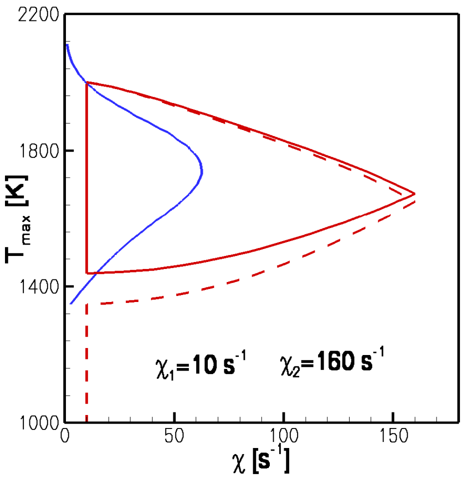

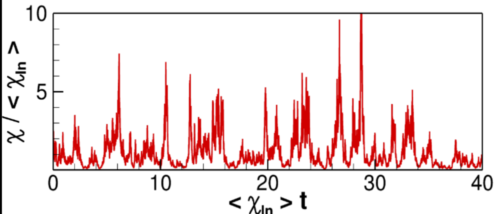

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Estimates of the unsteady extinction frequency in turbulent non-premixed flames with simple stochastic models

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NNSA National Nuclear Security Administration

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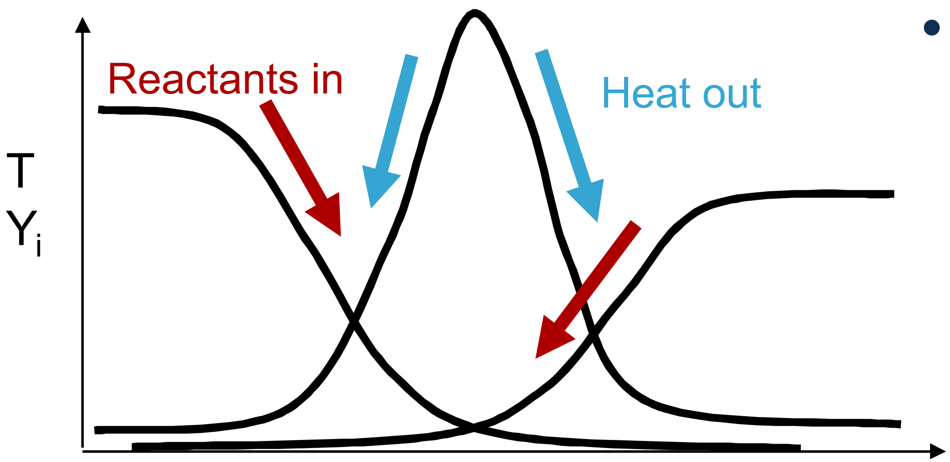
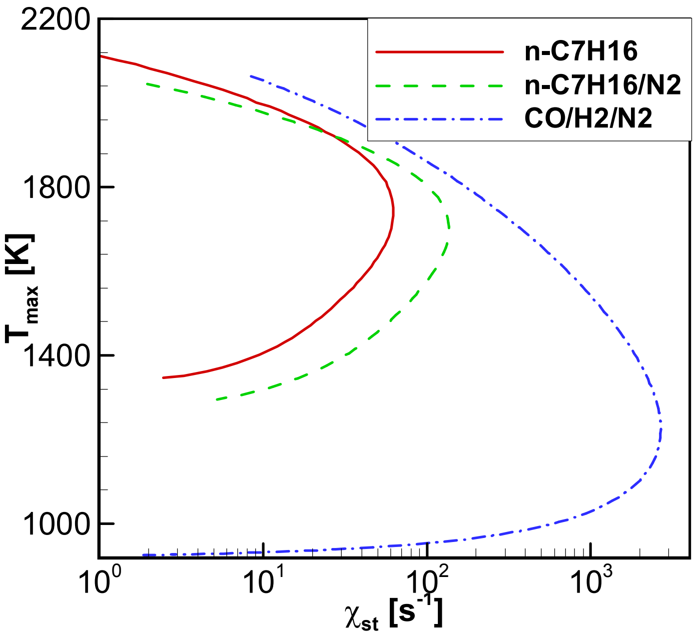
Extinction in steady-laminar flames



- Extinction occurs when chemistry is slower than mixing.
  - Heat losses exceed reactant consumption rates.

