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A Craig-Bampton Experimental Dynamic Substructure using the Transmission Simulator Method

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Nomenclature

CB	Craig-Bampton method of substructuring
CMIF	complex mode indicator function
FE	finite element model
FRF	frequency response function
MCFS	method of constraint for fixture and subsystem
MPC	multi-point constraint
TS	transmission simulator - the fixture attached to the experimental substructure of interest
dof	degree of freedom
sdof	single degree of freedom
p	modal dof of the experimental substructure with fixed boundary
q	modal dof of free modes extracted from experimental substructure with TS attached
s	free modal dof of the transmission simulator
x	physical displacement dof
ω	frequency in radians per second
ζ	modal damping ratio
K	stiffness matrix
L_{fix}	reduction matrix applying fixed boundary constraint to experimental equations of motion
M	mass matrix
T	transformation matrix to convert free modal model to modified CB model
Φ	mode shape matrix extracted for experimental substructure with TS attached
Ψ	free mode shape matrix of the TS
Γ	eigenvectors resulting from fixed boundary constraint of experimental equations of motion
b	subscript for the fixture or boundary
fix	subscript for the fixed boundary modes of the experimental substructure with TS as the boundary
$free$	subscript for the free modes obtained in the modal test of the experimental substructure with TS
+	superscript indicating the Moore-Penrose pseudo-inverse of a matrix

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1) Abstract

Experimental dynamic substructures in both modal and frequency response domains using the transmission simulator method have been developed for several systems since 2007. The standard methodology couples the stiffness, mass and damping matrices of the experimental substructure to a finite element (FE) model of the remainder of the system through multi-point constraints. This can be somewhat awkward in the FE code. It is desirable to have an experimental substructure in the Craig-Bampton (CB) form to ease the implementation process, since many codes such as Nastran, ABAQUS, ANSYS and Sierra Structural Dynamics have CB as a substructure option. Many analysts are familiar with the CB form. A square transformation matrix is derived that produces a modified CB form that still requires multi-point constraints to couple to the rest of the FE model. Finally the multi-point constraints are imported to the modified CB matrices to produce substructure matrices that fit in the standard CB form. The physical boundary degrees-of-freedom (dof) of the experimental substructure matrices can be directly attached to physical dof in the remainder of the FE model. This paper derives the new experimental substructure that fits in the CB form, and presents results from an analytical and an industrial example utilizing the new CB form.

Keywords – Experimental Dynamic Substructures, Substructuring, Craig Bampton

2) Introduction and Motivation

Experimental dynamic substructuring has experienced a resurgence in the last ten years. Multiple groups have been motivated to couple experimental substructures with FE substructures to obtain full system response. In general, one cannot couple the physical connection dof of the experimental substructure to the physical dof of the FE model because small errors in the experimental model will cause the coupling to be so ill conditioned that the effort will fail. There are additional challenges including:

1. Rotational connection dof are difficult to measure but can be important;
2. Translation connection dof may not be measurable either (often the connection dof are in a joint interface where transducers cannot be installed);
3. The connection dof may not actually be discrete, i.e. the connection may be a large surface contact;
4. The basis vectors from a standard free modal test may not span the space of the true connected motion well;
5. The joint stiffness and damping are often uncharacterized and usually ignored.

A method using an instrumented fixture known as a transmission simulator (TS), originally dubbed the method of constraint for fixture and subsystem (MCFS)[1], mitigates these problems. By attaching the fixture to the desired substructure in the same way it will be attached to the rest of the system, which will be modeled as a FE substructure, the joint stiffness and damping are captured. The TS can be instrumented at only translational dof that capture the motion of the connected TS in a truncated set of the free modes of the TS. Generally, the TS is a relatively simple structure that can be modeled with FE to help plan where to mount the instrumentation. The fixture is designed so that accelerometers may be mounted in convenient locations and directions. Ultimately the generalized dof of the TS are used to couple the experimental substructure to the FE model of the rest of the system. Because the generalized dof inherently contain the rotational dof, these are no longer being neglected. As long as the retained modes of the TS capture the connected motion, the method can even capture continuous, not just discrete, connections. For this reason, the method becomes a tremendous tool for providing a reduced order model. Originally, the TS method utilized multi-point constraints (MPC's) to couple the experimental substructure generalized coordinates to the FE model of the rest of the system, which removed most of the ill conditioning that is seen when one attempts to couple the measured physical dof directly. This improvement is due to a least squares fitting of the physical motion to the generalized dof that does not require that the errors in the experimental measurements have perfect continuity with the physical FE dof to which they will ultimately be attached.

