SAND2015-7120PE

Model Predictive Control Strategies for Resource Management

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This Presentation Describes MPC and HMPC applications to Resource Management

- Model Predictive Control (MPC) has Three Parts
  - Discrete-Time Prediction model of the system
  - Cost / Value function definition
  - Algorithm for determining a control input at t_{k+1} based on measurement of state at t_k
  - MPC Numerical Example

- Hybrid Model Predictive Control (HMPC)
  - Same three parts
  - Includes discrete-valued decision variables
  - Microgrid Example with Generators that switch *'on-off'*

Consumption of Resources are Modeled Effectively using Classical Methods

- Affine-linear Dynamic Resource Model (DRM)

$$\dot{x} = Ax + Bu + b$$

where

- Non-negative resource quantities: $x \in \Re^n : x \geq 0$
- Inputs relating to resource consumption: $u \in \Re^m$
- Resource consumption that is linear in time: $b \in \Re^n$
- A matrix is negative semidefinite to model “leakage”

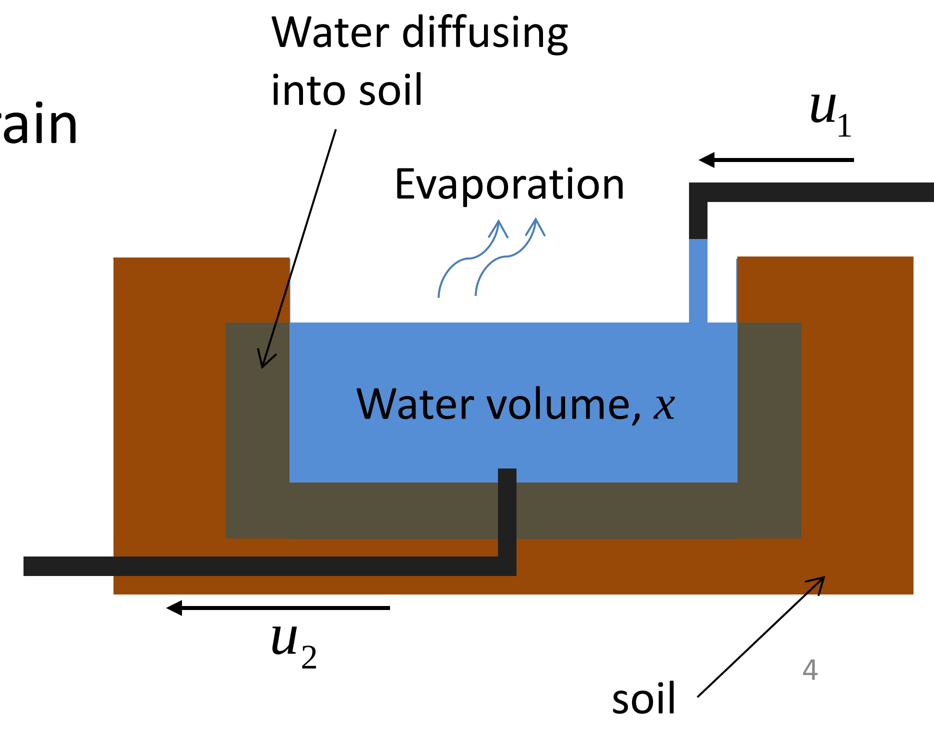
A Water Reservoir Example is Considered to Illustrate

- Example 1 - Water resource modeled as:

$$\dot{x} = Ax + Bu + b$$

  - x is water volume in m^3
  - u_1 is water added, u_2 is water drained m^3/sec
  - A is rate of diffusion into soil
  - $b(t)$ is rate of evaporation or rain

$$\dot{x} = [-0.01]x + [1 \quad -1]\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} - 0.02$$



A Discrete-Time Model is Created

- A discrete-time (DT) DRM is generated to perform as a prediction model

- A time step: $$x_{k+1} = Ax_k + Bu_k + b$$

where $x_k = x(t_k), \quad t_k = t_0 + kT$ and A, B, b are reformulated for DT and T is time step

