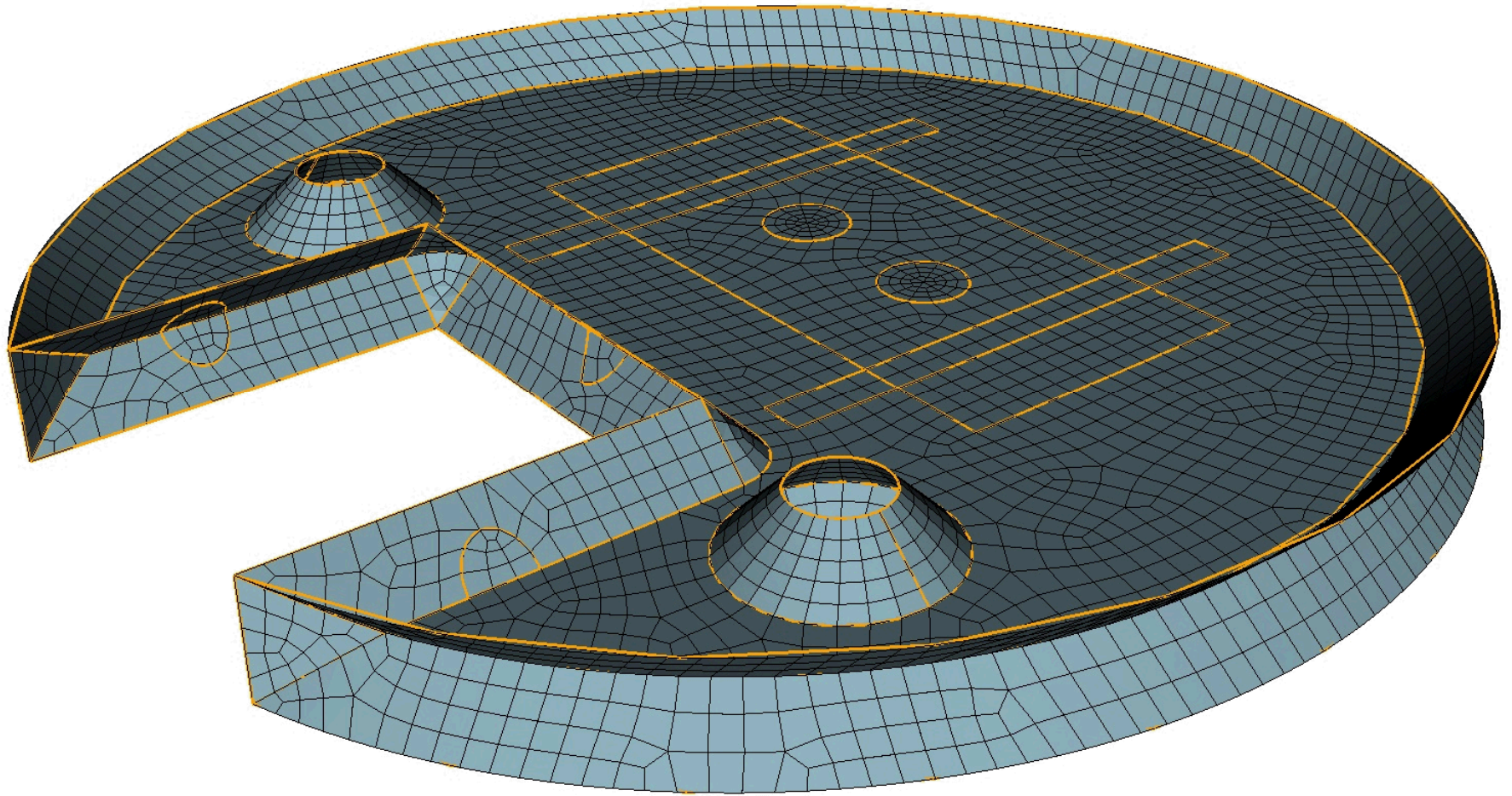


# Automatic Geometry Decomposition using Imprints and Medial Junctions



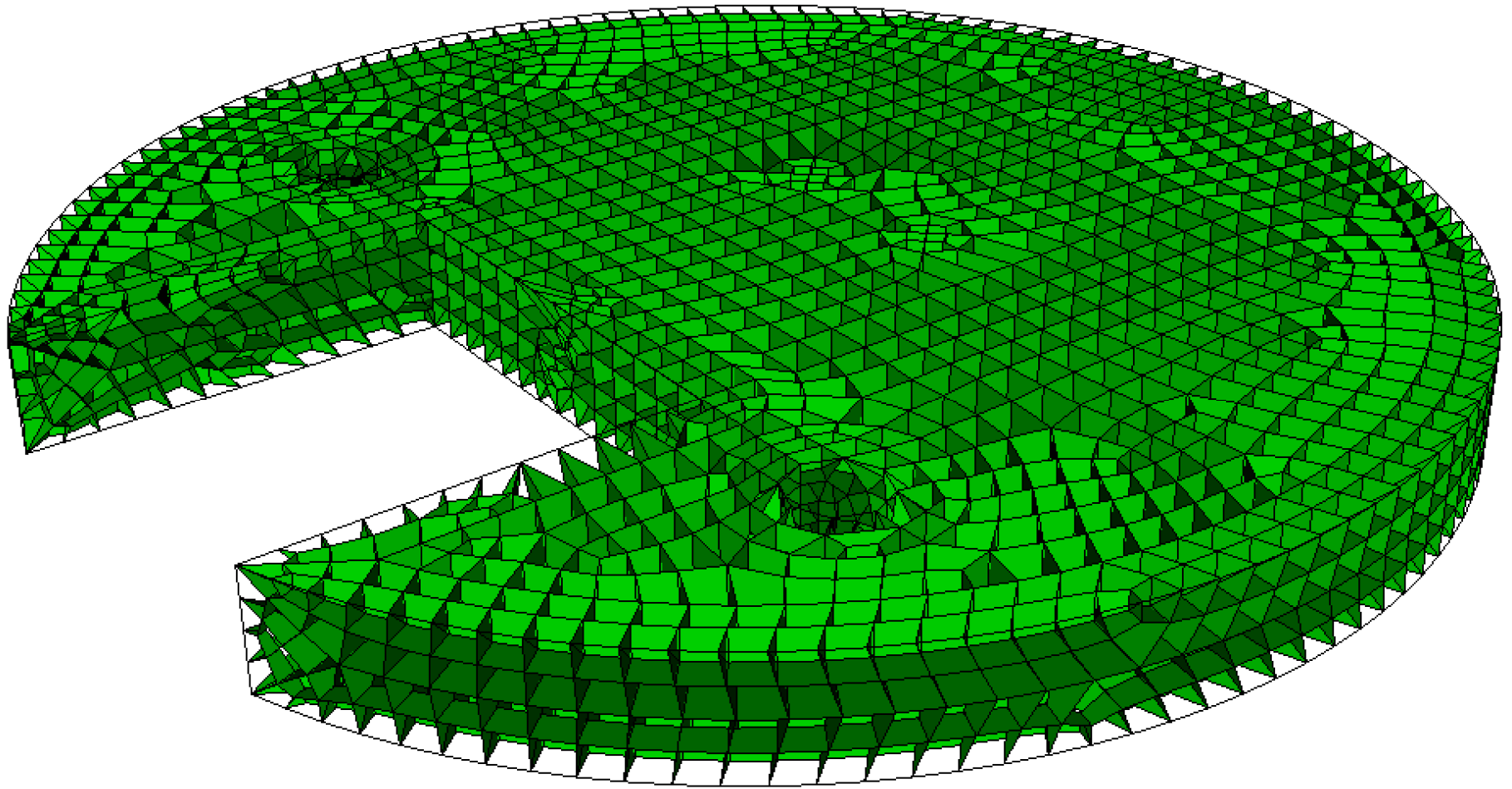
Corridors

# Quad Mesh on Imprinted Medial

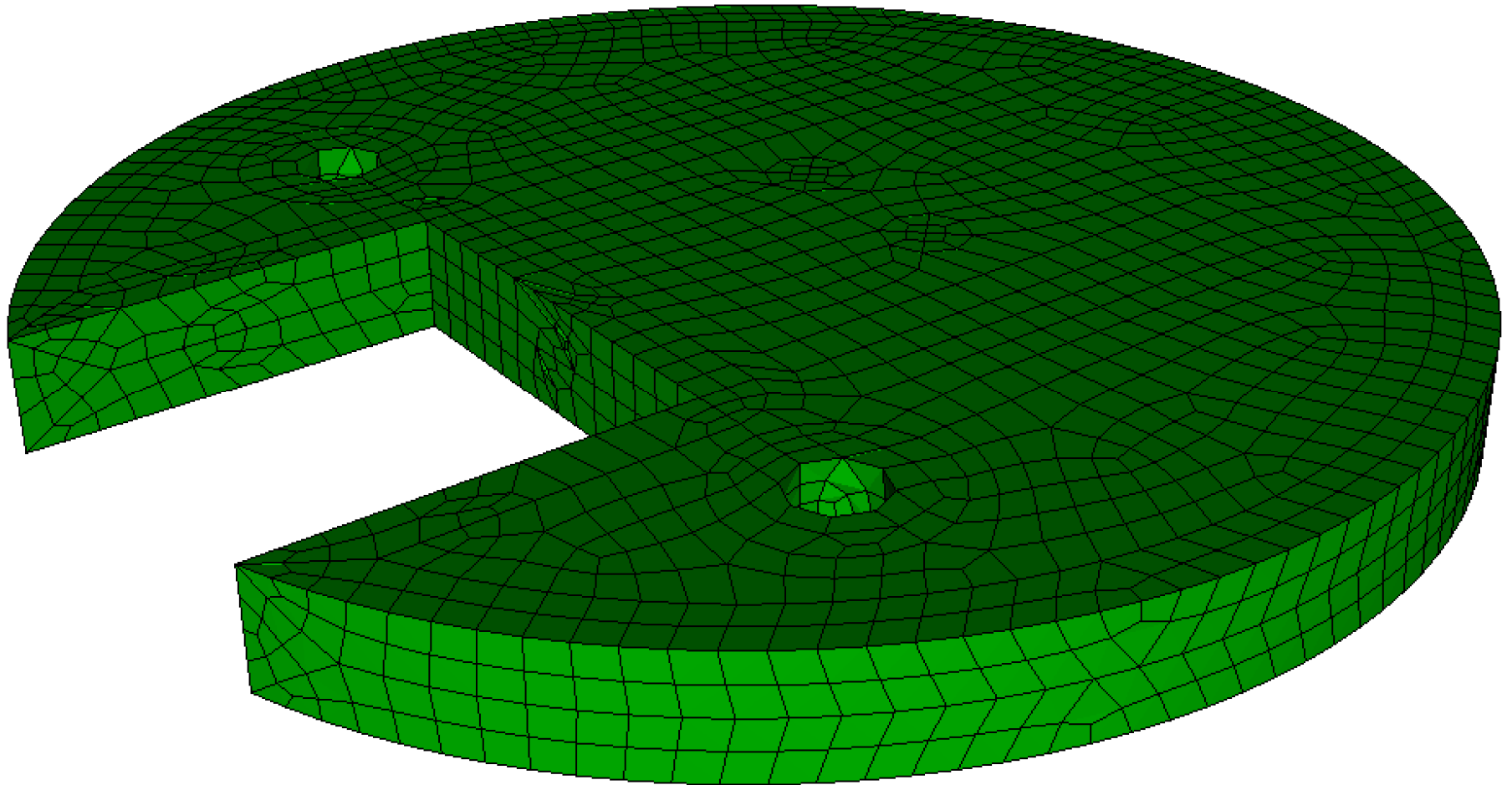