However, this approach has been utilized mostly in third party codes such as MATLAB for the coupling, since FE codes often do not allow MPC's to couple generalized to generalized or generalized to physical dof. This makes it awkward to implement the experimental model directly in the FE code, which would be the ideal approach for the FE analyst. However, the Craig Bampton substructure is already implemented into several FE codes such as NASTRAN, ABAQUS, ANSYS and the Sierra Structural Dynamics code at Sandia National Laboratories. Researchers have developed a couple of methods to utilize the CB form of the TS method in FE codes[2]. Here, another transformation is developed, dubbed the Craig-Mayes method, which transforms the free modes from the experimental model with the TS mode shapes into a modified CB form. This method preserves the experimentally extracted modal parameters exactly.

This paper will present the theory first, an analytical problem applying the method next, and finally an industrial problem applying the method. Some discussions on maintaining good conditioning for the matrices follows, and then conclusions are presented.

3) Theory

Consider an experimental substructure tested with the TS fixture attached. An experimental substructure that can be implemented in the Craig-Bampton form is desired. An example, which will be developed fully hereinafter, is the beam pictured in Figure 1. The red beam is the experimental structure for which a substructure in the CB form is desired. It is tested in a free-free modal test with the TS fixture, the magenta beam, attached. The goal is to transform the test results so that there is a substructure of the red beam that fits in the CB form. To achieve this, the magenta TS must be subtracted. The modal test will produce modal parameters associated with the *q* dof. The TS has free modal parameters associated with the *s* dof, and the final desired substructure in the CB form will have stiffness, mass and damping matrices associated with the physical boundary dof, x_b, and the fixed boundary modal dof *p*.

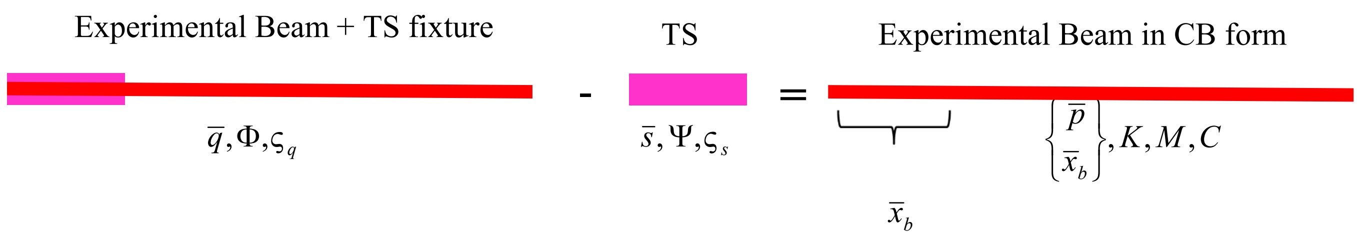

Figure 1 - Example Experimental Substructure - Tested Structure - TS = Experimental Substructure

Generally, there is a FE model of the TS. The FE model is used in test planning to define measurement locations that will achieve independent mode shape measurements for all free modes of the TS slightly beyond the frequency band of interest. The TS fixture is thus instrumented. The transmission simulator hardware is attached to the experimental substructure and the free TS mode shapes are assumed to span the space of the motion when connected to the experimental substructure. How well it spans the actual connection motion space affects the fidelity of the substructure model. Ultimately, the TS stiffness, mass and damping will be subtracted from the experimental substructure, so that the experimental substructure may then be coupled with the FE model of the rest of the system. The modal parameters from a free modal test of the experimental substructure with the TS attached can be used to produce the following equations of motion as

$$\left[\omega^2{}_{free} + j2\omega\omega_{free}\zeta_{free} - \omega^2 I\right]\bar{q} = 0 \tag{1}$$

where the subscript *free* represents the set of modes obtained from the experimental modal test of the experimental substructure attached to the TS in which there are generally no additional constraints added to the structure (The structure is typically suspended by bungee cords or some very soft suspension whose mass, stiffness and damping are considered negligible). The mass-normalized mode shapes derived from the test will be contained in the measured mode shape matrix, Φ. For convenience, the rest of this derivation will drop the damping matrices, but they may easily be included. Now we wish to derive a square matrix transformation, *T*, that will convert eqn. (1) to a modified CB form. Here we consider the generalized coordinates, *p* as the fixed-boundary modal coordinates and the generalized coordinates, *s* as the coordinates that account for the motion of the TS, which is considered the boundary of the experimental substructure as

$$\bar{q} = T\begin{Bmatrix}\bar{p}\\ \bar{s}\end{Bmatrix} \quad . \tag{2}$$

The first constraint ties the TS to the tested structure. Use the modal approximations to set the motion of the experiment on the boundary (TS dof) to match the free modal motion of the TS as

$$\Phi_b\bar{q} \approx \Psi_b\bar{s} \tag{3}$$

where the subscript *b* dof will actually be a subset of the boundary dof where the measurements are made, Φ is the experimental mode shape and Ψ is the chosen truncated set of free modes of the TS. Ψ usually comes from a TS FE model, but could also be measured. Then the relation between *q* and *s* is

$$\bar{q} = \Phi_b^+ \Psi_b \bar{s} \tag{4}$$

where the + sign represents the Moore-Penrose pseudo inverse. This provides the *s* portion of the transformation, *T*.