- Critical dissipation rate, χ_q, well known.

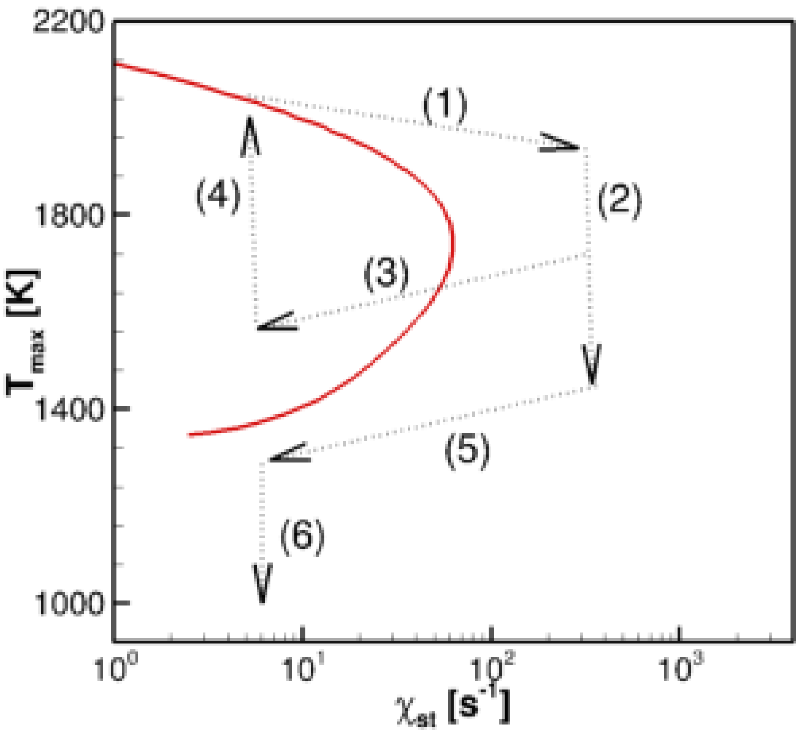
- Steady-state T-χ 'S-curve.'
  - Upper (and lower) branches stable.
  - Middle branch unstable.

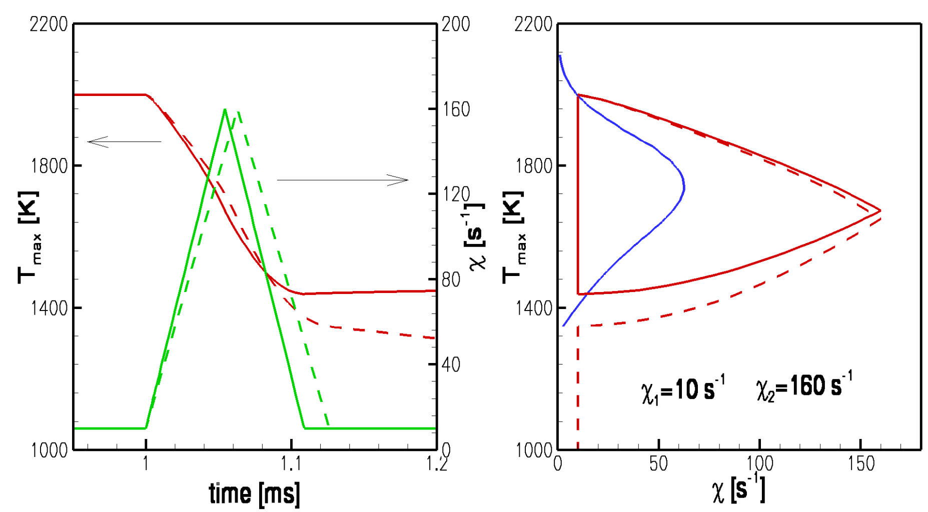


Flame state during dissipation rate fluctuations

Fluctuations in dissipation rate move flame state away from steady-state S-curve.

