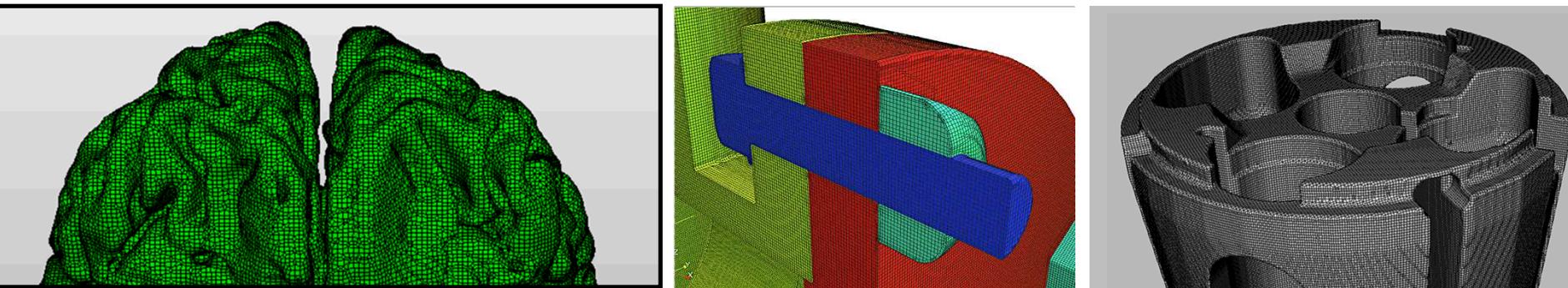


Exceptional service in the national interest

SAND2015-6194C



Sandia
National
Laboratories



Parallel Mesh Optimization for Grid-based Hex Meshes

US National Congress on
Computational Mechanics

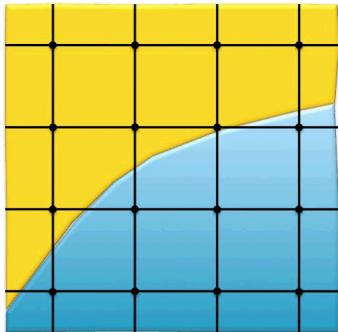
San Diego, CA
July, 26th 2015

Steven J. Owen
Sandia National Laboratories,
Albuquerque, NM

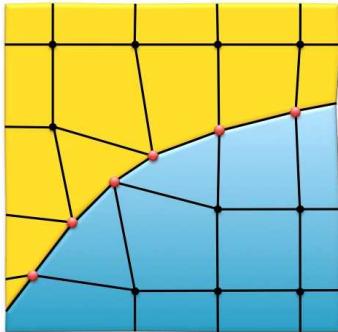


Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. SAND NO. 2011-XXXXP

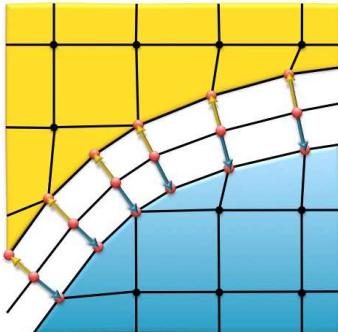
Overlay Grid Hex Meshing Method



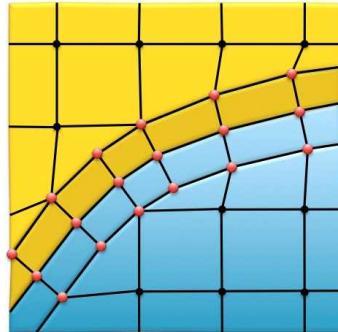
Overlay
Cartesian
Grid on
Geometry



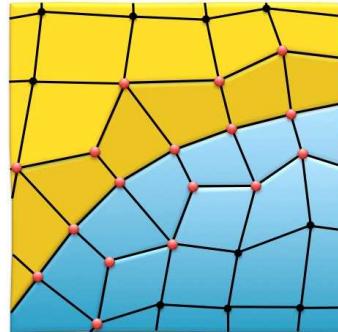
Nodes
projected to
interfaces



Nodes
duplicated at
interfaces and
moved
orthogonally

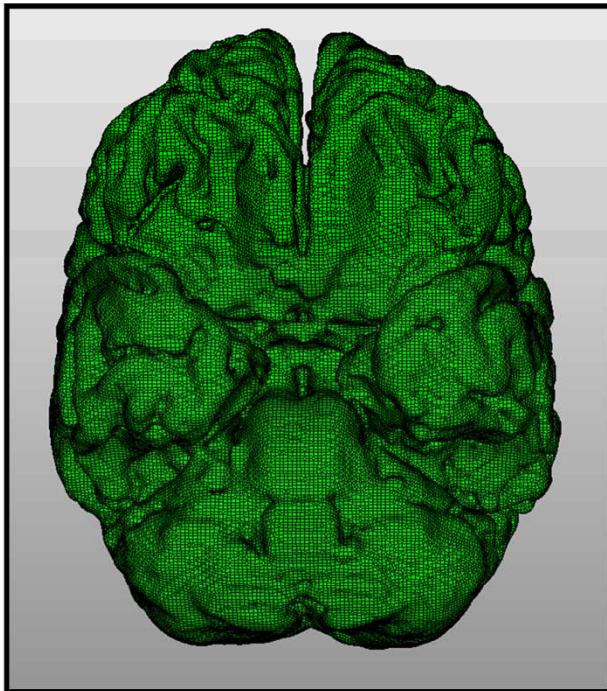


Layers of
hexes created
at interfaces

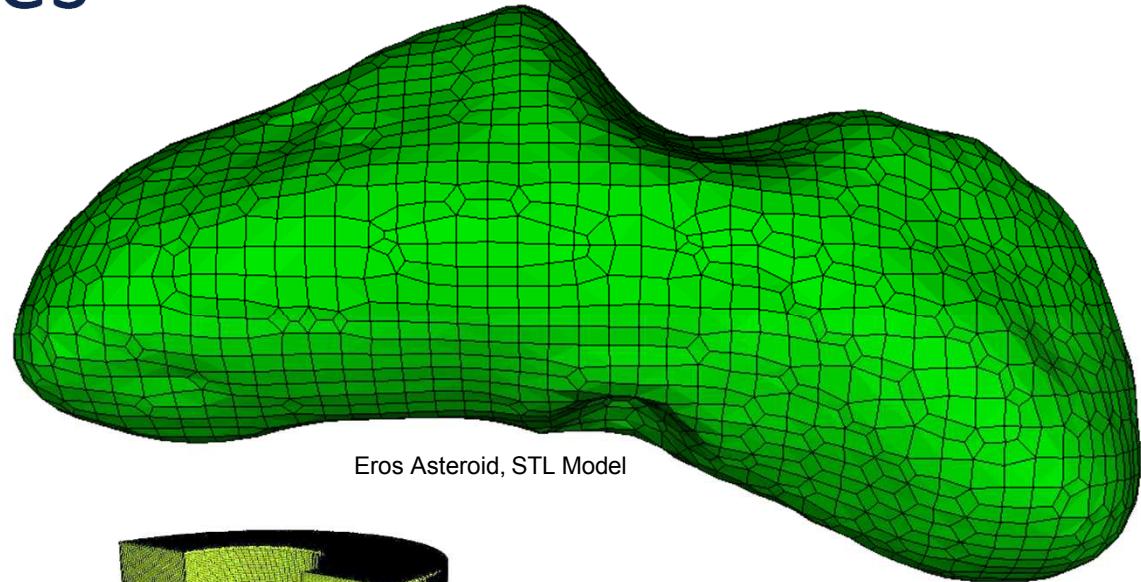


Smoothing
performed

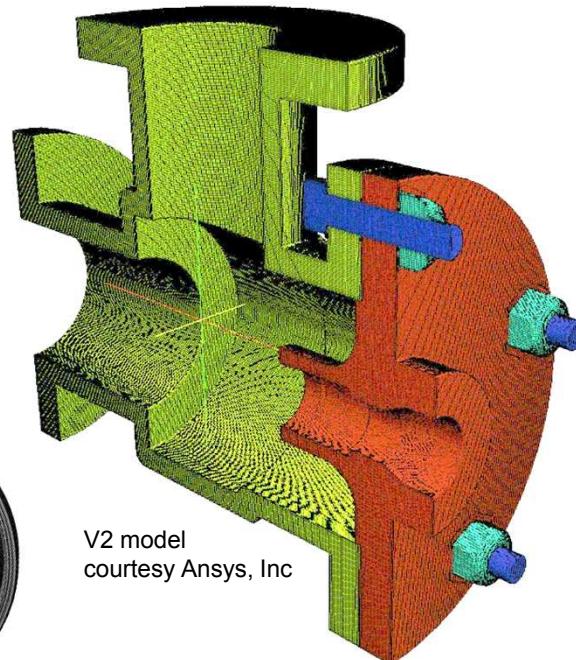
Sculpt Examples



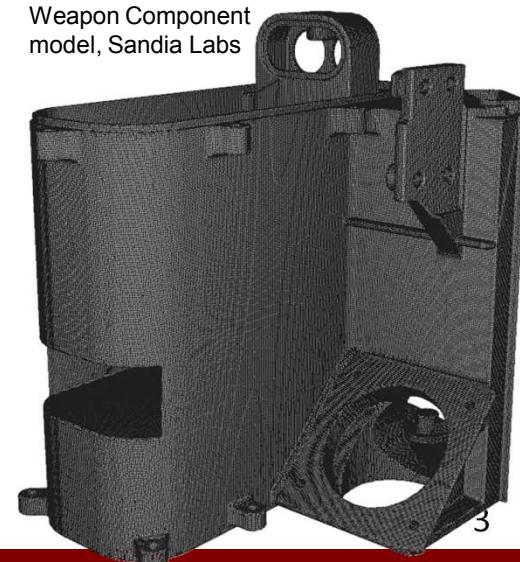
STL MRI Brain Model, Courtesy Bryce Owen
Brigham Young University, Provo, UT



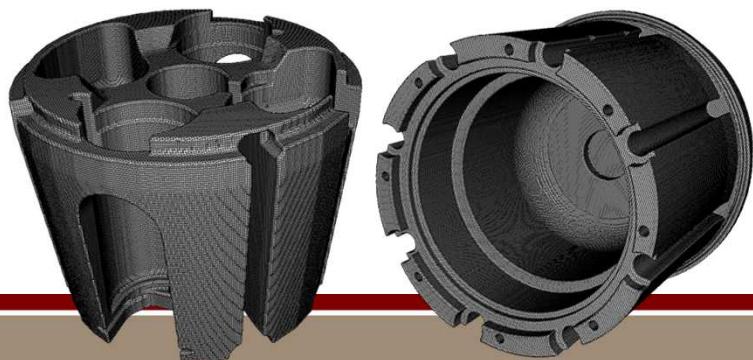
Eros Asteroid, STL Model



V2 model
courtesy Ansys, Inc

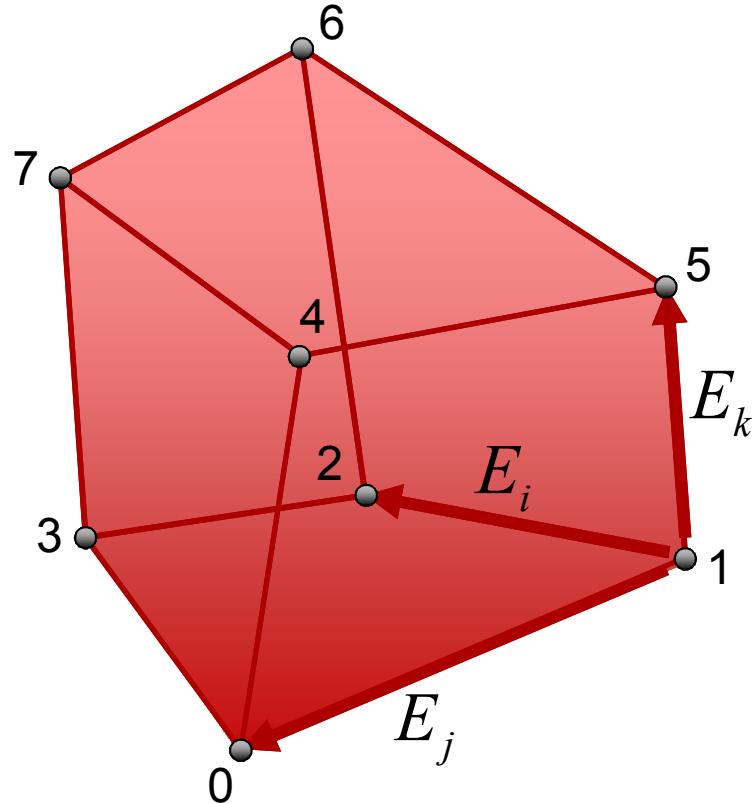


Weapon Component
model, Sandia Labs



Weapon Component Models Courtesy Stephen Recchia,
US Army, Picatinny. Used with Permission

Scaled Jacobian



$$(J_s)_I = \det \left\{ \hat{E}_i \hat{E}_j \hat{E}_k \right\}^\top$$

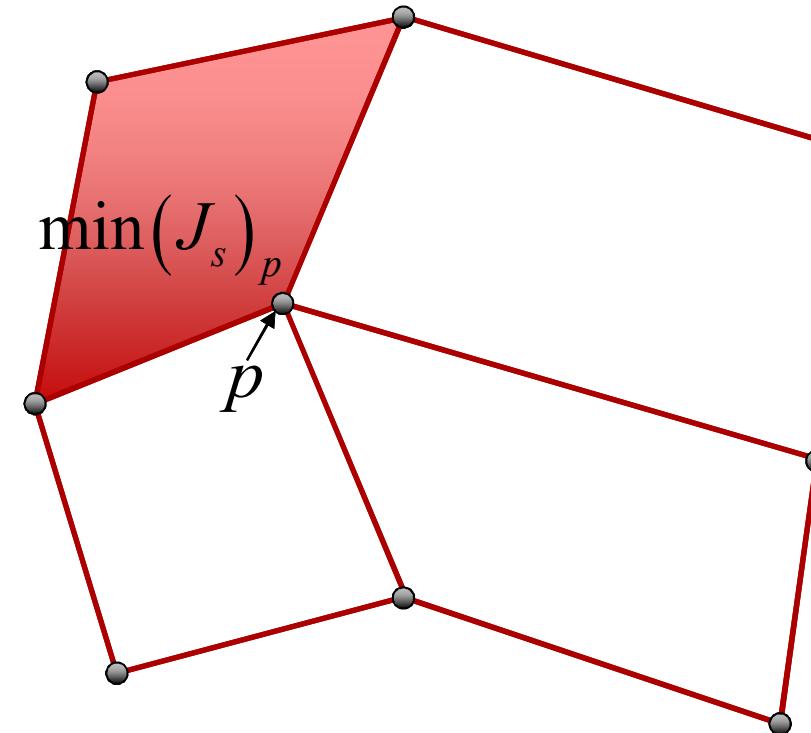
$$J_s = \min ((J_s)_I, I = 0, 1, \dots 7)$$

