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Scaling of Induction-Cell Transverse Impedance: Effect on Accelerator Design

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***Abstract*—The strength of the dangerous beam breakup (BBU) instability in linear induction accelerators (LIAs) is characterized by the transverse coupling impedance $Z_\perp$. This note addresses the dimensional scaling of $Z_\perp$, which is important when comparing new LIA designs to existing accelerators with known i BBU growth. Moreover, it is shown that the scaling of $Z_\perp$ with the accelerating gap size relates BBU growth directly to high-voltage engineering considerations. It is proposed to firmly establish this scaling though a series of AMOS calculations.**

***Index Terms*— Accelerators, Electron beams, Instability, High-voltage breakdown**

I. Introduction

THE most dangerous instability in high-current electron linear induction accelerators (LIAs) is the beam breakup (BBU) [1, 2, 3, 4]. This instability is the result of beam excitation of electromagnetic cavity modes that have a transverse magnetic field, in particular the TM_{1n0} modes. In an LIA the cavities are connected by lengths of beam pipe that form a waveguide beyond cutoff for these modes, so the cavities only communicate via RF oscillations of the beam centroid. This is known as cumulative BBU. It has been shown theoretically [2, 3, 4], through simulations [5], and experimentally [6, 7], that BBU growth depends exponentially on the transverse impedance, $Z_\perp$, which characterizes the strength of the interaction between the beam and the TM cavity modes. Since $Z_\perp$ is generally an increasing function of the width of the accelerating gap, g , it is usually thought one should reduce g to better suppress the BBU. However, there are serious constraints on this approach, not the least of which is high-voltage breakdown. Moreover, when the entire accelerator design is taken into account, it is not clear that reducing the gap is actually advantageous for BBU suppression.

The purpose of this note is to provide some insight into this topic, especially with respect to increasing the required width of gaps in new LIAs in order to prevent high-voltage breakdowns.

II. Theory

Specifically, the maximum amplitude of the BBU has been shown to asymptote after a large number of cells (N) to $\max \xi(z) = \xi_0 \left[\gamma_0 / \gamma(z)\right]^{1/2} \exp\left(\Gamma_m\right)$ where subscript zero denotes initial conditions, and γ is the relativistic mass factor [refs]. Here, the maximum number of e-foldings is

$$\Gamma_m\left(z\right) = \frac{I\, N\, Z_\perp}{300}\left\langle \frac{1}{B} \right\rangle \quad , \tag{1}$$

where I is the beam current in kA, $Z_\perp$ is the transverse coupling impedance in Ohms/cm, B is the solenoidal focusing field in kG, and $\langle\ \rangle$ indicates an average over the cells. (In this formula, $Z_\perp$ is understood to be the maximum value of the real part of the transverse impedance, $Z_\perp(\omega)$, which occurs at the resonant frequency.)

In order to compare the susceptibility to BBU of a new accelerator design with existing LIAs, it is useful to know how the transverse impedance scales with dimensions. It has been suggested that

$$Z_\perp = Z_0 \frac{g}{\pi b^2}\eta \ , \tag{2}$$

where $Z_0 = 120\pi\ \Omega$ is the impedance of free space, b is the beam pipe radius, g is the accelerating gap width, and η is a non-dimensional form factor of order unity [8, 9]. However, this is somewhat misleading, because it implies that η is independent of b and g , which is not the case, even for a simple pillbox cavity [10].

In fact, it has been shown that in the high frequency limit defined by,

$$\frac{\omega\left(R-b\right)^2}{cg} >> 1 \tag{3}$$

where c is the speed of light, the transverse impedance of a simple pillbox cavity with outer radius R is independent of R and given by

$$\operatorname{Re} Z_{\perp}(\omega) = Z_0 \frac{\sqrt{g}}{b^3}\left(\frac{c}{\pi\,\omega}\right)^{3/2} \tag{4}$$

which implies that η is a function of both b and g [11, 12, 13]. (Theoretical work also suggests that this expression is largely independent of the exact details of the cavity shape [12].) The exact scaling of $Z_{\perp}$ with g has implications for the design of a new LIA, because it provides the link between high-voltage engineering constraints and BBU growth.

As a practical example, Table I compares these expressions for $Z_{\perp}$ with measured values for the DARHT accelerators [14, 15, 16, 17, 18, 19, 20, 8]. It is seen that neither of the above expressions for $Z_{\perp}$ agrees very well with the measured DARHT-I impedance.

Table I. Transverse impedance of the DARHT LIAs

	symbol	units	D-I	D-II
Gap Width	g	cm	2	2.54
Beam Pipe Radius	b	cm	7.5	12.5
Outer Radius	R	cm	23	87.6
Shape Factor	η		1	1
Resonant Frequency	f_0	GHz	0.8	0.17
	ω_0	radian/ns	5.03	1.06
Eq. (3)	LHS		>20	>78
Impedance Eq. (2)	$Z_{\perp}$	Ω/cm	4.3	1.9
Impedance Eq. (4)	$Z_{\perp}$	Ω/cm	3.3	8.4
Measured Impedance	$Z_{\perp}$	Ω/cm	7.0	1.8

III. Engineering

At face value, Eq. (1) argues for decreasing $Z_{\perp}$ as much as possible by decreasing g (e.g., through Eq. (2) or Eq. (4)) and increasing the external focusing field B as much as possible. However, there are practical constraints. For example, B cannot be increased indefinitely, because that also increases the growth of corkscrew motion [21], which is proportional to the total phase advance [22]. Furthermore, the gap size cannot be decreased without limit, because of electrical breakdown across the insulator, and/or emission from field enhancement at convex cathodic surfaces.

