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Sensitivity Analysis Techniques for System Dynamics Models of Human Behavior

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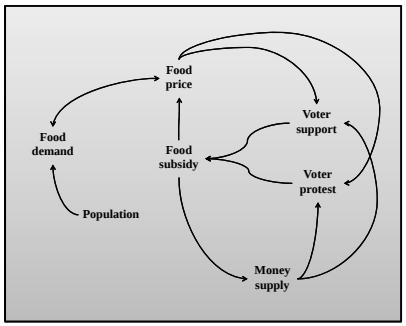
Sensitivity Analysis

The goal of this work was to gain insight into which sensitivity analysis techniques are most appropriate for models that simulate human behavior. **Sensitivity analysis determines which model inputs have the largest impact on model response**. The results of a sensitivity analysis can be used to:

- Identify where data collection resources should be directed
- Find leverage points
- Understand model robustness
- Find areas where a model can be simplified

Food Subsidy Model



The example model used for this study is a system dynamics model that simulates government and voter behavior. The government tries to earn support by implementing a food subsidy. If oil revenues are insufficient to pay for the subsidy, the government must print money. When the government prints money, inflation increases, which decreases voter satisfaction.

The model includes 12 uncertain inputs, shown in the table below.

Variable	Distribution	Details
Expected Voter Protests (EVP)	Uniform	[0.05,0.15]
Expected Voter Support (EVS)	Uniform	[0.6,0.8]
Oil Price (OP)	Log-normal	μ=4, σ=0.55
Price Adjustment (PA) Fraction of indicated change in price	Uniform	[0.05,0.5]
Food Demand β (FDβ) How much food price affects demand	Uniform	[0,10]
Government Food Subsidy β (GFSβ) How much voter support affects GFS	Uniform	[0,5]
Government Food Subsidy γ (GFSγ) How much voter protest affects GFS	Uniform	[0,10]
Voter protest β (VPβ) How much food price affects protests	Uniform	[-10,0]
Voter protest γ (VPγ) How much general prices affect protests	Uniform	[-10,-1]
Voter support β (VSβ) How much food price affects support	Uniform	[0,10]
Voter support δ (VSδ) How much protests affect support	Uniform	[0,10]
Voter support γ (VSγ) How much general prices affect support	Uniform	[0,10]

Implementation of Methods

Different promising methods were applied to the food subsidy model, tested, and compared.

- Scatterplots
- Correlation coefficients
- Stepwise regression
- Elementary effects
- Sensitivity indices

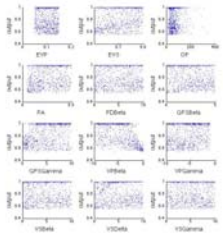
While different outputs are certainly of interest in this model, results presented here focus on one output: voter support. The metrics used were:

- Static sensitivity analyses: highest value of voter support over the time horizon
- Dynamic sensitivity analyses: voter support at each point throughout the time horizon

Each sensitivity analysis described here used a sample size of 1,000, except sensitivity indices for which N=10,000.

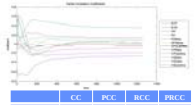
Scatterplots

- Used to look for patterns
  - Especially unusual/unanticipated patterns (thresholds, etc.)
- Results
  - Patterns are apparent for some inputs
  - No inputs are obviously dominant



Correlation Coefficients

- Strength of linear relationship
- Variations
  - Partial: corrects for linear effects of other inputs
  - Rank: monotonic (rather than linear) relationships
- Varies from -1 to 1
- Small p-value indicates significance



	CC	PCC	RCC	PRCC
EVP	-0.0115	-0.0411	-0.0194	-0.0558
EVS	0.2108	0.3630	0.2126	0.3753
OP	-0.0860	-0.0456	-0.0784	-0.0731
PA	0.2875	0.4079	0.3235	0.4681
FDBeta	-0.0216	-0.0348	-0.0144	-0.0332
GFSBeta	-0.1399	-0.1609	-0.1377	-0.1647
GFSGamma	0.3825	0.5483	0.3648	0.5563
VPBeta	-0.5191	-0.6457	-0.5284	-0.6758
VPGamma	0.1604	0.2199	0.1494	0.2185
VSBeta	0.1026	0.1580	0.1577	0.2481
VSDelta	0.0209	0.0705	0.1011	0.2012
VSGamma	-0.0085	-0.0205	-0.0214	-0.0495

Stepwise Regression

- Creates linear regression model by repeatedly adding the most important variable
- Excludes insignificant inputs
- R squared measures how much of the output variance is explained by the regression model

