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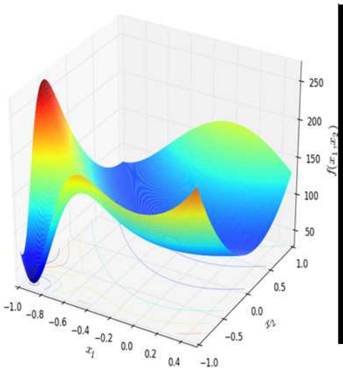

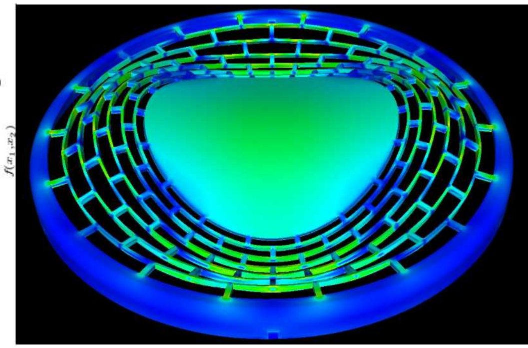

A Second-Order Accurate Mathematical Optimization Algorithm Based on Gradient Information

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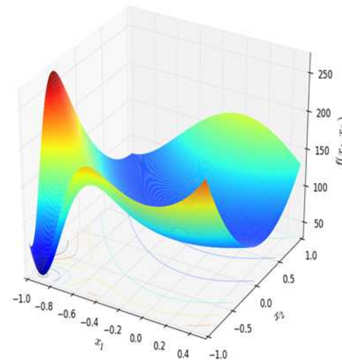
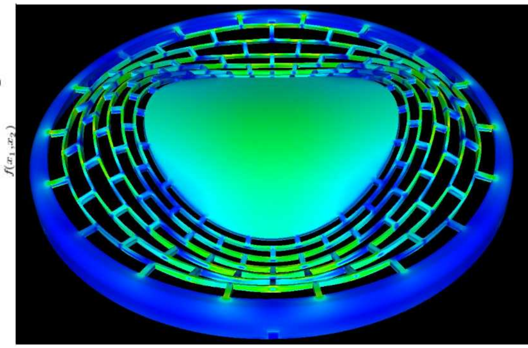
Agenda

- Motivation
- Proposed Methodology
- Optimization Algorithm
  - Pseudocode
  - Hessian Information Approximation
- Examples
- Future Research Directions
  - Conclusions





Motivation

Motivation

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- **Optimization Algorithmic Research**
  - Reduce computational expense of topology optimization problems
  - Minimize tuning parameter
  - Investigate effectiveness of multiple optimization algorithms in the context of topology optimization
- **Questions**
  - Can second-order information reduce computational expense of topology optimization problems?
  - Can we minimize the computational cost associated with computing second-order information?

Proposed Methodology

- **Objective:**
  - Develop a general optimization algorithm that enables the computation of *second-order (i.e. Hessian)* information from the available *first-order (i.e. gradient)* information
- **Requirements:**
  - Second-order derivative operators are not required
  - Computational cost does not increase with the number of design variables
  - Computational cost is the same as solving the topology optimization problem with just gradient information

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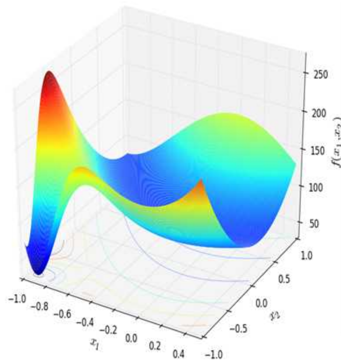

