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Comparison of FRF Correlation Techniques

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ABSTRACT

Mode shape correlation techniques have proven to be an excellent method for assessing the degree of similarity between a finite element model and a set of test data. Because mode shapes inherently contain spatial information on the mass and stiffness matrices of the model and are easily extracted from test data, this allows an overall similarity assessment as well as providing indications of localized areas of discrepancy. Although frequency response function (FRF) correlation contains the same information as used in mode shape correlation (both contain a collection of measurement points), FRF correlation has been found to be more difficult to use successfully. This difficulty is primarily because of the high sensitivity of frequency response functions to slight perturbations in both the boundary condition and in the excitation and response. As a result, the globally extracted mode shapes minimize the contamination due to testing perturbations, while frequency response functions maintain this sensitivity This sensitivity makes it possible to measure similarity, or lack there of, in so called identical components or assemblies.

Sandia National Laboratories wishes to develop a quick testing process that could identify assembly errors in a complex structure with many joints while minimizing the structure's required downtime prior to its intended purpose. A frequency response function similarity approach was used where the structure was mounted on a test fixture and a limited number of responses were recorded with one single drive point. Because mode shape correlation techniques are widely used, the corresponding frequency response function correlation techniques have not been studied as much. For this paper, several different frequency response function correlation methods were assessed on a simplified structure to identify which technique(s) are best able to detect any assembly errors while being insensitive to other errors.

Keywords: modal testing, correlation, frequency response functions, similarity

Nomenclature

$\omega_{k,l}$	Specific (kth) Frequency at degree of Freedom L
H_1, H_2	Frequency Response Function (1) or (2)
A^H	Complex conjugate transpose (Hermitian) of A
FRF	Frequency Response Function

CSF Cross Signature Scale Factor

FRFSF Frequency Response Function Scaling Factor

RMS Root Mean Square

CSAC Cross Signature Assurance Criterion

1. INTRODUCTION

At Sandia National Laboratories, many of the structures tested are complex multi-bodies with many assembled joints. Sandia wants to define a quick testing process that could identify assembly errors while minimizing the downtime required on the unit. Similarity testing at Sandia has traditionally been performed on a vibration table where the unit is exposed to a low level random input in all three principal directions; however this is very timely and costly due to the party's involved and the hardware required. This testing is also less than ideal as the force inputs are unknown, and visual comparison of transfer functions leave a lot of room for interpretation as the most common metric used visually is peak response or resonant frequency. There is also a large variability of the dynamic response introduced by the vibration table as well. Due to these reasons, Sandia chose modal testing as the test method for developing a unit-to-unit similarity metric. Unfortunately, the physical footprint of the assemblies tested often allows for only limited sets of internal instrumentation, which would make mode shape correlation difficult and time consuming. In comparison, a single FRF taken at a strategic external location gives an indicator of the test article's similarity to a previously tested unit. As a result, the engineer can perform a simple modal test with limited instrumentation to obtain useful FRFs. These measurements can be used in the similarity metrics presented below instead of requiring a full experimental modal model.

Because structural dynamic correlation is commonly done using mode shapes, there has been significantly less research done on frequency response function correlation techniques. Allemang [1] provided a thorough overview of various correlation techniques, although most of the paper's focus is on the various mode shape correlation metrics. Pascual, et al. [2] used both the Frequency Domain Assurance Criterion (FDAC) and Frequency Response Scale Factor (FRSF) in order to perform model updating. Heylen et al [3] investigated multiple case studies using the Frequency Response Assurance Criterion (FRAC), although the majority of the paper focused on modal correlation metrics. Fotsch et al [4] also examined the FDAC and FRAC while introducing a new tool called Modal FRF Assurance Criterion (MFAC), which used the combination of mode shapes, and FRFs to assess the correlation. Finally Dascotte et al [5] employed the Cross Signature Assurance Criterion (CSAC) and Cross Signature Scale Factor (CSF) in order to study sensitivity based model updating.

Although there has been a lot of research done on the various FRF correlation techniques, very few case studies have been written where the different techniques are compared. Much of these correlation methods were developed for comparing experimental data to analytical or finite element analysis data for model correlation. In this paper, a case study will compare seven different metrics for discerning unit-to-unit variability and incorrect assembly using experimental modal data.

2. THEORY

A summary of the different correlation techniques that were examined is provided below. Details on the mathematical development can be found in the respective references where appropriate.

<u>Technique 1 : Frequency Peak Pick</u>

The easiest way to compare frequency response functions is by simply picking the frequency of the peak(s) on an FRF. This is traditionally how Sandia has compared the similarity of the two transfer functions; this technique is very visual and easy to understand. Because natural frequencies are characteristic of the structure, comparing the discrepancy between the frequency corresponding to the same peak on the FRF of two different units can provide an initial assessment of the similarity. This technique has proven difficult in the past due to a large dependence on engineer interpretation, usually of vibration data. While the boundary condition replicated in a free-free modal test is fairly repeatable, there is a lot of unknowns when comparing vibration data between units. Typically the auto spectrum of each response is compared, which can be rather noisy due the unknown force input. It has also been found that the more complicated the assembly, the more likely it is that the resonant peaks wont align. While this would indicate a different structure, Sandia is relaxing the word "Similar" to include small shifts in modes that would lead to a large error using this metric while exhibiting similar responses.