# Tracks in 3D

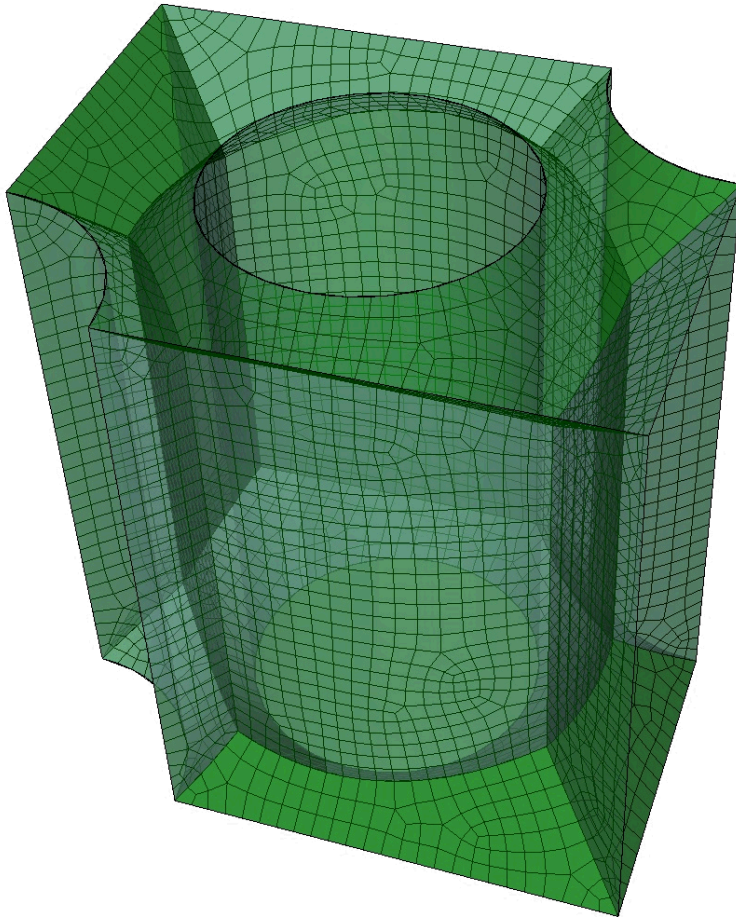


# Hex-Dominant Mesh

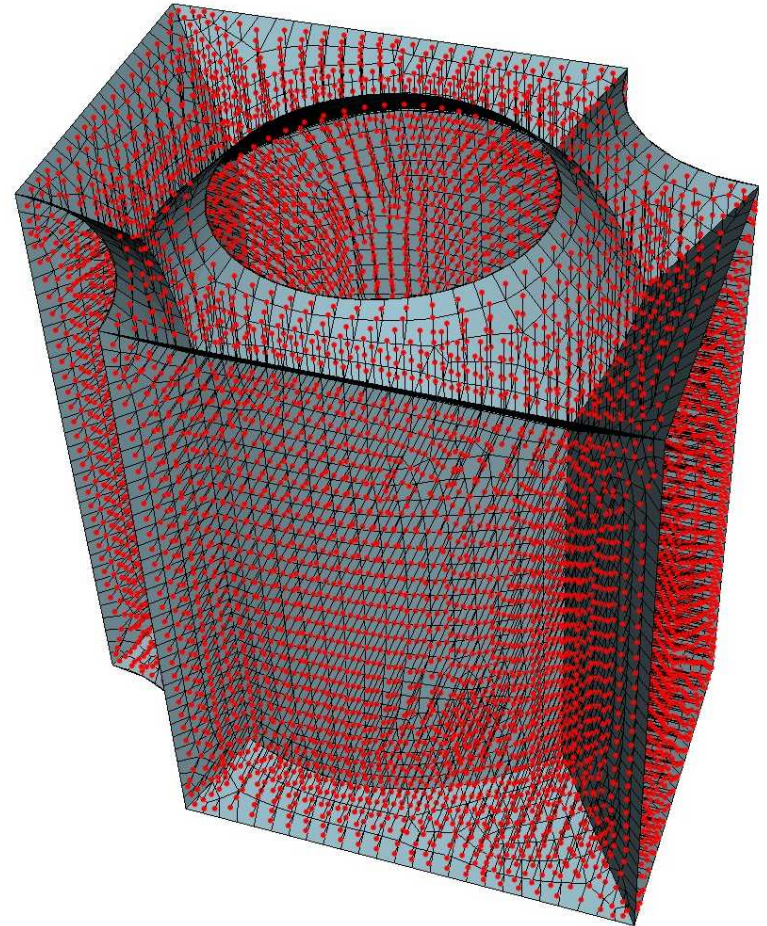


# Improve Quality at Common 3-manifold MA Curve

# All-Quad Mesh on Medial is not Ideal!



