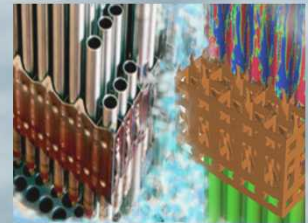
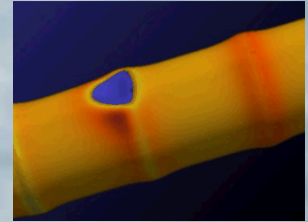
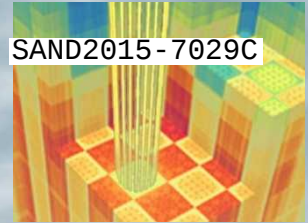


Assessment of Models for Near Wall Behavior and Swirling Flows in Nuclear Reactor Sub-systems

European Turbulence Conference 15
Delft Technical University
Delft, Netherlands
August 25-29, 2015

Thomas M. Smith
Center for Computing Research
Sandia National Laboratories
Albuquerque, New Mexico USA
Mark A. Christon
Emilio Baglietto, Ben Magolan, MIT



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. SAND NO. 2011-XXXXP

Background

- CASL Program Goals and Challenges
 - Deploy software for advanced Simulation of PWR and BWR
 - Coupled high-fidelity Thermal Hydraulics, Neutronics, CHT
 - UQ, Parameter Sensitivities, Optimization
 - Objectives for Thermal Hydraulics (TH)
 - Full reactor core coupled physics simulations
 - Fuel rod performance on existing systems and new designs
 - Grid-to-rod-fretting, mechanical wear due to flow induced vibration
 - Corrosive residual unidentified deposits "hot spots"
 - QoI dictate level of fidelity for simulations
 - *“Extensive application of computational fluid dynamics (CFD) in the last two decades has clearly revealed turbulence modeling to be the **weakest link** in the development of a reliable numerical methodology.”* – Emilio Baglietto, MIT, CASL/THM Lead
- Near wall turbulence behavior Identified as area of concern
 - Wall damping or wall functions or both?
 - Accurate prediction of wall gradients desired
 - Map variability along a rod spacer span
 - High fidelity skin friction and heat flux to improve lower-fidelity zonal based TH models

Computational Tools

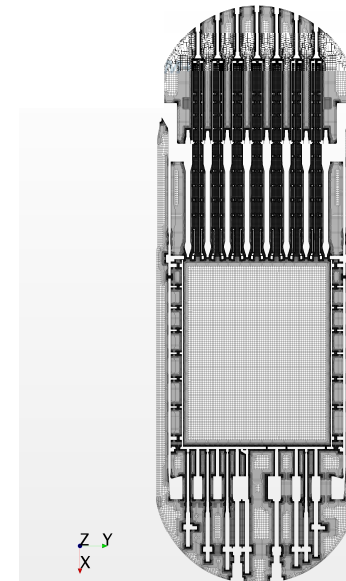
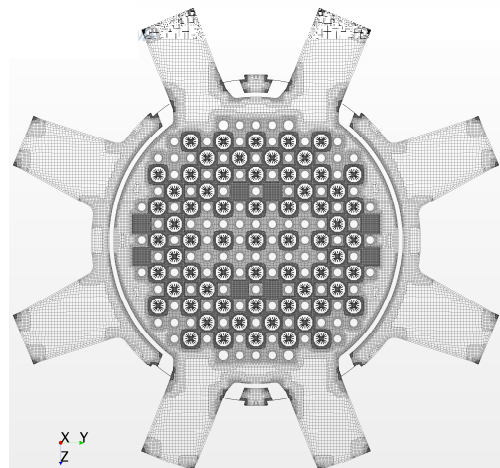
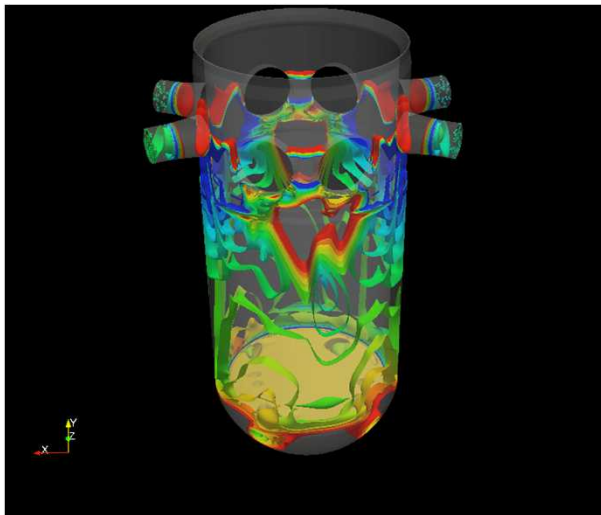
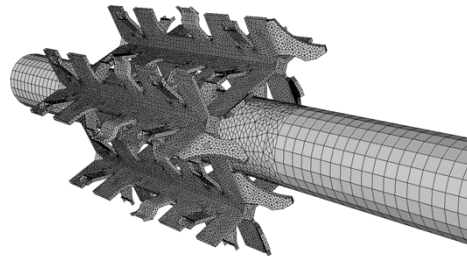
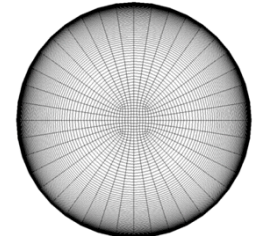
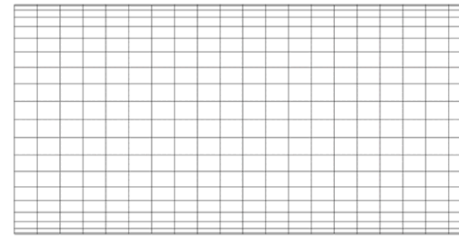
- Hydra-TH
 - Hydra-TH is a hybrid finite-volume/finite-element incompressible/low-Mach number CFD code
 - Multi-physics CHT toolkit being developed for turbulent reactor core simulations.
 - surface-based output.
 - Linear algebra is handled through an abstract interface that permits use of popular libraries such as PetSC and Trilinos ML
 - The toolkit supports semi- and fully-implicit solvers for time-dependent and steady-state
 - Incompressible Navier-Stokes equations with heat conduction and transport on heterogeneous unstructured meshes.

Assessment of Turbulence Models

- RANS Eddy Viscosity Models
 - Spalart-Allmaras Eddy Viscosity Model with wall damping
 - Requires normal-distance to walls at every integration cell
 - Integrate to the wall $y^+ \leq 5$
 - Simple wall boundary condition $\tilde{\nu}=0$
 - Surface gradients computed using finite-differences
 - k- ϵ Eddy Viscosity Models with y^* insensitive wall function
 - Requires normal distance in wall adjacent cells
 - First cell in log layer, $y^+ = 20-40$
 - ν_t , ϵ , ν_t production modified in wall adjacent cells
 - Surface gradients and temperature inferred from wall function
- Turbulence Sub-System Test Problems
 - Detailed examination of model accuracy and robustness
 - Known expected outcomes
 - Contain important flow features present in reactor cores
 - Simple to complex

Background Cont.

- From simple to complex
 - Flow structures
 - geometry
 - coupled physics



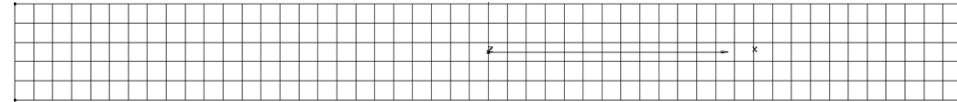
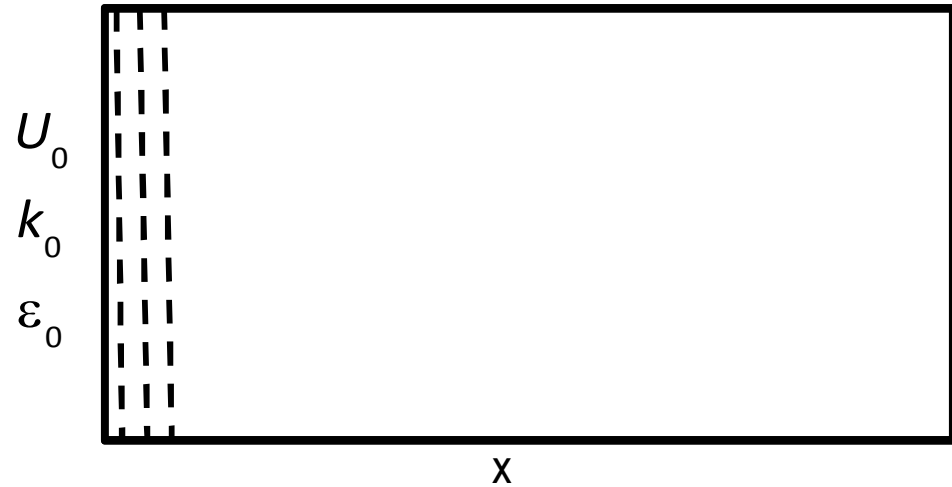
Torture Test: Grid Turbulence Geometry

- Elems: 50x5x1, 100x10x1, 200x20x1
- Domain: 1.0 x 0.1 x 0.01
- rho=1
- mu=1.5e-6
- $C_2=1.92$ for STD k-eps model
- $U_0 = 1, k_0 = 0.01, \varepsilon_0 = 0.01$
- Analytical Solution: Mohammadi and Pironneau, "Analysis of the K-Epsilon Turbulence Model," Wiley New York, (1993).
- What's tested: time evolution, advection, diffusion, production, dissipation
- What's not tested: **near wall behavior**

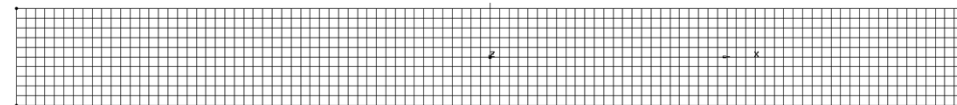
Analytical Solution

$$k(x) = k_0 \left(1 + (c_2 - 1)x \frac{\varepsilon_0}{k_0 U_0} \right)^{\frac{1}{1-c_2}}$$

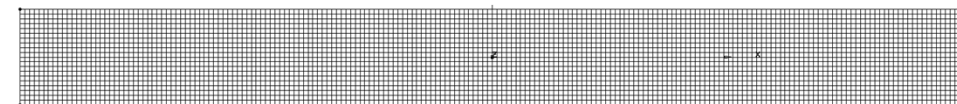
$$\varepsilon(x) = \varepsilon_0 \left(1 + (c_2 - 1)x \frac{\varepsilon_0}{k_0 U_0} \right)^{\frac{c_2}{1-c_2}}$$



