

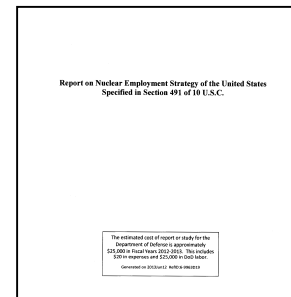
Hedge Math

Theoretical Limits on Minimum Stockpile Size across Nuclear Hedging Strategies

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*Exceptional service
in the national interest*



Deployed



Warhead
Type 1



Warhead
Type 2



Warhead
Type 3

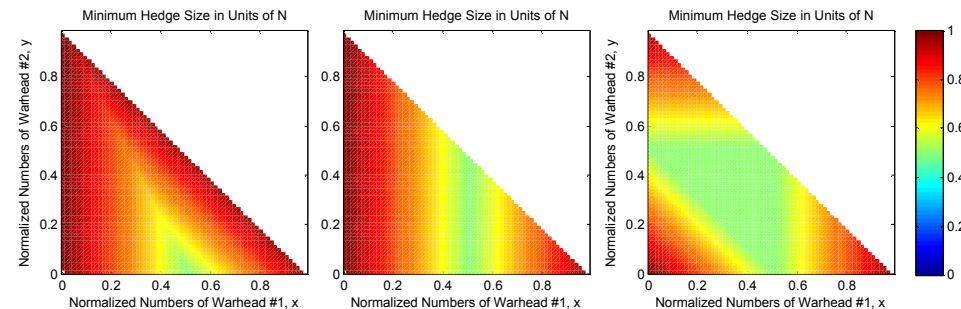
Total Deployed:

$$N = x + y + z$$

x

y

z



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What is the Stockpile Hedge?

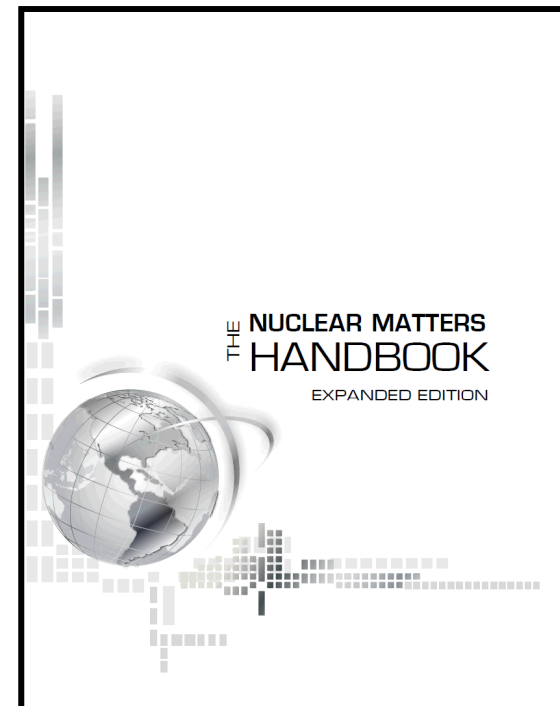


Stockpile-looking Hedges

Image Credit: Tricia Mitchell,
<http://triciaannemitchell.com/2011/09/21/a-september-day-at-versailles-%E2%80%93-paying-tribute-to-the-palace-of-the-sun-king/>

What is the Stockpile Hedge?

- Various risks have the potential to threaten the stability of the U.S. nuclear deterrent, including:
 - **Unforeseen catastrophic failure** of a class of delivery vehicles or warhead type/family
 - **Reversal of geopolitical situation** that would require more weapons available for use
- Without an active nuclear weapon production capability, the United States relies on **maintenance of a stockpile “hedge” to reduce risk** to acceptable levels.



Office of the Assistant Secretary of Defense for Nuclear, Chemical, and Biological Defense Programs, *The Nuclear Matters Handbook* (Washington, DC: Department of Defense, 2011), 37-38. Available at http://www.acq.osd.mil/ncbdp/nm/nm_book_5_11/docs/NMHB2011.pdf.

DoD Nuclear Hedge Guidance

June 2013



Report on Nuclear Employment Strategy of the United States Specified in Section 491 of 10 U.S.C.