To obtain the fixed boundary modal dof, *p*, fix the boundary dof with

$$\bar{x}_b = \Phi_b \bar{q} = 0 \quad . \tag{5}$$

Previous work[3] has shown that a practical way to accomplish eqn. (5) is to fix the TS dof with

$$\Psi_b^+ \Phi_b \bar{q} = s = 0 \quad . \tag{6}$$

With Rixen's primal assembly[4], the modal dof are replaced with

$$\bar{q} = L_{fix} \bar{\eta} \tag{7}$$

which is substituted back into eqn. (6) to obtain

$$\Psi_b^+ \Phi_b L_{fix} \bar{\eta} = 0 \quad . \tag{8}$$

Since $\bar{\eta}$ can be anything, depending on the forcing motion, L_{fix} is chosen to guarantee satisfaction of the constraint as

$$L_{fix} = null(\Psi_b^+ \Phi_b) \quad . \tag{9}$$

Pre and post-multiply eqn.(1) using the transformation L_{fix} appropriately to give

$$L_{fix}^T \left[\omega^2{}_{free} - \omega^2 I \right] L_{fix} \bar{\eta} = 0 \quad . \tag{10}$$

Solve eqn.(10) to get the eigenvectors, Γ, and the eigenvalues to uncouple the dof, *p*. Then the relationship between *q* and the fixed boundary dof, *p,* is

$$\bar{q} = L_{fix} \Gamma \bar{p} \tag{11}$$

which provides the rest of the transformation written from eqn.(4) and (11) as

$$T = \left[L_{fix} \Gamma \quad \Phi_b^+ \Psi_b \right] \quad . \tag{12}$$

Pre multiplying eqn.(1) by the transpose of *T* and substituting eqn.(2) into eqn.(1) for *q* yields the following transformed equations of motion for free vibration

$$\left[\begin{bmatrix} `\omega^2_{fix\backslash} & K_{ps} \\ K_{ps}^T & K_{ss} \end{bmatrix} - \omega^2 \begin{bmatrix} I & M_{ps} \\ M_{ps}^T & M_{ss} \end{bmatrix} \right] \begin{Bmatrix} \bar{p} \\ \bar{s} \end{Bmatrix} = 0 \tag{13}$$

for which the eigenvalue and eigenvector solution have not changed from eqn.(1). It has exactly as many dof as eqn.(1), but now they have been transformed to the fixed base modes associated with *p* and the TS modes which were on the boundary as modal dof *s*. The upper left portion of the matrices is diagonal. Now there are coupling terms between the fixed base modes and the TS motion. The shapes associated with *p* are $\Phi L_{fix} \Gamma$ which one can see by pre-multiplying eqn. (11) by Φ. To obtain the experimental substructure without the TS attached, simply subtract the TS stiffness and mass from the lower right partition which corresponds to the boundary motion, as

$$\left[\begin{bmatrix} `\omega^2_{fix\backslash} & K_{ps} \\ K_{ps}^T & K_{ss} - `\omega^2_{TS\backslash} \end{bmatrix} - \omega^2 \begin{bmatrix} I & M_{ps} \\ M_{ps}^T & M_{ss} - I \end{bmatrix} \right] \begin{Bmatrix} \bar{p} \\ \bar{s} \end{Bmatrix} = 0 \quad . \tag{14}$$

This is almost in the form of CB matrices, but the generalized dof, *s*, must be converted to physical dof, x_b, to couple it with the FE model of the rest of the system in codes with CB substructure capability.
Since

$$\bar{x}_b = \Psi_b \bar{s} \tag{15}$$

write a transformation

$$\begin{Bmatrix} \bar{p} \\ \bar{s} \end{Bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \Psi_b^+ \end{bmatrix} \begin{Bmatrix} \bar{p} \\ \bar{x}_b \end{Bmatrix} \tag{16}$$

and, similar to eqn. (10), pre and post-multiply eqn.(14) appropriately by this transformation to produce the modified CB form as

$$\left[\begin{bmatrix} \omega_{fix}^2 & K_{ps} \\ K_{ps}^T & \Psi_b^{+^T}\left[K_{ss} - \omega_{TS}^2\right]\Psi_b^+ \end{bmatrix} - \omega^2 \begin{bmatrix} I & M_{ps} \\ M_{ps}^T & \Psi_b^{+^T}\left[M_{ss} - I\right]\Psi_b^+ \end{bmatrix}\right]\begin{Bmatrix} \bar{p} \\ \bar{x}_b \end{Bmatrix} = 0 \quad . \tag{17}$$