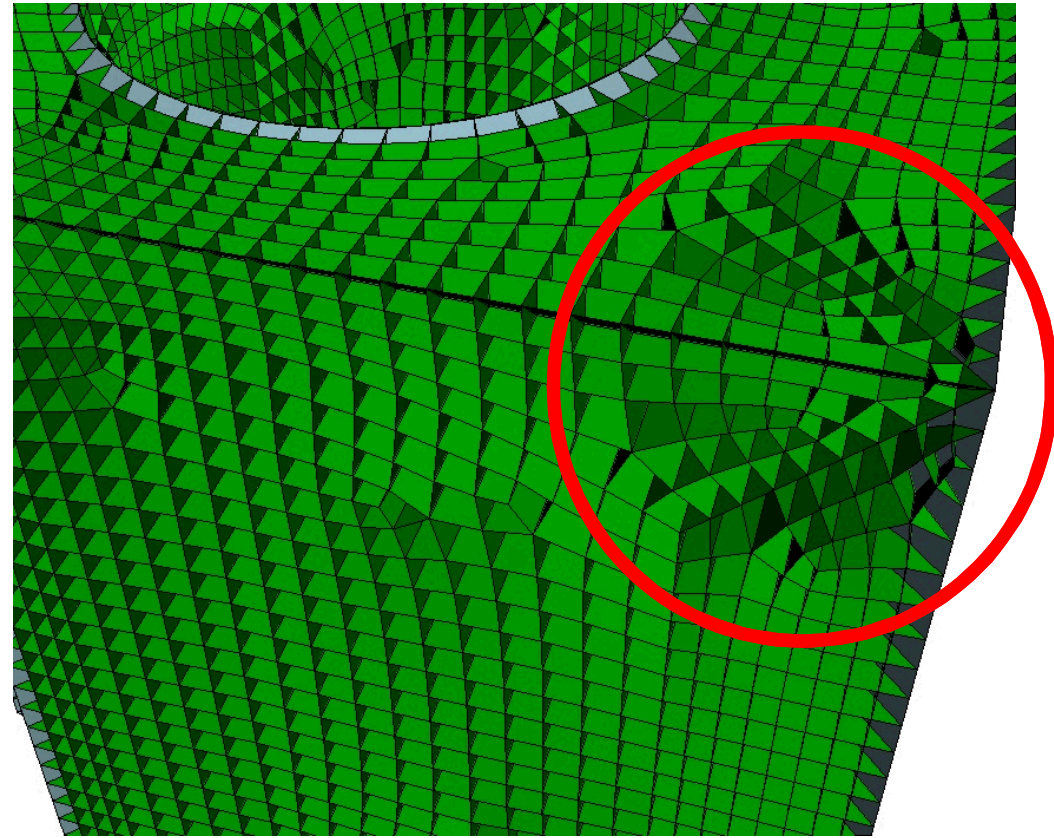
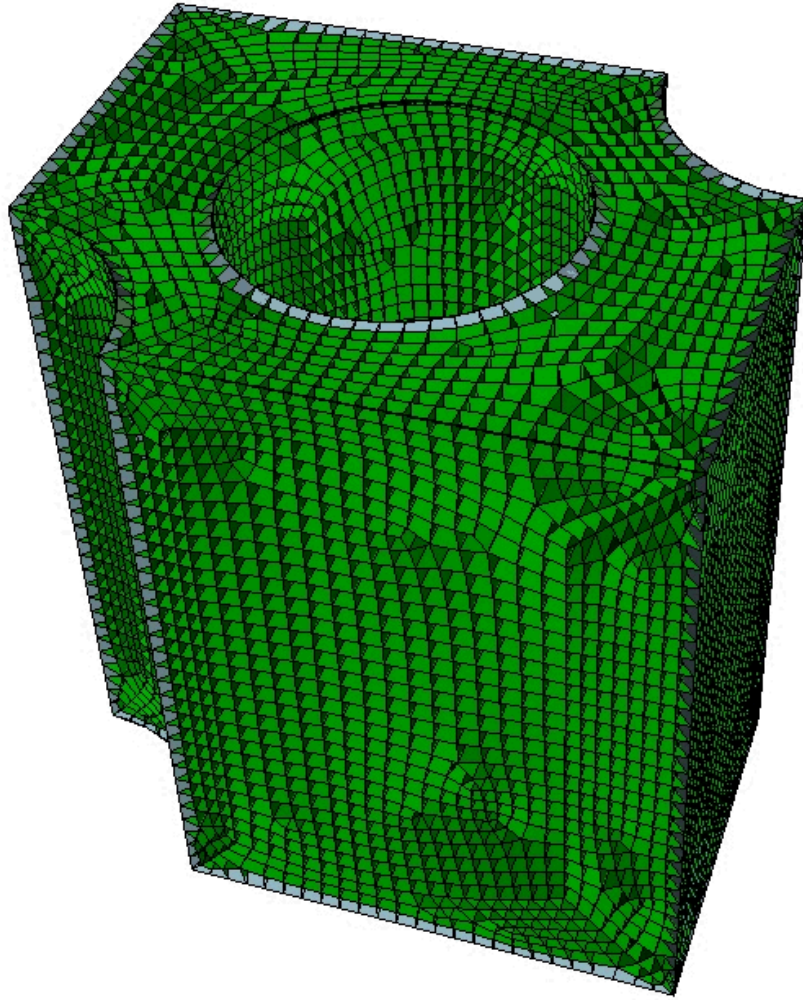
All-Quad Mesh on  
Medial Surface



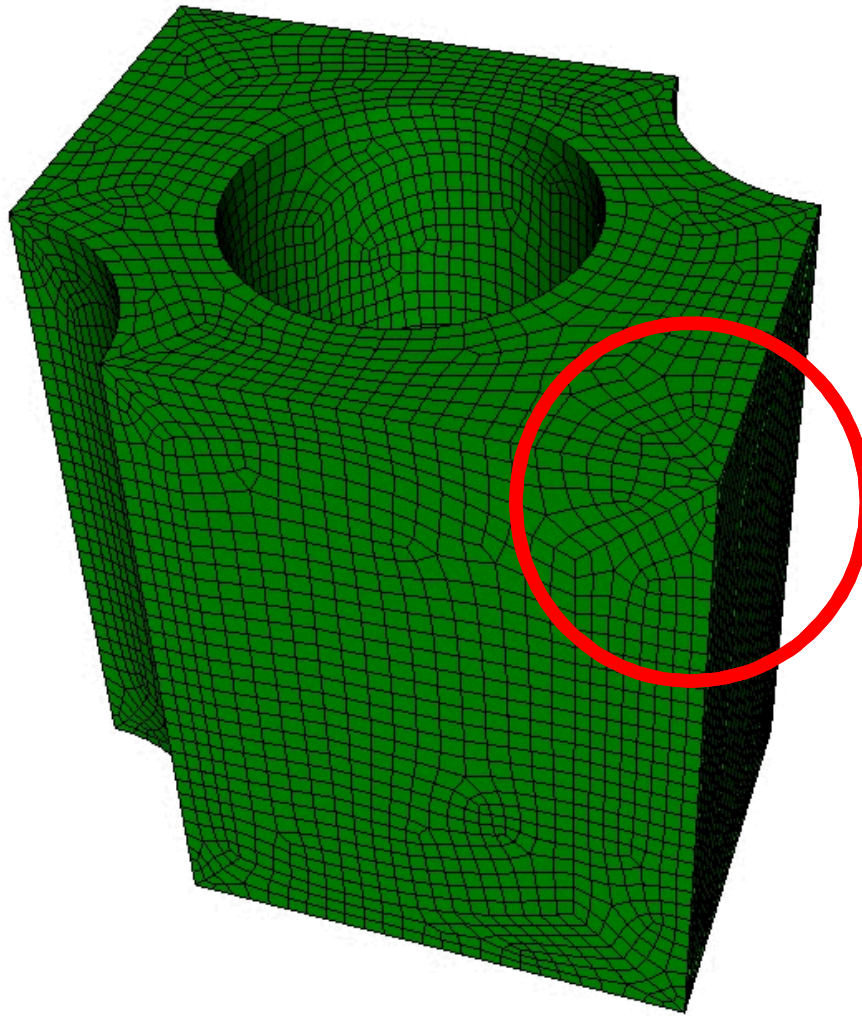
Rails Generated using  
Projection Operator



