

Overview: Incorporating microstructural scale damage into continuum models for performance of structural materials

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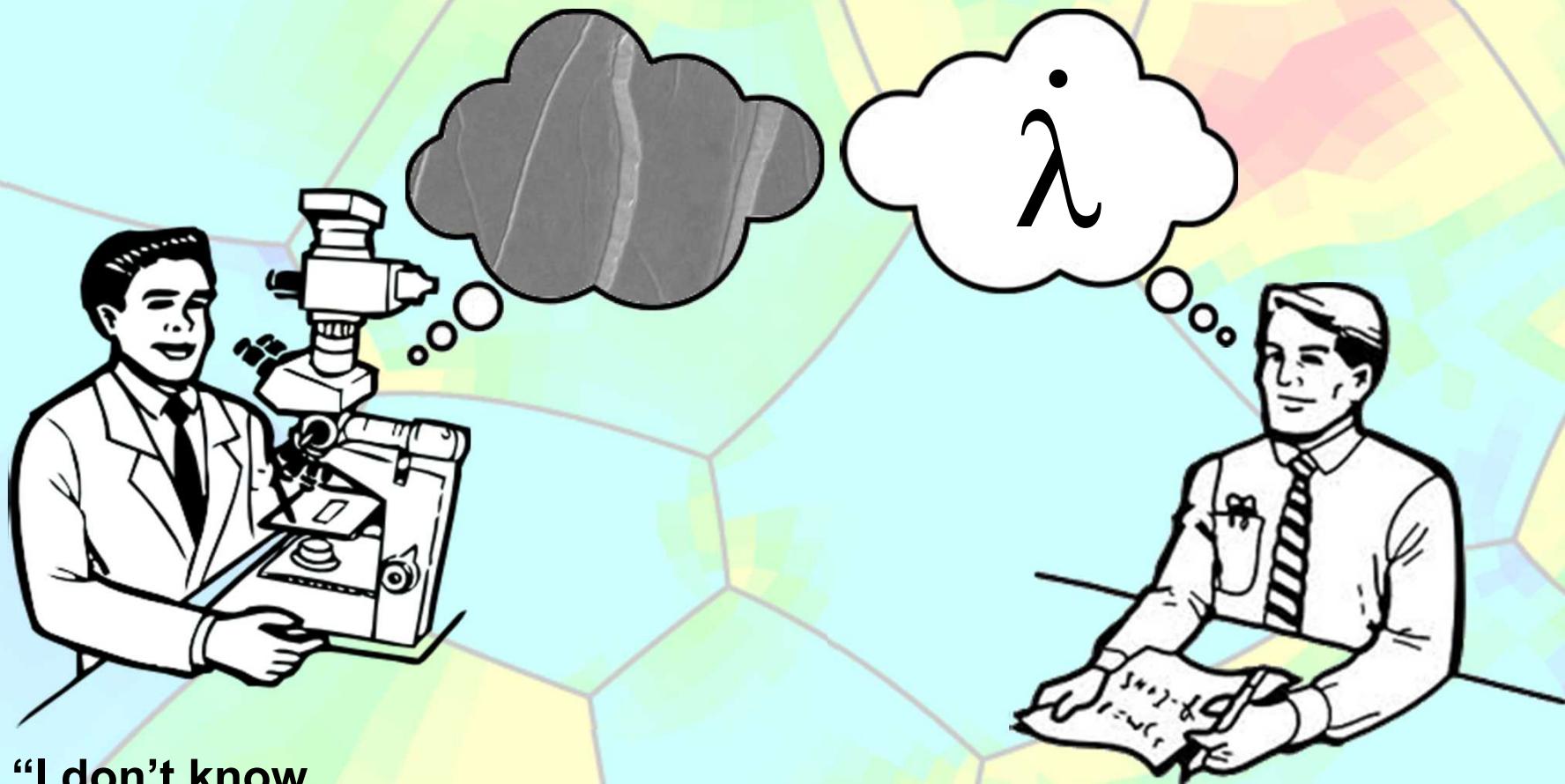
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What is damage?



**“I don’t know.
But I know it when I see it.”**

**“I don’t know.
But I can parameterize it.”**

What are the microstructural features associated with damage?

Depends on your point-of-view...

Deformation-induced:

- Dislocations
- Pileups
- Void nucleation
- Pore coalescence
- Microcracks
- Fracture
- Shear banding
- Particle shearing
- ...

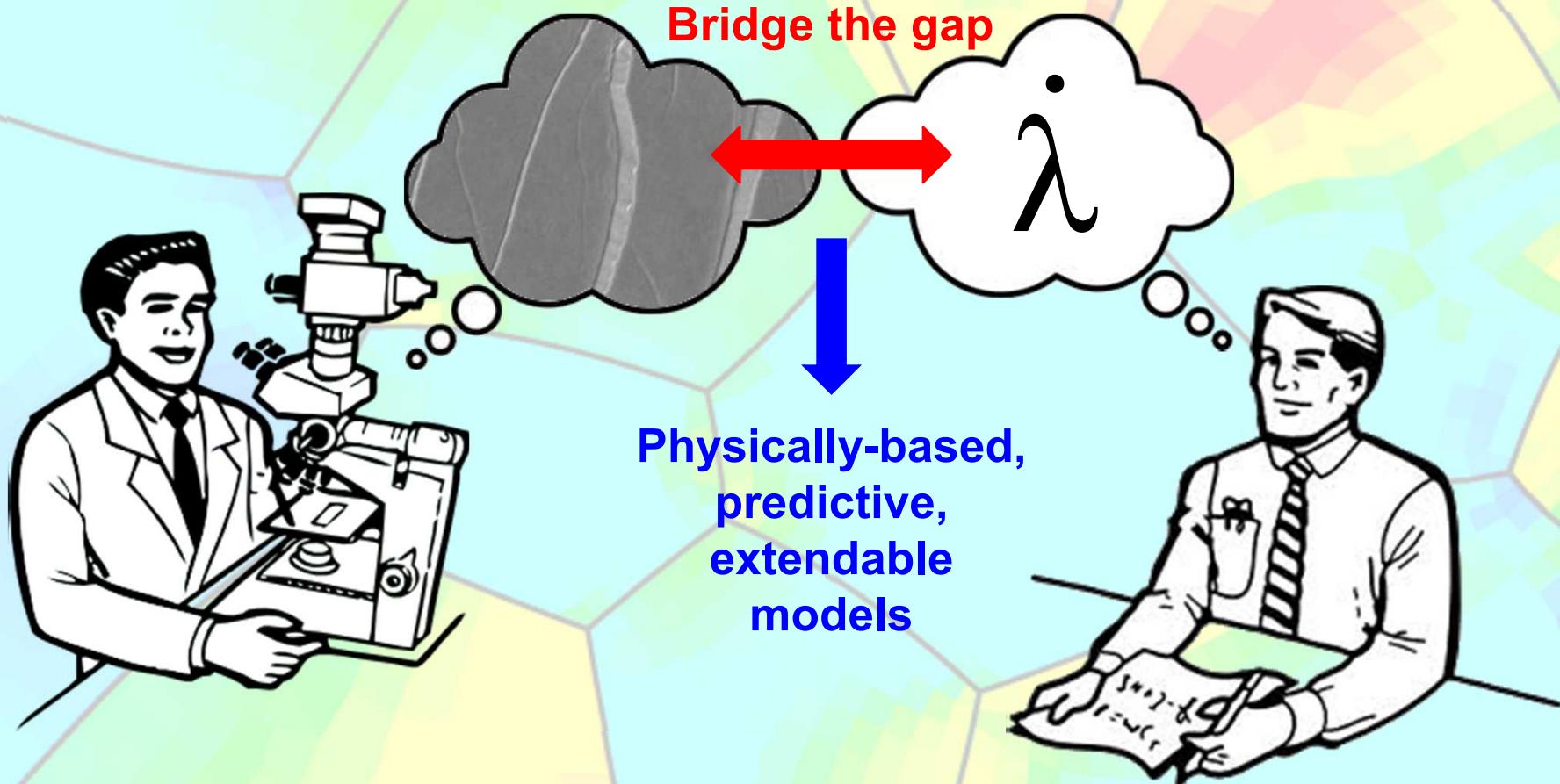
Microstructure-based:

- Grain size
- Grain boundaries
- Porosity
- Gas bubbles
- Precipitation
- Coarsening
- Densification
- Phase transformation
- Segregation
- Freckling
- ...

Other:

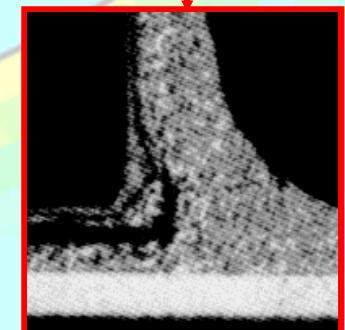
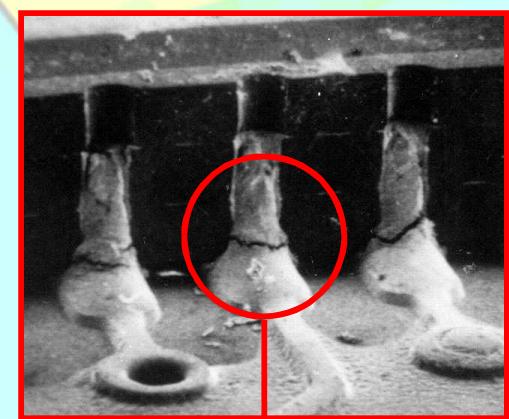
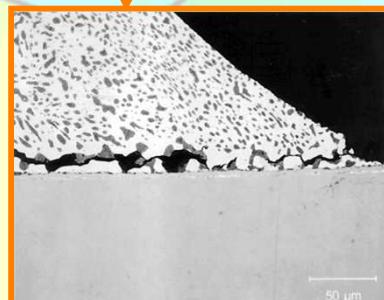
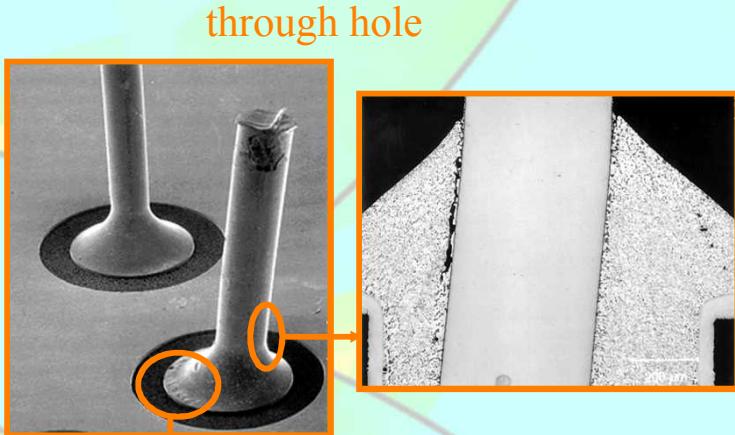
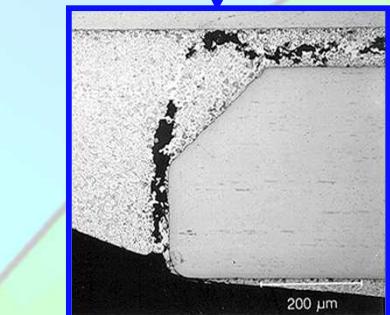
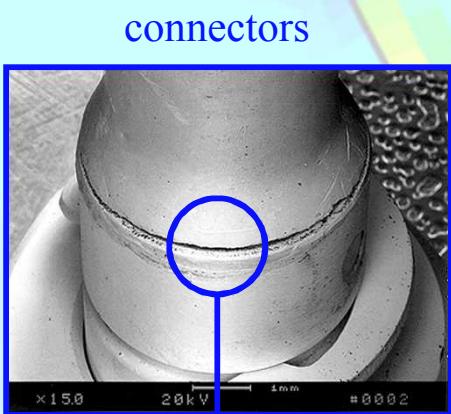
- Corrosion
- Etching
- Irradiation defects
- ...

What is the goal of damage modeling?



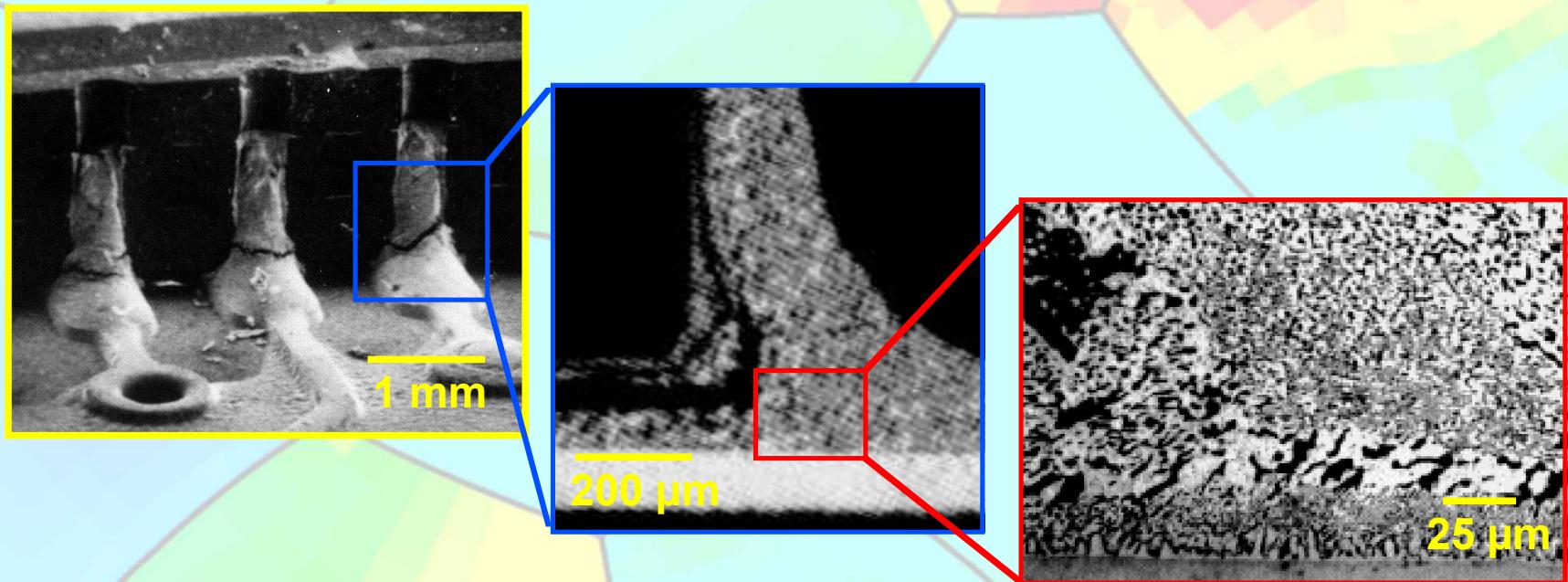
Early example of predictive damage modeling: Predicting Pb-Sn solder joint lifetime

- Typical circuit boards contain thousands of solder joints, functioning at a high homologous temperature under thermomechanical fatigue conditions.
- Studies indicate that **at least 48% of electronics failures** are likely due to solder joint failure.



What is the damage mechanism?