<u>Technique 2 : RMS Percent Error</u>

Rather than assessing the similarity at each frequency peak as described in technique 1, the overall similarity of the FRF can be assessed. Using the RMS value of the FRF and comparing it to the RMS value of another measurement assesses the overall energy of the function. Although this allows slight perturbations between the functions to be minimized, the RMS calculation also allows the possibility of two different functions to have the same RMS value even though they are clearly visually different.

Technique 3: R^2 coefficient of the Imaginary Response
Although transfer functions are normally viewed as an absolute magnitude, more information about the transfer function can often be obtained by converting the function amplitude into a complex number. By plotting the imaginary component of the response of the 1st FRF with respect to the imaginary component of the 2nd FRF, a linear fit can be obtained. Based on the R^2 term of this linear fit, the similarity of the measurements can be evaluated.

Technique 4: Nyquist Comparison
Another common way to view the transfer function is by plotting the real component of the FRF with respect to the imaginary part. Because damping heavily influences the real component of the FRF, the Nyquist Comparison can indicate structural changes in the assembly or part. By averaging the maximum discrepancies between the real and imaginary legs of the Nyquist plots and the control assembly, this difference can be utilized for similarity quantification.

The next 3 techniques use a more rigorous calculation in order to obtain a correlation metric. All techniques use the assumed orthogonality of the FRFs to compute a ratio from 1.0 (identical) to 0.0 (no similarity). They also evaluate each FRF on a line by line basis in the frequency domain, but are sensitive to different aspects of the FRF.

Technique 5: Cross Signature Scale Factor (CSF)
The CSF is used to evaluate discrepancies between amplitudes and is defined in Eq. 1. The CSAC ranges from 0 to 1 and is sensitive to changes in damping. Often the CSF is used with the CSAC in model correlation efforts.

$$CSF(\omega_k) = \frac{2\left|H_1^H(\omega_k)H_2(\omega_k)\right|}{\left(H_1^H(\omega_k)H_1(\omega_k)\right)+\left(H_2^H(\omega_k)H_2(\omega_k)\right)} \tag{1}$$

Technique 6: Frequency Response Function Scaling Factor (FRFSF)
The FRFSF is used to evaluate discrepancies between amplitudes of the opposing FRF's, and is defined in Eq. 2.

$$FRFSF = \frac{\sum_{j=1}^{k}\sum_{n=1}^{L}\left|H_1(\omega_{k,L})\right|}{\sum_{j=1}^{k}\sum_{n=1}^{L}\left|H_2(\omega_{k,L})\right|} \tag{2}$$

Technique 7: Cross Signature Assurance Criterion
The CSAC is used to evaluate discrepancies between the shape of the function and is defined in Eq. 3. Similar to the traditional Modal Assurance Criterion (MAC), the CSAC is adapted for the frequency domain. Since the position and number of resonant peaks mainly determine the shape of an FRF, the CSAC function is sensitive to changes in mass and stiffness.

$$CSAC(\omega_k) = \frac{\left|H_1{}^H(\omega_k)H_2(\omega_k)\right|^2}{\left(H_1{}^H(\omega_k)H_1(\omega_k)\right)*\left(H_2{}^H(\omega_k)H_2(\omega_k)\right)} \tag{3}$$

Because there are many different frequency response function correlation techniques, a case study was performed to evaluate the different techniques and determine which technique(s) were the most useful for this particular application. These techniques will be narrowed down to the most useful, and used on more complicated assemblies in the near future as more hardware becomes available to test.

3. Case Study: Vibration Fixture Plates with Attached Assembly

To assess the similarity metrics listed above, a mass mock assembly consisting of multiple components connected with bolted joints was used as a test article. The variability on which the similarity metrics were assessed was introduced by individually attaching multiple vibration test fixtures to the test article. The mass mock assembly attaches to each fixture via four ¼-28 steel alloy bolts around the perimeter of the base. In addition to the evaluation of similarity metrics, this data also serves to check unit to unit variability on all of the vibration fixtures built by an outside entity are suppose to be identical and will lead to a duplicate response during environmental testing. The vibration fixtures were a square titanium plate with special features cut to adapt to the mass mock test article. The mass mock was conical in nature, and is represented as a general shape henceforth to protect proprietary information as shown in Figure 1. The experimental setup consisted of four total vibration fixtures with the mass mock assemblies sitting upon three foam pieces to best represent a free-free boundary condition. The vibration fixture was instrumented with three PCB 356A03 triaxial accelerometers with a sensitivity of 10 mv/G and a PCB 288D01 impedance head attached to a 50lb MB modal shaker. The modal shaker was oriented to input a 45 degree skewed burst random force input in all three perpendicular axis between 20-2000 Hz.