This form is slightly modified from the normal CB in that there are non-zero stiffness coupling terms, K_{ps}, which are zero in the normal CB form. However, this can now be implemented directly in the FE model as a CB type substructure. The author calls this form Craig-Mayes. Note the damping can be carried along in an analogous way. One disturbing issue about the Craig-Mayes form is that eqn. (17) has become rank deficient, unlike eqn. (14), so it is not useful to solve eqn. (17) by itself. This is because the length of vector $\bar{x}_b$ is greater than the length of vector $\bar{s}$. However, as pointed out by Simmermacher[5], when coupled with another FE substructure, the entire system will not be rank deficient because of the stiffness and mass added by the FE substructure to the $\bar{x}_b$ dof. As a final note, the basis shapes chosen in eqn. (3) need not necessarily be the free shapes of the TS. Other basis shapes may prove to give more accurate or robust solutions. If one uses the free shapes, the result in coupling the Craig-Mayes substructure of eqn.(17) with the FE model is exactly what is obtained with the standard TS method when eqn. (1) is coupled to a FE substructure through MPC's.

4) Beam Example

In this analytical example, a beam is the experimental structure. A short beam is attached at one end which is the TS. This system is converted to the Craig-Mayes substructure and coupled to a FE model of a second beam to produce the response of two beams attached to one another. The results are compared to the FE model of the entire system which acts as the truth model. Figure 2 shows the beam substructures. In this problem the right beam is 15 units long and the TS simulator is a short beam 4 units long that overlaps the left most 4 units of the right beam. The experimental structure is the right beam with TS beam attached. The FE substructure is the left beam that is 20 units long and is ultimately to be coupled with the right beam in the substructuring process. The FE substructure overlaps the right beam by four units. Figure 3 shows the first four elastic bending modes of the "truth" assembly. The circle/asterisk dof in the middle are where the two beam overlap and are connected.