“Acceptable”

$$J_s \geq 0.2$$

Optimization

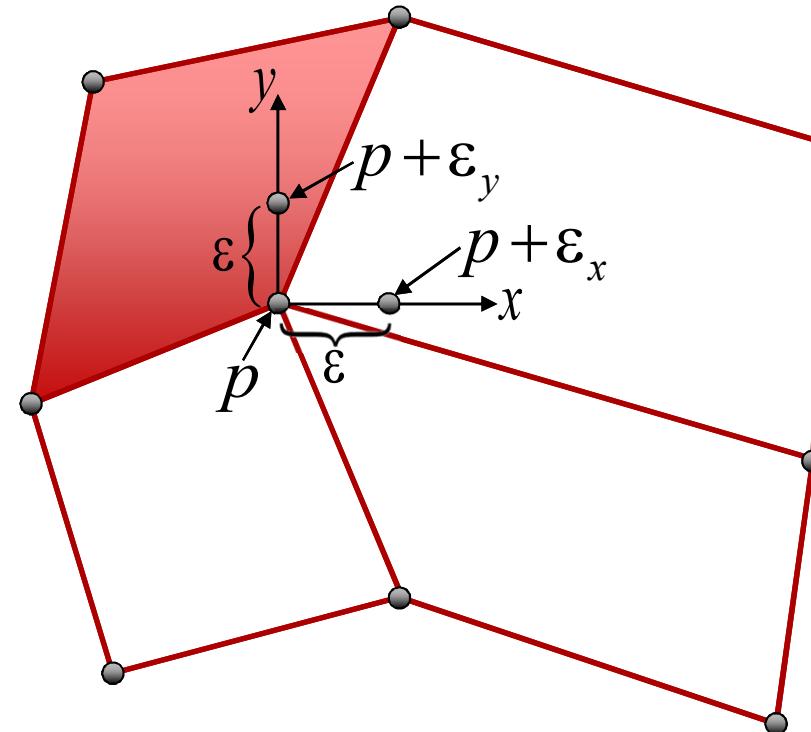
Compute minimum scaled
Jacobian, $(J_s)_p$, of node p in all
attached hexes



Optimization

Compute minimum scaled Jacobian, $(J_s)_p$, of node p in all attached hexes

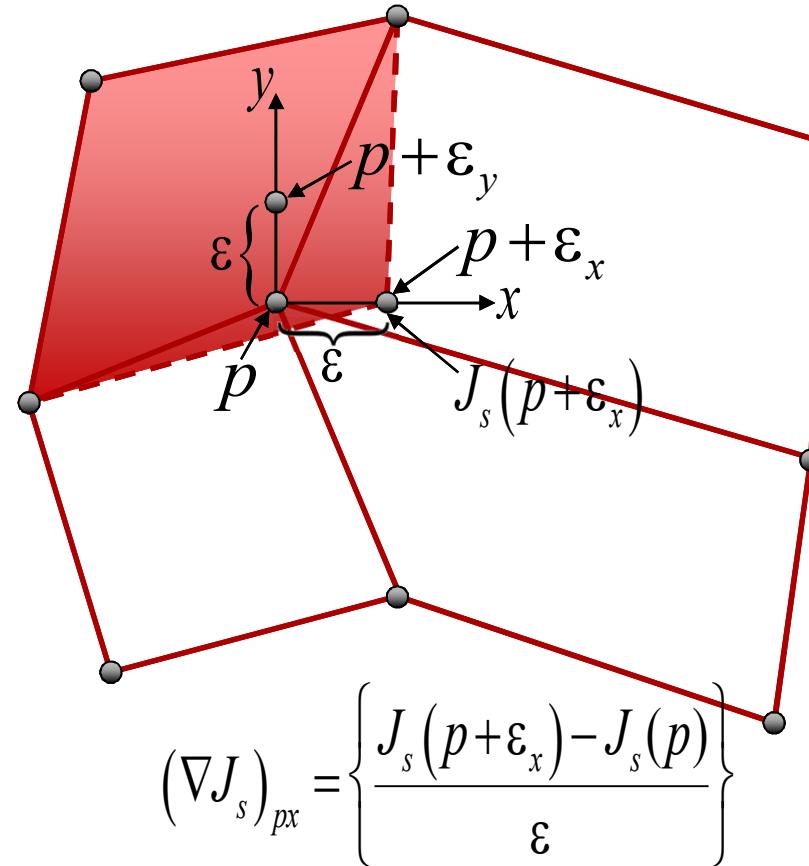
Compute numerical gradient $(\nabla J_s)_p$



Optimization

Compute minimum scaled Jacobian, $(J_s)_p$, of node p in all attached hexes

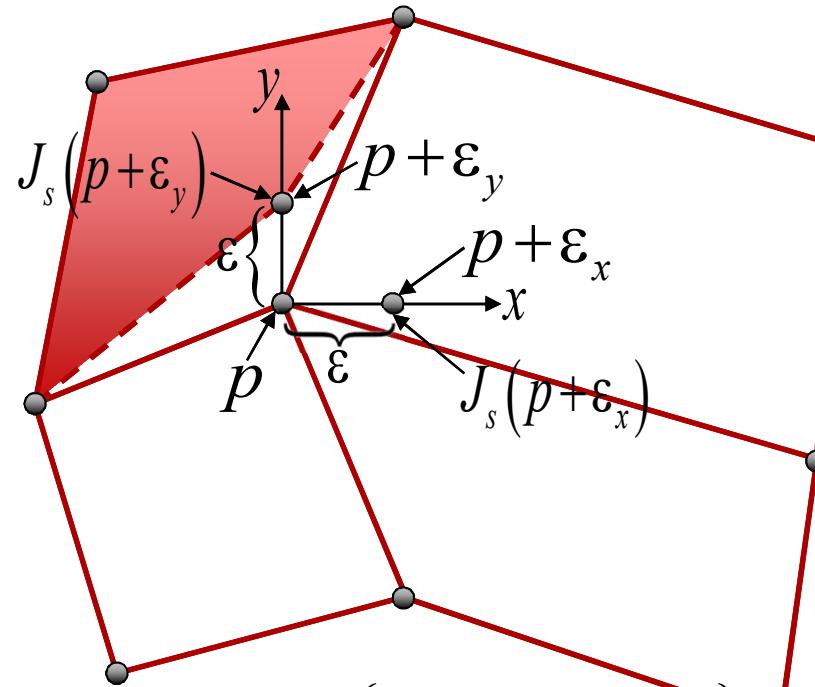
Compute numerical gradient $(\nabla J_s)_p$



Optimization

Compute minimum scaled Jacobian, $(J_s)_p$, of node p in all attached hexes

Compute numerical gradient $(\nabla J_s)_p$



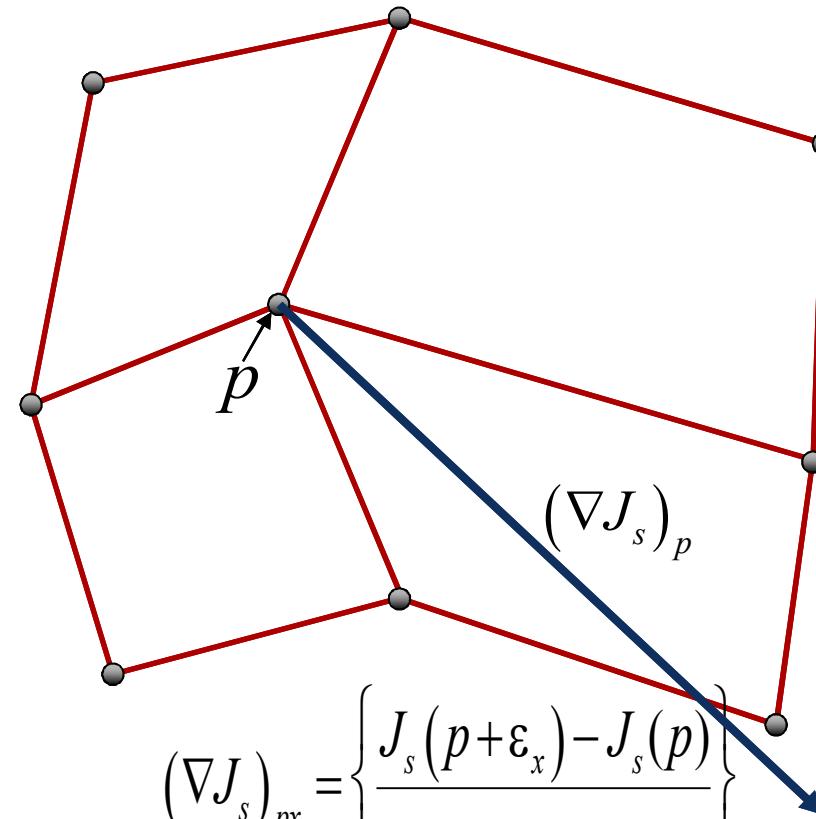
$$(\nabla J_s)_{px} = \left\{ \frac{J_s(p + \epsilon_x) - J_s(p)}{\epsilon} \right\}$$

$$(\nabla J_s)_{py} = \left\{ \frac{J_s(p + \epsilon_y) - J_s(p)}{\epsilon} \right\}$$

Optimization

Compute minimum scaled Jacobian, $(J_s)_p$, of node p in all attached hexes

Compute numerical gradient $(\nabla J_s)_p$



$$(\nabla J_s)_{px} = \left\{ \frac{J_s(p + \varepsilon_x) - J_s(p)}{\varepsilon} \right\}$$

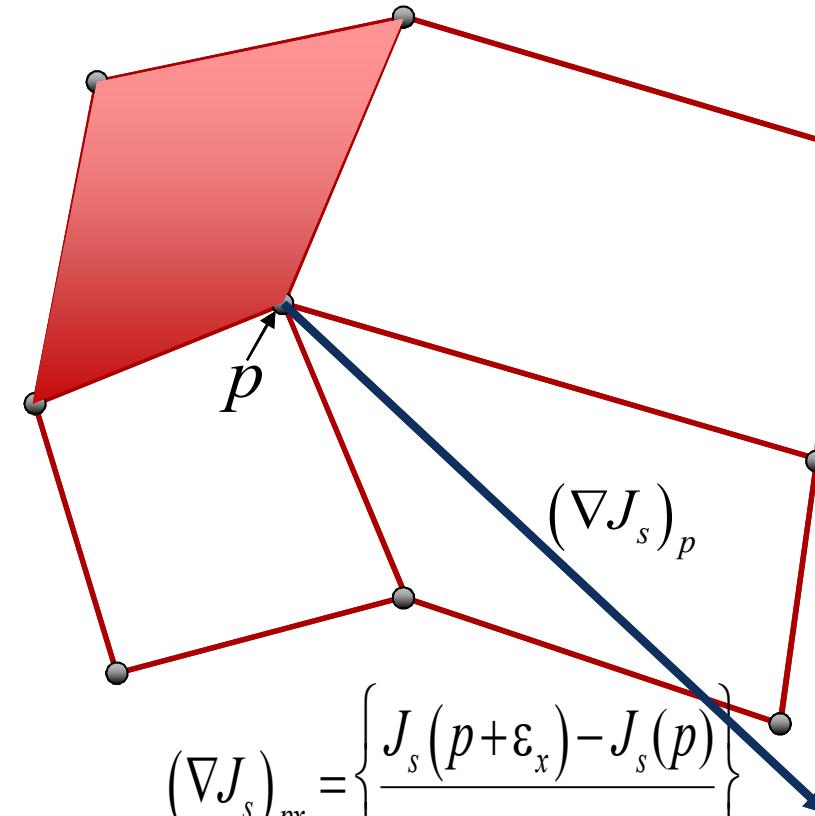
$$(\nabla J_s)_{py} = \left\{ \frac{J_s(p + \varepsilon_y) - J_s(p)}{\varepsilon} \right\}$$

Optimization

Compute minimum scaled Jacobian, $(J_s)_p$, of node p in all attached hexes

Compute numerical gradient $(\nabla J_s)_p$

Find improved $(J_s)_p$ by searching along vector $(\nabla J_s)_p$



$$(\nabla J_s)_{px} = \left\{ \frac{J_s(p + \varepsilon_x) - J_s(p)}{\varepsilon} \right\}$$

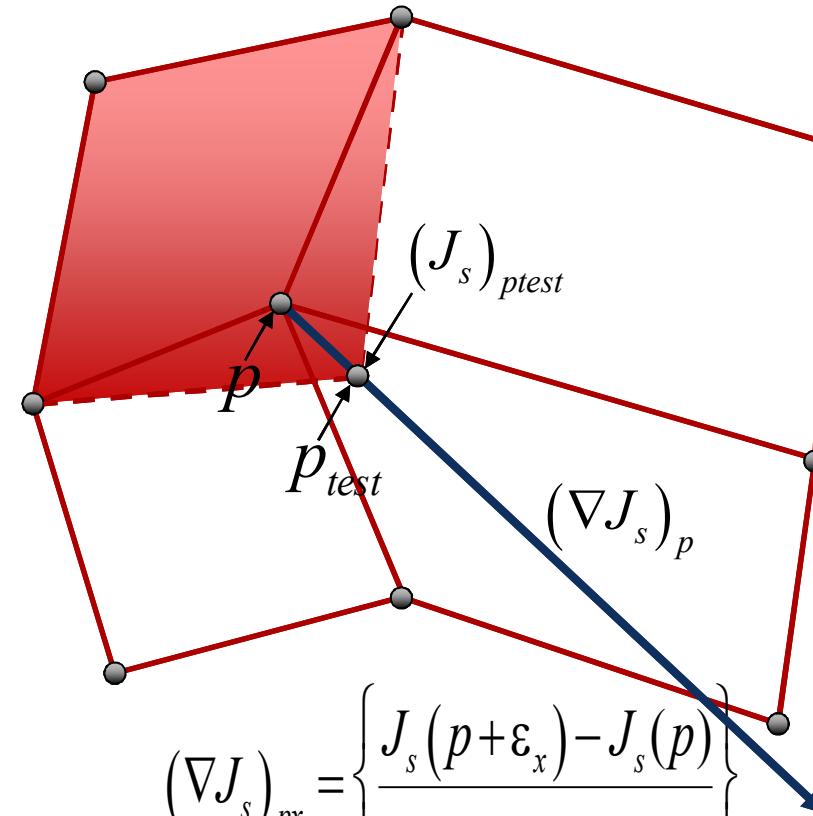
$$(\nabla J_s)_{py} = \left\{ \frac{J_s(p + \varepsilon_y) - J_s(p)}{\varepsilon} \right\}$$

Optimization

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$$(\nabla J_s)_{px} = \left\{ \frac{J_s(p + \varepsilon_x) - J_s(p)}{\varepsilon} \right\}$$

