



# A Maximum Likelihood Expectation Maximization Iterative Image Reconstruction Technique for Mask/Anti-Mask Coded Aperture Data

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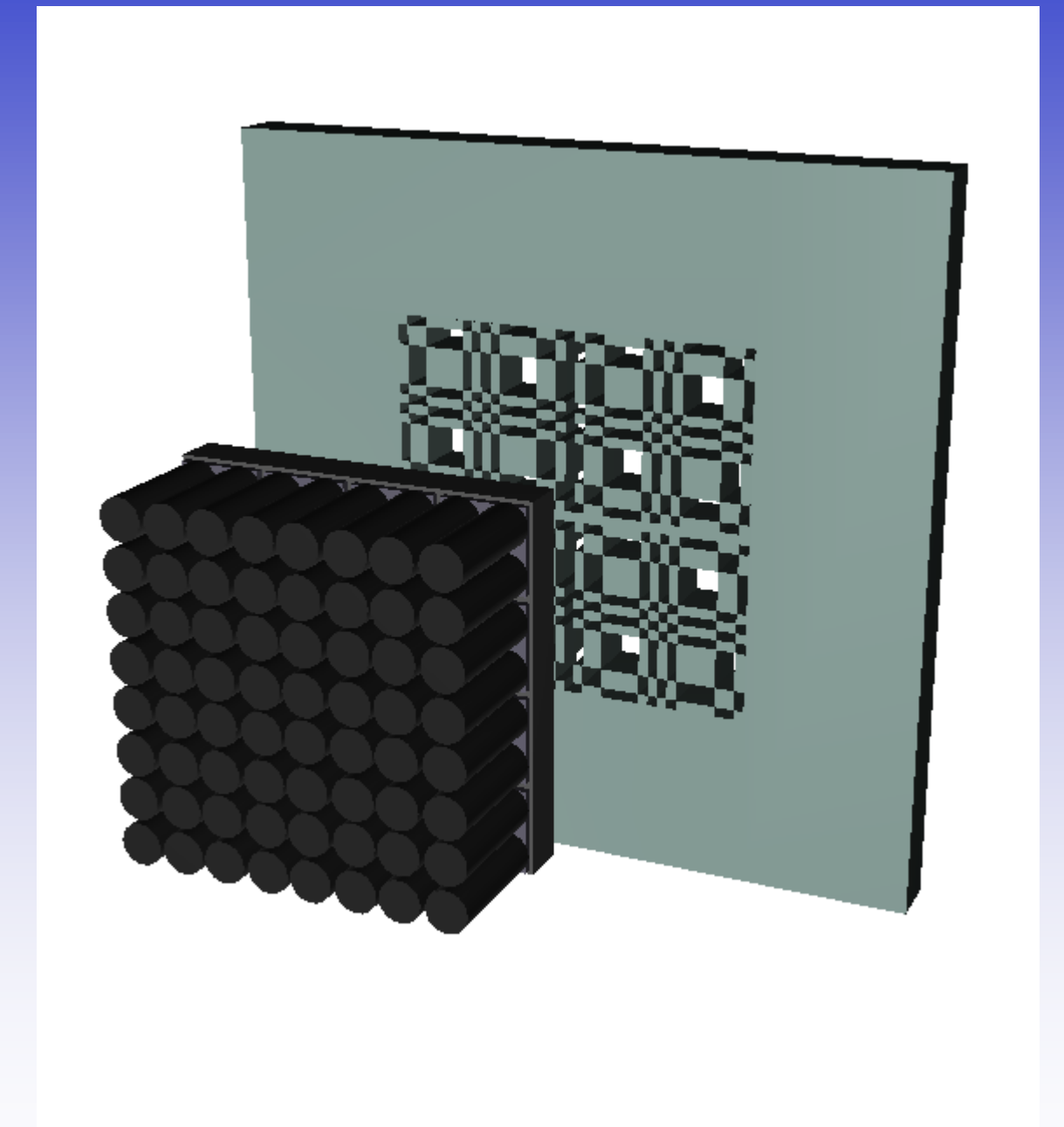
## Introduction

We present a method to use coded aperture mask/anti-mask data with maximum likelihood expectation maximization (MLEM) image reconstruction. The mask/anti-mask approach eliminates “unmodulated” data, improving image quality when backgrounds, room scatter, or noisy detectors are significant. MLEM permits complex detector response models, desirable in gamma-ray or fast neutron imaging with thick masks, near-field imaging, or tomographic reconstruction. Subtracted mask/anti-mask data is not Poisson distributed, and cannot be used with MLEM. Instead, we treat unmodulated data as generated by source terms indexed by detector pixel, so that MLEM converges to simultaneous estimates of the true image and the unmodulated event rates.

