

Diagnostic Constraints on the Amount of Cold Mass in Imploded Argon Pinches on Z*



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Outline



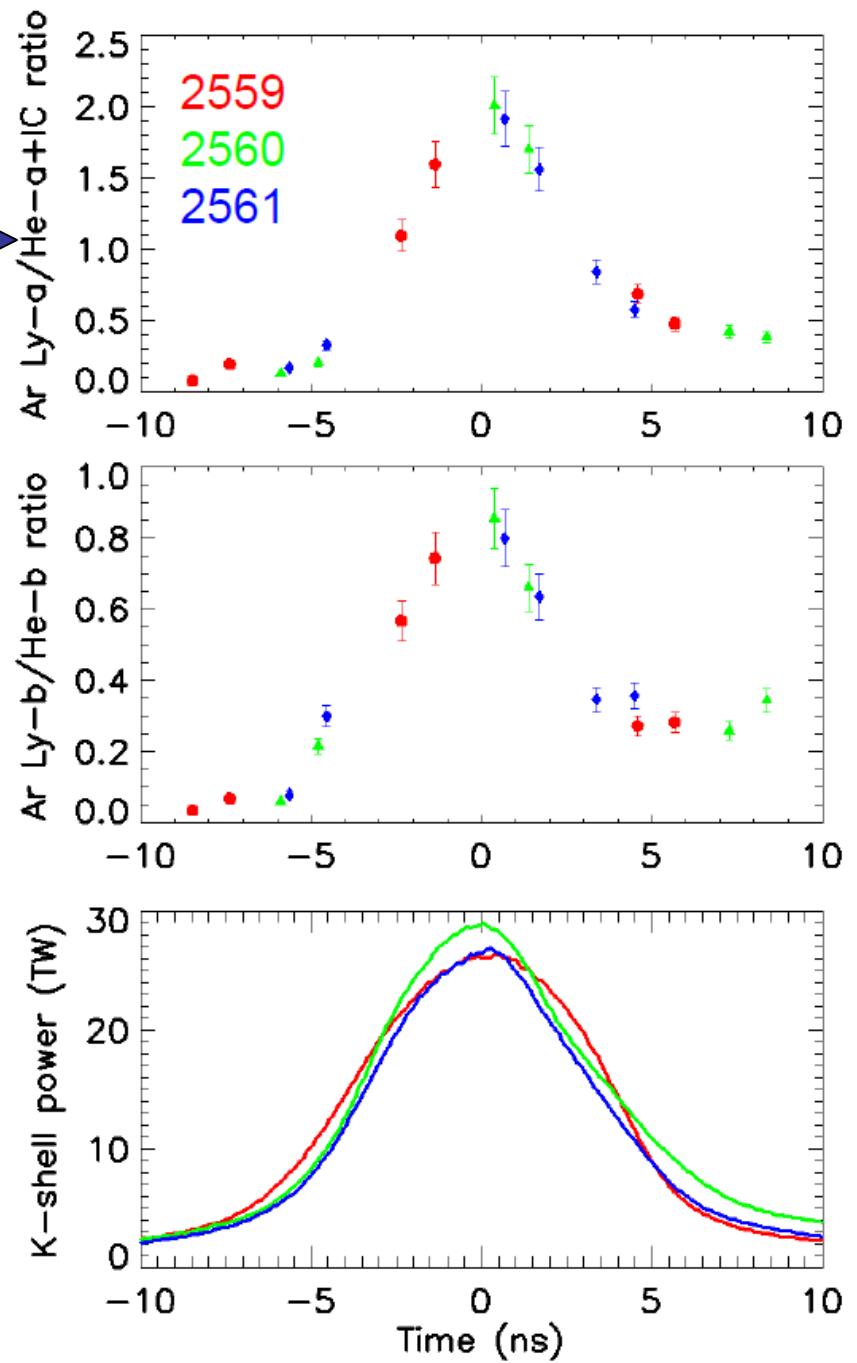
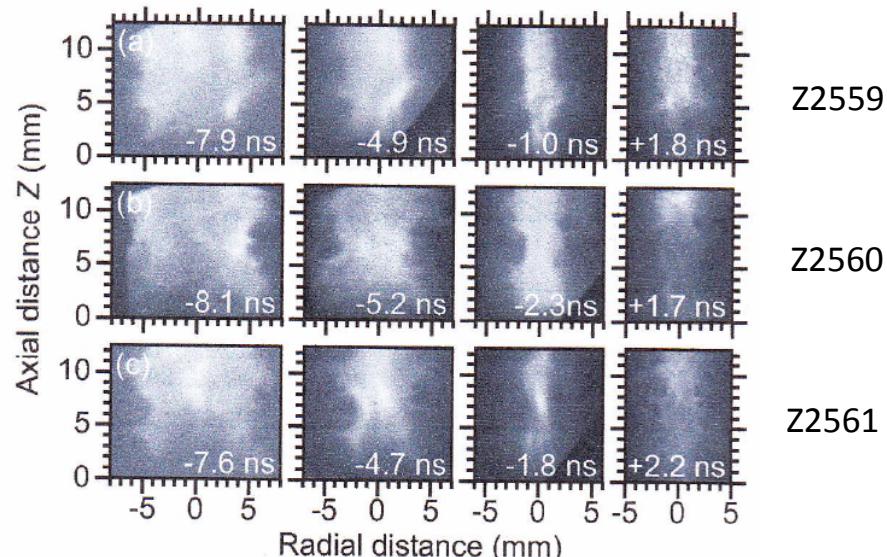
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- Description of experiments and x-ray data
- Modeling approach, χ^2 minimization
- How the data constrains the pinch properties
- Best-fit conditions
- Conclusions

