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SSPX Simulation Model

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Abstract

An analytical approximation to an R-L-C circuit representing SSPX is shown to reproduce the observed capacitor bank efficiency and gun optimization data. As in the SPICE code, the spheromak gun is represented by a fixed resistance chosen to balance energy transfer to the gun. A revised estimate of the magnetic decay time in SSPX Shot 1822 then brings our estimate of the gun efficiency itself in line with the observed spheromak magnetic field for this shot. Prompted by these successes, we present a turbulence-based theoretical model for the spheromak resistance that can be implemented in the SPICE code, of the form:

$$R_S = \kappa I (1 - I_o / I)^2$$

where I is the gun current, $I_o = (\lambda_o / \mu_o)\Phi$ with bias flux and Taylor eigenvalue λ_o, and κ is a coefficient based on the magnetic turbulence model employed in Dan Hua's spheromak simulation code. The value of κ giving a good energy balance (around 0.1 mΩ/KA) implies substantial turbulence levels. Implementing our model in SPICE would provide a calibration for theoretical calculations of the turbulence.

Our analytic approximation to the SPICE code provides guidance to optimize future performance in SSPX, the greatest benefit appearing to come from reducing or eliminating the protective resistor to increase bank efficiency. Eliminating the resistor altogether doubles the bank efficiency and the spheromak magnetic energy.

1. Turbulent Resistance Model

We first present a theoretical model of the spheromak as a variable resistance representing "dynamo" helicity injection, thus justifying the approximation of a resistive load used to achieve an energy balance in the SPICE simulation code. In Section 2 we derive an analytic approximation to the code, which we apply to SSPX Shot 1822 in Section 3 and to the optimization of future performance in SSPX in Section 4.

Following Reference [1], we take as the gun power into the flux core:

$$I\,V_{GUN} = \int dz\, 2\pi\, a_C\, (v_A B^2/\mu_o)\, \langle \delta B^2/B^2 \rangle \quad (1)$$

$$\langle \delta B^2/B^2 \rangle = \gamma\delta t\, (\lambda' a/\lambda)^2 \approx \gamma\delta t\, (1 - I_o/I)^2 \quad (2)$$

where in Eq. (1) the integration runs over the flux core of length ≈ R, the flux conserver radius, and a_C is the flux core radius that varies along the length. Also $v_A = 2 \times 10^{16}$ B $(1/\sqrt{n})$ is the Alfven speed at density n. In Eq. (2), g is a "duty factor" giving the time average with island growth constant γ and duration δt, and the final factor represents the magnetic free energy approximated on the right in terms of the gun current I and the threshold for helicity injection:

$$I_o = (\lambda_o /\mu_o)\Phi \quad (3)$$

with bias flux Φ and Taylor eigenvalue $\lambda_o \approx 10$ in SSPX.

Introducing $B = \mu_o I/2\pi a_C$ into Eqs. (1) and (2) gives, after some algebra:

$$V_{GUN} = I\, R_S \quad (4)$$

$$R_S = \kappa I\, (1 - I_o /I)^2 \quad \text{for } I > I_o \text{ (0 otherwise)} \quad (5)$$

$$\kappa = g(\mu_o^2/4\pi R)(v_A/B) = g(0.16/\sqrt{10^{-20} n}) \quad \text{m}\Omega/\text{KA} \quad (6)$$

$$g = \gamma\delta t \int dz R/a_C^2 \quad (7)$$

In Hua's 1 D transport code, a quantity corresponding to g is determined self-consistently to pump in power while maintaining a Taylor state in competition with ohmic decay. Here we treat g as a parameter.

For SSPX Shot 1822, the calculated value of I_o is 200 KA for a bias flux of 0.025 webers, to be compared to a measured threshold of 278 KA at which buildup of the spheromak poloidal field begins and below which the gun voltage drops to zero during decay. The value of κ giving the measured maximum gun voltage of 1300 volts, using the measured threshold current and measured maximum current of 400KA, is:

$$\kappa_{EXP} \quad = \quad 1300/(400 - 280)^2 \quad = \quad 0.09 \text{ m}\Omega/\text{KA} \quad . \qquad (8)$$

At probable densities in SSPX, Eqs. (6) and (8) agree for g of order unity, indicating strong turbulence early in the shot when $I/I_o > 1$. Estimates of the integral in Eq. (7) give a duty factor $\gamma\delta t \approx 0.1 \approx 1/\sqrt{S}$ for this low temperature shot, suggestive of the likely theoretical maximum turbulence levels for resistive modes [2].

The variable resistance in Eq. (5) can be substituted directly for the fixed resistance now used in SPICE. Though the code also allows for gun inductance, it is sufficient to set this inductance to zero, keeping only the external inductance which appears to explain the L/R decay of the current after the bank is crowbarred. Both κ and t_Ω can be treated as adjustable constants to be compared with the theoretical estimates later. Implementation of the model in SPICE would give a better calibration for theoretical estimates of the turbulence than our crude estimate above.

