Applying semiconductor-laser modeling tricks to atomtronic circuits

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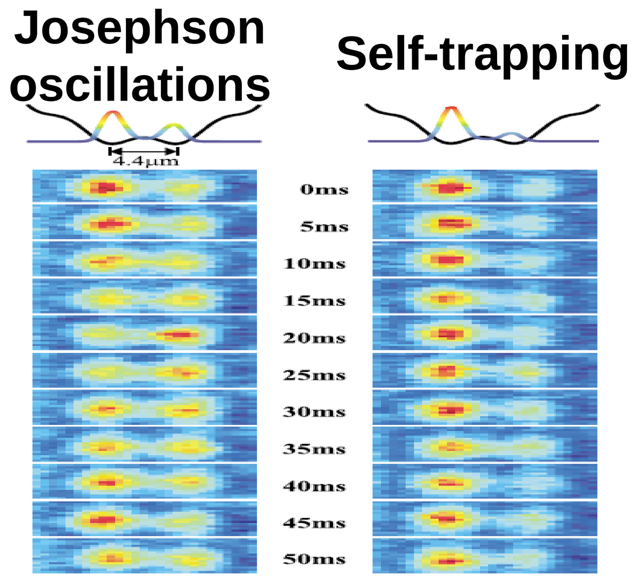
Motivation: Engineering design tool is needed to transition Bose-Einstein Condensation (BEC) from laboratory to devices

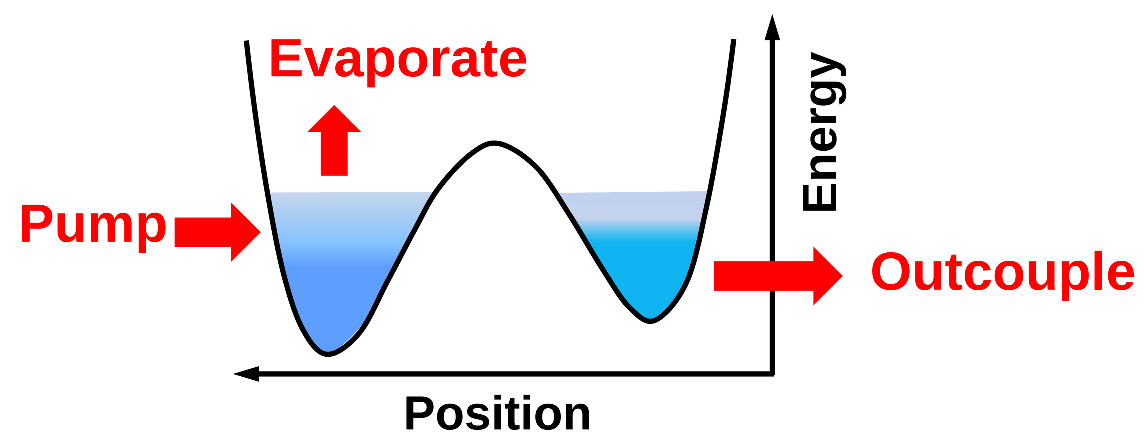
Example: Double-potential well



Albiez, et al, PRL **95**, 010402 (2005)

Typically studied as closed system (Fixed total population)

Atomtronic building block (like p-n junction)



- Open system (see red items)
- Finite temperature Bose-condensed gases with atom injection, extraction and relaxation
- Question example: Will system settle into single condensate or two different ones? Answer: Both.

Atomtronics model setup

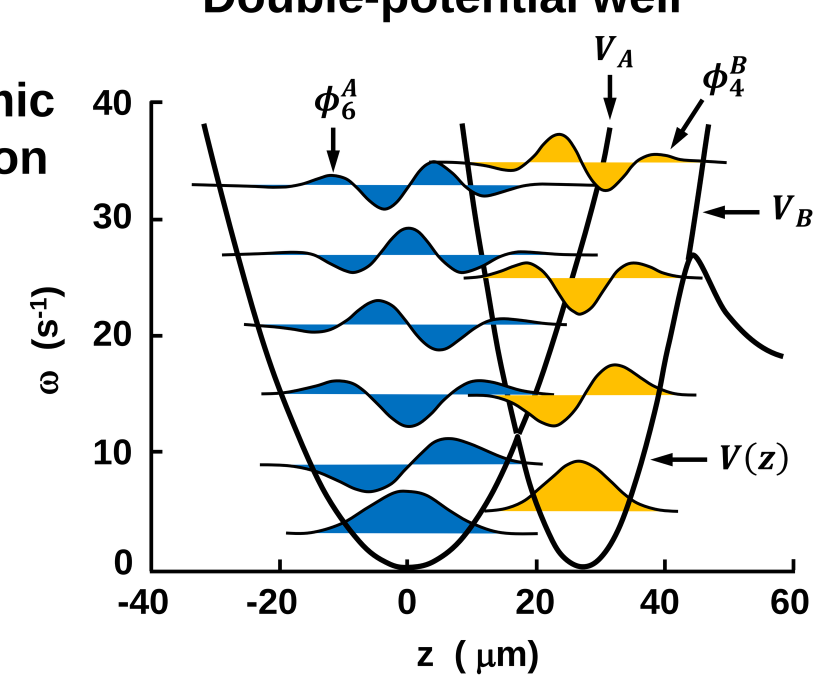
System Hamiltonian: $H = \sum_i \left[\frac{p_i^2}{2m} + V(r_i)\right] + \sum_{i<j} V(r_i, r_j)$