- Consider state after period of high dissipation-rate:
  - If state is above middle branch, return to fully burning state.
  - If state is below middle branch, move to extinguished state.





Formulating unsteady extinction criterion.

- Criterion motivated by trajectories in T-χ phase space (S-curve).
- Estimate heat loss to get temperature decrement:

$$\frac{dT}{dt} = \frac{\chi}{2}\frac{d^2T}{dZ^2} - \left(\sum_i \frac{\omega_i h_i}{\rho c_p}\right)_{\max} \approx \frac{(\chi_q - \chi)\left(T - T_\infty\right)}{Z_{st}\,(1 - Z_{st})\ \varepsilon}$$

- Reaction rate is at χ_q rate.
- Consider time $\chi > \chi_q$.
- Estimate 2nd derivative using stoichiometry, reaction zone thickness.

- Integrate:

$$\frac{T_2 - T_\infty}{T_1 - T_\infty} = \exp(-\Xi) \ \text{with}\ \ \Xi \approx \frac{A \int_{\chi > \chi_q} (\chi - \chi_q)dt}{Z_{st}\,(1 - Z_{st})\ \varepsilon}$$

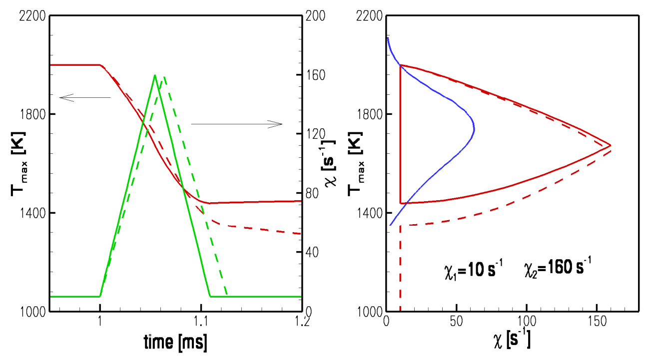
- Critical value for T_2 on middle branch

$$\Xi_q = \ln\left(\frac{T_1 - T_\infty}{T_m - T_\infty}\right)$$

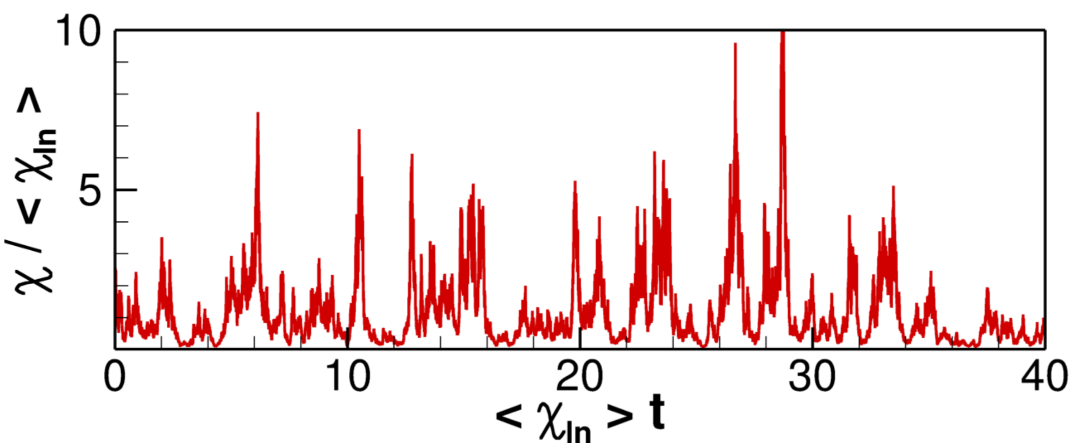
Hewson, *Comb Flame*, 160: 887-897, 2013

Unsteady extinction frequency

- How often does $\Xi > \ln\left(\dfrac{T_1 - T_\infty}{T_m - T_\infty}\right)$ occur where $\Xi \propto \displaystyle\int_{\chi > \chi_q} (\chi - \chi_q)\, dt$



- How often does each of these happen in intermittent turbulence (below)?



- Statistics of $\in_q$?