Grid 1: 50x5x1

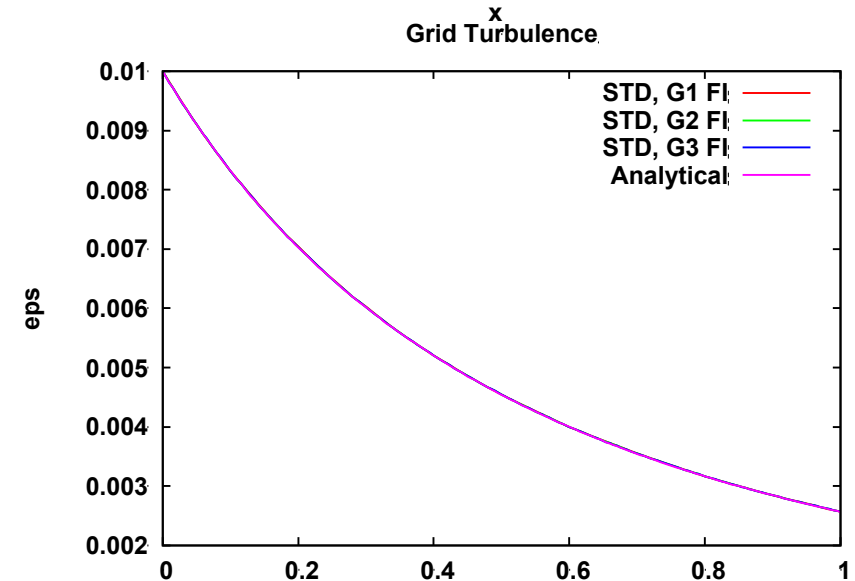
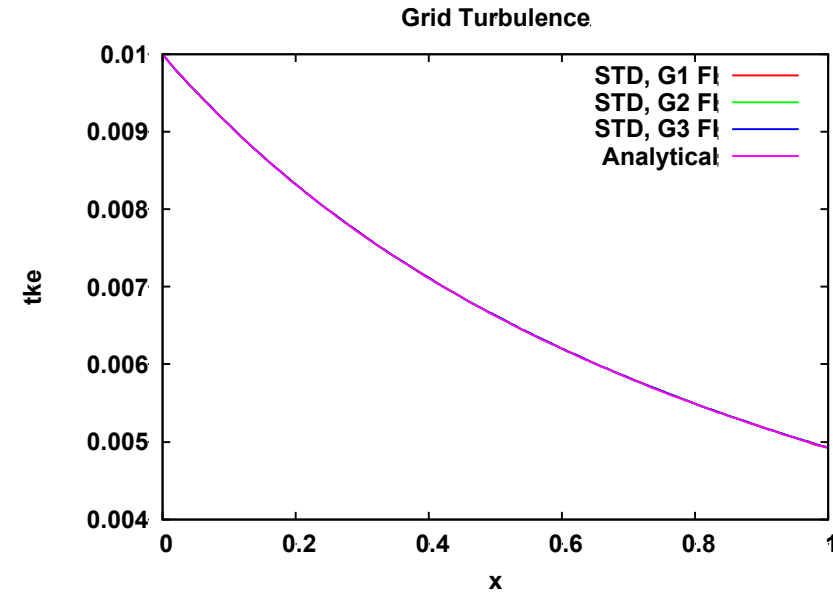
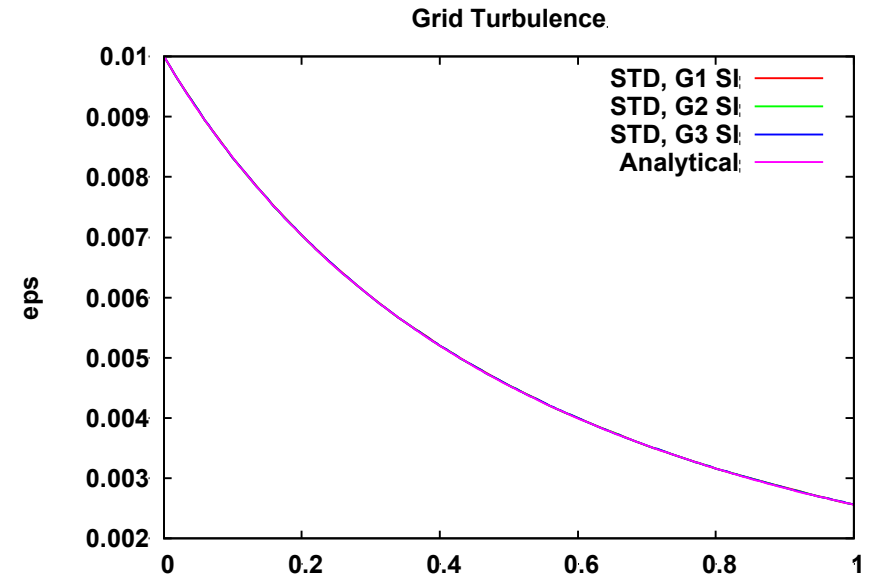
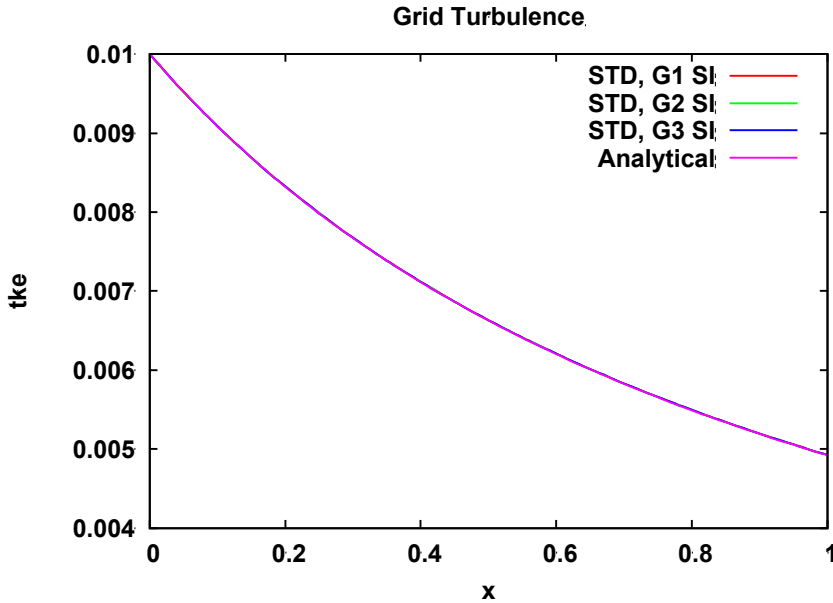


Grid 2: 100x10x1



Grid 3: 200x20x1

Torture Test: Grid Turbulence: STD k-eps



Wall Functions y^* insensitive Model

(Launder&Spalding, 1974; Grotjans&Menter, 1989; Craft et al. 2002)

- STD, RNG, NL use 2-Layer " y^* -insensitive" or "*scalable*" WF
- Implementation in Hydra-TH

Theory

$$y_p^* = \frac{\rho C_\mu^{1/4} y_p k^{1/2}}{\mu}$$

$$y_v^* = \frac{\rho C_\mu^{1/4} y_v k^{1/2}}{\mu} = 11.225$$

$$Pk = \begin{cases} 0 & y_p^* < y_v^* \\ \frac{\tau_w^2}{\kappa C_\mu^{1/4} \rho k^{3/2} y_p} & y_p^* > y_v^* \end{cases}$$

$$\tau_w = \frac{C_\mu^{1/4} \rho u_p k^{1/2}}{\kappa \ln(Ey_p^*)}$$

$$\varepsilon = \begin{cases} \frac{2\mu k}{y_v^2} & y_p^* < y_v^* \\ \frac{\rho C_\mu^{3/4} k^{3/2}}{\kappa y} & y_p^* > y_v^* \end{cases}$$

$$y_{p,lim}^* = \max(y_p^*, y_v^*)$$

$$y_{p,lim} = \frac{y_{p,lim}^* \mu}{\rho C_\mu^{1/4} k^{1/2}}$$

Numerical Implementation

$$Pk_{ave} = \frac{1}{y_n} \int_0^{y_n} Pk dy = \frac{\tau_w^2}{\kappa C_\mu^{1/4} \rho k^{1/2} y_n} \ln\left(\frac{y_n}{y_v}\right)$$

$$Dk_{ave} = \frac{1}{y_n} \int_0^{y_n} Dk dy = \frac{2\mu k}{y_n y_v} + \frac{\rho C_\mu^{3/4} k^{3/2}}{\kappa y_n} \ln\left(\frac{y_n}{y_v}\right)$$

$$\varepsilon_p = \frac{C_\mu^{3/4} k^{3/2}}{\kappa y_p}$$

$$\mu_{eff} = \begin{cases} \mu & y_p^* < y_v^* \\ \frac{\rho C_\mu^{1/4} k^{1/2} y_p}{\kappa \ln(Ey_p^*)} & y_p^* > y_v^* \end{cases}$$

$$\kappa_{eff} = \frac{\mu C_p y^*}{T^*}$$

$$T^* = \begin{cases} Pr y_p^* & y_p^* < y_v^* \\ Prt \left(\frac{1}{\kappa} \ln(Ey_p^*) + P \right) & y_p^* > y_v^* \end{cases}$$

Wall Functions Cont. – Post Processing

(Defraeye, Blocken and Carmeliet JIHMT, 2010)

- Two different velocity scales

Theory (y^+)

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

$$y^+ \equiv u_\tau y / \nu$$

$$u^+ \equiv \frac{u}{u_\tau} = \frac{1}{\kappa} \ln(Ey^+)$$

$$T^+ \equiv \frac{(T_w - T_p) \rho C_p u_\tau}{\dot{q}''_w} = \text{Pr}t \left[\frac{1}{\kappa} \ln(Ey^+) + P_j \right]$$

$$\dot{q}''_w = \frac{\rho C_p u_\tau [T_w - T_p]}{\text{Pr}t \left[\frac{1}{\kappa} \ln(Ey^+) + P_j \right]}$$

$$T_w = T_p + \frac{\dot{q}''_w \text{Pr}t \left[\frac{1}{\kappa} \ln(Ey^+) + P_j \right]}{\rho C_p u_\tau}$$

Theory (y^*)

$$y^* = \frac{\rho C_\mu^{1/4} y k^{1/2}}{\mu}$$

$$u^* = \frac{\rho C_\mu^{1/4} u k^{1/2}}{\tau_w}$$

$$T^* \equiv \frac{[T_w - T_p] \rho C_\mu^{1/4} C_p k^{1/2}}{\dot{q}''_w} = \text{Pr}t \left(\frac{1}{\kappa} \ln(Ey_p^*) + P_j \right)$$

$$\dot{q}''_w = \frac{[T_w - T_p] \rho C_\mu^{1/4} C_p k^{1/2}}{\text{Pr}t \left(\frac{1}{\kappa} \ln(Ey_p^*) + P_j \right)}$$

$$T_w = T_p + \frac{\dot{q}''_w \text{Pr}t \left(\frac{1}{\kappa} \ln(Ey_p^*) + P_j \right)}{\rho C_\mu^{1/4} C_p k^{1/2}}$$

Wall Functions Cont.

Hydra surface delegates, H&L 2011

- Desire an estimate of wall shear

Theory – output delegates

$$u^* = \frac{u k^{1/2} C_\mu^{1/4}}{(\tau_w / \rho)} = \frac{1}{\kappa} \ln(E y_p^*)$$

$$\frac{\tau_w}{\rho} = \frac{C_\mu^{1/4} \kappa u_p k_p^{1/2}}{\ln(E y_p^*)}$$

$$\tau_w = \mu_w^{eff} \frac{u_p - u_w}{y_p}$$

$$\mu_w^{eff} = \frac{\rho \kappa C_\mu^{1/4} k_p^{1/2} y_p}{\ln(E y_p^*)}$$

$$\mu_w^{eff} = \begin{cases} \mu & y_p^* < y_v^* \\ \frac{\rho \kappa C_\mu^{1/4} k_p^{1/2} y_p}{\ln(E y_p^*)} & y_p^* > y_v^* \end{cases}$$

$$\kappa_w^{eff} = \kappa + \frac{\mu_w^{eff} C_p}{Pr t}$$

Fully Developed Pipe Flow

Test Conditions

$$L/D = 20$$

$$\Delta P = 1.0, 4.0, 14.0$$

$$T_{in} = 300$$

$$T_w = 350$$

$$q_w'' = 250$$

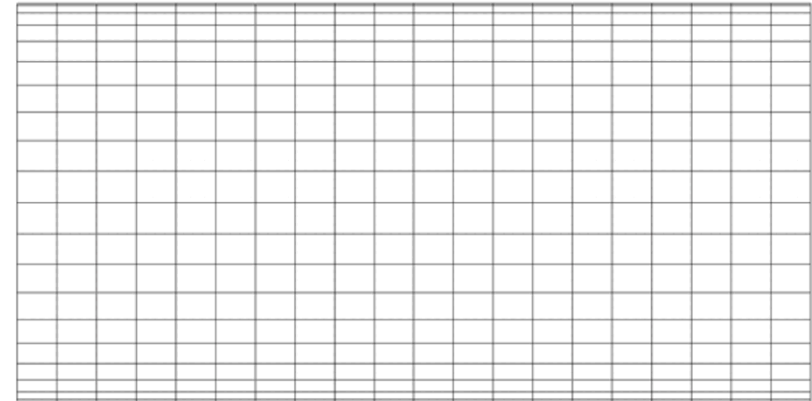
$$\rho = 1$$

$$\text{Pr} = 1$$

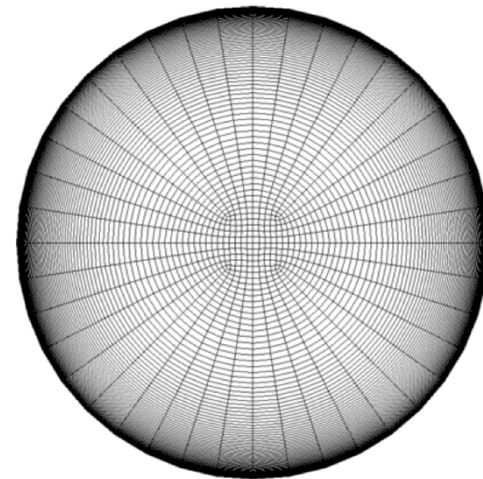
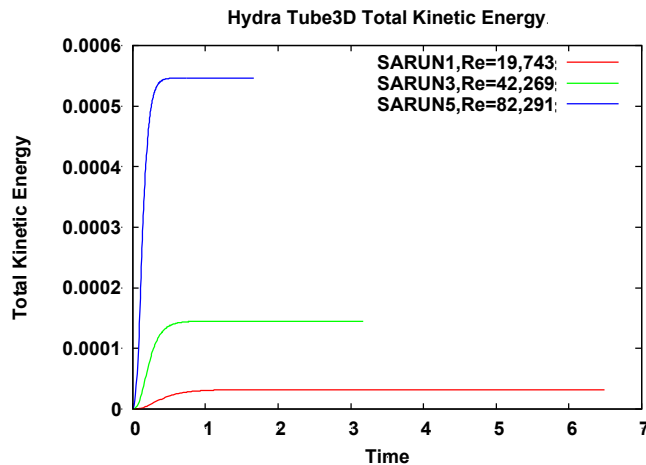
$$\text{Pr}_t = 1$$