Ar puff shots Z 2559-2561: basic properties

- 8-cm diameter shell-on-shell Ar gas puffs, 1.0 mg/cm mass load. 1.6:1 inner-to-outer shell mass ratio. 2.5 cm height.
- Very consistent x-ray data among the 3 shots: time-resolved K-shell spectroscopy, filtered PCD's, bolometers, and a calorimeter. Also, time-gated K-shell pinhole imaging.
- K-yields were \sim 309-363 kJ, peak K-shell powers: 10.6-11.4 TW/cm. Peak soft x-ray powers: 4.6-7.1 TW/cm.
- Marx charge: 85 kV, feed current \sim 16 MA (significant convolute losses).
- Z2560 is modeled in detail. The consistency of the data indicates that results of this analysis will also likely apply to Z2559 and Z2661.

The remarkable consistency of the x-ray data is illustrated at right: key line ratios and K-shell powers vs. time for the 3 shots. This despite the differing structure of the pinhole images during the implosion (below).



Chi-squared fitting is a well-established method to determine a “confidence level” that a dataset is consistent with model predictions.

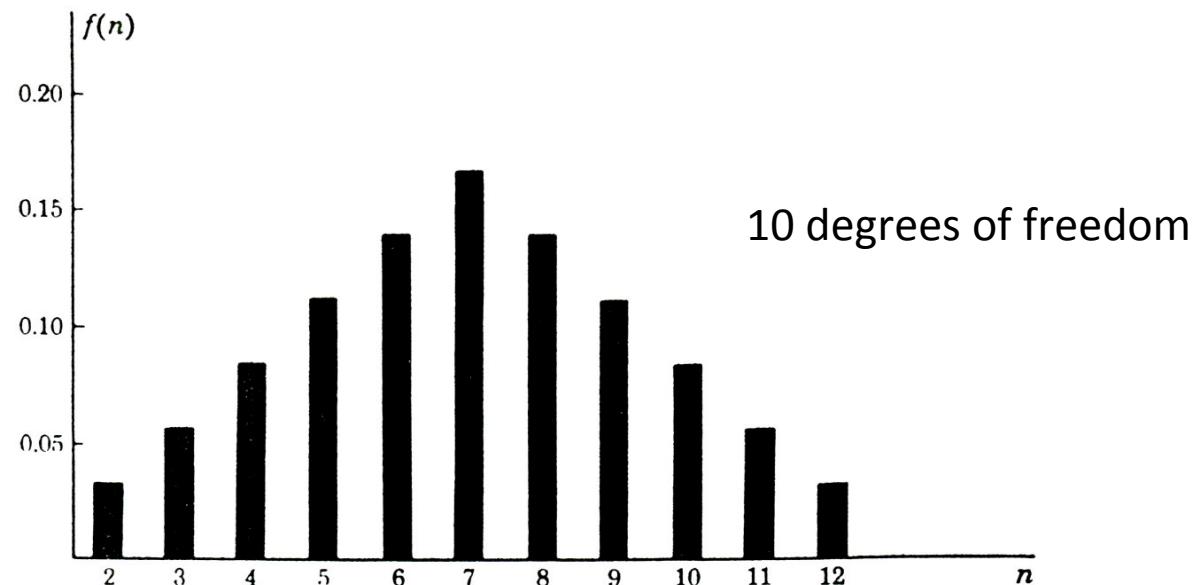


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Consider a simple physical system: a rolling pair of dice.

A “physics model” for this system assumes (1) They are cubic, (2) They are of uniform density, (3) They are not magnetized, nor electrically charged nor polarized, (4) Rolling randomizes their positions when they come to rest.

This “physics model” predicts the following probability spectrum*



* H. D. Young, *Statistical Treatment of Experimental Data*, McGraw-Hill, New York, 1962, p. 41.

The experiment consists of many rolls of the dice. If number n is observed with a measured probability $p(n)$ and standard deviation $\sigma(n)$, the quantity chi-squared is



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$$\chi^2 = \sum_{n=2}^{n=12} \frac{[p(n) - f(n)]^2}{\sigma^2(n)}$$

Note that if each $p(n)$ is measured to be within one standard deviation of the model prediction, χ^2 is \leq the number of data points.