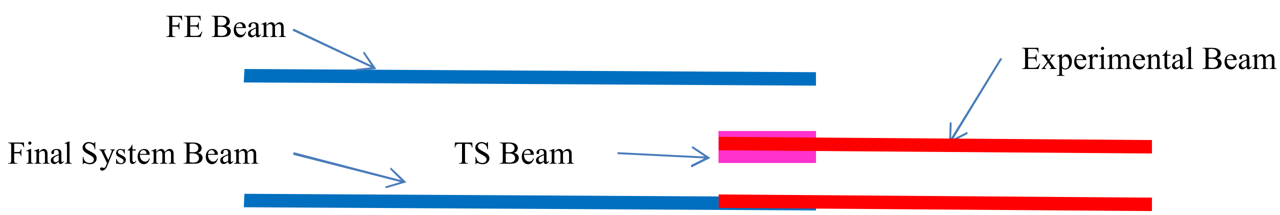

Figure 2 - Beam Substructures and Final Assembled Beam

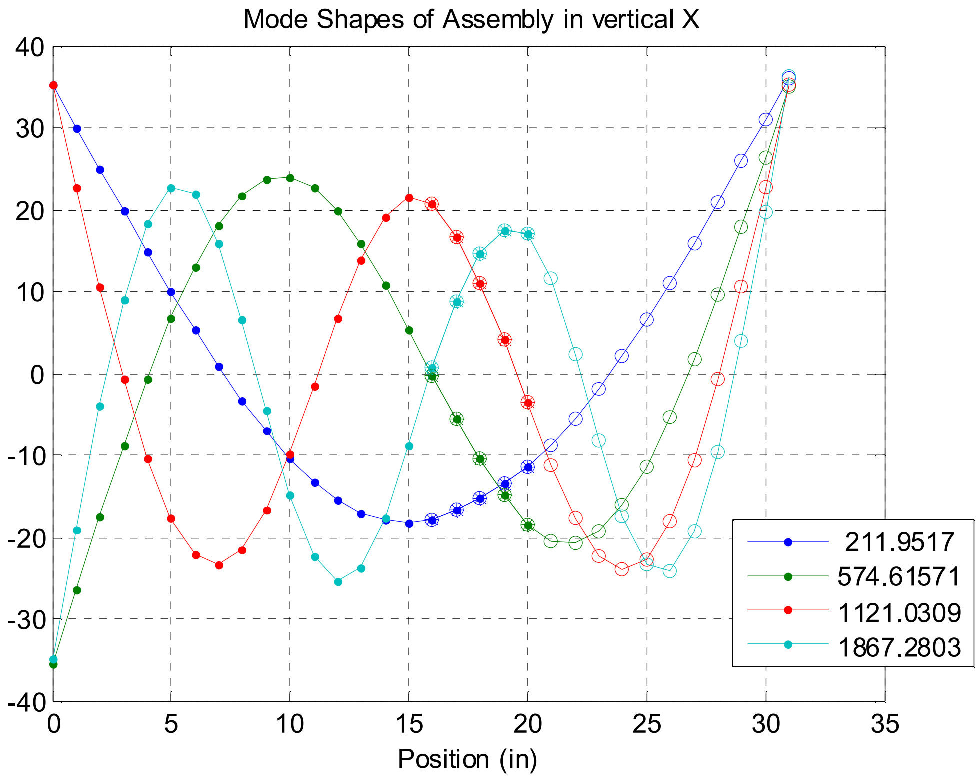

Figure 3 - Bending Mode shapes of Truth Beam

The FE model of seven modes (up 5876 Hz) of the right beam with the short TS attached was used to create the virtual test and the resulting experimental structure. The TS had six measured dof, three vertical translations and three horizontal translations at the three nodes located at the TS beam left end, center, and right end. Four modes of the TS were retained (three rigid body modes and one elastic bending mode). The Craig-Mayes substructure was created using the TS shapes and the seven virtual test shapes. It had three fixed base modes and six connection dof. This was coupled to the FE model of the 20 unit long left beam at the six "measured" connection dof. The frequency comparisons of the truth beam and the substructured beam are given in Table 1.

Table 1 - Comparison of Beam Truth Frequency and Craig-Mayes Substructure Frequency (Elastic Modes Only)

Truth Frequency (Hz)	Substructured Frequency (Hz)	Error in Frequency (%)
212.0	209.7	-1.1
574.6	571.5	-0.5
1,121.0	1,131.4	0.9
1,867.3	1,877.4	0.5
2,750.2	2,782.4	1.2
3,341.7	3,398.4	1.7
3,949.6	4,034.7	2.2
5,115.9	5,167.6	1.0
5,965.5	5,946.9	-0.3

* Highest frequency experimental substructure mode retained was 5,876 Hz

5) Industrial Example

The industrial hardware consisted of a shell with dozens of internal components. The shell is chosen as the TS, and a FE model of the shell exists. The shell is relatively easy to model, but the internal parts are not easily modeled with FE. Dozens of internal accelerometers measured response of internal components of interest. Figure 4 shows a schematic representation of the test setup.

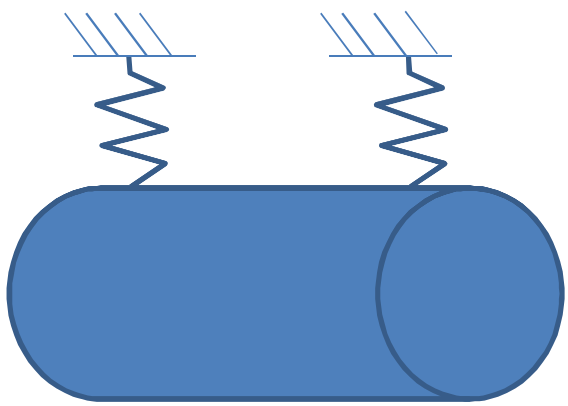

Figure 4 - Schematic of Free Modal Test of Shell with Internal Components

5.1) Description of Transmission Simulator Model

A FE eigenvalue analysis of a large number of the external translation dof of the empty shell for the first 200 free modes was performed. From this analysis, 38 modes of the TS were chosen to attempt to obtain response out to 2,000 Hz. Analysis to select measurement dof on the outside of the shell was performed. The algorithm selected measurement dof by attempting to keep the condition number of the mode shape matrix to a minimum. The condition number for the selected 84 measured dof and 38 modes was 3.54. When one more mode was added the condition number jumped to 7.2. The frequency of the 38th mode was 2,285 Hz. Seven modes had frequencies above the desired 2,000 Hz. After the dof selection was performed, the external shell was instrumented per the dof selection analysis. Optimal driving points based on the mode shapes of the free TS were also chosen.