$$\mu = 1.0E-6$$

Cubit Mesh

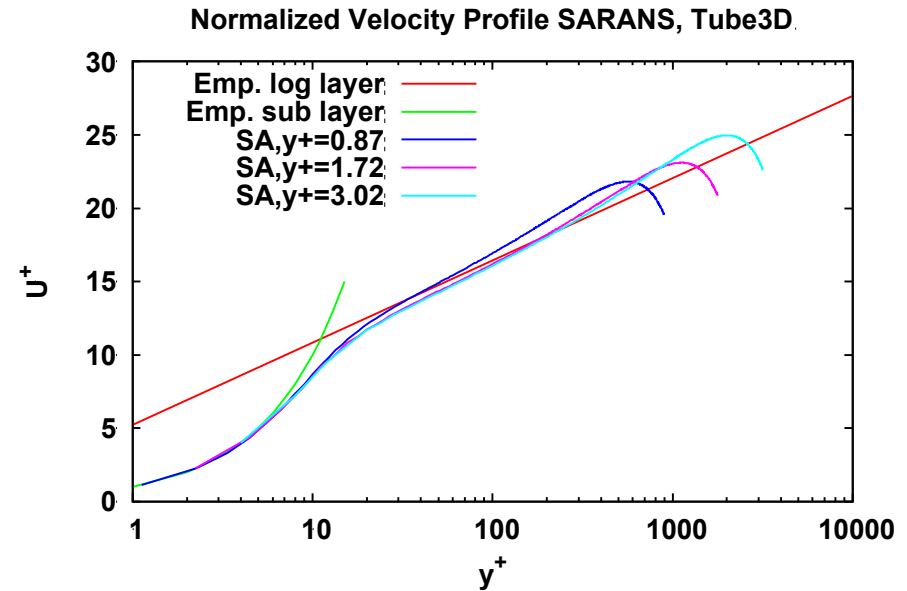
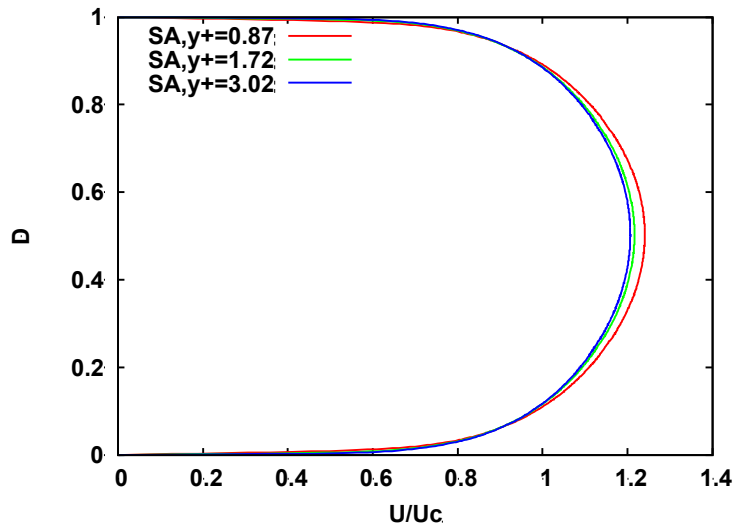


Flow



Fully Developed Pipe Flow

- Domain: $L/D=20$
- Same mesh for all three cases
- $Re_D=19,743; 42,269; 82,291$



Post Processing

$$\tau_w \approx \mu \left. \frac{\partial u(y)}{\partial n} \right|_w \quad u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

Have to approximate τ_w

Prediction

$$y^+ = u_\tau y / \nu$$

$$U^+ = \frac{u(y)}{u_\tau}$$

Theory

$$U^+ = y^+$$

$$U^+ = \frac{1}{0.41} \ln(y^+) + 5.2$$

Fully Developed Pipe Flow

- Domain: $L/D=20$
- Re_D : 18,100; 39,200; 78,300
- y^* -insensitive wall model makes computing τ_w difficult due to coarse wall normal grid spacing

Replace: $\tau_w \approx \mu \left. \frac{\partial u(y)}{\partial n} \right|_w$ $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$

$$c_f / 2 = (2.236 \ln(Re_D) - 4.639)^{-2} \quad (\text{EQ. 12-14, Kays\&Crawford, 1993; Petukhov, 1970})$$

$$u_\tau = V \sqrt{c_f / 2}$$

$$y^* = \frac{u^* y}{\nu}$$

$$u^* = C_\mu^{1/4} \sqrt{k}$$

Theory

$$U^+ = y^+$$

$$U^+ = \frac{1}{0.41} \ln(y^+) + 5.2$$

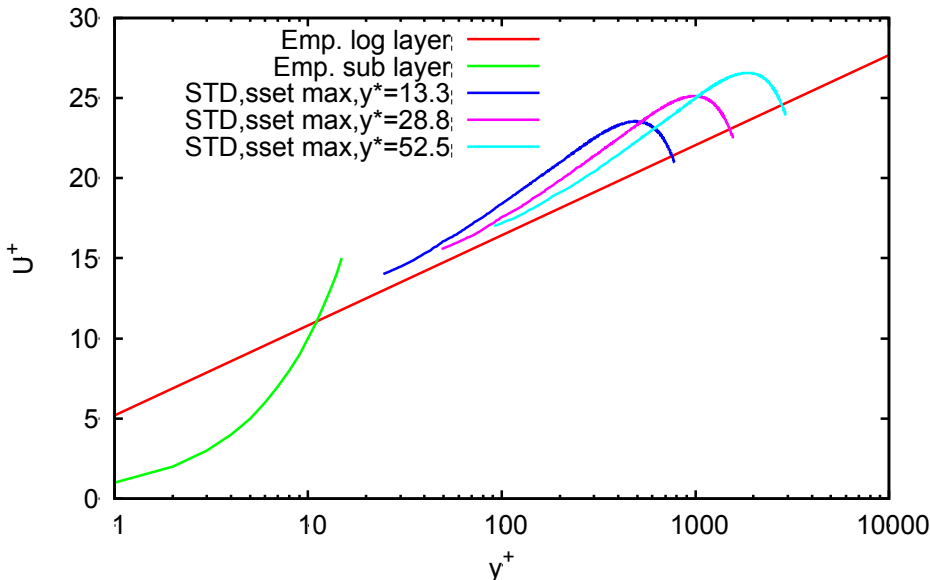
Prediction

$$y^+ = u_\tau y / \nu$$

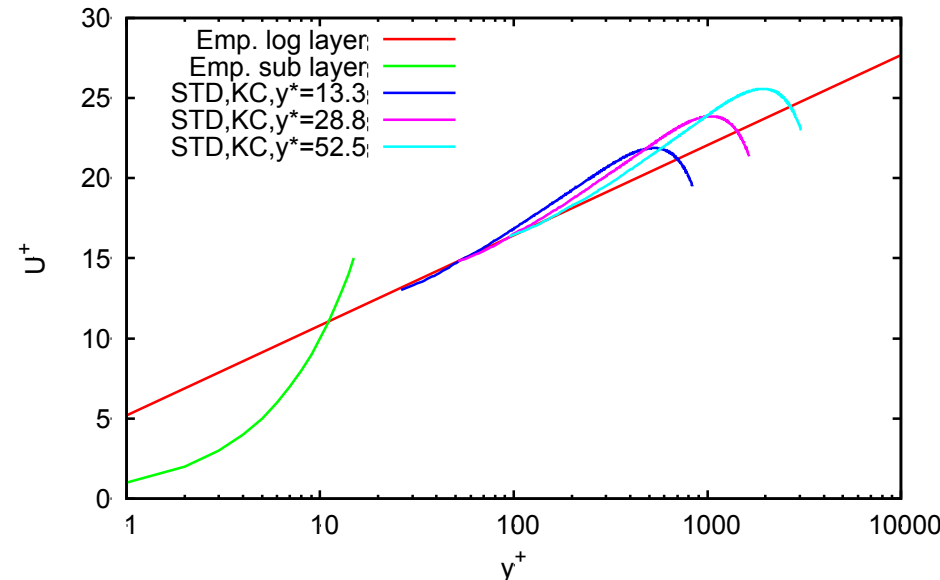
$$U^+ = \frac{u(y)}{u_\tau}$$



Normalized Velocity Profile STD Model, Tube3D.



Normalized Velocity Profile STD Model, Tube3D.



Fully Developed Pipe Flow

- Domain: $L/D=20$
- Entry Length: $L/D=20$
- $ReD=19,743; 42,269; 82,291$

Mixing V_m and T_m

$$V_m = \frac{1}{\rho A} \int_A \rho \mathbf{u} \cdot \mathbf{n} dA$$

$$T_m = \frac{1}{\rho V_m A} \int_A \rho \mathbf{u} \cdot \mathbf{n} T dA$$

$$Nu = \frac{hD}{\kappa} = \frac{\dot{q}_w'' D}{\kappa(T_w - T_m)}$$

Newton's Law of Cooling

$$\dot{q}_w'' = h(T_w - T_m)$$

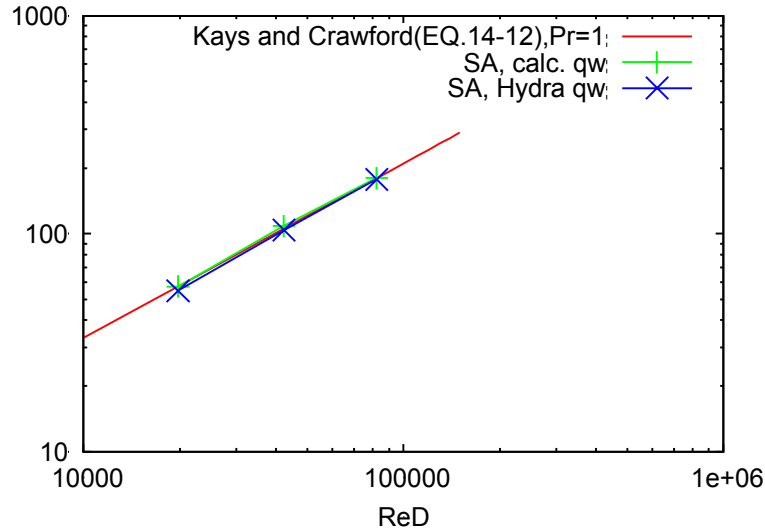
Wall heat flux and Temperature

$$\dot{q}_w'' = \frac{1}{L} \oint_L \kappa \frac{\partial T}{\partial n} dl$$

$$T_w = \frac{1}{L} \oint_L T dl$$

Constant wall temperature

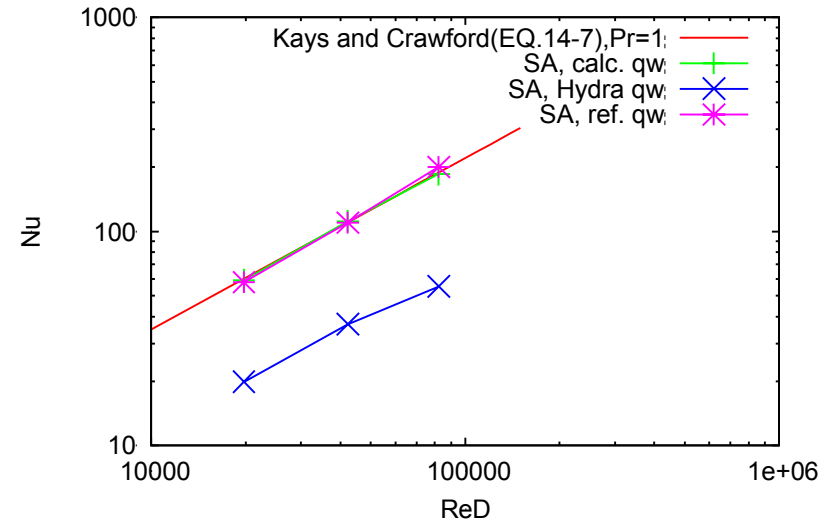
Nusselt Number, Hydra, $Pr=1$, $Prt=1$, tube3d, Const. T wall.



$$Nu = 0.021 Pr^{0.5} Re_D^{0.8} \quad (\text{EQ. 14-12, Kays\&Crawford, 1993})$$

Constant wall heat flux

Nusselt Number, Hydra, $Pr=1$, $Prt=1$, tube3d, Const. qw.