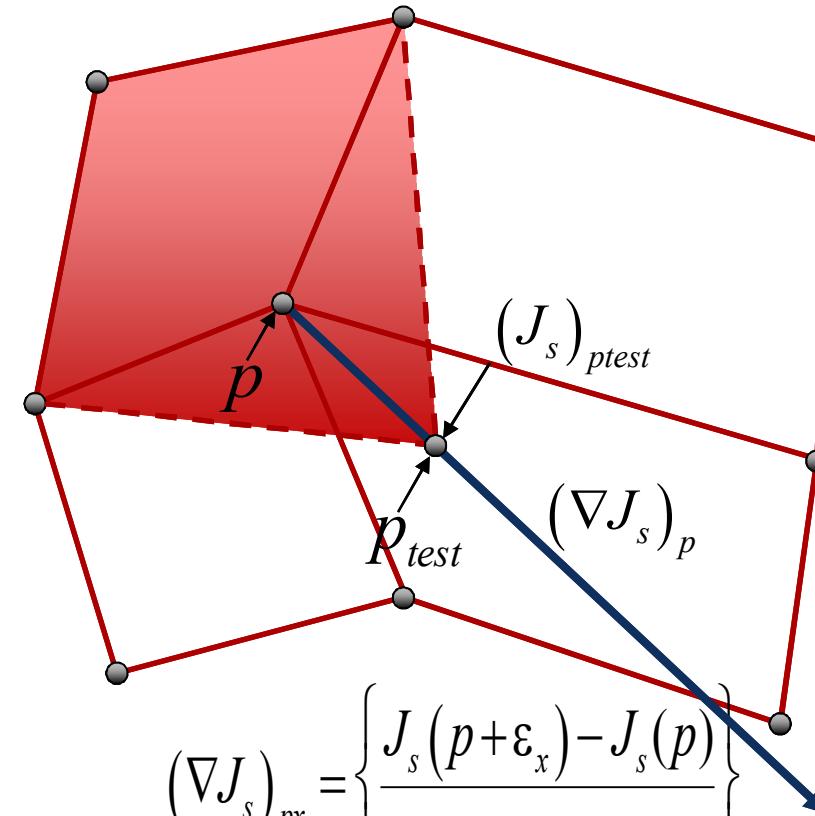
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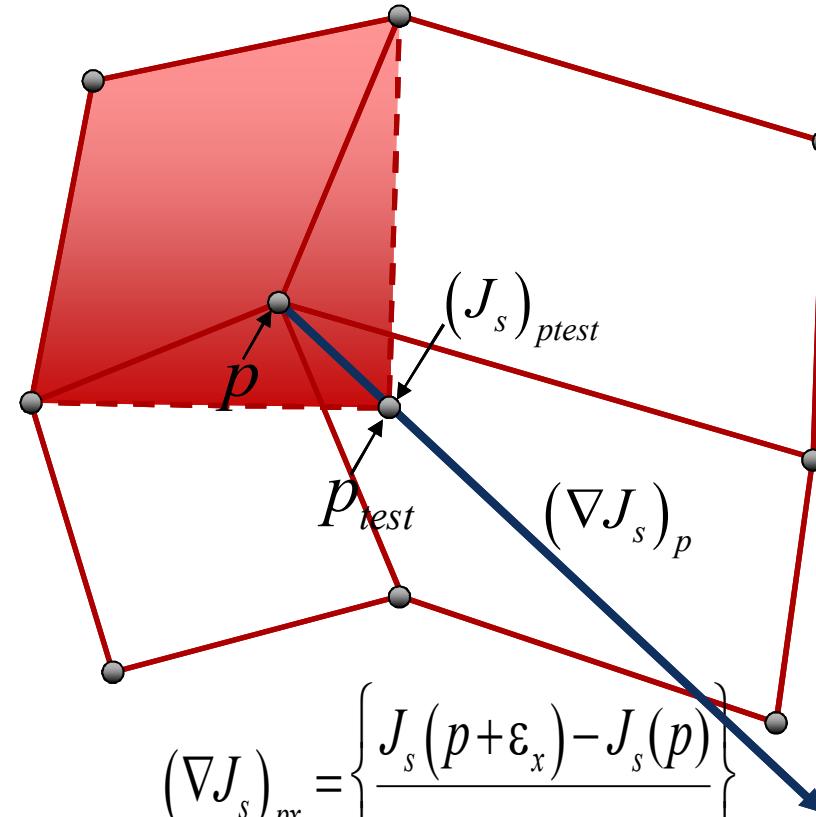
$$(\nabla J_s)_{py} = \left\{ \frac{J_s(p + \varepsilon_y) - J_s(p)}{\varepsilon} \right\}$$

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$$(\nabla J_s)_{px} = \left\{ \frac{J_s(p + \varepsilon_x) - J_s(p)}{\varepsilon} \right\}$$

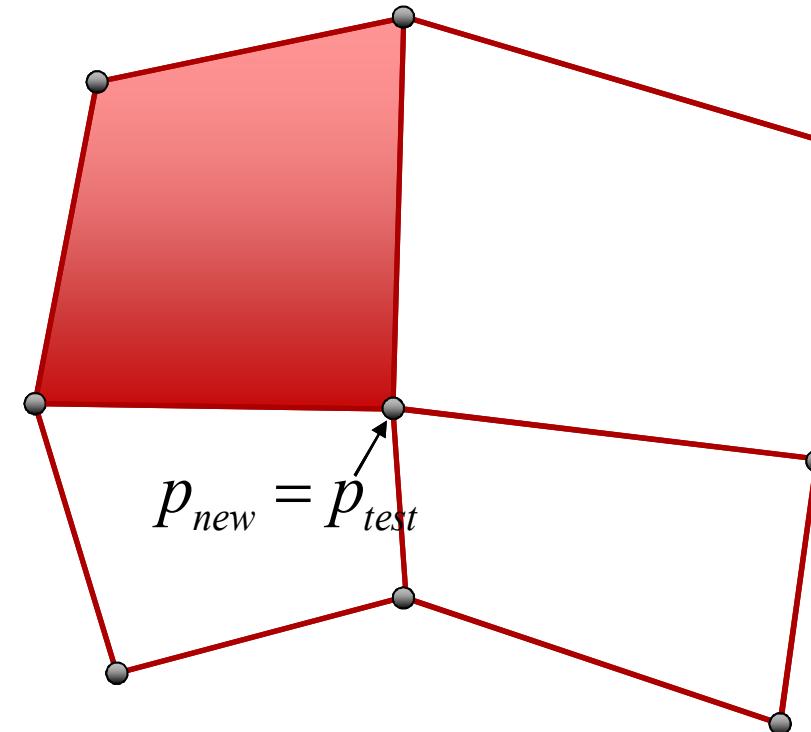
$$(\nabla J_s)_{py} = \left\{ \frac{J_s(p + \varepsilon_y) - J_s(p)}{\varepsilon} \right\}$$

Optimization

Compute minimum scaled Jacobian, $(J_s)_p$, of node p in all attached hexes

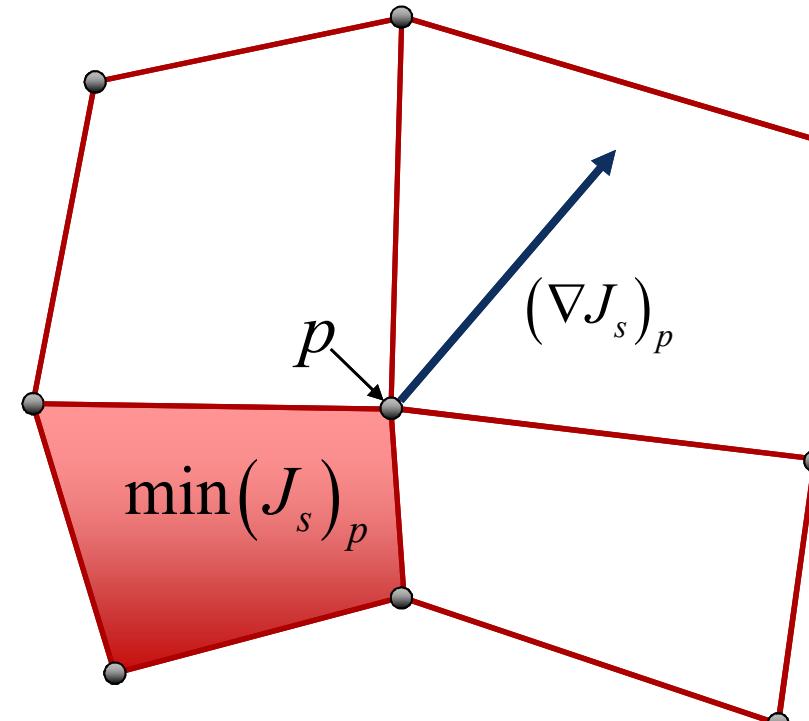
Compute numerical gradient $(\nabla J_s)_p$

Find improved $(J_s)_p$ by searching along vector $(\nabla J_s)_p$



Optimization

- Compute minimum scaled Jacobian, $(J_s)_p$, of node p in all attached hexes
- Compute numerical gradient $(\nabla J_s)_p$
- Find improved $(J_s)_p$ by searching along vector $(\nabla J_s)_p$



Stopping Criteria

$$(J_s)_p > 0.2$$

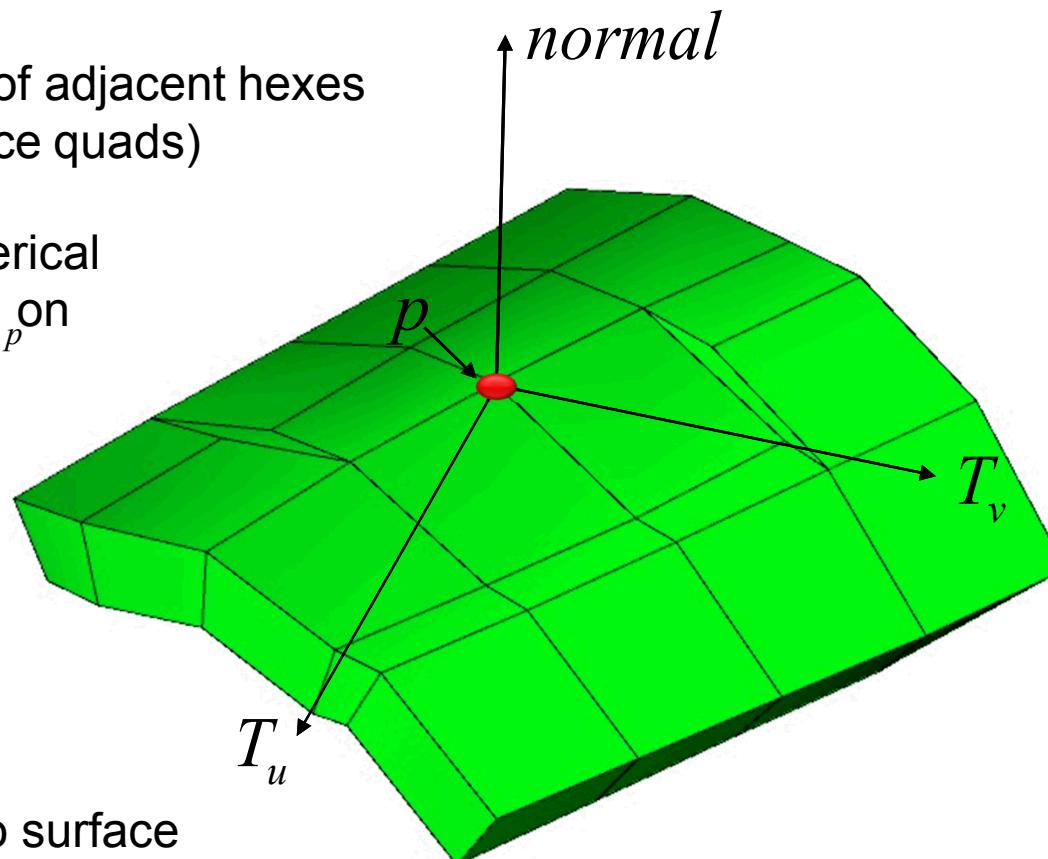
Maximum 3 iterations

Optimization for Surface Nodes

Optimize $(J_s)_p$ of adjacent hexes
to p (not surface quads)

Compute numerical
gradient $(\nabla J_s)_p$ on
tangent plane

Project p_{new} to surface

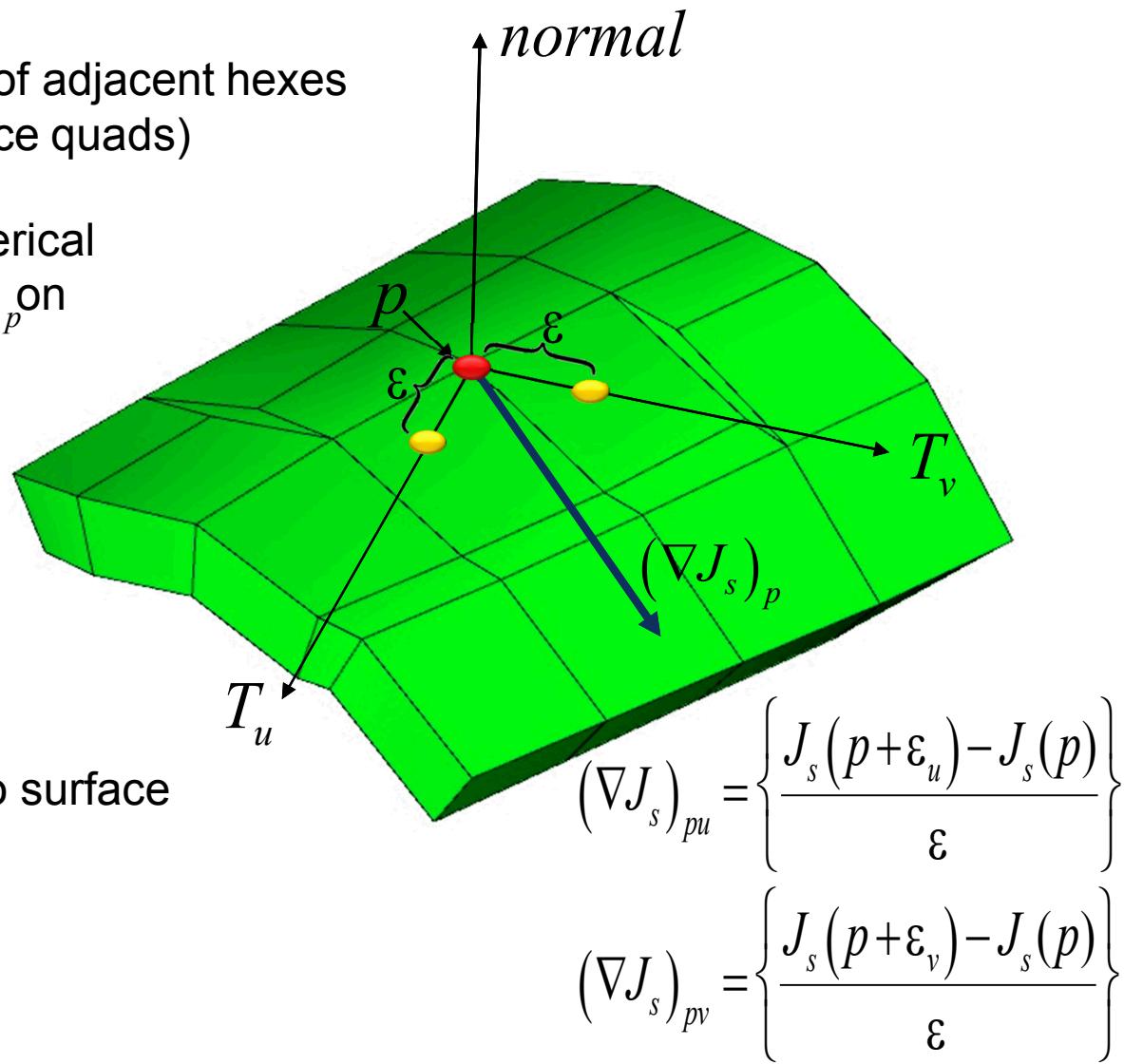


Optimization for Surface Nodes

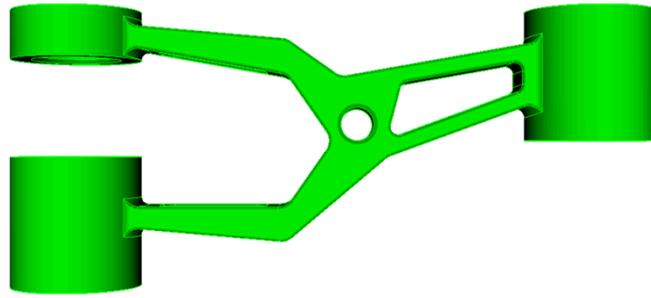
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gradient $(\nabla J_s)_p$ on
tangent plane