2. An Analytic Approximation to SPICE

To anticipate results of using our resistive model in SPICE, we examine an analytical approximation to the code, using a fixed value for the resistance R_S representing its maximum value during helicity injection.

The SPICE code simulates SSPX by an equivalent circuit in which the spheromak is represented by a resistance R_S together with an external resistance R_X , inductance L and capacitance C. For small $R = R_X + R_S$, the current is given by:

$$I \quad = \quad I_C \exp(-t/2\tau) \sin \omega t \qquad (9)$$

where $\omega = 1/\sqrt{LC}$, $\tau = L/R$ and $I_C = V_o (C/L)^{1/2}$, V_o being the charging voltage. Eq. (9) is accurate to order $(2\omega\tau)^{-2}$.

The maximum current occurs around $\omega t = \pi/2$, giving:

$$I_{MAX} \quad = \quad I_C \exp(-\pi/4\omega\tau) \qquad (10)$$

$$V_{MAX} = I_{MAX} R_S \quad . \tag{11}$$

For SSPX formation bank parameters ($L = 0.8\mu H$, $C = 0.01$ f, $R_X = 3.25 m\Omega$), and taking $R_S = R_X$ as Stallard found to satisfy energy balance, the calculated values are $I_{MAX} = 438$ KA and $V_{MAX} = 1424$ volts in reasonable agreement with measured values of approximately 400 KA and 1300 volts, respectively.

3. Efficiencies

The injected helicity is given by integrating:

$$dK/dt = 2\,V\,\Phi - K/t_{\Omega} \tag{12}$$

which gives approximately:

$$K = \alpha\, f_D\, 2\, V_{MAX} \Phi\, \Delta t \tag{13}$$

with dissipation efficiency given by:

$$f_D = 1/[1 + (\Delta t/t_{\Omega})] \quad . \tag{14}$$

where t_{Ω} is the helicity and energy ohmic decay time. The averaging parameter α is discussed below. Setting $I = I_o$ gives two values of t which determine the time interval during which helicity is injected, giving approximately:

$$\Delta t = (4/\omega)(1 - I_o/I_{MAX})^{1/2} \quad . \tag{15}$$

From helicity we can calculate the magnetic energy in the spheromak:

$$E_{MAG} = f_{SC}\,(\lambda_o/2\mu_o)\,K = \alpha\, f_{SC}\, f_D\, V_{MAX}\, I_o\, \Delta t \tag{16}$$

where f_{SC} is the fraction of magnetic energy in the flux conserver calculated by Corsica (the short-circuit effect). Eq. (16) is usually accurate to 10% even though the spheromak is not in an exact Taylor state [3]. We also calculate the energy input to the gun and the total bank energy:

$$E_{GUN} = \int I\, V = \alpha\, V_{MAX}\, I_{MAX}\, \Delta t \qquad (17)$$

$$E_{BANK} = 1/2\, CV_o^2 = 1/2\, I_C^2\, L \quad . \qquad (18)$$

We write the overall efficiency of delivering bank energy to spheromak magnetic field energy as:

$$\text{Efficiency} = E_{MAG}/E_{BANK} = f_{BANK}\, f_{GUN} \qquad (19)$$

where, using the relations above, we find:

$$f_{GUN} = E_{MAG}/E_{GUN} = \{f_{SC}\, f_D\}\, f \qquad (20)$$

$$f_{BANK} = E_{GUN}/E_{BANK} = \alpha\, (8R_S/\omega L)\, (1 - I_0/I_{MAX})^{1/2} \exp(-\pi/2\omega\tau) \qquad (21)$$

where in Eq. (20) f is the "fundamental efficiency" given by:

$$f = I_o / I_{MAX} = \lambda_o / \lambda_{GUN} \quad . \qquad (22)$$

For estimation purposes, we choose the parameter α to insure energy conservation for the case of zero external resistance, in which case we require:

$$f_T\, E_{BANK} = E_{GUN} + 1/2\, L\, I_o^2 \qquad (23)$$

where the last term accounts for residual inductive storage when the bank is crowbarred and the gun voltage drops to zero. The factor f_T represents additional bank inefficiencies due to transient losses early in the shot that are not accounted for by the known resistance and inductance listed above.

Note that, because f_{GUN} goes up with increasing bias flux while Δt and hence f_{BANK} go down, for a given system and charging voltage the overall efficiency is maximized by a bias flux such that $I_0 = 2/3\, I_{MAX}$, giving:

$$f \quad = \quad I_o/I_{MAX} \quad = \quad 2/3 \quad . \qquad (24)$$

An optimization similar to Eq. (24) is observed in SSPX, Shot 1822 having been near optimum (see Figure 1).

