SAND2011-4962C

On the Utility of the Boris Approach for Dealing with Fast Magnetosonic Wave Speeds in an ALE MHD Modeling Context

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Computational Shock and Multiphysics, 1443
Sandia National Laboratories

11th U.S. National Congress on Computational Mechanics
July 25-28, 2011
Minneapolis, Minnesota

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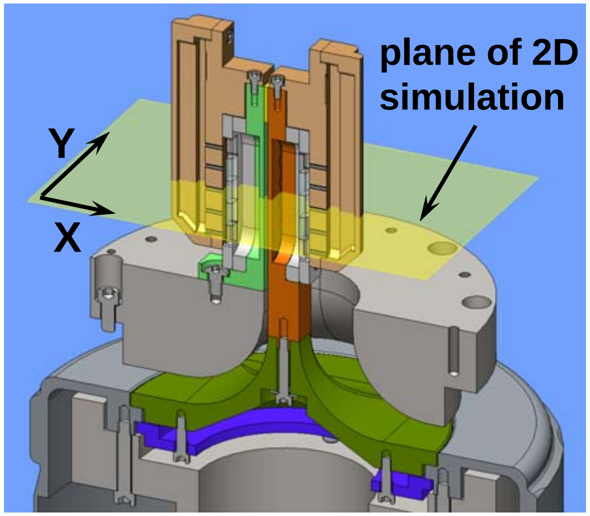
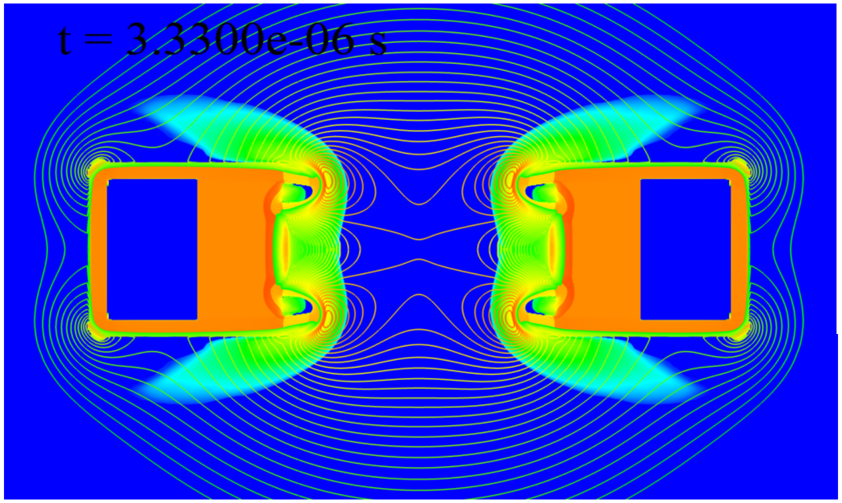
ALEGRA magnetohydrodynamic (MHD) modeling provides for predictive design of flyer plate experiments (Ray Lemke)

Two-sided Strip-line Flyer Plate Experiment



2D Simulation Plane of Two-sided Strip-line



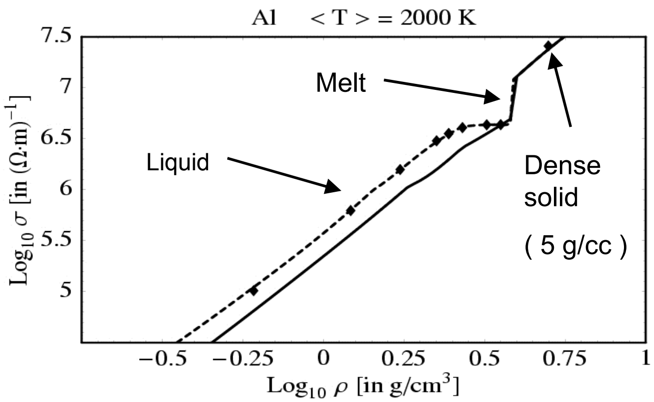
Resistive MHD

Accurate electrical conductivities (Desjarlais QMD/LMD).

Sesame EOS for materials.