The Discrete-Time Model is used as a Prediction Model

- Multiple time steps may be computed for determining the resource consumption over a time horizon

$$x_{k+1} = Ax_k + Bu_k + b$$
$$x_{k+2} = Ax_{k+1} + Bu_{k+1} + b$$
$$= A^2 x_k + ABu_{k+1} + Bu_{k+2} + 2b$$
$$***$$

Using algebra, all future outputs are expressed using the initial condition and the inputs

- The MPC model requires knowledge of the Horizon N

$$\begin{bmatrix} x_k \\ x_{k+1} \\ x_{k+2} \\ \vdots \\ x_{k+N} \end{bmatrix} = \begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 & 0 & 0 \\ B & 0 & 0 & 0 \\ AB & B & 0 & 0 \\ \cdots & \cdots & \ddots & \cdots \\ A^{N-1}B & A^{N-2}B & \vdots & B \end{bmatrix} \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+N-1} \end{bmatrix} + \begin{bmatrix} 0 \\ b \\ 2b \\ \vdots \\ Nb \end{bmatrix}$$

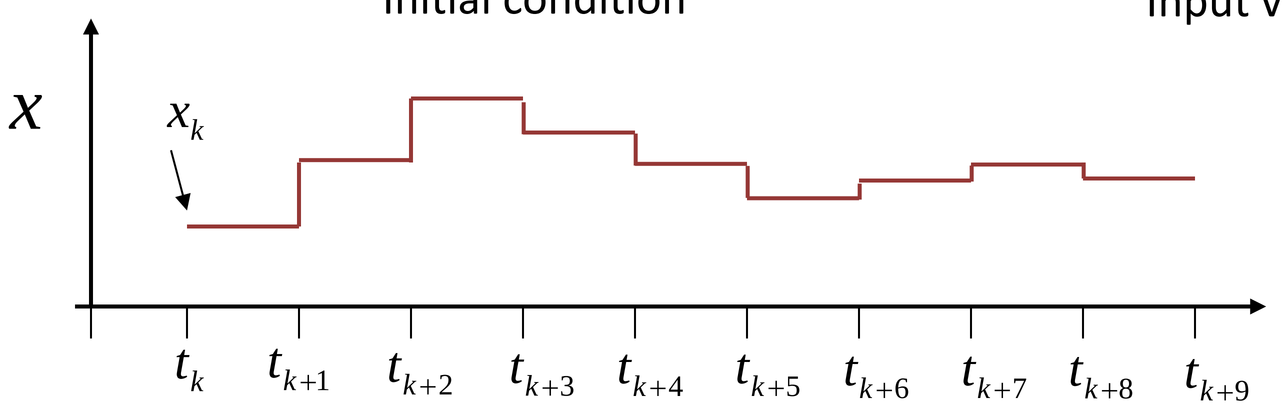
The Prediction Model and Horizon are Captured in one Expression

- When the initial condition and input sequence is known, this formulation predicts the trajectory

$$
\begin{bmatrix} x_k \\ x_{k+1} \\ x_{k+2} \\ \vdots \\ x_{k+N} \end{bmatrix} = \begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 & 0 & 0 \\ B & 0 & 0 & 0 \\ AB & B & 0 & 0 \\ \cdots & \cdots & \ddots & \cdots \\ A^{N-1}B & A^{N-2}B & \vdots & B \end{bmatrix} \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+N-1} \end{bmatrix} + \begin{bmatrix} 0 \\ b \\ 2b \\ \vdots \\ Nb \end{bmatrix}
$$

Initial condition

Input vector



State Reference Values are Easily Incorporated into the Model

- For some resources, such as battery energy storage, we may have a set point x^* (i.e. keeping a battery at 60% state-of-charge)
- Define <u>error state</u> $\Delta x_k = x_k - x^*$

$$\begin{bmatrix} \Delta x_k \\ \Delta x_{k+1} \\ \Delta x_{k+2} \\ \vdots \\ \Delta x_{k+N} \end{bmatrix} = \begin{bmatrix} x_k - x^* \\ x_{k+1} - x^* \\ x_{k+2} - x^* \\ \vdots \\ x_{k+N} - x^* \end{bmatrix} = \begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 & 0 & 0 \\ B & 0 & 0 & 0 \\ AB & B & 0 & 0 \\ \cdots & \cdots & \ddots & \cdots \\ A^{N-1}B & A^{N-2}B & \vdots & B \end{bmatrix} \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+N-1} \end{bmatrix} + \begin{bmatrix} -x^* \\ b - x^* \\ 2b - x^* \\ \vdots \\ Nb - x^* \end{bmatrix}$$