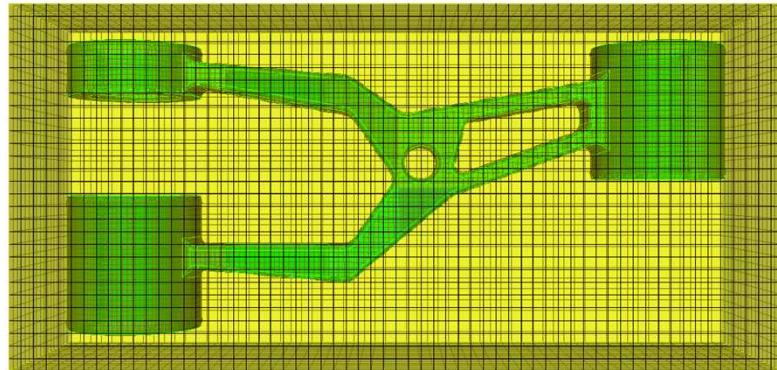
Project p_{new} to surface



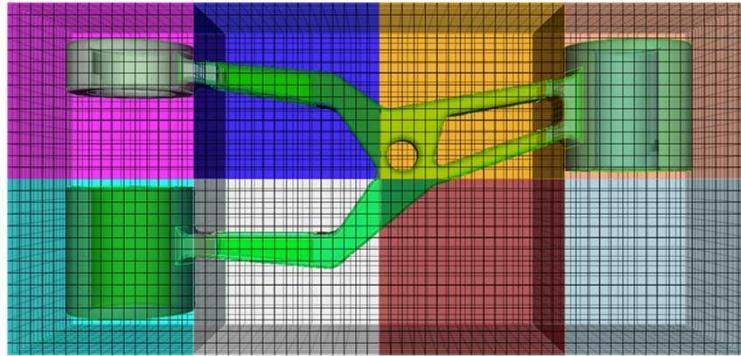
Distributed Meshing



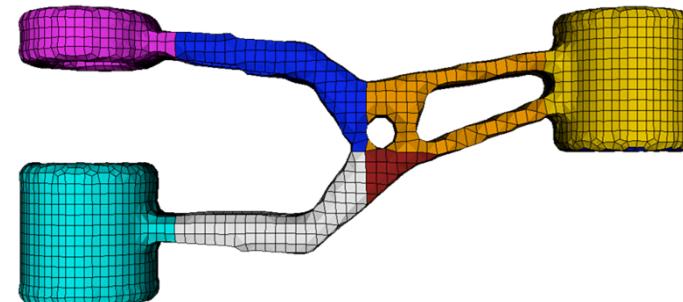
CAD geometry



Global overlay Cartesian grid

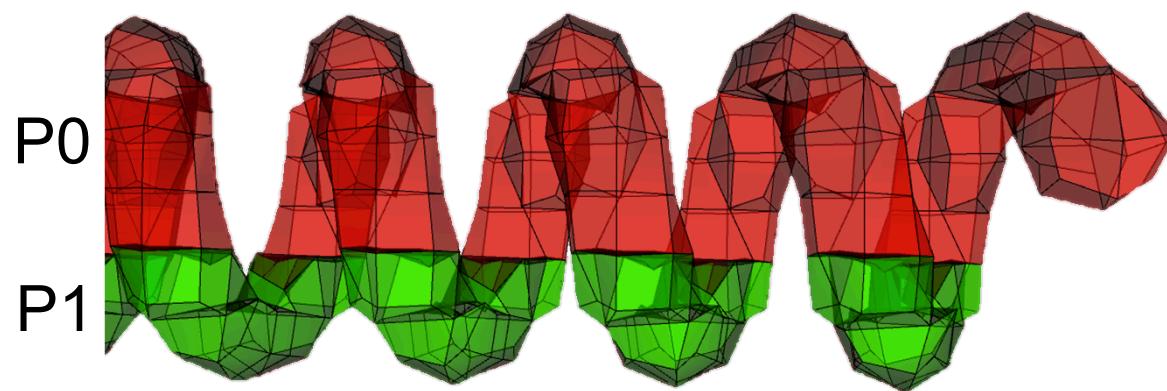


Cartesian grid decomposed and distributed amongst many processors

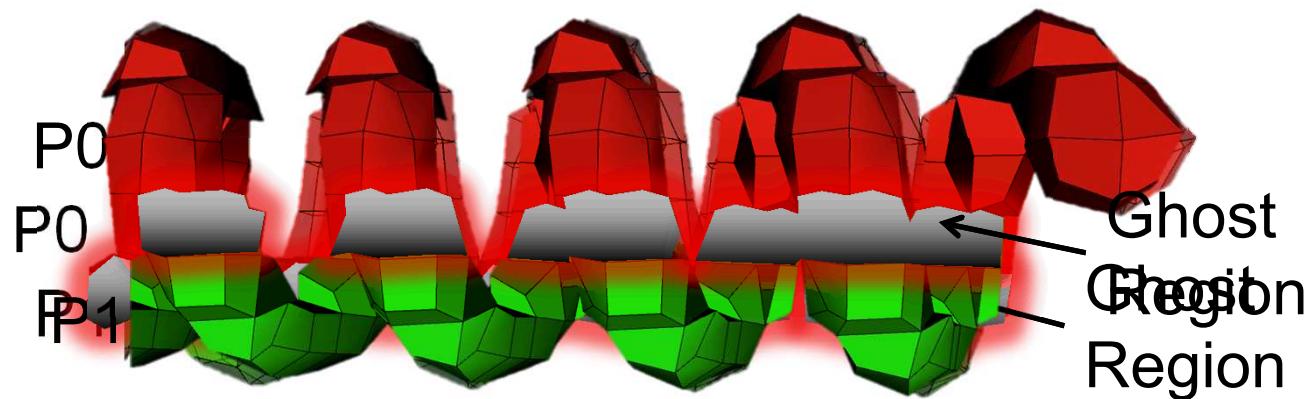


Each processor independently meshes its portion of Cartesian grid

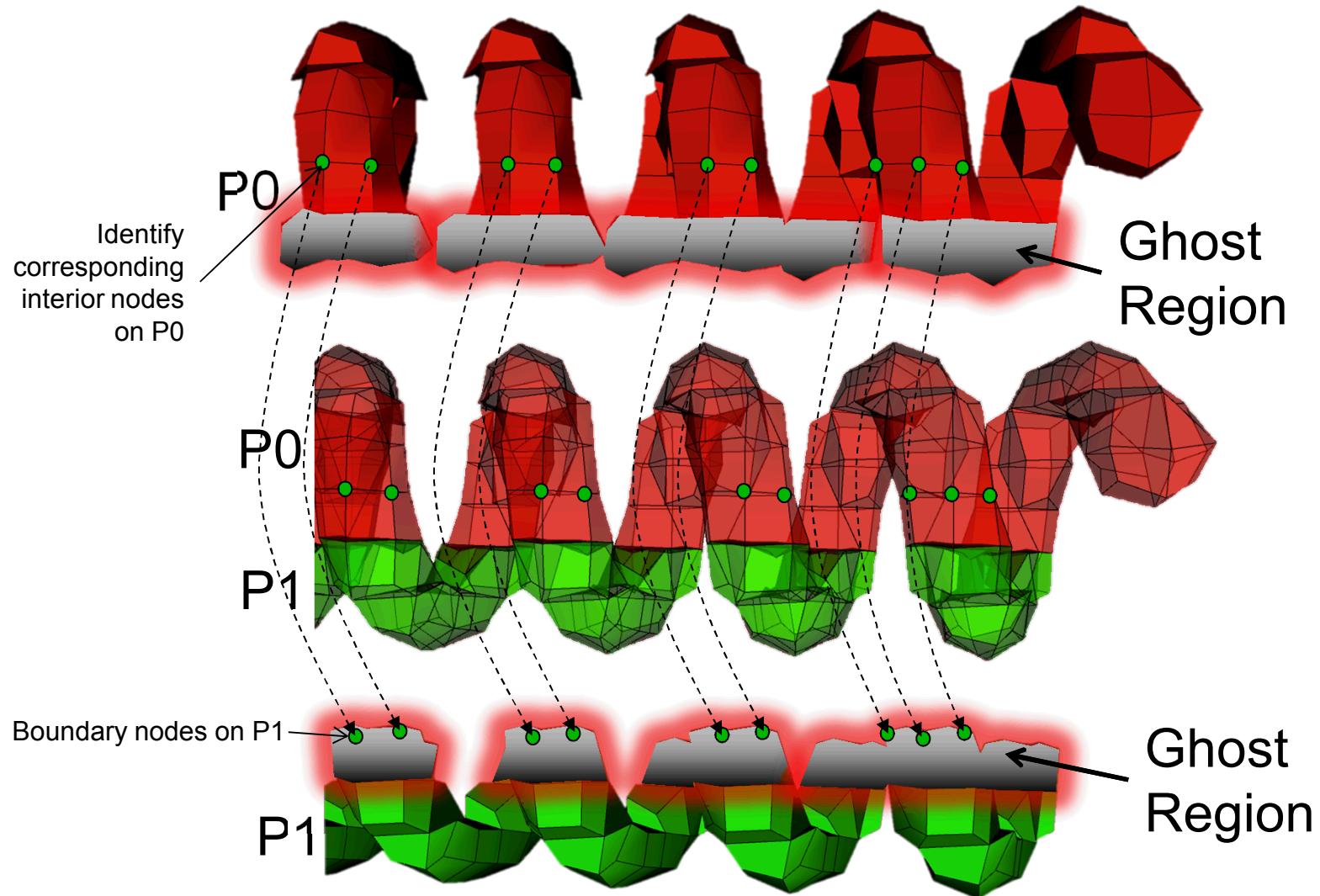
Communication for Smoothing



Communication for Smoothing



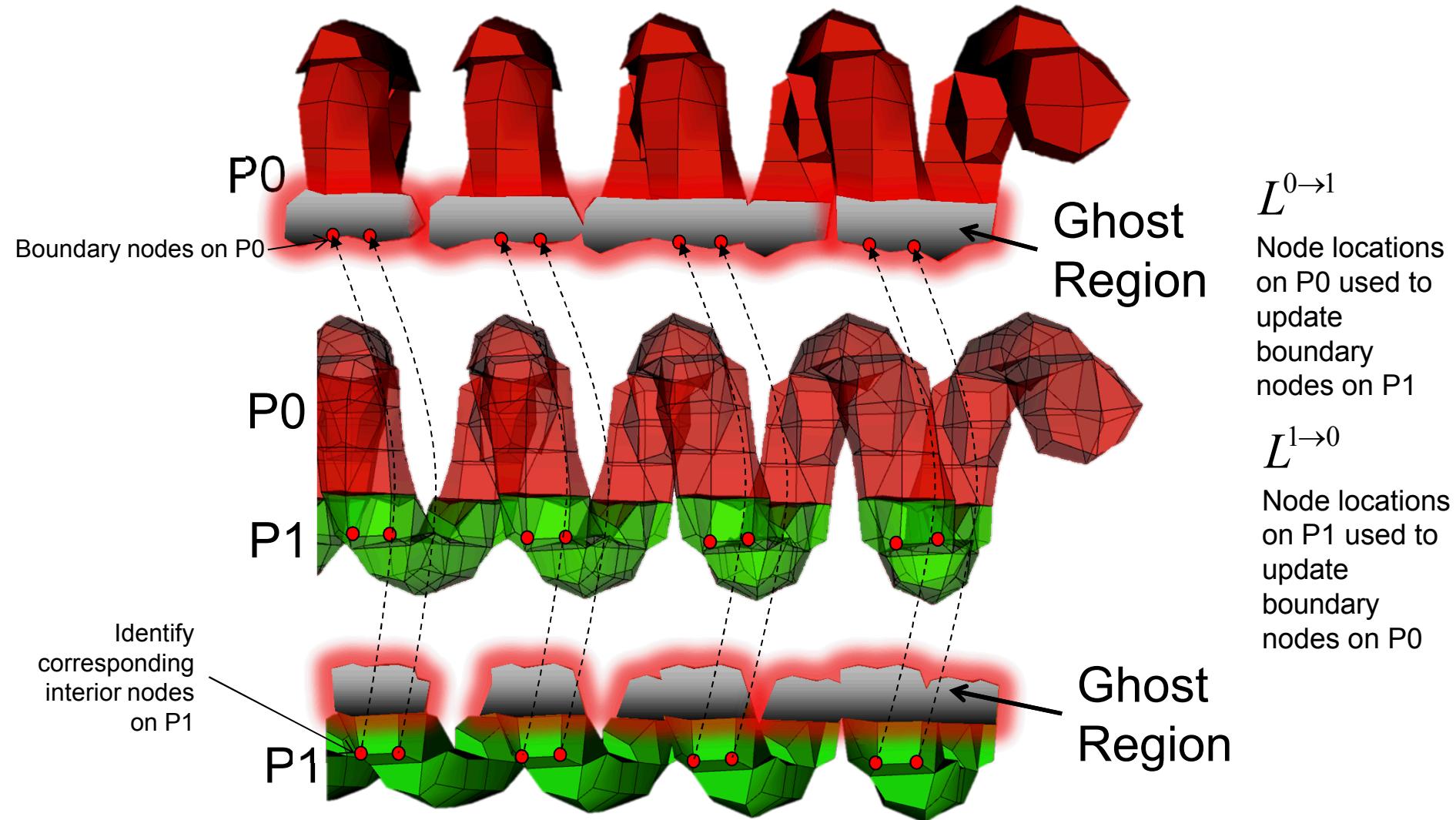
Communication for Smoothing



$$L^{0 \rightarrow 1}$$

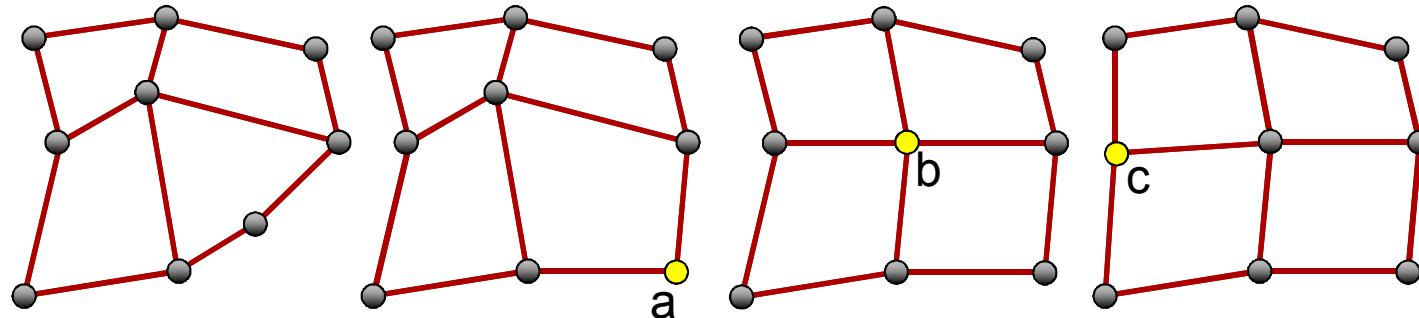
Node locations
on P_0 used to
update
boundary
nodes on P_1