Circuit model for self-consistent coupling.

Dakota optimization loops



Balance laws for Resistive MHD

$$\dot{\rho} + \rho \nabla \cdot \mathbf{v} = 0$$

Balance of mass

$$\rho \dot{\mathbf{v}} = \nabla \cdot \left(\mathbf{T} + \mathbf{T}^M \right) + \mathbf{f}$$

Balance of momentum
including JxB forces

$$\rho \dot{e} = \rho s + \mathbf{T} : \mathbf{L} - \nabla \cdot \mathbf{q} + \frac{1}{\sigma} \mathbf{J} \cdot \mathbf{J}$$

Balance of internal energy
including Joule heating

$$\overset{*}{\mathrm{B}} = \frac{\partial \mathrm{B}}{\partial t} + \nabla \times (\mathrm{B} \times \mathbf{v}) + \mathbf{v}(\nabla \cdot \mathrm{B}) = -\nabla \times \frac{1}{\mu_0 \sigma} (\nabla \times \mathrm{B})$$

Balance of magnetic flux
Including resistive diffusion

$$\nabla \cdot \mathbf{B} = 0 \qquad \mathbf{T}^M = \frac{1}{\mu_0} \left(\mathbf{B}\mathbf{B}^T - \frac{1}{2} \mathbf{B}^2 \mathbf{I} \right)$$

In addition, closure relations for the stress, electrical conductivity and heat flux are required.

ALEGRA-MHD Algorithm

The strategy used in ALEGRA for MHD is a time operator-split algorithm for ideal and resistive MHD with 3 distinguishing phases:

1. Ideal MHD step: integration of equations of motion in moving (Lagrangian) frame.
   - Compatible node/edge/face centering of magnetic quantities
   - Central difference time integrator (time-staggered)
   - Artificial viscosity for shocks
   - Maxwell stress tensor for magnetic forces
   - Magnetic flux conservation.

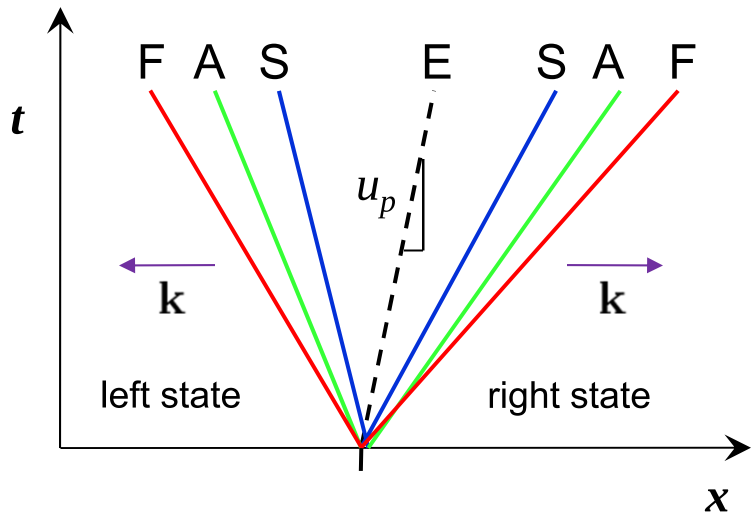
$$\frac{d}{dt}\int_{S_t} \mathrm{B}\cdot d\mathbf{a} = \int_{S_t} \overset{*}{\mathrm{B}} \cdot d\mathbf{a} = 0$$

2. Optional remesh/remap operation which respects the div **B** constraint for magnetic flux. (Constrained transport)

3. Magnetic diffusion at the new coordinates: transient magnetics solution for resistive component of field evolution.

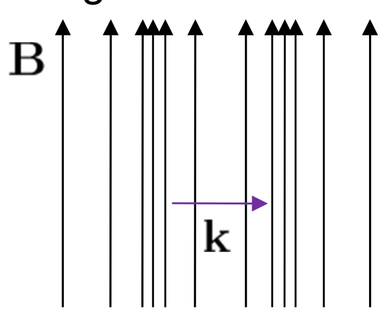
Ideal MHD wave propagation

The equations of ideal MHD are hyperbolic and have seven real eigenvalues, corresponding to seven characteristics or modes:

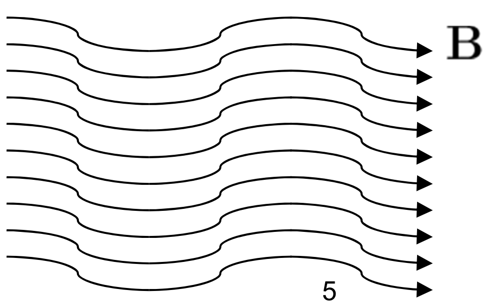


- 2 "fast" magnetosonic modes
- 2 Alfvén modes
- 2 "slow" magnetosonic modes
- 1 contact or "entropy" mode
- [In pure gas dynamics: only 3 real eigenvalues → only 3 modes.]

Magnetosonic waves:

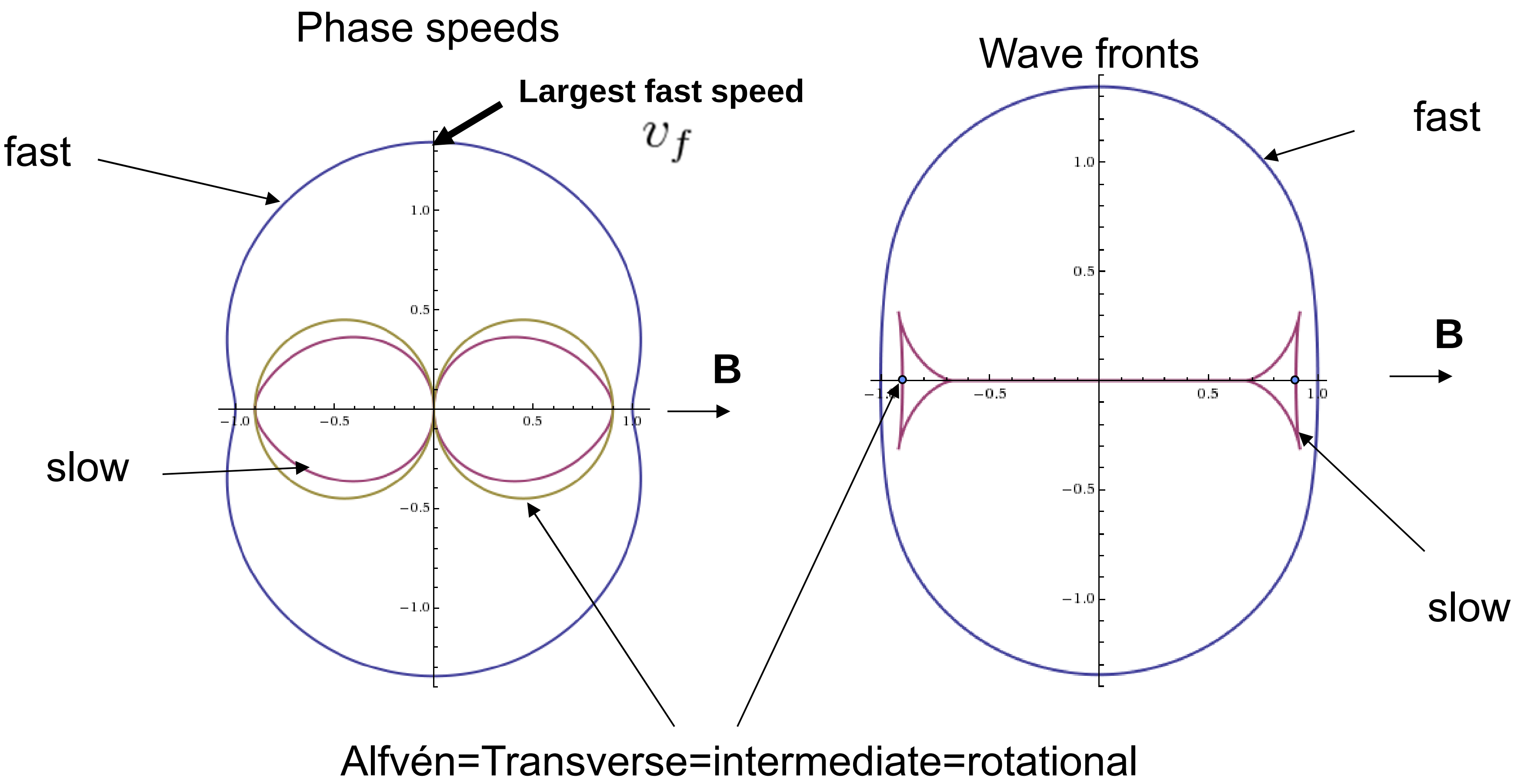


Alfvén waves:



Features of ideal MHD

The wave speeds are dependent on the wave front normal relative to the magnetic field (example with Alfven speed=.9 * sound speed)



The fastest “fast” wave speed controls time step

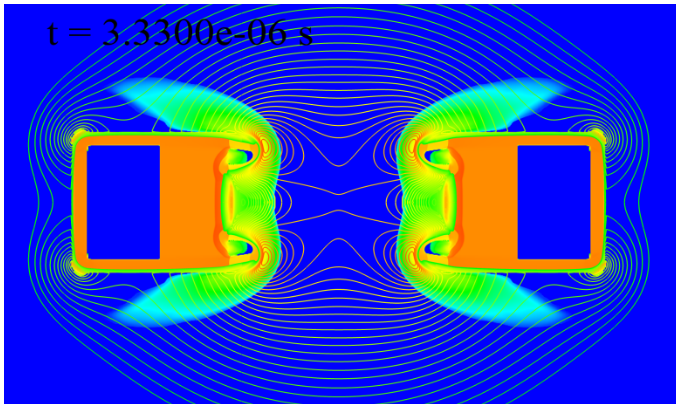
For MHD, the CFL-based time step control used in solid dynamics and hydrodynamics is modified to account for magnetoacoustic wave propagation

$$\Delta t \leq \frac{h}{\sqrt{a^2 + v_A^2}} = \frac{h}{v_f} \qquad v_A = \frac{B}{\sqrt{\mu_0 \rho}}$$

Sound speed **Alfvén speed**

The explicit hydro step drops the time step as the fast wave speed increases. What if the density goes to zero where the field is large? We will be spending resources tracking waves in regions which can have little mechanical effect.

For practical reasons explicit MHD codes need some approach to deal with the fast wave speed problem.



Boris's Semi-relativistic Correction

Boris, J. P., "A physically Motivated Solution of the Alfven Problem," NRL Memorandum Report 2167, 1970.