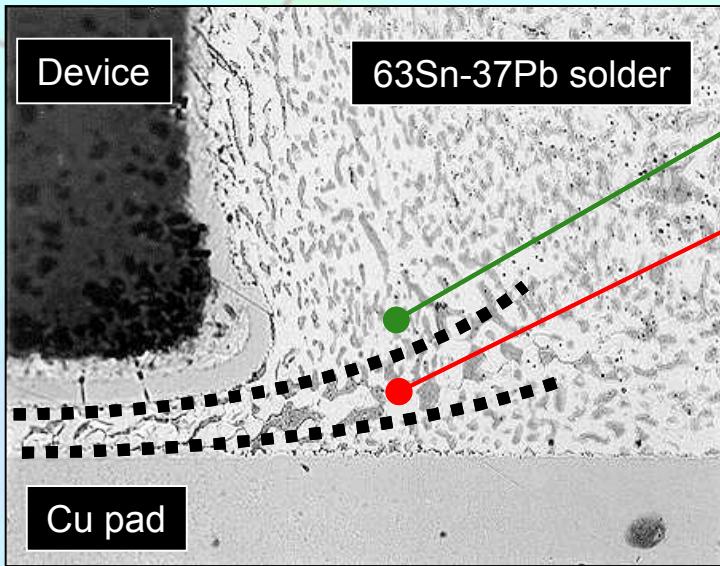
- In eutectic Pb-Sn solders, the coupling between microstructure and mechanical response causes failure.



strain localization \rightarrow local coarsening \rightarrow further softening \rightarrow failure

Defining the damage metric

- Local microstructure determines the local mechanical properties.
- ⇒ The damage metric is λ , the mean Pb-rich phase particle diameter.



$$[\dot{\gamma}/dt]_{\text{fine}} = f(\sigma, T, \lambda_{\text{fine}})$$

$$[\dot{\gamma}/dt]_{\text{coarse}} = f(\sigma, T, \lambda_{\text{coarse}})$$

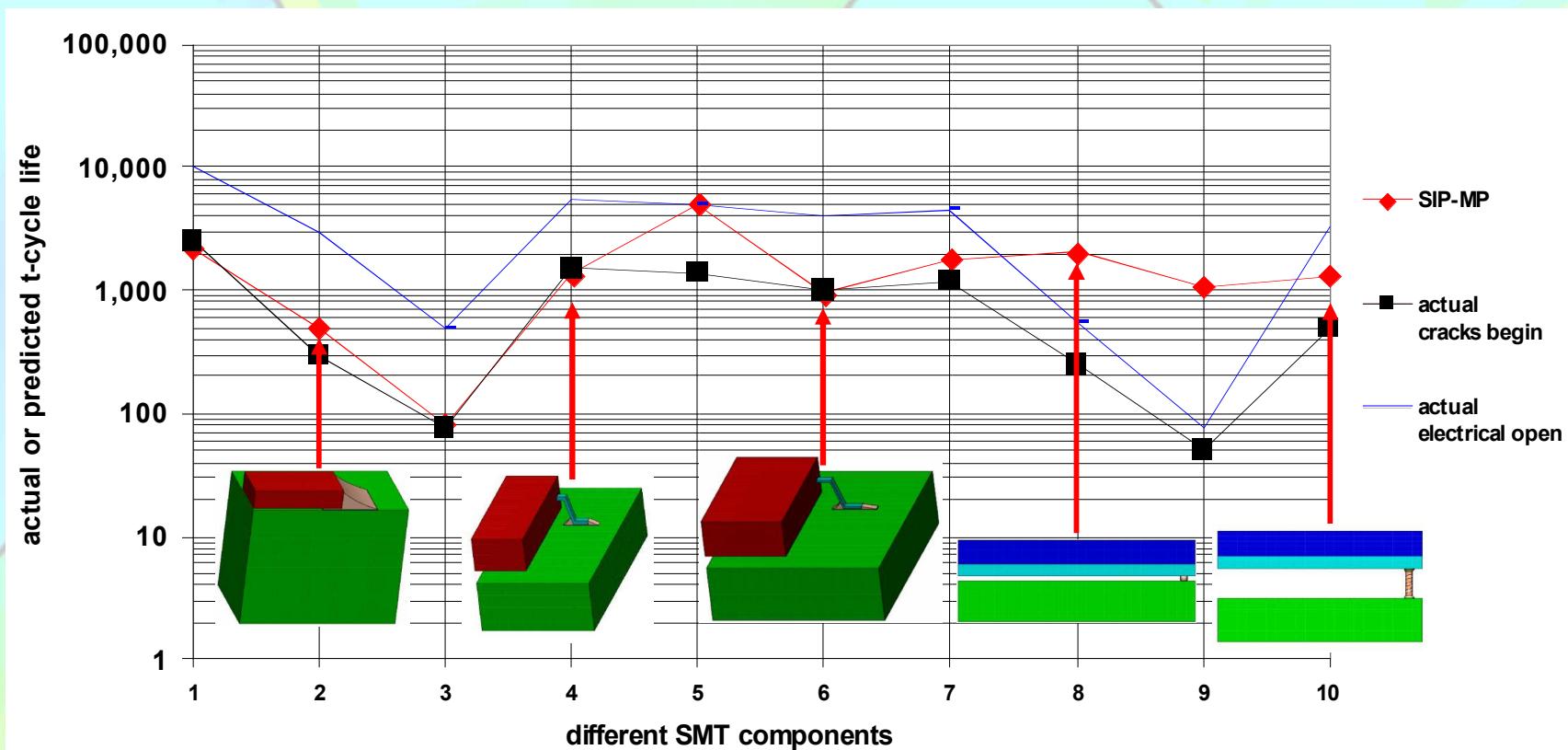
$$\begin{aligned}\sigma &= E : (d - d^{\text{in}}) \\ d^{\text{in}} &= \frac{3}{2} \gamma n = \frac{3}{2} f \exp\left(\frac{Q}{kT}\right) \left(\frac{\lambda_O}{\lambda}\right)^p \sinh^m\left(\frac{\tau}{\alpha(c+\bar{c})}\right) n \\ n &= \frac{\left(s - \frac{2}{3}B\right)}{\tau} \\ \tau &= \sqrt{\frac{3}{2}\left(s - \frac{2}{3}B\right) : \left(s - \frac{2}{3}B\right)} \\ c &= A_1 \dot{\gamma} - (A_2 \dot{\gamma} + A_3) \sqrt{\left(\frac{2}{3}B : B\right)} B \\ B &= A_4 d^{\text{in}} - (A_5 \dot{\gamma} + A_6) \sqrt{\left(\frac{2}{3}B : B\right)} B \\ \bar{c} &= A_7 \left(\frac{\lambda_O}{\lambda}\right)^{A_8} A_9 \\ \lambda &= \frac{(A_9 + A_{10} \dot{\gamma})}{(A_{11} - \lambda_O)}\end{aligned}$$

- Our approach:
 - Include λ and $d\lambda/dt$ in the solder constitutive model:
 - Use a microstructurally-based failure criterion:

failure when $\lambda = \lambda_{\text{critical}}$

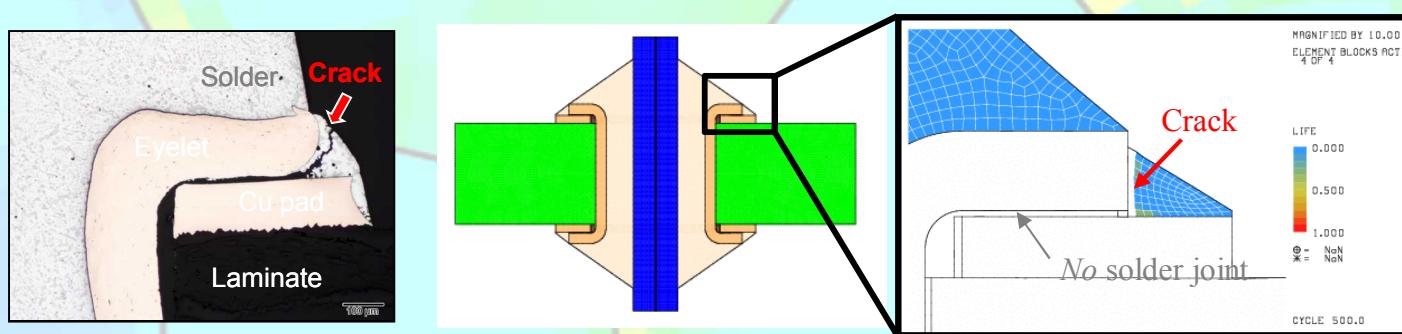
The microstructurally informed model predicts crack initiation accurately

- A Lockheed-Martin sponsored round robin offered the chance to validate the model in a blind study against experimental data for 10 different solder joints.
- For most joint types, our model (**red** line) agreed very well with experiment (**black** line) in predicting crack initiation.

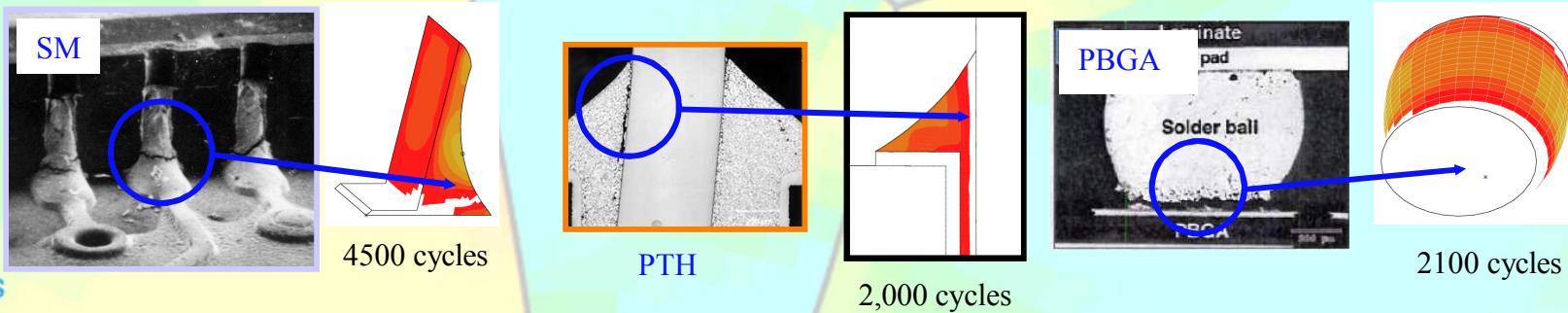


Benefits of a predictive damage model

- Modeling saves time and money.
 - Unexpected failures were observed in eyelet solder joints.
 - Lifetime modeling had predicted an order of magnitude longer fatigue life.
 - Failure mode analysis identified non-symmetric fillets (due to process variations) as the root cause of this failure, allowing efficient mitigation.



- Because it works, the solder model is used throughout Sandia and by the US Navy, Lockheed-Martin, Boeing, and other customers.

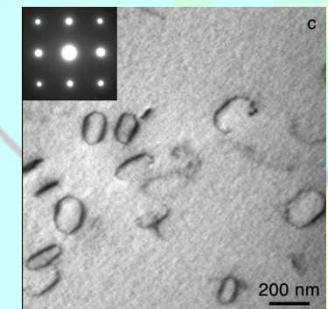
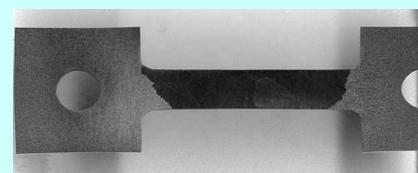
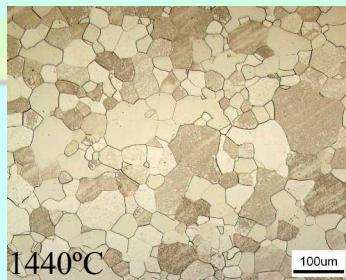
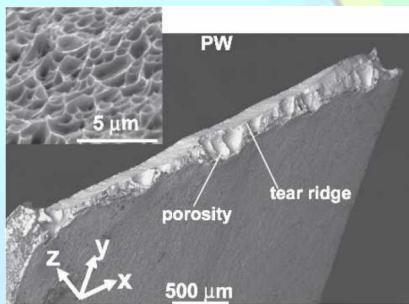
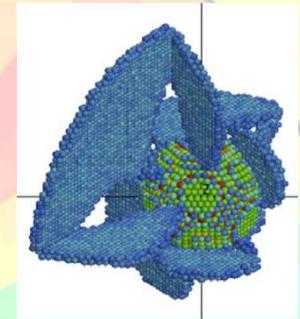
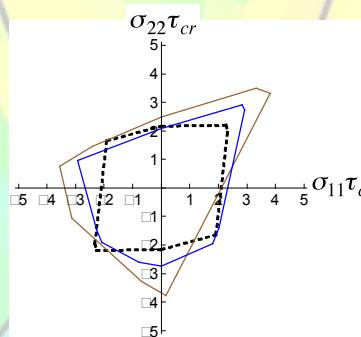
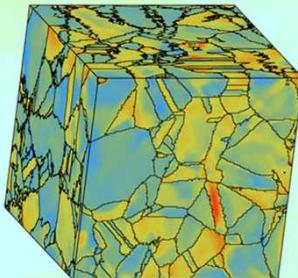
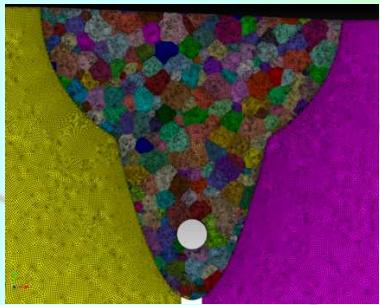


BUT...Such an approach is not always possible

- Legislation and industry standard mandate a change to Pb-free solder alloys.
 - We developed a constitutive framework for Pb-free solder lifetime prediction.
 - Unified Creep Plasticity Damage (UCPD) model.
 - Failure when the microscale damage parameter $w = w_{critical}$
- However, the physical meaning of w is undefined; the damage mechanism is unknown.
- Happily, the model works.
- Sadly, predictive confidence is limited to the fit regime.
- **Lesson learned: Physical understanding of the damage mechanism is critical to predictive failure modeling.**

$$\dot{\sigma} = \mathbf{E} : \dot{\epsilon}^e = \mathbf{E} : (\dot{\epsilon} - \dot{\epsilon}^{in})$$
$$\dot{\epsilon}^{in} = \frac{3}{2} \dot{\gamma} \mathbf{n} = \frac{3}{2} f \sinh^p \left(\frac{\tau}{D(1-cw)} \right) \mathbf{n}$$
$$\dot{D} = \frac{A_1 \dot{\gamma}}{(D-D_0)^{A_3}} - A_2 (D-D_0)^2$$
$$\dot{\mathbf{B}} = \frac{A_4 \dot{\epsilon}^{in}}{b^{A_6}} - A_5 b \mathbf{B}$$
$$\mathbf{n} = \frac{\mathbf{s} - \frac{2}{3} \mathbf{B}}{\tau} \quad b = \sqrt{\frac{2}{3} \mathbf{B} : \mathbf{B}}$$
$$\tau = \sqrt{\frac{3}{2} \left(\mathbf{s} - \frac{2}{3} \mathbf{B} \right) : \left(\mathbf{s} - \frac{2}{3} \mathbf{B} \right)}$$
$$N_f \approx \left(\frac{\alpha}{\Delta \gamma_{EQPS}} \right)^\beta$$
$$\Delta w = \frac{1}{N_f} \approx \left(\frac{\Delta \gamma_{EQPS}}{\alpha} \right)^\beta$$
$$\dot{w} = \frac{\beta}{\alpha^\beta} \left(\gamma_{EQPS}^i \right)^{\beta-1} \dot{\gamma}$$

How can we incorporate physical damage mechanisms in material models?



