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Modeling Transitions from Laminar to Turbulent Flow by Level Cuts of Gaussian Random Fields

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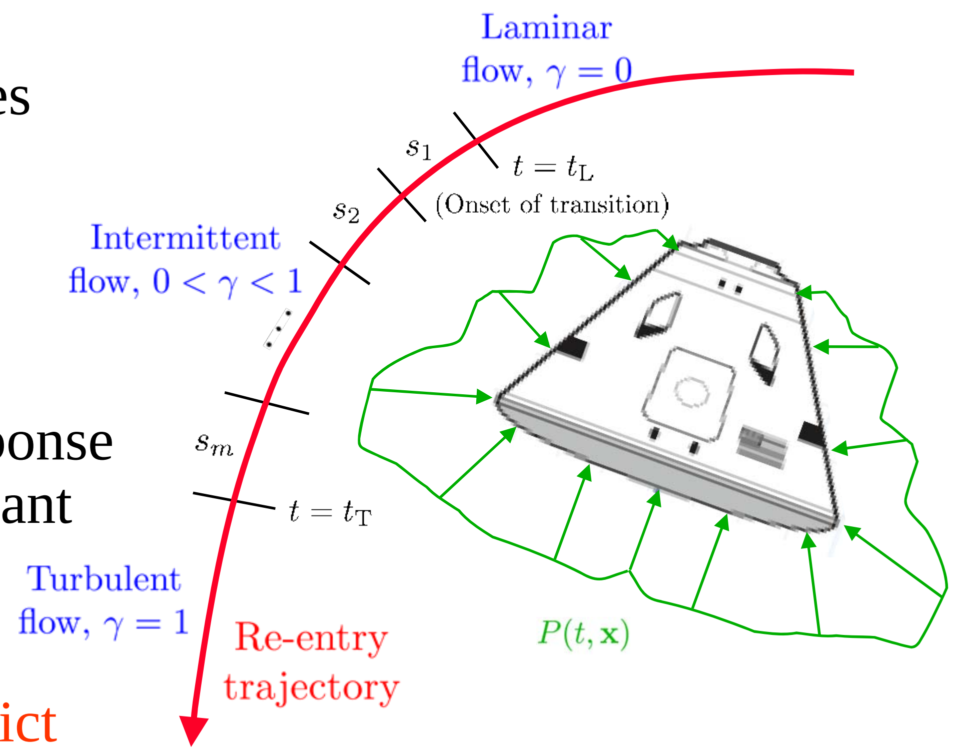
Outline

- Overview of laminar-to-turbulent transition
  - Intermittent formation of localized turbulent bursts
  - The formation and movement of these bursts cause flow-induced structural vibration response
- Model development
  - Intermittency random field (random oscillation between states “0” and “1”) to model location of turbulent bursts
  - Intermittent pressure field: a physics-based model
- Application
  - Beam with attached oscillator, a 1-D model for a flight vehicle
- Concluding remarks

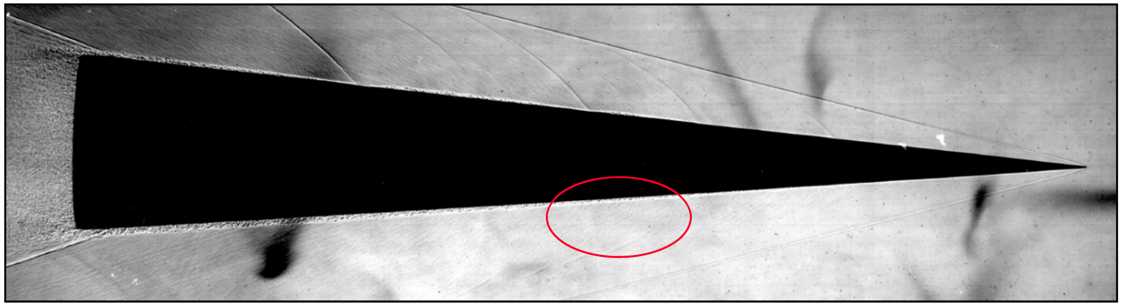
Laminar-to-Turbulent Transition During Atmospheric Re-Entry

- Flow in boundary layer transitions from laminar ($\gamma = 0$) to turbulent ($\gamma = 1$)
  - Fluctuating pressure field excites structure when $\gamma > 0$
- When $0 < \gamma < 1$, have (random) intermittent formation of localized turbulent bursts
  - Flight data shows vibration response during transition can be significant
  - Strongly non-Gaussian
- Objective: develop a model for the fluctuating pressure field *P* to predict vibration response during transition

