

# Comparison of Multi-Axis and Single Axis Testing on Plate Structures

85<sup>th</sup> Shock and Vibration Symposium

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October 29<sup>th</sup>, 2014



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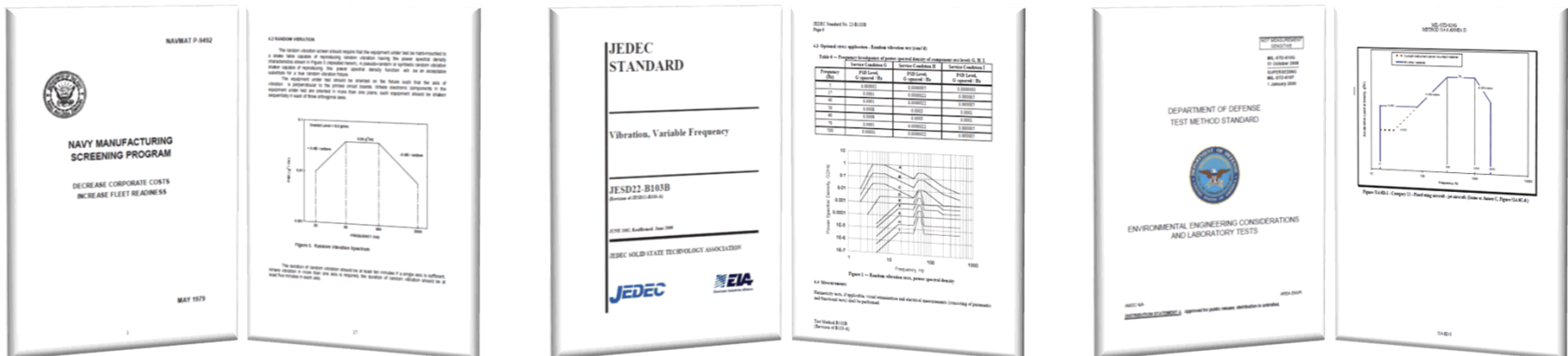
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# Outline

- Motivations & Objective
- Equipment
- Test Article
- Sensor Selection & Configuration
- Test Sequence
  - Axes
  - Levels
  - Coherence and Phase
- Overview of Test Results
  - Energy Levels
  - Peak Accelerations
  - Modal Contributions

# Motivations

- Sequential single axis testing has been firmly established as the preferred test method for environmental vibration characterization and analysis
  - MIL STD 810G: U.S. Department of Defense Environmental Test Standard
  - NAVMAT P-9492: U.S. Navy Manufacturing Screening Test Standards
  - JESD22-B103B: JEDEC Environmental Test Standards for Microelectronics



# Motivations

- Unfortunately, vibrations in real world environments are 3-dimensional and these vibrations can result in different failure modes and component lifecycles [1-5]
- Recent developments in electrodynamic shaker capabilities have enabled reliable and controllable simultaneous multi-axis testing [6,7]
- Multi-axis control makes possible true single axis testing by allowing control of off axis and rotational vibrations that may be present in uniaxial shakers [8]
- Model validation and system identification via single axis test results cannot account for off axis affects [9]

# Objective

- Through a collaboration of experimental and modeling work conducted on a given test article investigate:
  - The relationship between single axis and multi-axis vibration testing
    - How are fatigue life estimates influenced by stimulation from more than one axis?
    - Can results from a single axis test be used to predict multi-axis results for a given part structure?
    - What's the effect of multi-axis inputs on a test article's modal response?
  - The effect of coherence and/or phase relationships on the energy levels experienced by a part during biaxial or triaxial vibration testing
  - The benefit of single axis testing conducted on equipment capable of mitigating off-axis (and rotational) vibration

# Test Equipment

- Shaker System: Team Corporation Tensor™ 900



- Simultaneous or sequential excitation of X, Y, and/or Z axes
- Complete control of rotations around all axes

## Specifications

Table First Frequency	5,000 Hz
Test Frequency Range	10 - 5,000 Hz
Max Payload	9 lbs
Max Displacement	0.5 in
Max Acceleration (w/max payload)	10 g



- Controller Software: Spectral Dynamics JAGUAR Shaker Control and Analysis System

- Multi-Input and Multi-Output Control
- Input and Output Transformation for 6DOF Control



- Data Acquisition: National Instruments™ LabVIEW and NI PXIe-4496 Data Acquisition Modules

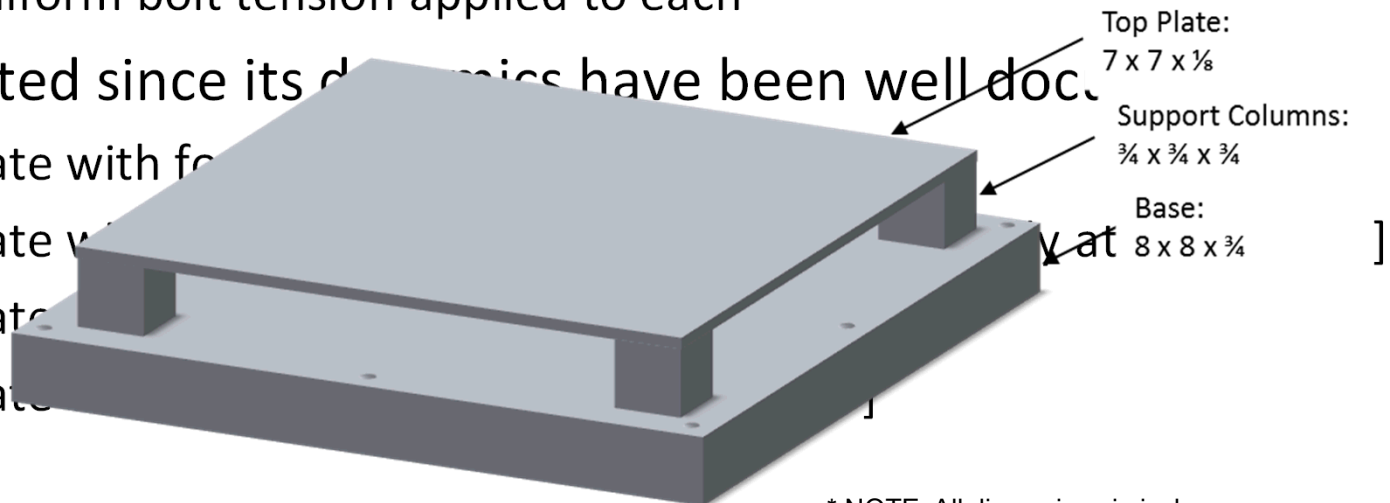


# Test Article

- A column supported square aluminum 6061 plate
  - Base, columns, and top plate made of a continuous piece of material
  - Both homogenous and isotropic
  - Eliminates added dynamics due to support boundaries
- Evenly spaced mounting holes
  - Eight (8) circumferential and one (1) central
  - Uniform bolt tension applied to each