Material performance
 $10^0 \text{ m } 10^6 \text{ s}$

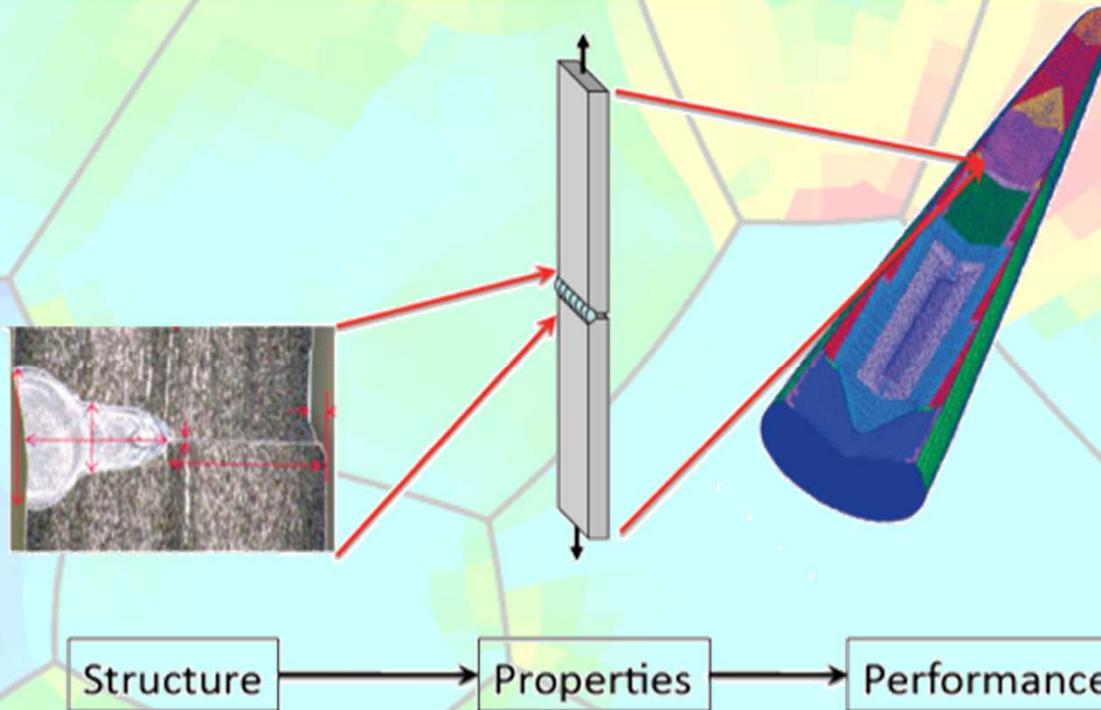
Microstructural effects
 $10^{-3} \text{ m } 10^3 \text{ s}$

Single crystal behavior
 $10^{-6} \text{ m } 10^0 \text{ s}$

Atomic scale phenomena
 $10^{-9} \text{ m } 10^{-9} \text{ s}$

Atoms-up: Develop physics-based models to provide scientific insight

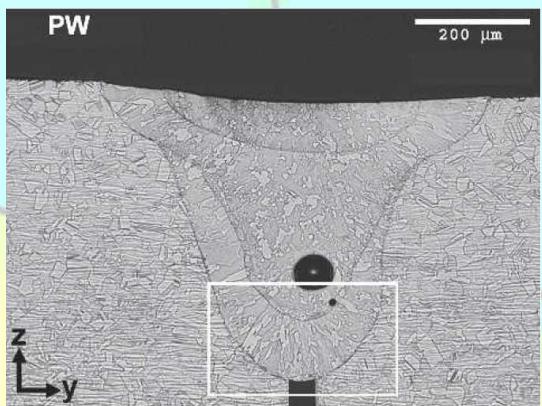
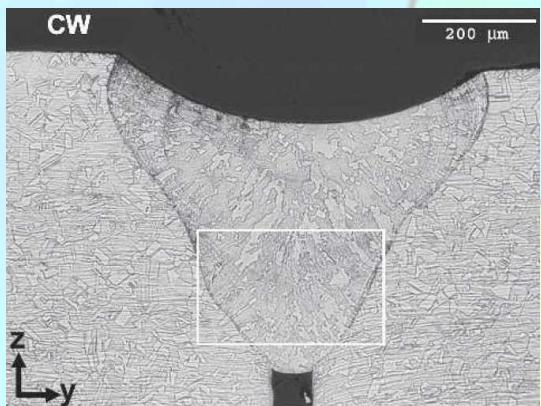
Continuum-down approach to incorporating damage into engineering models



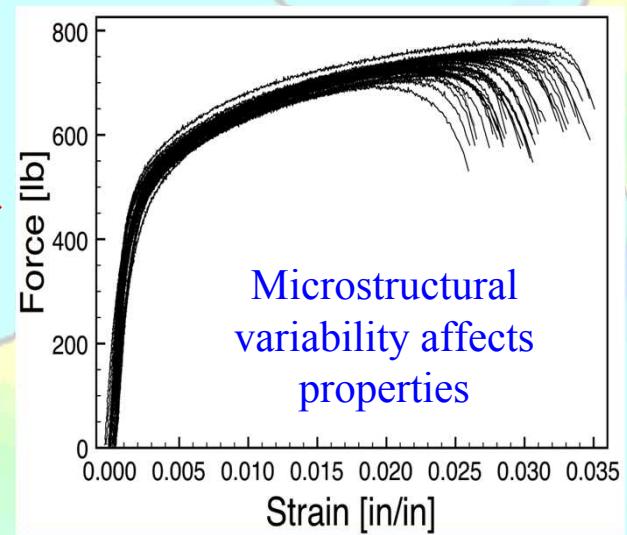
- Welds often form the weakest region in mechanical components.
- Predicting performance requires an accurate model for the variability in mechanical response of 'typical' production welds.
- Current analysis models do not capture this variability, necessitating expensive full-system mechanical tests.

Materials variability arises from microstructure

- Certain microstructural features can act as damage/failure sites.
- The relationship between the variability and distribution of these features and property statistics is unknown.
- To predict materials performance, we require a paradigm shift, from the idealistic view that all parts are created equal, to the realistic view that structure, properties, and performance are probabilistic.

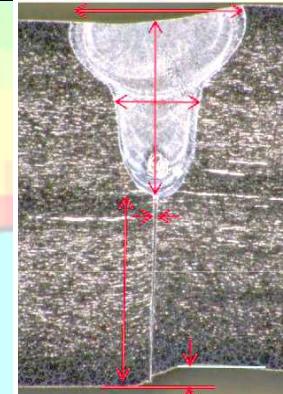
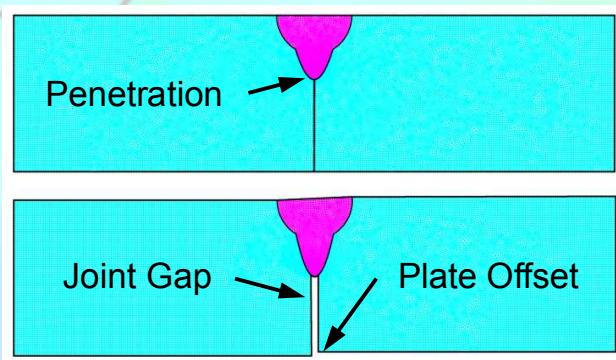


Microstructural details vary
among weldments

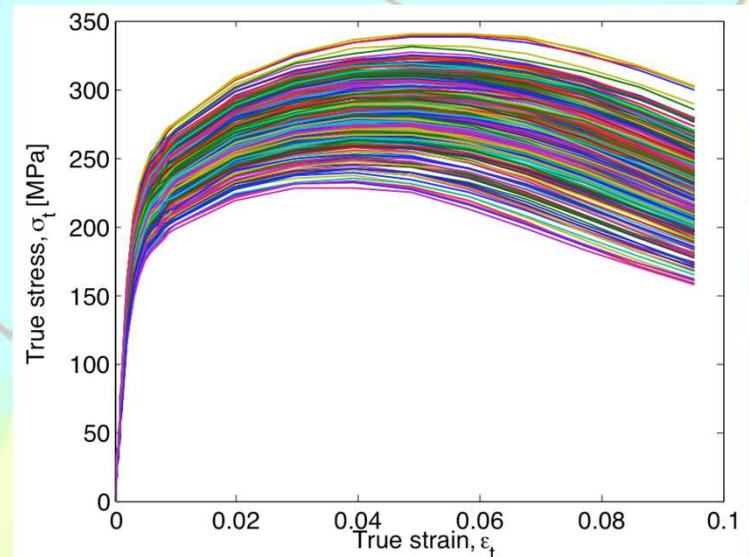
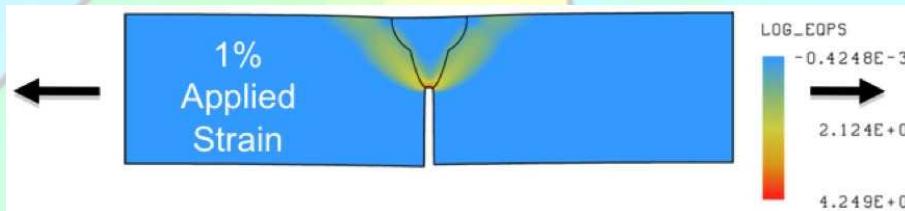


Initial suggestion: Geometric variability causes variable response

- Production welds exhibit significant geometric variability
- We created a data set of 250 model welds, varying penetration, joint gap and plate offset

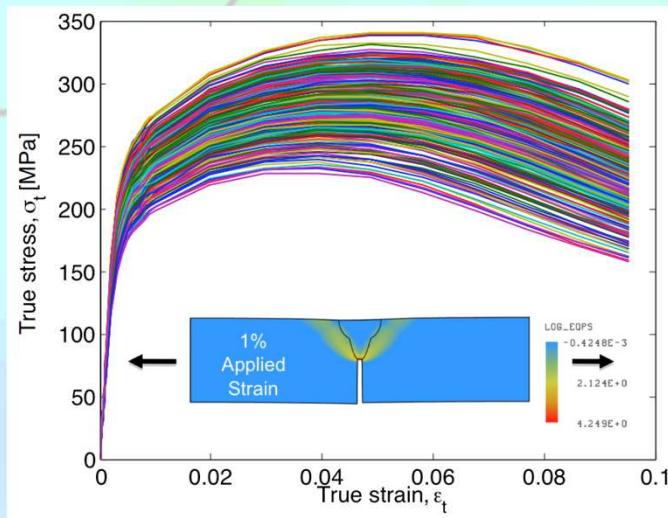


- FEM tensile tests show variability in weld properties due to geometric variation

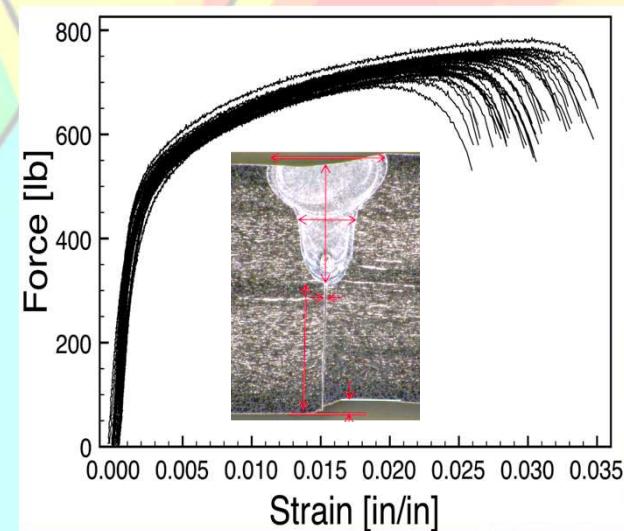


Geometric variability alone is insufficient

- The initial weld model does not capture the strain localization to failure behavior of real welds:



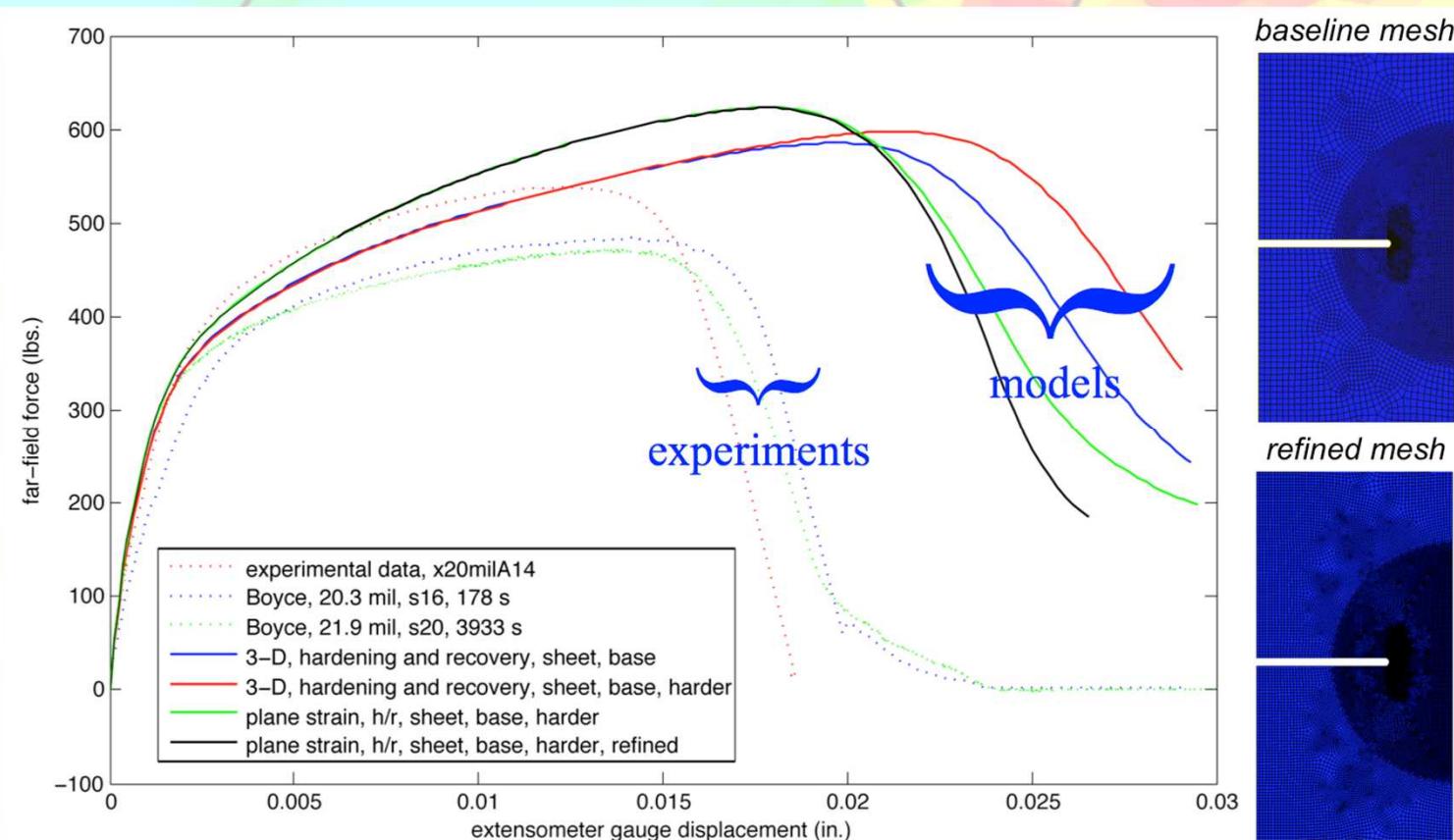
VS.



- While the spread in properties is similar, the qualitative behavior differs.
- The precipitous decline in force seen in experiments is typically attributed to damage (i.e. necking).

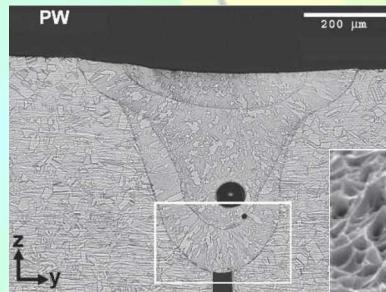
Necking alone does not account for material response

- Including necking in the weld model matches qualitative behavior, but lacks quantitative agreement: The weld is too strong.
- Are there additional localization mechanisms?