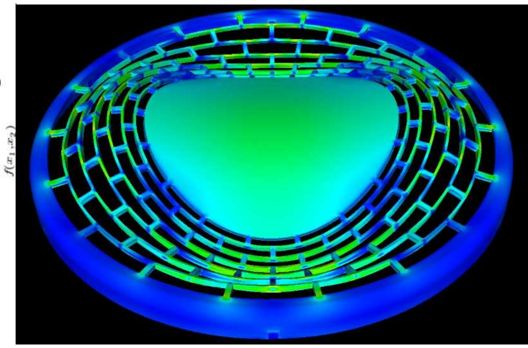

Optimization Algorithm

Algorithm Pseudocode

Outer Loop

Set initial $x_{i=0}$

Compute $J(x_{i=0})$

while(not converge)

{

Solve Sub-problem

Check convergence

i=i+1

}

Legend

x = Control

J = Objective function

G = Gradient

H = Hessian

d = Trial step

Line Search Sub-problem

{

Compute $G(x_i)$

Apply filter $F(G_i)$

Solve $H(x_i)d_i=-G(x_i)$

Line search scaling s_i

$x_{i+1} = x_i + s_i d_i$

}

Trust Region Sub-problem

while(TR criteria met)

{

Compute $G(x_i)$

Apply filter $F(G_i)$

Solve $H(x_i)\bar{d}_i=-G(x_i)$, $\bar{d}_i=s_i d_i$

Check TR sub-problem convergence

- $x_{i+1} = x_i + \bar{d}_i$

}

Hessian Approximation: Finite Differences (FDiff)

Finite Difference

Given h > 0, the d^{th} order derivative of an univariate function f(x) can be approximated by finite differences as follows:

Forward $$\frac{h^d}{d!}F^d(x) + O(h^{d+p}) = \sum_{i=i_{\min}}^{i_{\max}} C_i F(x+ih)$$

Backward $$(-1)^d \frac{h^d}{d!}F^d(x) + O(h^{d+p}) = (-1)^d \sum_{i=i_{\min}}^{i_{\max}} C_i F(x+ih)$$

Centered = *Forward - Backward*

where p > 0 denotes the order of accuracy, i_{min} and i_{max} denote some choice of extreme indices , and coefficients C_i

Cont: Finite Difference

Example

Given gradient operator $G(x_i)$, approximate the application of the trial step to the Hessian operator, i.e. $H_i(x_i)d_i$ through finite differences as follow

Backward $$G'(x_i) = \frac{3G(x_i) - 4G(x_i - hd_i) + G(x_i - 2hd_i)}{2h} + O(h^2)$$

Key Features

- Full Hessian operator is not assemble
  - Interested in the application of the trial step to the Hessian operator
- Each evaluation is independent and thus parallelizable
  - Task parallelism can be effectively employ to expedite computation

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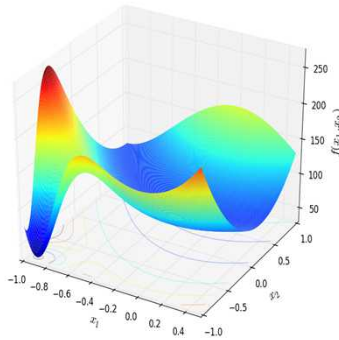

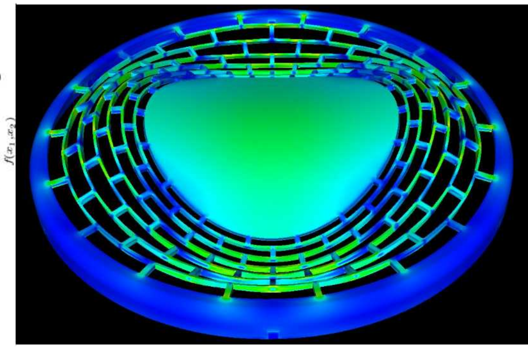

Examples

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Case Study: Compliance Minimization