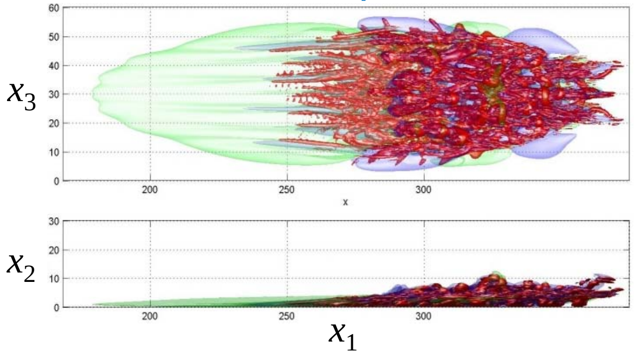


The Physics of Transition: Turbulent Bursts

Turbulent Spot on 5° Half-Angle Cone at Mach 4



DNS simulation of turbulent burst



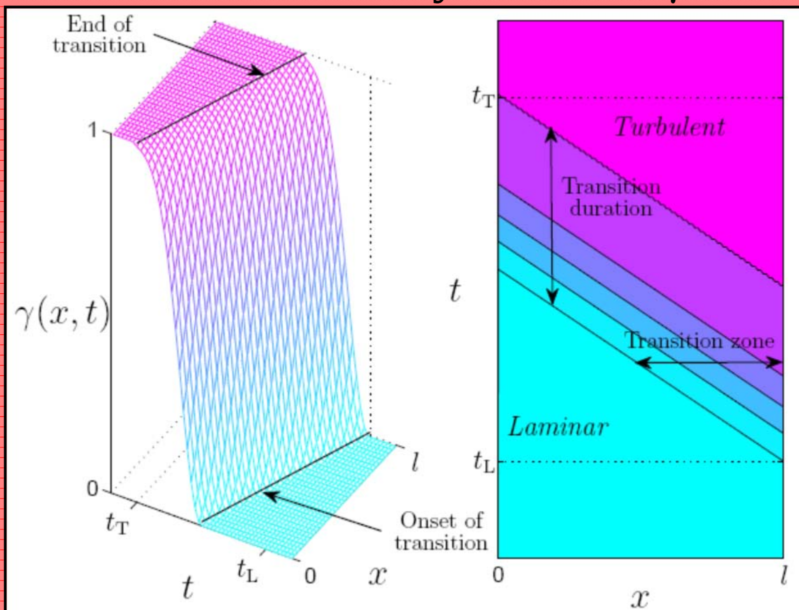
- Narsimha (1958) first proposed that transition is characterized by the intermittent appearance of turbulent spots which move downstream
- The existence of turbulent spots in boundary layers has been shown experimentally by Mitchener (1954), Schubauer & Klebanoff (1955) and others
- The key variables in describing a boundary layer during transition from laminar to turbulent flow are intermittency & burst rate
- Pressure fluctuations are highest in an intermittently turbulent flow

Intermittent Pressure Field Model Development

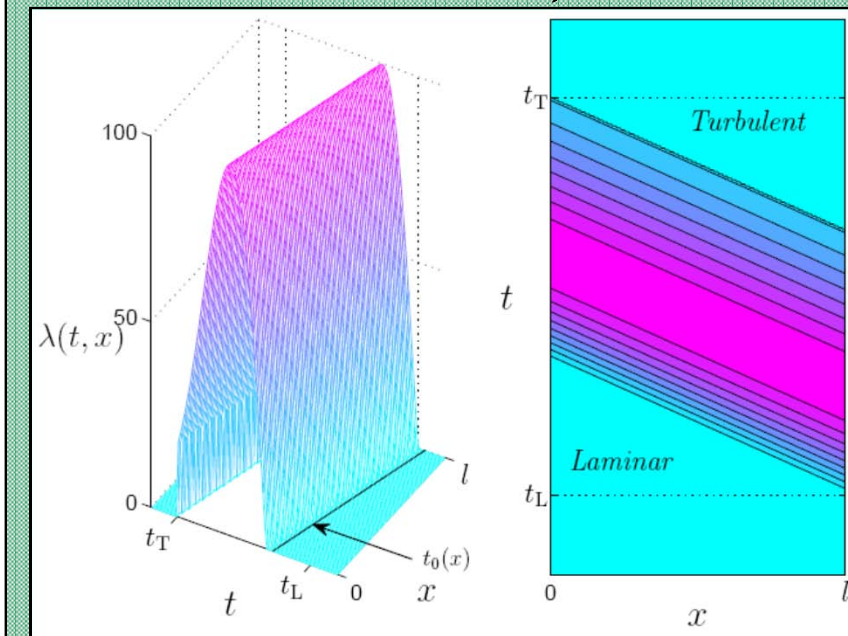
- Part 1: The intermittency random field, $A(t, \boldsymbol{x})$
  - Oscillates randomly between $A = 0$ to $A = 1$
  - Available information
    - Intermittency fraction, γ, describing the fraction of time the flow is turbulent ($A = 1$) at any location
    - Burst rate, λ, describing the expected number of turbulent bursts (number of times A transitions from 0 to 1) per unit of time, per unit volume of space
  - Two models for A have been considered
    - Poisson random field (shape control)
    - Level-cut Gaussian random field (very efficient)
- Part 2: Intermittent fluctuating pressure field
  - $P(t, \boldsymbol{x}) = \sigma(t, \boldsymbol{x}) \cdot A(t, \boldsymbol{x}) \cdot P_T(t, \boldsymbol{x})$
    - $\sigma(t, \boldsymbol{x})$ represents RMS of fluctuating pressures
    - $P_T(t, \boldsymbol{x})$ models fully turbulent flow