Taking a holistic approach to the LIA design provides some insight into the tradeoffs between BBU suppression and high-voltage engineering. For example, for a simple geometry, the maximum average electric field that can be sustained might be expressed as

$$\bar{E}_{\max} = V_g / g \tag{5}$$

where V_g is the acceleration voltage across the gap. Also, take the required energy gain for the LIA to be

$$\Delta KE = N V_g \tag{6}$$

Then, from Eq. (1), Eq.(5) , and Eq. (6) one has

$$\Gamma_m(z) = \frac{I\,\Delta KE}{300\bar{E}_{\max}}\frac{Z_{\perp}(g)}{g}\left\langle\frac{1}{B}\right\rangle_{\min} , \tag{7}$$

where the first term on the right is fixed by the LIA requirements ($I, \Delta KE$) and high-voltage engineering constraints ($\bar{E}_{\max}$), and the magnetic field is constrained by maximum allowable uncorrected corkscrew growth.

Thus, in this simple model, according to Eq. (2), the BBU growth would be independent of the gap size, and the gap widths can be increased as needed to minimize risk of breakdown without fear of increasing the number of BBU e-foldings. Moreover, according to Eq. (4), the number of BBU e-foldings would be proportional to $1/\sqrt{g}$, and increasing gap width would both reduce probability of breakdown and help suppress the BBU.

Unfortunately, due to field enhancement the maximum field in the gap does not actually roll off as fast as Eq. (5) would suggest (see Appendix), and as a result the number of e-foldings is indeed an increasing function of the gap width; $\Gamma_m = \Gamma_m(g)$. For example, by the scaling of Eq. (2) one has $\Gamma_m(g) \propto g^{0.6}$. Thus, doubling the gap to reduce the maximum field (by ~32%) would increase the number of BBU e-foldings by ~51%. To mitigate this with increased magnetic field would require ~50% increase in field strength, and attendent increases in the magnet current supply power and solenoid heating.

On the other hand, by the scaling of Eq. (4), one would have only a weak dependence of BBU growth on gap width; $\Gamma_m(g) \propto g^{0.1}$. That is; doubling the gap width would reduce the maximum field by ~32% while only increasing the number of BBU e-foldings by ~ 7%. This might be an acceptable trade to reduce risk of breakdown in the gap.

Thus, it is imperative to have accurate knowledge of the functional form of $Z_{\perp}(g)$ in order to ascertain the dependence of BBU growth on gap size under these engineering constraints.

IV. Conclusions

The number of BBU e-foldings in an LIA is a function of the accelerating gap width. Wider gaps reduce the risk of breakdown, and it is vital to have accurate knowledge of impedance as a function of gap width when assessing the trade-off of breakdown vs BBU growth.

One way to obtain the necessary dependence of impedance on gap-width would be to use a reliable code, such as AMOS [23], to calculate the DARHT-1 transverse impedance with different gap widths. From these results, the accurate scaling of $Z_{\perp}$ with gap size g can be deduced. I propose that this be undertaken in a timely fashion, so that the requirements for the accelerating gaps can be evaluated.

APPENDIX

In this Appendix, the results of calculations of the maximum electric field in the DARHT-I cell gap are presented. These simulations were performed using the 2-D Estat finite-element code, which is a component of TriComp [24]. All simulations were performed with 250 kV applied to the 2-cm gap. As shown in Fig. 1, the maximum field occurs on the convex part of the cathode, and it is much greater than $V_0 / g = 125$ kV/cm. The scaling of the maximum field with increasing gap size is shown in Fig. 2, based on several simulations with different separation of the cathode and anode shapes shown in Fig. 1.

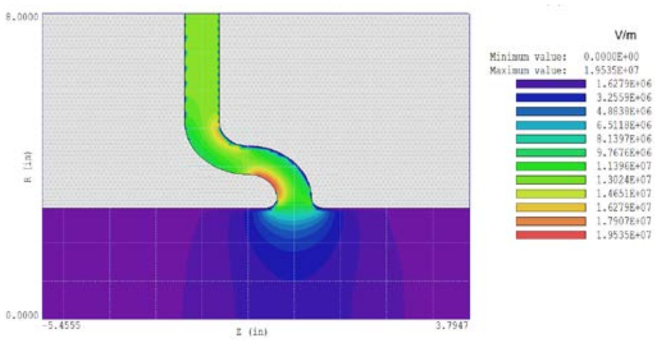

Fig. 1: Electric field in the 2-cm wide DARHT-I gap as simulated with the Estat finite-element code. The left-hand is negatively charged (cathodic) to 250 kV and the right-hand side is ground. The maximum field on the convex surface (shown in dark orange) is 195 kV/cm.

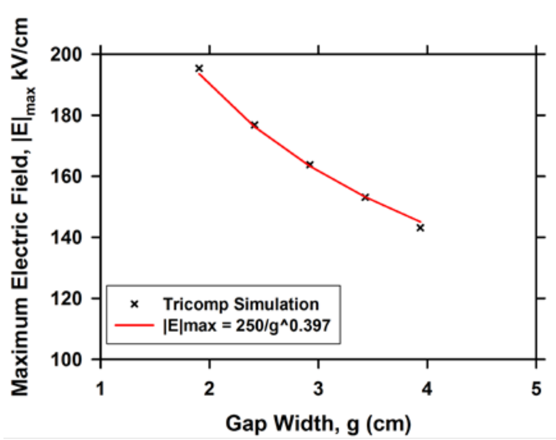

Fig. 2: Maximum field in DARHT-I gap as function of gap width. The x's are results of Estat simulations, and the red line is a power-law fit showing that the field only decreases as $\sim 1/g^{0.4}$ as the gap is increased by a factor of two.

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