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A Survey of Techniques to Estimate the Uncertainty in Material Parameters

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Outline

- Motivation
- Experimental Data
- Model Discussion
- Optimization Results
- Bayesian Results
- Comparisons
- Conclusions

Motivation

- Material parameter uncertainty needed for
  - Model validation
  - Uncertainty quantification
  - Quantifying margin and margin uncertainty
- Typically uncertainty in parameters is "estimated"
  - Assume Gaussian or uniform distribution
  - Bounds based upon engineering judgment
  - Little justification available
- Need a quantified and defendable uncertainty estimate for material parameters

Experiment Description

- Foam samples 1.5 inches by 0.5 inches thick
- Nine total samples
- A steel ball baring dropped on sample
- Acoustic emissions measured and analyzed for three natural frequencies
- Density estimated by measuring dimensions of sample and weighing.

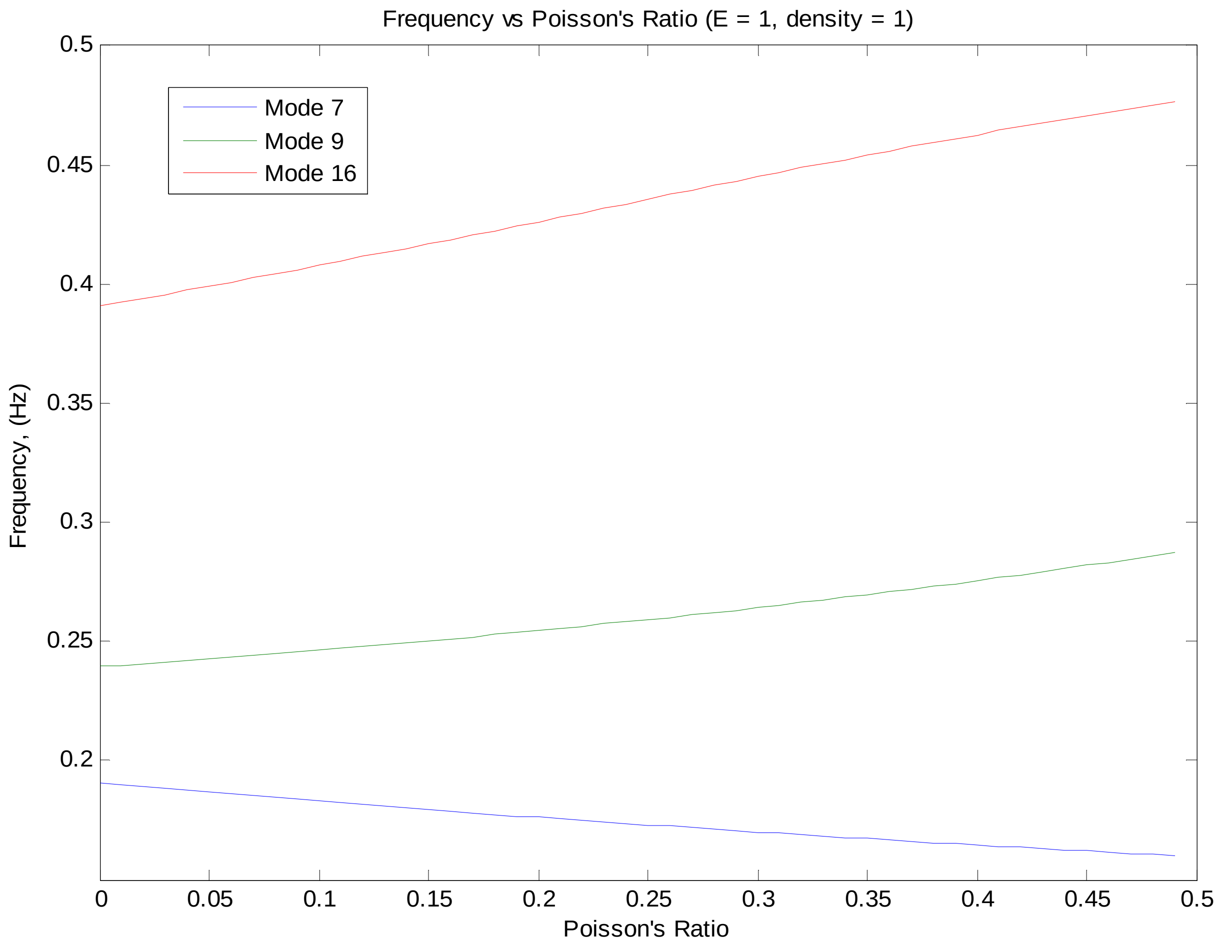
Model Description

- Two models developed
  - Finite Element
  - Physically based surrogate model

$$f_i = \sqrt{\frac{E}{\rho}} g_i(\nu)$$

  - Frequency is a function of modulus, density, and a function of Poisson's ratio
  - The function of Poisson's ratio is determined from the FE model
    - Modulus, density set to one
    - Vary Poisson's ratio and save resulting frequencies.

Frequency vs. Poisson's Ratio



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Optimization Procedure

- Four cost functions used
  - Least squares on *two* frequencies
    - All samples used in single optimization
    - Single Modulus and Poisson's ratio identified
  - Least squares on *two* frequencies
    - Performed on each sample individually
    - Nine different pairs of modulus and Poisson's ratio identified
  - Least squares on *three* frequencies