A Compact Formulation is Defined

- It is convenient to create a more compact formulation.

$$
\begin{bmatrix} \Delta x_k \\ \Delta x_{k+1} \\ \Delta x_{k+2} \\ \vdots \\ \Delta x_{k+N} \end{bmatrix}
= \underbrace{\begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}}_{\bar{A}} x_k
+ \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ B & 0 & 0 & 0 \\ AB & B & 0 & 0 \\ \cdots & \cdots & \ddots & \cdots \\ A^{N-1}B & A^{N-2}B & \vdots & B \end{bmatrix}}_{\bar{B}}
\begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+N-1} \end{bmatrix}
+ \underbrace{\begin{bmatrix} -x^* \\ b - x^* \\ 2b - x^* \\ \vdots \\ Nb - x^* \end{bmatrix}}_{\bar{b}}
$$

$$
\boxed{\Delta \bar{x} = \bar{A} x_k + \bar{B} \bar{u} + \bar{b}}
$$

These matrices are often big

$$\bar{x} \in \Re^{(N+1)n} : x \geq 0$$

$$\bar{A} \in \Re^{(N+1)n \times n}$$

$$\bar{B} \in \Re^{(N+1)n \times Nn}$$

$$\bar{u} \in \Re^{Nn}$$

$$\bar{b} \in \Re^{(N+1)n}$$

MPC Requires a Performance Index (i.e. a Cost Function)

- The resource use is evaluated using a cost function

$$J = \int_{t_0}^{t_f} F(x,u)\,d\tau$$

- In discrete time

$$J = T\sum_{k=1}^{N} F(x_k, u_k)$$

- Typical formulation

$$J = \frac{T}{2}\sum_{k=1}^{N}\left(\Delta x_k^T Q \Delta x_k + u_k^T R u_k\right)$$

Cost Function and Model are Combined to Give Variational Terms

- The resource use is evaluated using a cost function

$$J = \frac{T}{2}\sum_{k=1}^{N}\left(\Delta x_k^T Q \Delta x_k + u_k^T R u_k\right)$$

$$= \frac{T}{2}\left(\Delta\overline{x}^T \overline{Q}\,\Delta\overline{x} + \overline{u}^T \overline{R}\,\overline{u}\right)$$

$$= \frac{T}{2}\left(\left(\overline{A}x_k + \overline{B}\,\overline{u} + \overline{b}\right)^T \overline{Q}\left(\overline{A}x_k + \overline{B}\,\overline{u} + \overline{b}\right) + \overline{u}^T \overline{R}\,\overline{u}\right)$$

- With gradient

$$\left[\nabla_u J\right]_k = \left[\frac{\partial J}{\partial u}\right]_k = T\left(\overline{B}^T \overline{Q}\,\overline{A}x_k + \overline{B}^T \overline{Q}\,\overline{B}\,\overline{u} + \overline{B}^T \overline{Q}\,\overline{b} + \overline{u}_k^T \overline{R}\right)$$

New measurement
each time step

Variational Terms are Used for Iterative Solution

- And Hessian

$$H_{i,j} = \left[\frac{\partial^2 J}{\partial u_i \partial u_j} \right] = T\overline{B}^T \overline{Q}\,\overline{B}$$

- Iterative solution in α to compute optimal control sequence each time step k
- Newton's Method: $$\boxed{{}^{\alpha+1}\overline{u}_k = {}^{\alpha}\overline{u}_k - H^{-1} \cdot \left[\nabla_u J\right]_k}$$
- Levenberg-Marquard: $$\boxed{{}^{\alpha+1}\overline{u}_k = {}^{\alpha}\overline{u}_k - \left(\beta I + H\right)^{-1} \cdot \left[\nabla_u J\right]_k}$$