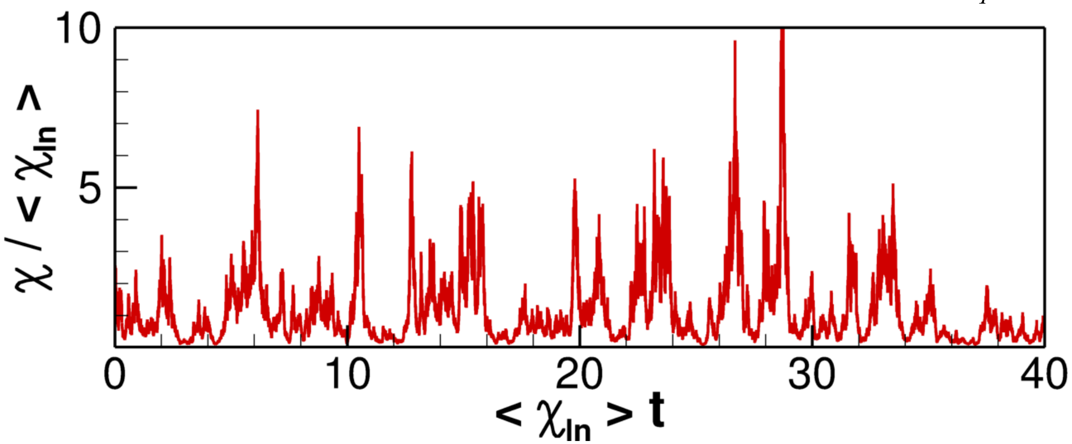
- Dissipation rate intermittent (log normal).

Simple stochastic model for dissipation rate

- Ornstein-Uhlenbeck process can be used to simulate lognormal dissipation rate fluctuations.

$$d \ln \chi^* = -\left(\ln \chi^* + \frac{\sigma^2}{2} \right) dt^* + \sqrt{2}\sigma\, dW \qquad \chi^* = \chi / \langle \chi \rangle, \quad t^* = t/\tau_\chi$$

- Normalized diss. impulse: $\Xi^* = \int_{\chi > \chi_q} \left(\chi^* - \chi_q^* \right) dt^* = \left[\frac{2 Z_{st} \left(1 - Z_{st} \right)}{A \langle \chi \rangle \tau_\chi} \right] \Xi$



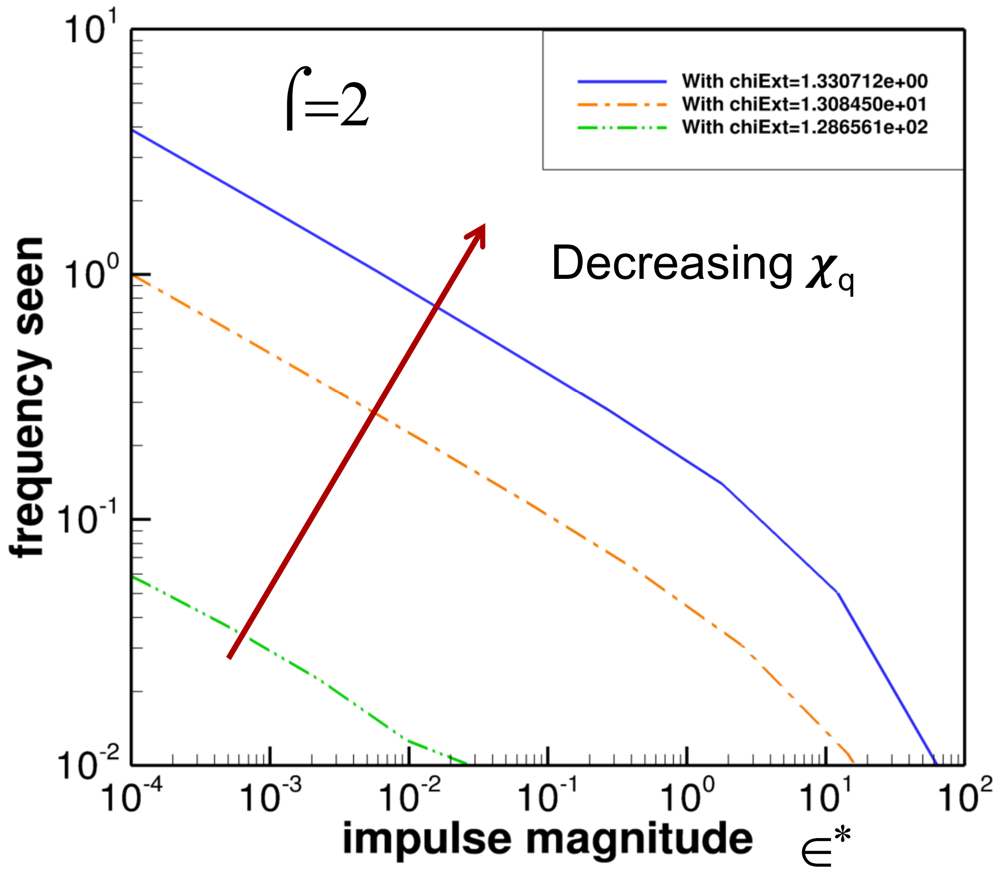
- Statistics for $\in^*$: area under peaks and above χ_q