# Complex Tracks at a Convex Vertex

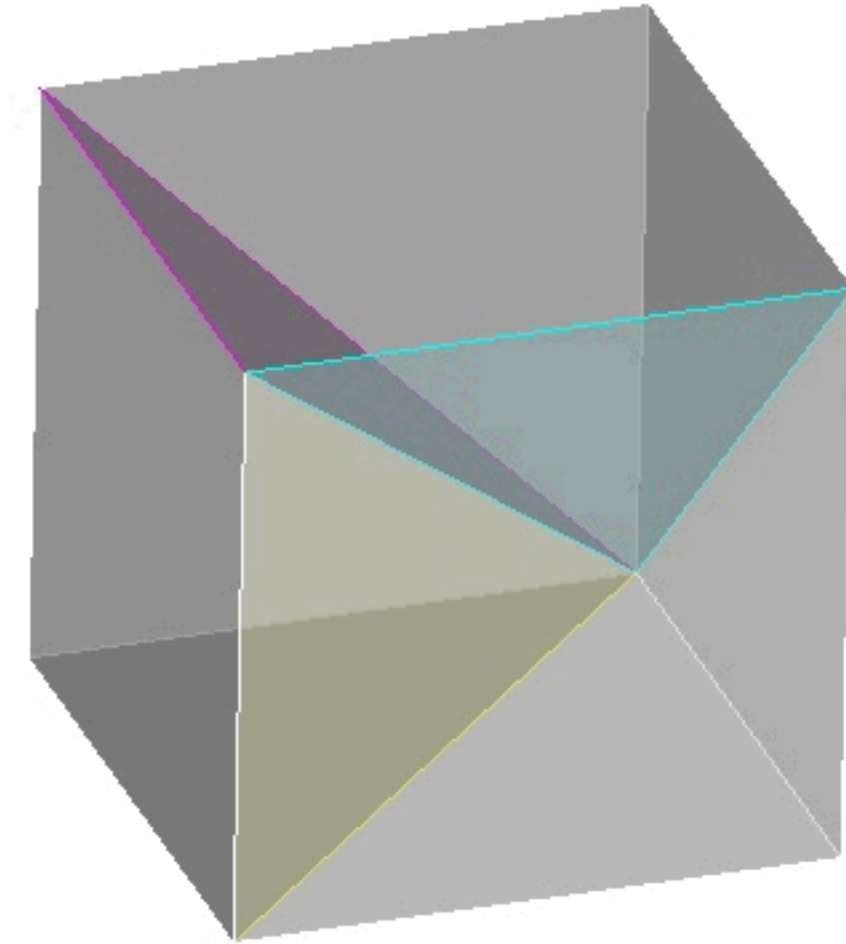


# Hex-dominant Mesh at Convex Vertex



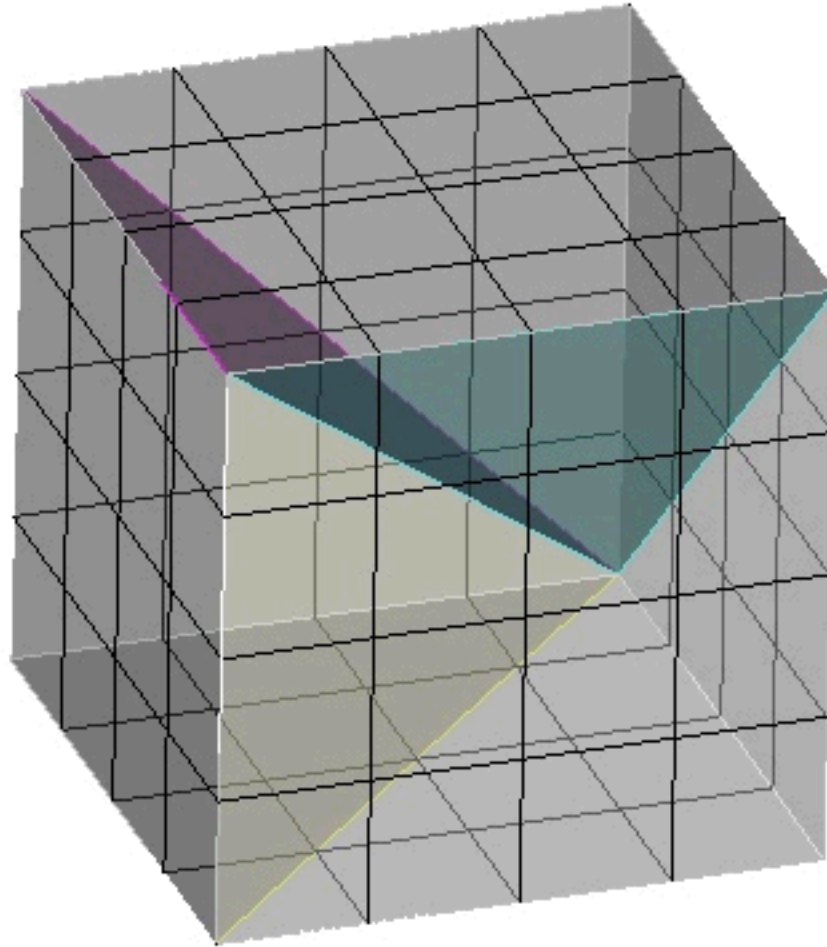
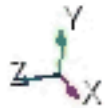
# How to Improve Hex Quality at Convex Vertex?

# Typical 3-manifold Medial at a Convex Vertex

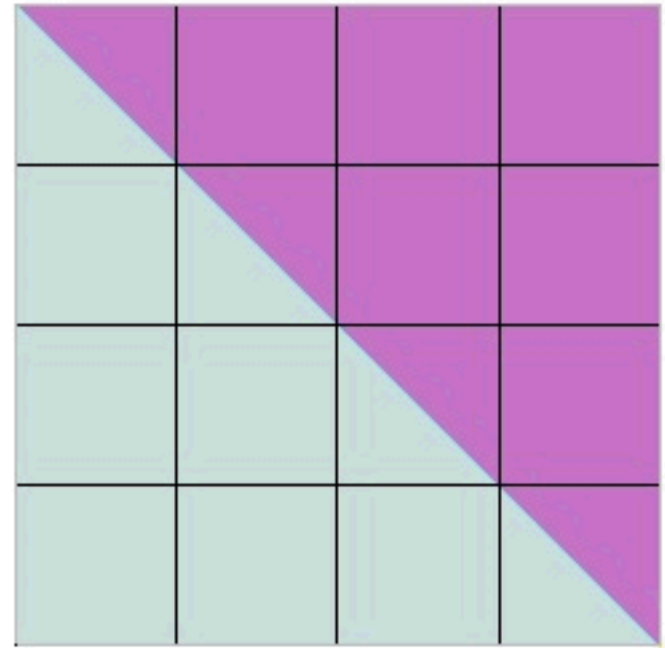
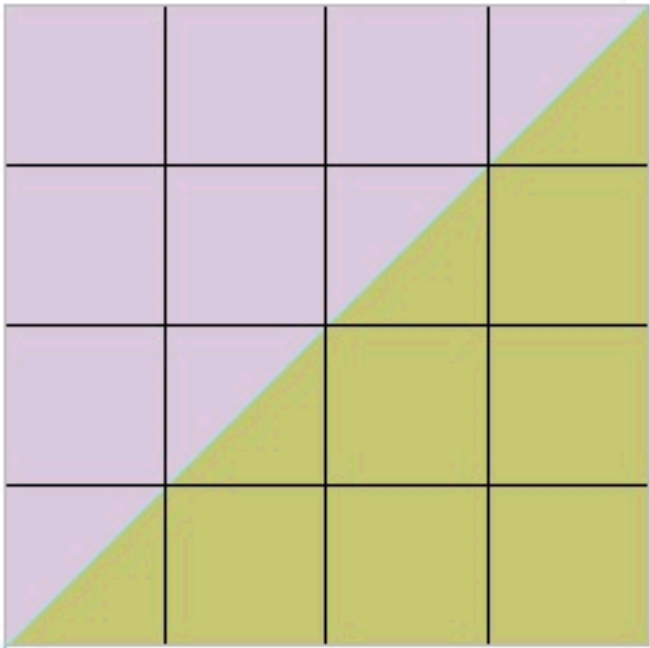




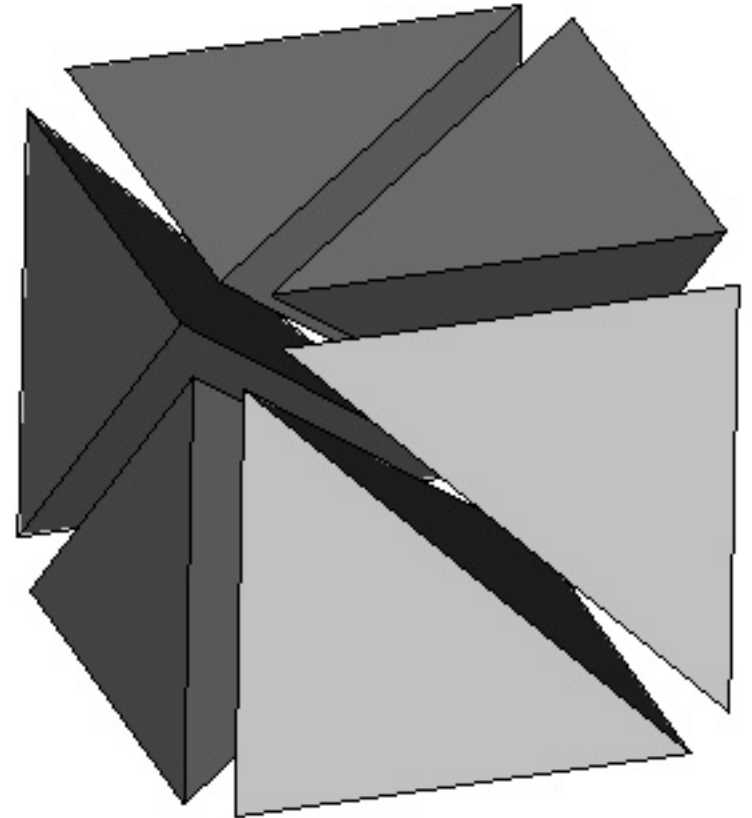
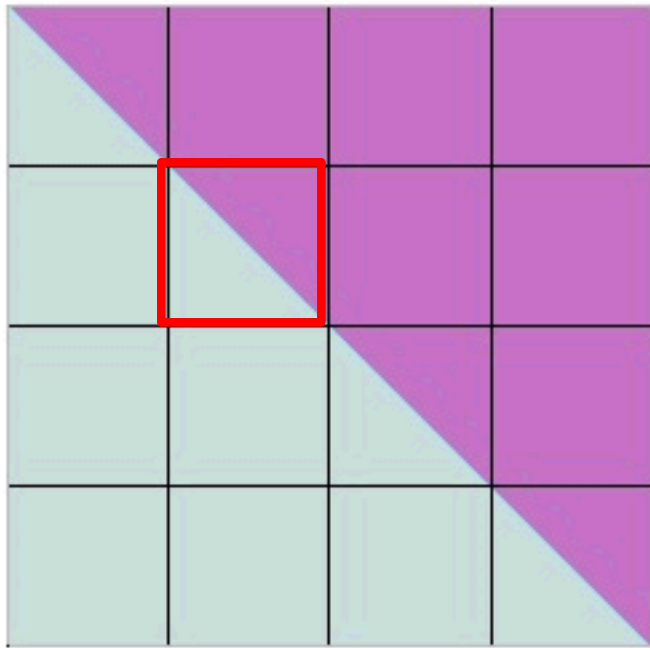
# Ideal Hex Mesh at a Convex Vertex