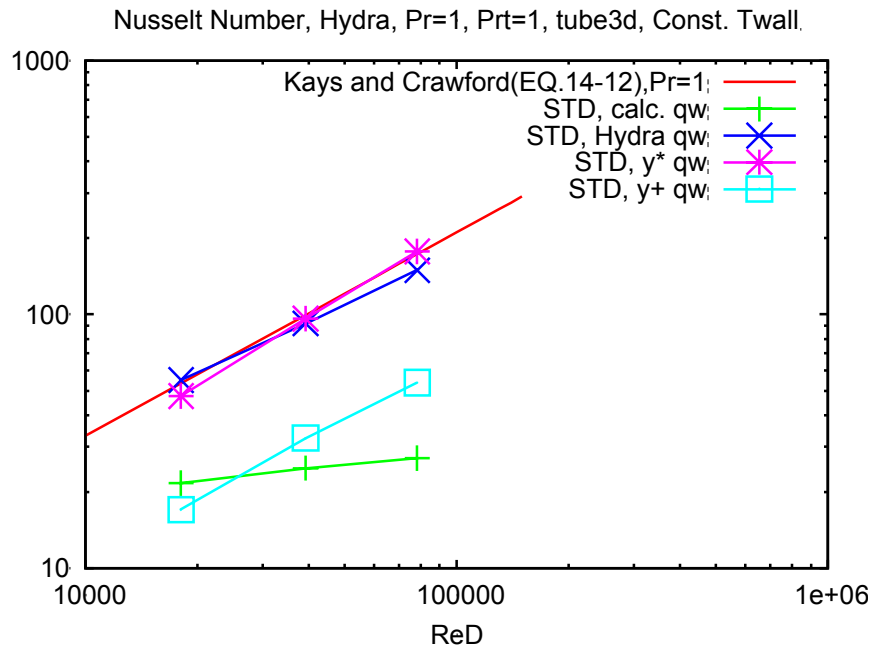


$$Nu = 0.022 Pr^{0.5} Re_D^{0.8} \quad (\text{EQ. 14-7 Kays\&Crawford, 1993})$$

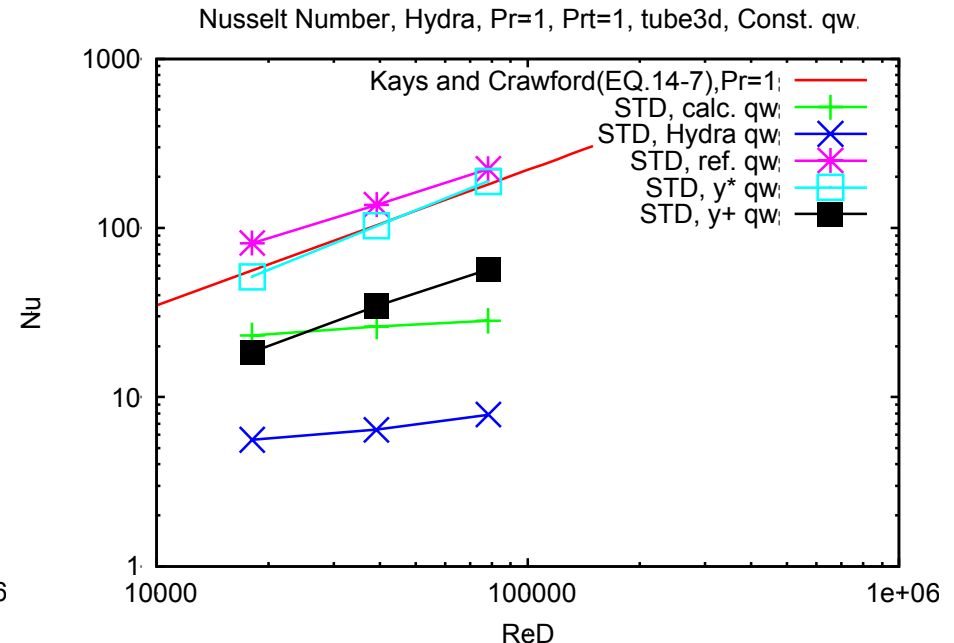
Fully Developed Pipe Flow

- Domain: $L/D=20$
- Re_D : 18,100; 39,200; 78,300

Constant wall temperature



Constant wall heat flux



Impinging Jet

Post Processing using Law of the Wall

(with E. Baglietto and B. Magolan, MIT)

Test Conditions

$$Re_D = 23,000$$

$$Pr = 0.71$$

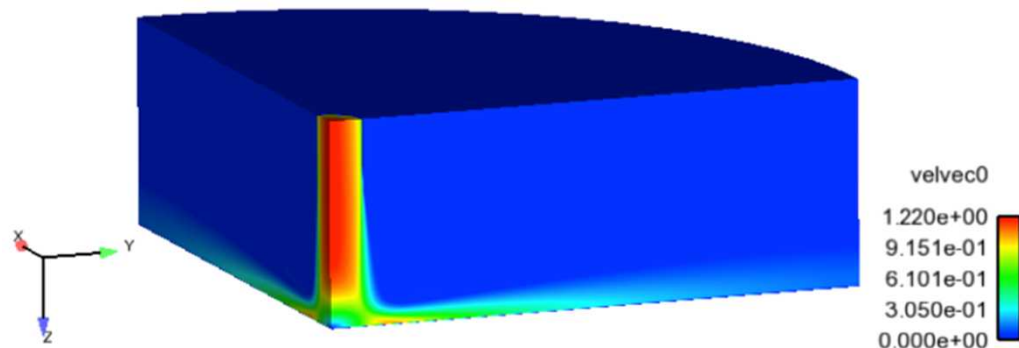
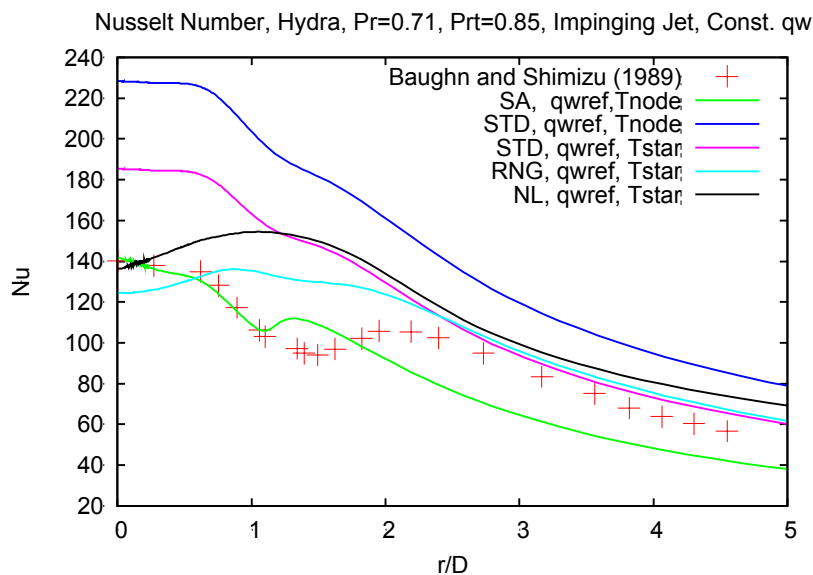
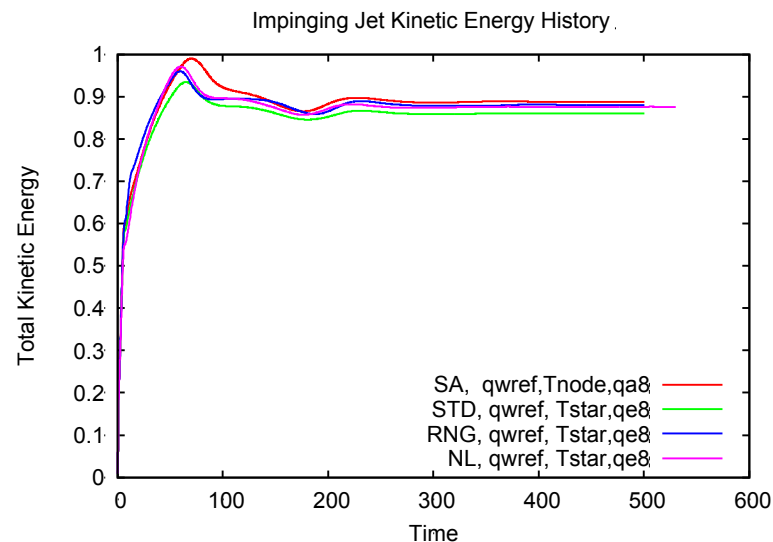
$$Prt = 0.85$$

$$H/D = 2$$

$$r/D = 8$$

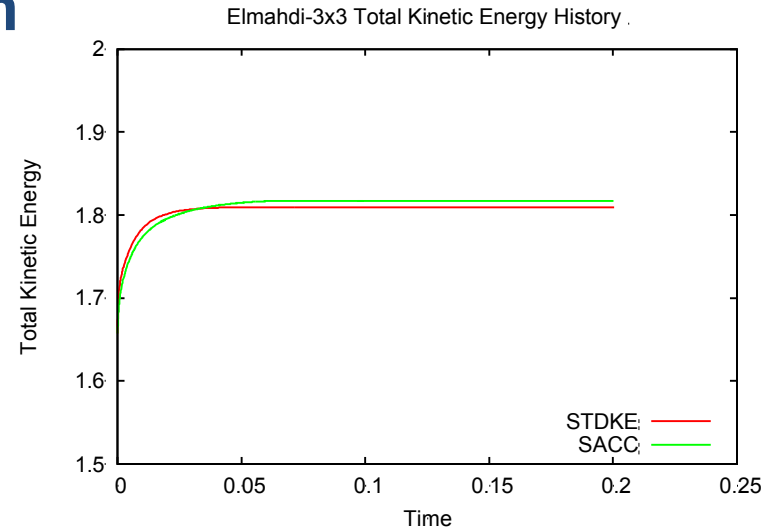
$$V_{intrain} = 0.01U_z$$

$$p_{out} = 0$$



Elmahdi-3x3 Spacer-Grid Problem

- Three estimates of heat flux from hot rods
 - BC value in cntl file
 - Post process using finite-difference to compute grad T
 - Hydra-TH delegate "nheatflux"
- Boundary conditions
 - ss3 – inflow (Hss3)
 - ss2 – outflow (Hss2)
 - ss9 center rod (qss9)
 - ss10 outer rods (qss10)
- Quantities of Interest
 - SST energy balance – rate of energy coming in equals the rate of energy leaving the domain
 - Heat flux – Is the right amount of energy being exchanged into the flow from the rods



$$\dot{q}_{ss}'' = \kappa \frac{\partial T}{\partial n} = \frac{\rho C_p v}{Pr} \frac{\partial T}{\partial n} \bigg|_{ss}$$

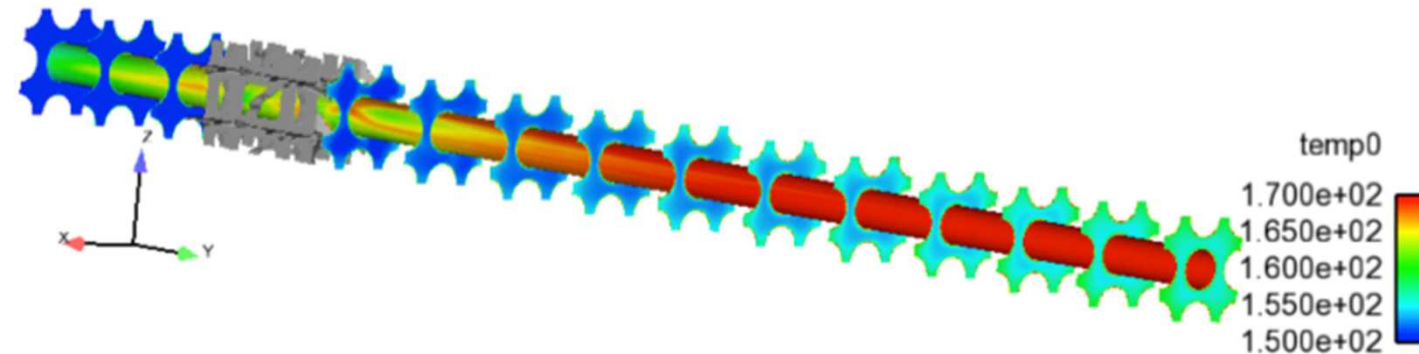
$$Q_{bc} = \dot{q}_{bc}'' A$$

$$h = \rho C_p Tu$$

$$H = \rho C_p TuA$$

$$H_{ss2} = H_{ss3} + \frac{\rho C_p v}{Pr} \frac{\partial T}{\partial n} \bigg|_{ss9} + \frac{\rho C_p v}{Pr} \frac{\partial T}{\partial n} \bigg|_{ss10}$$

$$H_{ss2} \neq H_{ss3} + Q_{bc9} + Q_{bc10}$$



3x3 Rod/Spacer Grid Sub-Assembly-Energy Balance

ReD=218,025

Pr=1

Prt=0.9

Tin=150.0

uin=5

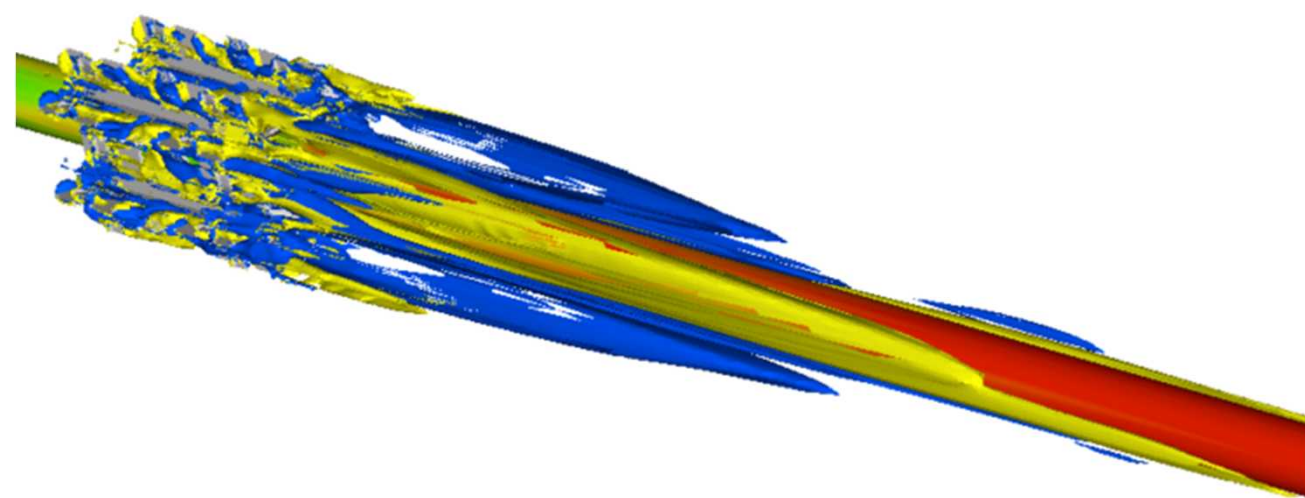
qwin=1.0e6

A_rod=0.047679

A_channel=0.000358497

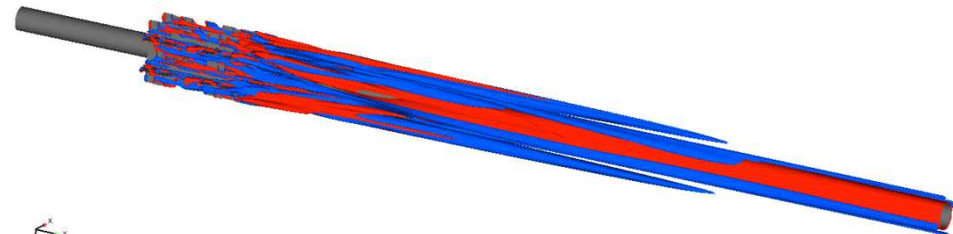
L=0.40132

	qw_BC	qw_delegate (hydra)	qw_in (fd) rods	H_in	total	H_out	%diff
KE	1.0E6	???	1,655	1,019,647	1,021,302	1,038,597	1.7
SA	1.0E6	???	23,769	1,056,631	1,080,400	1,084,112	0.3