Single-particle   Interatomic interaction

Basis
$$\left[\frac{p^2}{2m} + V_A(r)\right]\phi_n^A(r) = \varepsilon_n^A\,\phi_n^A(r)$$
$$\left[\frac{p^2}{2m} + V_A(r)\right]\phi_n^A(r) = \varepsilon_n^A\,\phi_n^A(r)$$

2nd-quantize: $\hat{\psi}(r) = \sum_i a_i\,\phi_i^A(z) + \sum_i b_i\,\phi_i^B(z)$

Double-potential well



Two-site Bose Hubbard Hamiltonian

$$H = \sum_i \varepsilon_i^A\, a_i^\dagger a_i + \sum_i \varepsilon_i^B\, b_i^\dagger b_i + \sum_{i,j}\left(g_{ij}\, a_i^\dagger b_j + g_{ji}^*\, b_j^\dagger a_i\right) \longleftarrow \textit{Hopping}$$
$$+ \sum_{i,j,k\neq 0}\left(v_{ijk}^{AA}\, a_{i-k}^\dagger a_{j+k}^\dagger a_j a_i + v_{ijk}^{BB}\, b_{i-k}^\dagger b_{j+k}^\dagger b_j b_i + 2v_{ijk}^{AB}\, a_{i-k}^\dagger b_{j+k}^\dagger b_j a_i + h.a.\right)$$

$\gamma = \gamma_1 = 4s^{-1}$

Matrix elements
$$g_{ij} = \int dz\, \phi_i^A(z)\left[\frac{p^2}{2m} + V(z)\right]\phi_j^B(z)$$
$$v_{ijk}^{\alpha\beta} = \int dz' \int dz\, \phi_{i-k}^\alpha(z')\phi_{j+k}^\beta(z)\, V(z,z')\phi_j^\beta(z)\phi_i^\alpha(z')$$

Equations of motion   *Bose-Einstein distribution*

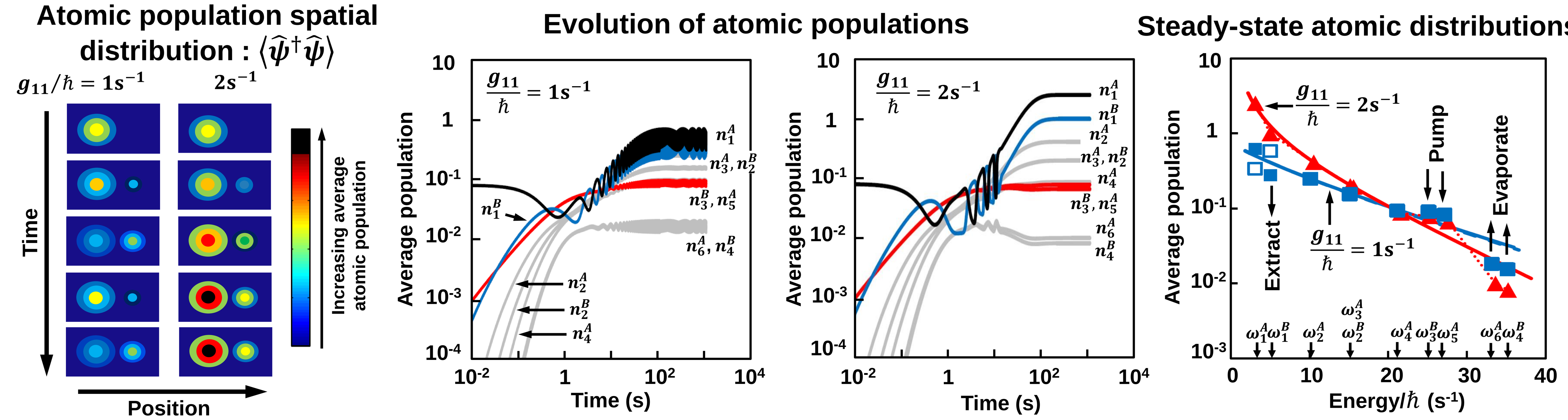
$$i\hbar\frac{d}{dt}\langle a_i^\dagger a_i\rangle = -\sum_j g_{ij}\left(\langle b_j^\dagger a_i\rangle - \langle a_i^\dagger b_j\rangle\right) - \gamma_1\left[\langle a_i^\dagger a_i\rangle - f(\varepsilon_i^A,\mu,T)\right] + P_i^A - \gamma_i^A\langle a_i^\dagger a_i\rangle$$
$$i\hbar\frac{d}{dt}\langle b_j^\dagger b_j\rangle = \sum_i g_{ij}\left(\langle b_j^\dagger a_i\rangle - \langle a_i^\dagger b_j\rangle\right) - \gamma_1\left[\langle b_j^\dagger b_j\rangle - f(\varepsilon_j^B,\mu,T)\right] - \gamma_j^B\langle b_j^\dagger b_j\rangle$$
$$i\hbar\frac{d}{dt}\langle a_i^\dagger b_j\rangle = \left[\Delta_{ji} + \sum_{k\neq 0}\left(\sigma_{jk}^B\langle b_{j+k}^\dagger b_{j+k}\rangle - \sigma_{ik}^A\langle a_{i+k}^\dagger a_{i+k}\rangle\right)\right]\langle a_i^\dagger b_j\rangle - g_{ij}\left(\langle b_j^\dagger b_j\rangle - \langle a_i^\dagger a_i\rangle\right) - \gamma\langle a_i^\dagger b_j\rangle$$