Available Information: Statistics of Turbulent Bursts

Intermittency fraction, γ



Burst rate, λ



– γ = the fraction of time the flow is turbulent at any location

– λ = the expected number of bursts per unit of time

– Flow becomes "more turbulent" as time increases

– Transition begins at the aft end ($x = l$) and moves forward

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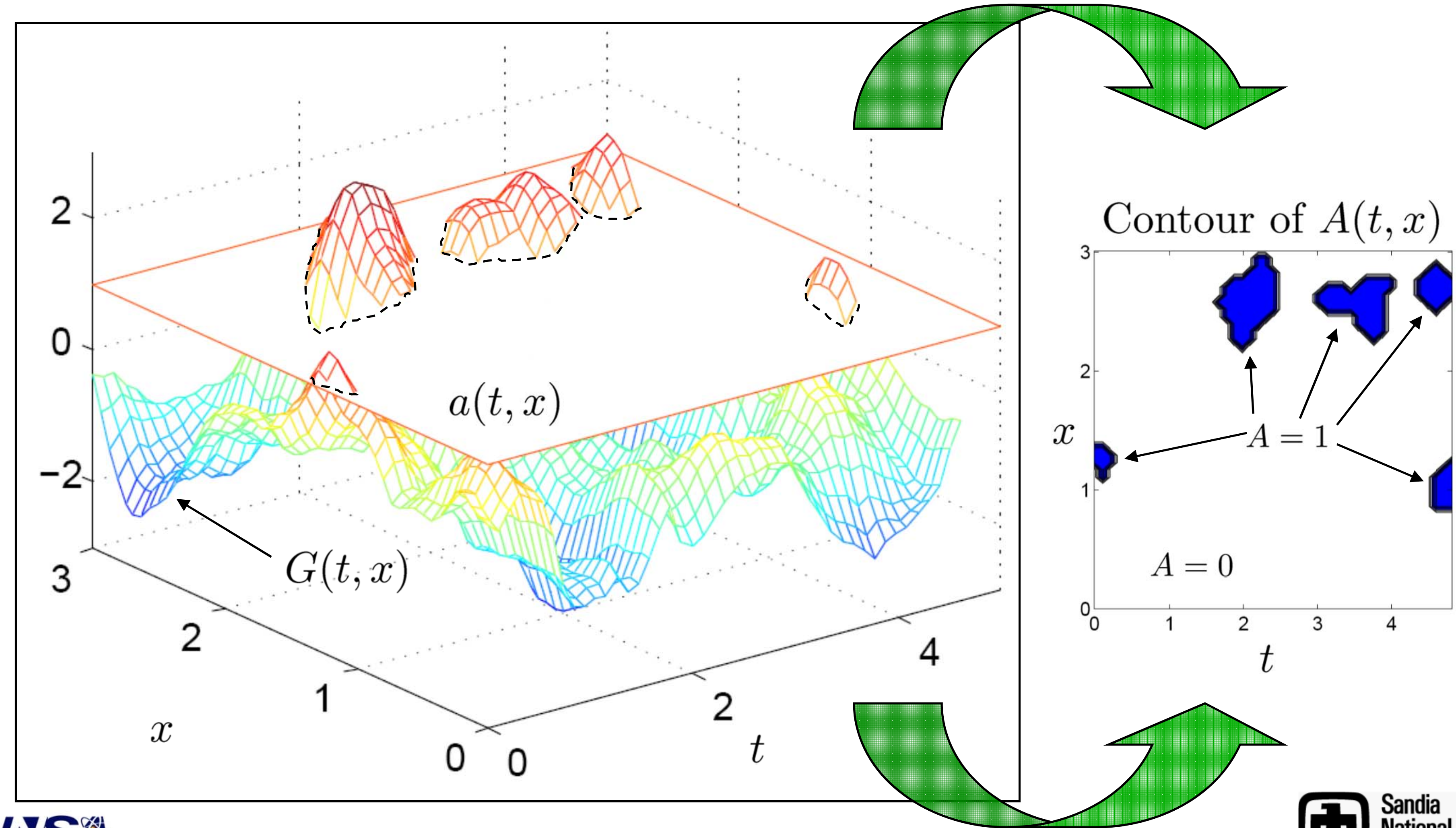
Level-Cut Gaussian Field to Model Location of Turbulent Burst

$$A(t,x) = \mathbf{1}\left(|G(t,x)| > a(t,x)\right), \; t \geq 0, \; 0 \leq x \leq l$$