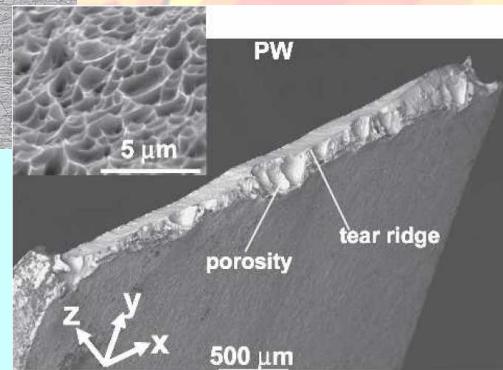


The role of porosity in laser welds

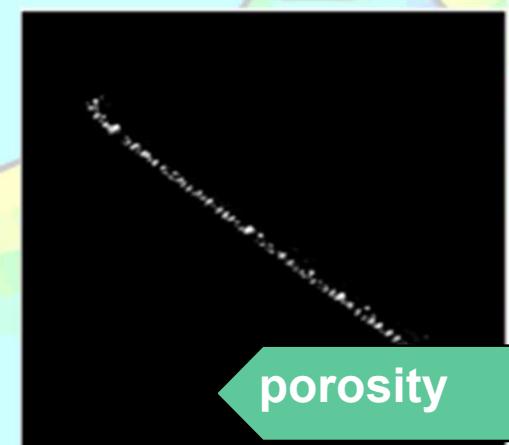
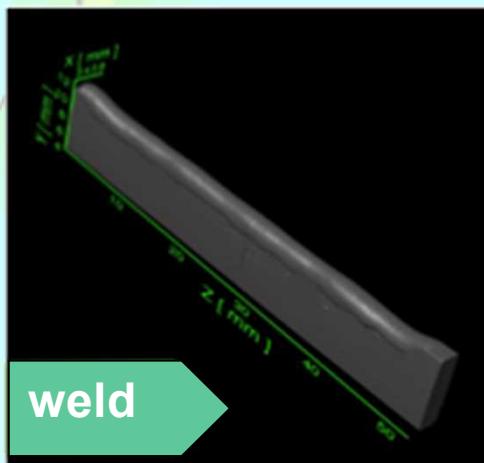
- Pores are widely observed in welds.
 - Porosity is a manifestation of microscale damage.
 - Fracture surfaces often follow porosity.



BL Boyce, PL Reu, CV Robino, *Mater. Metall. Trans. A*, **37A**[8] (2006) 2481.



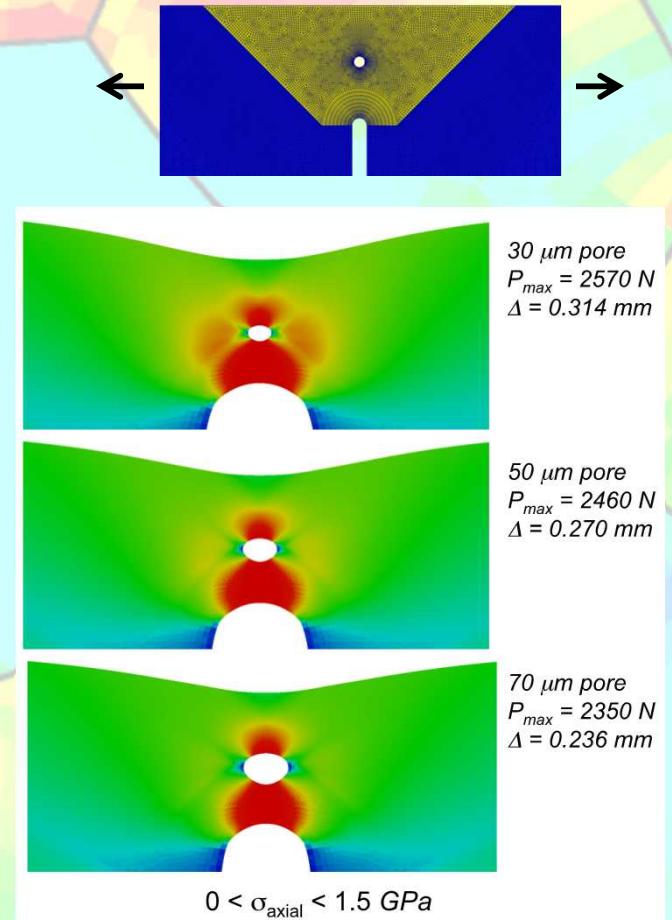
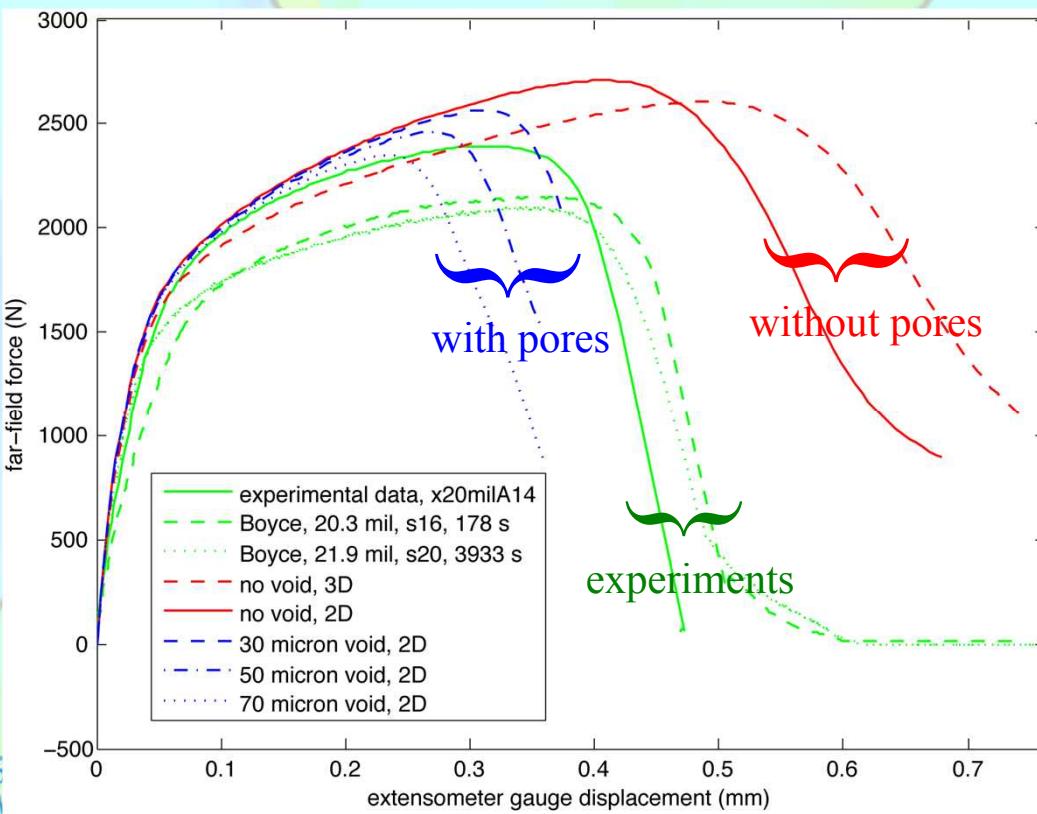
- New weld micro-tomography capability can resolve pores in three dimensions.
 - Significant porosity can occur.
 - Pores align in the weld direction.



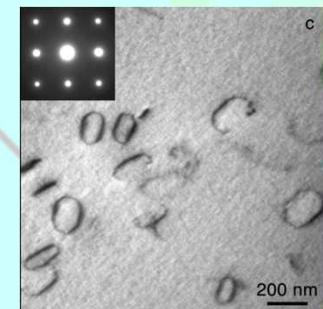
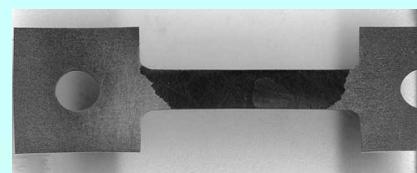
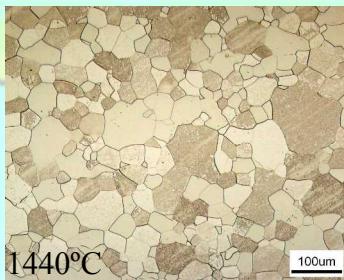
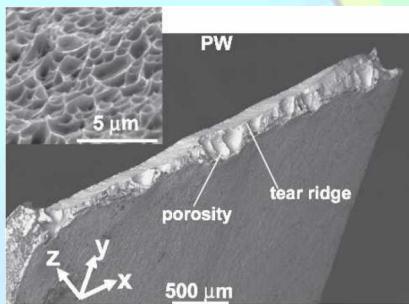
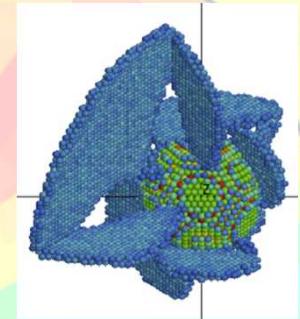
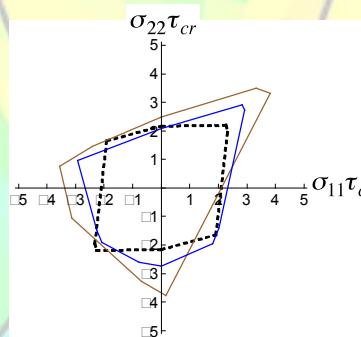
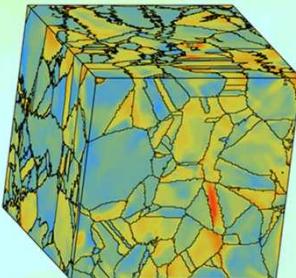
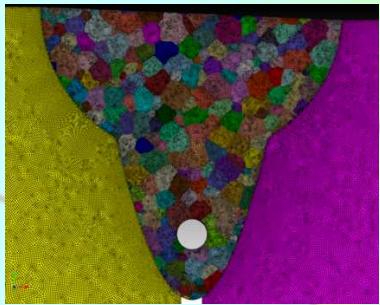
J. Madison and D. Susan, *in preparation* (2012).

Including pores in the weld model improves model fidelity

- Weld pores interact strongly with the neck region to weaken the weld; the effect is size and placement dependent.
- With pores as the damage mechanism, the model achieves qualitative and quantitative agreement with experiments.



How can we incorporate physical damage mechanisms in material models?



Material performance
 $10^0 \text{ m } 10^6 \text{ s}$

Microstructural effects
 $10^{-3} \text{ m } 10^3 \text{ s}$

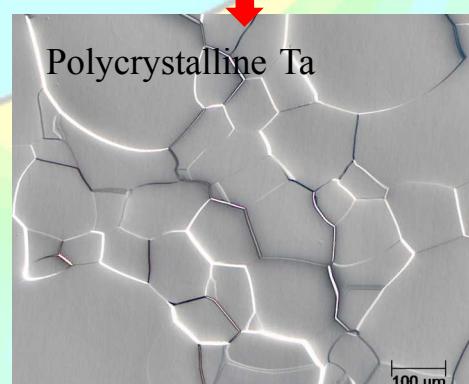
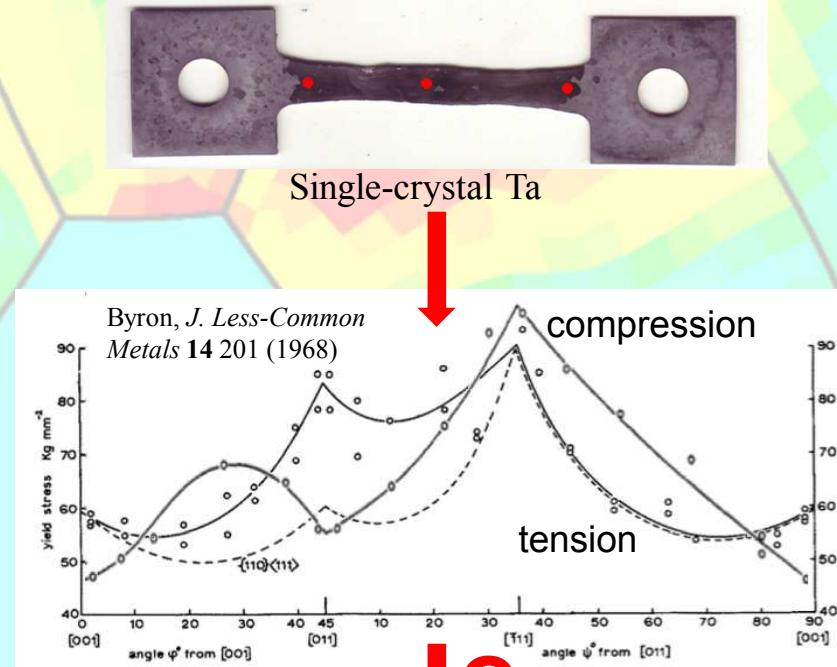
Single crystal behavior
 $10^{-6} \text{ m } 10^0 \text{ s}$

Atomic scale phenomena
 $10^{-9} \text{ m } 10^{-9} \text{ s}$

Atoms-up: Develop physics-based models to provide scientific insight

Atoms-up Foundational Engineering Problem: Plasticity in BCC metals

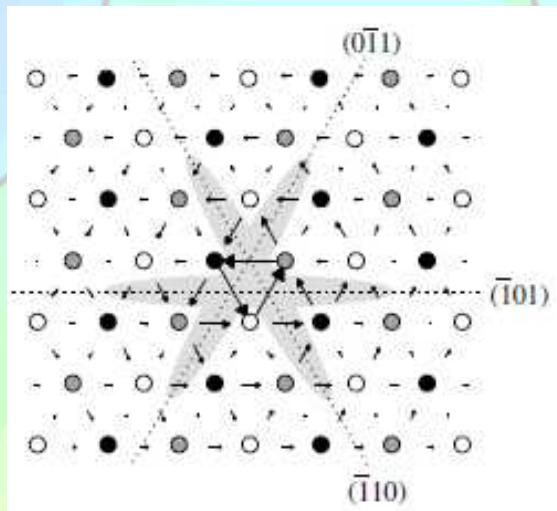
- BCC metals are technologically important (W, Mo, Ta, steel...)
- BCC single crystals exhibit considerable tension/compression asymmetry at room temperature.
 - Polycrystalline materials are made up of single crystals.
 - Tension/compression asymmetry **influences dislocation damage processes** in polycrystalline BCC metals.
- Tension/compression asymmetry arises due to atomic-scale processes, but current computational models are strictly **phenomenological**.



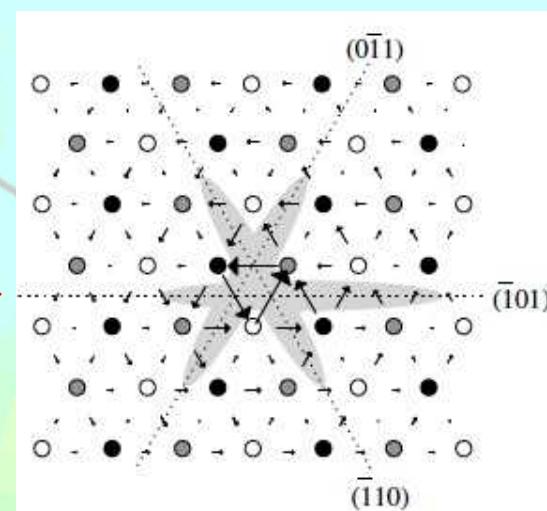
Atomic Scale: Physical model for dislocation motion in BCC metals

- Atomic scale simulations show dislocation core spreading onto adjacent (110) planes in BCC metals.
 - Core spreading creates a **significant Peierls barrier** to dislocation motion.
 - Because the dislocation spreads onto three planes, motion can be affected by stress components outside the preferred slip plane, i.e. **non-Schmid stresses**.