5.2) Modal Test of Industrial Structure with Transmission Simulator

The structure was supported by bungee cords, and a modal test was performed with an impact hammer. Twelve reference input locations were used in the analysis. Each reference was analyzed separately because the structure was slightly nonlinear, so multi-reference algorithms could not handle the frequency shifts of like modes extracted from one reference to another. The SMAC algorithm[6] in automated mode was utilized to extract the modes. The option to extract real modes was utilized. Almost 500 modes were extracted in the twelve data sets. Many of these were redundant extractions of the same mode already in another data set, and some modes were poorly excited. The modal parameters were culled to 110 elastic modes with the six rigid body modes (calculated analytically from mass properties) for the experimental model with the TS associated with eqn. (1).

5.3) Craig-Mayes Experimental Substructure Coupled to FE Model - Comparison with Free Modal Model

The TS fixture is the actual part that the internal components are mounted within. This allows us to have a convenient "truth" comparison. A Craig-Mayes substructure was developed by taking the experimental model and subtracting the 38 mode FE TS. The Craig-Mayes substructure was then added to a 200 mode modal substructure of the shell derived from the FE model of the shell. This was coupled together in MATLAB. This result is utilized to attempt to reproduce the original modal test FRF data. In Figure 5 the top level complex mode indicator function (CMIF) is plotted for the experimental data data (blue), the extracted modal model (green), and the Craig-Mayes substructure coupled with the 200 mode FE model of the shell (magenta). The CMIF plots incorporate all the FRF data together in one plot. Differences between the experimental and modal model CMIFs show errors in the modal fitting. Differences between the modal model and the substructured CMIF show errors due to the truncated modal model used for the TS and errors due to the FE model. To the extent the TS mode shapes do not span the space of the true experimental motion, constraining errors are introduced which can move the resonant frequencies and change the amplitudes of certain mode shapes. If one compares the modal model (green) and substructured CMIFs (magenta), one can see that the results below 1000 Hz are nearly identical, but the substructured CMIF results above 1000 Hz are not quite as good as the original modal extraction for the experiment. The constraining process pollutes the higher frequency modes either because the 38 TS mode shapes did not perfectly reproduce the motion that was actually experienced in the modal test on the shell, and the FE model of the shell is not perfect.