$$\text{where } a(t,x) = \Phi^{-1}\left(1 - \frac{\gamma(t,x)}{2}\right)$$

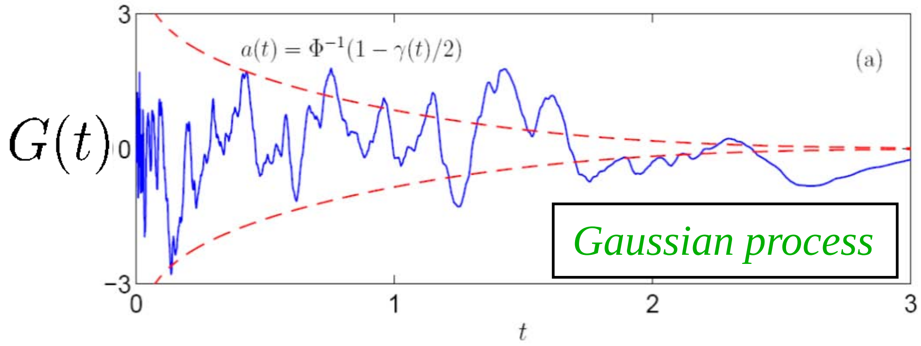
- $1(E) = 1$ if event E is true; 0 otherwise
- a = level surface defined by intermittency fraction γ
  - A will satisfy target γ
- G = a space-time Gaussian stochastic process with zero mean, marginal CDF Φ, and covariance function c
  - A will satisfy target λ by proper choice of c
- This type of stochastic model has been used extensively in mechanics applications (e.g., material microstructure)

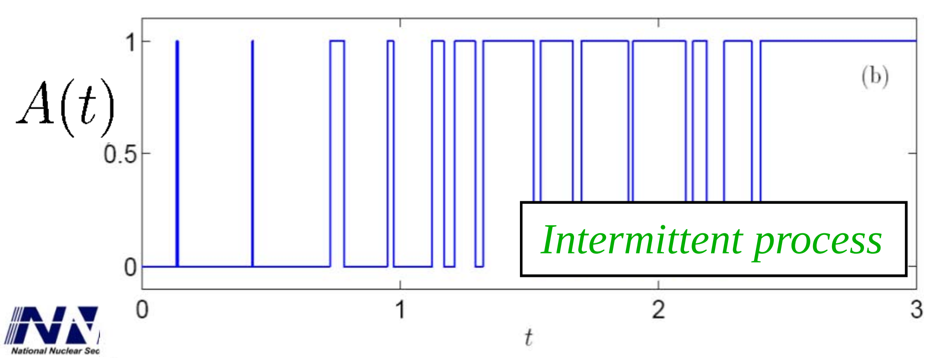
Schematic of a Level-Cut Gaussian Random Field

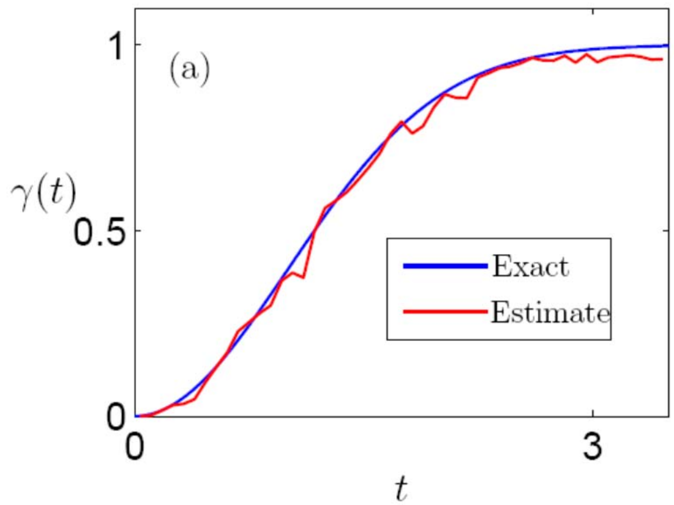


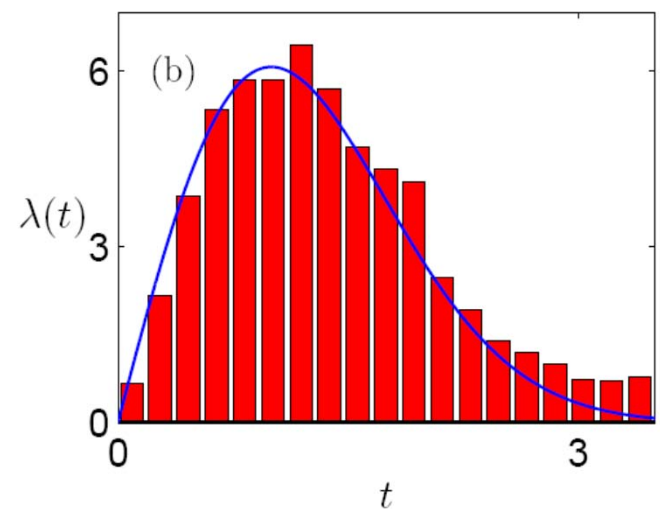
Example: Intermittency at a Point

- Consider: $$\gamma(t) = 1 - e^{-t^2/2}$$ $$\lambda(t) = 10\, t\, e^{-t^2/2}$$
- Estimates for γ and λ based on 200 samples of $A(t)$