- U.S. policy is to achieve a credible deterrent, with the lowest possible number of nuclear weapons, consistent with our current and future security requirements and those of our Allies and partners.
- The United States will maintain a sufficient number of non-deployed weapons to hedge against the technical failure of any single weapon type or delivery system at a time. Where possible, the United States will provide intra-leg hedge options – i.e., uploading another warhead type from within a leg of the Triad in the event that a particular warhead fails. In instances where the current stockpile will not allow intra-leg hedging, the United States will be prepared to hedge adequately using inter-leg hedging – uploading additional warheads on another leg of the Triad to compensate for the failure of a given type of warhead.

“Report on Nuclear Employment Strategy of the United States Specified in Section 491 of 10 U.S.C.,” Department of Defense, Washington, 12 June 2013, pp. 4, 7. Available at http://www.defense.gov/pubs/ReporttoCongressonUSNuclearEmploymentStrategy_Section491.pdf

A Model for Inter-Leg Hedging

Given a treaty-negotiated number of deployed weapons N , what are mathematical consequences of the first two pieces of guidance?

- Minimize number of weapons.
- Maintain enough non-deployed weapons to hedge against the technical failure of any single weapon type.

Modeling Assumptions

Theoretical Minimum Hedge Size

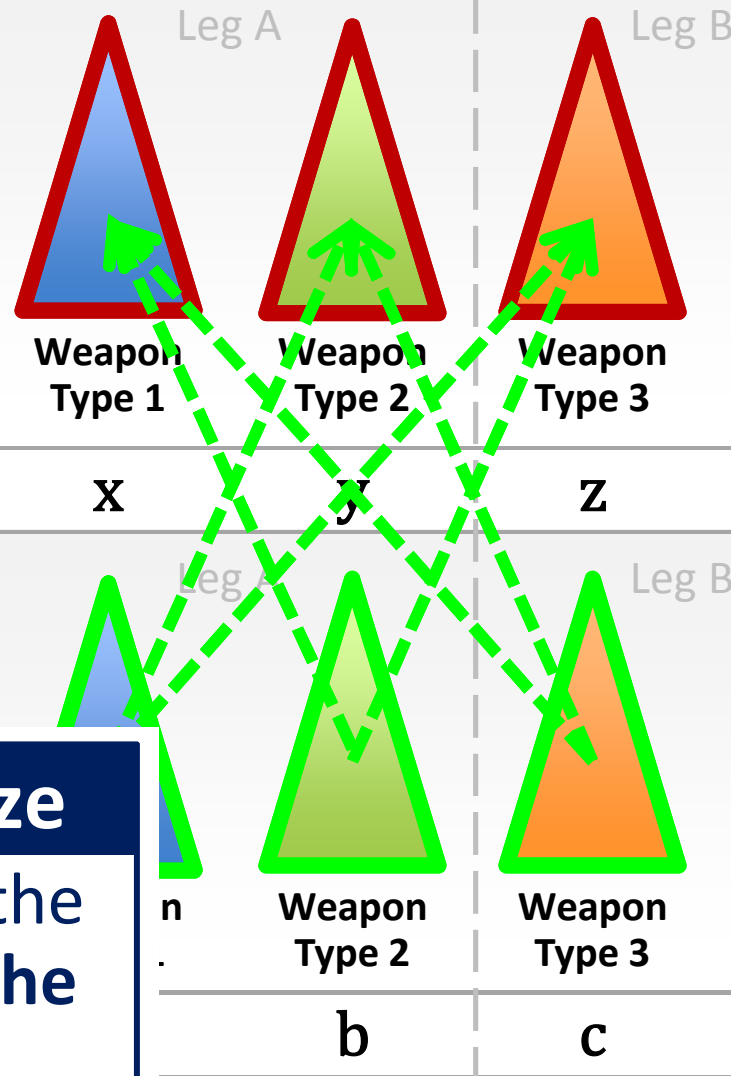
For a three-weapon-type stockpile, the theoretical minimum hedge is **50% the size of the deployed stockpile**.