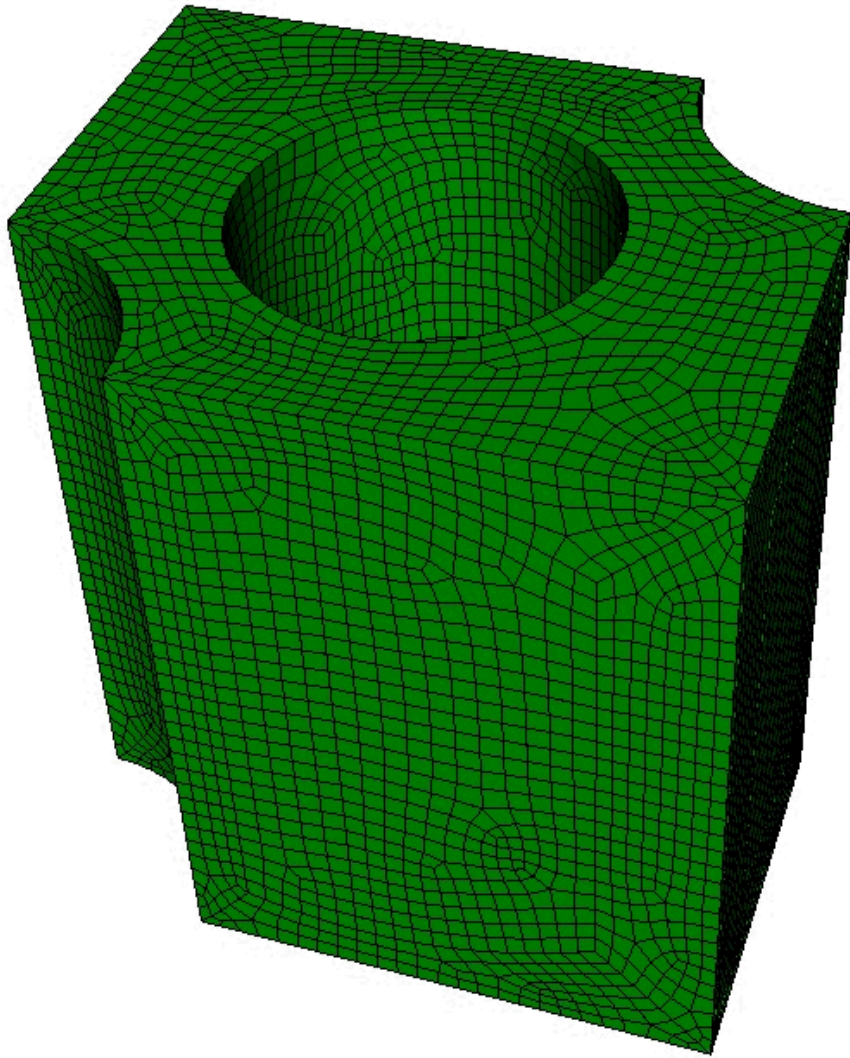
# Ideal Mesh Should Contain Tri Along 3-manifold Medial Curve!!!



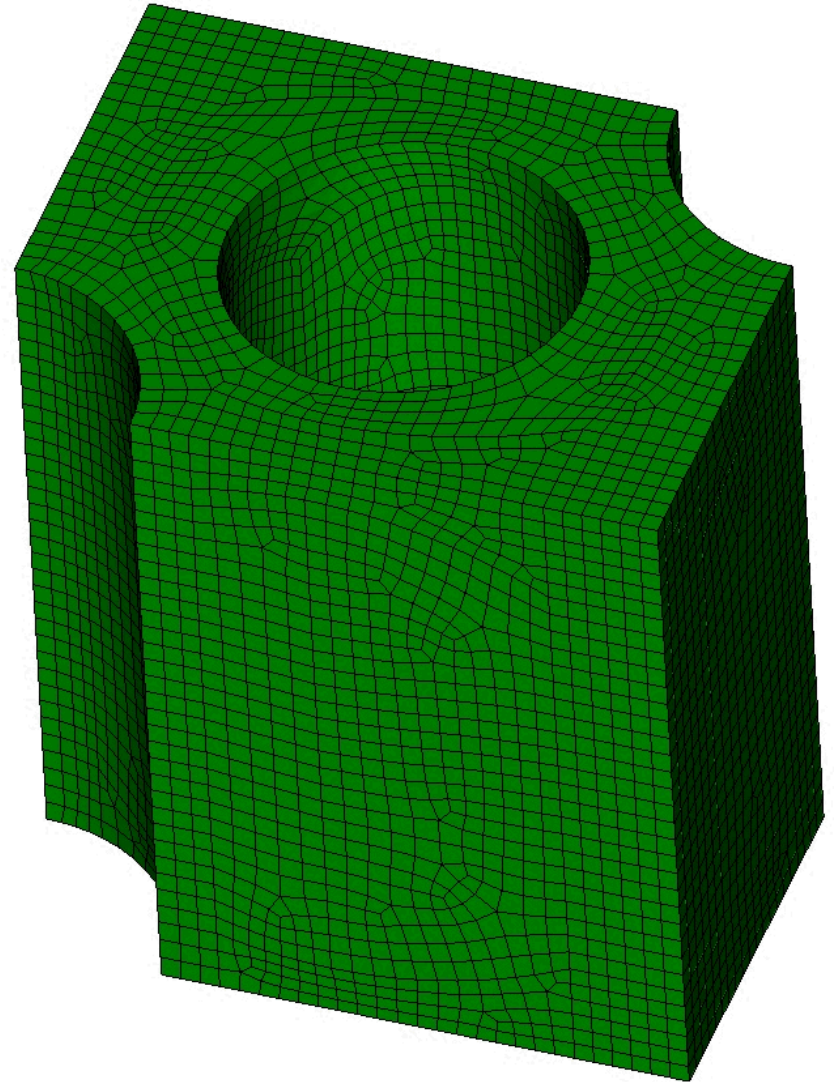
# Merge Six Tets to form a Hex along 3-Manifold Medial Edge!!!