Summary

- Enhanced turbulence model development is required for predictive core reactor flows
- Special attention needs to be devoted to near wall behavior modeling
- Low Re K-epsilon models are under development
- Development plan and execution of the plan can be found in CASL documents
 - “Multi-Year Plan for Enhancing Turbulence Modeling in Hydra-TH,” (**L3.THM.CFD.P10.02**), 2014
 - Findings from this first year will be reported later this year - “Enhanced Turbulence Model Capabilities in Hydra-TH,” (**L3.THM.CFD.P11.04**), 2014

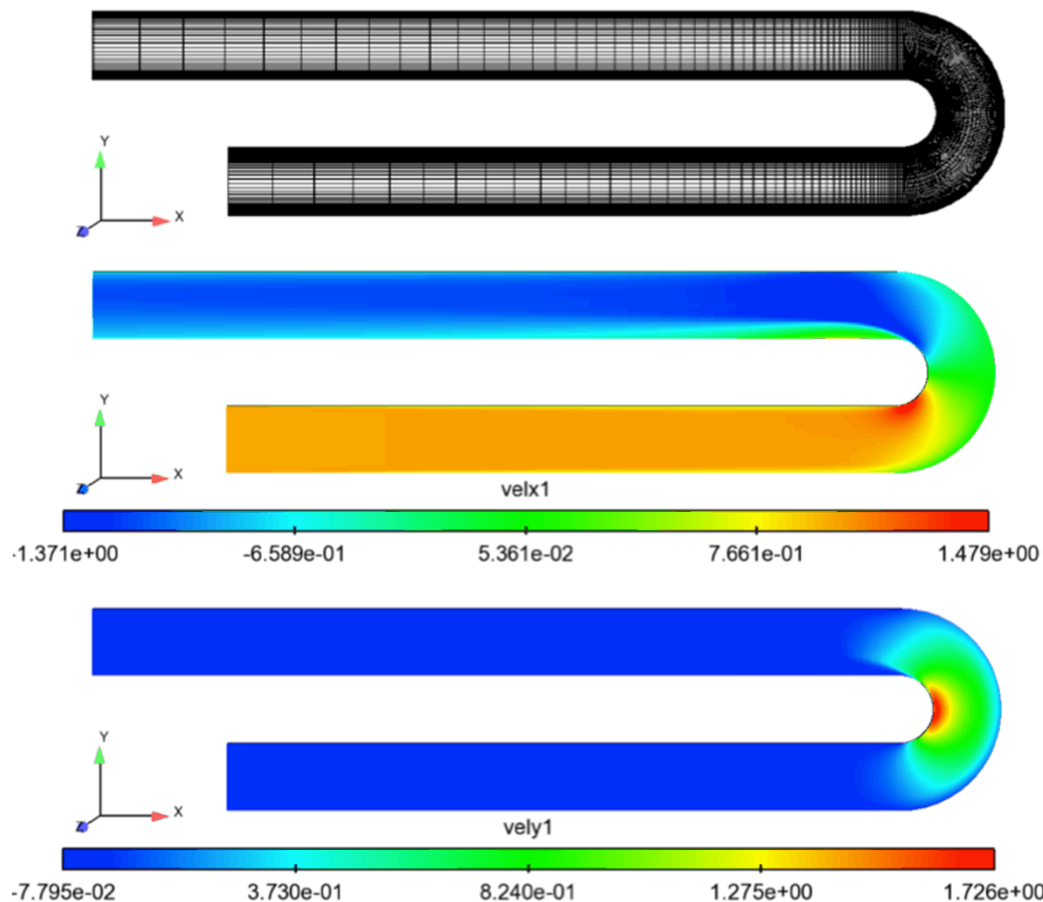


Backup Slides

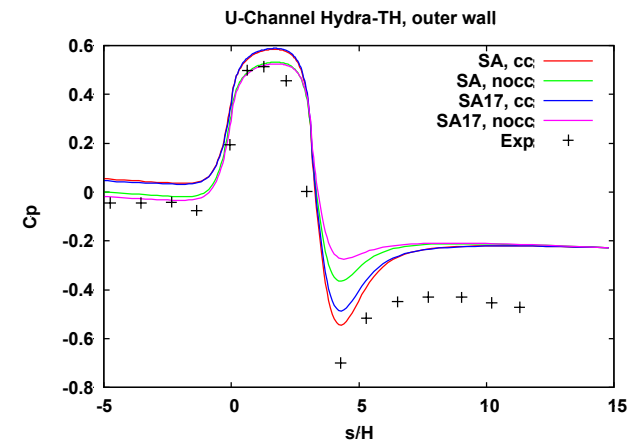
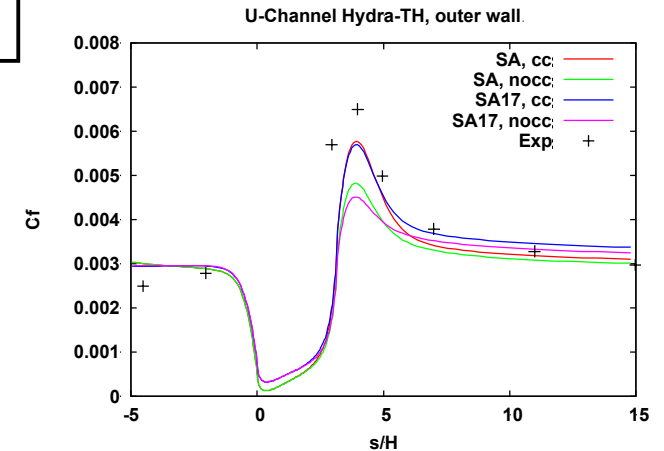
Turbulence Torture Tests

Test	Dimensions	Objectives	Meshes	Status	Documented	Regression
Back Step	2D	Reattachment	Cubit	regression	Some	yes
Channel	2D	Law-of-wall	Cubit	SA, RNG	Verification	no
Grid Turbulence	2D	Decay rate	3 Levels	RNG, STD	Verification	yes
Couette Flow	2D	Mean Velocity profiles	Cubit	SA, RNG, STD, NL	Some	no
Mixing Layer	2D	Scale Similarity	Cubit	SA, RNG, STD	Some	no
Jets	2D-3D	Spreading Rate	none	Not Started	None	no
Pipe Flow	3D	Law-of-wall, Nu	Cubit	SA, RNG, STD	Some	no
U-Channel	2D	Curvature effects	Cubit	SA, RNG, STD	L3.THM.CFD.P.06	no
Circ. Cylinder	2D	Strouhal No.	Cubit	Not Started	none	no
Tri. Cylinder	2D	Strouhal No.	none	Not started	none	no
Sq. Cylinder	2D	Strouhal No.	none	Not started	none	no
Asym Diffuser	2D	Separation	none	Not Started	none	no
Impinging Jet	2D	Stagnation point	none	Not Started	none	no
Jet in Crossflow	2D	Complex vortex	none	Not started	none	no
Mounted Cube	3D	Mass. Separation	none	Not Started	none	no
Natural Conv.	2D	Buoyancy effects	none	Not started	none	no
Sub-Channel	3D	Secondary flow	Cubit	Extensive NL	Some	no
3x3 Rod/Spacer	3D	Pressure drop	Hexpress, Cubit	SA, RNG, LES	L2.THM.CFD.P4.01	no
T-Junction	3D	Velocity Profiles	???	Some LES	Some	no

- Eddy-Viscosity models predict un-naturally high eddy viscosity when vorticity is larger than strain (e.g., swirling flows)
- Correction applied to production term



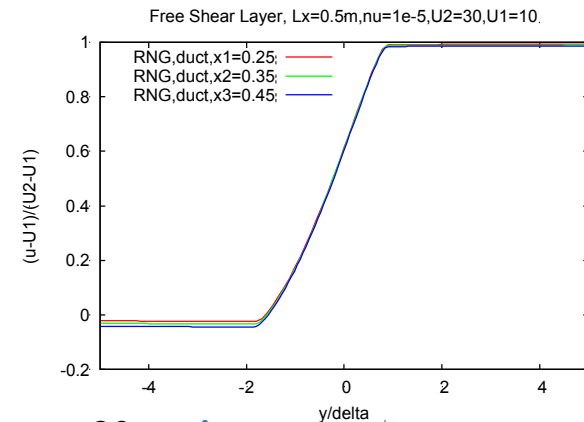
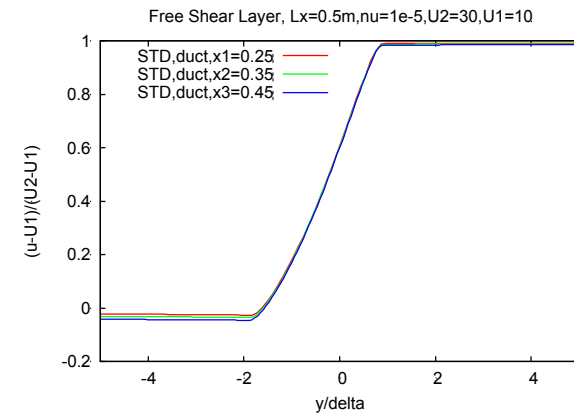
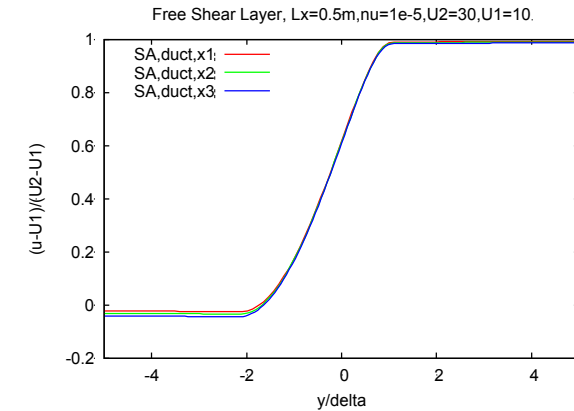
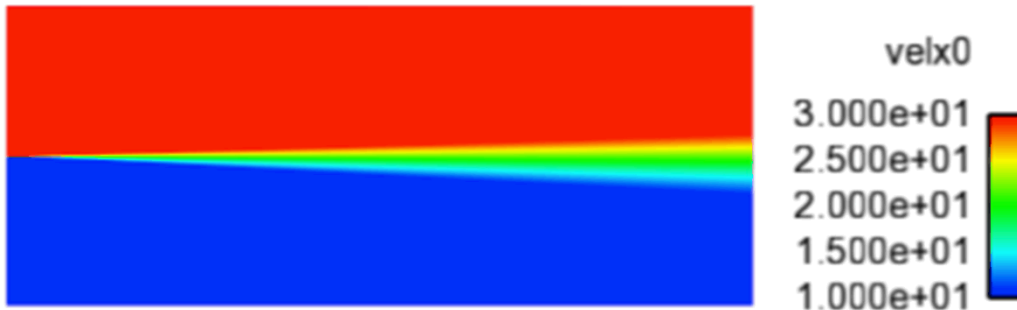
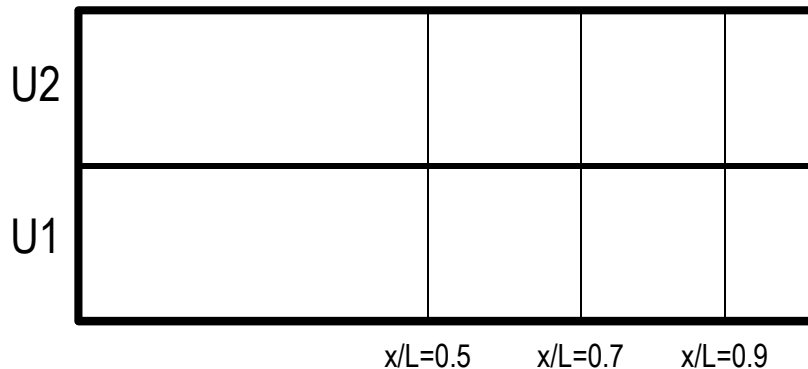
2D U-Channel
mesh: 204x111x1
Re=1.0E6
Uniform inflow profile