Deployed

Total Deployed:

$$N = x + y + z$$

Technical



A Model for Inter-Leg Hedging

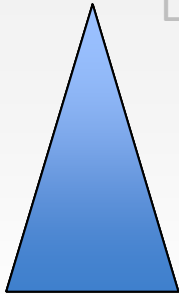
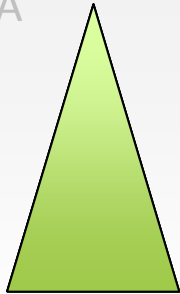
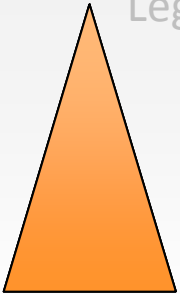
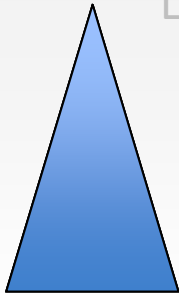
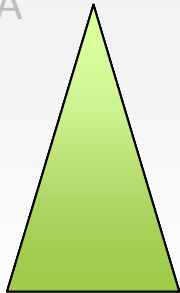
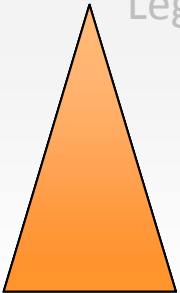
General Solution for Theoretical Minimum:

If $a = \frac{1}{2}N - x$
 $b = \frac{1}{2}N - y$
 $c = x + y - \frac{1}{2}N$

then
 $a + b + c = \frac{1}{2}N$

Example Solutions:

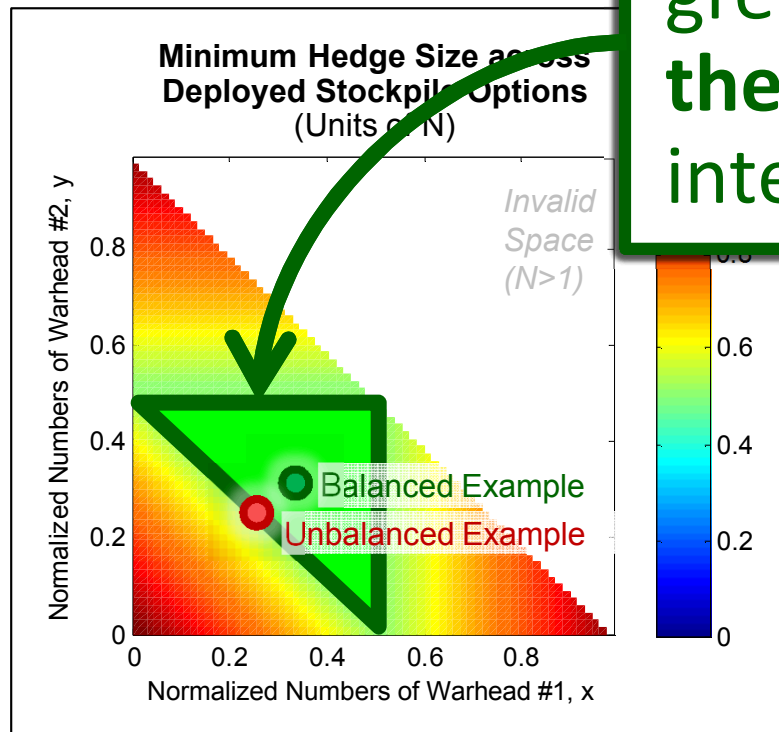
- Unbalanced
- Balanced

Deployed			Technical Hedge		
Leg A		Leg B	Leg A		Leg B
					
Weapon Type 1	Weapon Type 2	Weapon Type 3	Weapon Type 1	Weapon Type 2	Weapon Type 3
Total Deployed: $N = x + y + z$					
$\frac{1}{4}N$	$\frac{1}{4}N$	$\frac{1}{2}N$	$\frac{1}{4}N$	$\frac{1}{4}N$	0

A Model for Inter-Leg Hedging

General Solution for Theoretical Minimum:

If $a = \frac{1}{2}N - x$
 $b = \frac{1}{2}N - y$
 $c = x + y - \frac{1}{2}N$



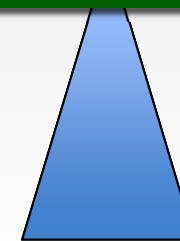
Deployed

Leg A

Leg B

Many stockpile options (in the green triangle) can achieve the theoretical minimum hedge if inter-leg hedging is allowed.