# Improved Hex-dominant Mesh



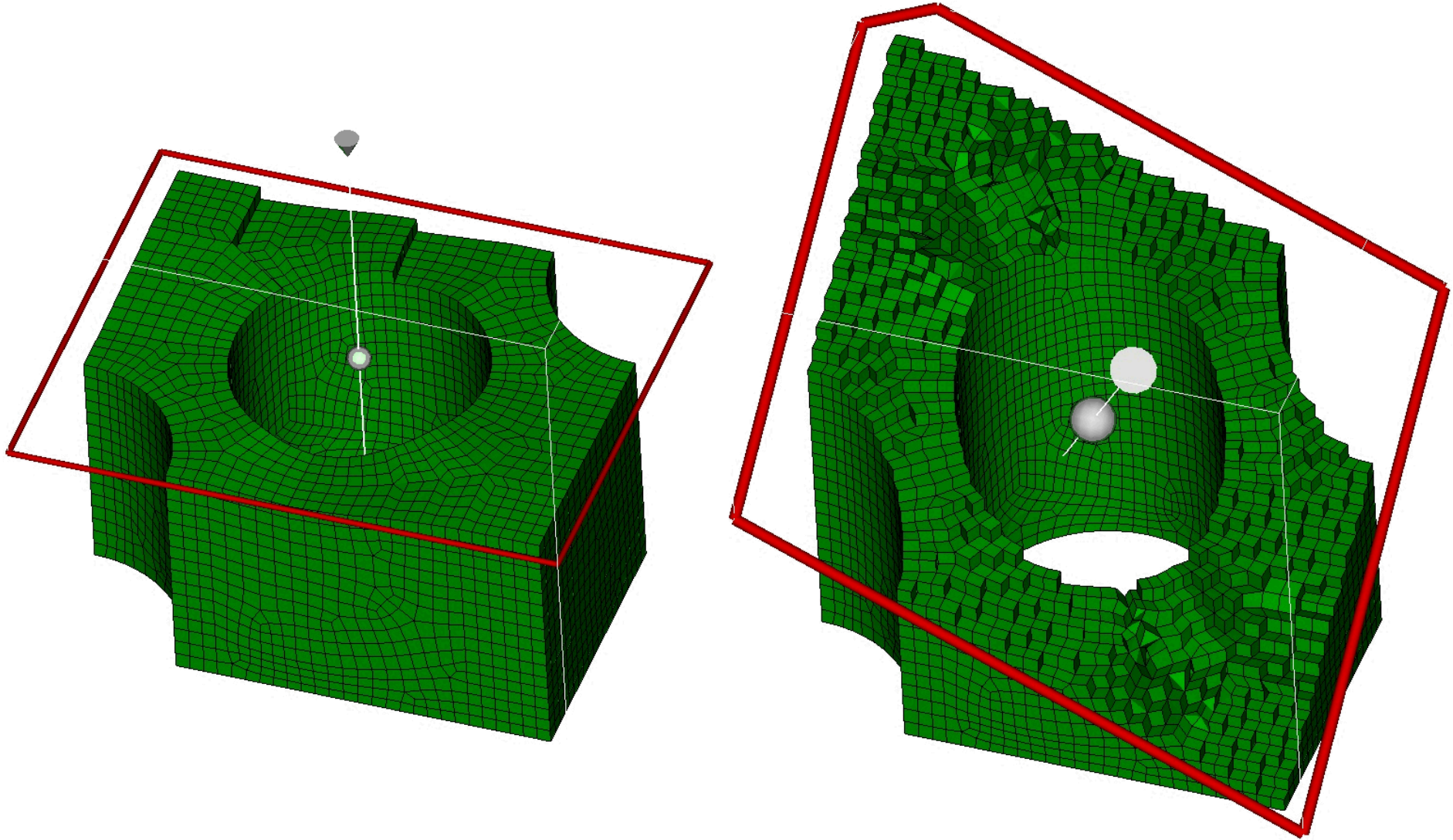
All-Quad Mesh on Medial



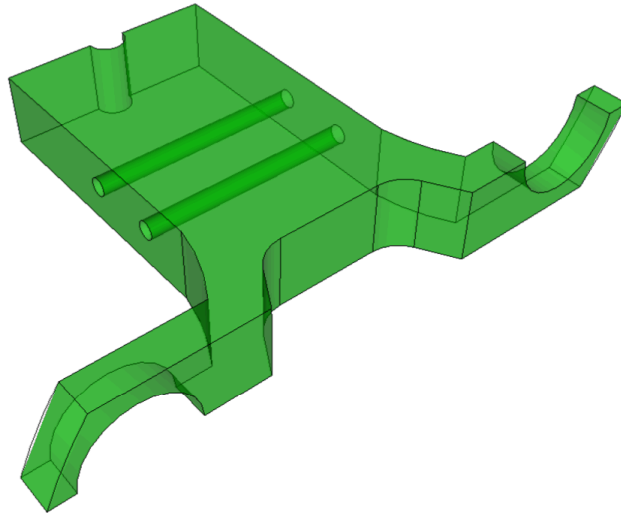
Tri at 3-Manifold Medial Curve



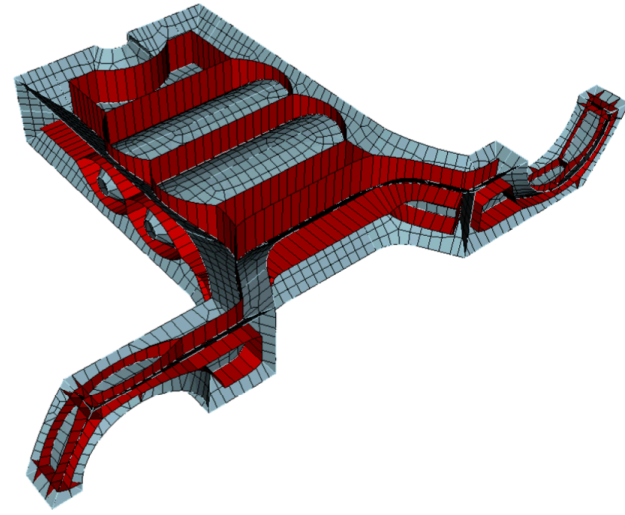
# Sectional View



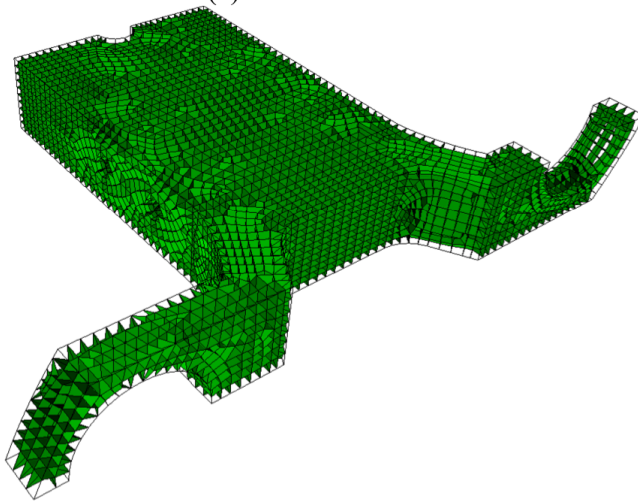
# Demo



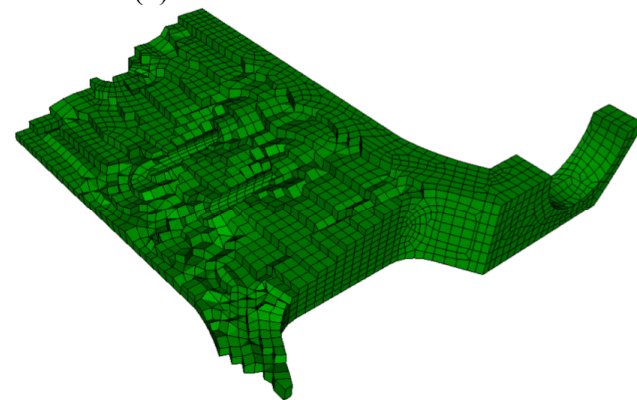
(a) Solid with holes



(b) Mesh on MA inside corridors



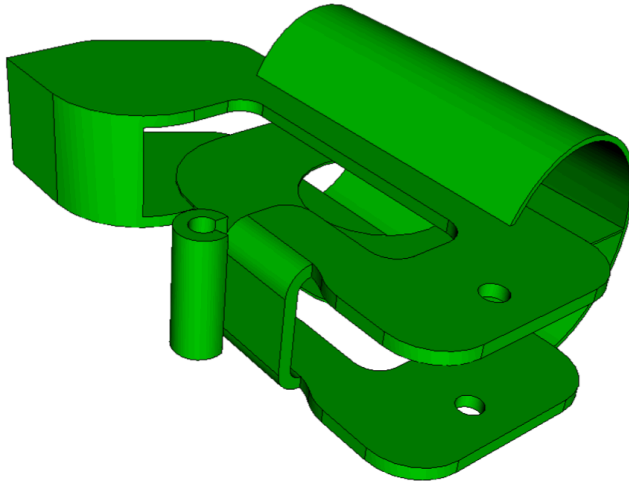
(c) Tracks in 3D



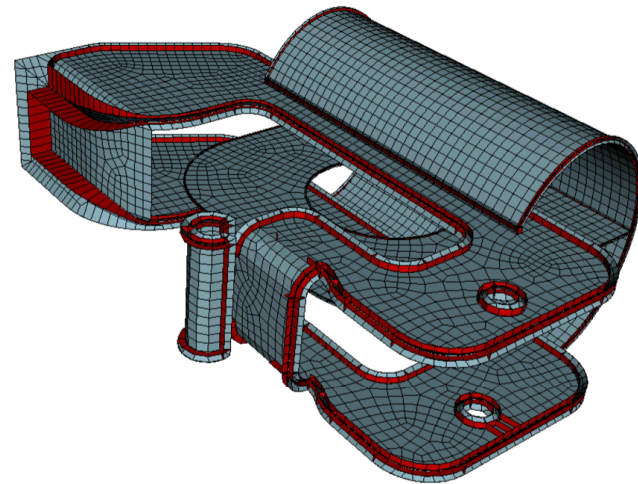
(d) Mesh cross section