Step	Variable	Regression Coefficient	R squared
1	GFSGamma	0.0254	0.1998
2	EVS	0.6017	0.2605
3	VPBeta	-0.0289	0.5314
4	PA	0.2663	0.5743
5	VPGamma	0.0074	0.5907
6	GFSBeta	-0.0103	0.6001
7	VSBeta	0.0050	0.6080
8	VSDelta	0.0032	0.6113
9	EVP	-0.2453	0.6134

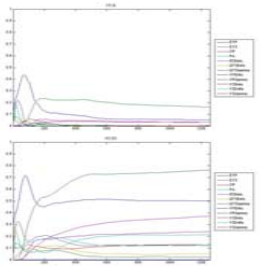
Elementary Effects

- Measures average difference in output when one input is perturbed
- Similar to a derivative
- μ* = average of absolute values of derivatives over the input domain

	Mu	Mustar	Sigmaq
EVP	0.0091	0.0915	0.0002
EVS	-0.0077	0.0855	0.0006
OP	-0.0038	0.0809	0.0001
PA	-0.0003	0.0987	0.0014
FDBeta	0.0005	0.0916	0.0001
GFSBeta	-0.0031	0.0865	0.0002
GFSGamma	-0.0003	0.1127	0.0006
VPBeta	0.0071	0.1028	0.0006
VPGamma	0.0073	0.0915	0.0012
VSBeta	-0.0030	0.0870	0.0009
VSDelta	0.0080	0.1008	0.0003
VSGamma	-0.0054	0.0927	0.0006

Sensitivity Indices

- S_i: Main effect
  - Proportion of output variance that can be attributed to input (i)
- S_{Ti}: Total effect
  - Includes S_i plus variance attributed to i and its interactions
  - ex: $S_{T1} = S_1 + S_{12} + S_{13} + S_{123}$
- Can use to identify areas where reduced uncertainty would cut variance in output



	Si	STi
EVP	-0.0001	0.0005
EVS	0.0621	0.1210
OP	0.0000	0.0000
PA	0.0633	0.1247
FDBeta	0.0013	0.0013
GFSBeta	0.0112	0.1110
GFSGamma	0.2255	0.3628
VPBeta	0.3773	0.5154
VPGamma	0.0097	0.0381
VSBeta	0.0146	0.0577
VSDelta	0.0154	0.1142
VSGamma	0.0008	0.0107

Comparison of methods

No one, or few, inputs dominated the results of the food subsidy model. Interactions between inputs, however, did play a large role, particularly at the beginning of the simulation. The table below shows how the various methods would rank the uncertain inputs by relative importance.

	Correlation Coefficient	Partial Correlation Coefficient	Rank Correlation Coefficient	Partial Rank Correlation Coefficient	Stepwise Regression	Elementary Effects Mustar	Sensitivity Indices Si	Sensitivity Indices STi
EVS	1-4	1-7	1-4	1-8	2	11	4	4
PA	1-4	1-7	1-4	1-8	4	4	3	3
GFSGamma	1-4	1-7	1-4	1-8	1	1	2	2
VPBeta	1-4	1-7	1-4	1-8	3	2	1	1
VPGamma	5	1-7	7	1-8	5	8	8	8
VSBeta	6	1-7	5	1-8	7	9	6	7
GFSBeta	7	1-7	8	1-8	6	10	7	6
EVP	8	11	9	12	9	7	12	11
FDBeta	9	8	11	9	X	6	9	10
OP	10	10	12	11	X	12	11	12
VSGamma	11	12	10	10	X	5	10	9
VSDelta	12	9	6	1-8	8	3	5	5

Conclusion

Each of the methods implemented with the food subsidy model gave insight into the influence of the uncertain inputs. However, some of the methods were more useful than others. The table below describes the main points learned about each method from this investigation.

Method	What is measured	Comparison and implications
Scatterplots	Subjective relationship between inputs and outputs	•Good first method for identifying patterns •No very obvious patterns
Correlation Coefficients	Strength of linear (or monotonic) relationship	•Useful in ranking inputs •Results not that different for different types of CC
Stepwise Regression	Coefficients for linear model that best predicts output	•Most inputs were significant •Results similar to correlation coefficients
Elementary Effects	Average derivative when one input is perturbed over different points in its domain	•Little variation in μ* between inputs, but high between output times
Sensitivity Indices	Proportion of output variance attributed to input variance	•Interactions were significant, but not for all uncertain variables

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