[111] zone depiction of a relaxed screw dislocation core in Mo



Distortion of the dislocation core under an applied shear stress



Single crystal behavior: BCC crystal plasticity model

The atomic results can be fit to a yield criterion given by:

We use this form to derive the generalized stress state on a slip system:

$$\tau^{(s)} = \mathbf{P}_{\sigma}^{(s)} : \boldsymbol{\sigma}^{app}$$

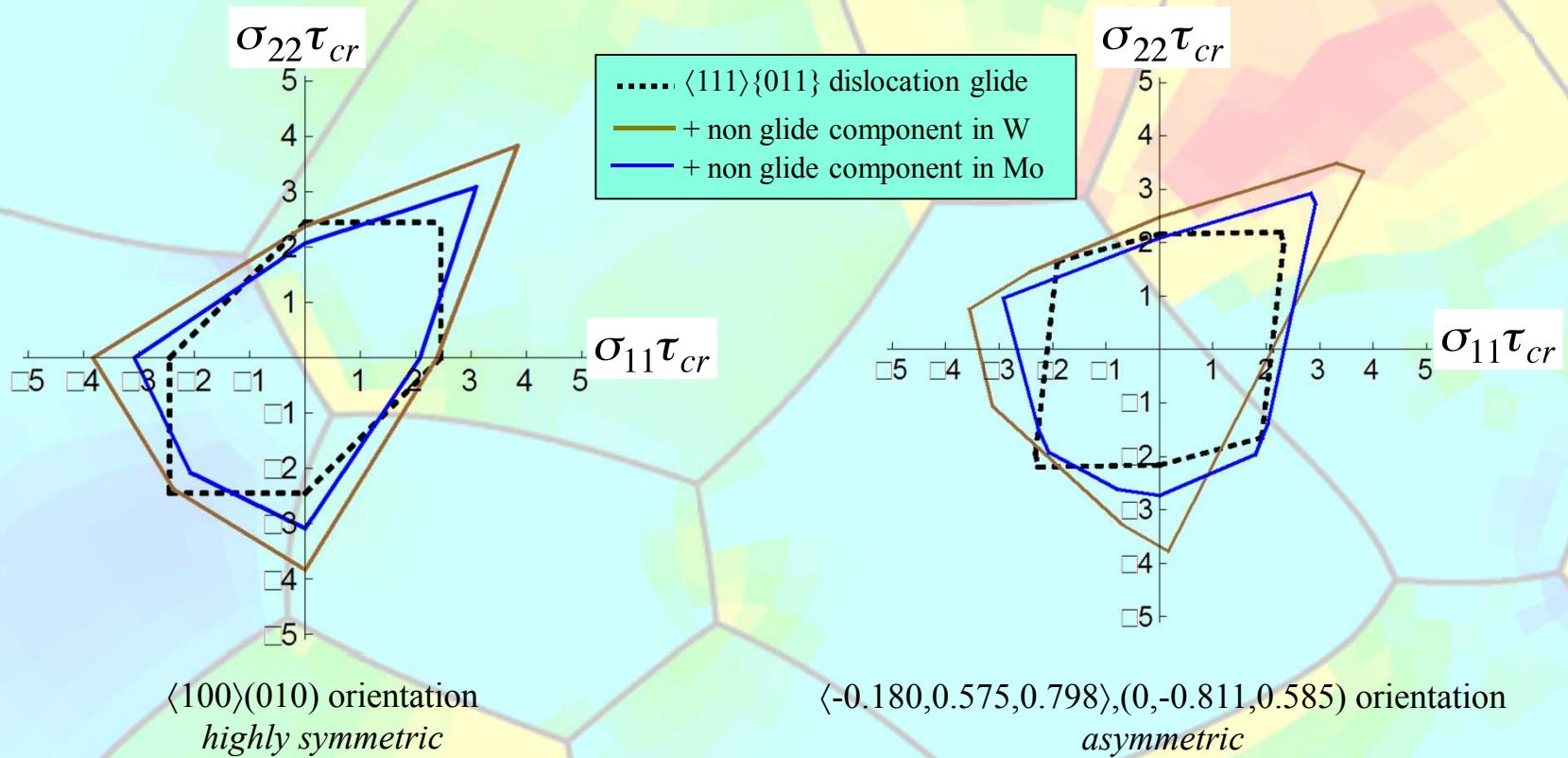
Which leads to a single-crystal constitutive law:

$$\dot{\gamma}^{(s)} = \frac{\tau^{(s)}}{\tau_{cr}} \left| \frac{\tau^{(s)}}{\tau_{cr}} \right|^{\frac{1}{m}-1}$$

Which gives the plastic strain rate:

$$\mathbf{D} = \sum_s \dot{\gamma}^{(s)} \mathbf{m}^{(s)}$$

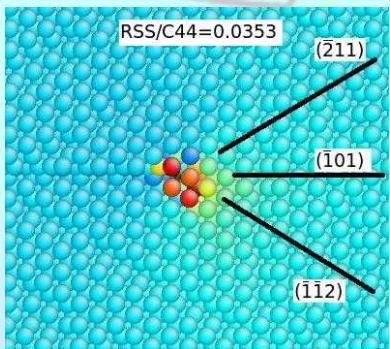
Single Crystal Results: BCC single crystal yield surfaces



- BCC yield surfaces are considerably different from FCC yield surfaces.
- The yield surfaces of W and Mo are quite distinct.
- Tension/compression asymmetry is apparent.

Atoms-up Approach: Plasticity in BCC metals

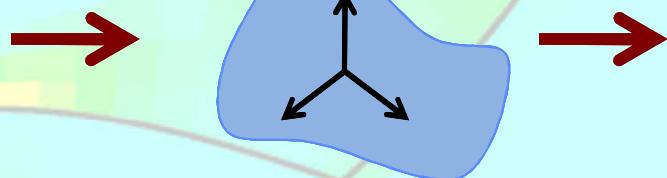
Atomic phenomenology:
Fundamental deformation
mechanisms



Yield criterion

$$\begin{aligned}\sigma_{cr}^{app} [a_0 \mathbf{m}^{(s)} \mathbf{n}^{(s)} + a_1 \mathbf{m}^{(s)} \mathbf{n}^{(s')} \\ + a_2 (\mathbf{n}^{(s)} \times \mathbf{m}^{(s)}) \mathbf{n}^{(s)} \\ + a_3 (\mathbf{n}^{(s)} \times \mathbf{m}^{(s)}) \mathbf{n}^{(s')}] = \tau_{cr}\end{aligned}$$

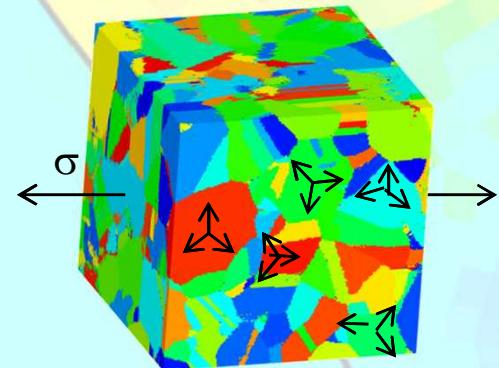
Single crystal
plasticity:
Deformation of one,
isolated crystal



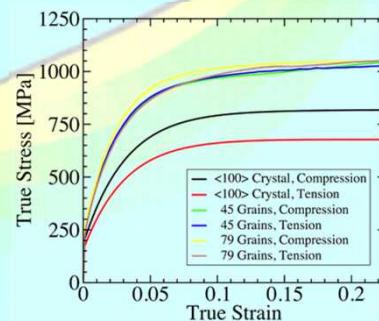
Constitutive law

$$\dot{\gamma}^{(s)} = \frac{\tau^{(s)}}{\tau_{cr}} \left| \frac{\tau^{(s)}}{\tau_{cr}} \right|^{m-1}$$

Polycrystal plasticity:
Assemble single crystals into
polycrystalline ensemble

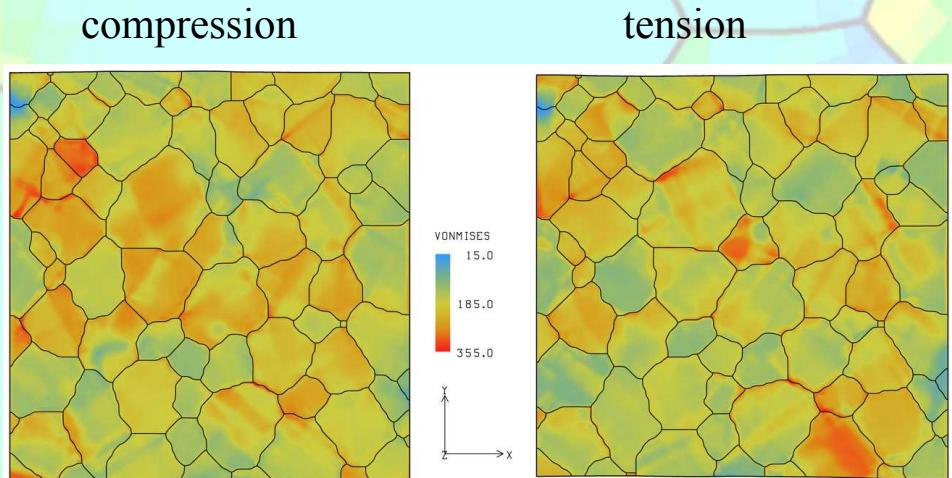
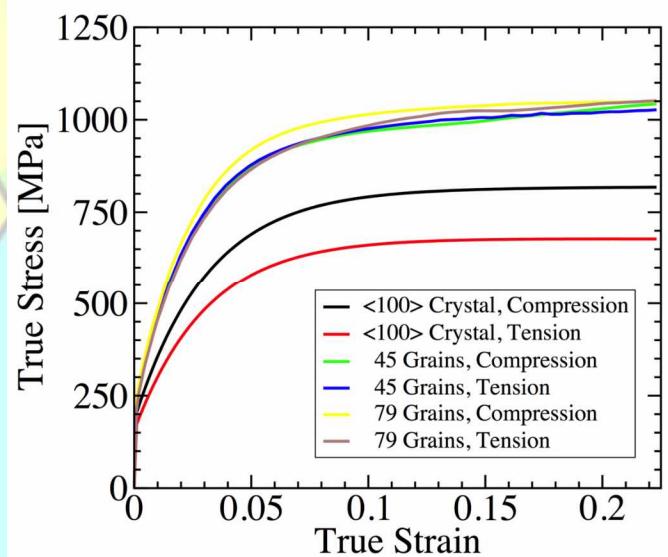


Collective deformation
behavior



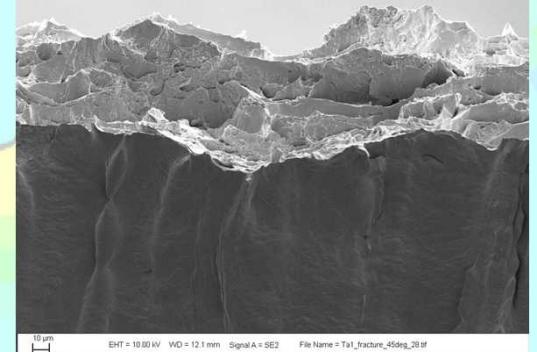
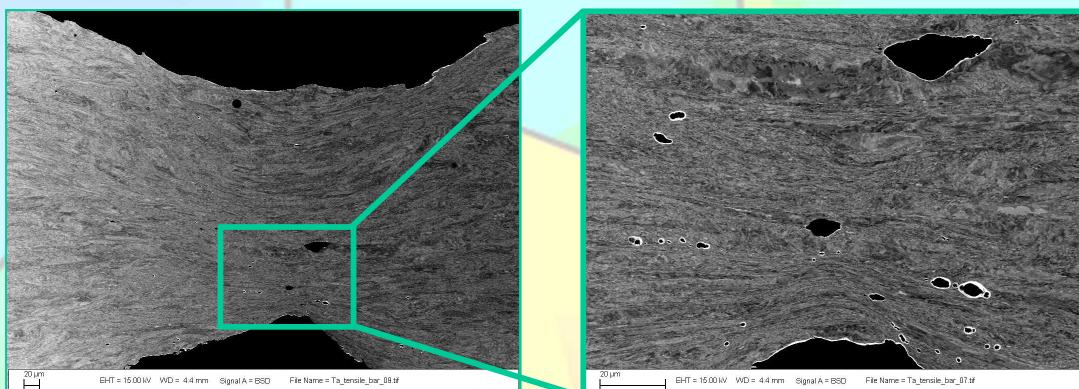
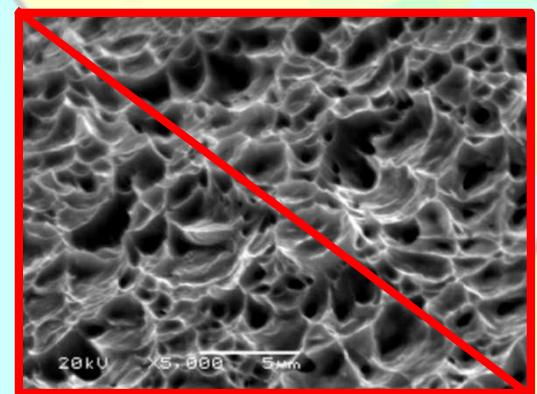
Microstructural Effects: Mechanical response of BCC polycrystals

- Microstructural simulations give **continuum behavior**:
 - Single crystal tension/compression asymmetry.
 - Polycrystalline averaging.
- Microstructural simulations also give **local behavior**:
 - The local stress concentrations indicate **potential damage initiation** sites.
 - Locations of highest stress varies with stress state and microstructure.



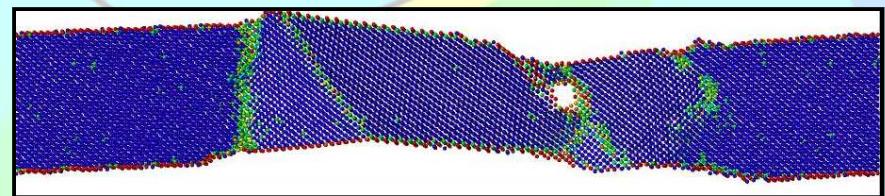
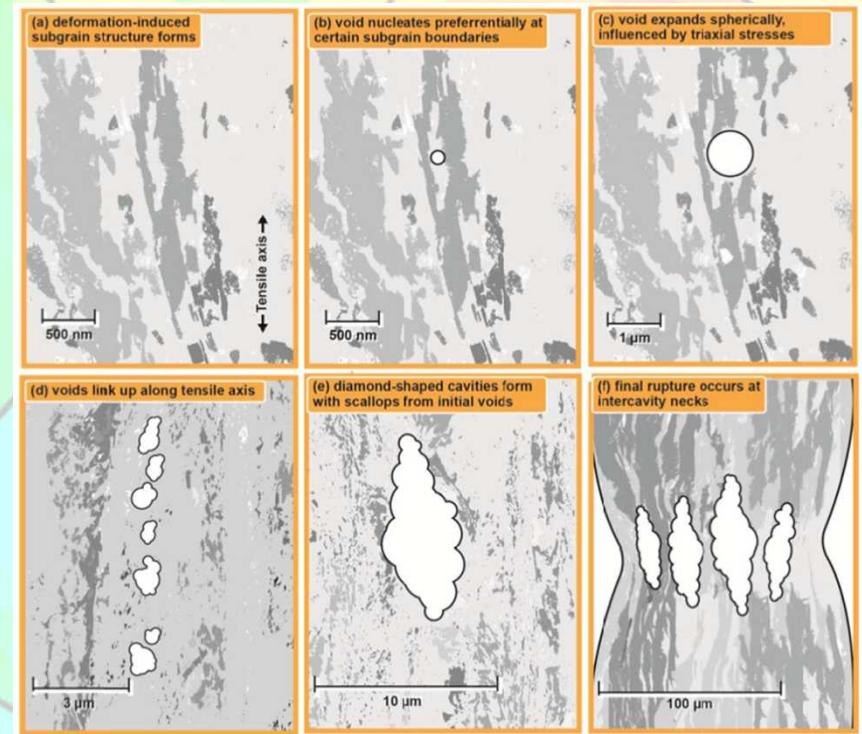
Next steps: Develop a damage-based failure model for ductile fracture in Ta

- After some amount of deformation, damage due to dislocation plasticity nucleates the failure mechanism.
- In Ta tensile tests, we observe a characteristic, serrated fracture surface.
- Closer examination shows both large, lenticular micropores and strings of smaller, spherical pores.
- At this point, the primary damage mechanism has transitioned from dislocations to micropore nucleation and coalescence.