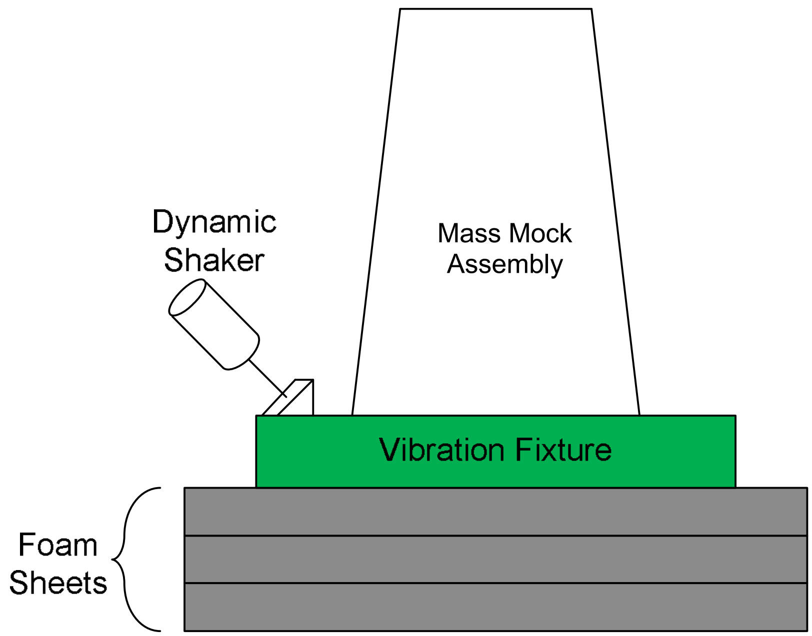

Figure 1: Experimental Test Setup

A total of four fixtures were tested each machined to the same drawing and were assumed identical. These fixtures will be referred to as “RT1”, “AM1”, “AM2”, and “JG8” in the rest of the paper. The test setup was replicated for all cases so that the same shaker input, sensors and boundary condition were maintained. Measurement nodes were measured precisely, although some instrumentation discrepancies are sure to exist due to user error.

For the purpose of evaluating the similarity of the fixtures, RT1 was used as the standard. Each fixture with attached mass mock assembly was tested and compared to the standard fixture assembly. For the majority of the metrics described above, a compiled average FRF of all locations were used to best filter out any unintended test assembly errors such as misaligned sensor placement, skewed shaker, etc. This is test setup was designed to best capture the assemblies dynamics utilizing minimum instrumentation on the fixture and zero instrumentation on the unit. Ideally, a larger number of accelerometers would be placed throughout the unit to capture modes that may not be picked up at the fixture locations. During testing, the burst random force input was averaged around 2.6 lbrms after a small non-linearity test was ran with varying force levels. At the frequency range and force levels tested, no non linearity’s were detected, however an improved fixture with improved transmissibility could lead to different results.

The compiled FRF’s below in Figure 2 compare the four fixtures correctly assembled to the mass mock assembly. While the results from the fixtures are generally similar, there is a mode around 940 Hz that contains a noticeable discrepancy. Although the fixtures are manufactured from the same drawing, they were made at different times and may have slightly different mass properties due to tolerances.

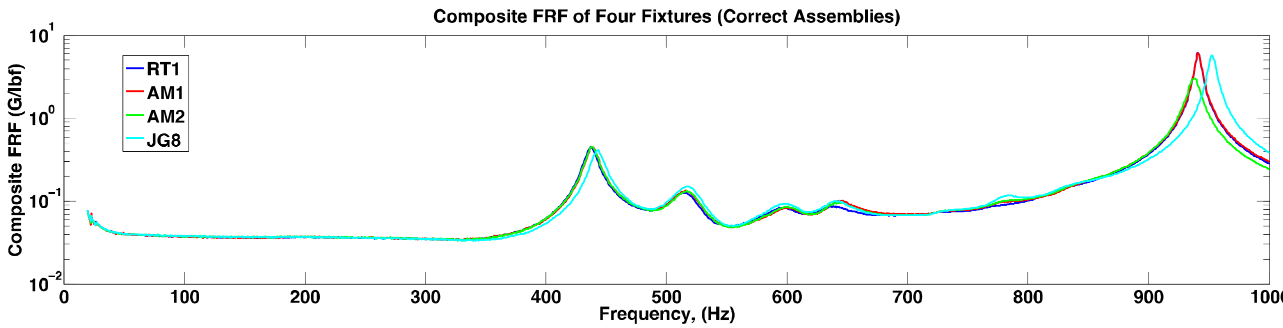

Figure 2: Compiled FRF Comparison of Four Fixtures Correctly Assembled

After this testing was completed, two different assembly errors were studied. The first consisted of torqueing the set of 4 fasteners between the mass mock and fixture to an incorrect torque value (approximately 1/3 of the specified value). The second assembly error consisted of properly installing the fasteners, but removing one of the four. These two assembly errors were conducted on all four fixtures and the resulting data sets were then compared to the corresponding correctly-assembled system. A comparison of these compiled FRF's is seen in Figure 3 below. These plots highlight the discrepancy when the assembly is missing a bolt, but does not obviously indicate when the bolt is incorrectly-torqued. The goal of this paper is to find a metric that is sensitive to small changes in the structures dynamics; even as subtle as this incorrectly torqued bolt. Some of the metrics will be presented in more detail below with a detailed summary of the results of all metrics at the end of the paper.