Communication for Smoothing



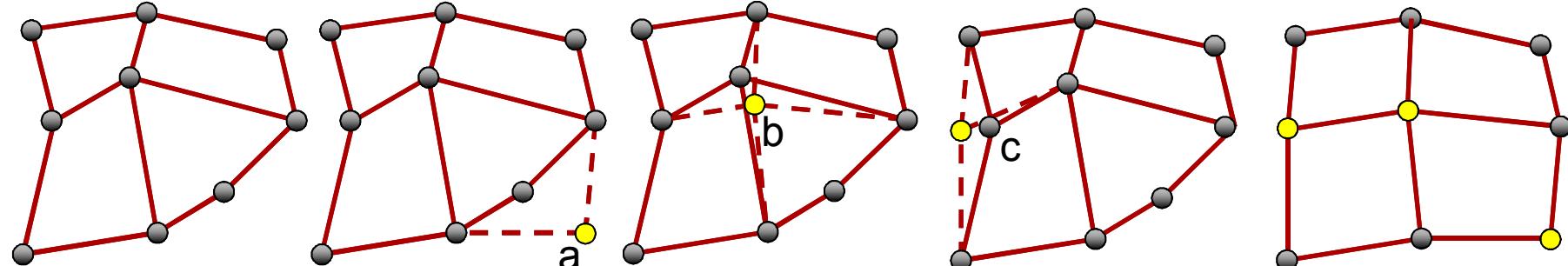
Smoothing Strategies

Gauss-Siedel



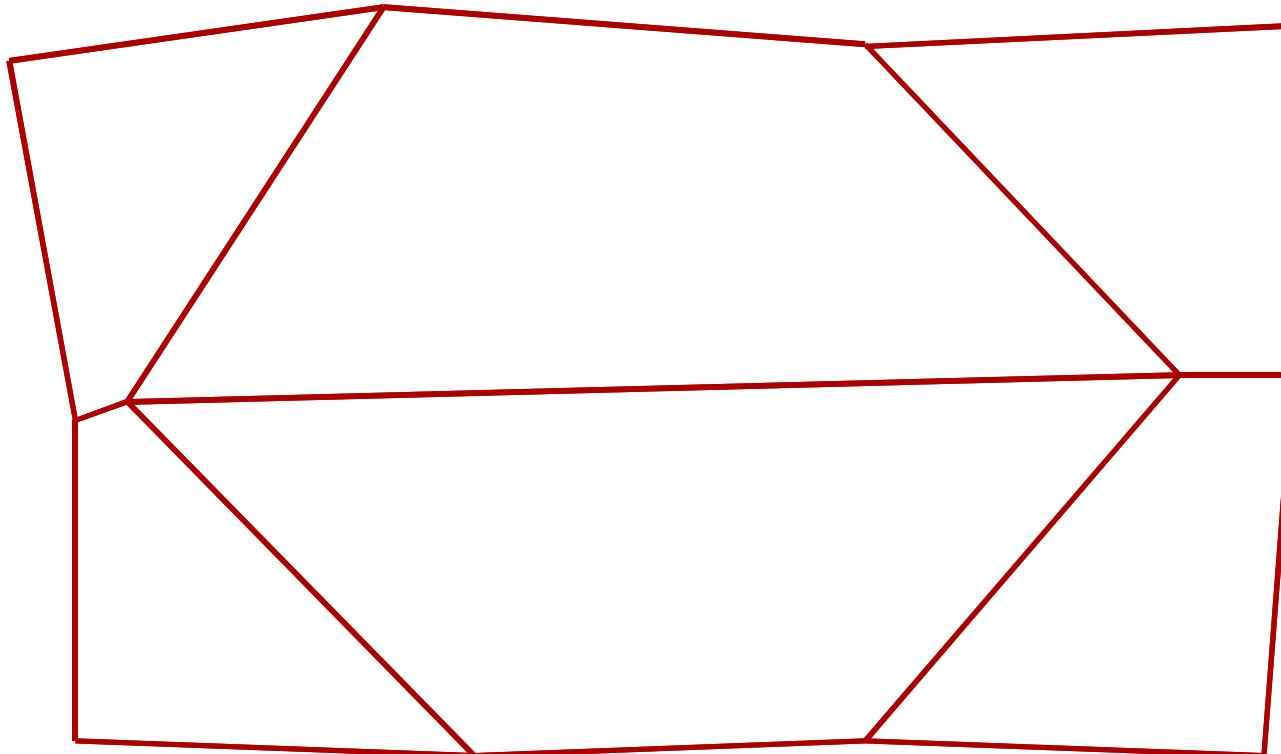
Order-dependent

Jacobi

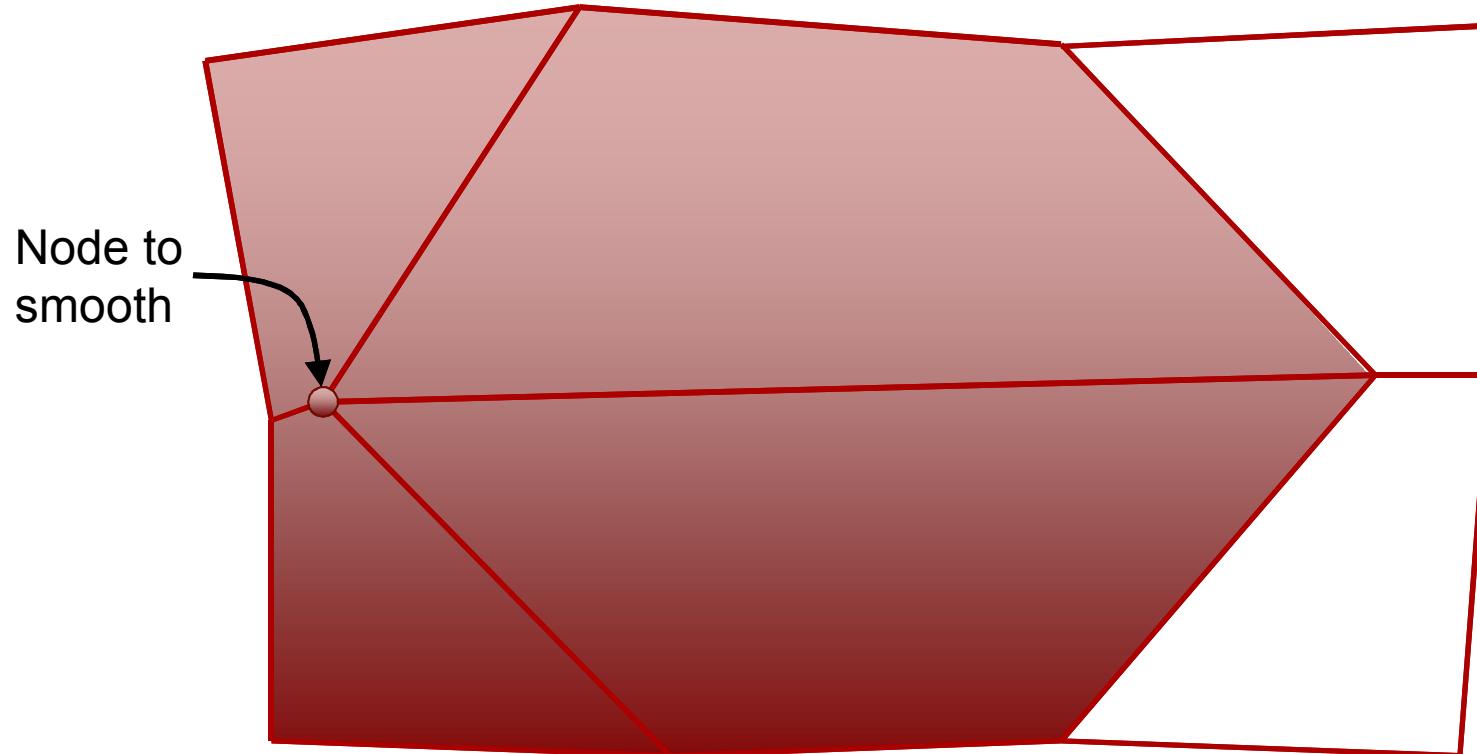


Order-independent

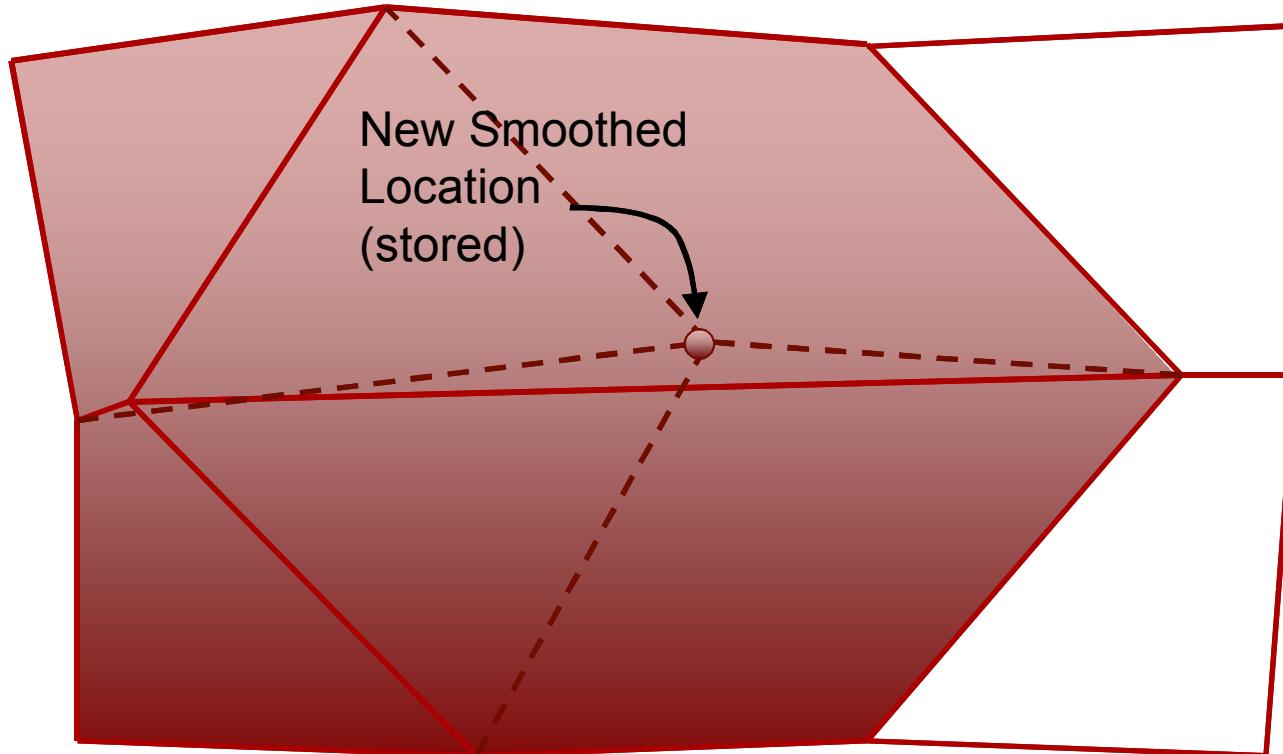
Jacobi Smoothing



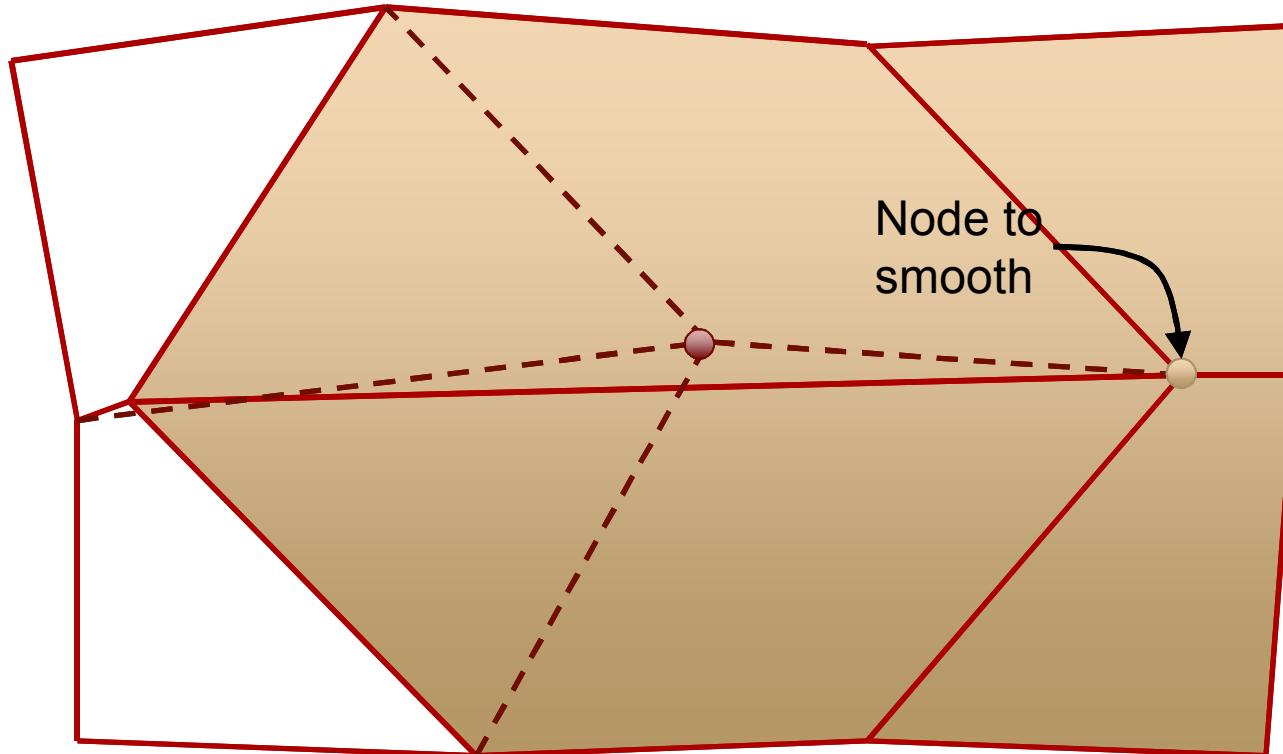