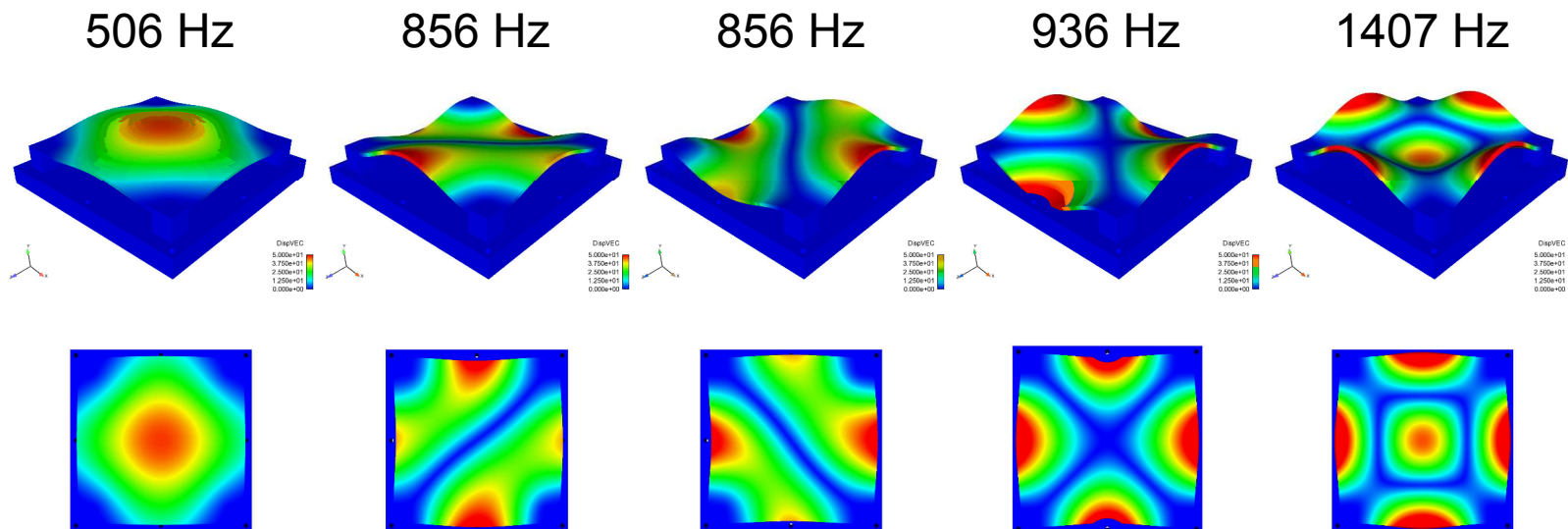
- Selected since its dynamics have been well doc.

- Plate with four
- Plate with four
- Plate with four
- Plate with four



\* NOTE: All dimensions in inches

- Finite Element Model
  - Used to predict dominant mode shapes and frequency components
  - Confirms that support columns are not dynamically active in frequency range of interest
- Primary Mode Shapes Identified



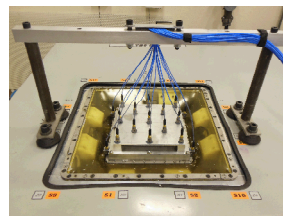
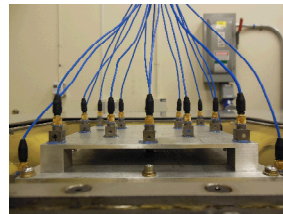
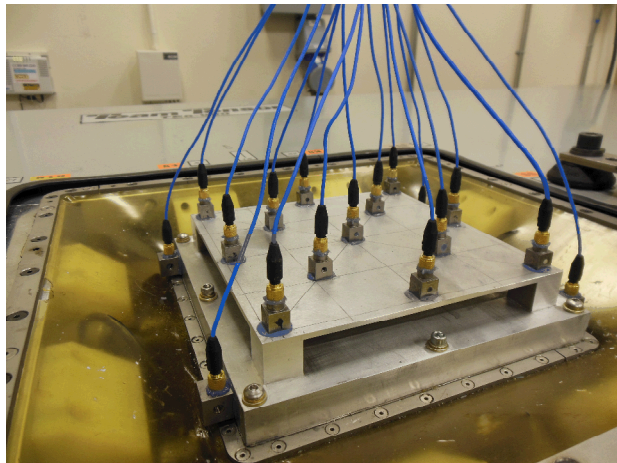


# Sensor Selection & Configuration

## ■ Control Accelerometers

### ■ PCB 356A15

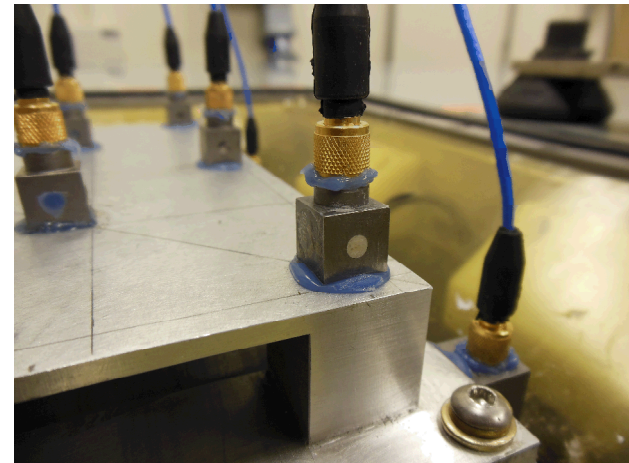
- Triaxial ICP Accelerometer
- Nominal Sensitivity: 100 mV/g
- Weight: 0.37 oz