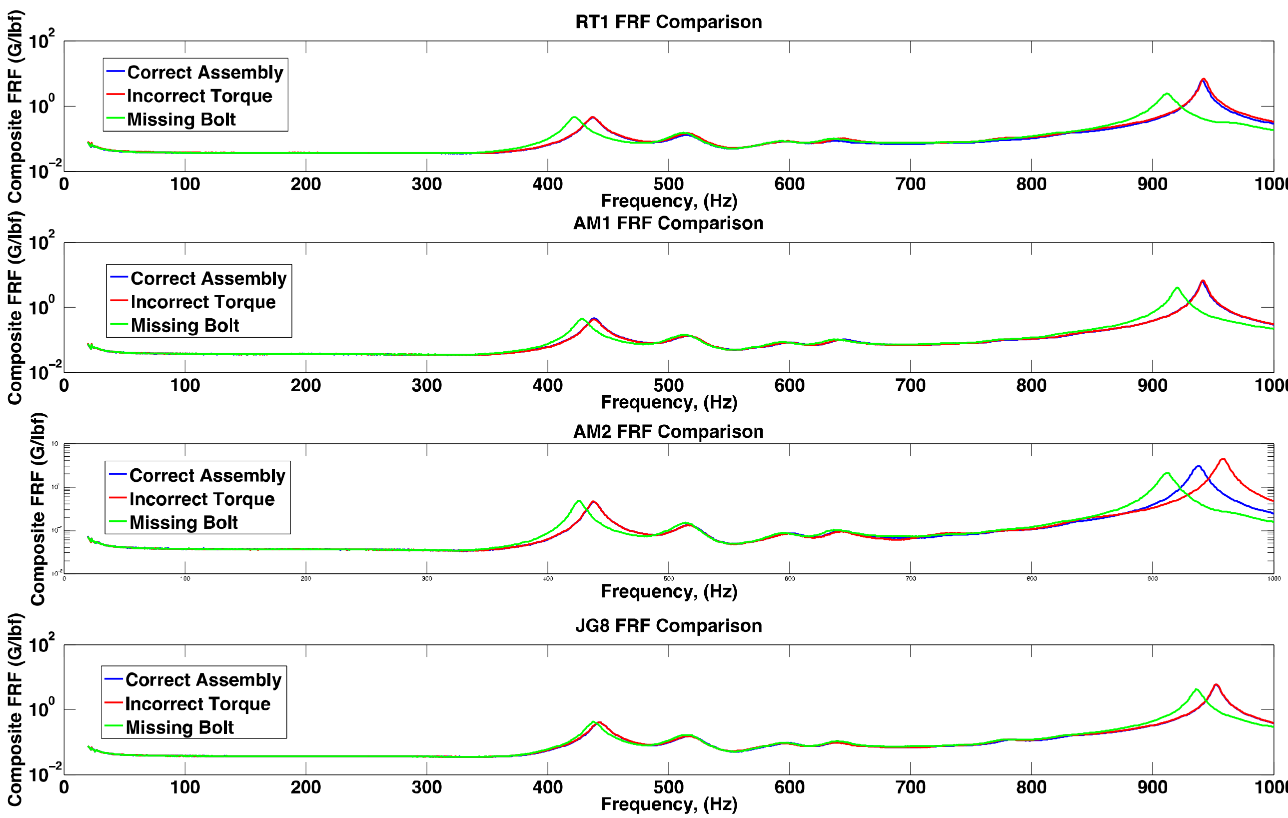

Figure 3: Compiled FRF Comparison of All Four Fixtures and Assemblies

Nyquist Comparison

Examining the Nyquist plots for the drivepoint response on the correct and incorrect assemblies lead to some interesting discrepancies. As seen in Figure 4 below, the diameter of the Nyquist plot at resonance (940 Hz mode) varies greatly depending on the assembly tested. Across all four fixtures, we see an increase in damping in the 940 Hz mode (characterized by a larger diameter circle) when the bolt is incorrectly torqued, and a decrease in damping (smaller diameter circle) when the bolt is left out altogether. This metric thus has a large sensitivity to changes in damping and can be quantified and normalized for comparison as shown in the summary.

In a similar fashion, the correct assemblies of the fixture / mass mock components were compared to get a baseline measurement on the difference in the fixtures themselves. Based on this method there exists a rather large variation between the "AM2" fixture and the other three fixtures as seen in Figure 5 below.