Optimality Criteria (OC) Formulation

$$\min \quad \frac{1}{2}\mathbf{u}^\intercal(z^p\mathbf{K})\mathbf{u} + R(z)$$

$$s.t. \quad (z^p\mathbf{K})\mathbf{u} - f = 0$$

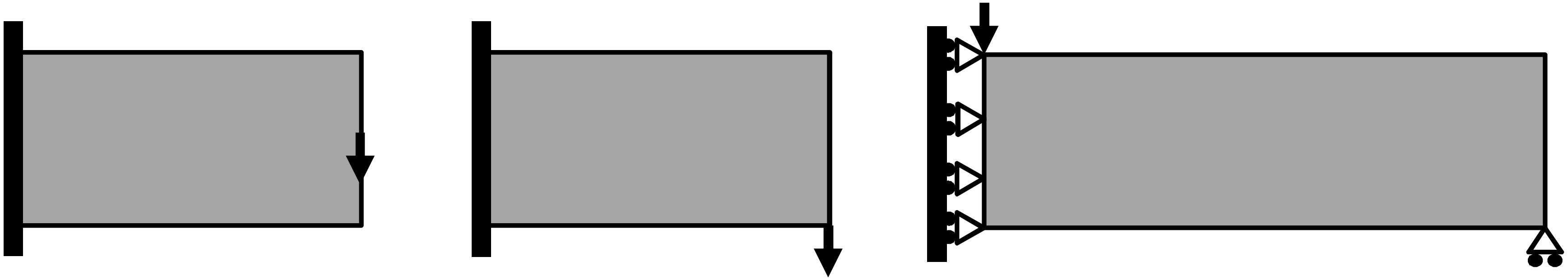
$$\frac{V(z)}{V_o} \leq \gamma$$

Nonlinear Programming Solver's Formulation

$$\min \quad \frac{1}{2}\mathbf{u}^\intercal(z^p\mathbf{K})\mathbf{u} + R(z) + \frac{1}{2}(V(z) - \gamma V_o)^2$$

$$s.t. \quad (z^p\mathbf{K})\mathbf{u} - f = 0$$

Test Problems



Mitchell Cantilever MBB

Example 1: 60x40 FE Mesh

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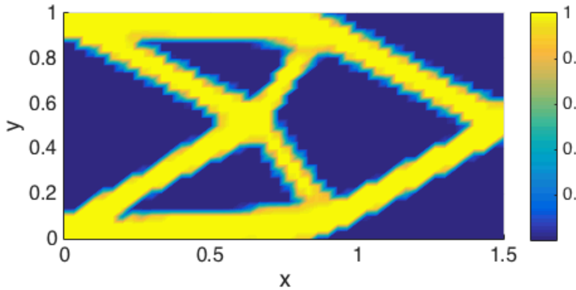
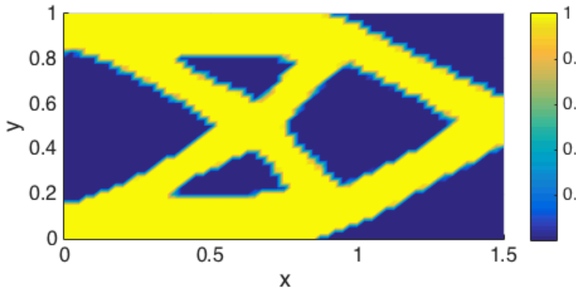
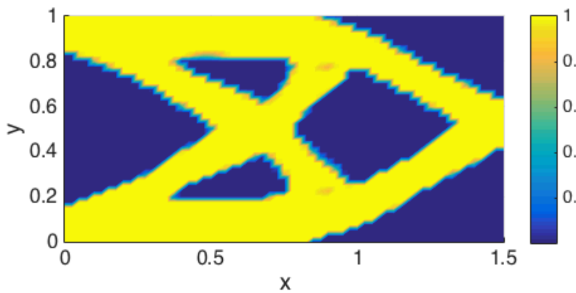
OC