χ^2 tables based on standard statistics give the probability that the measured distribution is consistent with the model, rather than being a random excursion due to statistical fluctuation.



Determining confidence level from χ^2

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Degrees of Freedom (df)	Probability (p)											
	0.95	0.90	0.80	0.70	0.50	0.30	0.20	0.10	0.05	0.01	0.001	
1	0.004	0.02	0.06	0.15	0.46	1.07	1.64	2.71	3.84	6.64	10.83	
2	0.10	0.21	0.45	0.71	1.39	2.41	3.22	4.60	5.99	9.21	13.82	
3	0.35	0.58	1.01	1.42	2.37	3.66	4.64	6.25	7.82	11.34	16.27	
4	0.71	1.06	1.65	2.20	3.36	4.88	5.99	7.78	9.49	13.28	18.47	
5	1.14	1.61	2.34	3.00	4.35	6.06	7.29	9.24	11.07	15.09	20.52	
6	1.63	2.20	3.07	3.83	5.35	7.23	8.56	10.64	12.59	16.81	22.46	
7	2.17	2.83	3.82	4.67	6.35	8.38	9.80	12.02	14.07	18.48	24.32	
8	2.73	3.49	4.59	5.53	7.34	9.52	11.03	13.36	15.51	20.09	26.12	
9	3.32	4.17	5.38	6.39	8.34	10.66	12.24	14.68	16.92	21.67	27.88	
10	3.94	4.86	6.18	7.27	9.34	11.78	13.44	15.99	18.31	23.21	29.59	

χ^2

For example, if the dice experiment yielded a χ^2 of 3.94, that would indicate a 95% confidence level that our data conformed to the “physics model”, i.e, the dice are likely not “loaded”. Stated more precisely, 95% of the time, χ^2 would be higher than 3.94 for experiments using fair dice.

If each measurement was exactly one sigma from the model

prediction, χ^2 would be 11, giving a much smaller $\sim 36\%$ confidence level. [JA](#) 4 Aug 2014 7

6 data points are fitted for Z2560. Powers and ratios are time-resolved near peak K-shell power. Diameter at stagnation is ~ 2.8 mm.



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peak K-shell power	11.4 TW/cm \pm 10%
peak soft x-ray power	4.6 TW/cm \pm 20%
Ly- α /(He- α +IC)	2.00 \pm 10%
Ly- β /He- β	0.85 \pm 10%
Ly- γ /He- γ	1.20 \pm 10%
mass load	1.0 mg/cm \pm 10%

Chi-square minimization is used to select the “most probable” fit using a detailed CRE atomic-radiation transport model. Note: since the model is 1D, a 95% confidence level fit is not expected and may be unachievable.