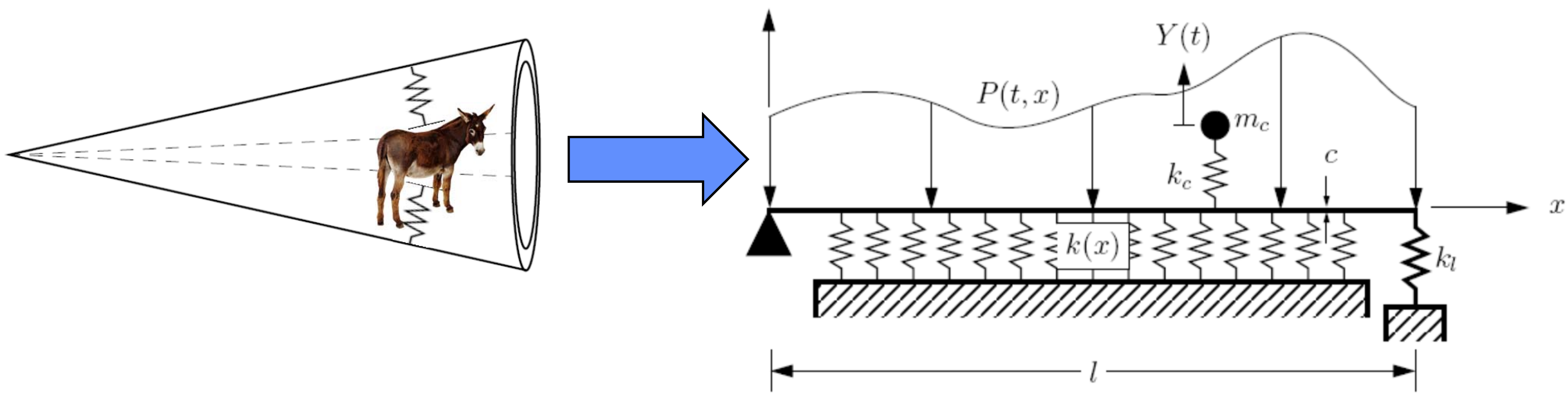


Extension of Intermittency Models to an Intermittent Pressure Field

$$P(t,x) = \sigma(t,x) \cdot A(t,x) \cdot P_{\mathrm{T}}(t,x),\ t \geq 0,\ 0 \leq x \leq l$$

- σ = a modulation (scaling) function
  - $\sigma \geq 1$ because "pressure fluctuations are highest in an intermittently turbulent flow"; empirically-based
- A = model for intermittency discussed previously
- P_{T} = Gaussian pressure field assuming fully turbulent flow
  - Abundance of models in the literature; any can be used here
- Therefore P is a scaled version of P_{T}, that turns "on" and "off" intermittently
  - When $A = 1$, a burst appears; laminar flow when $A = 0$

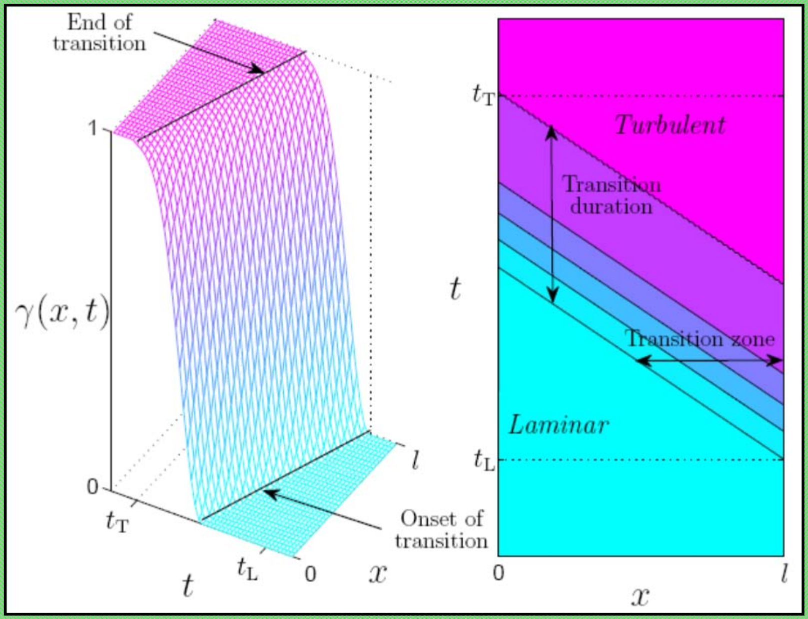
Simple 1-D Model for Flight Vehicle: Flexible Beam on Elastic Foundation