## ■ Response Accelerometers

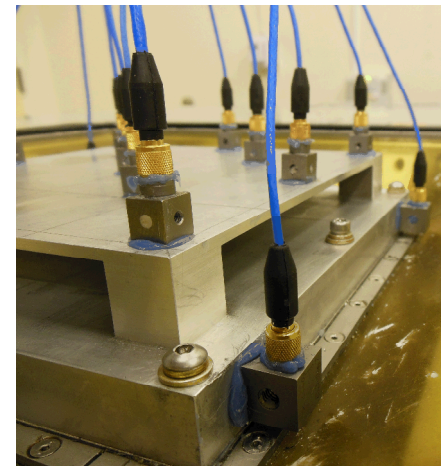
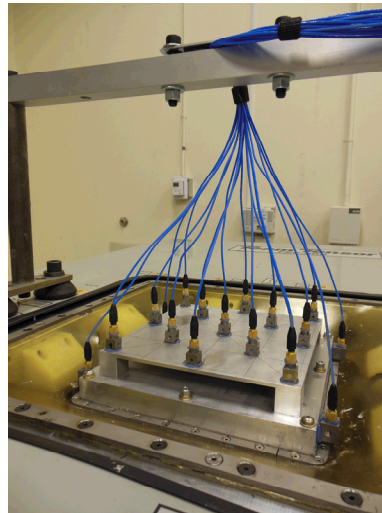
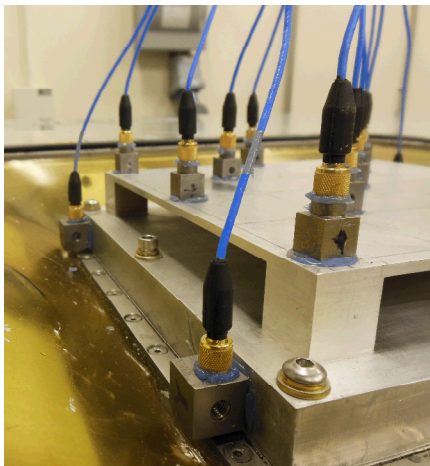
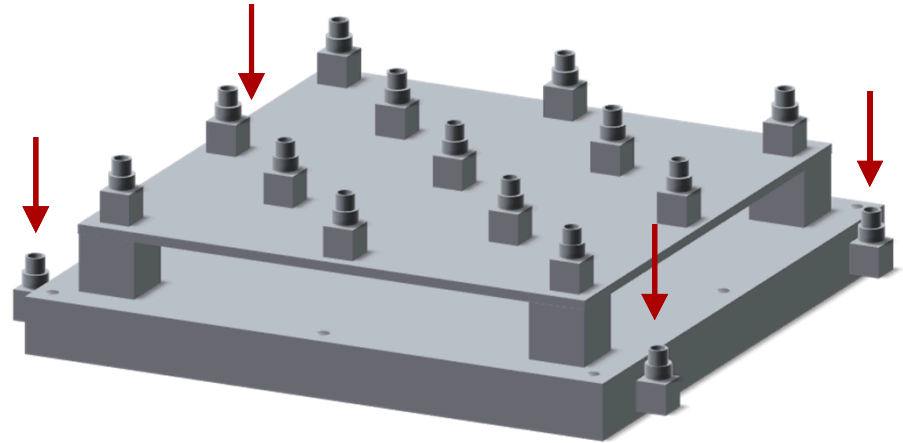
### ■ PCB 356A33

- Triaxial ICP Accelerometer
- Nominal Sensitivity: 10 mV/g
- Weight: 0.19 oz



# Sensor Selection & Configuration

- Control Sensor Placement
  - Symmetric about both lateral axes on base
  - Allows calculation of all base translational and rotational degrees of freedom

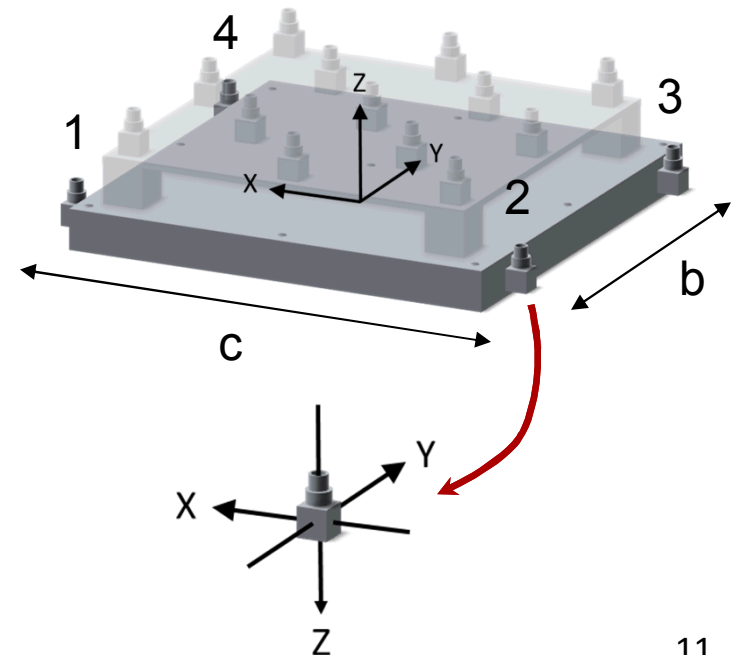


# Sensor Selection & Configuration

## ■ Control Transformation

- Given the acceleration from triaxial control accelerometers
  - $a_{x_i}$ ,  $a_{y_i}$ , and  $a_{z_i}$
- Calculate the base input translational and rotational acceleration [14]
  - $\vec{A} = H\vec{a}$