Iterative image reconstruction techniques such as maximum likelihood expectation maximization (MLEM) are attractive for coded aperture systems because they allow the use of arbitrarily complex detector response functions, which are needed for non-ideal situations (often encountered in gamma-ray or fast neutron imaging) such as semi-transparent masks, thick masks, and near-field imaging, as well as for advanced imaging tasks such as multi-view tomographic reconstruction.

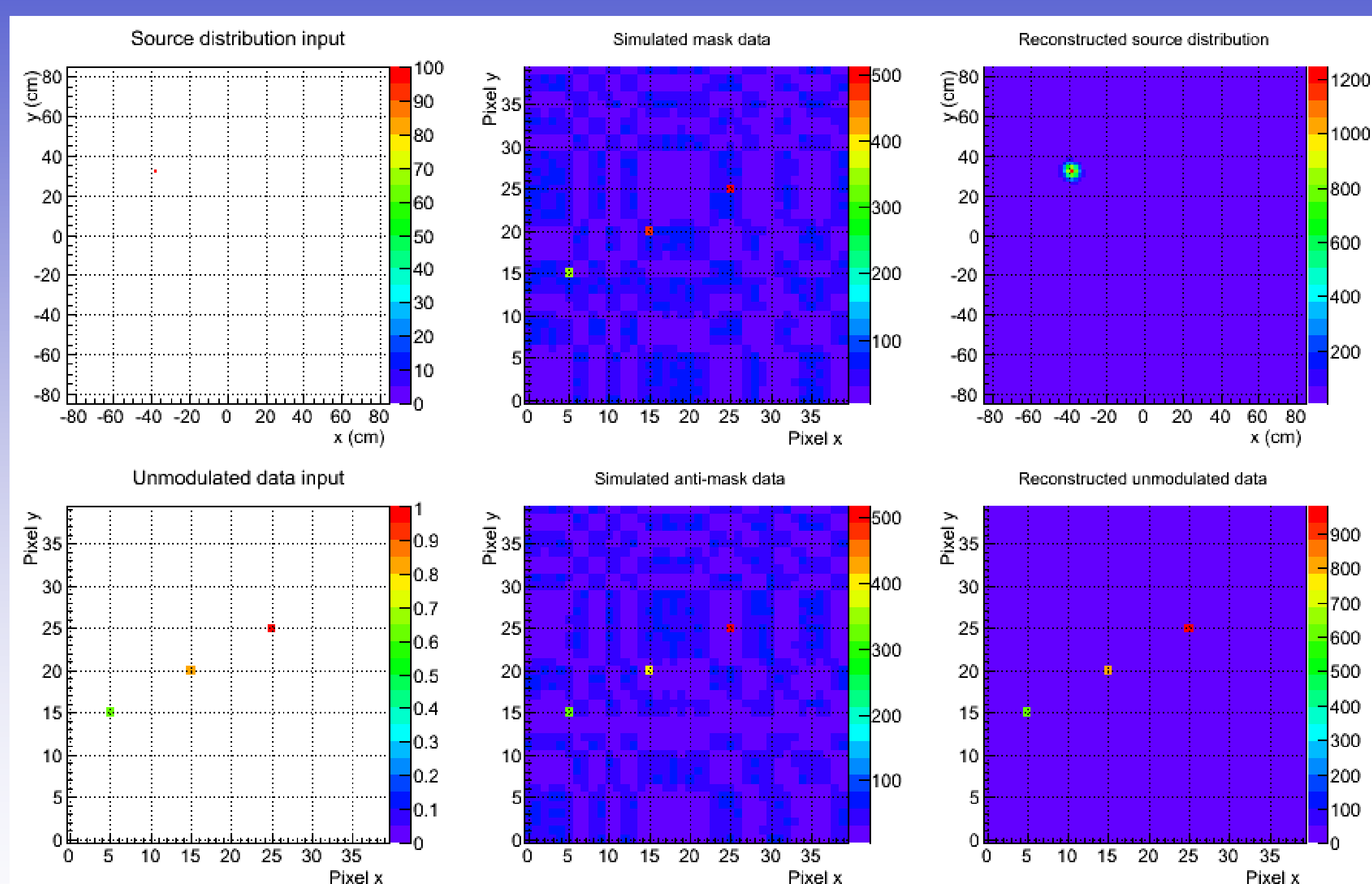
In some scenarios, a significant fraction of detected events are “unmodulated” by the mask, meaning that they lack the characteristic shadow pattern of the mask because they are due either to detector effects such as hot pixels or to particles that enter the detection elements without passing through the mask (e.g. backgrounds or room scatter). In these scenarios, a standard technique is to image using both the mask and its inverse, the anti-mask. The two datasets can be subtracted to isolate the relevant data by removing unmodulated events, which contribute equally to mask and anti-mask data.