Besides the experimental uncertainties shown above, additional errors may arise from uncertainties in the underlying atomic rates (10%) and extracting calculated line ratios from the composite spectra (10%).



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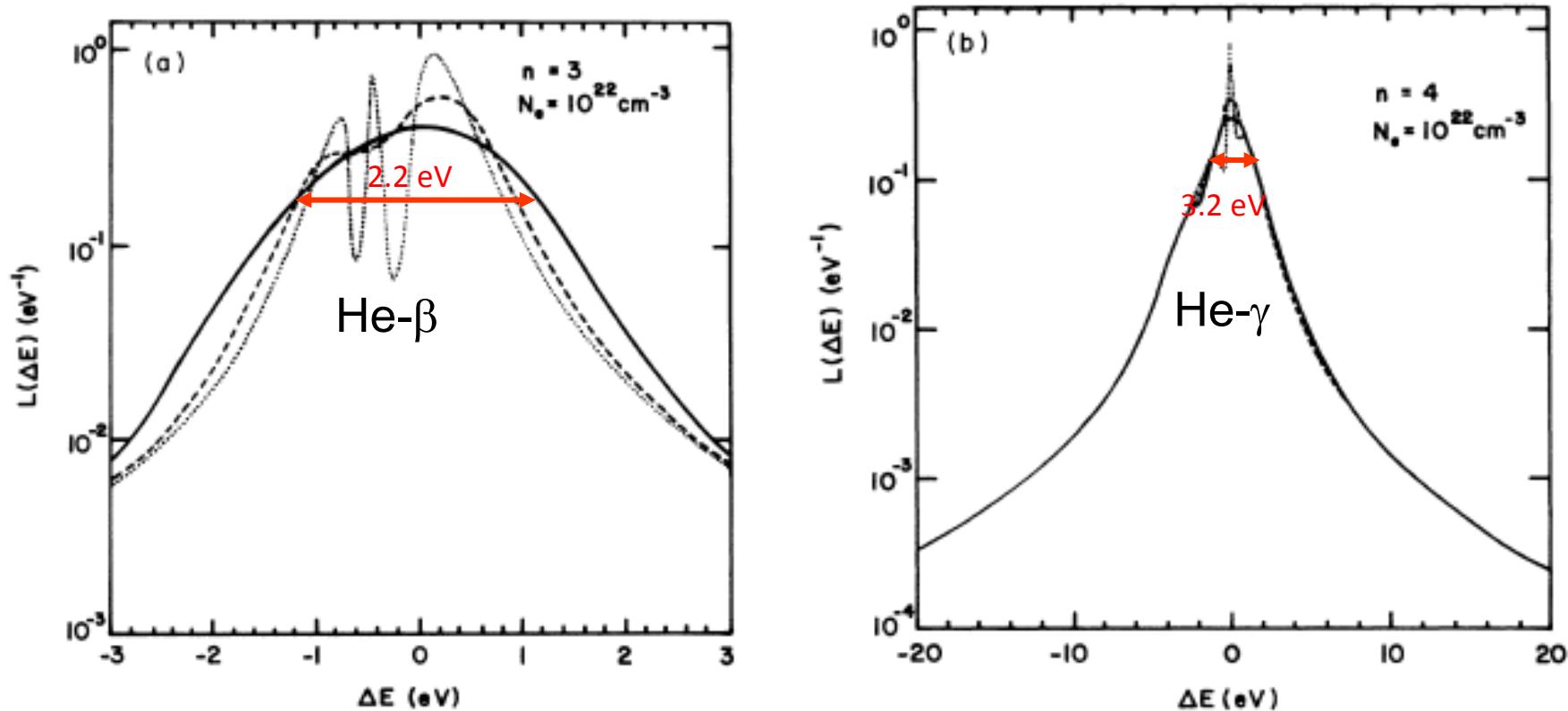
Fitting procedure to infer pinch conditions

- First step: a fast (37 Ar atomic levels, 51 lines), probability-of-escape based CRE model¹ was run for about 10^5 density-temperature combinations. Those yielding the lowest χ^2 were selected for further evaluation.
- Second step: the best-fits from the fast model were recalculated and fine-tuned using a more detailed, 186-level, 611-line model that transports 15488 photon energies to resolve the line profiles. The best overall fit (minimum χ^2) is selected.
- The effective ion temperature (50 keV) was determined by fitting the width of the He- γ line, whose Stark width (next VG) is much less than the observed 14 ± 1 eV FWHM, and is less affected by unmodeled satellites.