Statistics of the dissipation impulse

- Based on assumptions of OU evolution, statistics for $\in^*$ as function of $\sigma_{\ln\chi}$, $\chi_q/\langle\chi\rangle$, $\in^*$

- Reporting **cumulative frequency** $\in^*$ **exceeds given value** in t^* units as function of $\chi_q/\langle\chi\rangle$.

- Frequency increases with easier extinction (as expected).



Statistics of the dissipation impulse

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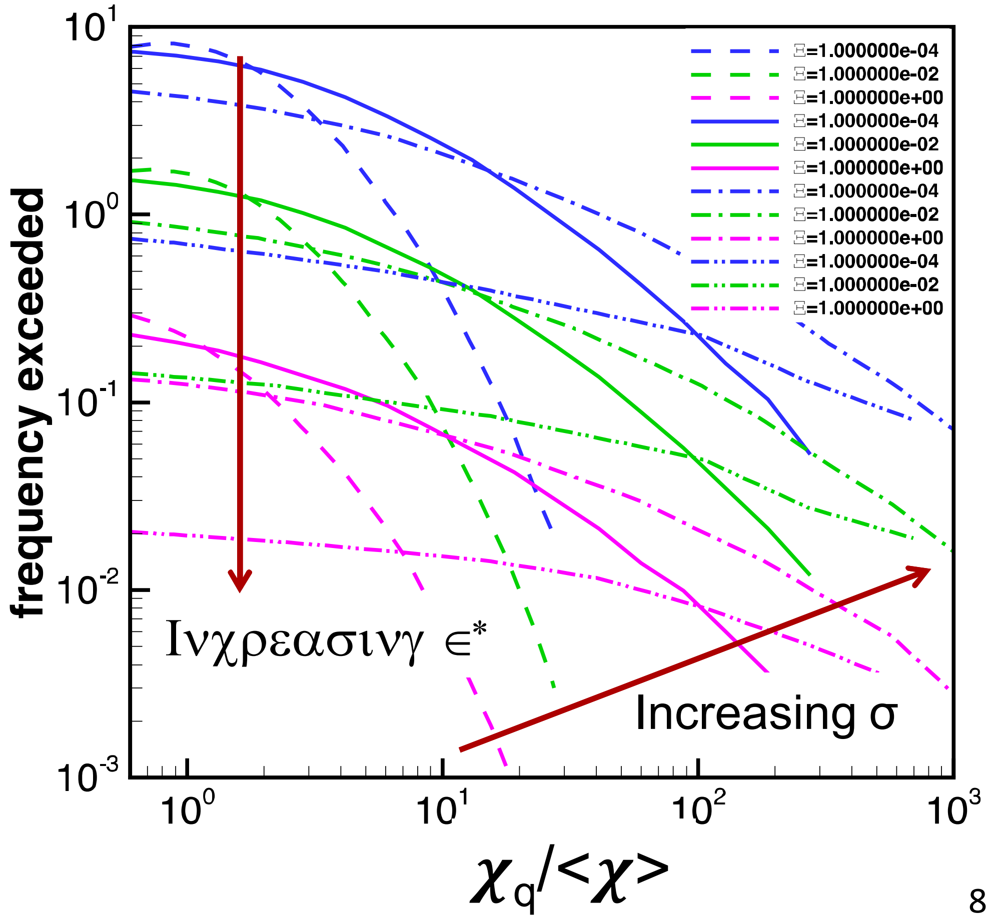
- Based on assumptions of OU evolution, statistics for $\in^*$ as function of $\sigma_{\ln\chi}$, $\chi_q/\langle\chi\rangle$, $\in^*$