Number conservation: $\sum_i\langle a_i^\dagger a_i\rangle + \sum_j\langle b_j^\dagger b_j\rangle = \sum_i f(\varepsilon_i^A,\mu,T) + \sum_j f(\varepsilon_j^B,\mu,T)$

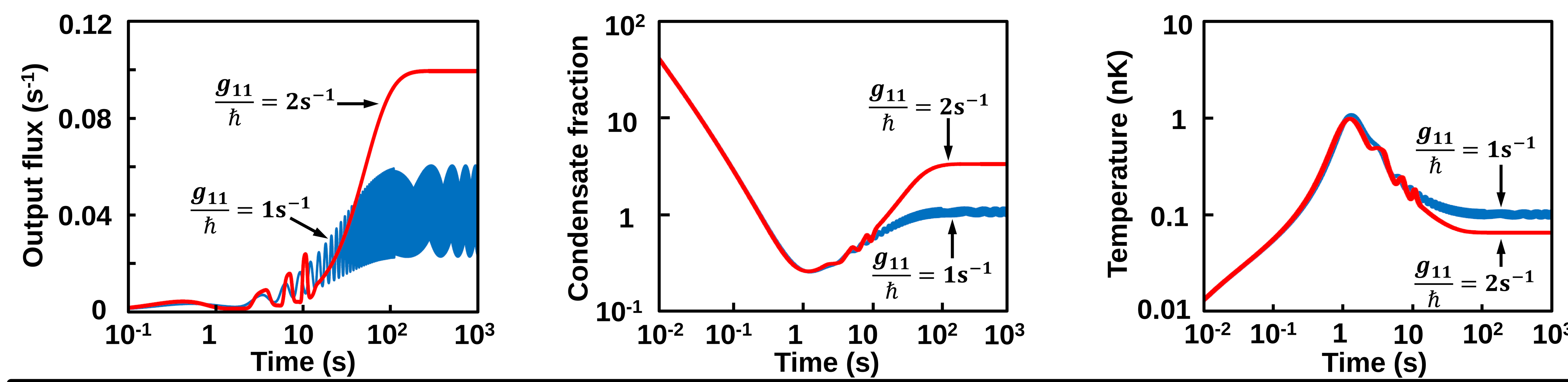
Energy conservation: $\sum_i \varepsilon_i^A\langle a_i^\dagger a_i\rangle + \sum_j \varepsilon_j^B\langle b_j^\dagger b_j\rangle = \sum_i \varepsilon_i^A f(\varepsilon_i^A,\mu,T) + \sum_j \varepsilon_j^B f(\varepsilon_j^B,\mu,T)$

Details: Chow, Straatsma and Anderson, Phys. Rev. A 00, 003600 (2015)