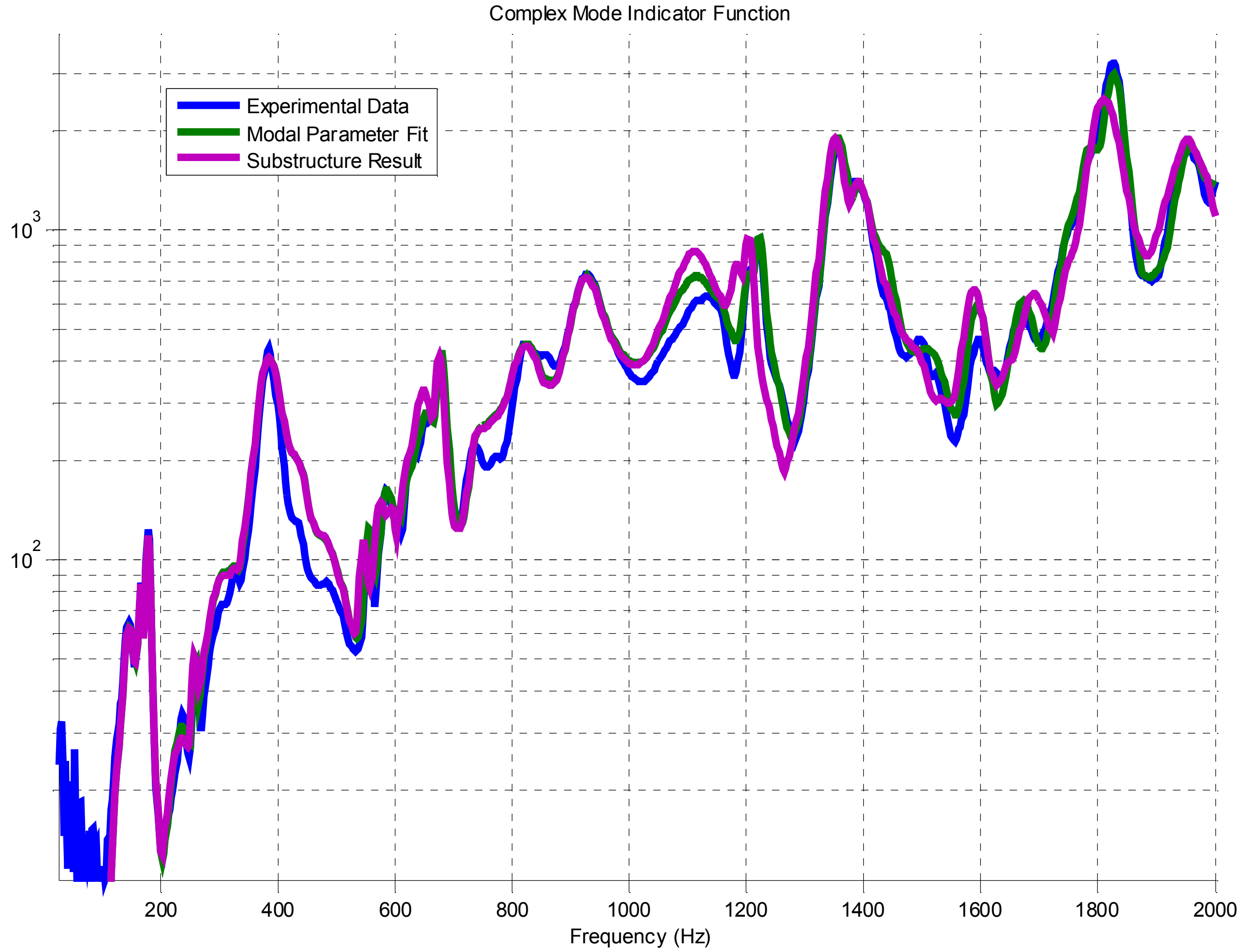

Figure 5 - CMIF Experiment (Blue) vs Modal Model (Green) vs Craig-Mayes Substructure Added to 200 modes of FE Shell (Magenta)

Figure 6 shows sample experimental FRFs (blue), FRFs synthesized from extracted modal parameters (green), and FRFs synthesized from the Craig-Mayes substructure coupled to the 200 mode FE model of the shell (magenta). The pattern is similar to the CMIFs in that the accuracy of the substructured FRFs deteriorates some with higher frequency. However, if one considers only the amplitude for defining specification envelopes, and one is willing to accept a factor of two in the uncertainty of the amplitude at certain frequencies, even the high frequency results in the magenta curves of the substructured model might be considered "useful". The responses are all from different forcing input locations. The first row shows two responses on the outside shell. The second row shows two responses on substantial internal components. The third row shows two responses from small internal components.