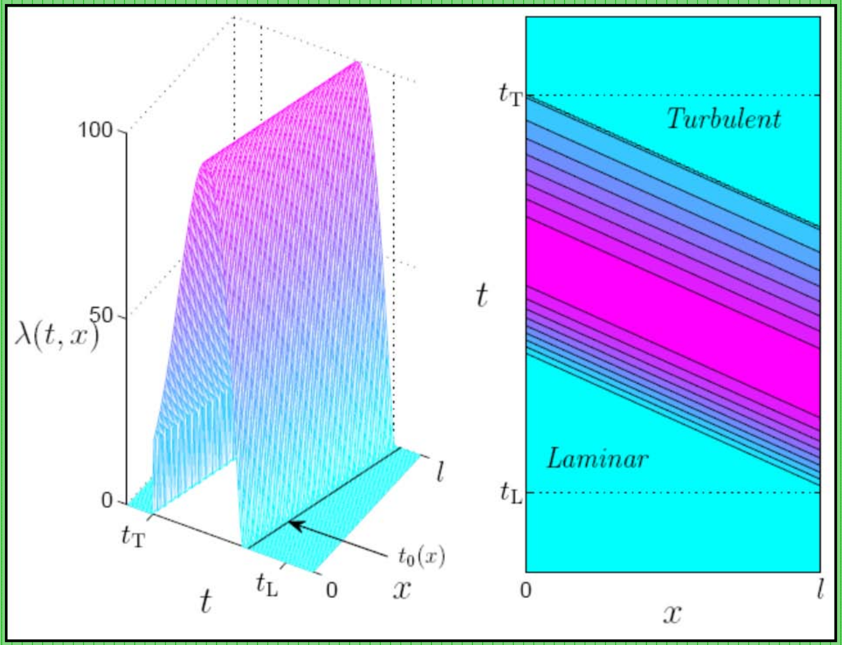
- Develop model for input
  - $P(t, x)$ = intermittent fluctuating pressure field
- Predict "component" vibration response, $\ddot{Y}(t)$
  - $Y(t)$ = displacement response of internal mass m_c
- On-going work: apply to high-fidelity 3-D FE models for various flight vehicles