Damage-based failure model: Subgrain-mediated pore coalescence

- Ductile fracture in Ta proceeds by a modified void coalescence mechanism:
 - Strings of spherical voids **nucleate and grow** along certain elongated subgrain boundaries
 - Voids **coalesce** into large, diamond shaped cavities
 - Cavities **link** across the neck, leading to a serrated fracture surface.
- This process is amenable to statistical modeling (in progress)
- Preliminary atomistic models show development of similar features; may inform the nucleation and growth model.



J. Zimmerman

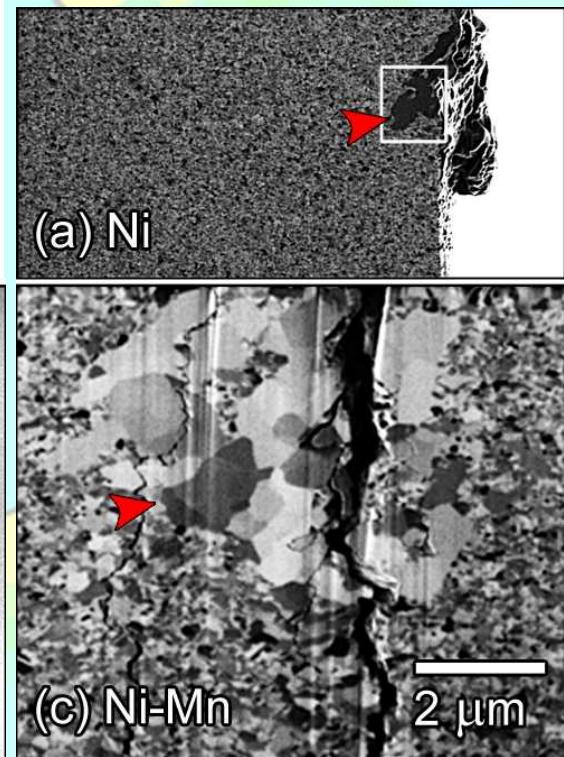
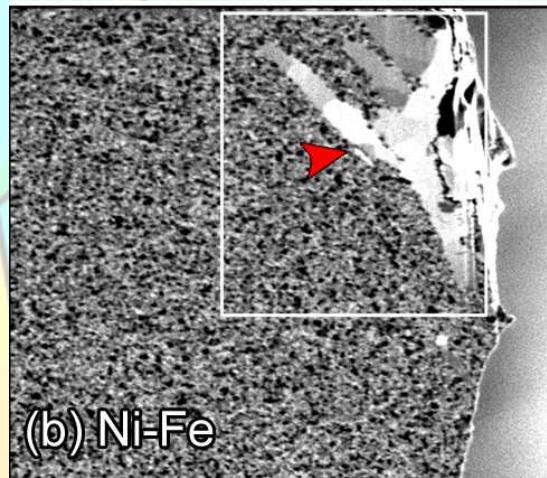
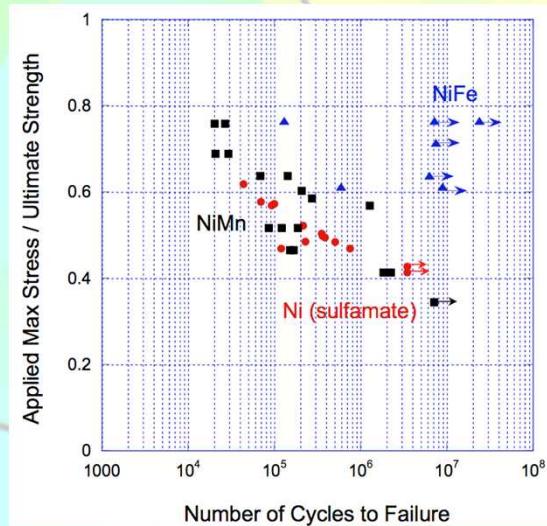
Beyond conventional wisdom

- Conventional concepts of fracture involve:
 - Void nucleation → coalescence → ductile fracture
 - Microcrack nucleation → link-up → brittle fracture
- However, always keep in mind that entirely different damage processes may be the critical ones.

Fatigue failure: Abnormal grain growth as damage mechanism

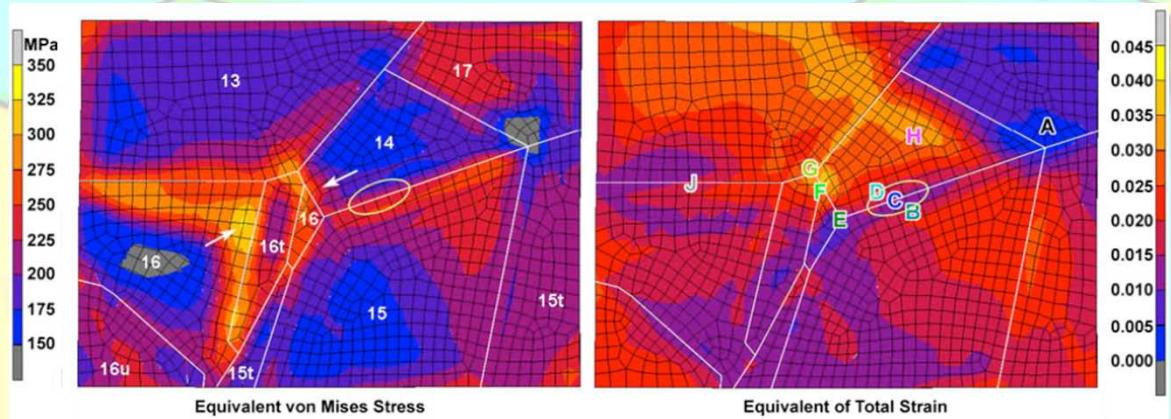
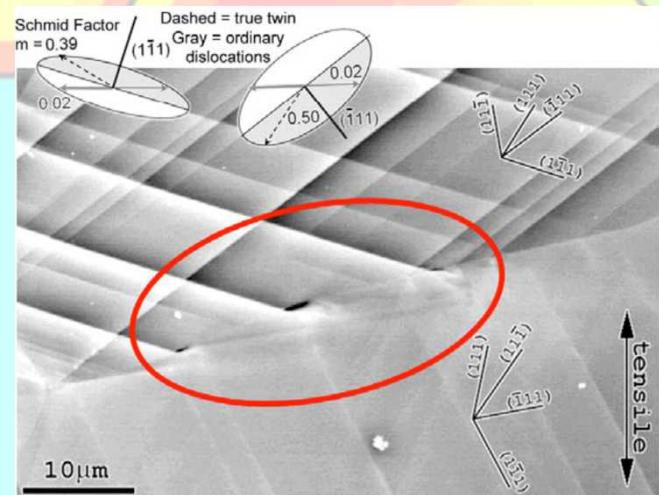
- Nanocrystalline Ni alloys have anomalously long fatigue lives.
- However, when they fail, the fatigue crack always initiates at a cluster of very large, crystallographically related grains.
- These grains are not present in the initial microstructure.

➤ Understanding the abnormal grain growth process is key to modeling the damage mechanism.



Microcrack nucleation: Grain boundary character as damage parameter

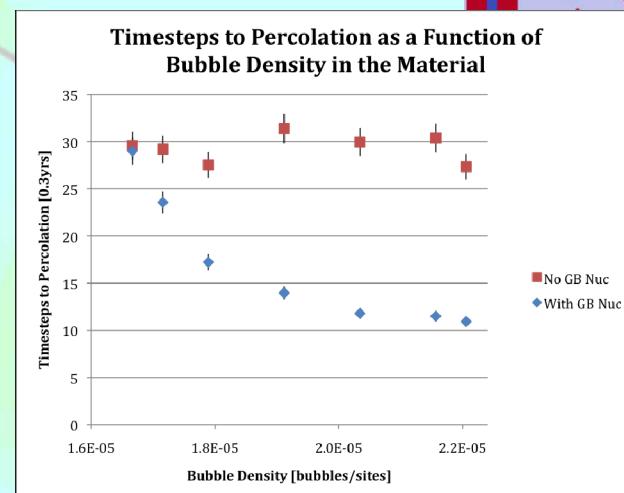
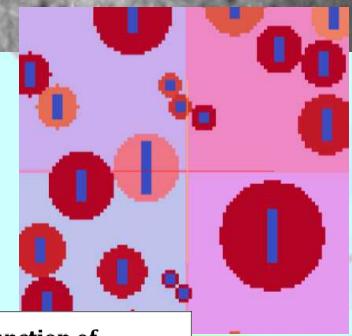
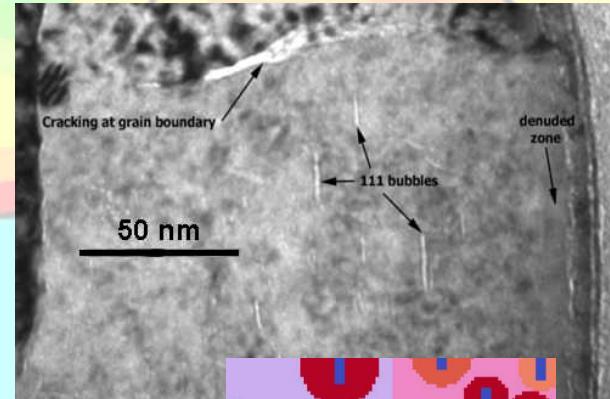
- In TiAl, grain boundary microcracking is a critical damage mechanism.
- Comparing FEM models to experiments, we find:
 - Damage does **not** nucleate at locations having the highest local stress, strain, or strain energy.
 - Damage nucleation occurs in particular boundaries where imperfect slip transfer leaves residual dislocation content in the boundary.
- Understanding how grain boundary character interacts with dislocations and twins is key to modeling the damage mechanism.



Fission gas release: Pressurized bubbles as damage mechanism

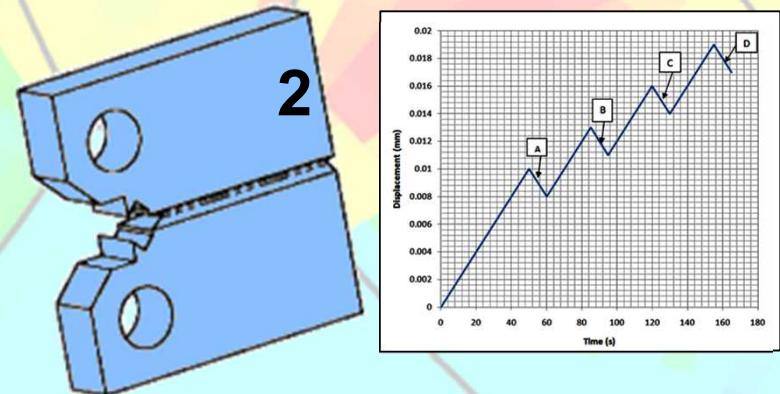
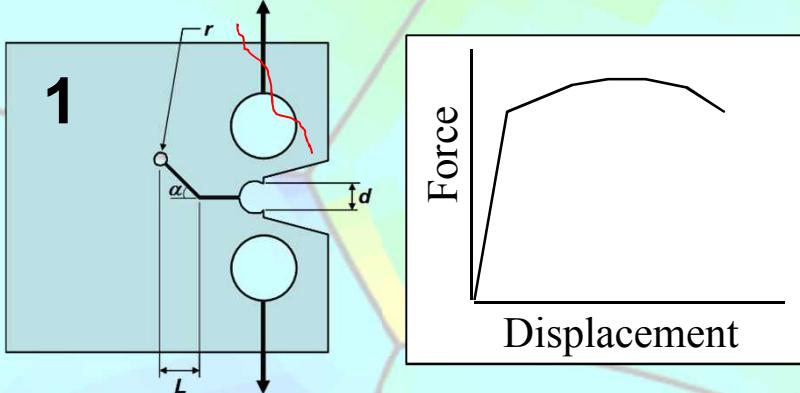
- In ErT₂, He bubble nucleation and growth is the critical damage mechanism.
- Bubbles are flat and pressurized.
- Percolation models of stress-field overlap show:
 - In the absence of grain boundaries, for a given He density, fission gas release rate is independent of bubble nucleation density.
 - The presence of grain boundary bubbles significantly increases fission gas release .

➤ Understanding bubble nucleation, growth, and link-up is key to modeling the damage mechanism.



Why study microscale damage? The Sandia X-Prize: Ductile Fracture Challenge

- **The system:** Standardized ductile fracture specimens.

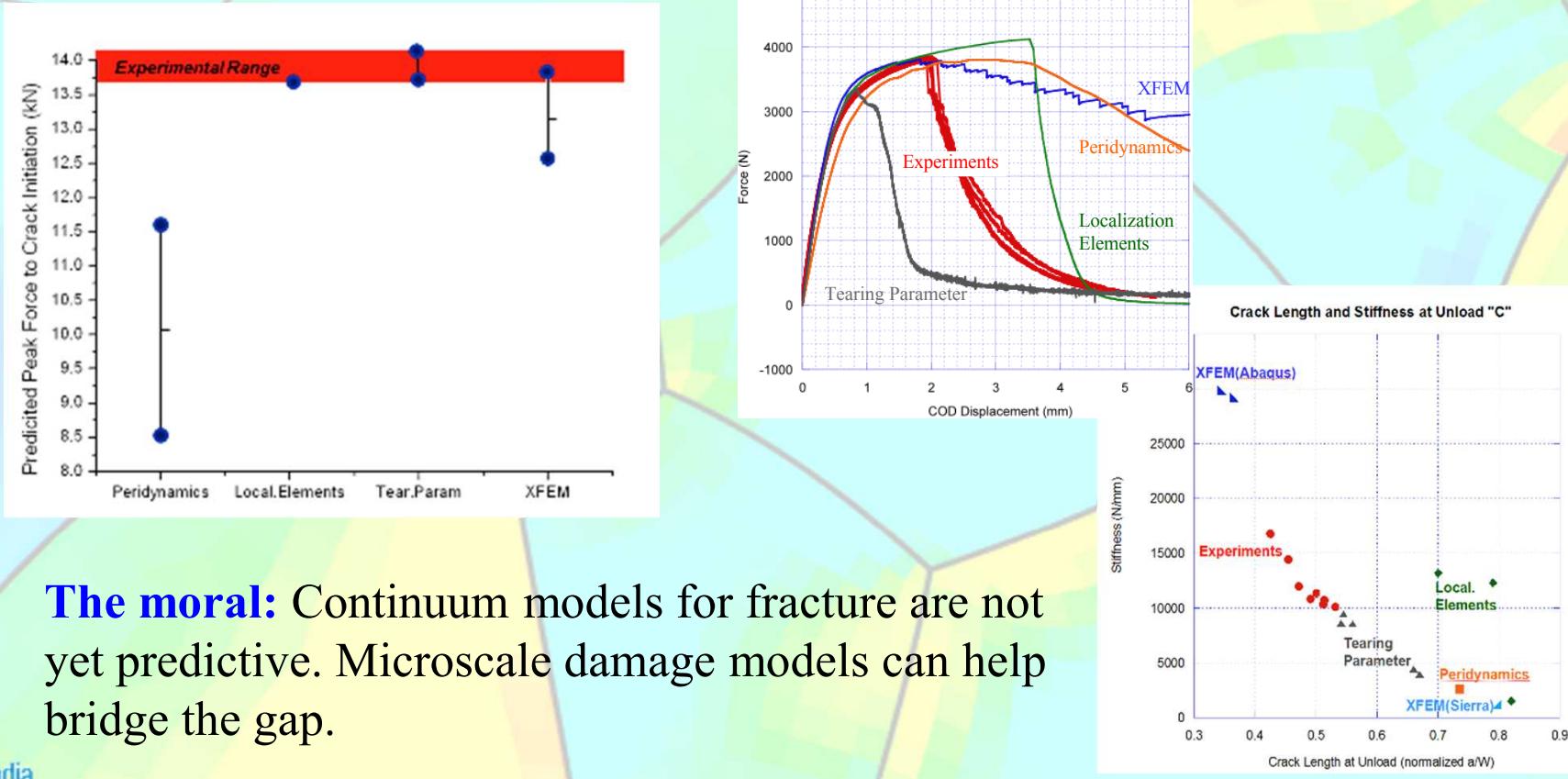


- **The challenge:** Predict fracture behavior.
 - *Sample 1:* d and F at several crack locations, crack path
 - *Sample 2:* crack length and unloading compliance at each displacement peak
 - *Blind study:* Experimental results were not released until after modeling results were submitted.
- **The competitors:** Four production fracture analysis codes.