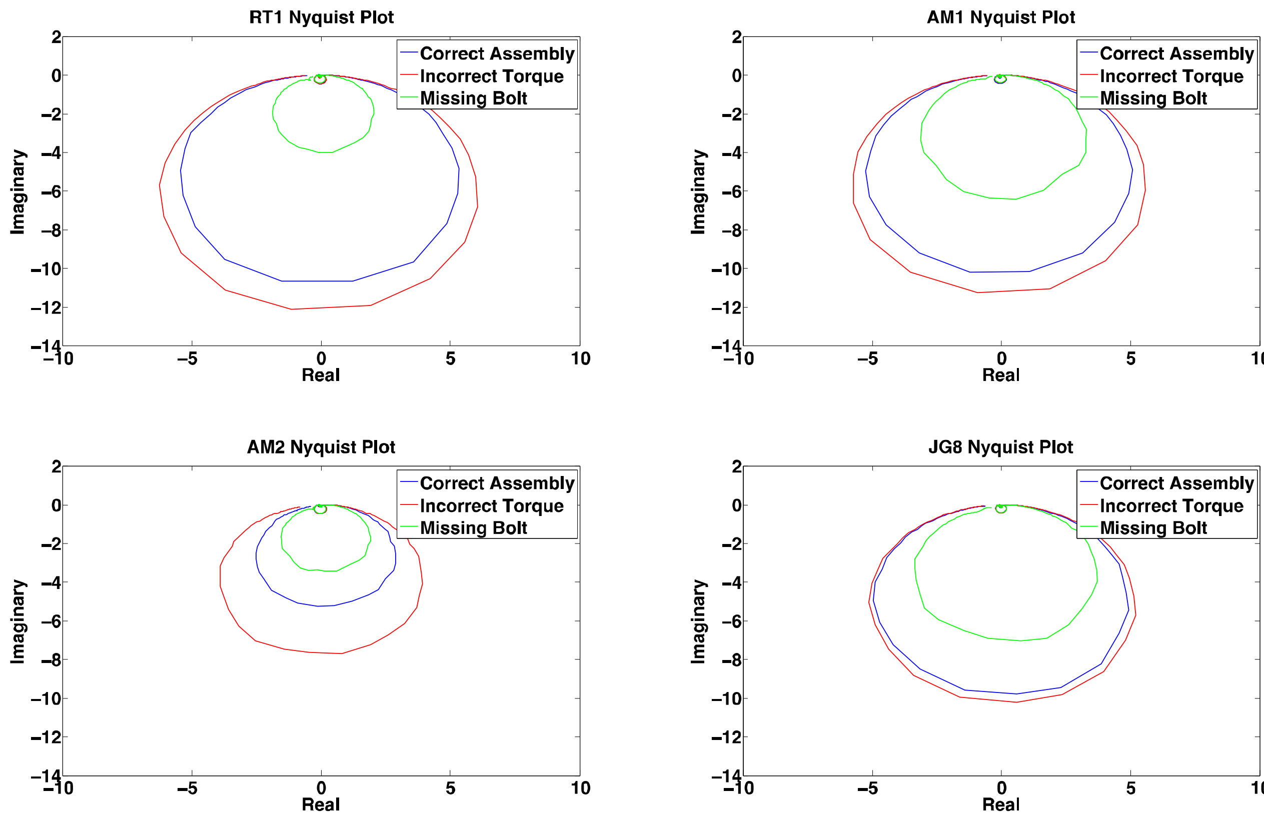

Figure 4: Nyquist Plot Comparison of Correct Vs. Incorrect Assemblies

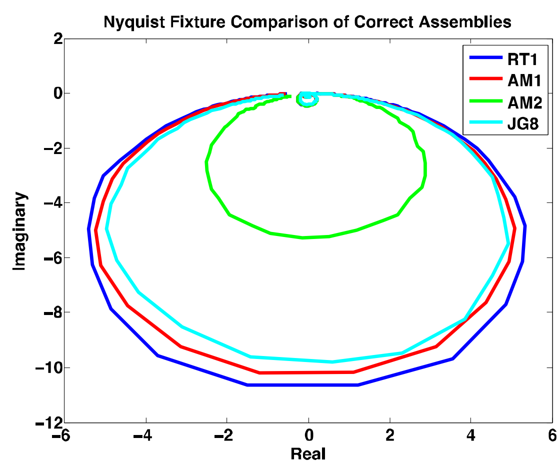

Figure 5: Nyquist Plot Comparison of All Four Correct Assemblies

R^2 coefficient of the Imaginary Response

Another visual method of comparing similarity is by plotting the imaginary response of one assembly versus the other. For identical assemblies, the relationship would be linear with a slope and R^2 value of 1. Although the results are tabulated in the following section, the plots shown below in Figure 6 compare these linear relationships and explain this methodology in a visual format. Similar to the results seen in the Nyquist plots, the discrepancy due to a missing bolt is more pronounced than that from the incorrectly torqued fasteners.

While at first glance, the linear relationship between the two assemblies appears to give a good representation of the units assembly, the linear regression tends to be biased towards the larges mode while the lower modes are condensed near the origin. To insure the R^2 value is a good representation of the comparison between correct and incorrect assemblies, the residuals from the linear regression are plotted against frequency in Figure 7 below. In this figure, we see a very small deviation in the lower frequencies giving confidence in the linear fit in the regime.