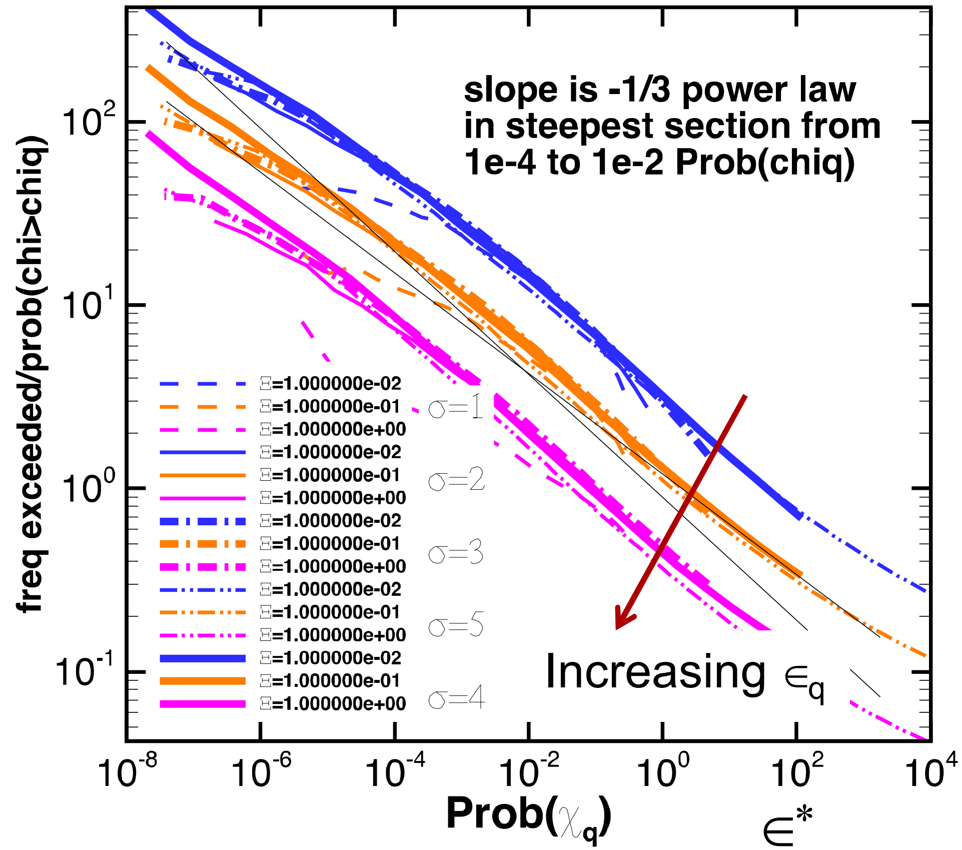
  - As $\chi_q/\langle\chi\rangle$ increases (harder to extinguish) the frequency exceeded decreases.
  - Reduction in frequency strongest for smaller σ (when large dissipation rates less frequent).



Normalized extinction frequencies

- Normalize rate by Prob($\chi > \chi_q$) (cumulative distribution) as measure of time and magnitude of $\chi - \chi_q$ when $\chi > \chi_q$.

- Normalized frequency then decreases approximately as power law in Prob($\chi = \chi_q$).