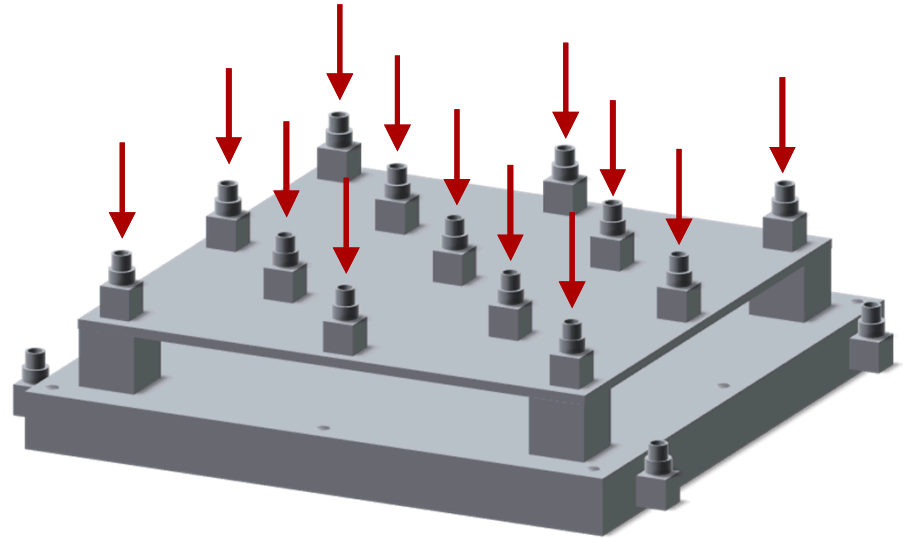
$$\underbrace{\begin{bmatrix} A_x \\ A_y \\ A_z \\ A_\psi \\ A_\theta \\ A_\phi \end{bmatrix}}_{\vec{A}} = \underbrace{\begin{bmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{1}{4} & 0 & 0 & -\frac{1}{4} & 0 & 0 & -\frac{1}{4} & 0 & 0 & -\frac{1}{4} \\ 0 & 0 & -\frac{g_0}{2b} & 0 & 0 & \frac{g_0}{2b} & 0 & 0 & \frac{g_0}{2b} & 0 & 0 & -\frac{g_0}{2b} \\ 0 & 0 & -\frac{g_0}{2c} & 0 & 0 & -\frac{g_0}{2c} & 0 & 0 & \frac{g_0}{2c} & 0 & 0 & \frac{g_0}{2c} \\ -\frac{g_0}{4b} & -\frac{g_0}{4c} & 0 & \frac{g_0}{4b} & -\frac{g_0}{4c} & 0 & \frac{g_0}{4b} & \frac{g_0}{4c} & 0 & -\frac{g_0}{4b} & \frac{g_0}{4c} & 0 \end{bmatrix}}_H \underbrace{\begin{bmatrix} a_{x_1} \\ a_{y_1} \\ a_{z_1} \\ a_{x_2} \\ a_{y_2} \\ a_{z_2} \\ a_{x_3} \\ a_{y_3} \\ a_{z_3} \\ a_{x_4} \\ a_{y_4} \\ a_{z_4} \end{bmatrix}}_{\vec{a}}$$



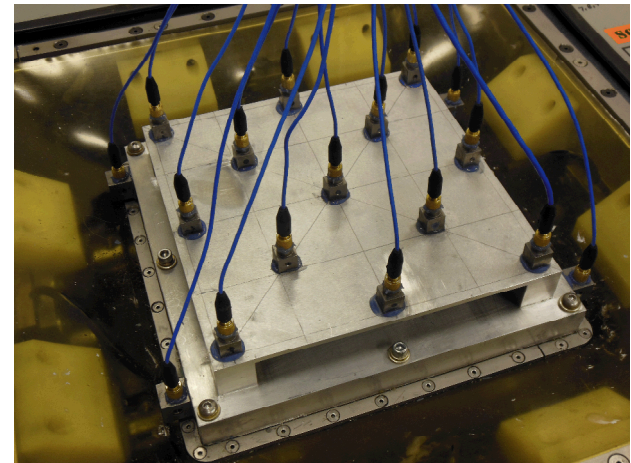
# Sensor Selection & Configuration

## ■ Reference Sensor Placement:

- Center of plate
- Each corner
- Mid-span of each side
- Along each diagonal placed evenly between the center and corner accelerometers



- The high number of accelerometers caused a mass loading affect on the top plate
- Their positions were selected to mitigate mode shape distortion
- FEA data confirms that modes are preserved although all frequencies were shifted lower

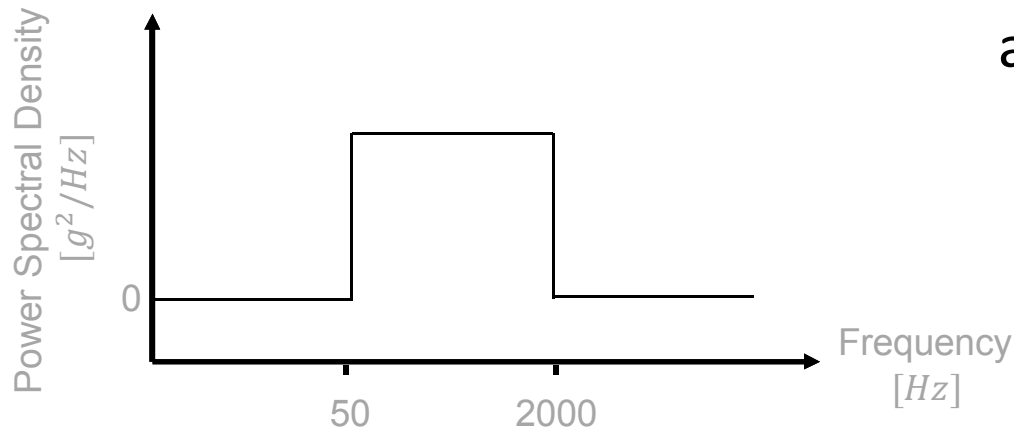


# Test Sequence

## ➤ Control Signal

- Axes
- Input Level
- Cross-Axis Coherence & Phase

- Band-limited white noise (50Hz – 2kHz)
- Causes simultaneous excitation of all frequencies within range
- All frequency dependence in stimulated responses can be attributed to plate dynamics