Jacobi Smoothing



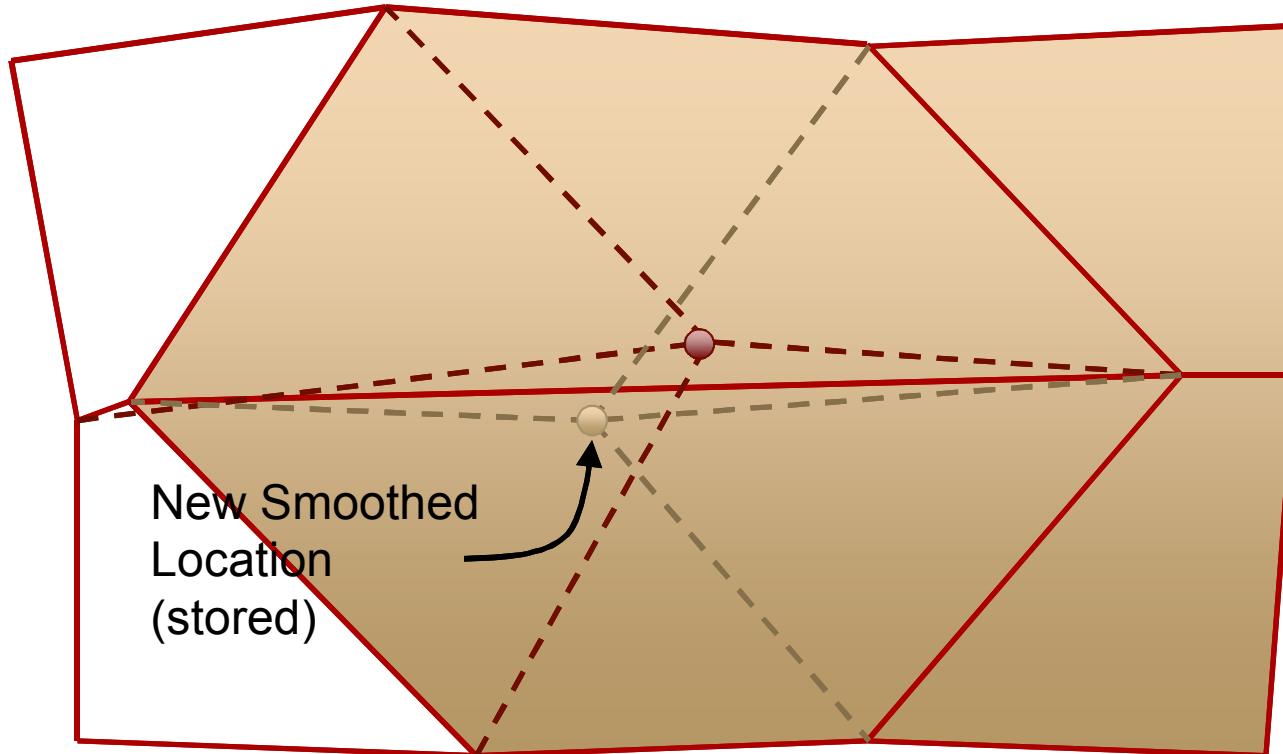
Jacobi Smoothing



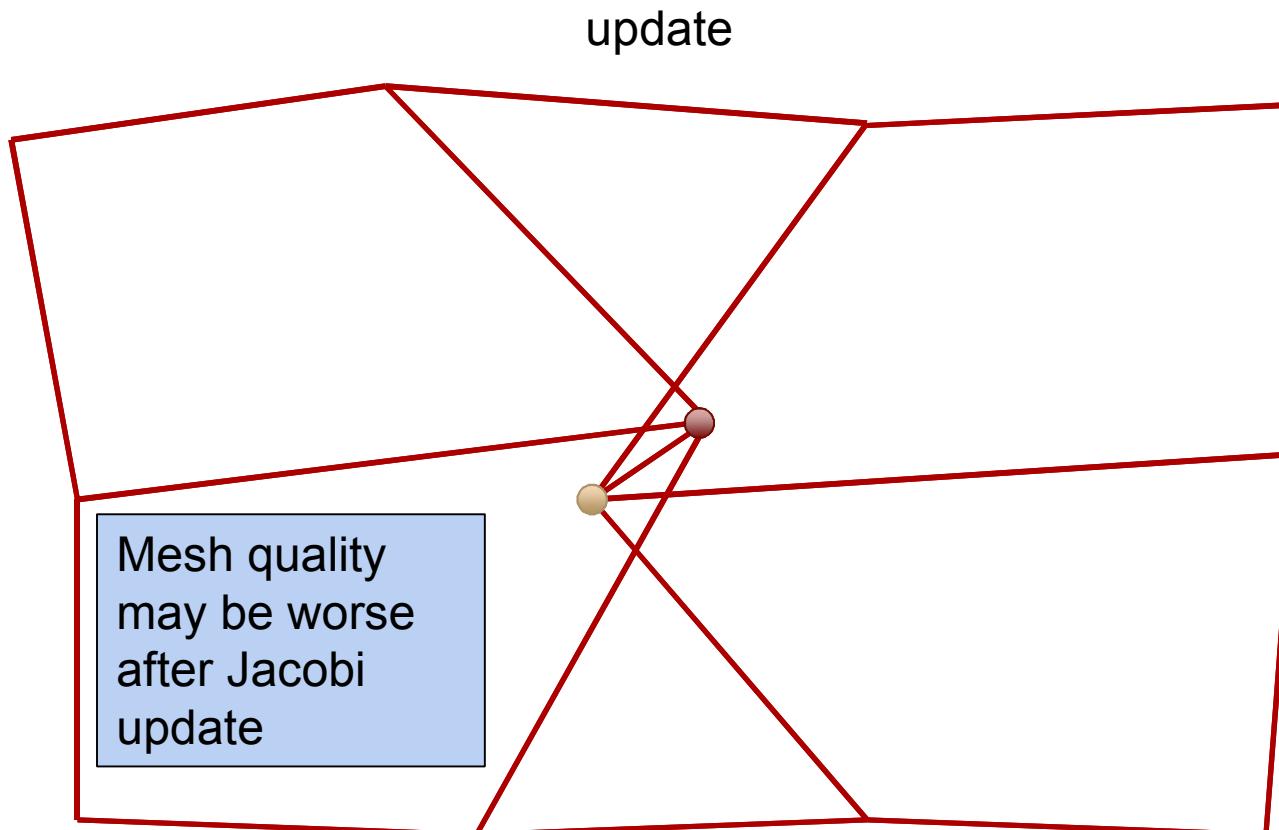
Jacobi Smoothing



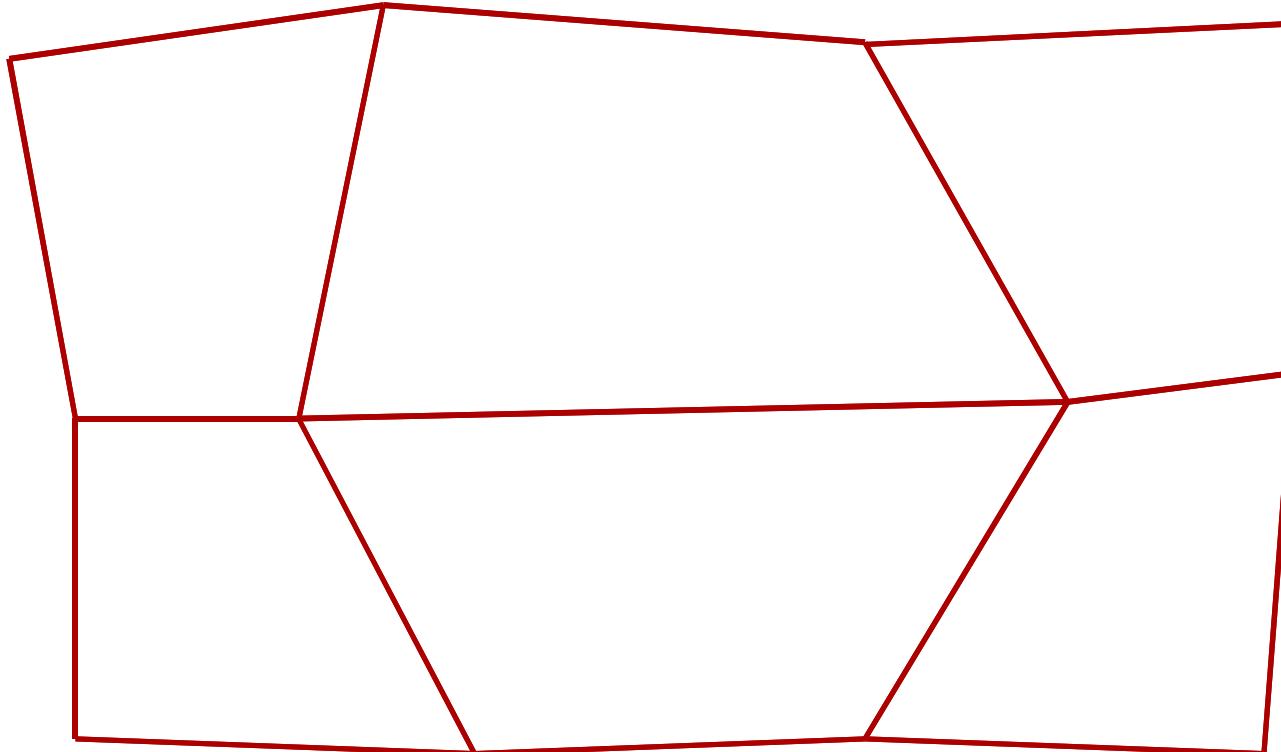
Jacobi Smoothing



Jacobi Smoothing

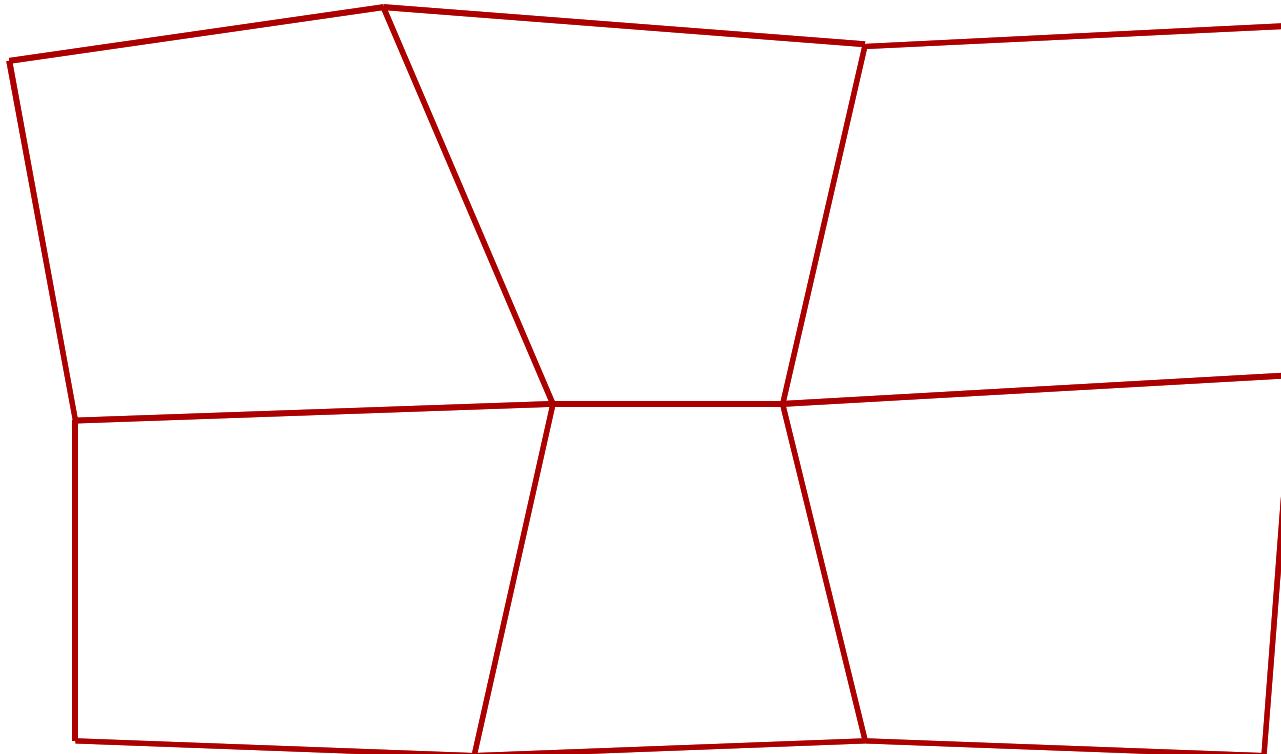


Jacobi Smoothing



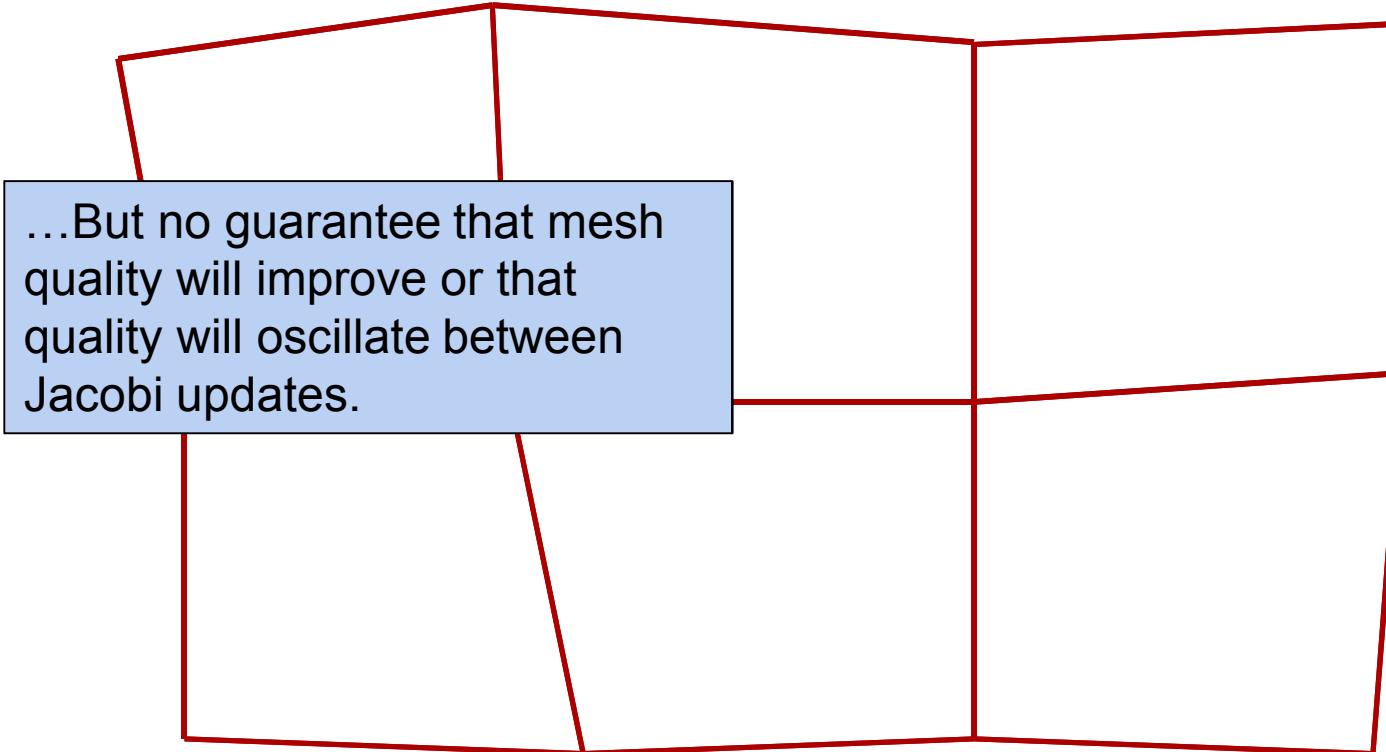