Stark widths of He-like Ar lines have been calculated by Griem, Kepple, and Blaha [Phys. Rev. A **41**, 5600 (1990)]. They are much smaller than the measured ~ 14 eV FWHM.



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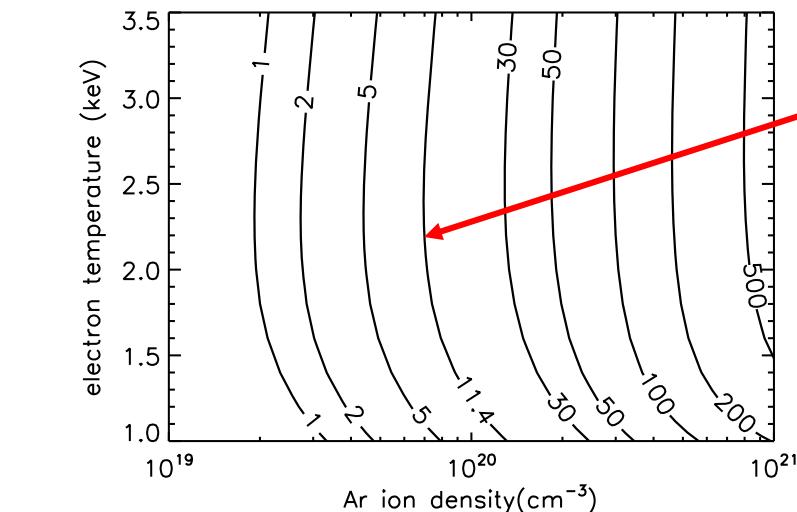


Solid lines are Stark profiles including ion dynamical effects.
 If the entire load is spread uniformly within a 2.8 mm diameter cylinder, the electron density for $\bar{Z} = 17$ would be $4.2 \times 10^{21} \text{ cm}^{-3}$.

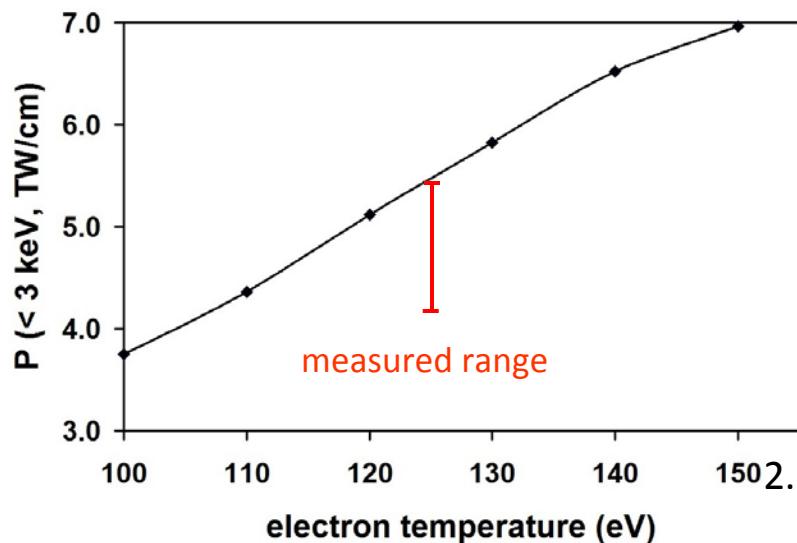
Components of the calculated 14 eV He- γ width

- FWHM of the 50 keV ion temperature **Doppler profile**: 10.6 eV
- **Instrumental broadening**: 5.5 eV. Doppler convolved with instrumental gives a width $\sim (10.6^2 + 5.5^2)^{1/2} = 11.9$ eV. The remaining ~ 7 eV is due to opacity broadening.
- **Opacity broadening**: the emitted profile is enhanced on the wings compared to the intrinsic profile. This is due to the fact that photon escape is more probable far from line center. The observed profile is thus wider than the intrinsic profile.