# Test Sequence

- Control Signal

- Axes

- Input Level

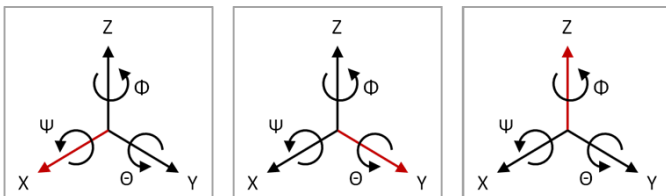
- Cross-Axis  
Coherence & Phase

- Single and Multi-Axis

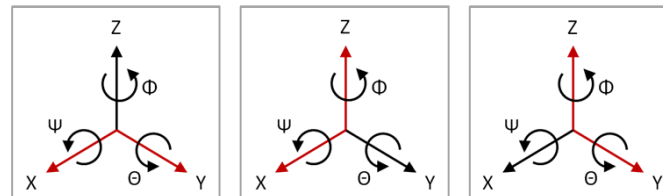
- Uniaxial: One translational axis at a time (X, Y, Z)
- Biaxial: Two translational axes at a time (XY,XZ,YZ)
- Triaxial: All three axes simultaneously (XYZ)

- All axes always controlled, but not all to full test levels

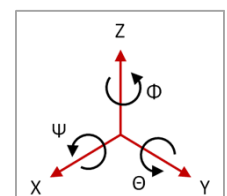
Uniaxial



Biaxial



Triaxial

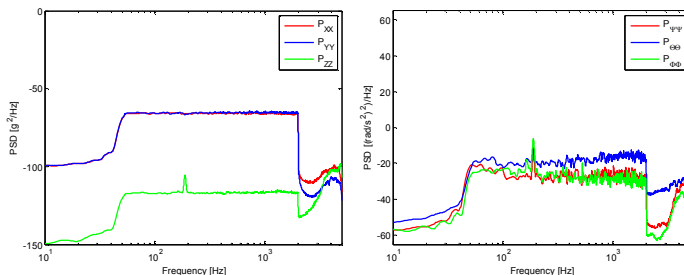




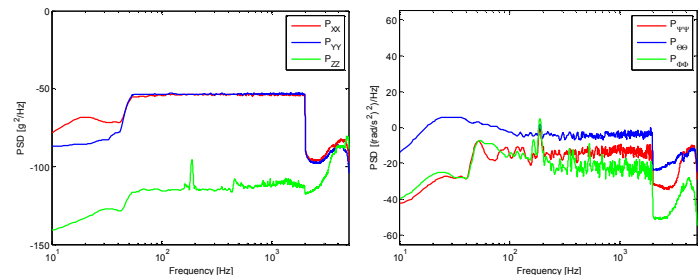
# Test Sequence

- Control Signal
- Axes
- Input Level
- Cross-Axis Coherence & Phase
- Input Acceleration Levels:  
low ( $1g_{rms}$ ) & high ( $2g_{rms}$ )
- Same acceleration level for all applicable axes
- All other DOFs controlled to low level

X & Y Translation Controlled (@ 1g)  
Z Translation & All Rotations Minimized



X & Y Translation Controlled (@ 2g)  
Z Translation & All Rotations Minimized

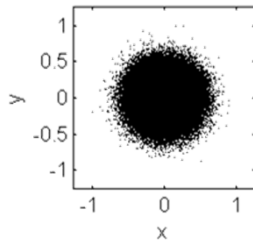


# Test Sequence

- Control Signal
  - Axes
  - Input Level
  - Cross-Axis Coherence & Phase
- Zero phase between all axes
  - Coherence is measure of relationship between two signals
  - Levels: low ( $\sim 0$ ), medium (0.50), and high ( $\sim 1$ )
    - Coherence with and between all other DOFs set at zero

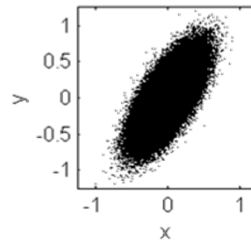
X,Y Unrelated

$$C_{XY} \cong 0$$



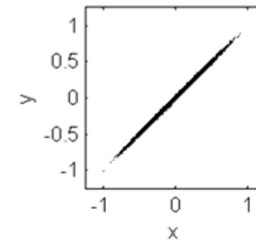
X,Y Partially Related

$$C_{XY} \cong 0.5$$



X,Y Highly Related

$$C_{XY} \cong 1$$





# Response Energy

- $e = e_x + e_y + e_z$ 
  - Where,  $e_n = \frac{1}{L} \sum_{j=1}^L v_n[j]^2 \quad \{\forall n = x, y, z\}$
- The energy of the  $i^{\text{th}}$  response accelerometer ( $e_{R_n[i]}$ ) was normalized by the energy of the control input ( $e_C$ )
  - $e'_{R_n[i]} = e_{R_n[i]} / e_C$
  - This accounts for the known input energy level differences between single and multi-axis test cases
- Then, the total normalized energy of the response accelerometers is given by
  - $E'_n = \sum_{i=1}^N e'_{R_n[i]} \quad \{N = 13 \text{ (total number of response accels)}\}$

# Response Energy

- Total normalized energy levels are comparable between multi-axis

This estimate was bounded by the maximum and minimum energy levels for multi-axis tests