Tearing parameter, Localization elements, Peridynamics, XFEM

The Sandia X-Prize: Results

- **The good news:** Respectable results for crack initiation and fracture path (system 1).
- **The bad news:** Poor predictions of force-displacement (system 1) and R-curve behavior (system 2).
- **The moral:** Continuum models for fracture are not yet predictive. Microscale damage models can help bridge the gap.



Summary and Conclusions

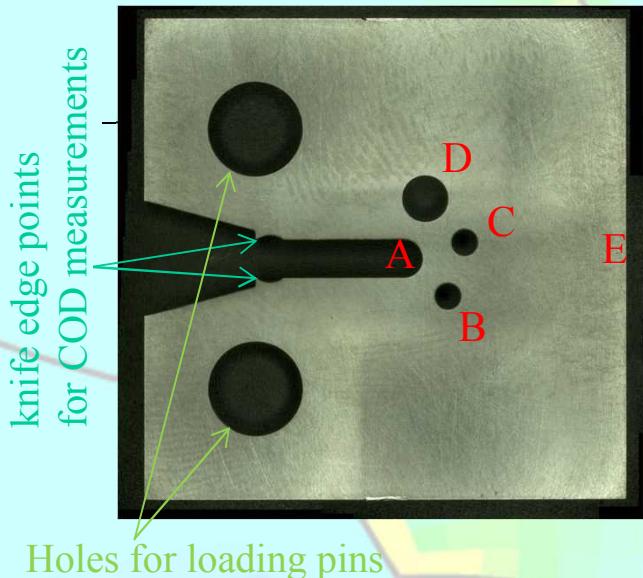
- Microstructural scale damage takes many forms.
- Understanding microscale damage offers important benefits:
 - Physical insight
 - Predictivity
 - Extendability
- Multiscale models and experiments elucidate microscale damage mechanisms and consequences.
 - Continuum-down
 - Atoms-up
- Damage processes at the microstructural scale are key to capturing fracture and failure.

AN OPEN INVITATION: Sandia Fracture Challenge

- The challenge scenario involves predicting deformation, crack nucleation, and propagation in a common engineering alloy.
- The challenge was issued by e-mail and posted on imechanica.org on May 15th, with predictions due 5pm EST Sept 15th.
- Any research institution can participate.
- Participants are offered an invited talk at a Symposium on Ductile Fracture at ASME IMECE 2012 in November in Houston to present their methodology.
- Participants can request to have their predictions be anonymous.

To confirm participation, please e-mail Brad Boyce blboyce@sandia.gov

The Sandia Fracture Challenge



The alloy. Precipitation hardened stainless steel, alloy 15-5 PH, purchased from AK-Steel and heat treated at 1100 °F for 4 hours. Resulting tensile data (included in subsequent slides) indicates properties that are consistent with the H1075 or H1100 condition.

The geometry. A sheet of nominally 0.125 inch thick (actual 0.124 inch thick) 15-5 PH was machined into the challenge geometry as well as tensile bars, fracture toughness samples, and metallurgical coupons. The challenge geometry, shown on the left, includes detailed engineering drawings in the subsequent slides.

The challenge. When loaded at room temperature at a loading rate of 0.0005 inches/sec,

(Q1) What is the force and COD displacement¹ at which a crack first initiates²?

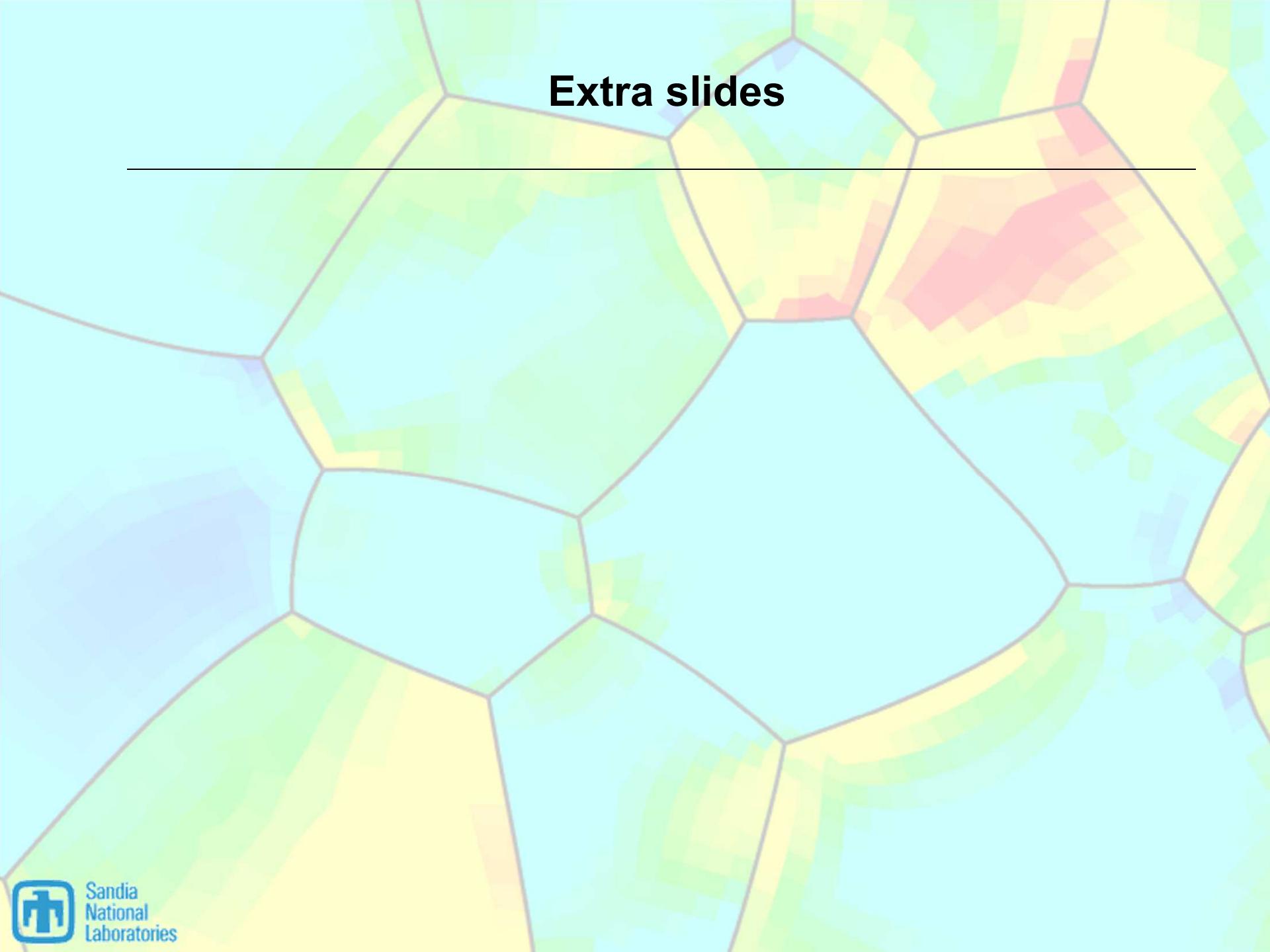
(Q2) The starter notch, A, holes B-D, and the backside edge, E are labeled in the drawing. What is the path of crack propagation? i.e. a crack that initiated on the backside and propagated to hole D and then to notch A would be labeled "E-D-A".

(Q3) If the crack does propagate to either holes B,C, or D, at what force and COD displacement does the crack re-initiate out of the first hole?

¹COD: A Crack opening displacement (COD) gage will be used to monitor load-line displacement at the point of the 'knife-edge' features, akin to fracture toughness testing. Only Δ COD will be measured (the test will begin with COD=0 inches).

²For the purposes of this challenge, crack initiation will be defined as a crack $\geq 100\mu\text{m}$ on the sidewall surface of the sample, so as to be witnessed by in-situ microscope.

Extra slides

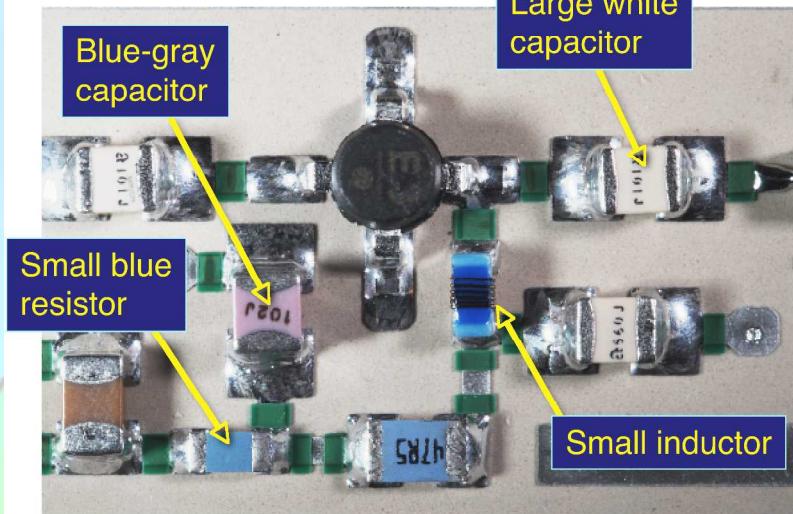


How can we add predictivity?

- The Round Robin results showed that crack initiation precedes failure, often by thousands of cycles (**blue** line).
⇒ *Our model was extended and validated to include cracking to open circuit.*

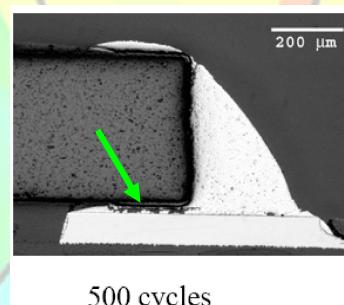
Test vehicle
accelerated aging
conditions

-55°C ... 125°C, 20 min holds;
0, 500, 1000, and 1500 cycles



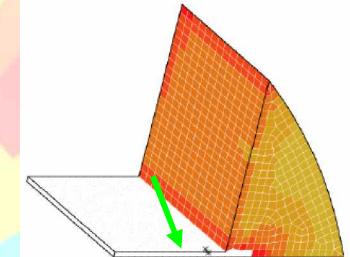
Small blue resistor:

experiment

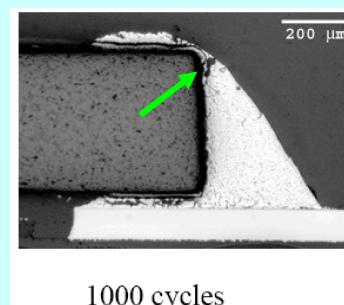


500 cycles

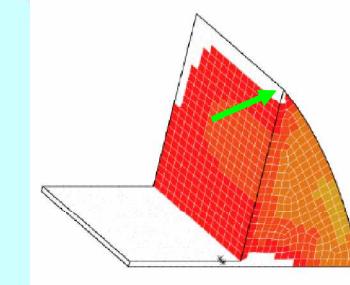
model



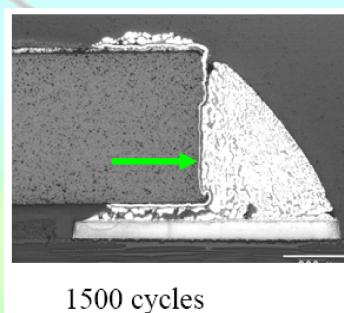
500 cycles



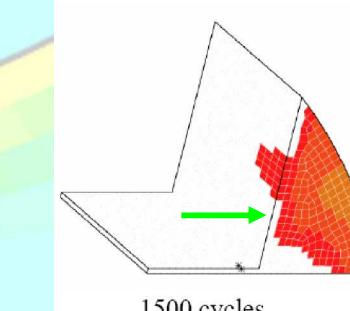
1000 cycles



1000 cycles



1500 cycles

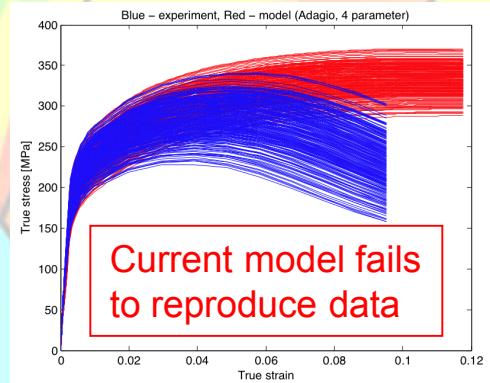
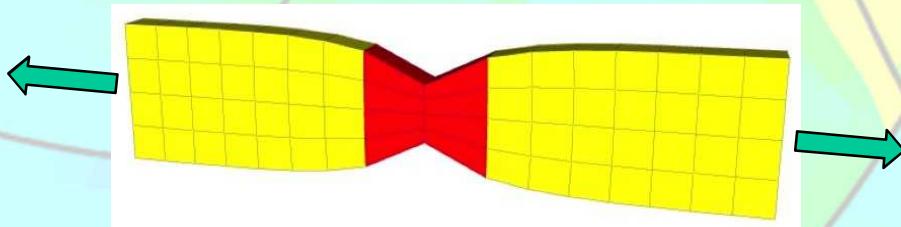


1500 cycles

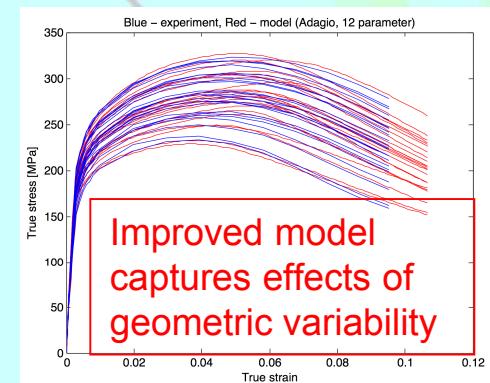
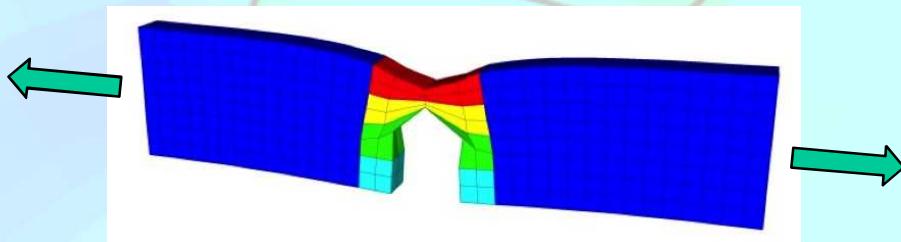
White elements are cracks.

