Final Project Report for DOE Grant NO.: DE-SC0010534
Period: Sept 2013-March 31, 2016[1]
PI: Murat Gunaydin

Work on Higher Spin Theories:

Higher spin theories has been an active area of research in recent years. One of the main research activities of the PI Murat Gunaydin over the period of this grant has been the application of quasiconformal methods to construct and study higher spin (HS) algebras and superalgebras in various dimensions. Below is a summary of main results obtained in the field of higher spin theories:

- **Deformed Twistors and AdS_5/CFT_4 Higher Spin Algebras and Super-Algebras:**

  The minimal unitary representation (minrep) of conformal group SU(2,2) describes a massless conformal scalar field in four space-time dimensions and it admits a one-parameter family of deformations that describe massless conformal fields of arbitrary helicity[1]. These results were obtained by quantization of the geometric nonlinear realization of SU(2,2) as a quasiconformal group. Quasiconformal realizations of non-compact groups were first discovered in [2] and later developed and extended to non-compact Lie superalgebras[3, 1, 4, 5].

  In a joint work with Karan Govil, Gunaydin showed that the generators of the unitary irreducible representations of the $4d$ conformal group $SU(2,2)$ obtained in [1] can be written as bilinears of deformed twistorial oscillators which transform nonlinearly under the Lorentz group and applied them to define and study higher spin algebras and superalgebras in five dimensional anti-de Sitter space AdS_5[6]. The standard higher spin (HS) algebra of Fradkin-Vasiliev type in AdS_5 is simply the enveloping algebra of SU(2,2) quotiented by a certain two-sided ideal (Joseph ideal) which annihilates the minrep. This ideal vanishes identically as an operator for the quasiconformal realization of the minrep and its enveloping algebra leads directly to the HS algebra in AdS_5. Furthermore, Govil and Gunaydin showed that the enveloping algebras of the deformations of the minrep define a one parameter family of HS algebras in AdS_5 for which certain 4d covariant deformations of the Joseph ideal vanish identically. They extended these results to superconformal algebras $SU(2,2|N)$ with even subalgebras $SU(2,2) \oplus U(N)$ and showed that there exists a one parameter family of HS superalgebras defined by the enveloping algebras of the minimal unitary realization of $SU(2,2|N)$ and its deformations. Their results imply the existence of a family of (supersymmetric) HS theories in AdS_5 which are dual to free (super)conformal field theories (CFTs) or to interacting but integrable (supersymmetric) CFTs in 4d. These results were published in JHEP [6].

[1] Irina Mocioiu was a co-PI in this Grant during the period Sept 2013-March 31, 2015.

- **Deformed Twistors and AdS_7/CFT_6 Higher Spin Algebras and Super-Algebras:**

  The minimal unitary representation (minrep) of the $6d$ conformal group $SO(6,2)$ describes a massless conformal scalar field in six dimensions. The minrep of $SO(6,2)$ admits a discrete infinite family of "deformations" labelled by the spin t of an $SU(2)_T$ subgroup of the little group $SO(4)$ of massless particles , which is the $6d$ analog of helicity in four dimensions. These deformations of the minrep of $SO(6,2)$ describe massless conformal fields that are symmetric tensors whose indices carry the spinorial representation of the $6d$ Lorentz group $SO(5,1)$. The minrep and its deformations were obtained by quantization of the geometric nonlinear realization of $SO(6,2)$ as a quasiconformal group in [4]. In his joint work with Govil, Gunaydin gave a novel reformulation of the generators of $SO(6,2)$ for these representations as bilinears of *deformed* twistorial oscillators which transform *nonlinearly* under the Lorentz group $SO(5,1)$ and apply them to define higher spin algebras and superalgebras in AdS_7. The higher spin (HS) algebra in AdS_7 is simply the enveloping algebra of $SO(6,2)$ quotiented by the Joseph ideal which annihilates the minrep. Govil and Gunaydin showed that the Joseph ideal vanishes identically for the quasiconformal realization of the minrep and its enveloping algebra leads directly to the HS algebra in AdS_7. Furthermore, the enveloping algebras of the deformations of the minrep define a discrete infinite family of HS algebras in AdS_7 for which certain $6d$ Lorentz covariant deformations of the Joseph ideal vanish identically. They show that these results extend to superconformal algebras $OSp(8^*|2N)$ and find a discrete infinite family of HS superalgebras as enveloping algebras of the minimal unitary supermultiplet and its deformations. Their results suggest the existence of a discrete family of (supersymmetric) HS theories in AdS_7 which are dual to free (super)conformal field theories (CFTs) or to interacting but integrable (supersymmetric) CFTs in $6d$. These results were published in JHEP [7].