The subtracted mask/anti-mask data cannot be used as input to MLEM and similar techniques, which rely on Poisson-distributed raw data. A method has been developed to use mask/anti-mask data with MLEM to reap the benefits of both techniques. This method treats the unmodulated data as a source term indexed by detector pixel. The unmodulated data is then reconstructed by MLEM in parallel with the “true” image in the physical space. As a result, MLEM converges to a simultaneous estimate of the true image and the unmodulated event rates per pixel.



## Simulated Example

For illustration purposes, a simplified example scenario was simulated for a fast neutron coded aperture imager, in which the source distribution is a single point source and the unmodulated data consists of three hot pixels with different strengths. Real scenarios have more subtle unmodulated distributions, including both detector effects and e.g. room scatter.



**Top Left:** Input source distribution (point source). **Bottom Left:** Input hot pixels (unmodulated data). **Top Middle:** Simulated mask data: modulated pattern from the point source plus hot pixels. **Bottom Middle:** Simulated anti-mask data. **Top Right:** Reconstructed point source. **Bottom Right:** Reconstructed hot pixels.

## Implementation

Maximum likelihood techniques rely on a firm grasp of the system response function. In the case of a binned (e.g. pixilated) source space  $\mathbf{S}$  and observation space  $\mathbf{O}$ , the system response is a matrix  $\mathbf{M}$ :

$$\mathbf{O} = \mathbf{M} \cdot \mathbf{S}$$

Typically for coded aperture reconstruction,  $\mathbf{S}$  is the pixilated source space on an imaging plane at some fixed distance from the detector,  $\mathbf{O}$  is the observed counts on each pixel of the position-sensitive detector array ( $40 \times 40 = 1600$  pixels in the ORNL/SNL neutron coded aperture imager), and  $\mathbf{M}$  is the response matrix that connects  $\mathbf{S}$  and  $\mathbf{O}$ , i.e. each element of the matrix gives the probability to observe a particle emitted from a particular source pixel in a particular detector pixel.

For the technique described here, the source and observation spaces are both enlarged, and the response matrix is correspondingly more complex:

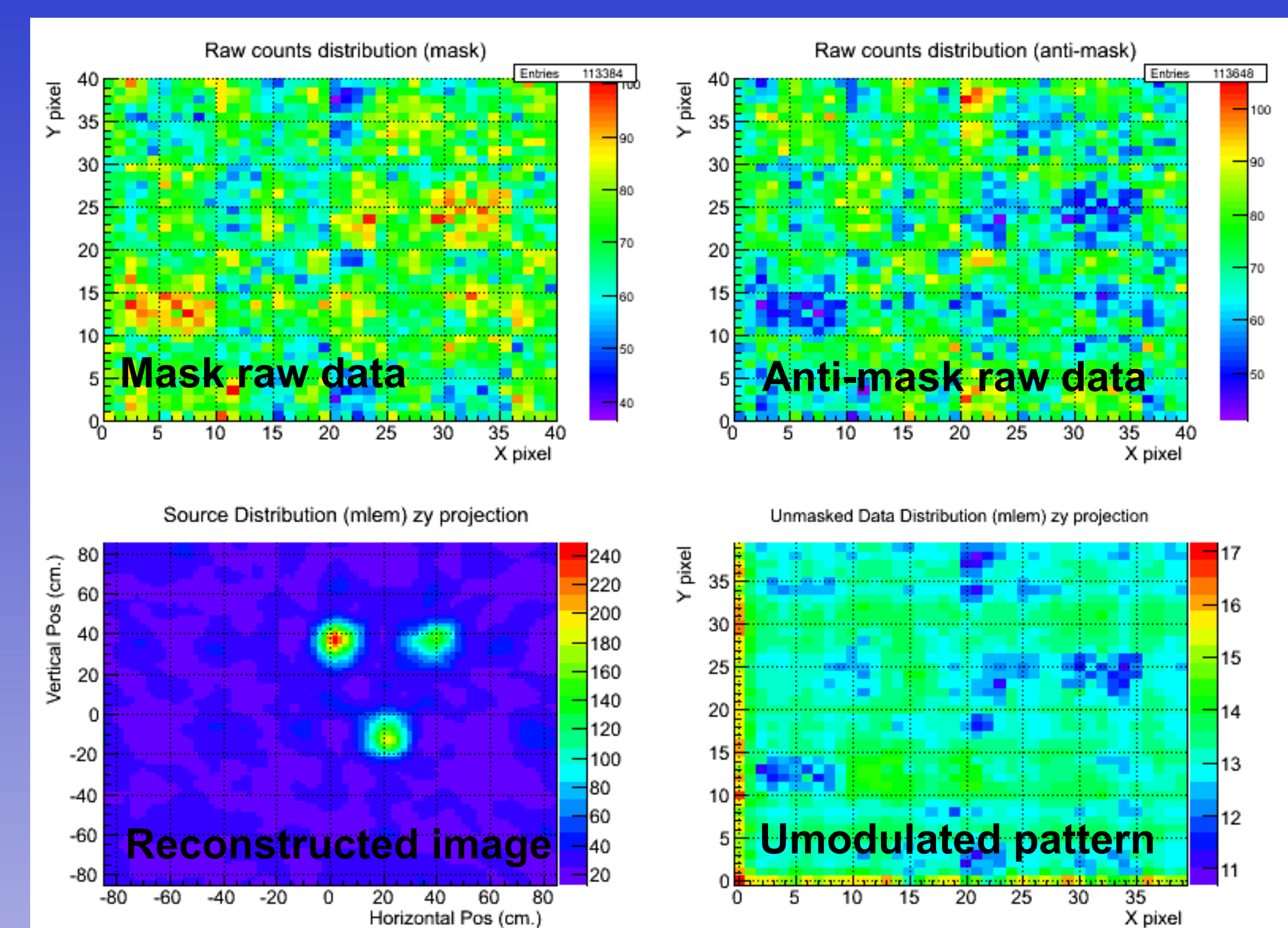
$$\mathbf{O} = \mathbf{O}_{\text{mask}} \oplus \mathbf{O}_{\text{anti-mask}}$$

$$\mathbf{S} = \mathbf{S}_{\text{image}} \oplus \mathbf{S}_{\text{unmodulated}}$$