Framework to incorporate variability in weld models: Geometric variability

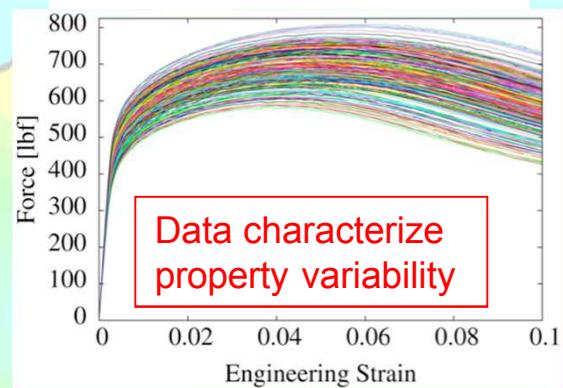
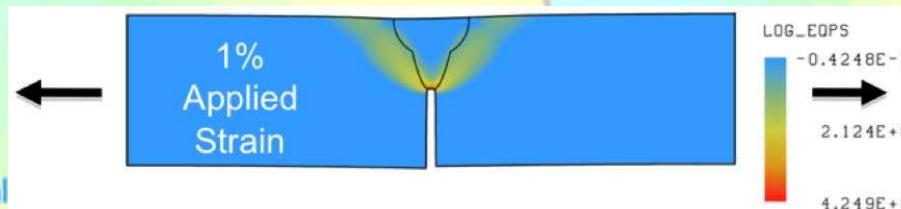
Current design approach: Use a single-material model with 4 parameters; 8 elements per cross-section.



Bridging the gap: Homogenize to 3 material model with 12 parameters; 8 elements with notch per cross-section

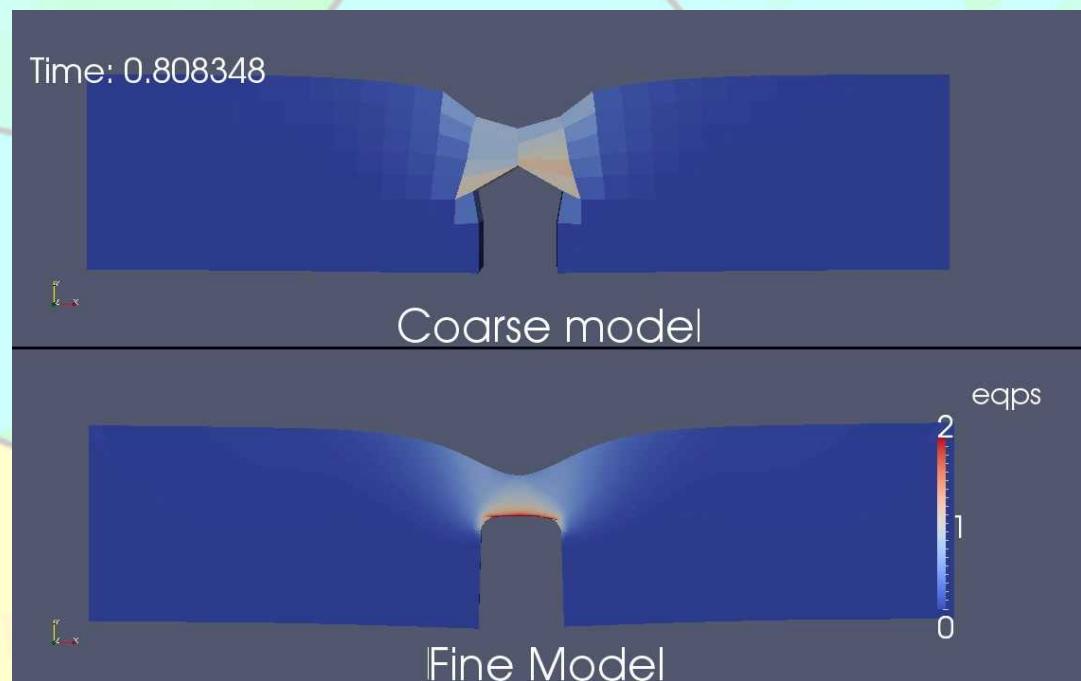
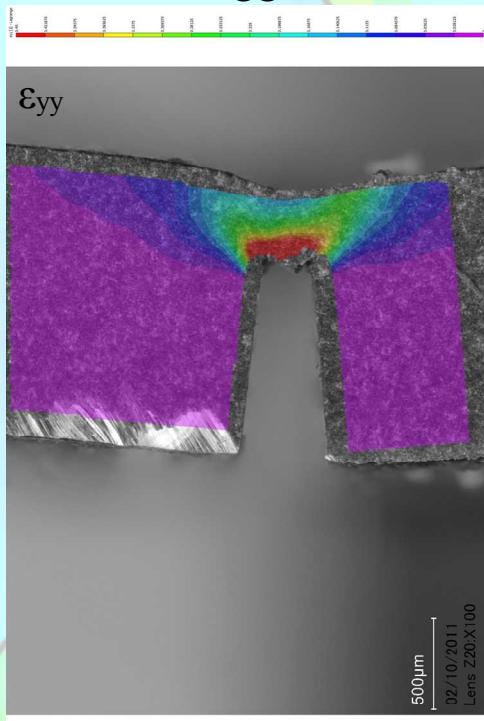


Materials science approach: Computational survey of weld geometries, varying weld depth, plate gap and offset



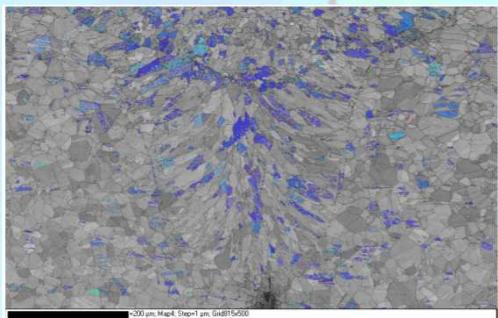
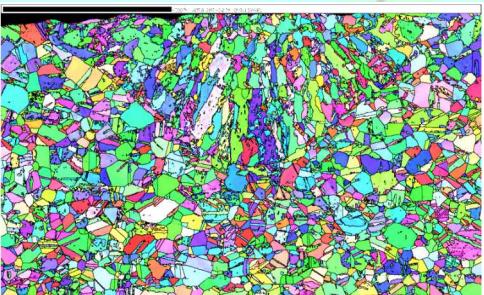
Experiments validate and inform weld models

- Digital image correlation (DIC) experiments track strains at the micron-scale during a tensile test of a 304L stainless steel weld.
 - Reveal deformation mechanisms
 - Validate computational results
 - Suggest modeling approaches

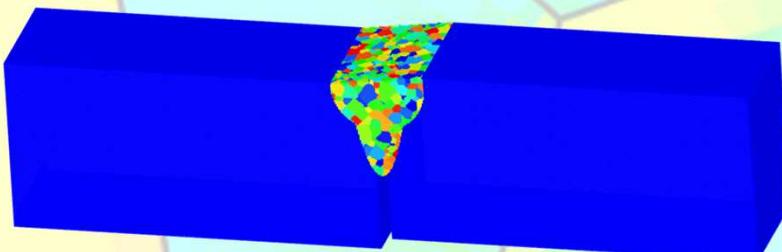
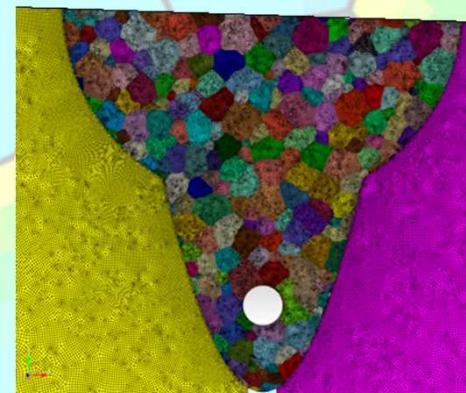
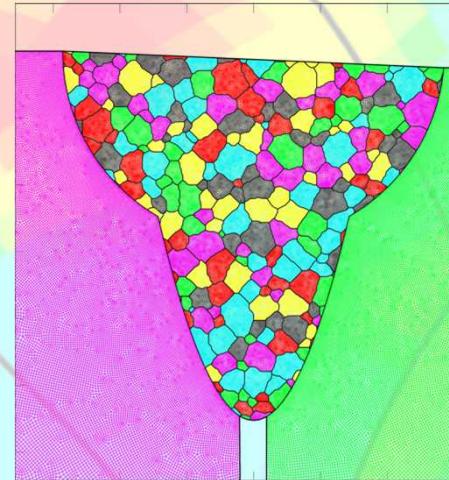


Putting it all together: Adding grains to the mix

- The current model includes geometry and pores; the final addition will be grain structure.
- We will utilize the same paradigm:

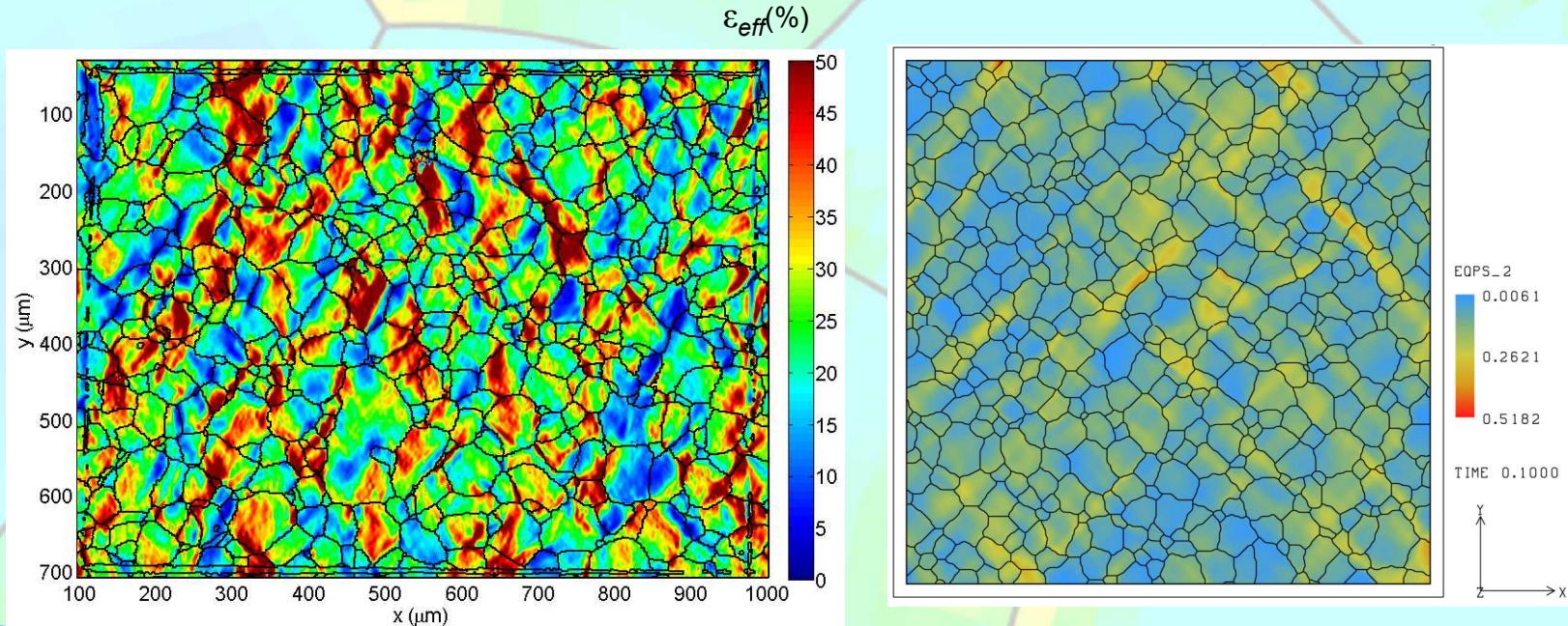


Experimentally-inspired
computational survey of microstructures
↓
Homogenize results
↓
Deploy engineering model



DIC observations validate BCC plasticity models

- Digital image correlation (DIC) coupled with electron-backscatter diffraction microscopy (EBSD) provide strain maps that can validate microstructural results in Ta polycrystals:
 - Reveal characteristic features, distributions, and trends.
 - Validate simulation results (i.e. strain bands at 45° to the tensile axis)
 - Suggest areas for future study (i.e. additional slip modes)



National
Laboratories

J. Carroll, B. Boyce, C. Battaile

Four teams were chosen to represent the breadth of Sandia failure modeling approaches.

Team	<u>Numerical Implementation</u>	<u>Crack Physics</u>	<u>Brief Description of Key Model Attributes</u>	<u>Failure Equation</u>
<i>Tearing Parameter</i>	K. Dion, G. Wellman	Crack Band FEA	<i>Tearing parameter with critical crack opening strain</i>	$T = \int_0^{\bar{\varepsilon}_f} \left\langle \frac{2\sigma_T}{3(\sigma_T - \sigma_m)} \right\rangle^4 d\bar{\varepsilon}$
<i>Localization Elements</i>	J. Foulk, A. Mota, J. Emery, J. Ostien	Localization Elements	<i>BJC_mem damage model with Cocks-Ashby Void Growth</i>	A BCJ damage model is implemented in a regularized subgrid describing surface elements at a crack. $\dot{\phi} = \left\{ \frac{1}{(1-\phi)^m} - (1-\phi) \right\} \sinh \left[\frac{2(2m-1)}{2m+1} \frac{\langle \sigma_h \rangle}{\bar{\sigma}} \right] \dot{\varepsilon}_p$
<i>Peridynamics</i>	J. Foster, J. Bishop, S. Silling, D. Littlewood	Peridynamics	<i>Critical Stretch</i>	Bond-node based meshless reformulation of continuum mechanics, particularly suitable for discontinuous displacement fields. $w_\xi = \int_0^{\eta(t_{final})} \{ \underline{\mathbf{T}}[\mathbf{x}, t] \langle \mathbf{x}' - \mathbf{x} \rangle - \underline{\mathbf{T}}[\mathbf{x}', t] \langle \mathbf{x} - \mathbf{x}' \rangle \} \cdot d\eta$
<i>XFEM</i>	J. Cox, D. Littlewood, B. Spencer	XFEM	<i>Max Princ. Stress, EQPS, tearing parameter</i>	Crack-like asymptotic displacement fields and discontinuities enrich the finite element approximation. No explicit meshing of crack surfaces is needed. $\dot{\bar{\varepsilon}}^p = \sqrt{\frac{2}{3} \dot{\varepsilon}_{ij}^p \dot{\varepsilon}_{ij}^p} = \sqrt{\frac{2}{3} \left\{ (\dot{\varepsilon}_{xx}^p)^2 + (\dot{\varepsilon}_{yy}^p)^2 + (\dot{\varepsilon}_{zz}^p)^2 + 2 \left[(\dot{\varepsilon}_{xy}^p)^2 + (\dot{\varepsilon}_{xz}^p)^2 + (\dot{\varepsilon}_{yz}^p)^2 \right] \right\}^{1/2}}$