Constraints on the cold mass are provided by the measured K-shell power (11.4 TW/cm), soft x-ray power (4.6 TW/cm), and total load mass (1 mg/cm).



K-shell power of 11.4 TW/cm suggests an ion density near $7 \times 10^{19} \text{ cm}^{-3}$. At observed pinch diameter of 2.8 mm, the radiating mass would be 0.29 mg/cm. The remaining 0.71 mg/cm is too cold to radiate in the K-shell.

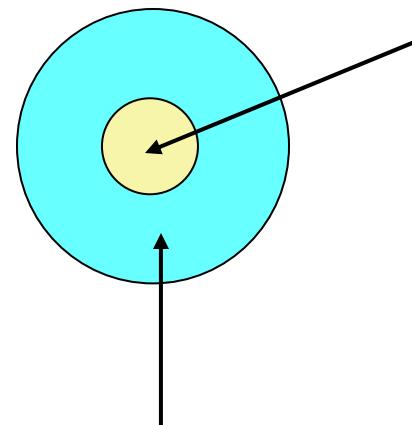


Assuming that the peripheral mass extends to about 8 mm (Ref. 2), calculations with the large Ar model show that its temperature can't exceed 125 eV without exceeding the observed soft x-ray power.

2. J. W. Thornhill *et al.*, High Energy Dens. Phys. 8, 197 (2012)



Properties of the best-fit case



K-shell core:

radius: 1.4 mm

$T_e = 2.2 \text{ keV}$

$N_i = 6.5 \times 10^{19} \text{ cm}^{-3}$

mass: 0.27 mg/cm

Outer blanket:

radius: 4.0 mm

$T_e = 0.11 \text{ keV}$

$N_i = 2.5 \times 10^{19} \text{ cm}^{-3}$

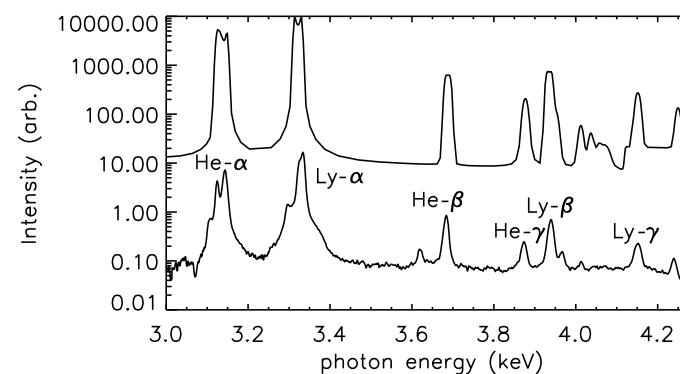
mass: 0.73 mg/cm

data

model fit

	data	model fit
K-shell power (TW/cm)	11.4	10.0
Soft x-ray power (TW/cm)	4.6	4.4
$\text{Ly-}\alpha/\text{He-}\alpha$	2.00	1.52
$\text{Ly-}\beta/\text{He-}\beta$	0.85	1.07
$\text{Ly-}\gamma/\text{He-}\gamma$	1.20	1.27
Mass load (mg/cm)	1.0	1.0

χ^2 is 5.3: a 51% confidence level fit



calculated spectrum

experimental spectrum
(at peak power)



Conclusions

- Ar gas puffs have produced $330 \text{ kJ} \pm 9\%$ of K-shell radiation ($> 3 \text{ keV}$), with remarkable time-resolved spectral reproducibility, despite quite different morphology at stagnation.
- The K-shell power of 11.4 TW/cm , combined with the observed pinch size at stagnation, implies that no more than $1/3$ of the load mass participated in the K-shell radiation (consistent with previous results³).
- The relatively low soft x-ray power of 4.6 TW/cm places an upper limit of about 125 eV on the temperature of the non-K-shell-radiating mass which is $2/3$ of the total load.
- The low-temperature outer blanket has minimal opacity to the K-shell x-rays emitted from the interior.