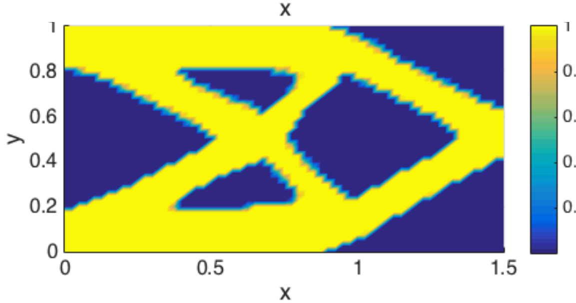
Liu-Storey

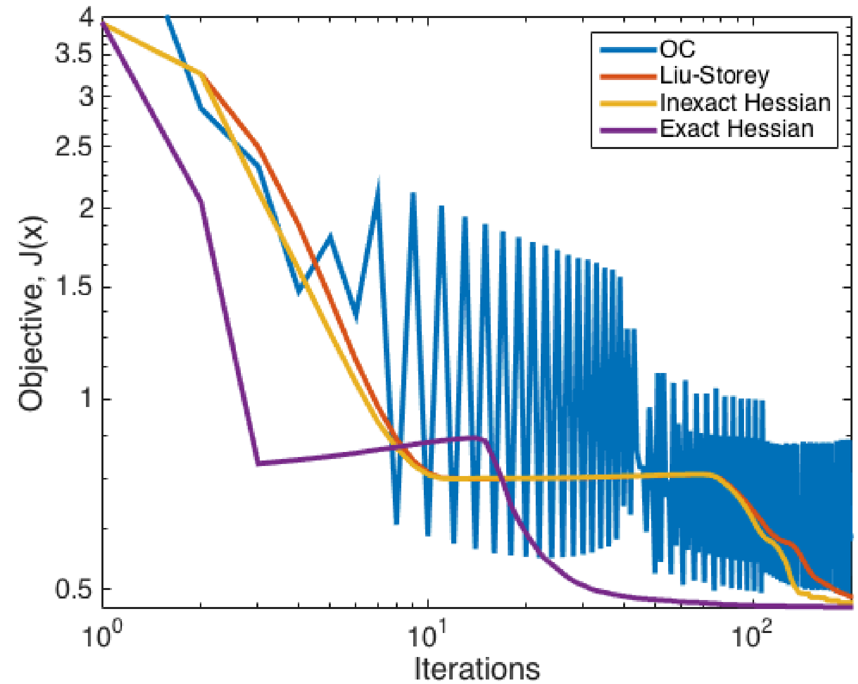


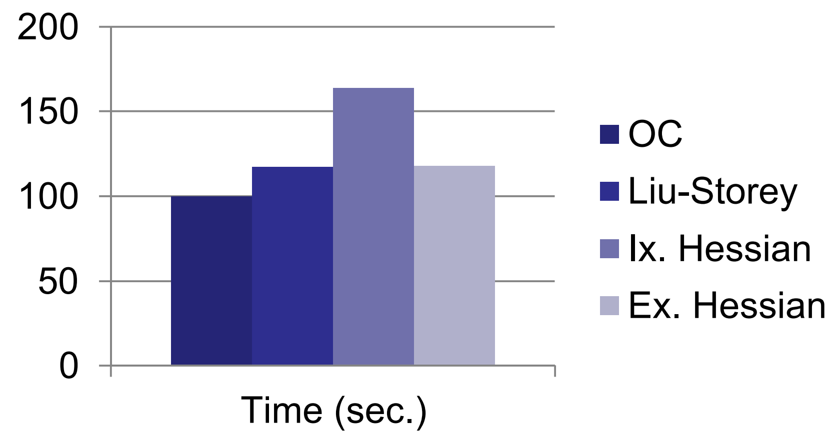
Ix. Hessian



Ex. Hessian

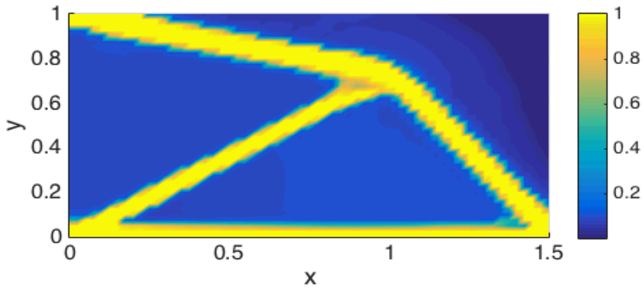






Example 2: 60x40 FE Mesh