Torture Test: Free Shear Layer

- Mesh: 500x300x1
- Domain: 0.5 x 0.2 x 0.001
- rho=1
- mu=1.0e-5
- U2=30, U1=10
- Scale Similarity (F.M. White 1991) $\longrightarrow$
- Rex=4x10⁵
- Tests behavior away from walls

$$\left\{ \begin{aligned} u^* &= \frac{u - U_1}{U_2 - U_1} \\ \delta &= \frac{U_2 - U_1}{(\partial u / \partial y)_{\max}} \end{aligned} \right.$$



Torture Test: Grid Turbulence Geometry

- Elems: 50x5x1, 100x10x1, 200x20x1
- Domain: 1.0 x 0.1 x 0.01
- rho=1
- mu=1.5e-6
- $C_2=1.92$ for STD k-eps model
- $U_0 = 1, k_0 = 0.01, \varepsilon_0 = 0.01$



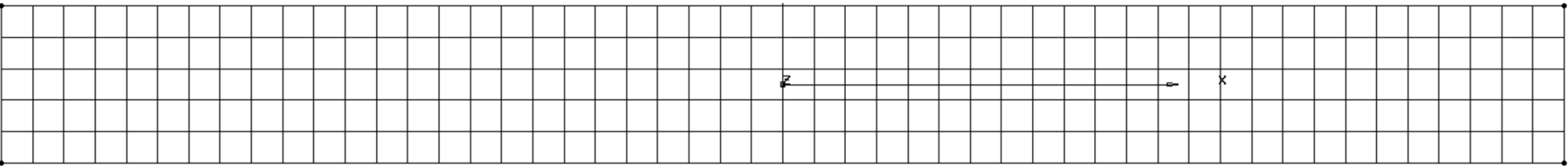
- Analytical Solution: Mohammadi and Pironneau, "Analysis of the K-Epsilon Turbulence Model," Wiley New York, (1993).
- What's tested: time evolution, advection, diffusion, production, dissipation
- What's not tested: **near wall behavior**

Analytical Solution

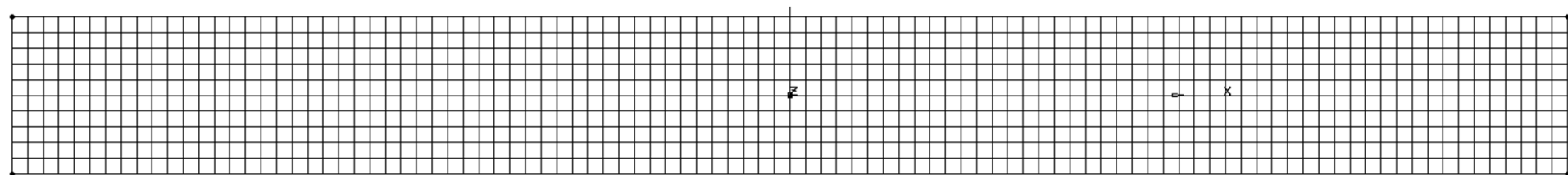
$$k(x) = k_0 \left(1 + (c_2 - 1)x \frac{\varepsilon_0}{k_0 U_0} \right)^{\frac{1}{1-c_2}}$$

$$\varepsilon(x) = \varepsilon_0 \left(1 + (c_2 - 1)x \frac{\varepsilon_0}{k_0 U_0} \right)^{\frac{c_2}{1-c_2}}$$

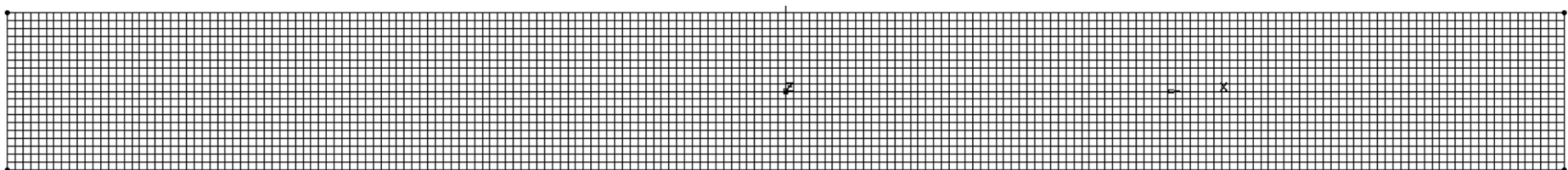
Torture Test: Grid Turbulence



Grid 1: 50x5x1



Grid 2: 100x10x1



Grid 3: 200x20x1

Wall Functions(2) General

- Desire an estimate of wall shear

Post Processing

$$\tau_w = \mu \left. \frac{\partial u(y)}{\partial n} \right|_w \approx \mu \left. \frac{\Delta u(y)}{\Delta y} \right|_w$$

Prediction

$$y^+ = u_\tau y / \nu$$

$$U^+ = \frac{u(y)}{u_\tau}$$

Theory

$$\frac{u}{u_\tau} = \frac{u_\tau y}{\nu}$$

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln \left(\frac{u_\tau y}{\nu} \right) + B$$

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

$$\frac{u}{u_\tau} = \frac{u_\tau y}{\nu}$$

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln \left(\frac{u_\tau y}{\nu} \right) + B$$

$$\dot{q}''_w \approx \frac{\rho C_p u_\tau [T_w - T_p]}{\text{Pr}t [U^+ + J]}$$

Wall Functions Cont. (2) - y^*

- Desire an estimate of wall shear

Post Processing

$$\tau_w = \mu \left. \frac{\partial u(y)}{\partial n} \right|_w \approx \mu \left. \frac{\Delta u(y)}{\Delta y} \right|_w$$

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

Prediction

$$y^+ = u_\tau y / \nu$$

$$U^+ = \frac{u(y)}{u_\tau}$$

Theory

$$u^* = \frac{\rho u_p k^{1/2}}{\tau_w}$$

$$u^* = \frac{1}{K^*} \ln(E y^*)$$

$$y^* = \frac{y_p k^{1/2}}{\nu}$$

$$T^* = \frac{[T_w - T_p] \rho C_p k^{1/2}}{\dot{q}''_w}$$

$$\dot{q}''_w = \frac{[T_w - T_p] \rho C_p k^{1/2}}{\text{Prt}(u^* + P^*)}$$

Wall Functions Cont. (3)

Defraeye, Blocken and Carmeliet JHMT, 2010

- Desire an estimate of wall shear

Theory

$$y^* = \frac{\rho C_\mu^{1/4} y k^{1/2}}{\mu}$$

$$u^* = \frac{\rho C_\mu^{1/4} u k^{1/2}}{\tau_w}$$

$$T^* = \frac{[T_w - T_p] \rho C_\mu^{1/4} C_p k^{1/2}}{\dot{q}_w''}$$

$$\sqrt{\frac{\tau_w}{\rho}} = C_\mu^{1/4} k^{1/2}$$

$$u_p^* = \frac{1}{K} \ln(E y_p^*) \quad y_p^* > y_v^* = 11.225$$

$$T_p^* = \text{Pr} t \left(\frac{1}{K} \ln(E y_p^*) + P_j \right) \quad y_p^* > y_T^* = 11.639$$

$$u_p^* = y^* \quad y_p^* < y_v^* = 11.225$$

$$T_p^* = \text{Pr} y^* \quad y_p^* < y_T^* = 11.639$$

$$E = 9.793 \quad P_j = -1.12$$

$$P_j = 9.0 \left[\left(\frac{\text{Pr}}{\text{Pr} t} \right)^{3/4} - 1 \right] \left[1 + 0.28 \exp \left(-0.007 \frac{\text{Pr}}{\text{Pr} t} \right) \right]$$

Wall Functions Cont. (5)

Zhang, Zhou and Wang, Building and Environment, 2013

- Desire an estimate of wall shear

Theory

$$T^+ = \frac{[T_w - T_p] \rho C_p u_\tau}{\dot{q}''_w} = \text{Pr}t \left(\frac{1}{\kappa} \ln(Ey^+) + P_j \right)$$

$$y^+ = \rho u_\tau y_p / \mu$$

$$P_j = 9.0 \left[\left(\frac{\text{Pr}}{\text{Pr}t} \right)^{3/4} - 1 \right] \left[1 + 0.28 \exp \left(-0.007 \frac{\text{Pr}}{\text{Pr}t} \right) \right]$$

$$y^* = \frac{\rho C_\mu^{1/4} y_p k_p^{1/2}}{\mu} \quad 30 < y^* < 300$$

$$T^* = \frac{[T_w - T_p] \rho C_\mu^{1/4} C_p k_p^{1/2}}{\dot{q}''_w} = \text{Pr}t \left(\frac{1}{\kappa} \ln(Ey_p^*) + P_j \right)$$

$$T^* = \text{Pr} y^* \quad y_p^* < y_T^*$$

$$h = \frac{\dot{q}''_w}{[T_w - T_{ref}]} = \frac{[T_w - T_p] \rho C_\mu^{1/4} C_p k_p^{1/2}}{[T_w - T_{ref}] \text{Pr}t \left(\frac{1}{\kappa} \ln(Ey^*) + P_j \right)}$$

$$\text{Pr}t = \frac{\dot{q}''_w}{[T_w - T_{ref}]} \frac{\text{Pr}t_{ref} [T_w - T_p] \left(\frac{1}{\kappa} \ln(Ey^*) + P'_j \right)}{h [T_w - T'_p] \left(\frac{1}{\kappa} \ln(Ey^*) + P_j \right)}$$

$$E = 9.793 \quad \kappa = 0.4187$$

$$\text{Pr} = 0.74 \quad \text{Pr}t_{ref} = 0.85$$

Torture Test: Fully Developed Pipe Flow

- Domain: $L/D=20$
- ReD: 18,100; 39,200; 78,300
- y^* -insensitive wall model makes computing τ_w difficult due to coarse wall normal grid spacing

Replace: $\tau_w \approx \mu \left. \frac{\partial u(y)}{\partial n} \right|_w$ $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$

$$c_f / 2 = (2.236 \ln(\text{Re}_D) - 4.639)^{-2} \quad (\text{EQ. 12-14, Kays\&Crawford, 1993; Petukhov, 1970})$$

$$u_\tau = V \sqrt{c_f / 2}$$

$$y^* = \frac{u^* y}{\nu}$$

$$u^* = C_\mu^{1/4} \sqrt{k}$$

Theory

$$U^+ = y^+$$

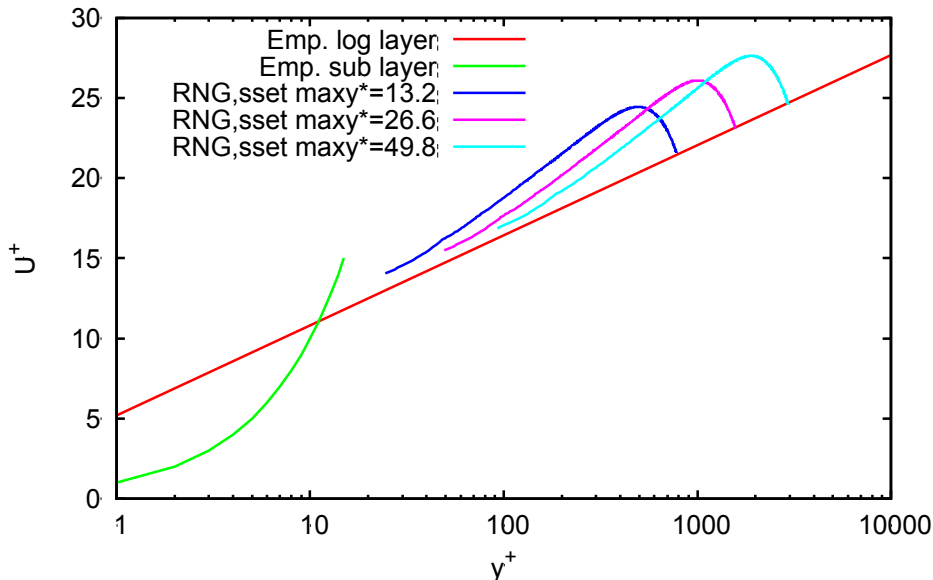
$$U^+ = \frac{1}{0.41} \ln(y^+) + 5.2$$

Prediction

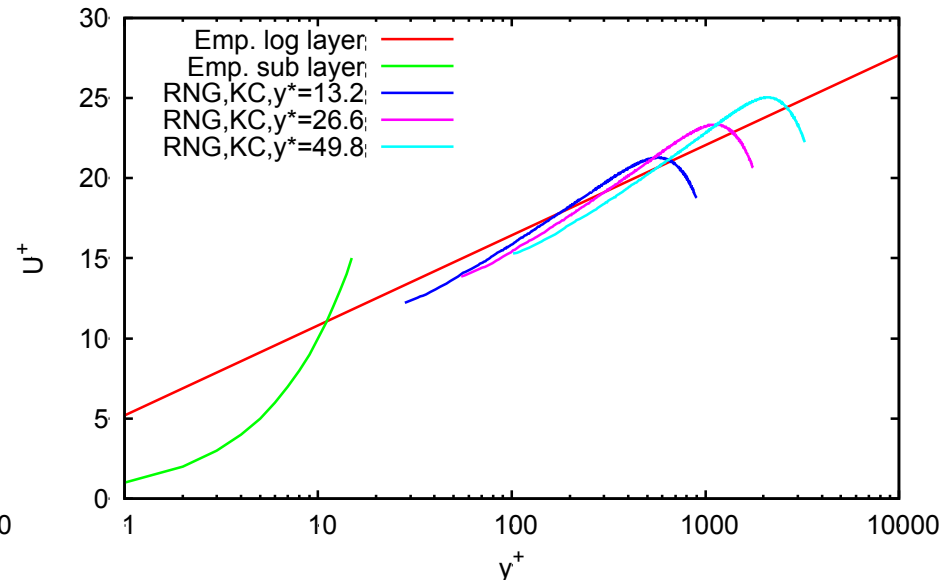
$$y^+ = u_\tau y / \nu$$

$$U^+ = \frac{u(y)}{u_\tau}$$

Normalized Velocity Profile RNG Model, Tube3D.