Boris proposed keeping the displacement current terms in the momentum equation to allow a user to manage the effect of the Alfven speed increasing beyond bounds.

This is termed a "semi-relativistic correction" and effectively results in a modified mass term in the momentum equation.

The user then has the option of artificially reducing the speed-of-light value in the modified mass term.

Lagrangian Step MHD Momentum Equation

$$\frac{\partial}{\partial t}(\rho\mathbf{v} + \epsilon_0 \mathbf{E}\times\mathbf{B}) + \nabla\cdot(\rho\mathbf{v}\otimes\mathbf{v} + \epsilon_0\mathbf{E}\times\mathbf{B}\otimes\mathbf{v})$$

$$= \nabla\cdot(-p\mathbf{I}) + \nabla\cdot\left(\frac{1}{\mu_0}\mathbf{B}\otimes\mathbf{B} - \frac{1}{2\mu_0}B^2\mathbf{I}\right)$$

$$+ \nabla\cdot\left(\epsilon_0\mathbf{E}\otimes\mathbf{E} - \frac{1}{2}\epsilon_0 E^2\mathbf{I}\right) + \nabla\cdot(\epsilon_0\mathbf{E}\times\mathbf{B}\otimes\mathbf{v})$$

Full semirelativistic momentum equation (Kovetz, p 223)

Boris suggested dropping RHS displacement current terms of $O(v^2/c^2)$

The advection terms associated with the Lagrangian form are of the same order

Ideal MHD: $\mathbf{E} = -\mathbf{v}\times\mathbf{B}$ $\qquad c = (\epsilon_0\mu_0)^{-1/2}$