OC



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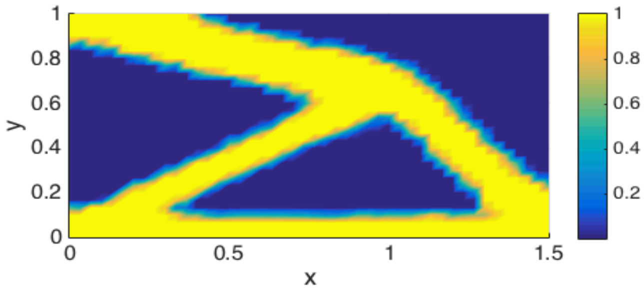
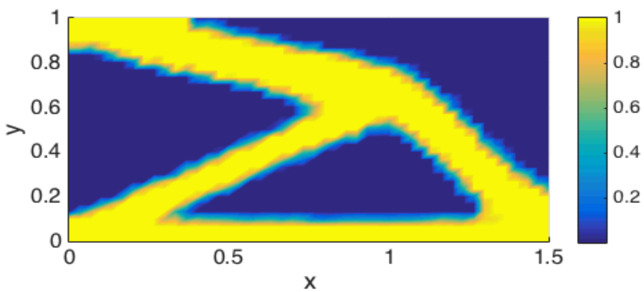
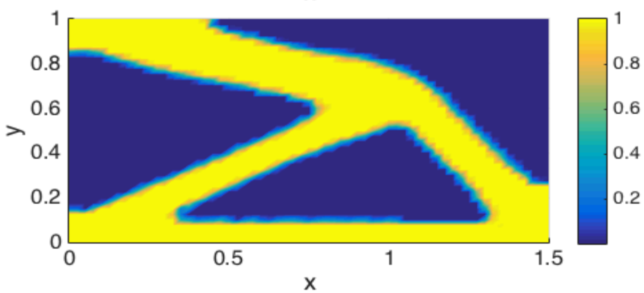
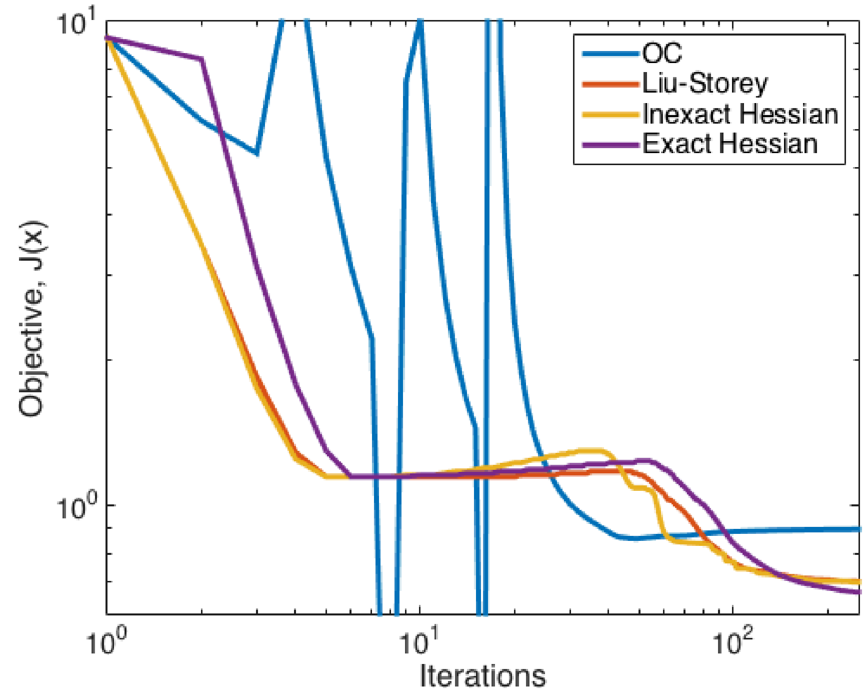