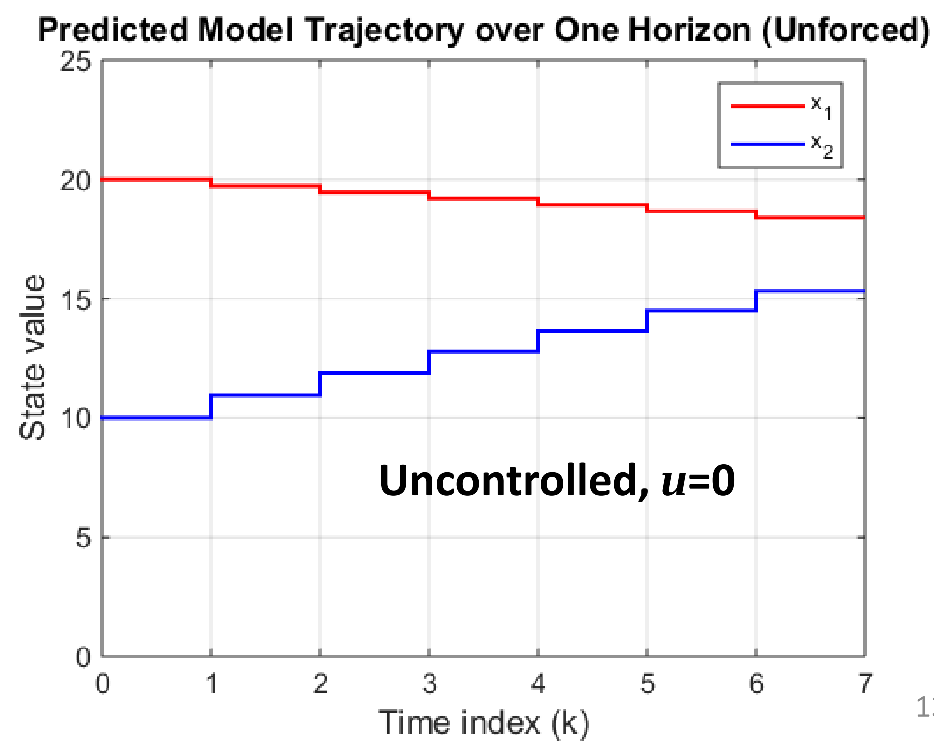
$$\beta > 0$$

A Simple MPC Example is Shown

- Example 2: $A = \begin{bmatrix} -0.1 & 0 \\ 0.5 & -0.2 \end{bmatrix},\ B = \begin{bmatrix} 0.5 & 1 \\ 1 & 0.25 \end{bmatrix},\ b = \begin{bmatrix} -0.7 \\ 1.5 \end{bmatrix}$

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix},\ R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ x^* = \begin{bmatrix} 12 \\ 4 \end{bmatrix},\ N = 6,\ T = 0.1$$