ge



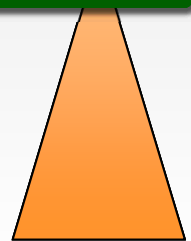
Weapon
Type 1

a



Weapon
Type 2

b

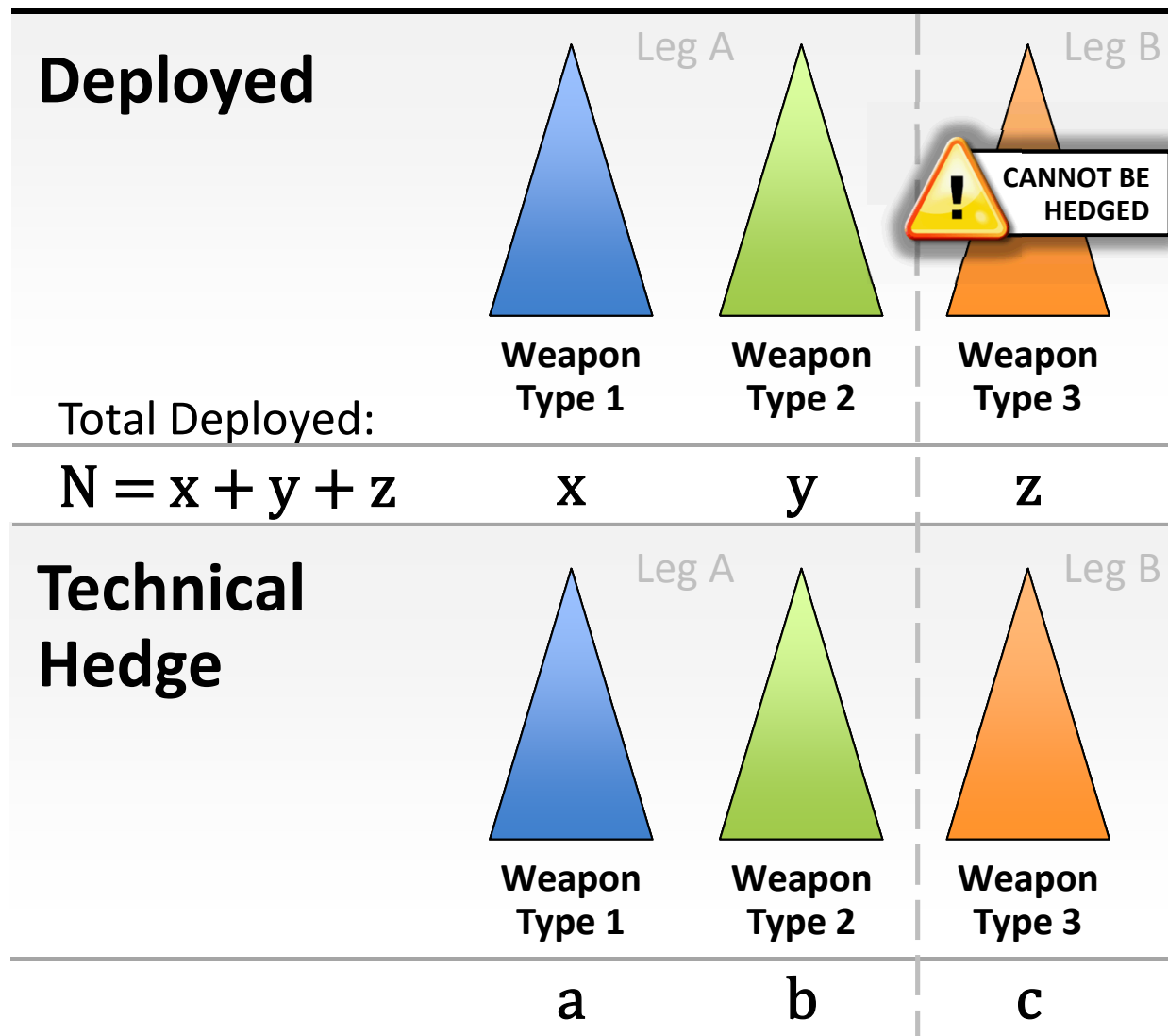


Weapon
Type 3

c

The Strategy of Intra-Leg Hedging

For a simple three-weapon-type stockpile with two legs, **at least one weapon type cannot be hedged** if only intra-leg hedging is allowed.



Modified Intra-Leg: 1 Adaptable Weapon

To solve Leg B's intra-leg hedging issue, we can introduce an **adaptable weapon** that can be deployed on either leg's delivery platform.