Summary

- Criterion defining magnitude of scalar-dissipation impulse leading to extinction defined: $\in > \in_q$.
  - Involves time integrated excess dissipation over χ_q.
  - Critical value, $\in_q$, related to shape of S-curve.
- Statistics for $\in$ based on Ornstein-Uhlenbeck process
  - Normalized extinction frequency increases with 'easier extinction' based on (all as expected)
    - average dissipation rate (larger $\langle\chi\rangle/\chi_q$).
    - shape of S-curve (smaller $\in_q$)
- Power law scaling of extinction frequency in Prob($\chi = \chi_q$) observed in relevant regime when frequency normalized by cumulative Prob($\chi > \chi_q$).

Thank you for your attention

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Backup material

An unsteady extinction criterion

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- Ξ estimates heat losses during unsteady extinction

$$\Xi \approx \frac{A \int\limits_{\chi > \chi_q} (\chi - \chi_q)\, dt}{Z_{st}(1 - Z_{st})\ \varepsilon}$$

- Ξ_q gives heat loss for extinction based on middle-branch crossing

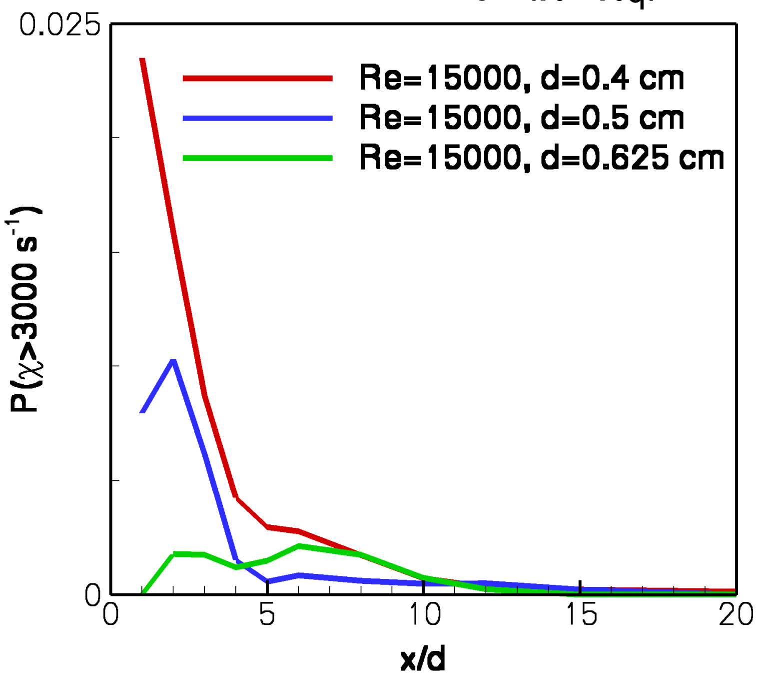
$$\Xi_q = \ln\left(\frac{T_1 - T_\infty}{T_m - T_\infty}\right)$$

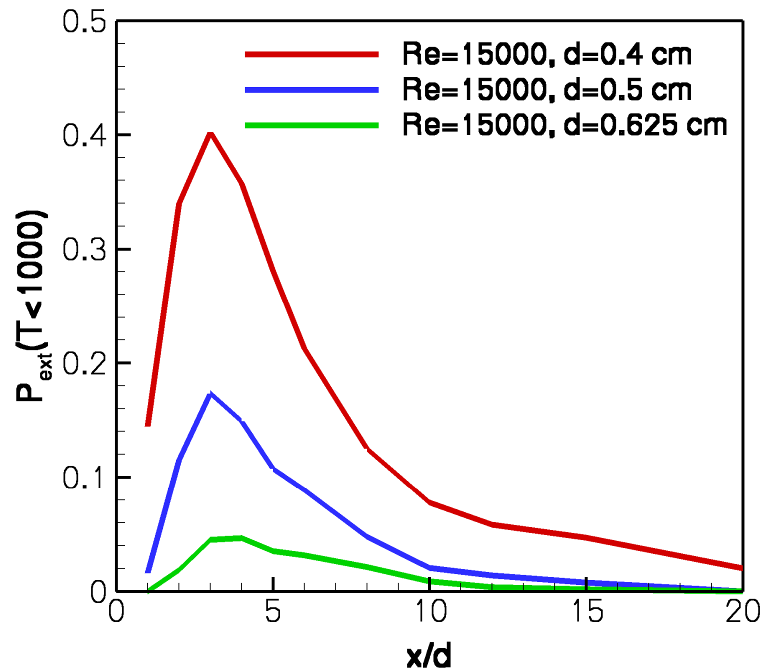
- Best agreement with extinction criterion for $\chi_2 \square \chi_q \quad \chi_1 \square \chi_q$
- Relevant for order of magnitude χ fluctuations.