After several iterations, quality in *most* cases will improve

Jacobi Smoothing



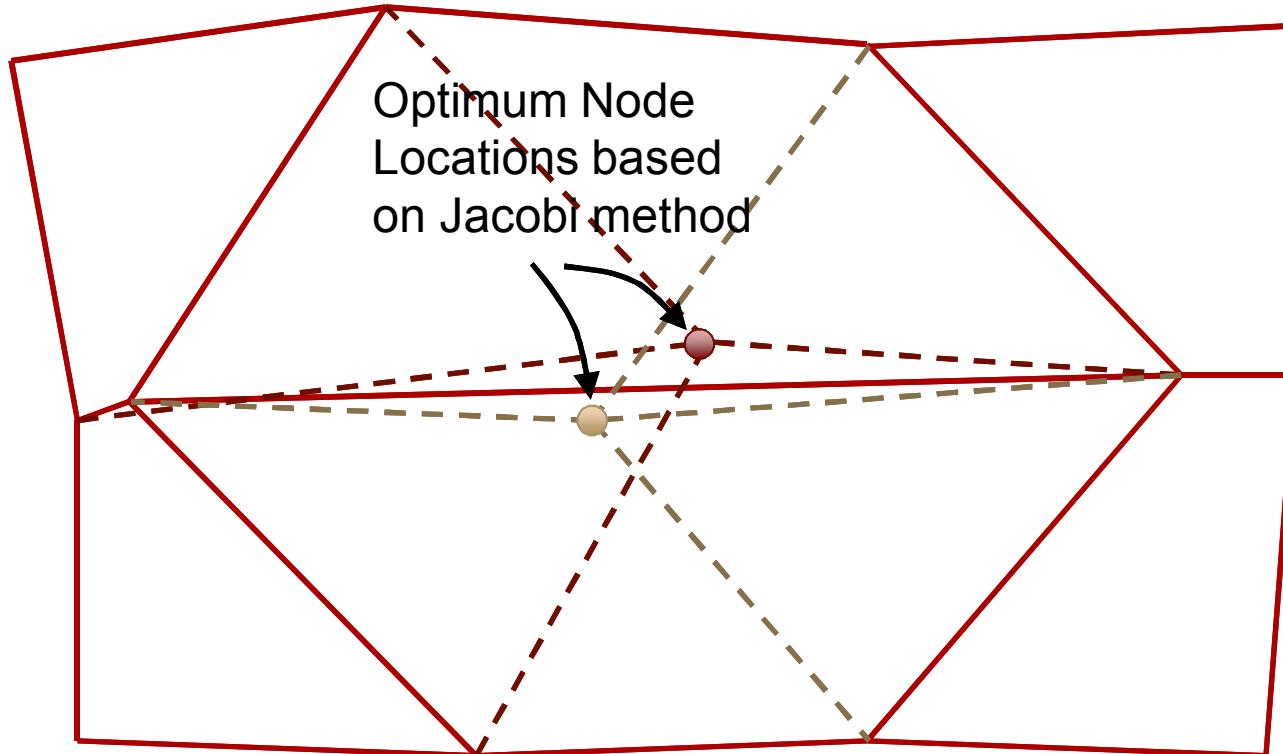
After several iterations, quality in *most* cases will improve

Jacobi Smoothing



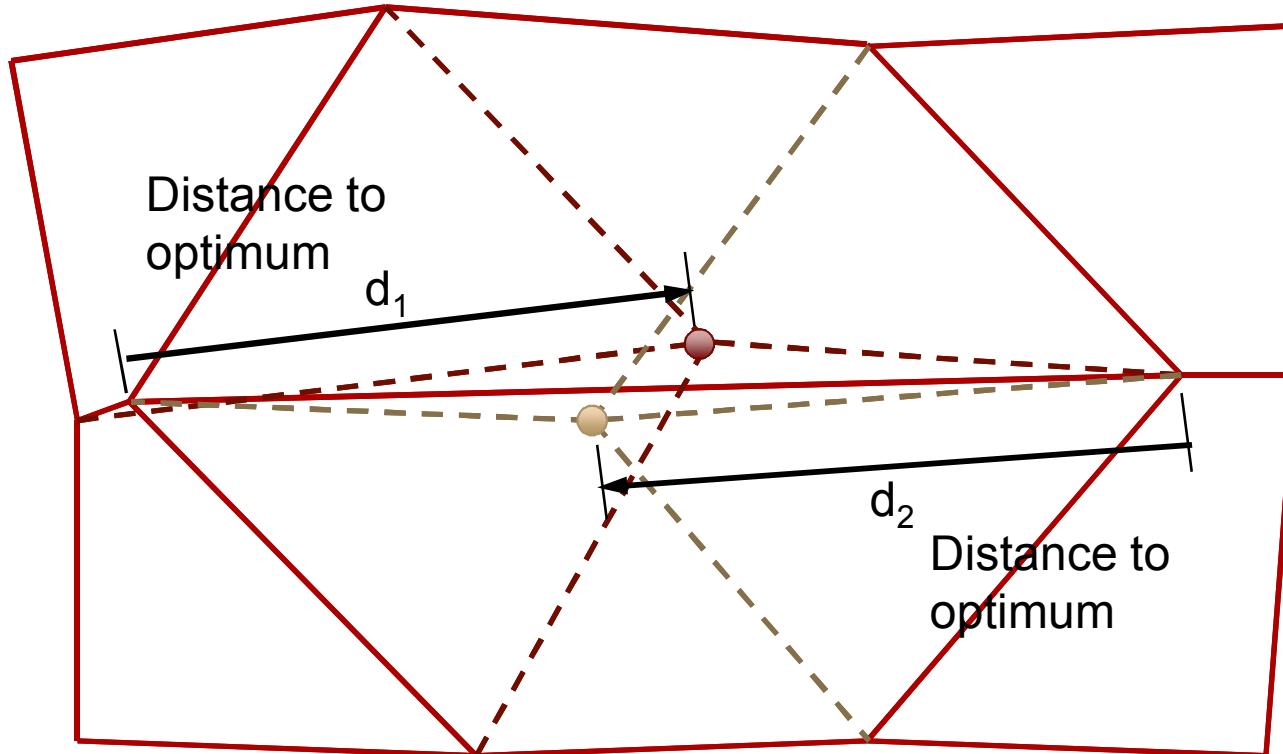
After several iterations, quality in *most* cases will improve