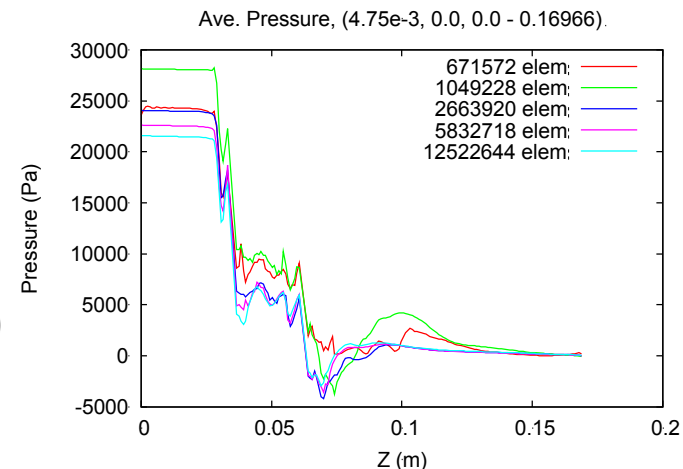
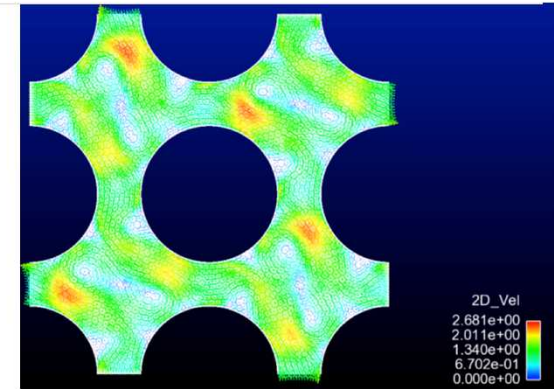
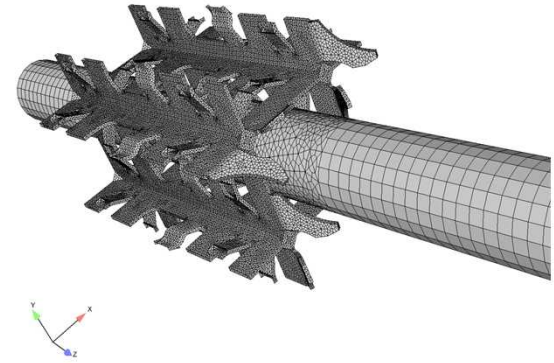
Normalized Velocity Profile RNG Model, Tube3D.



Summary of THM at Sandia During Phase I

(Shadid, Pawlowski, Cyr and Wildey)

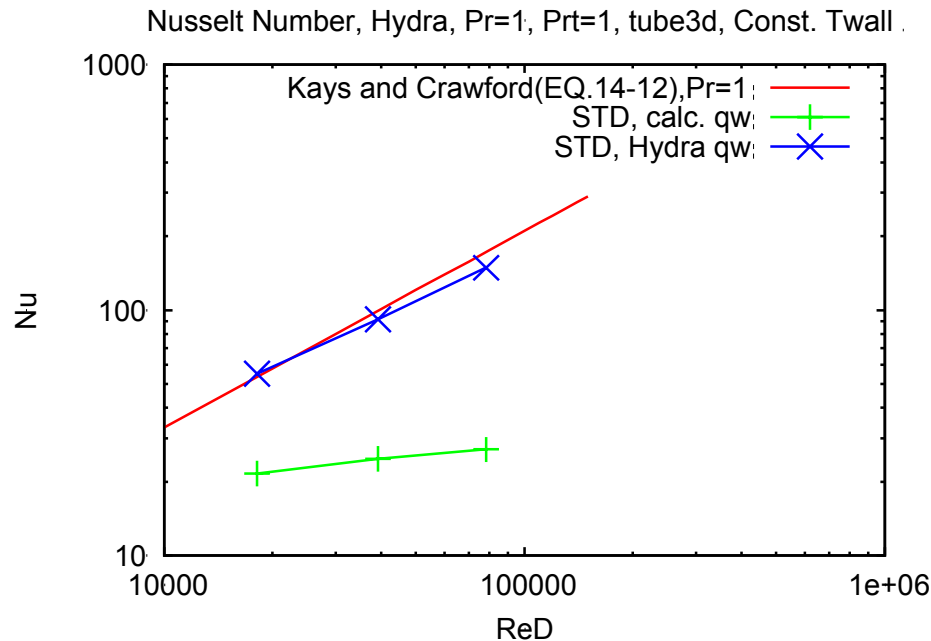
- **Description of Drekar (L3:THM.CFD.P4.02)**
 - *Drekar::CFD - A Turbulent Fluid-flow and Conjugate Heat Transfer Code: Theory Manual (Version 1.0)*
 - Contains discretization, model formulations and verification
- **3x3 rod/spacer LES for GTRF**
 - A summary of this work was provided referencing previous milestone reports and review presentations (L3:THM.CFD.P2.01, L3:THM.CFD.P4.02, “CASL THM CFD Review: FY12 Accomplishments”)
- **Reynolds Average Navier-Stokes (RANS) Turbulence Modeling**
 - Progression from previous isothermal work: L2:THM.CFD.P4.02, L3:THM.CFD.P5.01, L3:THM.CFD.P5.02
- **Thermal Hydraulics**
 - Demonstration of CHT on 3x3 rod/spacer grid model: **L3:VRI.PSS.P4.02**
- **Adjoint Based Error Estimation and Sensitivities**
 - Investigation using swirling flow through an axisymmetric sudden expansion tube (CASL benchmark #2, Pannala and Stagg, 2012) L3:THM.CFD.P4.02, **L3:THM.CFD.P5.02**
 - Builds on previous CASL VUQ work: L3:VUQ.SAUQ.P5.05, **L3:VUQ.SAUQ.P6.01**



Torture Test: Fully Developed Pipe Flow

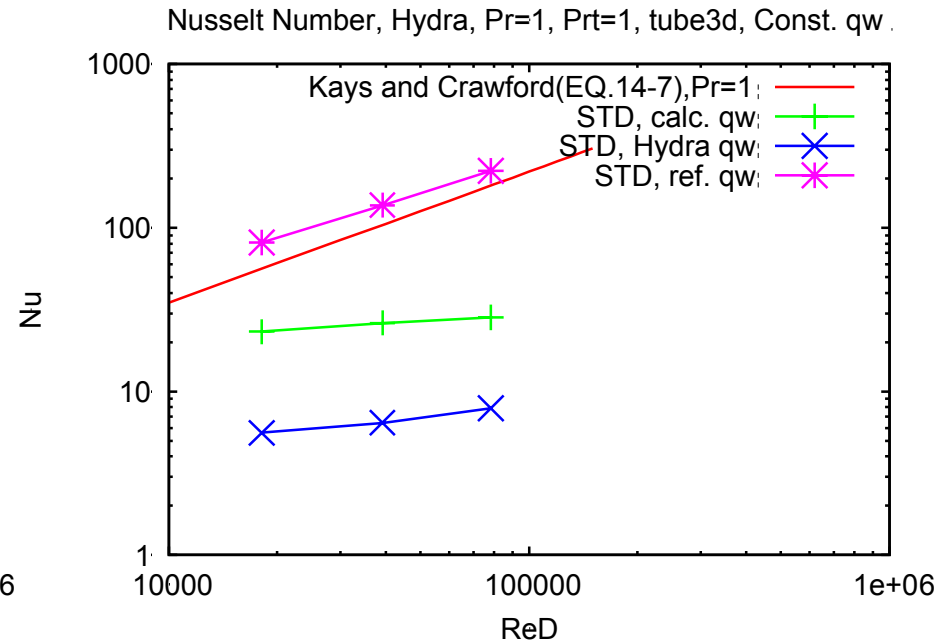
- Domain: $L/D=20$
- Re_D : 18,100; 39,200; 78,300

Constant wall temperature



$$Nu = 0.021Pr^{0.5}Re_D^{0.8} \quad (\text{EQ. 14-12, Kays\&Crawford, 1993})$$

Constant wall heat flux

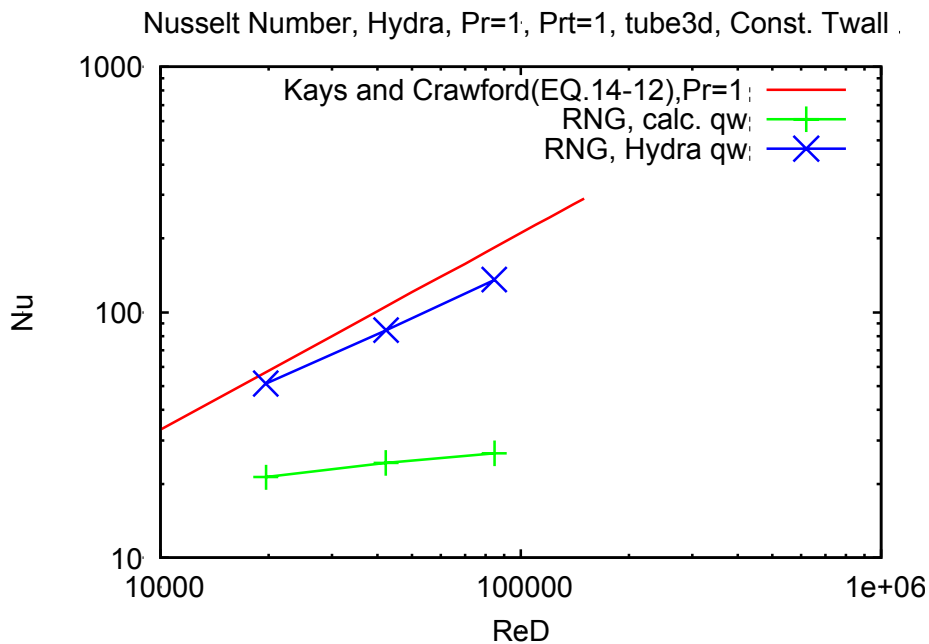


$$Nu = 0.022Pr^{0.5}Re_D^{0.8} \quad (\text{EQ. 14-7 Kays\&Crawford, 1993})$$

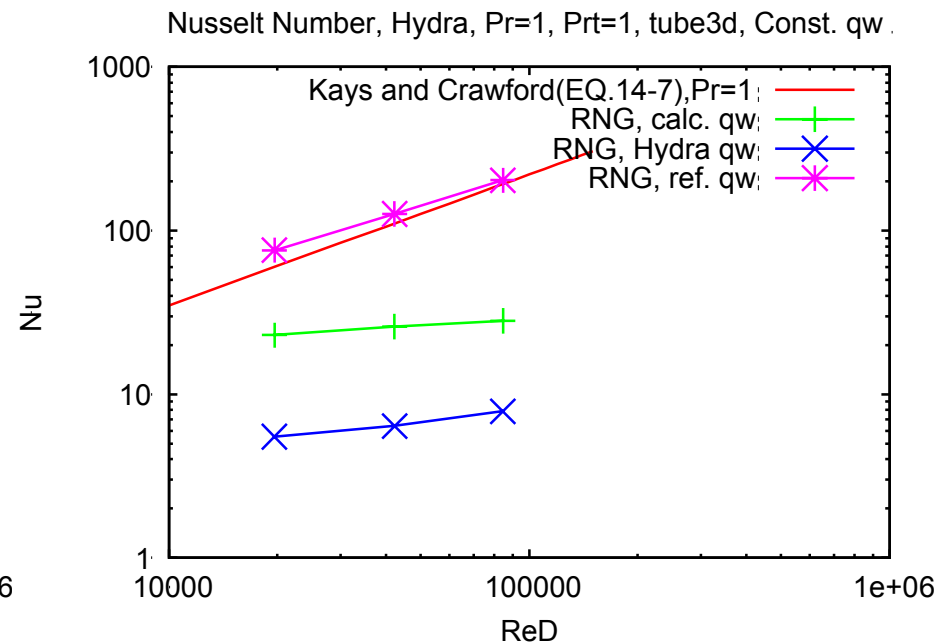
Fully Developed Pipe Flow Nusselt Number: RNG KE