    - All samples used in single optimization
    - Single Modulus and Poisson's ratio identified
  - Least squares on *three* frequencies
    - Performed on each sample individually
    - Nine different pairs of modulus and Poisson's ratio identified

Bayesian Estimation

- Bayesian estimation assumes that the estimated parameters are random rather than deterministic.
  - The optimization techniques presented here assumes the parameters are deterministic but unknown
  - Can incorporate prior knowledge into the parameter estimates
- Bayesian estimation has been around for a while
  - Not practical until computers became fast
  - Markov Chain Monte Carlo (MCMC) has made the method feasible

Basic Idea of Bayesian Updating

- Statement of Bayes Theorem

$$p(\theta|\delta) = \frac{p(\delta|\theta)p(\theta)}{p(\delta)}$$

- Need to define two quantities
  - Likelihood function (first term on right hand side)
  - Prior distribution (second term on right hand side)
- Likelihood function acts like a cost function
  - Includes an assumption on the form of the conditional distribution of δ (data)
- Prior incorporates any previous or expert knowledge
  - Can be uniform if no knowledge is available other than bounds

Define Likelihood and Priors

- Likelihood function (Gaussian, in this case)

$$p(\delta | f,\theta) = \prod_{i=1}^{9} \exp\left(\frac{1}{2\sigma^2} \left(\left(\frac{f_i^7 - f_{calc}^7}{f_{calc}^7} \right)^2 + \left(\frac{f_i^9 - f_{calc}^9}{f_{calc}^9} \right)^2 \right) \right)$$

- Both two and three frequencies used in likelihood function
- Parameters to be updated (θ)
  - Modulus, Poisson's ratio
  - Parameters are in the f_{calc} variable.
  - "Variance" (σ)
- Prior is assumed to be uniform
  - No previous knowledge or expectations for parameters other than bounds.

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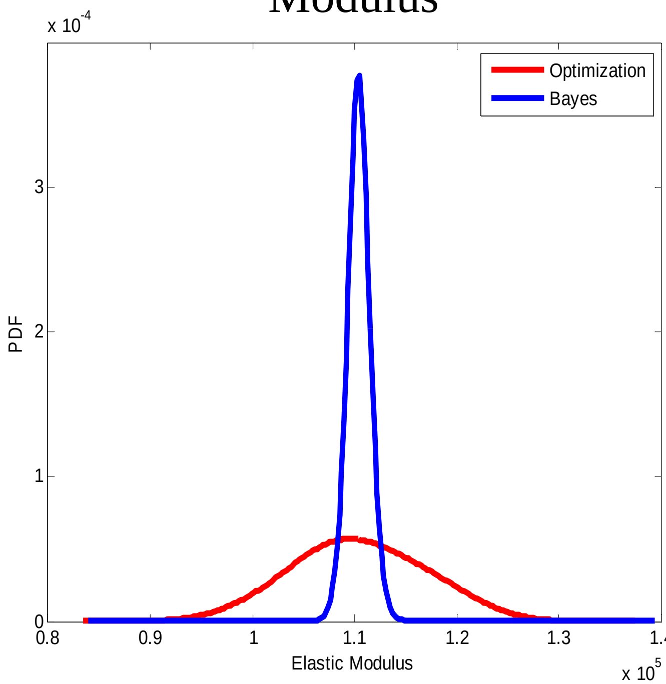
Updating is Performed Using MCMC