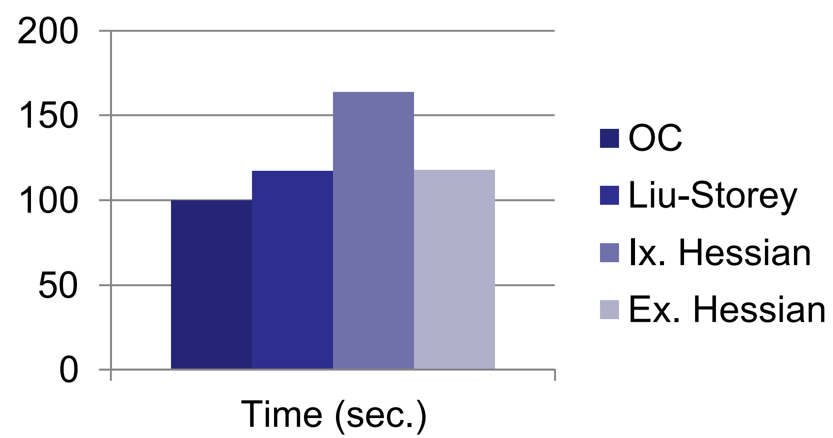
Ix. Hessian



Ex. Hessian

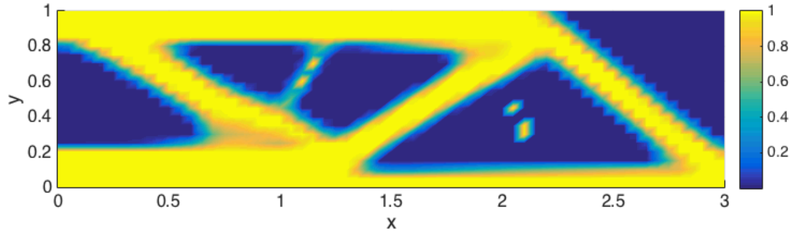






Example 3 (60x20)

OC



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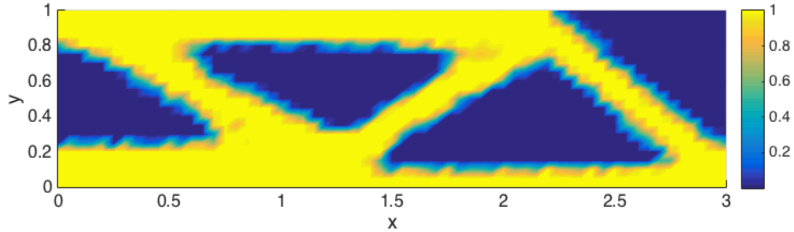
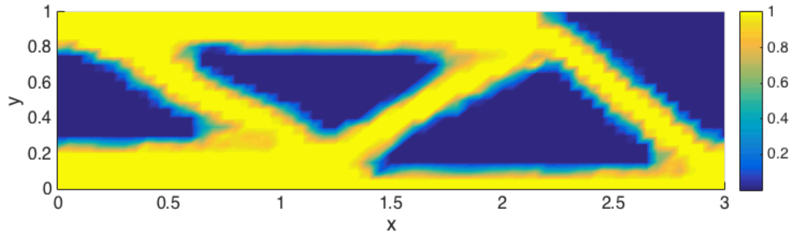
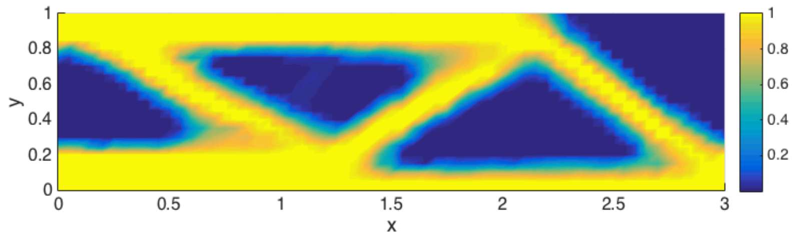
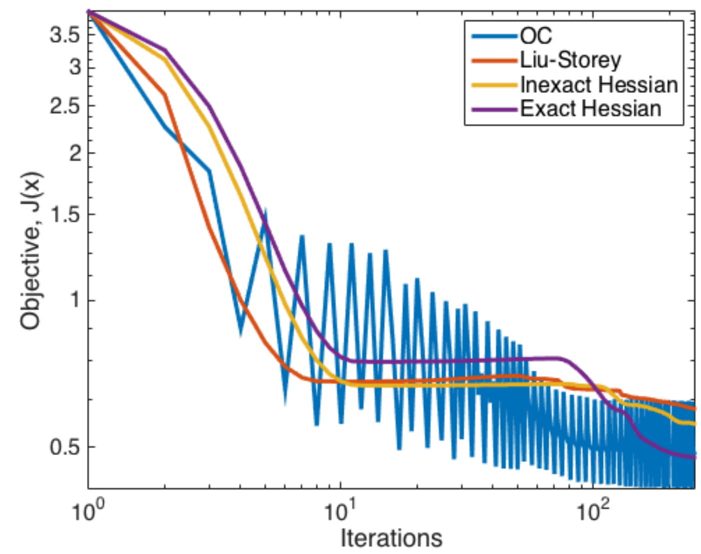