0

$$\frac{d}{dt}\int\left(\rho\mathbf{I} + \frac{1}{c^2\mu_0}(B^2\mathbf{I} - \mathbf{B}\otimes\mathbf{B})\right)\mathbf{v}dv =$$

$$= \int\nabla\cdot(-p\mathbf{I}) + \nabla\cdot\left(\frac{1}{\mu_0}\mathbf{B}\otimes\mathbf{B} - \frac{1}{2\mu_0}B^2\mathbf{I}\right)dv$$

Remove $O(v^2/c^2)$ terms but keep LHS tensor mass term

Boris also suggesting dropping the outer product term

Final Modifications to Lagrangian MHD Equations

$$\frac{d}{dt}\int(1+\frac{v_A^2}{c^2})\rho\mathbf{v}dv = \int \nabla\cdot(-p\mathbf{I}) + \nabla\cdot\left(\frac{1}{\mu_0}\mathbf{B}\otimes\mathbf{B} - \frac{1}{2\mu_0}B^2\mathbf{I}\right)dv$$

Mass multiplier

$$v_f^2 = \frac{a^2+v_A^2}{1+(v_A/c)^2}$$

The maximum fast speed is reduced by the square root of the inverse mass multiplier.

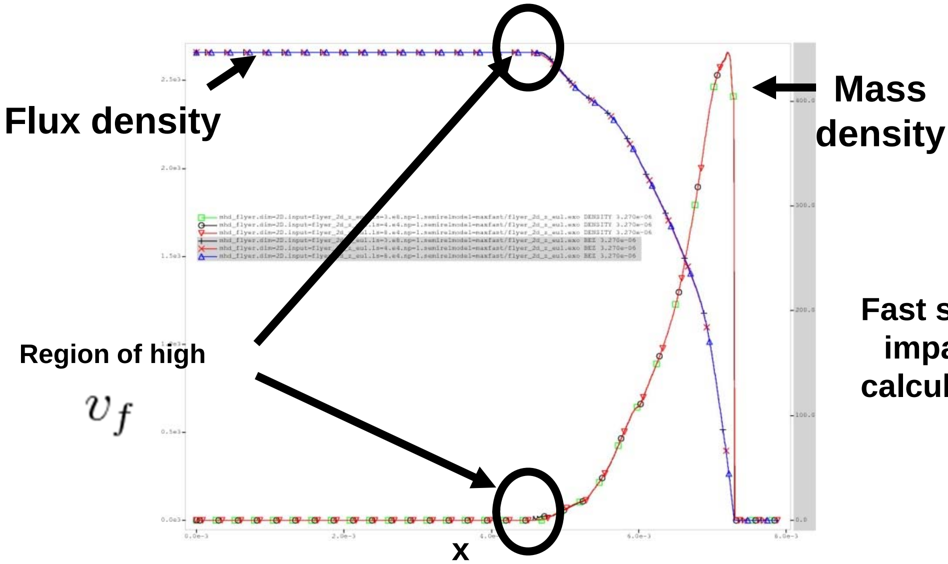
Generally speaking using the true light speed in the mass multiplier will have no effect or benefit. In order to reduce the overall impact on the physics, the correction should be applied only where the fast speed is undesirably too large.