The math problem is now:

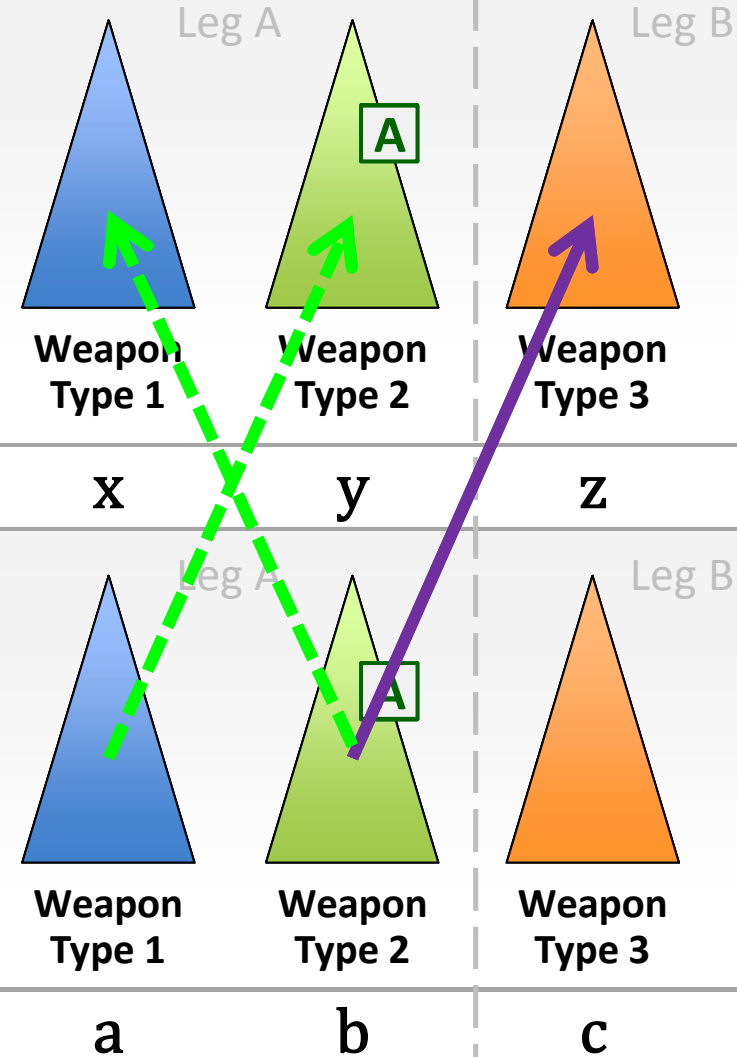
Minimize $a + b + c$
such that $b \geq z$
 $a \geq y$
 $b \geq x$
 $x + y + z = N$

Deployed

Total Deployed:

$$N = x + y + z$$

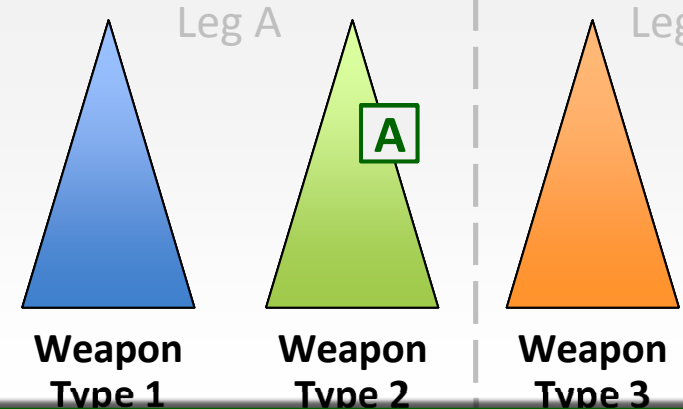
Technical
Hedge



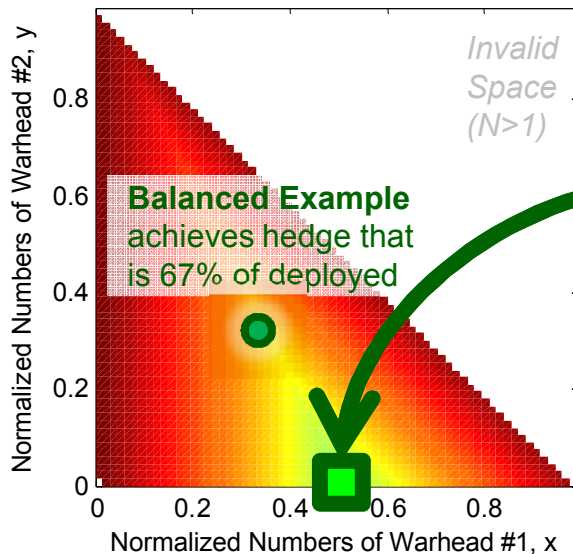
Modified Intra-Leg: 1 Adaptable Weapon

To solve Leg B's intra-leg hedging issue, we can introduce an **adaptable weapon** that can be deployed on either leg's delivery platform.