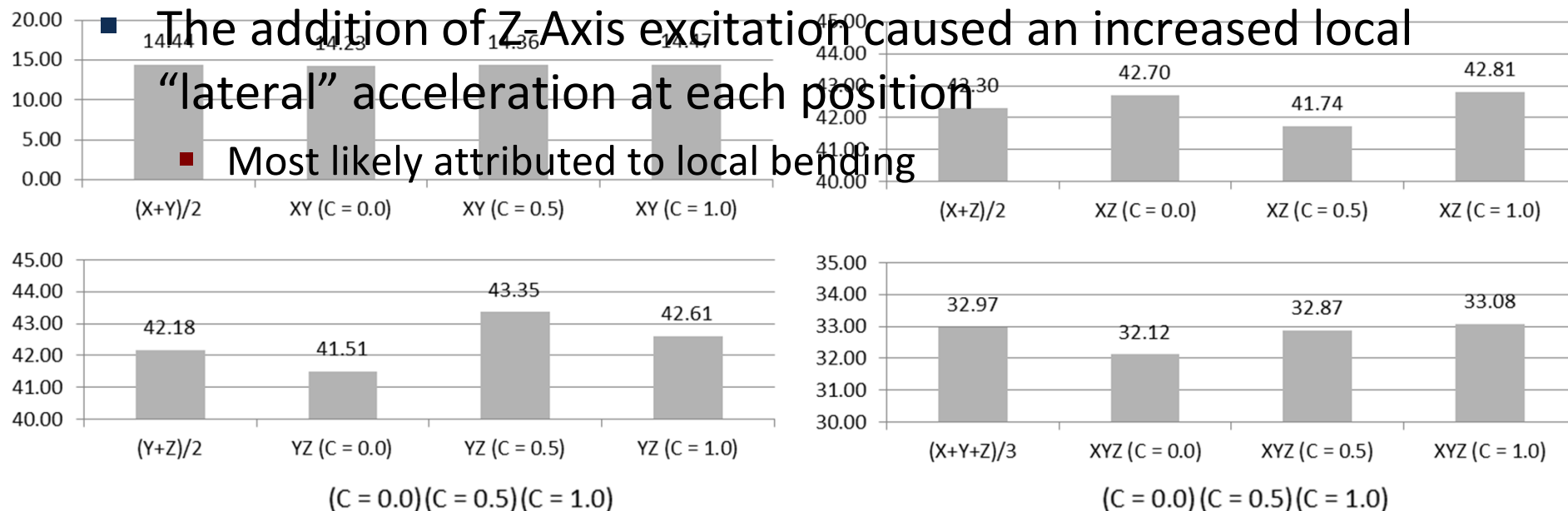
- Total no  
approxim  
single ax

∴  
It can't serve as either a reliable upper or lower bound for anticipated multi-axis energy levels

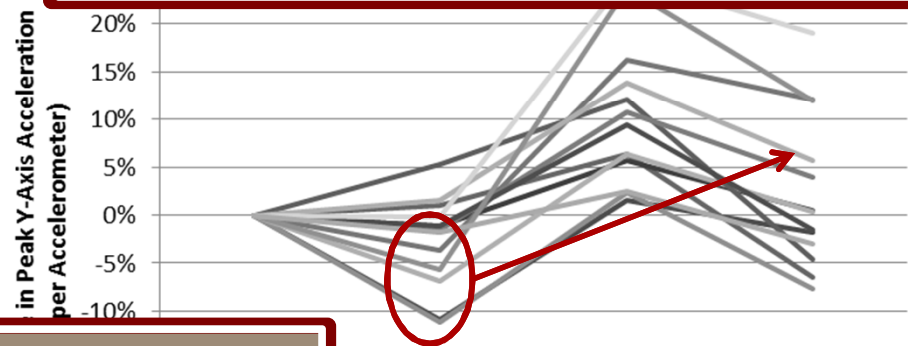
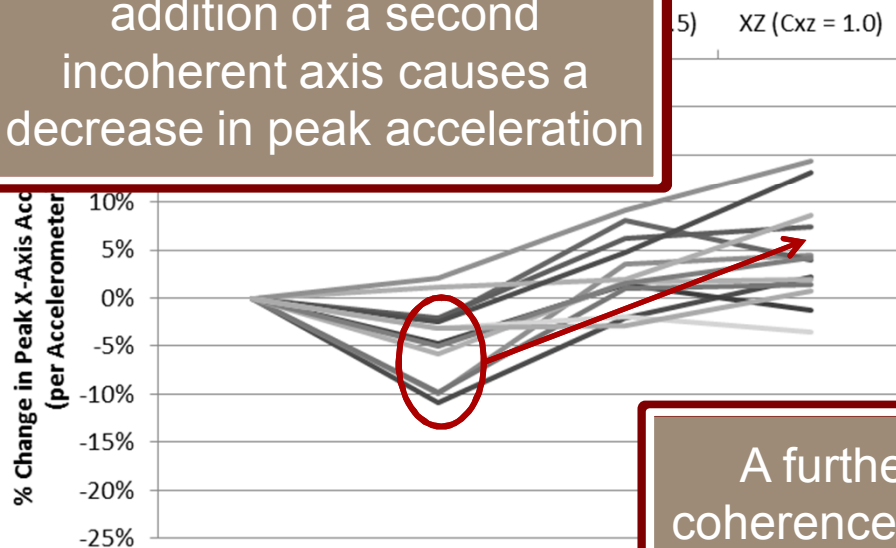
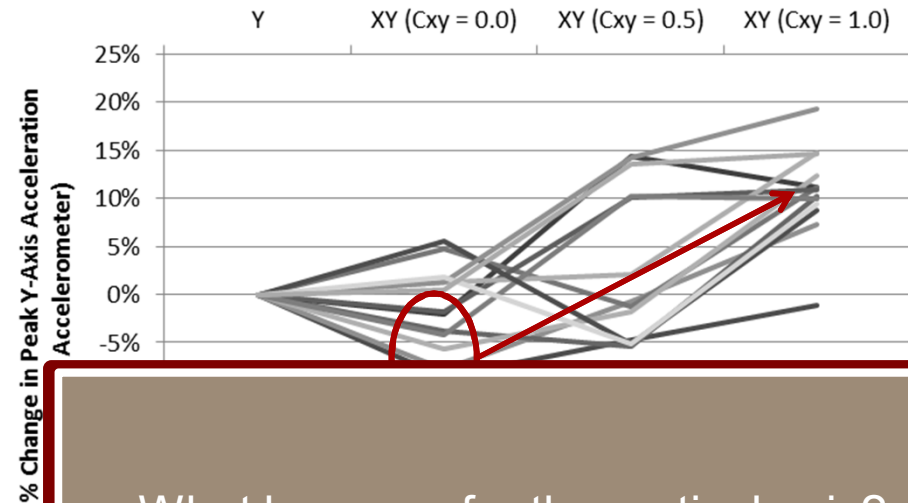
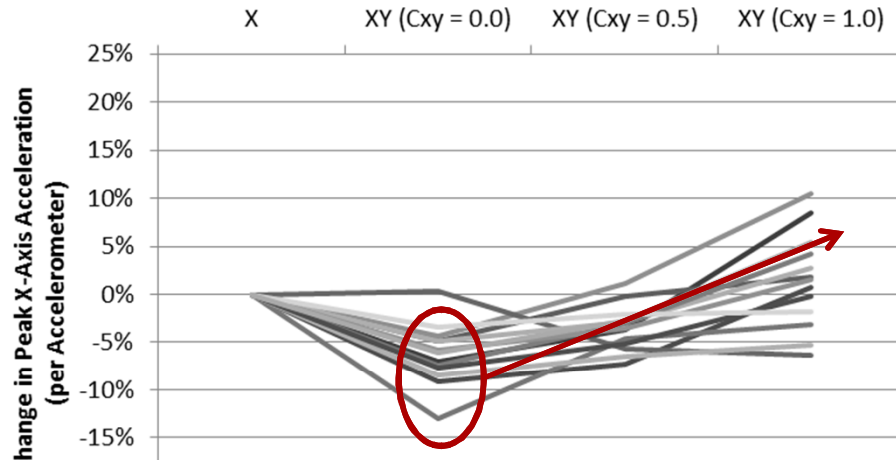
applicable