Results directly from simulations



Extraction of quantities of engineering interests

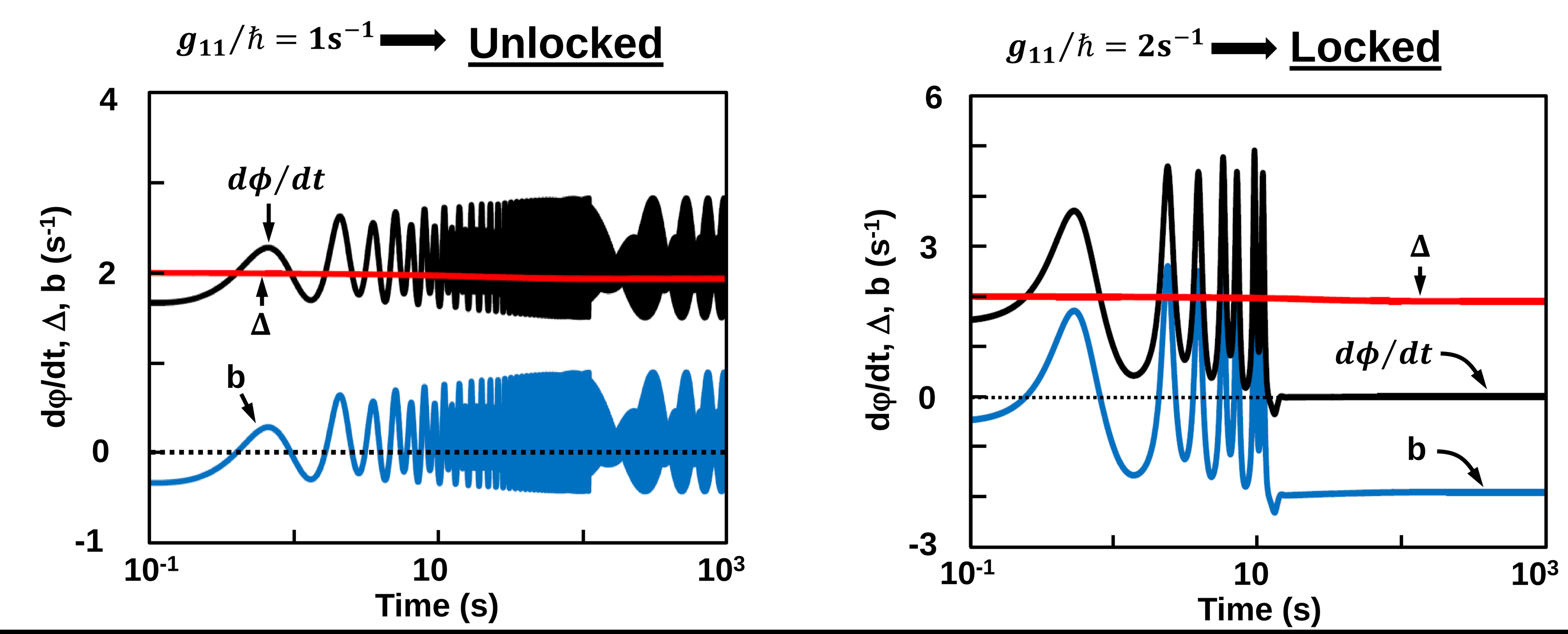


Understanding in terms of phase locking

Assume: $\langle a_1^\dagger b_1\rangle = \sqrt{n_1^A n_1^B}\,exp(-i\phi)$

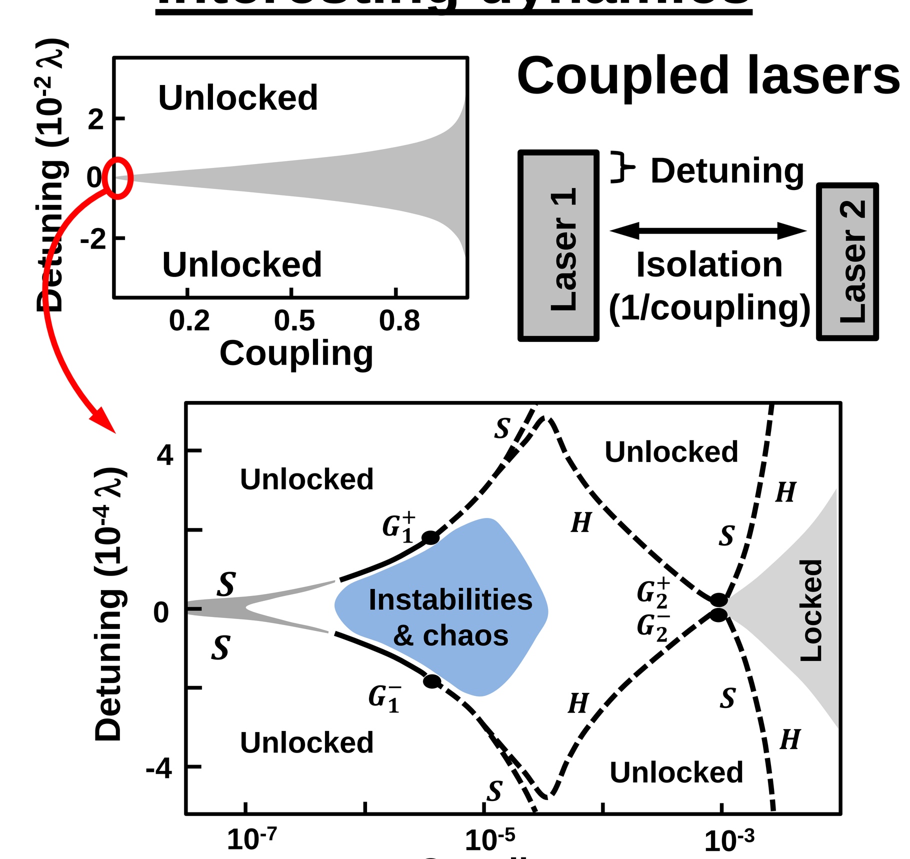
$$\underbrace{\frac{d\phi}{dt}}_{\text{Beatnote}} = \overbrace{\frac{1}{\hbar}\left[\Delta_{11} + \sum_{k\neq 0}\left(\sigma_{1k}^B n_{1+k}^B - \sigma_{1k}^A