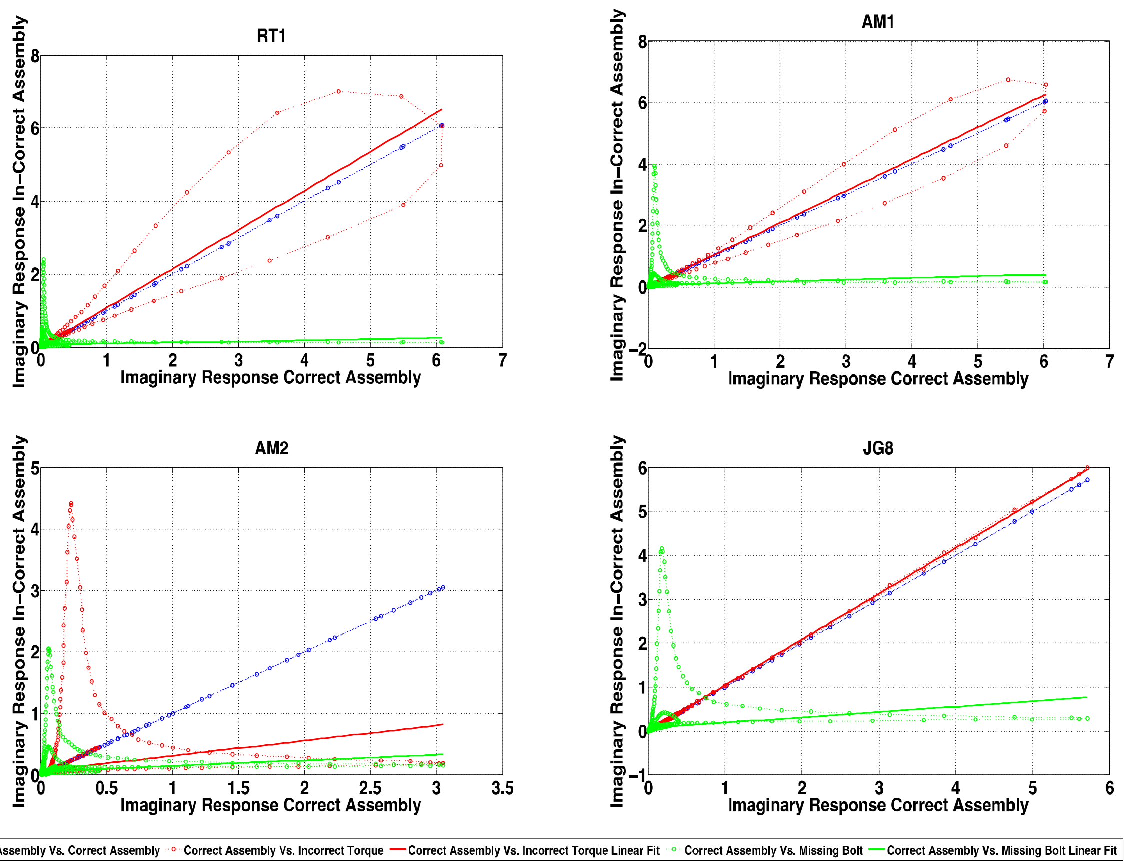

Figure 6: Linear Fit Plots for All Correct Vs. Incorrect Assemblies

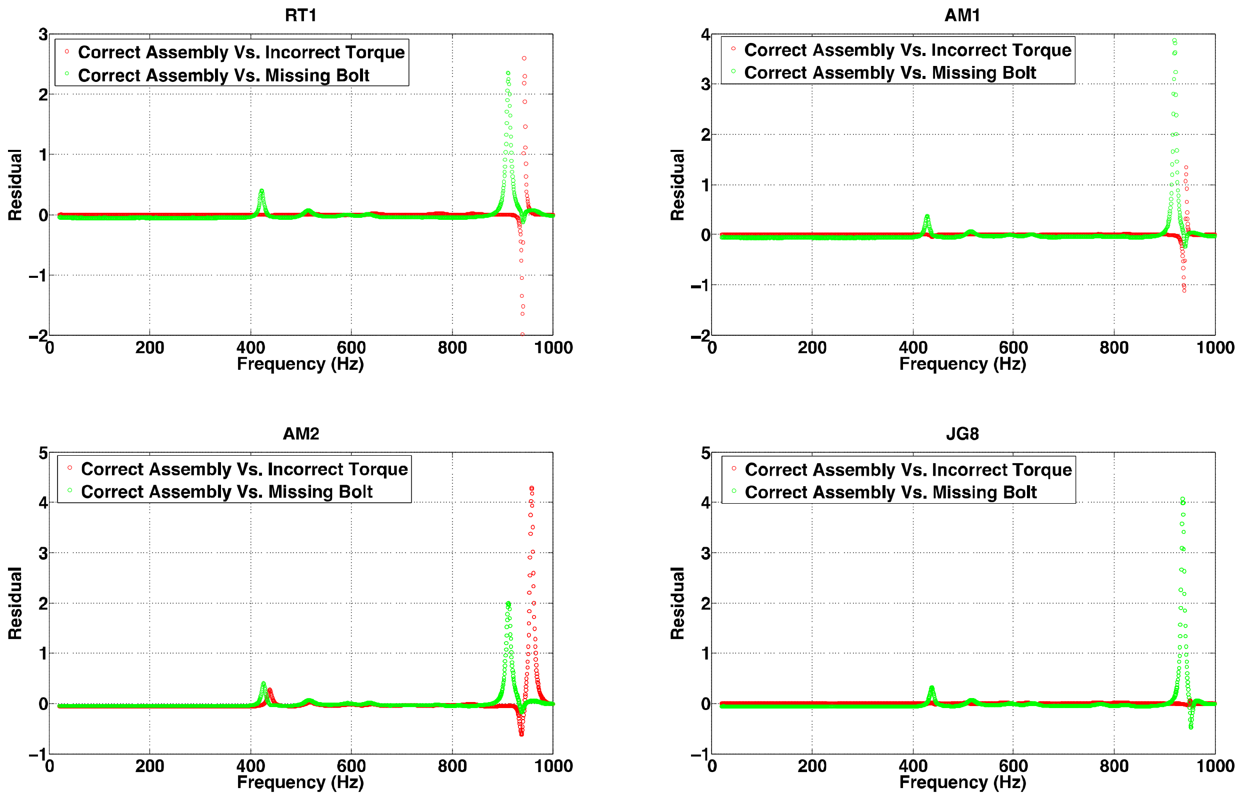

Figure 7: Residuals Vs. Frequency for Linear Regression Metric

Figure 8 displays the imaginary responses for the correct assemblies and gives an insight to the differences in the fixture. This metric demonstrates a large difference between the "RT1" fixture and the "JG8" fixture. This is not surprising as the "JG8" fixture is a few years old and has undergone wear and tear due to extensive vibration testing, while the other three fixtures are new and have had no time on the vibration table. Looking at the residual plot, we again gain confidence in the linear regression at lower frequencies. This also gives insight into the complexity of defining similarity as the 4 units tested were manufactured to the same drawing and should exhibit little to no unit to unit variability, yet the metrics explored demonstrate a large unit to unit variability.