Deployed



Minimum Hedge Size across
Deployed Stockpile Options
(Units of N)



Theoretical minimum of 50% can only be achieved if the adaptable weapon is used as a dedicated hedge and there are equal numbers of the remaining two weapon types.

a b c

Modified Intra-Leg: 2 Adaptable Weapons

The math problem is now:

Minimize $a + b + c$

such that $b \geq z$

$a + c \geq y$

$b + c \geq x$

$x + y + z \geq 1$

Deployed

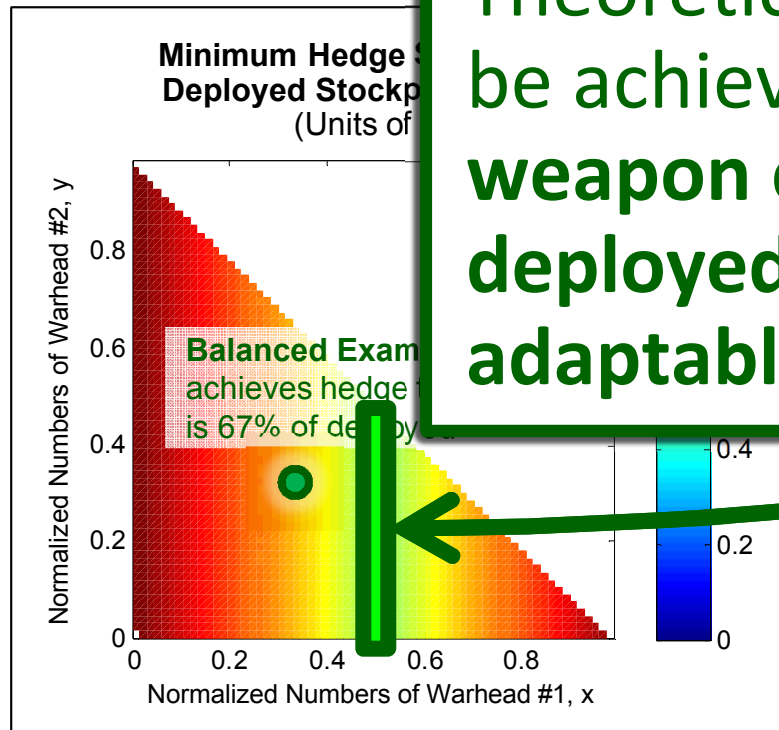
Leg A

Leg B

A

A

Theoretical minimum of 50% can only be achieved if the non-adaptable weapon comprises 50% of the deployed stockpile and only adaptable weapons are kept in hedge.