- Produces samples that converges to the posterior distribution
- Technique is dependent on the previous value
- Does not require proper probability density functions
  - Proper normalization of the PDF is not necessary
  - Normalization cancels out in MCMC formulation
  - This is the property that makes MCMC attractive with Bayes
- Issues
  - Resulting string of parameters are correlated to some extent
  - Requires large number (1000s) of samples to converge to the posterior distribution
  - Large number of initial samples are invalid (prior to convergence of the chain)

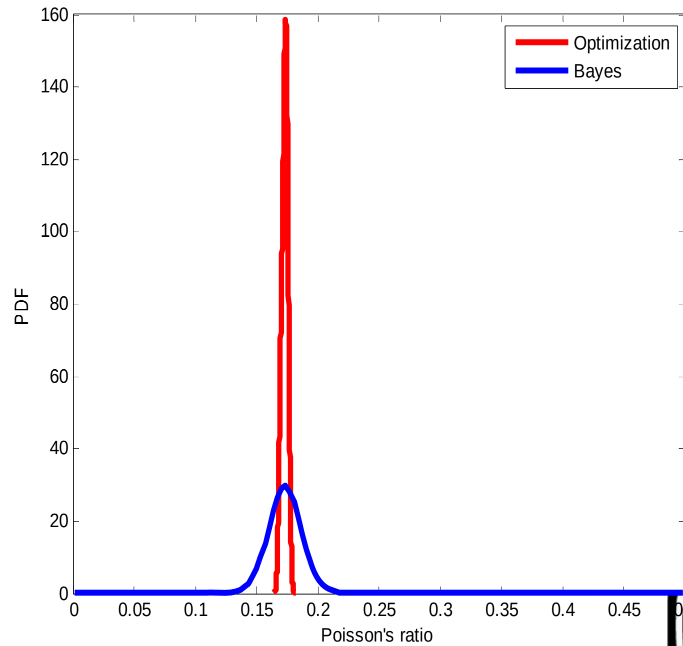
Two Frequency Comparison

- Distributions made from individual sample optimization and MCMC simulations
- Means are approximately the same
- Variance is different (not much other conclusions)