- The addition of Z-Axis excitation caused an increased local “lateral” acceleration at each position

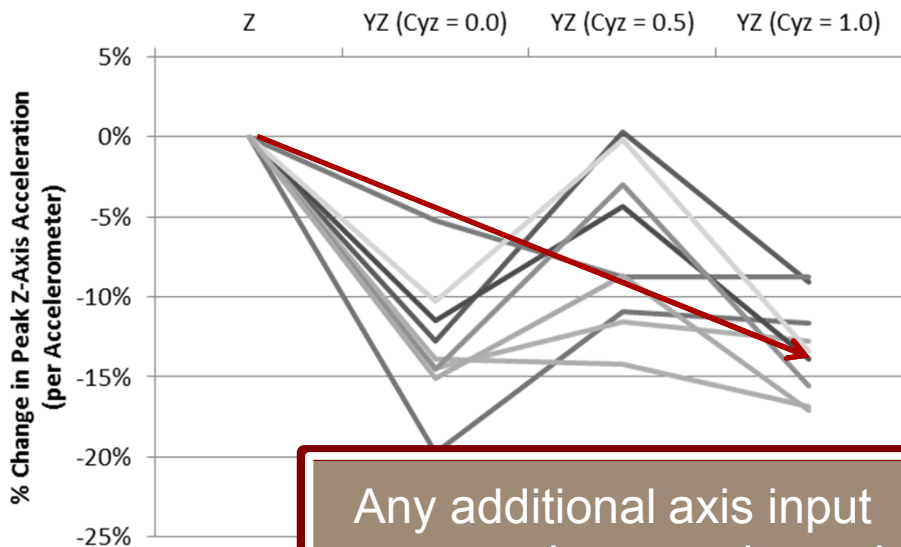
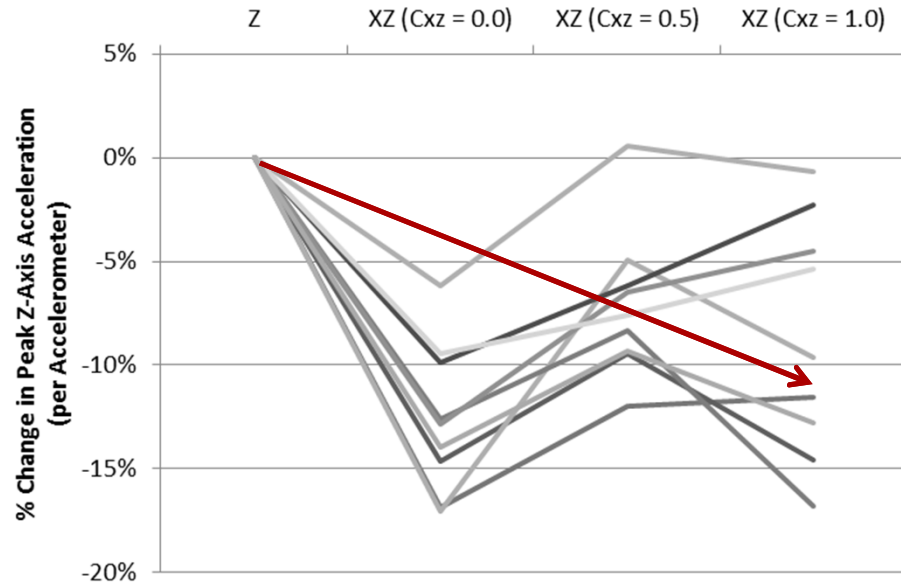
■ Most likely attributed to local bending



# Peak Acceleration

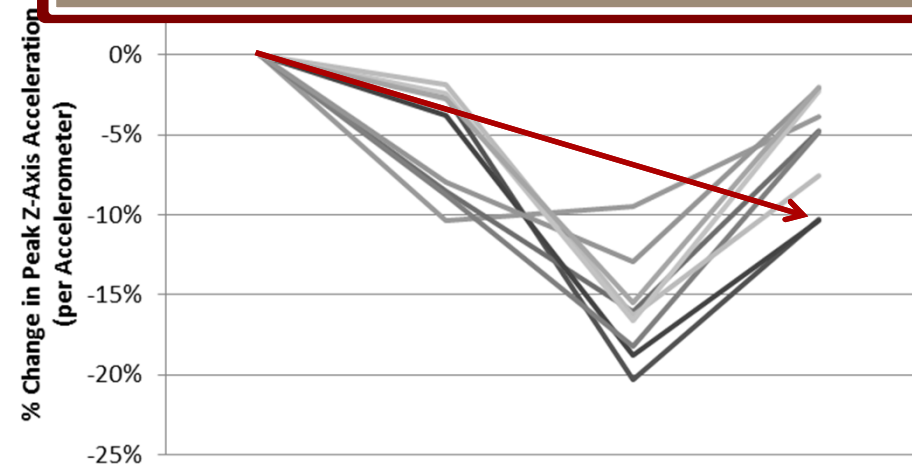


# Peak Acceleration



Any additional axis input causes a decrease in peak acceleration levels

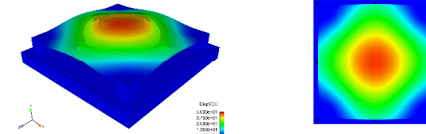
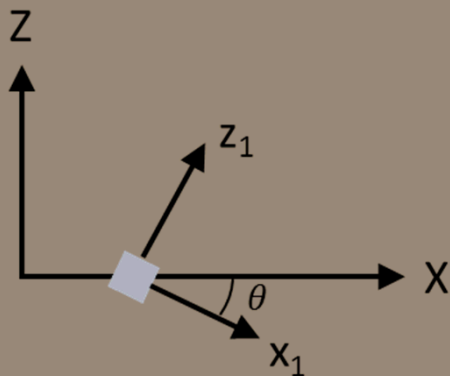
What happens for the vertical axis?