Specified Intermittency Fraction and Burst Rate Functions for 1-D Beam

Intermittency fraction, γ

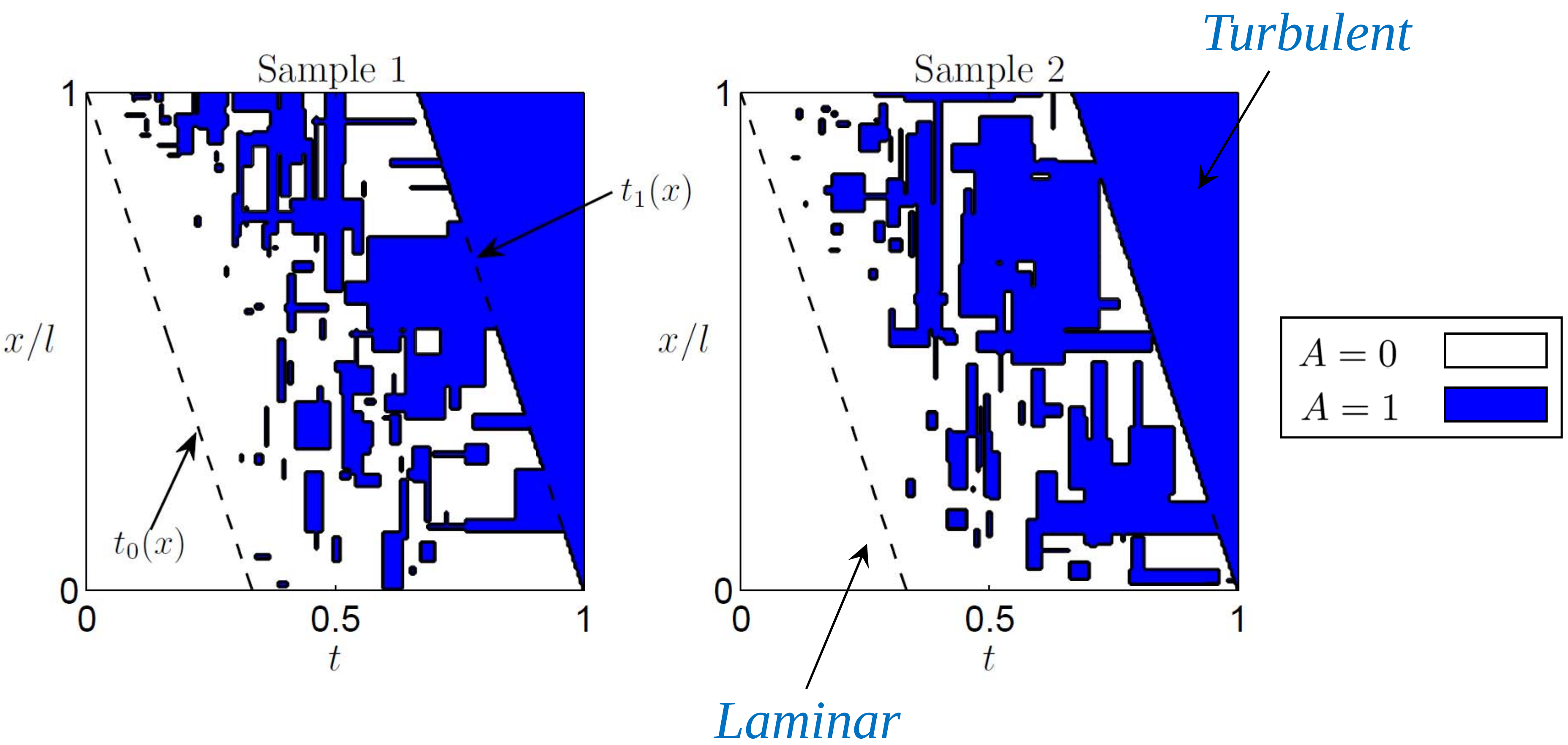


Burst rate, λ

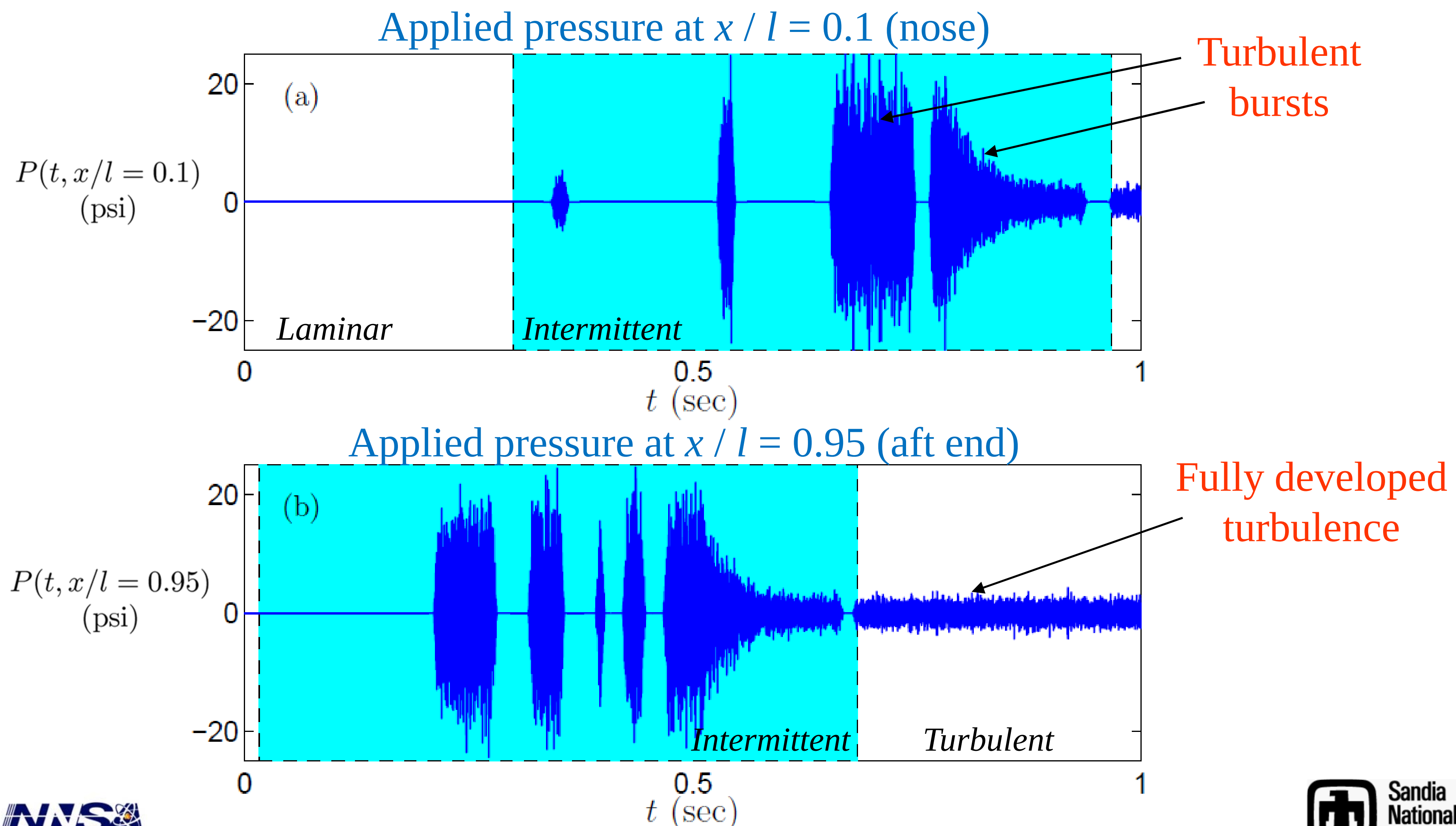


- Onset and end of transition occur at $t_0(x)$ and $t_1(x)$
- These are example γ and λ, but are consistent with literature