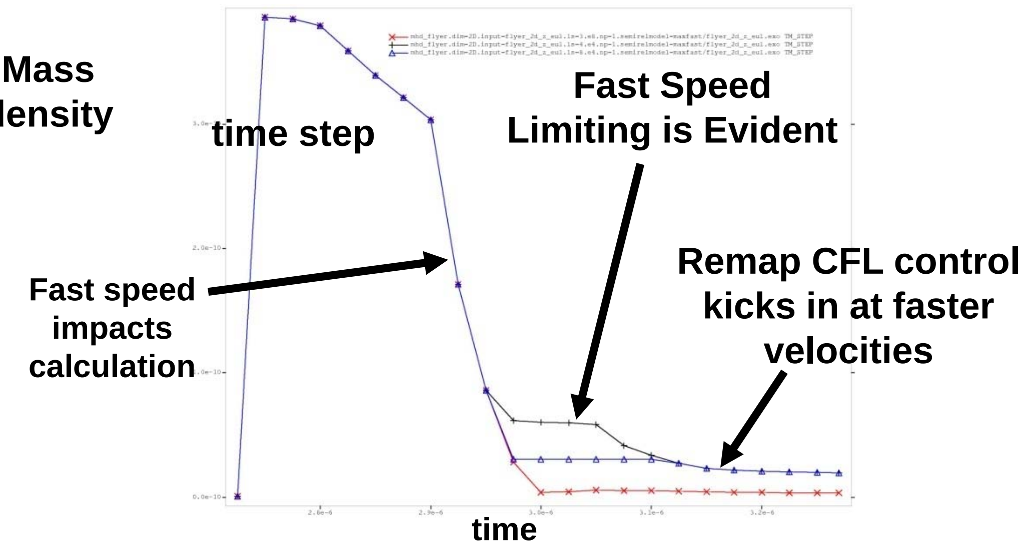
$$v_f = \min\left(1, \frac{v_f^{max}}{\sqrt{a^2+v_A^2}}\right)\sqrt{a^2+v_A^2} = \alpha\sqrt{a^2+v_A^2}$$

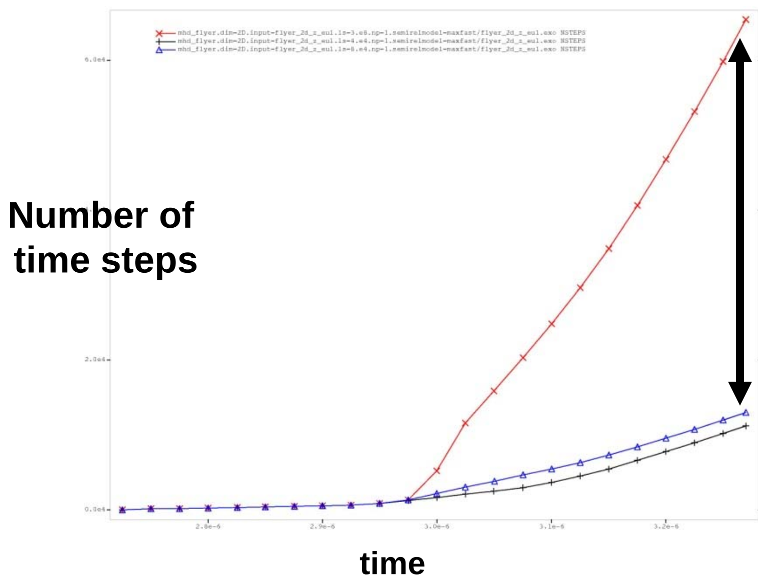
Also, reduce the accelerations by a factor proportional to α^2.

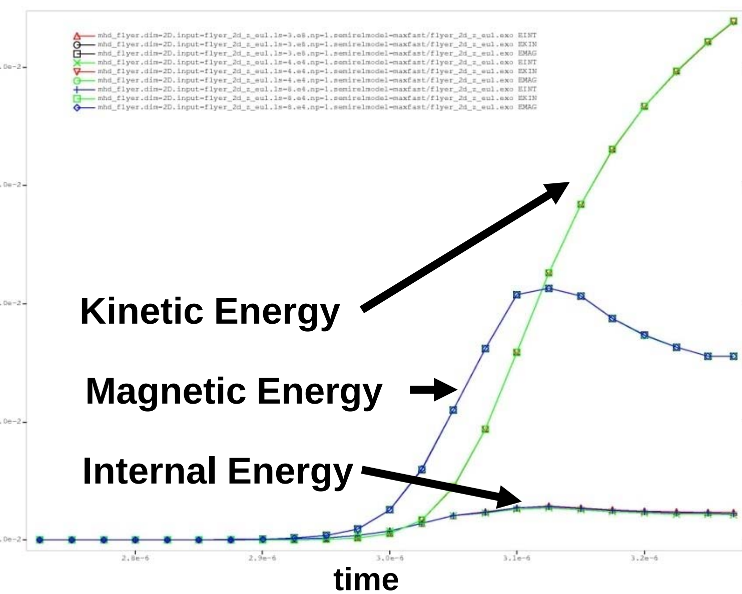
The algorithm is equivalent to a spatially varying "light speed."