n_{1+k}^A\right)\right]}^{\text{Detuning, }\Delta} + \underbrace{\frac{g_{11}}{\hbar}\frac{\left(n_1^B - n_1^A\right)}{\sqrt{n_1^A n_1^B}}\cos\phi}_{\text{Locking, b}}$$

$g_{11}/\hbar = 1s^{-1}$ ⟶ Unlocked $g_{11}/\hbar = 2s^{-1}$ ⟶ Locked



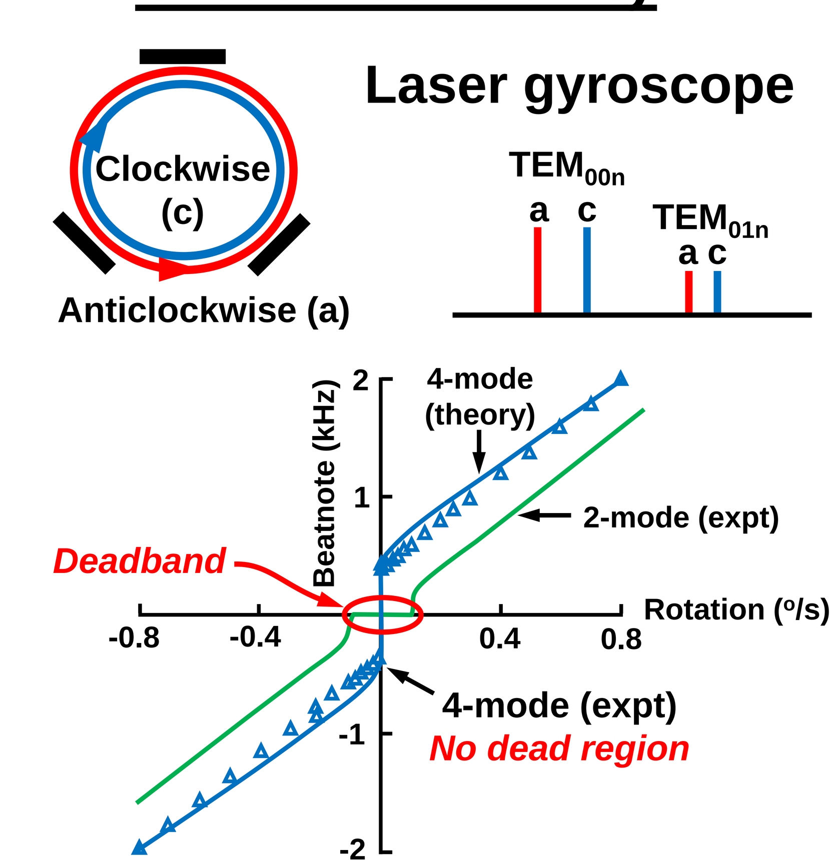
Why tunneling/phase-locking analogy?

Interesting dynamics



Wieczorek & Chow, Phys. Rev. Lett. **92**, 213901 (2004)

Greater sensitivity



Anderson, Chow, Scully & Sanders, Opt. Lett. **5** 413 (1980)

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