Over one Horizon … Uncontrolled …. $J = 919.5$



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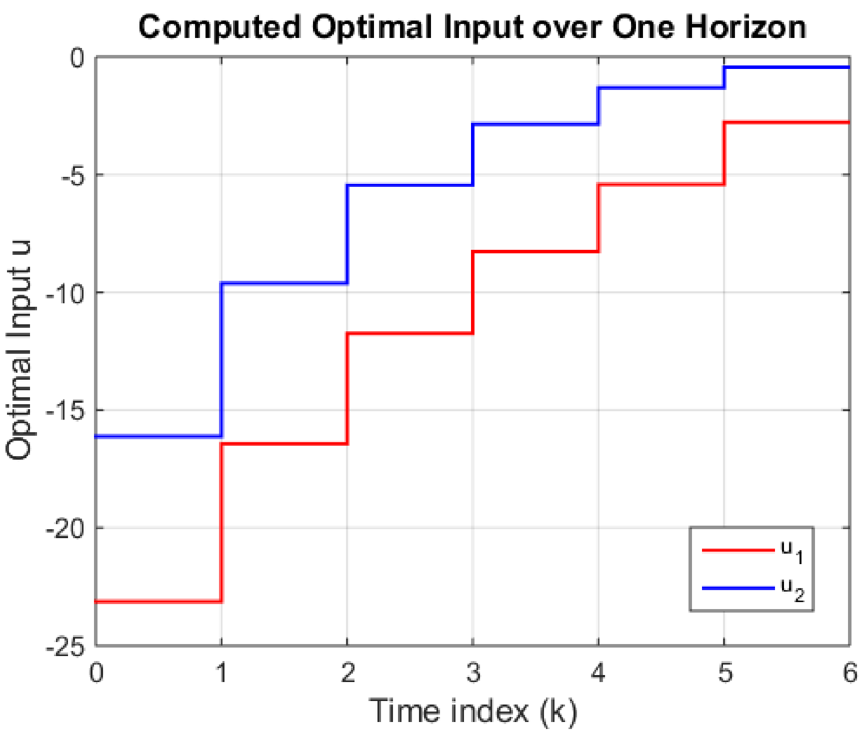
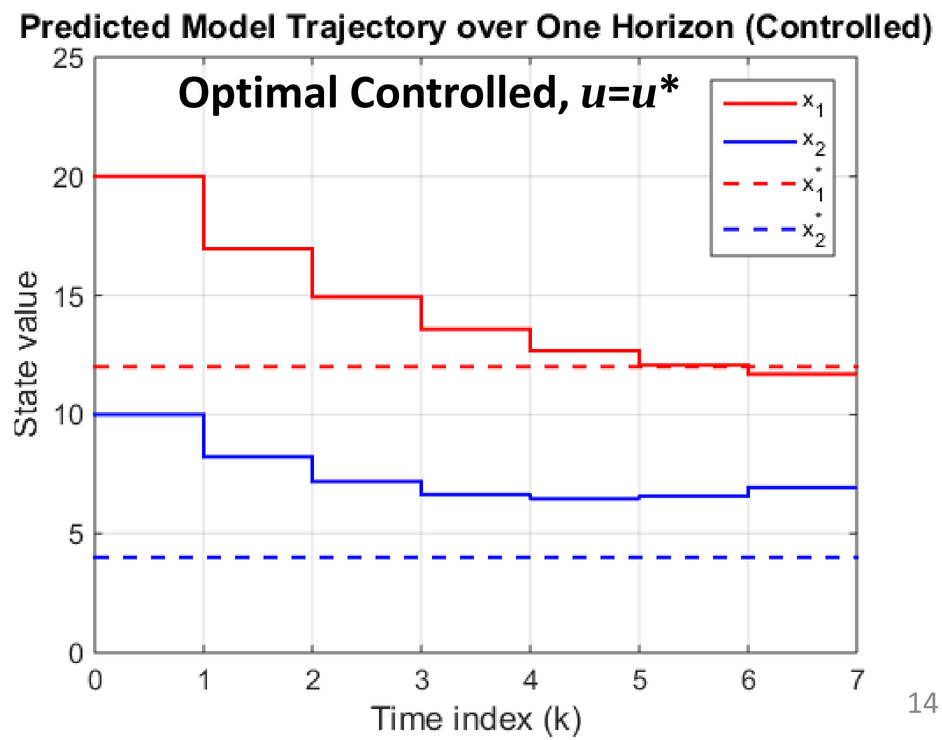
Over one Horizon … Optimal Control … $J=336.8$