- **Minimal unitary representation of 5d superconformal algebra $F(4)$ and AdS_6/CFT_5 higher spin (super)-algebras:**

  in [11] Fernando and Gunaydin studied the minimal unitary representation of $SO(5,2)$, obtained by quantization of its geometric quasiconformal action, its deformations and supersymmetric extensions. The minrep of SO(5,2) describes a massless conformal scalar field in five dimensions and admits a unique "deformation" which describes a massless conformal spinor. Scalar and spinor minreps of $SO(5,2)$ are the 5d analogs of Dirac's singletons of $SO(3,2)$. They also gave a construction of the minimal unitary representation of the unique 5d superconformal algebra F(4) with the even subalgebra SO(5,2) X SU(2). The minrep of F(4) describes a massless conformal supermultiplet consisting of two hypermultiplets and one symplectic Majorana spinor field. They further extended their results to the construction of higher spin AdS_6/CFT_5 (super)-algebras. The Joseph ideal of the minrep of SO(5,2) vanishes identically as operators

and hence its enveloping algebra yields the AdS_6/CFT_5 bosonic higher spin algebra directly. The enveloping algebra of the spinor minrep defines a "deformed" higher spin algebra for which a deformed Joseph ideal vanishes identically as operators. These results were then extended to the construction of the unique higher spin AdS_6/CFT_5 superalgebra as the enveloping algebra of the minimal unitary realization of F(4) obtained by the quasiconformal methods. These results were published in Nucl. Phys. B [11].

- **Higher spin theories in arbitrary dimensions and their deformations**

  Fernando and Gunaydin extended their earlier work on the minimal unitary representation of $SO(d,2)$ and its deformations for $d = 4, 5$ and 6 to arbitrary dimensions d. They showed that there is a one-to-one correspondence between the minrep of $SO(d,2)$ and its deformations and massless conformal fields in Minkowskian spacetimes in d dimensions. The minrep describes a massless conformal scalar field, and its deformations describe massless conformal fields of higher spin. Since the generators of Joseph ideal vanish identically as operators for the quasiconformal realization of the minrep its enveloping algebra yields directly the standard bosonic $AdS_{(d+1)}/CFT_d$ higher spin algebra. For deformed minreps the generators of certain deformations of Joseph ideal vanish as operators and their enveloping algebras lead to deformations of the standard bosonic higher spin algebra. In odd dimensions there is a unique deformation of the higher spin algebra corresponding to the spinor singleton. In even dimensions one finds infinitely many deformations of the higher spin algebra labelled by the eigenvalues of Casimir operator of the little group $SO(d-2)$ for massless representations. These results were published in Nuclear Physics B [9].

The results of the above reviewed works was presented by the PI at two international conferences, namely at the International Workshop on Higher Spin Gauge Theories" , Singapore, November 4-6, 2015 and at the "Aspects of Higher Spin theory" Conference in Garching, Germany, 23 - 25 May 2016. A written review of these results was submitted to hep-th arXiv by the PI [8] and will be published in the proceedings of the conference in Singapore.

Work on Amplitudes in matter coupled supergravities :

Over the past decade work on amplitudes in gauge theories, supergravity and string theories has been a very active area of research. Enormous progress has been made in the understanding of the structure of amplitudes in these theories. The novel methods and results obtained have made it possible to do calculations in gauge theories and supergravity theories that go well beyond the calculations one can do using the old fashioned Feynman diagram techniques. Work on amplitudes in matter coupled supergravity theories has been the second main focus of the PI during the funding period. The previous work of the PI on supergravity theories has played a fundamentally important role in the current work on amplitudes. Below is a summary of some of the main results obtained.

- **Scattering amplitudes in N=2 Maxwell-Einstein and Yang-Mills/Einstein supergravity**

  In a joint work with Chiodaroli, Johansson and Roiban, Gunaydin exposed a double-copy structure in the scattering amplitudes of the generic Jordan family of N=2 Maxwell-Einstein and Yang-Mills/Einstein supergravity theories in four and five dimensions. The Maxwell-Einstein supergravity amplitudes were obtained through the color/kinematics duality as a product of two gauge-theory factors, one originating from pure N=2 super-Yang-Mills theory and the other from the dimensional reduction of a bosonic higher-dimensional pure Yang-Mills theory. They showed that there exists a specific symplectic frame in four dimensions for which the on-shell fields and amplitudes from the double-copy construction can be identified with the ones obtained from the supergravity Lagrangian and Feynman-rule computations. The Yang-Mills/Einstein supergravity theories are obtained by gauging a compact subgroup of the isometry group of their Maxwell-Einstein counterparts. For the generic Jordan family this process is identified with the introduction of cubic scalar couplings on the bosonic gauge-theory side, which through the double copy procedure are responsible for the non-abelian vector interactions in the supergravity theory. As a demonstration of the power of this structure, they presented the results of explicit computations at tree-level and one loop. The double-copy construction allows one to obtain compact expressions for the supergravity superamplitudes, which are naturally organized as polynomials in the gauge coupling constant. These results were published in JHEP [12].