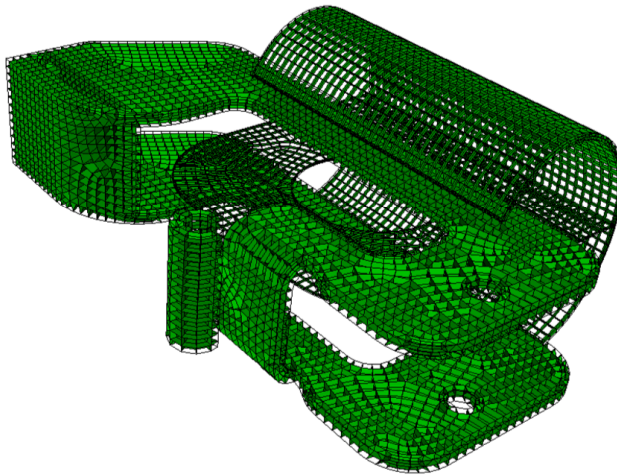
# Demo



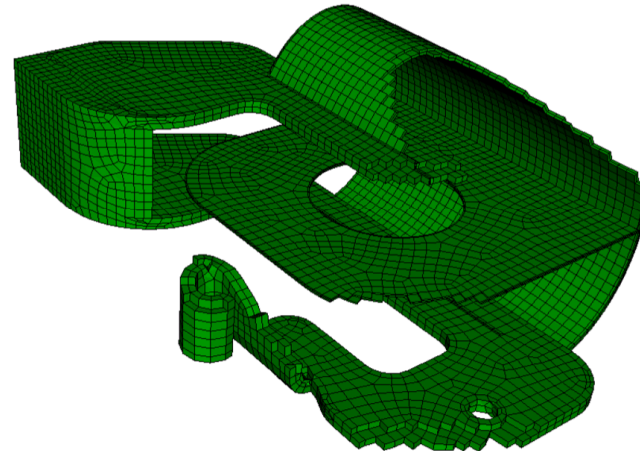
(a) Thin-wall solid



(b) Mesh on MA inside corridors



(c) Tracks in 3D



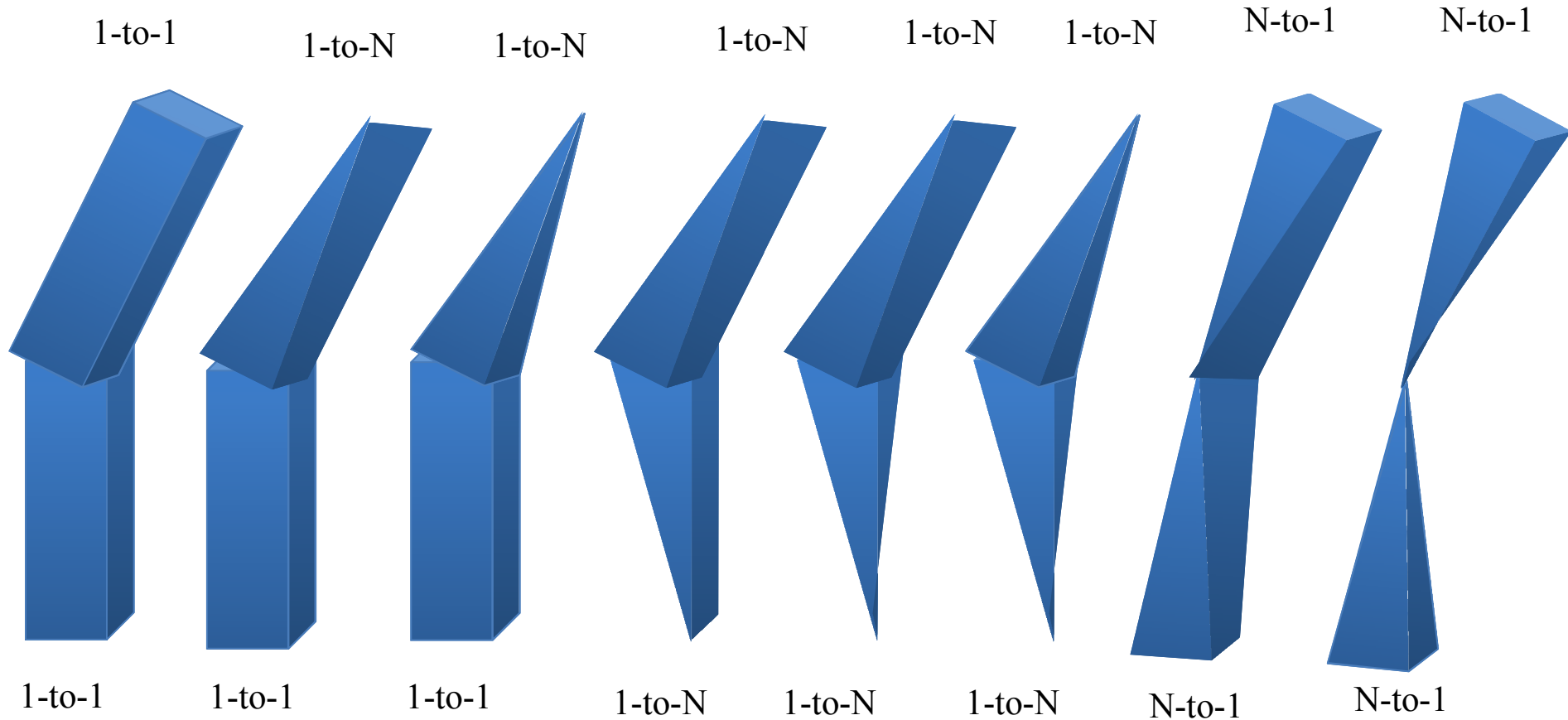
(d) Mesh cross section

# Extension to All-Hex Meshing

***Claim 1: A general solid can be decomposed into a set of connected quad cross-section 3D tracks***

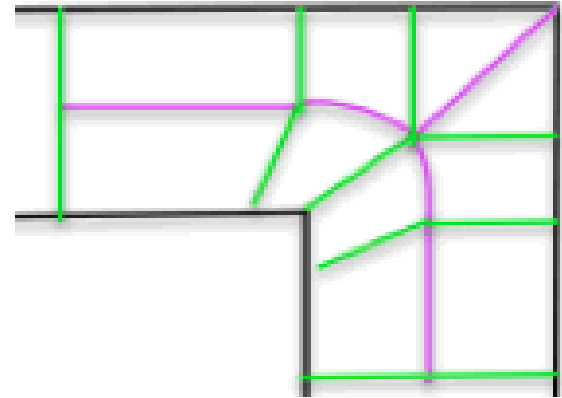
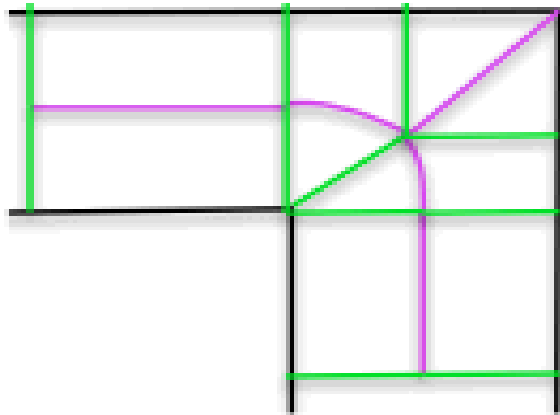
***Claim 2: A quad cross-section 3D track can be meshed with all-hex elements***