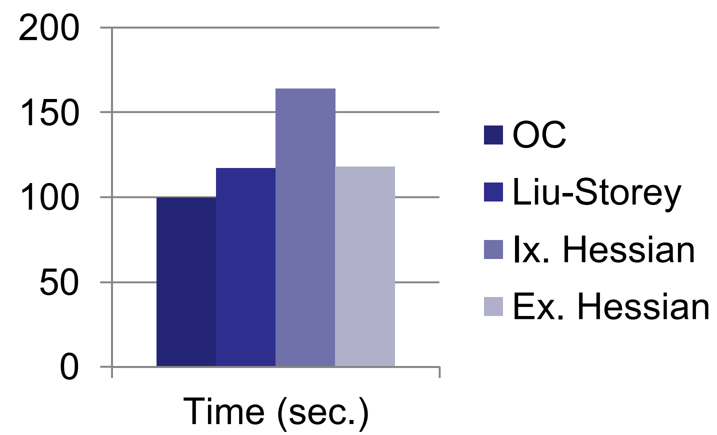
Ix. Hessian



Ex. Hessian







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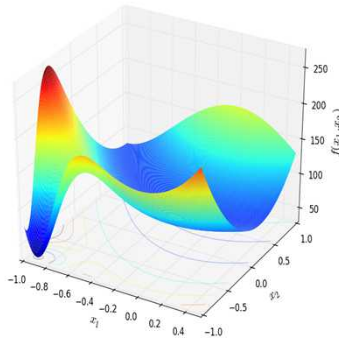

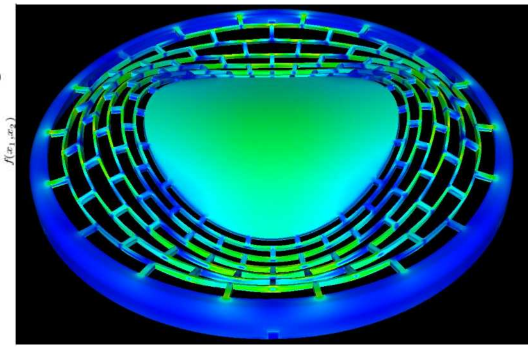

Future Research Directions

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Final Remarks

- **Additional tasks**
  - Compare performance against MMA and GCMMA algorithms
  - Exercise formulation based on filters and stress minimization problems
- **Algorithmic Improvements**
  - Research in primal-dual interior-point methods
  - Explore task parallelism of solves needed for inexact Hessian computation
  - Reduced order model based optimization algorithm
    - Allow optimization algorithm to automatically generate and update reduced order model for physics