Motivation

- Results for turbulent flames indicate local extinction can be prevalent even when the most typical dissipation rate is well below χ_q.

Probability ($\chi > \chi_q$) << Probability extinguished.



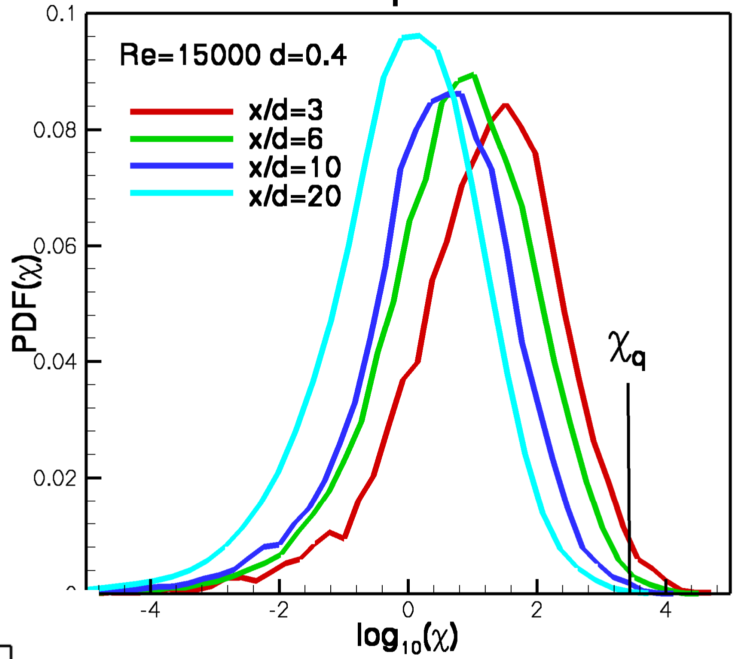


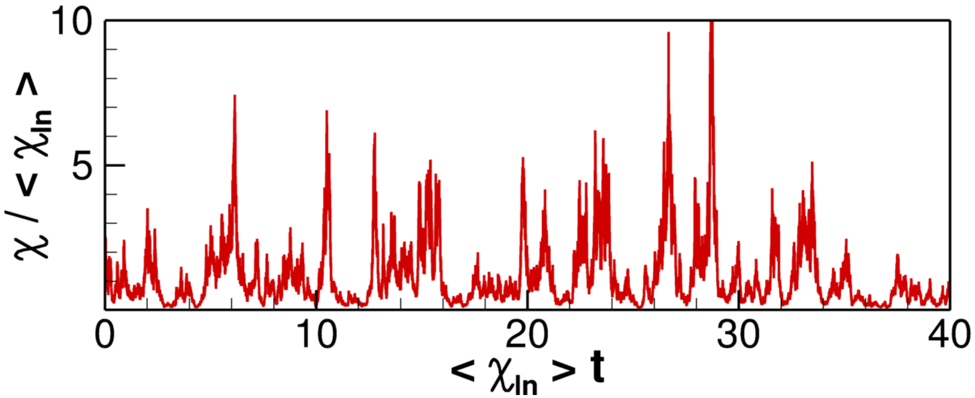
J. C. Hewson and A. R. Kerstein, "Local Extinction and Reignition in Nonpremixed Turbulent $CO/H_2/N_2$ Jet Flames," *Combust. Sci. Tech.*, 174 (5-6):35-66, 2002

Scalar dissipation rate fluctuations

- Dissipation rate PDF nominally lognormal.
    - Fluctuations are order of magnitude.
- Extinction associated with brief transients to large χ that must also be short-lived, being dissipated at Bachelor scales.

Scalar dissipation rate PDF





Scalar dissipation rate history generated using Ornstein-Uhlenbeck process