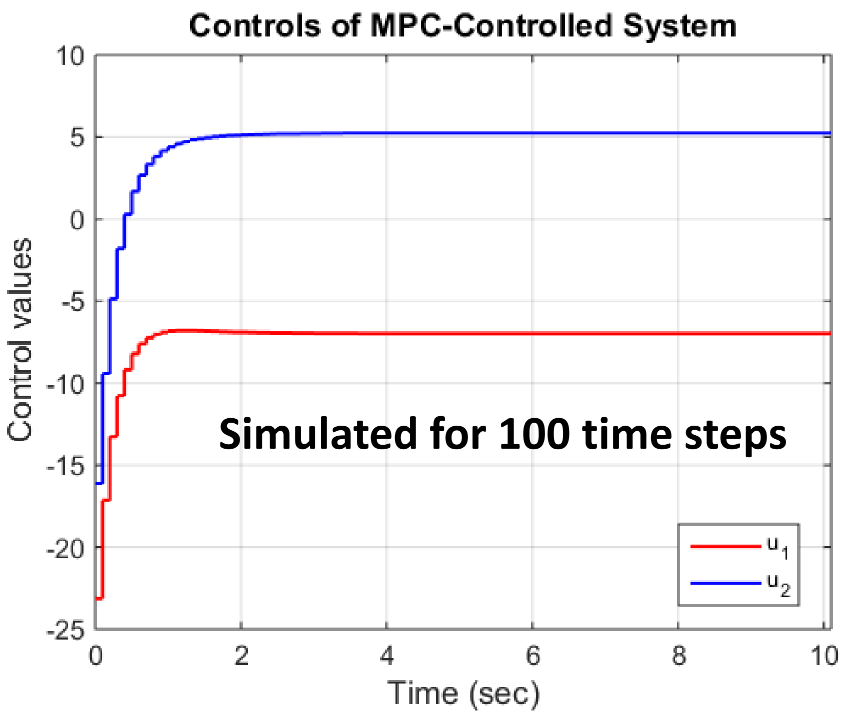


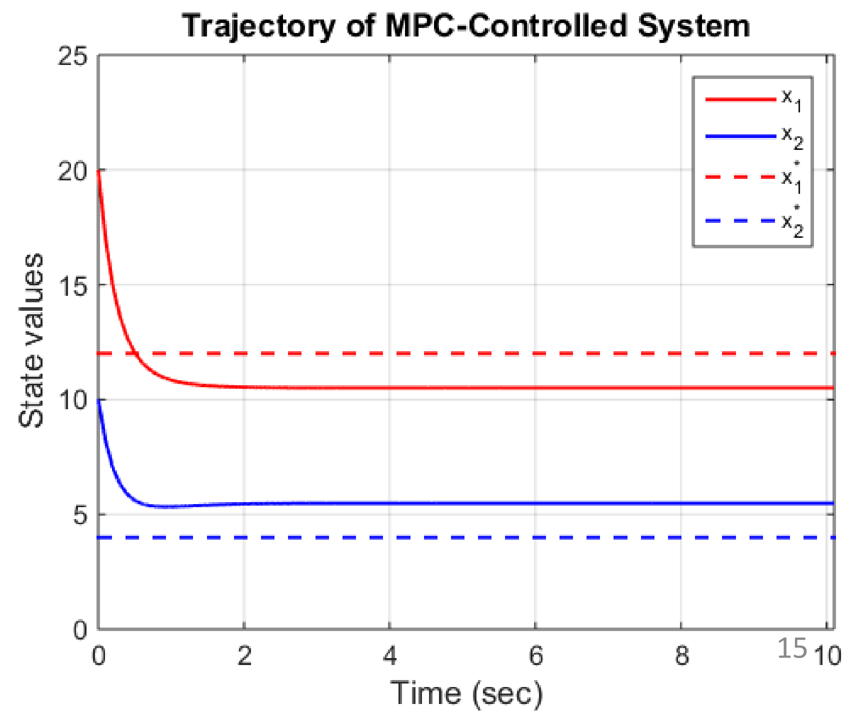


A Simple MPC Example is Shown

At each time step:

- x_k is measured
- An optimal control sequence is computed for horizon of 6 times steps
- The first control in the sequence is applied
- k is incremented





Hybrid Models Describe Systems with Continuous and Discrete-Valued Variables

- Hybrid Affine-linear DRM
  - *Generator 'on'* (s = 1): $\dot{x} = A_1 x + B_1 u + b_1$
  - *Generator 'off'* (s = 0): $\dot{x} = A_0 x + B_0 u + b_0$

- Switched model $$\dot{x} = s \cdot \left(A_1 x + B_1 u + b_1 \right) + \left(1 - s \right) \cdot \left(A_0 x + B_0 u + b_0 \right)$$

where $s \in \{0,1\}$