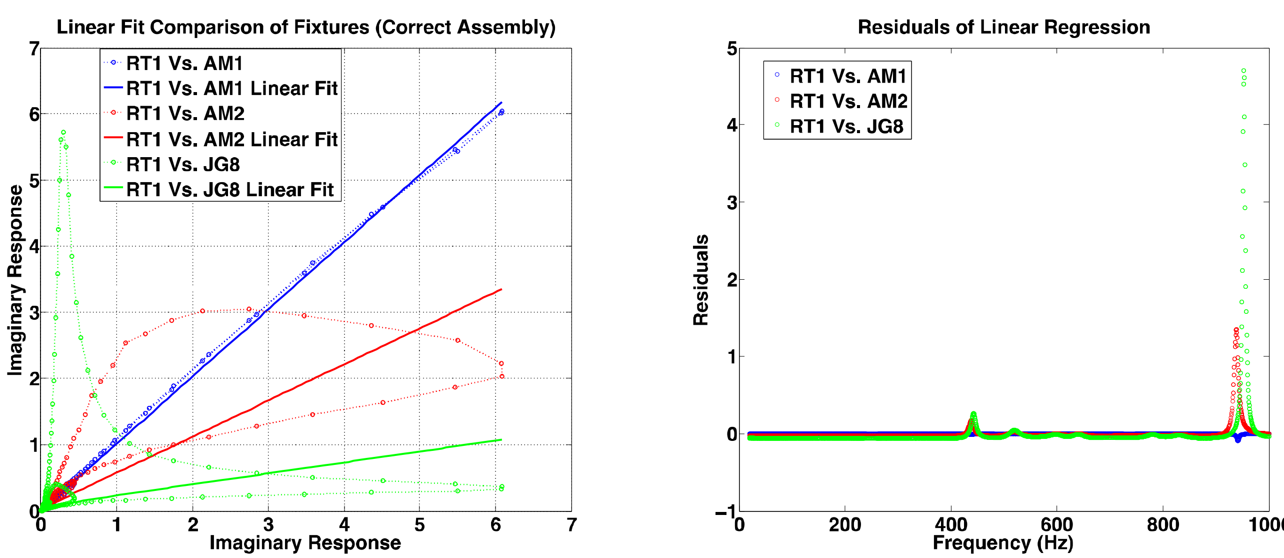

Figure 8: Linear Fit Plot for Fixture Comparison

As seen in the plots, both the Nyquist and imaginary response comparison metrics were able to visually identify the incorrectly assembled test data sets. Furthermore, the metrics were more sensitive to the missing bolt assembly error than they were to the incorrect torque error. Based on these results, any testing done in the future can use this information to help quickly identify assembly errors such as a missing bolt. More testing is still needed to determine how sensitive these metrics may be to other anomalies such as bad welds, missing washers, or deformed material.

4. EVALUATION OF METRICS

All seven metrics were computed for each fixture-assembly combination for both assembly errors and compared in Figures 9 and 10 below where all metrics were normalized to 1.0. Thus a metric of 1 would indicate the two assemblies were perfectly identical, while a metric of 0 would indicate no similarity. For metrics where a percent error was calculated, the metric was normalized to 1 by taking 1 minus the error thus a metric of 1 represents zero error. Thus a metric of 1 represents a similar FRF or an error of zero between the two metrics. All metrics identified assembly errors well except the Peak Picking metric which is most often used as a visual metric. Most of the metrics were able to correctly indicate the missing bolt assembly error whereas the metrics did not perform as well identifying the incorrectly torqued bolts. Because assembly errors may be caused by an incorrect torque, an ideal similarity metric would be sensitive to all assembly errors. For this case study, the Nyquist metric, RMS Error, and R^2 metric appeared to be the metrics most sensitive to the torque assembly error, however, the RMS error appears to be less sensitive to shifts in frequency and doesn't capture unit to unit variability reliably.

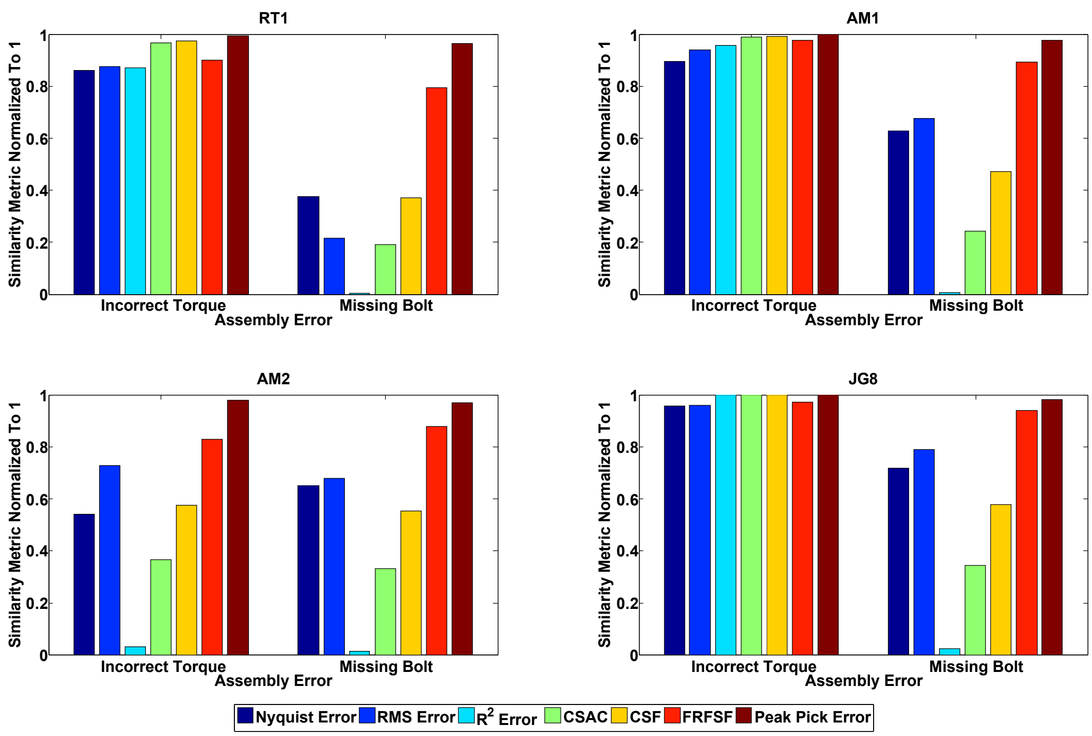

Figure 9: Similarity Metric Comparison for Incorrect Assemblies Normalized to One (1 = Identical to Correct Assembly)