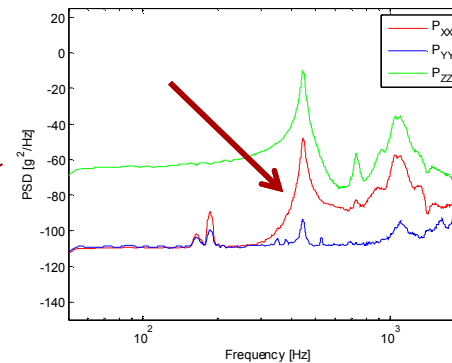
# Modal Response

- Responses for each test dominated by first mode shape

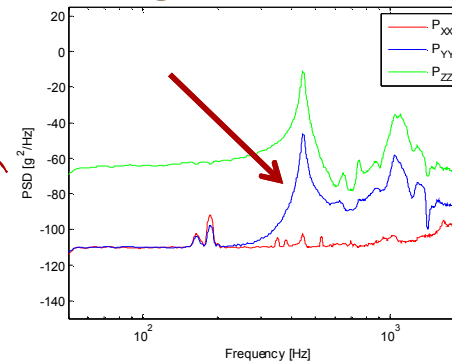
For edge accelerometers, the observed lateral response is due to plate bending. For a bent plate, a portion of the global z-axis acceleration appears locally as a x or y-axis acceleration



X Edge Accelerometer

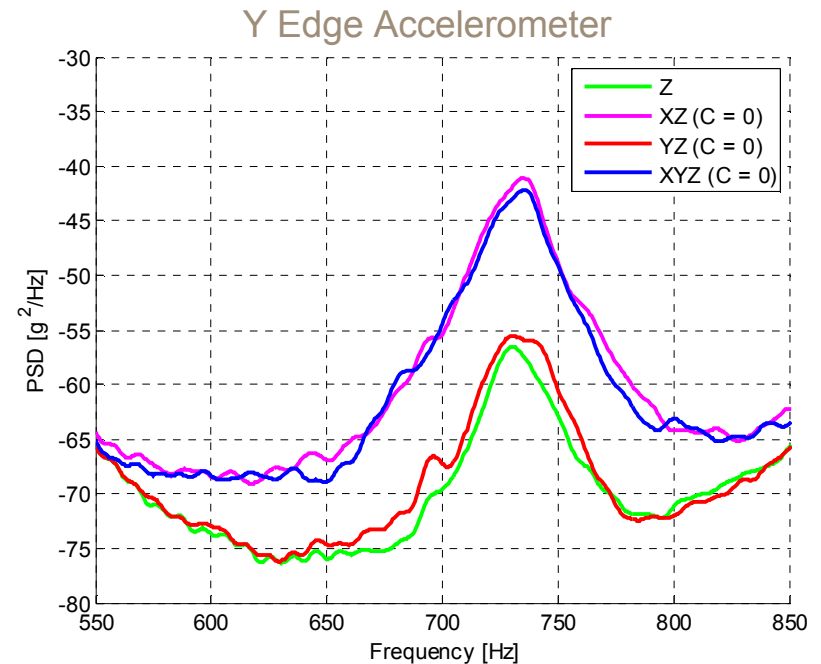
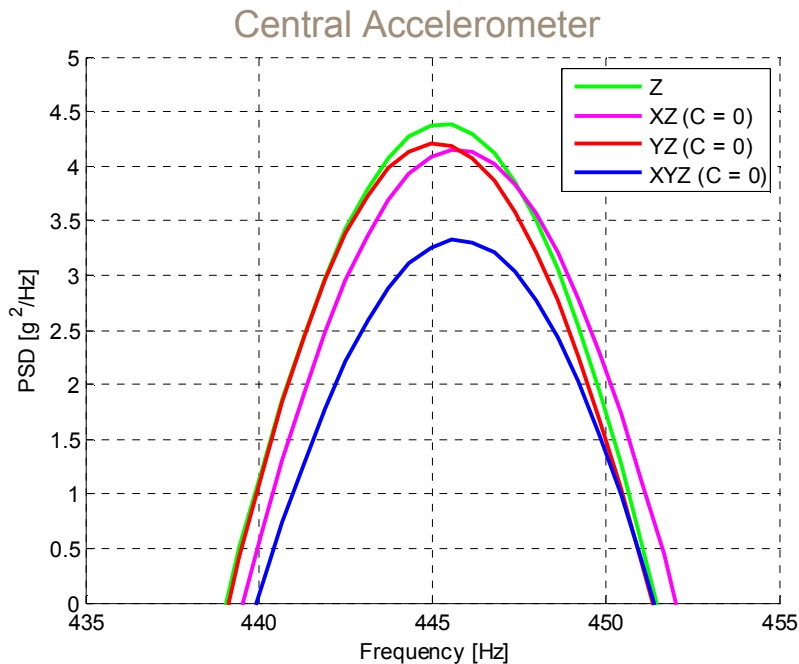


Y Edge Accelerometer



# Modal Response

- Dominant frequencies remain constant, but modal contribution changes depending on excitation level of additional axes



# Conclusions

- For a plate structure, uniaxial testing in line with the dominant axis results in worst case testing
- For conservative fatigue or life-cycle testing, peak response axis must be known a-priori
- Even for simple structures, modal contribution is altered by multi-axis testing
  - Corresponding stress state will never be captured by single axis tests
- On uniaxial shakers, presence of off axis stimuli due to internal coupling may distort results
  - True single axis testing can only be performed on multi-axis shaker
  - Control of off axis contributions can enable improved model validation
- Combined single axis response data cannot accurately be used to predict or bound multi-axis scenarios

# References

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- [14] M.A. Underwood and T. Keller, "Applying coordinate transformations to multi-DOF shaker control," *Sound and Vibration*, vol. 40, no. 1, pp. 22-27, Jan. 2006.

Thank you for your attention!

**QUESTIONS?**