- Domain: $L/D=20$
- $Re_D=19,612$; 42,268; 84,522

Constant wall temperature



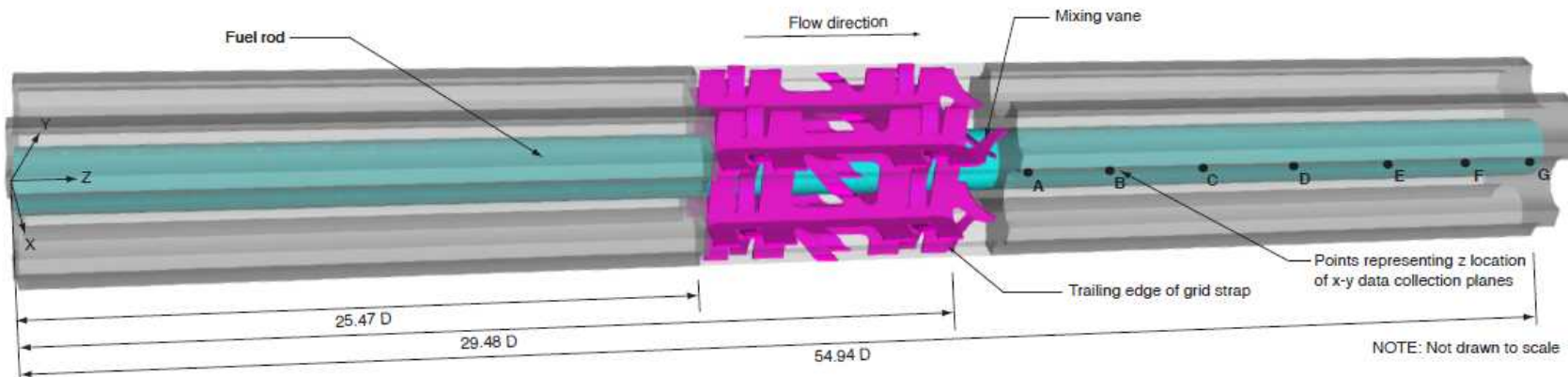
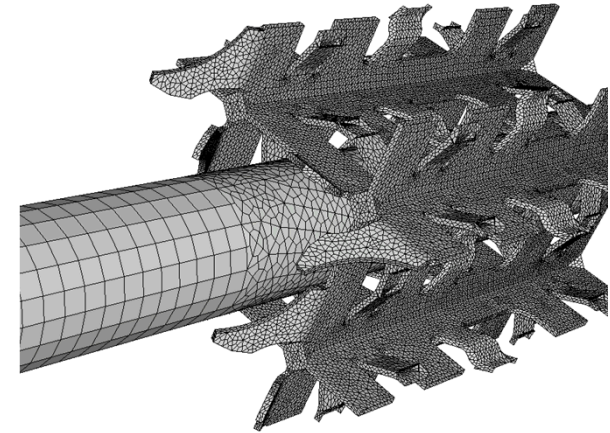
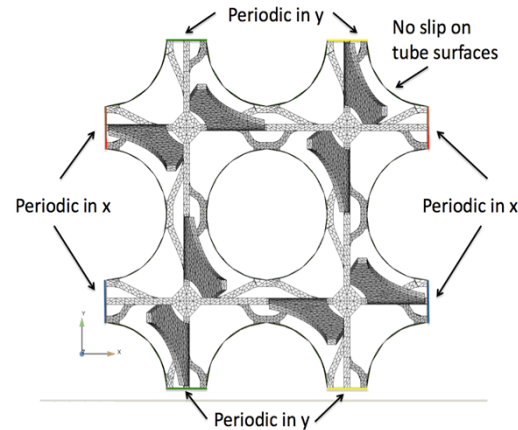
Constant wall heat flux



$$Nu = 0.021 Pr^{0.5} Re_D^{0.8} \quad (\text{EQ. 14-12, Kays\&Crawford, 1993})$$

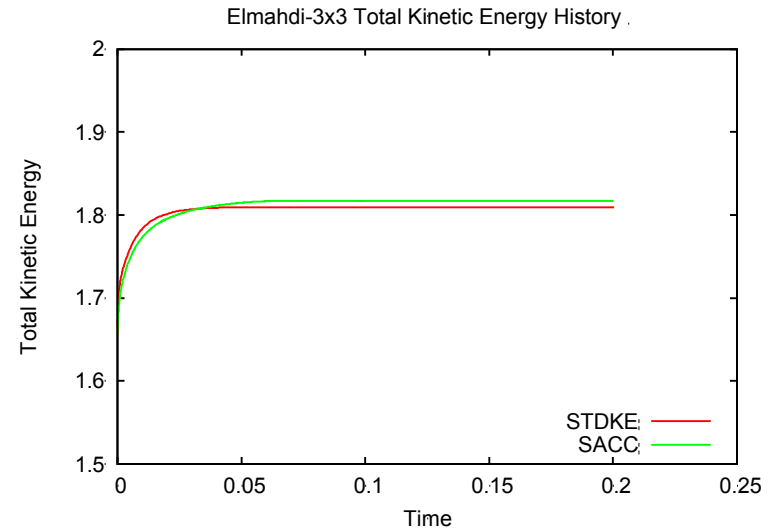
$$Nu = 0.022 Pr^{0.5} Re_D^{0.8} \quad (\text{EQ. 14-7 Kays\&Crawford, 1993})$$

- ⑩ 3x3 Rod Bundle
- ⑩ Isothermal
- ⑩ Fluid: Water
 - ⌘ T: 394K
 - ⌘ Viscosity: 2.32×10^{-4} Pa sec
 - ⌘ Density: 924 kg/m^3
- ⑩ $Re \sim 2 \times 10^5$
- ⑩ Periodic on sides
- ⑩ No slip ($\mathbf{v}=0$) on rods
- ⑩ Inflow on bottom
 - ⌘ 5 m/sec
 - ⌘ Uniform inflow
- ⑩ Outflow on top:



Elmahdi-3x3 Problem

- Three estimates of heat flux from hot rods
 - BC value in cntl file
 - Post process using finite-difference to compute grad T
 - Hydra-TH delegate "nheatflux"
- Boundary conditions
 - ss3 – inflow (Hss3)
 - ss2 – outflow (Hss2)
 - ss9 center rod (qss9)
 - ss10 outer rods (qss10)
- Quantities of Interest
 - SST energy balance – rate of energy coming in equals the rate of energy leaving the domain
 - Heat flux – Is the right amount of energy being exchanged into the flow from the rods
- Dilemma
 - Energy balances when I use compute heat flux from finite difference
 - Energy does not balance when I use cntl value
 - Note that Pr=1



$$\dot{q}_{ss}'' = \kappa \frac{\partial T}{\partial n} = \frac{\rho C_p v}{Pr} \frac{\partial T}{\partial n} \bigg|_{ss}$$

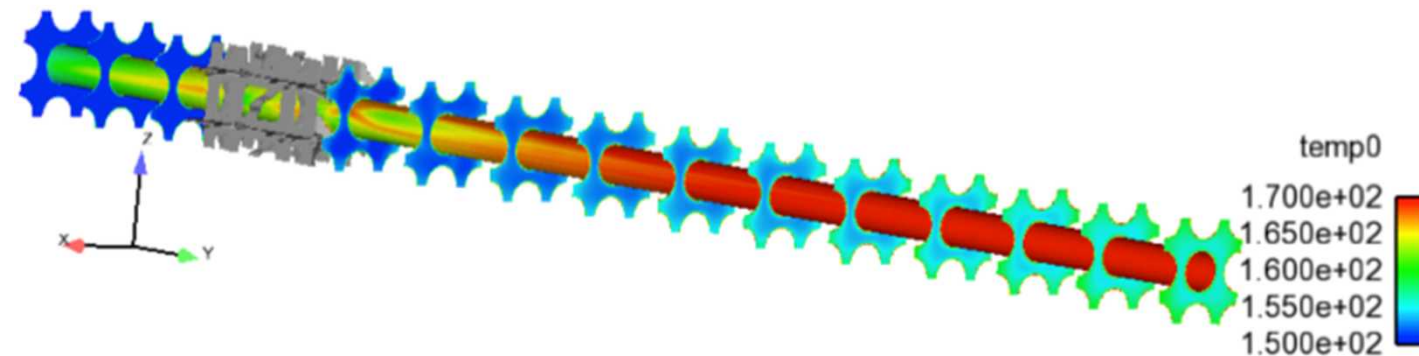
$$Q_{bc} = \dot{q}_{bc}'' A$$

$$h = \rho C_p Tu$$

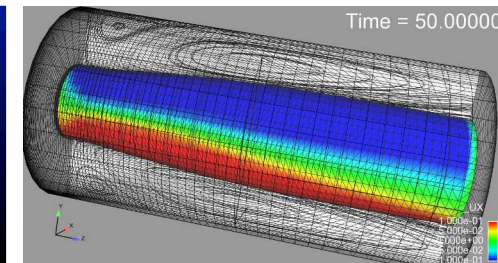
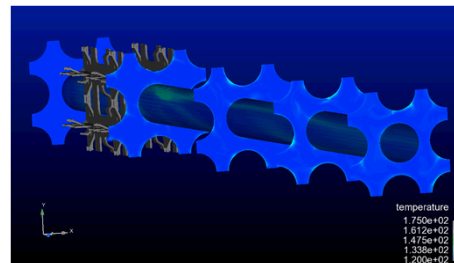
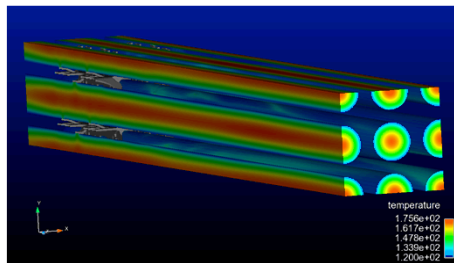
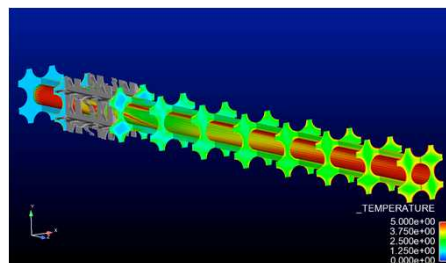
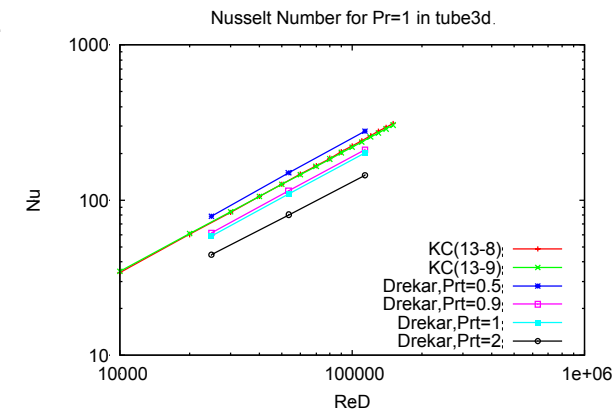
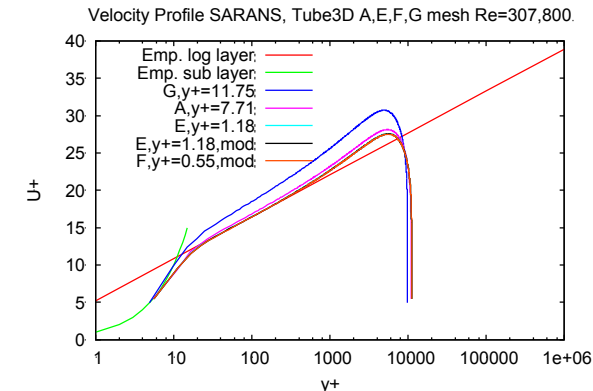
$$H = \rho C_p TuA$$

$$H_{ss2} = H_{ss3} + \frac{\rho C_p v}{Pr} \frac{\partial T}{\partial n} \bigg|_{ss9} + \frac{\rho C_p v}{Pr} \frac{\partial T}{\partial n} \bigg|_{ss10}$$

$$H_{ss2} \neq H_{ss3} + Q_{bc9} + Q_{bc10}$$



- **Demonstration of Isothermal RANS Capabilities**
- **Demonstration of Thermal Hydraulics**
 - Fully developed laminar and turbulent pipe flow heat transfer validation
 - 3x3 rod/spacer grid model with specified heat flux and wall temperature
 - Conjugate heat transfer on one span of a 3x3 rod/spacer grid model
- **Demonstration of Adjoint Based Error Estimation and Parameter Sensitivities**
 - Validation of methodology by way of comparison with analytic solution to Navier-Stokes in square duct
 - Investigation using high Re swirling flow through an axisymmetric sudden expansion tube
- **Demonstration of Direct-to-Steady-State Solutions**
- **Contained in L3.THM.CFD.P7.05 milestone and SAND Report**
- **The milestone work was submitted to CASL special issue of JCP**



Moving Forward in THM with Hydra-TH: Enhanced Turbulence Modeling

- **Planning Activities (L3.THM.CFD.10.02)**
 - Maturation of “standard” and “nonlinear” k-epsilon models
 - New model development: “Realizable” k-epsilon requested by WEC, k-omega SST
 - Closure models for buoyancy driven flows
 - A fresh look at near wall models
 - Turbulence Torture Tests
 - Fundamental examination of model accuracy and robustness
 - “Building Block” tests
 - Simple to complex
- **Development Activities this year (L3.THM.CFD.P11.04)**
 - Maturation of “standard” and “nonlinear” k-eps models is proceeding and “standard” model should be promoted to the master branch soon
 - Many torture tests have already been run and are producing useful information
 - Hydra-TH has proved to be robust/efficient solving these building block flows
 - Spalart-Allmaras model performs well in situations for which it is designed
 - For unbounded flows, k-epsilon models perform well
 - For wall bounded flows, these tests demonstrate the need to enhance near wall treatment
 - Need to take a closer look at how derived quantities are computed



www.casl.gov