4. Optimization

For Shot 1822 for which Stallard has done an energy balance, 50KJ is not accounted for with a 7 KV charging voltage (E_{BANK} =245 KJ) giving f_T = 0.8 . Using Corsica, Woodruff finds f_{SC} = 0.75 for this shot in order to match 0.2 T poloidal field at the wall. Other parameters for Shot 1822 give $\alpha = 0.58$, $\Delta t = 0.21$ ms (in good agreement with the data), $\omega = 1.1 \times 10^4 s^{-1}$ and $\tau = 1.23 \times 10^{-4}$s using $R_S = R_X = 1/2\,R$ as was found necessary to balance energy.

We assume a decay time of 200μs giving $f_D = 0.49$. Note that this decay time is less than my previous estimates. Dr. Nagata has pointed out that I over-estimated the magnetic decay time, due to recirculation of internal energy after the gun voltage drops to zero, and also I confused field decay and energy decay, which differ by a factor of 2. I now estimate the field decay time from dB/dt late in the shot to be about 400μs yielding an energy and helicity decay time of only 200μs.

Using these parameters and assuming optimized injection by Eq. (24), multiplying factors in the order they appear in the efficiency formulas gives for Shot 1822:

$$f_{GUN} = (0.75)(0.49)(2/3) = 0.25 \quad (25\%)$$

$$f_{BANK} = (0.58)(2.95)\,(1/\sqrt{3})\exp(-1.16) = 0.31 \quad (31\%)$$

$$E_{MAG}/E_{BANK} = (0.25)\,(0.31) = 0.07 \quad (7\%)$$

Multiplying the efficiency times the bank energy gives 17 KJ, which is about 50% higher than the spheromak field energy calculated by Corsica. The actual overall efficiency for Shot 1822 is nearer to 5%.

The best shots in CTX gave 17% efficiency compared to 5% in SSPX [4]. Since SSPX has a fundamental efficiency comparable to SSPX (> 50% by the above estimate), improvements must lie elsewhere.

While there has been much concern about the short circuit that might be remedied by new bias coils, the above analysis suggests that there is much to be gained by attention to the capacitor bank itself even before bias coils and additional capacitor bank capability are available.

Our analytical model suggests that the main factor that could be improved is the damping coefficient in Eq. (21), approximated here as:

$$\exp(-\pi/2\omega\tau) \quad = \quad \exp[\, - (\pi/2)(R\sqrt{C/L})] \quad = \quad \exp(-1.16) \;=\; 0.31 \qquad (24)$$

which accounts for the bank inefficiency in Shot 1822. Increasing this factor increases overall efficiency if also the bias flux is increased to maintain the optimization condition Eq. (24).

Perhaps the most practical way to increase this factor is to reduce the external resistance R_X, which is largely due to a protective resistor in the capacitor bank circuit. For example, eliminating R_X altogether would, for the same spheromak load, approximately double the bank efficiency. Improved bank efficiency would appear directly as an increase in spheromak field energy, from an actual 10 KJ in Shot 1822 to about 20 KJ.

Increasing the field by increasing the bank efficiency would provide early evidence as to how well confinement is working. According to Hua's simulations, at a fixed density the temperature rises directly with the field energy, with the following scaling:

$$n\, T_{KeV} \quad \approx \quad 10^{20}\, B^2 \qquad .$$

It is this scaling that allows buildup to continue, by increasing t_Ω so that the dissipation factor f_D continues to be order unity even as the helicity injection period is lengthened.

Increasing the temperature is imperative. With my revised estimate of a 200μs energy decay time, the temperature is probably only 15 eV, corresponding to a density of 2×10^{20} if resistance rather than radiation is the dominant core loss. Decreasing the density to increase the temperature might also improve efficiency, by decreasing the dissipative losses. However, according to Hua's simultaions, with sufficient power buildup to high field and high temperature is possible even at present densities, which we estimated to be around 2 x 10^{20}. High density may in fact be required to maintain the current at or near the ion saturation current as the field increases [5].

References

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Table 1. SSPX Shot # 1822
(V_o = 7000 v, E_{BANK} = 245 KJ)

	Data	SPICE	Theory(1)
V_{MAX}(V)	1300	1300	1424
I_{MAX}(KA)	400	400	431
$I_{THRESHOLD}$(KA)	278	300	292 (2/3 I_{MAX})
E_{GUN}(KJ)	75	75	76
f_{BANK}	0.3	0.3	0.31
f_{GUN}	0.13(=10/75)	--	0.25(2)
T(KeV)	?	--	0.015(3)
B(T)	0.18	--	0.26
n($\times 10^{-20}$m^{-3})	?	--	2(4)
Optimum λ_{GUN}/λ_o	≈ 2	?	1.5

(1) Treating spheromak as 3.25 mΩ resistance, (2) For optimum λ_{GUN}/λ_o = 2/3,

(2) From decay time 200μs, (4) To fit T = 0.015 Kev from Hua's code