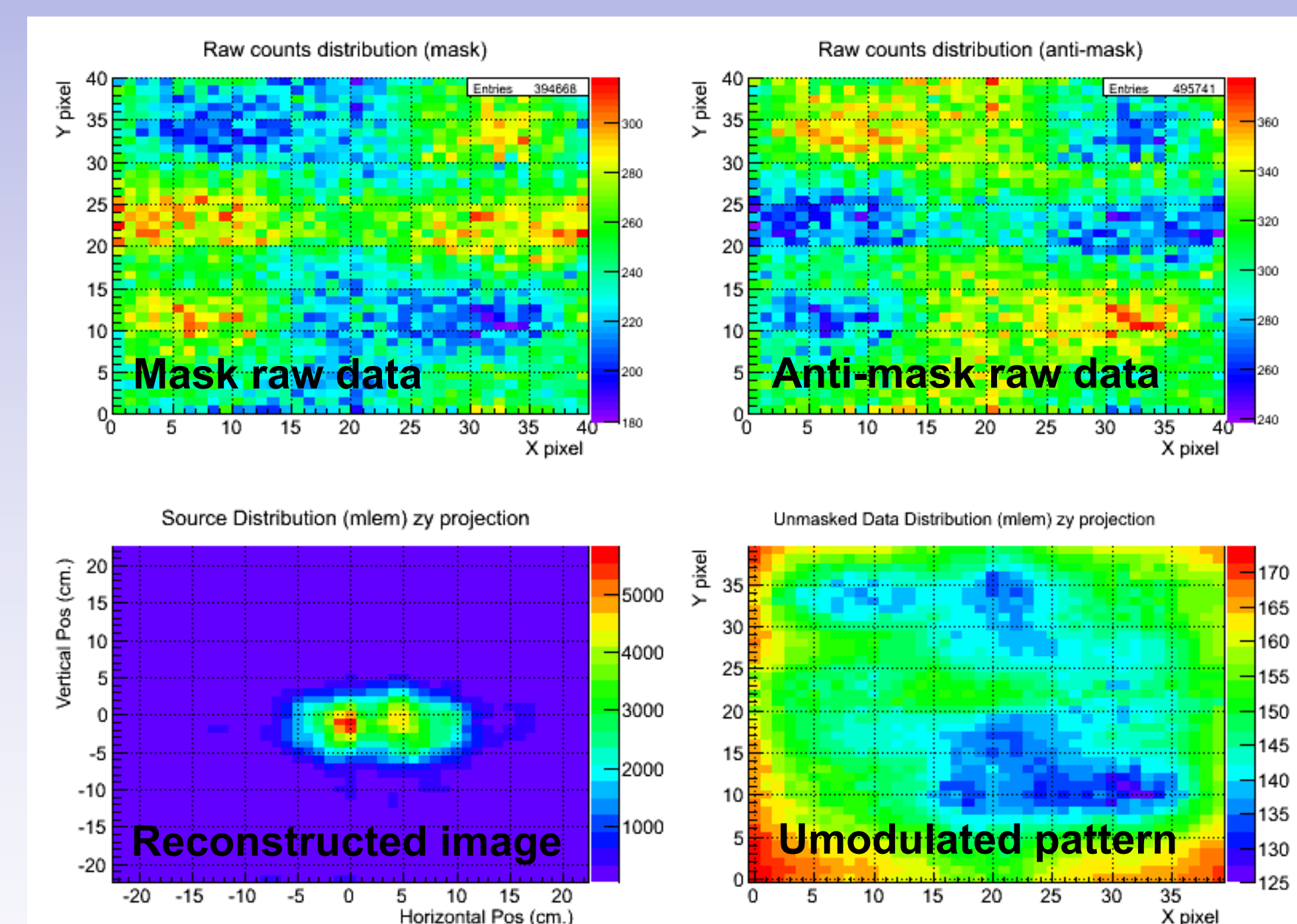
The  $\mathbf{S}_{\text{image}}$  subspace connects to the mask and anti-mask observations in the usual way, with the attenuation from the mask being the most important contributor to the response. The  $\mathbf{S}_{\text{unmodulated}}$  subspace has the same dimensions as the observation space, and each “pixel” in  $\mathbf{S}_{\text{unmodulated}}$  has non-zero probability only in the corresponding pixels of  $\mathbf{O}_{\text{mask}}$  and  $\mathbf{O}_{\text{anti-mask}}$ .

## Data Results

The technique has proven useful in measurements with the ORNL/SNL fast neutron coded aperture imager. Below is a reconstructed image with three neutron sources in the field of view.



The second image below is an extended neutron source imaged in a high-resolution configuration.



Any maximum likelihood reconstruction is only as good as the response function it uses. Patterns in the reconstructed “unmodulated” data can point to areas where the detector itself or the response function should be improved.

A related approach could be taken in the future to reconstruct multiple images from correlated datasets. For example, the liquid scintillator in the ORNL/SNL imager is sensitive to both neutrons and gammas, although the particle identification by pulse shape discrimination (PSD) is not perfect, especially at low energies. If the misidentification probabilities are well understood as a function of energy and pixel, all of the data (in bins of  $n/\gamma$  likelihood ratio for example) can be used to simultaneously reconstruct both gamma and neutron images.

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