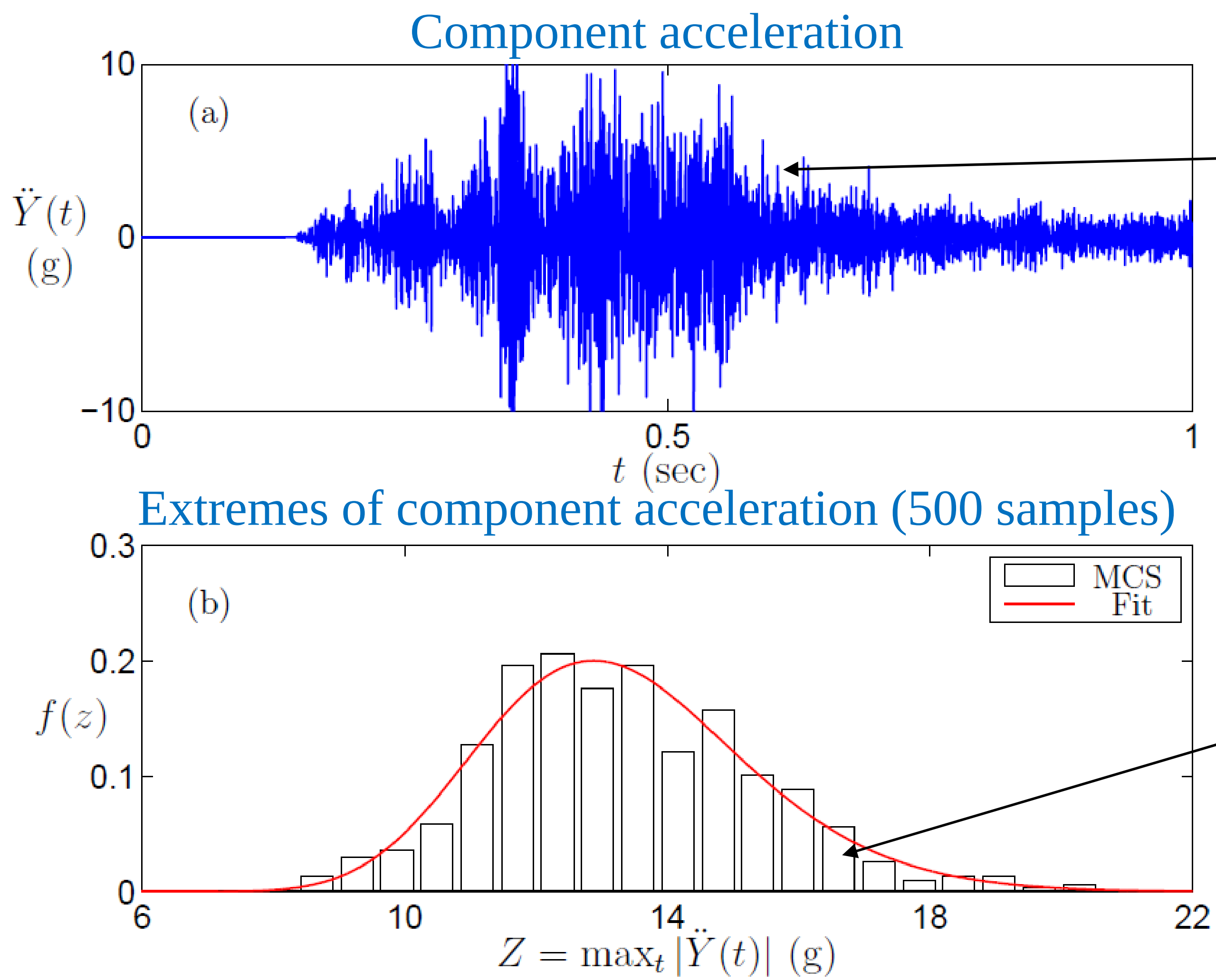
Two Samples of the Intermittency Field



Sample of Intermittent Pressure Field Applied to Beam



Response of Attached Mass During Laminar-to-Turbulent Transition



Vibration response due to multiple turbulent bursts

Can estimate component reliability during transition

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Concluding Remarks

- Developed physics-based model for the dynamic loading of a flight vehicle due to laminar-to-turbulent transition
- Intermittency random field
  - Represent the formation and movement of localized turbulent bursts
  - Available information: intermittency fraction and burst rate
- Intermittent pressure field
  - Model for fully-developed turbulence, modulated by intermittency random field
  - Qualitatively, beam "component" vibration response resembles measured flight data
- Future and on-going work
  - Extension to 3-D structures / complex geometries
  - Investigate alternative approach for the intermittency random field