AL 1D Eulerian flyer for 3 different levels of maximum fast wave speed (3.e8, 80.e3, 40e3)

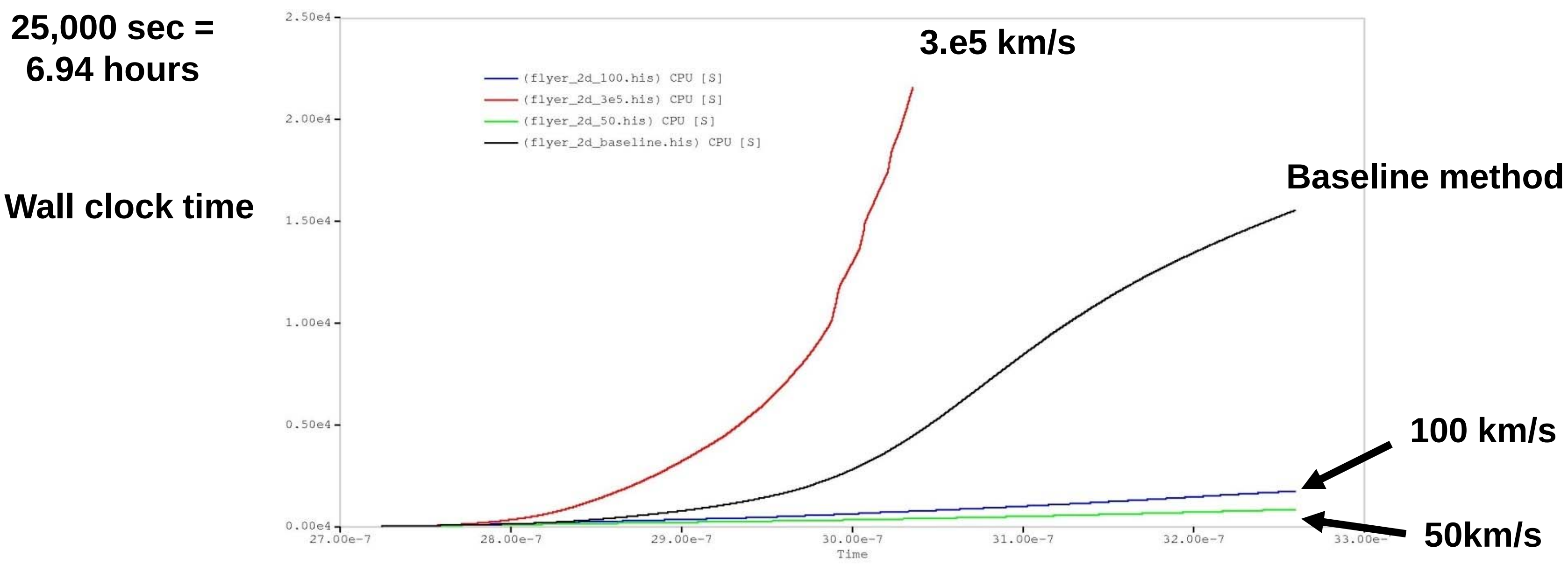








2D Flyer Calculation



Obviously, there is significant incentive and a practical requirement to control the very fast waves speeds.

The key is to carefully understand the consequences of utilizing this method relative to critical design metrics. The approach appears very promising.

Summary

The "physically motivated" approach suggested by Boris is a convenient way to view a method for adjustment to the MHD equations to deal with very fast MHD wave speeds.

In the Lagrangian MHD framework an additional advection term is neglected which is of the same order as the neglected displacement current terms.

Effective use relies on adjusting the equations only in regions of very high fast wave speeds. This is equivalent to a spatially varying "c" in the factor multiplying the mass.

Critical performance improvements can be gained for practical flyer plate configurations if used judiciously.