Modulus



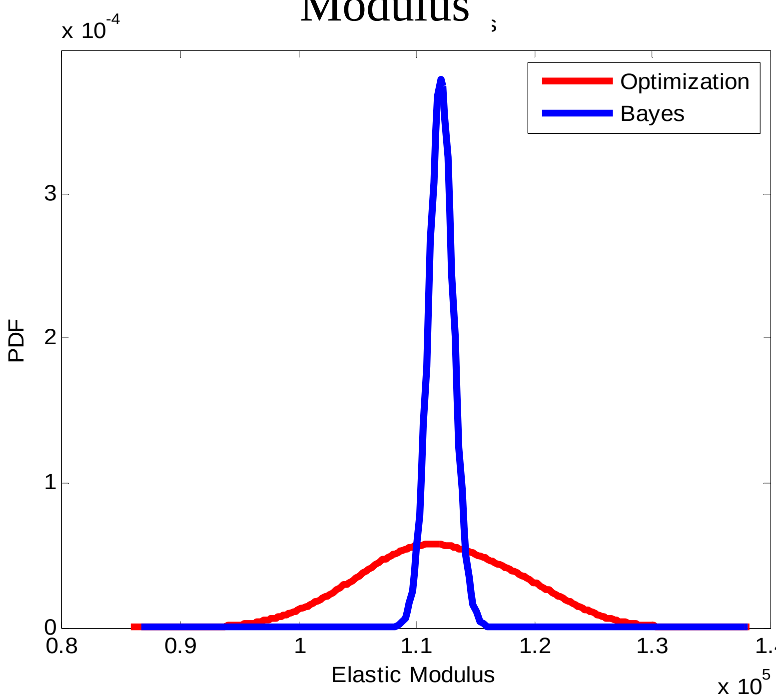
Poisson's Ratio



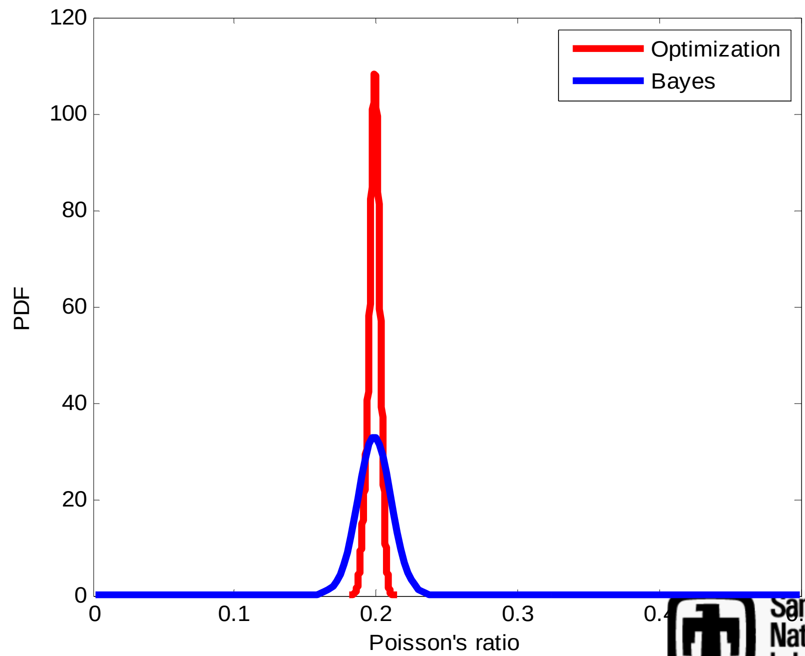
Three Frequency Comparison

- Distributions made from individual sample optimization and MCMC simulations
- Means are approximately the same
- Variance is different (not much other conclusions)

Modulus



Poisson's Ratio



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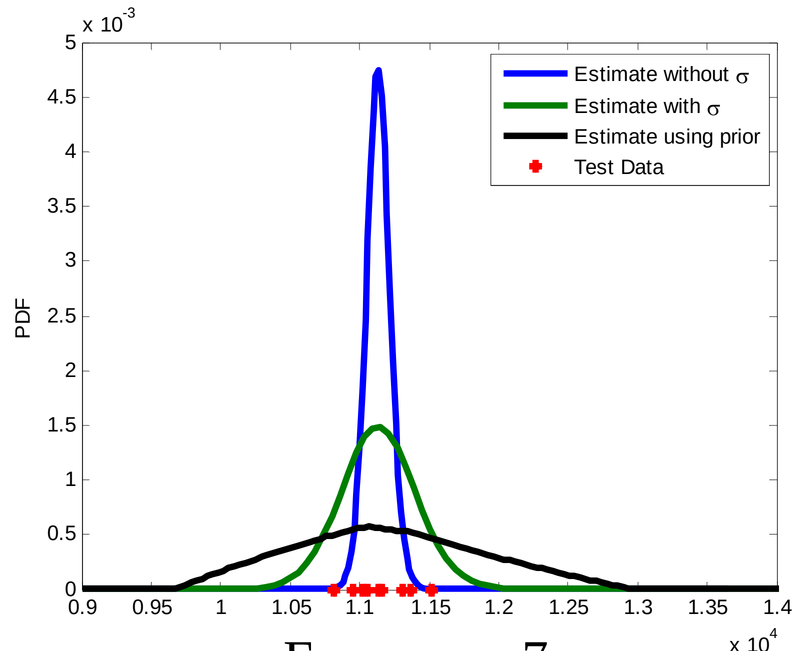
Forward Propagation of Posterior Distribution from Bayes Updating



Frequency 7

- All distributions shown above are from same data set
- Can analyze results by calculating the resulting frequencies from the ensemble of estimated parameters
- Forward propagation with error term should encompass data
- Note width of estimate from prior

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Conclusions

- Optimization and Bayes updating are used to estimate elastic parameters
- Differences between number of frequencies used are larger than differences between techniques
- No obvious conclusions can be drawn between trends between the different techniques
- Bayes provides uncertainty directly
- Optimization requires additional effort to estimate uncertainty.