Weapon Type 1

Weapon Type 2

Weapon Type 3

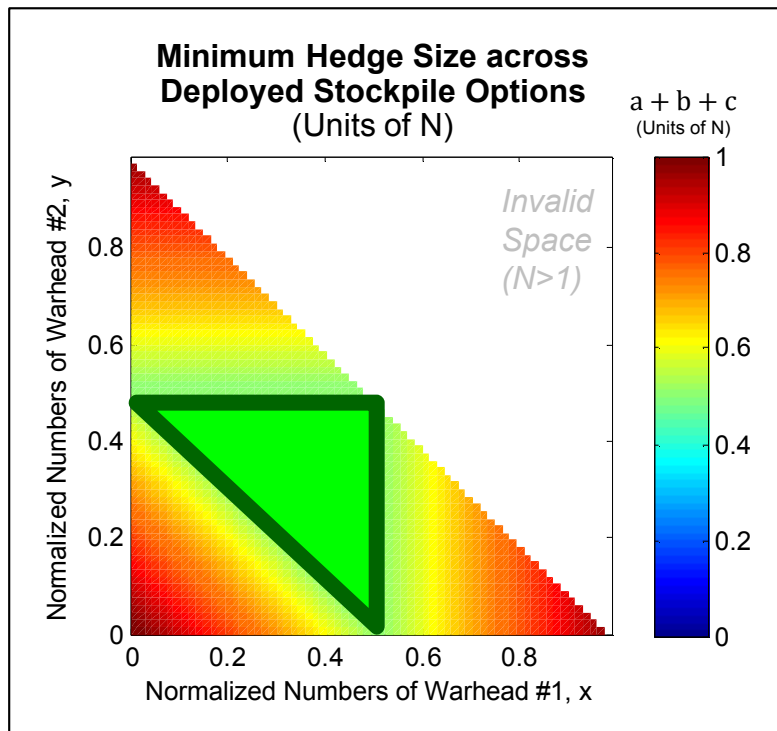
a

b

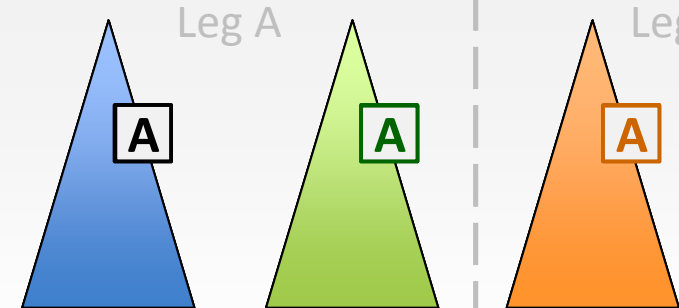
c

Modified Intra-Leg: 3 Adaptable Weapons

Three adaptable weapons permit the complete space of theoretical minimum hedging options.



Deployed



Weapon Type 1

Weapon Type 2

Weapon Type 3

Deployed:

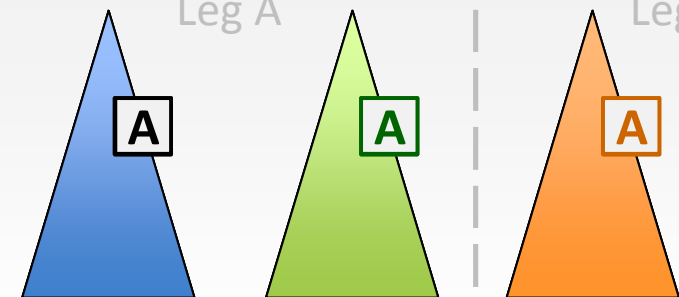
$x + y + z$

x

y

z

Technical
ge



Weapon Type 1

Weapon Type 2

Weapon Type 3

a

b

c

Summary of Results

Hedging Strategy	Complete Hedging Solution Possible?	Theoretical Minimum Deployed Stockpile Solution(s)	Min. Hedge Size for a Balanced Deployed Stockpile
Inter-Leg Hedging	Yes	Plane with no weapon comprising >50% of deployed stockpile	50%
Intra-Leg Hedging No Adaptable Weapon	No		
Modified Intra-Leg 1 Adaptable Weapon	Yes	Point with adaptable weapon as dedicated hedge and equal deployed numbers of non-adaptable weapons	67%
Modified Intra-Leg 2 Adaptable Weapons	Yes	Line with non-adaptable weapon comprising 50% of deployed stockpile and only adaptable weapons kept in hedge	67%
Modified Intra-Leg 3 Adaptable Weapons	Yes	Plane with no weapon comprising >50% of deployed stockpile	50%

Increasing Complexity



Conclusion

- Methodology allows **systematic and repeatable analysis of minimum hedge options** and is **extensible** to a variety of possible stockpiles.
- While the theoretical minimum hedge for a three-weapon-type stockpile is **50%** the size of the deployed stockpile, it can **only** be reached for **specific deployed stockpile compositions**.
- Adding **adaptable weapons** to an **intra-leg** strategy can make additional theoretical minimum hedge options **possible**. At best, this produces the same theoretical minimum results as inter-leg hedging.

What trades in resources are involved in making adaptable-weapon intra-leg hedging options possible?

- The United States will maintain a sufficient number of non-deployed weapons to hedge against the technical failure of any single weapon type or delivery system at a time. **Where possible, the United States will provide intra-leg hedge options – i.e.,**

“Report on Nuclear Employment Strategy of the United States Specified in Section 491 of 10 U.S.C.,” Department of Defense, Washington, 12 June 2013, p. 7. Available at http://www.defense.gov/pubs/ReporttoCongressonUSNuclearEmploymentStrategy_Section491.pdf