- **Spontaneously Broken Yang-Mills-Einstein Supergravities as Double Copies**

  Color/kinematics duality and the double-copy construction have proved to be powerful tools for gaining new insight into gravitational theories. Extending their earlier work Chiodaroli, Gunaydin , Johansson and Roiban obtained new double-copy constructions for large classes of spontaneously-broken Yang-Mills-Einstein theories with adjoint Higgs fields. One gauge-theory copy entering the construction is a spontaneously-broken (super-)Yang-Mills theory, while the other copy is a bosonic Yang-Mills-scalar theory with trilinear scalar interactions that display an explicitly-broken global symmetry. They showed that the kinematic numerators of these gauge theories can be made to obey color/kinematics duality by exhibiting particular additional Lie-algebraic relations. In particular they discussed in detail explicit examples with $N = 2$ supersymmetry, focusing on Yang-Mills-Einstein supergravity theories belonging to the generic Jordan family in four and five dimensions, and identified the map between the supergravity and double-copy fields and parameters. They also discussed the extension of their results to $N = 4$ supergravity theories. These results appeared in hep-th arXiv [10] and will be submitted for publication.

- **Complete construction of magical, symmetric and homogeneous $N = 2$ su-**

pergravities as double copies of gauge theories

In a recent paper titled "Complete construction of magical, symmetric and homogeneous $N = 2$ supergravities as double copies of gauge theories" and published in Phys. Rev. Lett. [13] Chiodaroli, Gunaydin , Johansson and Roiban showed that scattering amplitudes in magical, symmetric or homogeneous $\mathcal{N} = 2$ Maxwell-Einstein supergravities can be obtained as double copies of two gauge theories, using the framework of color/kinematics duality. The left-hand copy is $N = 2$ super-Yang-Mills theory coupled to a hypermultiplet, whereas the right-hand copy is a non-supersymmetric theory that can be identified as the dimensional reduction of a D-dimensional Yang-Mills theory coupled to P fermions. For generic D and P, the double copy gives homogeneous supergravities. For $P = 1$ and $D = 7, 8, 10, 14$, it gives the magical supergravities. They computed explicit amplitudes, discussed their soft limits and studied their UV-behavior at one loop.

Non-associativity in non-geometric string and M-theory backgrounds, the algebra of octonions, and missing momentum modes :

In a very recent paper titled "Non-associativity in non-geometric string and M-theory backgrounds, the algebra of octonions, and missing momentum modes" Gunaydin, Luest and Malek proposed a non-associative phase space algebra for M-theory backgrounds with locally non-geometric fluxes based on the non-associative algebra of octonions [14]. Their proposal is based on the observation that the non-associative algebra of the non-geometric R-flux background in string theory can be obtained by a proper contraction of the simple Malcev algebra generated by imaginary octonions. This algebra is isomorphic to the magnetic algebra that appeared in the work of Gunaydin and Zumino[15] as was pointed out in [16]. Furthermore, by studying a toy model of a four-dimensional locally non-geometric M-theory background which is dual to a twisted torus, Gunaydin , Luest and Malek showed that the non-geometric background is "missing" a momentum mode. The resulting seven-dimensional phase space can thus be naturally identified with the imaginary octonions. This allows one to interpret the full uncontracted algebra of imaginary octonions as the uplift of the string theory R-flux algebra to M-theory, with the contraction parameter playing the role of the string coupling constant g_s. This paper was submitted to JHEP for publication.

Work on the construction of unitary representations of noncompact superalgebras via the generalized covariant twistorial oscillator method :

During the funding period of this grant I continued my work on the construction of unitary representations of noncompact superalgebras. In particular with Dmytro Volin we are preparing a manuscript in which we show that all the unitary representations of the noncompact superalgebras $SU(n, m|k)$ with the even subalgebra $SU(n, m) \oplus U(k)$ can be obtained via a generalized twistorial oscillator construction. The results for the particular case of $SU(2, 2|k)$ have important for applications to AdS_5/CFT_4 dualities, integrable spin chains and the conformal bootstrap program in four dimensions.

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Publications resulting from the research supported by the DOE Grant NO.: DE-SC0010534 (PI: Murat Gunaydin):

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