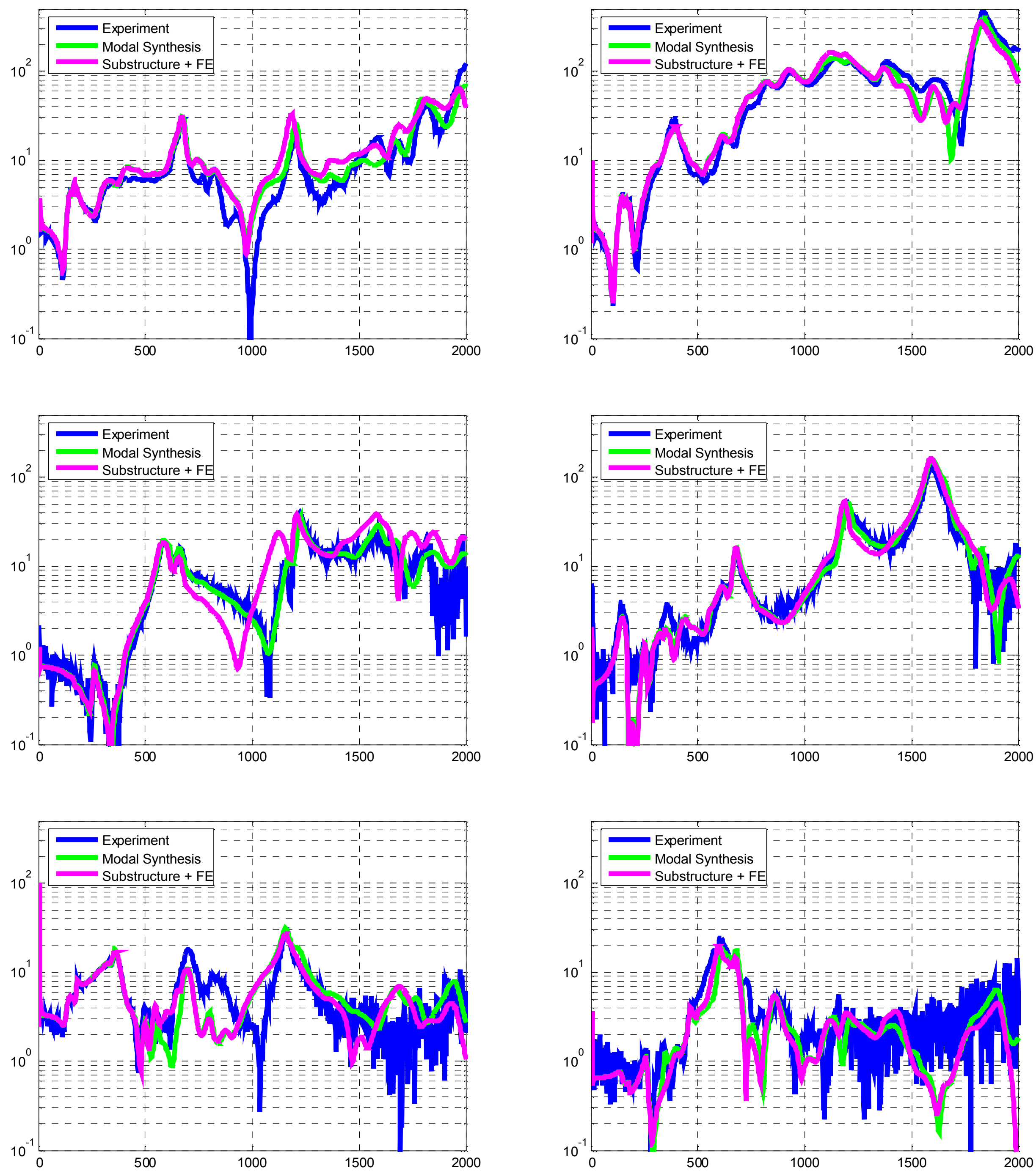

Figure 6 - Sample FRFs from Experiment (Blue), synthesized modal parameters (Green), and from the Craig-Mayes Substructure Plus 200 Mode FE Model of Shell (Magenta) - Row 1 external responses on shell, Row 2 substantial internal responses, Rows 3 small internal component responses

6) Discussion on Conditioning of the Matrices

As mentioned in Section 5.1, the condition of the TS mode shape matrix, Ψ , is kept low by using as few modes as possible to span the desired bandwidth and placing accelerometers at appropriate dof to keep the mode shapes independent. The number of measured dof is large enough so the least squares estimate of *s* is accurate (typically 1.5 to 2 times the number of TS modes retained). The author has not studied the effect of increasing the condition number significantly above 4.

It was discovered that the condition number of a matrix from eqn. (4), $\Phi_b^+\Psi_b$, was found to be important in maintaining the conditioning of the entire substructuring problem. In the beam problem, when Ψ_b had four shapes, a condition number of 10.5 was calculated for $\Phi_b^+\Psi_b$. However, when Ψ_b was increased to five shapes, the condition of $\Phi_b^+\Psi_b$ was 1.02x10^{14}, even though the condition of Ψ_b was 2.3. With the large condition number of $\Phi_b^+\Psi_b$, the coupling in the physical dof gave negative eigenvalues for the stiffness matrix, and the coupling with the FE beam failed. In the industrial problem, the condition number of $\Phi_b^+\Psi_b$ was 130, which may be near the limit of allowing a successful substructuring problem even with condition number of Ψ_b at 3.5 as it was here. Adding one more mode to Ψ_b caused the condition number of Ψ_b to double to 7.2 and the condition number of $\Phi_b^+\Psi_b$ to go up to 134, but then the coupling with the FE model of the shell produced a negative eigenvalue when the eigen analysis of the full system was performed in MATLAB. A negative eigenvalue is not desirable in a FE code, and can cause a fatal error. Negative eigenvalues can be removed from either a substructure's mass or stiffness matrix using methods described in previous work[7]. If the negative eigenvalues are not too large, they can be removed with only minor degradation of the resulting solution.

7) Conclusions

The standard free modes transmission simulator (TS) substructuring capability has been augmented by providing a transformation to convert the free modes substructure to a modified Craig-Bampton form called the Craig-Mayes substructure. This form can fit directly into a FE code with the Craig-Bampton substructure capability to couple the Craig-Mayes substructure directly with an FE model of the complement of the full system to provide full system response calculations. The experimental substructure includes the damping that occurred in the experimental substructure as well as its connection to the next substructure. The theory was presented along with results from an analytical example and an actual industrial substructure with 116 experimental modes. The effects of the constraining process were noted. New insight for the conditioning of certain important matrices was presented.

8) Acknowledgments

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5 Technical conversation with Todd Simmermacher, Sandia National Laboratories Albuquerque, Simmermacher suggested moving the multi-point constraint inside the experimental substructure, so that it could connect directly to physical dof as the normal Craig-Bampton substructure does. I pointed out that this would introduce more singularities in the CB stiffness matrix since there were more physical connection dof than generalized connection dof. Simmermacher reminded me that the FE substructure should provide adequate stiffness and mass for the physical dof so that the full system matrices would not be unduly singular. March 2014.
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