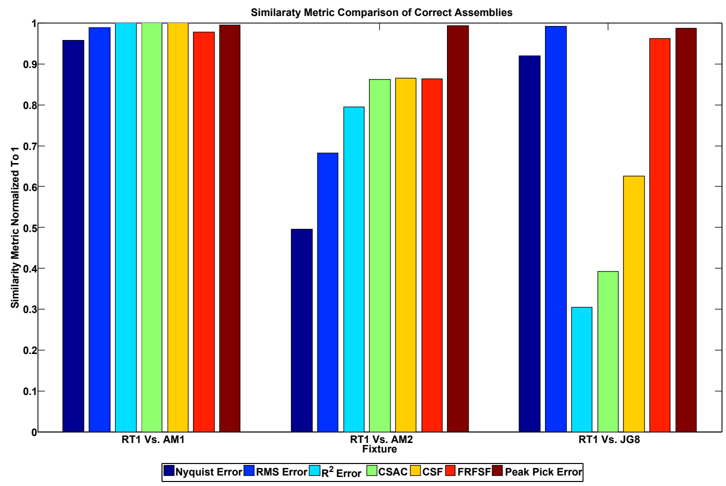

Figure 10: Similarity Metric Comparison of Fixtures for Correct Assemblies (1 = Identical to RT1)

Figure 9 displays the differences between all metrics when comparing the correct assemblies of the "AM1", "AM2", and "JG8" fixtures versus the "RT1" fixture as a reference. As expected from looking at the composite FRFs, RT1 vs AM1 shows a high degree of correlation across all metrics. In addition, both AM2 and JG8 are visually different from RT1 but not all of the metrics sufficiently capture this difference for both fixtures. Because only four fixtures were tested, additional fixtures would help identify the appropriate metrics to use for unit-to-unit variability.

In order for similarity testing to be useful, a threshold must be defined to indicate whether the two transfer functions are similar. This threshold value is difficult to define with the limited set of hardware, but averaging the similarity metric values gives some insight to what value might be acceptable for a given metric and assembly error. These early results indicate the need for various metrics to be used together to best identify any assembly errors or unit-to unit variability. From the values shown in Table 1, the FRFSF and Peak Picking method are not adequate metrics to measure similarity due to their high values for the incorrect assemblies. Considering the values for the three recommended metrics of Nyquist Error, RMS Error, and R^2 Error,it appears that an average similarity value above 0.9 would indicate a correct assembly. However, when evaluating unit-to-unit variability, this threshold may be much lower. More detailed testing will lead to an accurate threshold.

Table 2. Average Similarity Comparing Incorrect Assemblies for all 7 Similarity Metrics

	Nyquist Error	RMS Error	R^2 Error	CSAC	CSF	FRFSF	Peak Pick Error
Incorrectly Torqued Average Similarity	0.81	0.88	0.72	0.83	0.89	0.92	0.99
Missing Bolt Average Similarity	0.59	0.59	0.01	0.28	0.49	0.88	0.97

5. Conclusion

Sandia National Laboratories chose to use similarity testing to quickly identify any assembly errors encountered during production of complex multi body assemblies. Because mode shapes could not be measured due to classification, radiological contamination, or concern with damaging the hardware, a single drivepoint FRF measurement was combined with limited instrumentation on specialized test fixtures. In order to identify a metric that could be used to assess the similarity between assemblies, a study of various FRF correlation techniques was performed on a simplified test article under three different assembly configurations: correctly assembled, assembly-to-fixture bolts being incporrectly torqued, and correctly torqued bolts with one removed. The three techniques chosen: Nyquist Error, RMS Error, and R^2 Error were found to provide the best assessment of the FRF and were used on the actual unit to help identify the presence of assembly errors.

6. Future Work

Further studies on similarity are underway at Sandia National Laboratories as additional hardware becomes available. These studies will provide some statistical confidence on the presented metrics in order to verify that the metrics used are appropriate. This additional testing will also help identify the acceptable threshold for determining whether the two units are dynamically similar. During this work, the vibration test fixtures used contained unique and non-symmetric geometry, which provided some uncertainty in the results. Similarity testing in the future would benefit from designing test fixtures with simplified geometry. Sandia is also planning to use these similarity tests on larger and more complicated structures for actual production use in the future.

Sandia also has interest in comparing environmental vibration data for similarity purposes. A large majority of the hardware being developed at Sandia undergoes vibration testing, and the definition of a similarity metric for response only data could improve the ability to assess assembly on more units in production.

Sandia's primary goal with this work is to define a test procedure, metric and threshold for assessing similarity between units being produced either onsite or by an external contractor. This would allow similarity data to be acquired in a fast, reliable and non-intrusive manner. The results in this paper show promise in this methodology, and future work will further refine this process, which can prove useful not only to Sandia Laboratories, but to any industry where quality control is a concern.

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