# Claim 1: A general solid can be decomposed into a set of connected quad cross-section 3D tracks

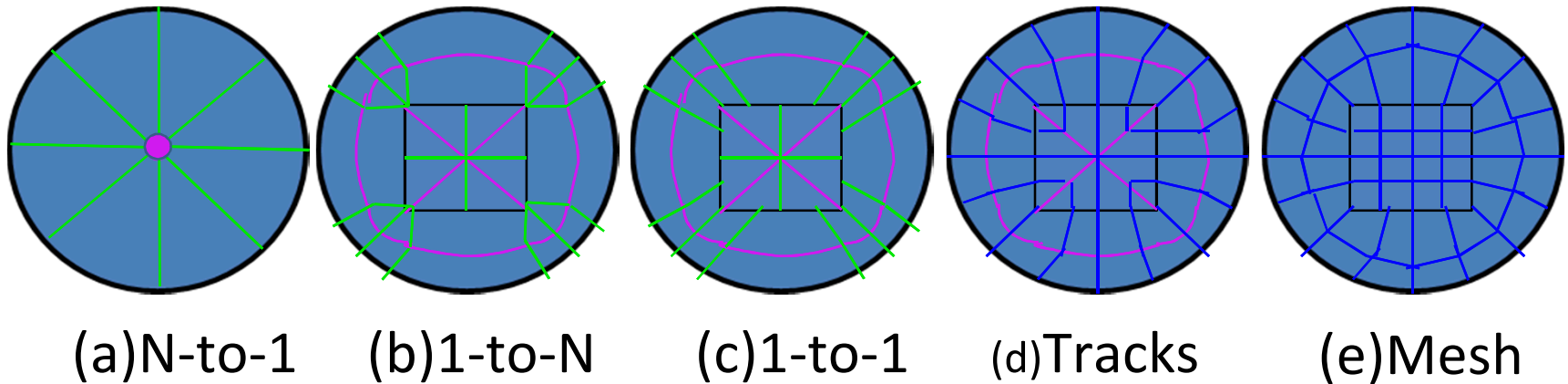


Types of Tracks in 3D

# 1-to-N Track Transformed to Quad Cross-Section 1-to-1 Track



# N-to-1 Track Transformed into Quad Cross-Section 1-to-1 Track



# Claim 2: A quad cross-section 3D track can be meshed with all-hex elements

In order to obtain an all-hex mesh inside a quad cross-section track,

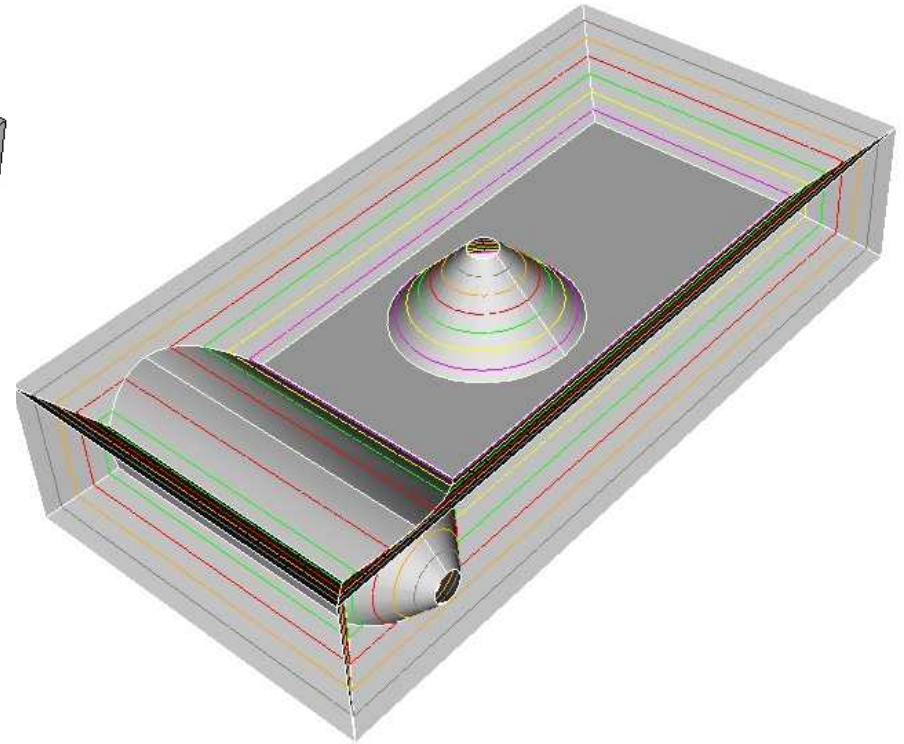
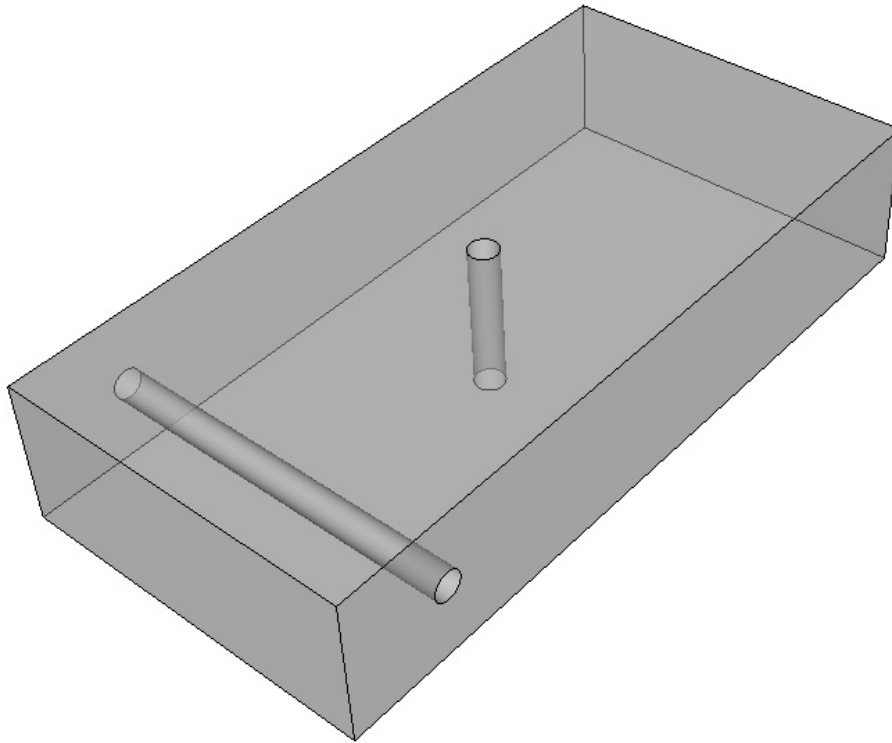
Case 1: all four rails must have the same intervals or Case 2: two rails must have  $2N$  intervals and the other two rails must have  $2(N+1)$  intervals.

With Case 1, equal number of hex elements are generated on both sides of MA and

with Case 2, two wedges at the MA are merged to form a hex.

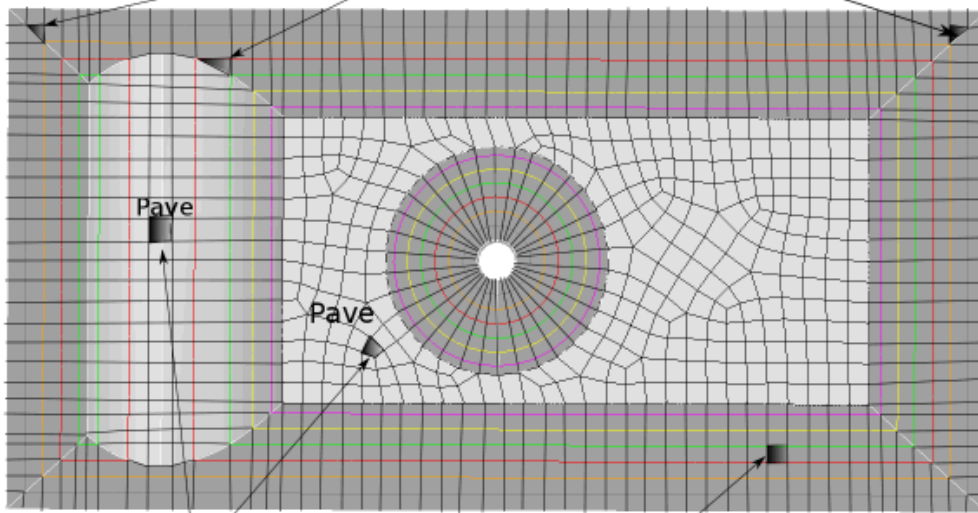


# Isocontours of Medial Radius Function Controls Mesh Intervals on Rails



# All-Hex Mesh Topology Visualization on Isocontours of MA

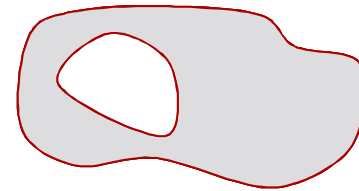
Tri along 3-manifold MA edges



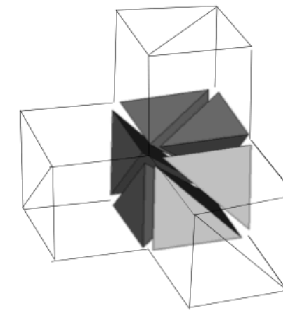
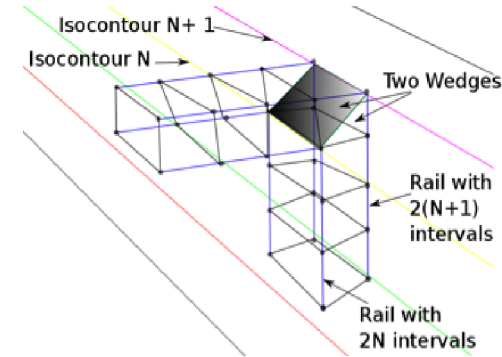
Case 1: quad in region bounded by one isocontour

Case 2: quad incident on N & N+1 isocontours

(c) Meshing MA using isocontours guarantees correct intervals on rails to satisfy Case 1 or Case 2 in each track



(a) Case 1: equal num of hexes exist on both sides of MA (b) Case 2: two wedges form a hex at a quad lying on N & N+1 isocontours



(d) Six tets form a hex at 3-manifold MA

# Acknowledgements

*Thanks to Henry Bucklow, Robin Fairey,  
and Mark Gammon at ITI TranscenData  
Europe Ltd for providing the CADfix medial  
object library.*

Thank You

Questions?

[wrquadr@sandia.gov](mailto:wrquadr@sandia.gov)