Jacobi Smoothing



Solution: Apply *damping* to node movement

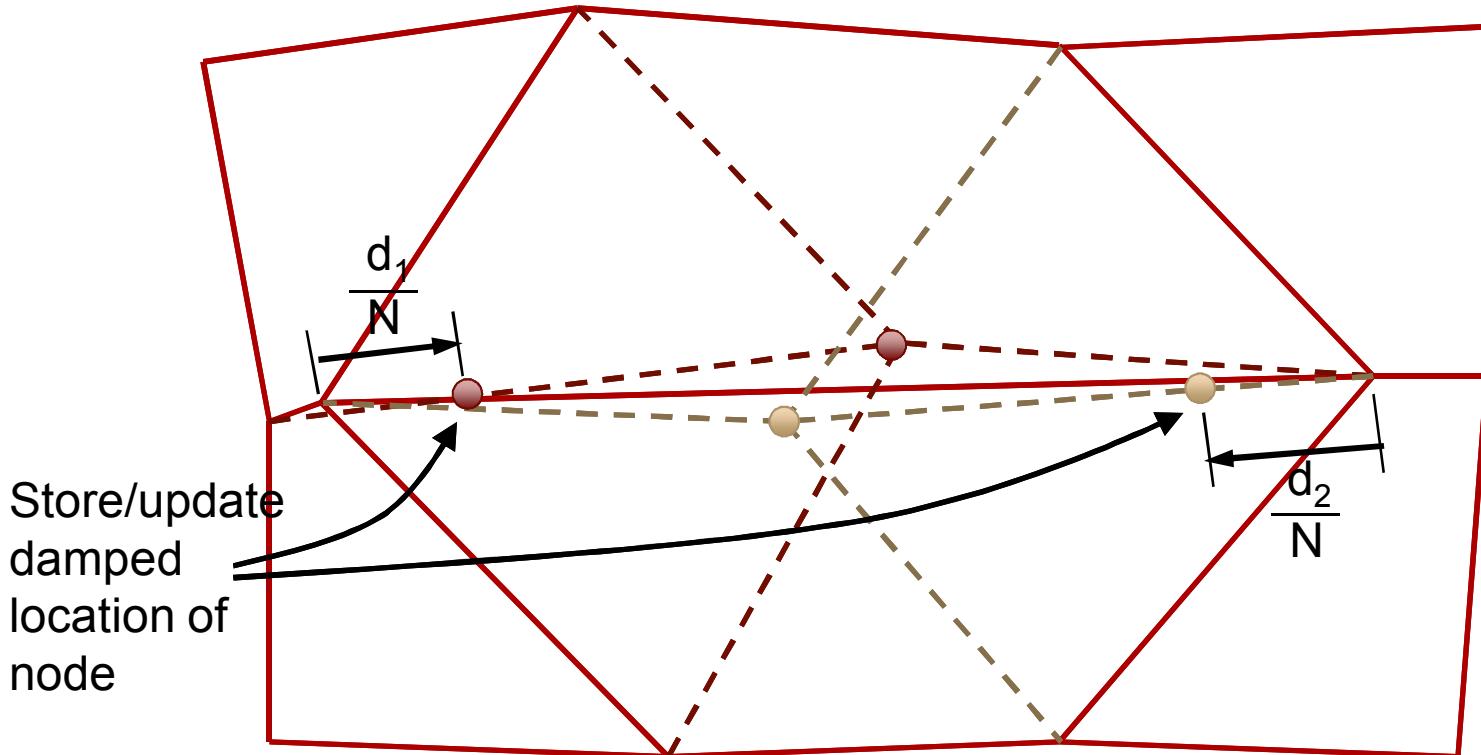
Jacobi Smoothing



Solution: Apply *damping* to node movement

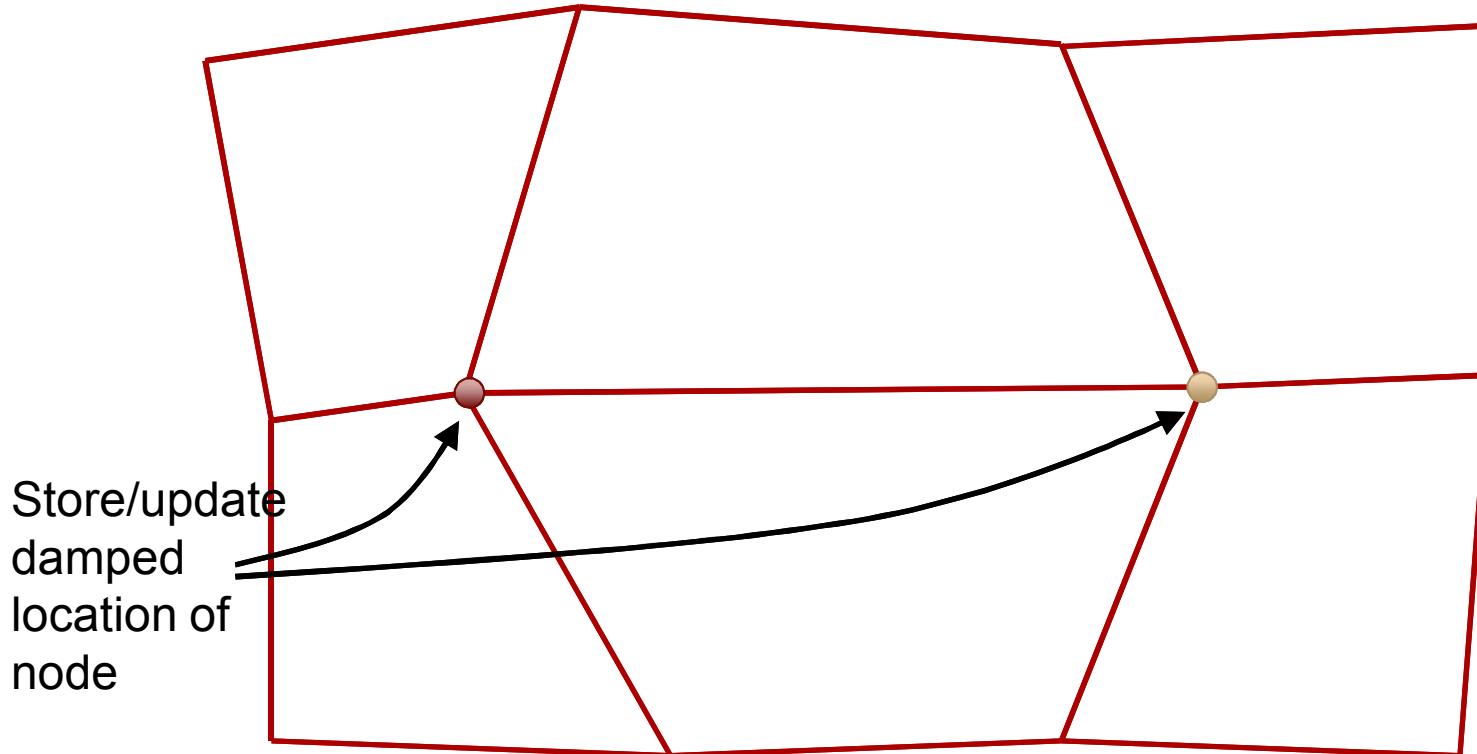
Jacobi Smoothing

N = number of remaining smoothing iterations



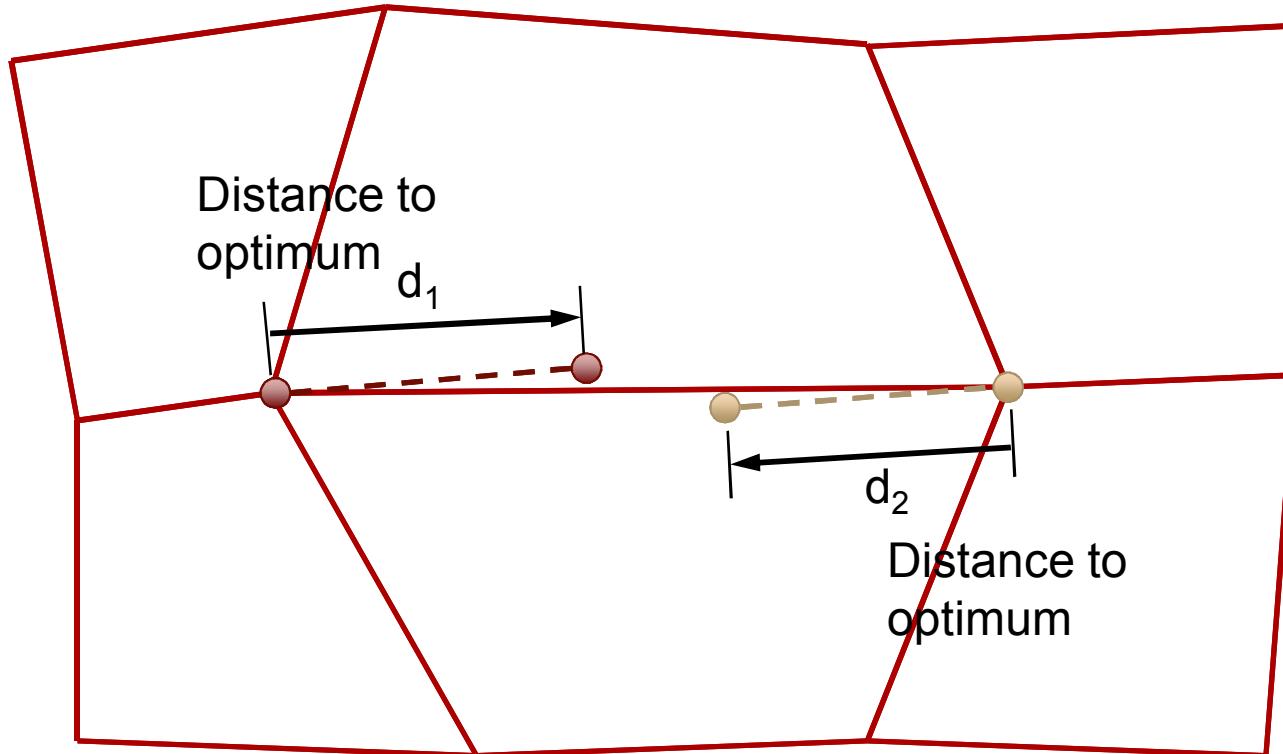
Solution: Apply *damping* to node movement

Jacobi Smoothing



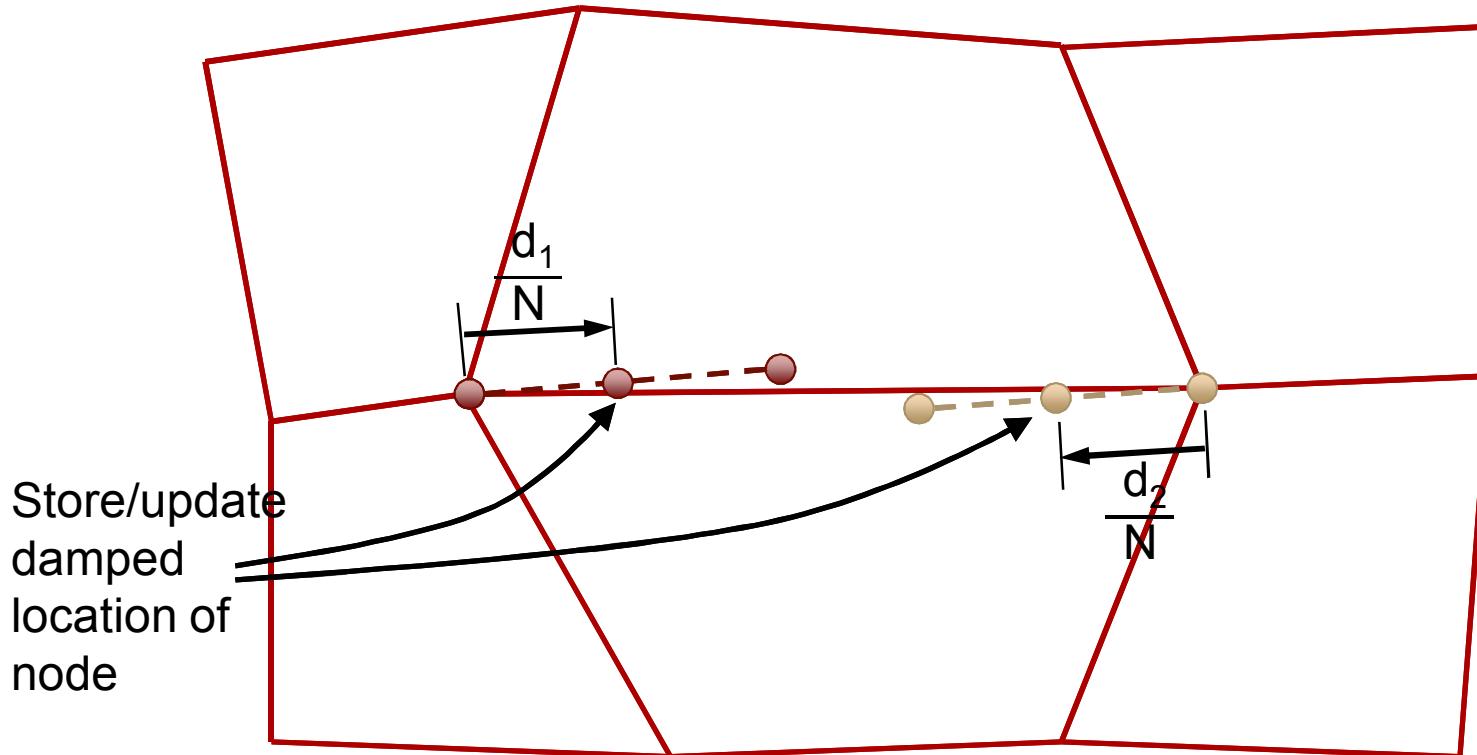
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Jacobi Smoothing



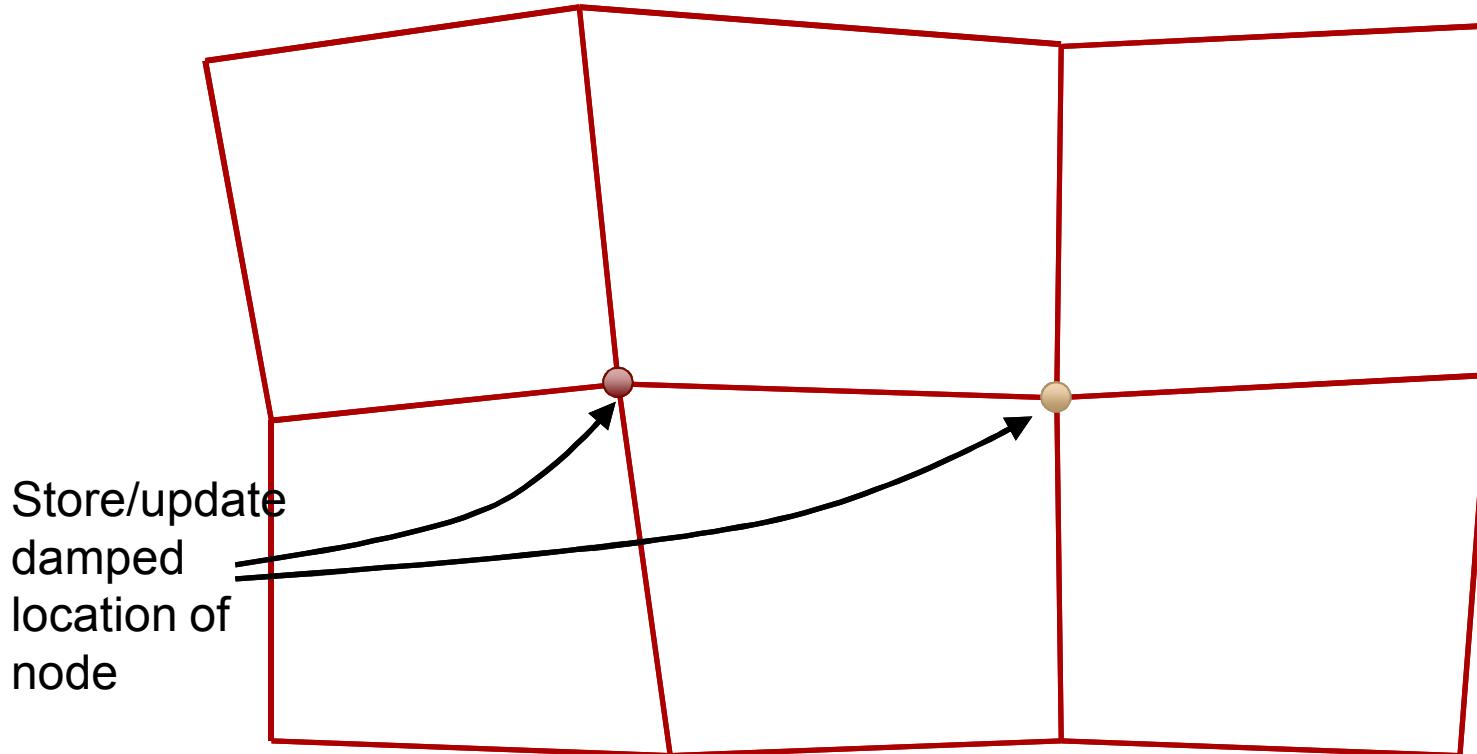
Jacobi Smoothing

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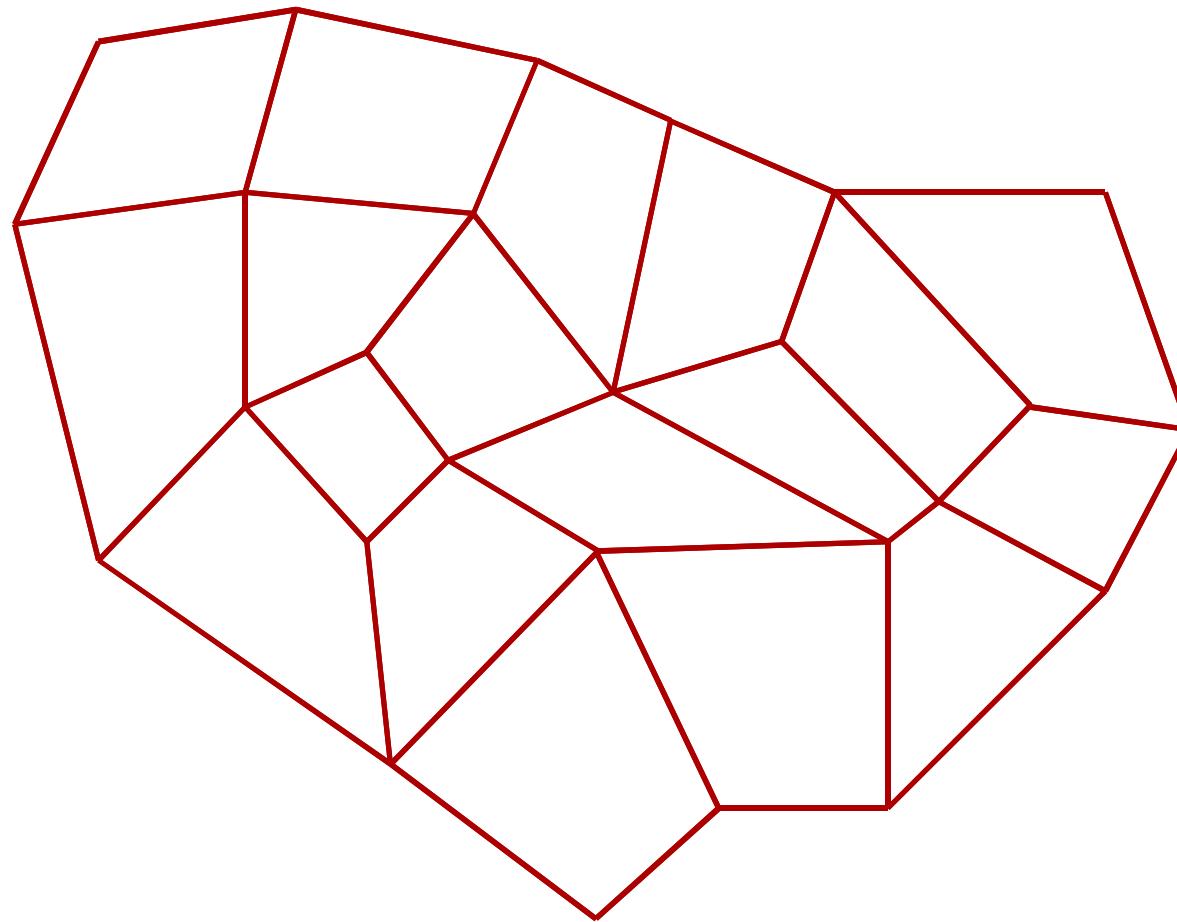
Solution: Apply *damping* to node movement

Jacobi Smoothing

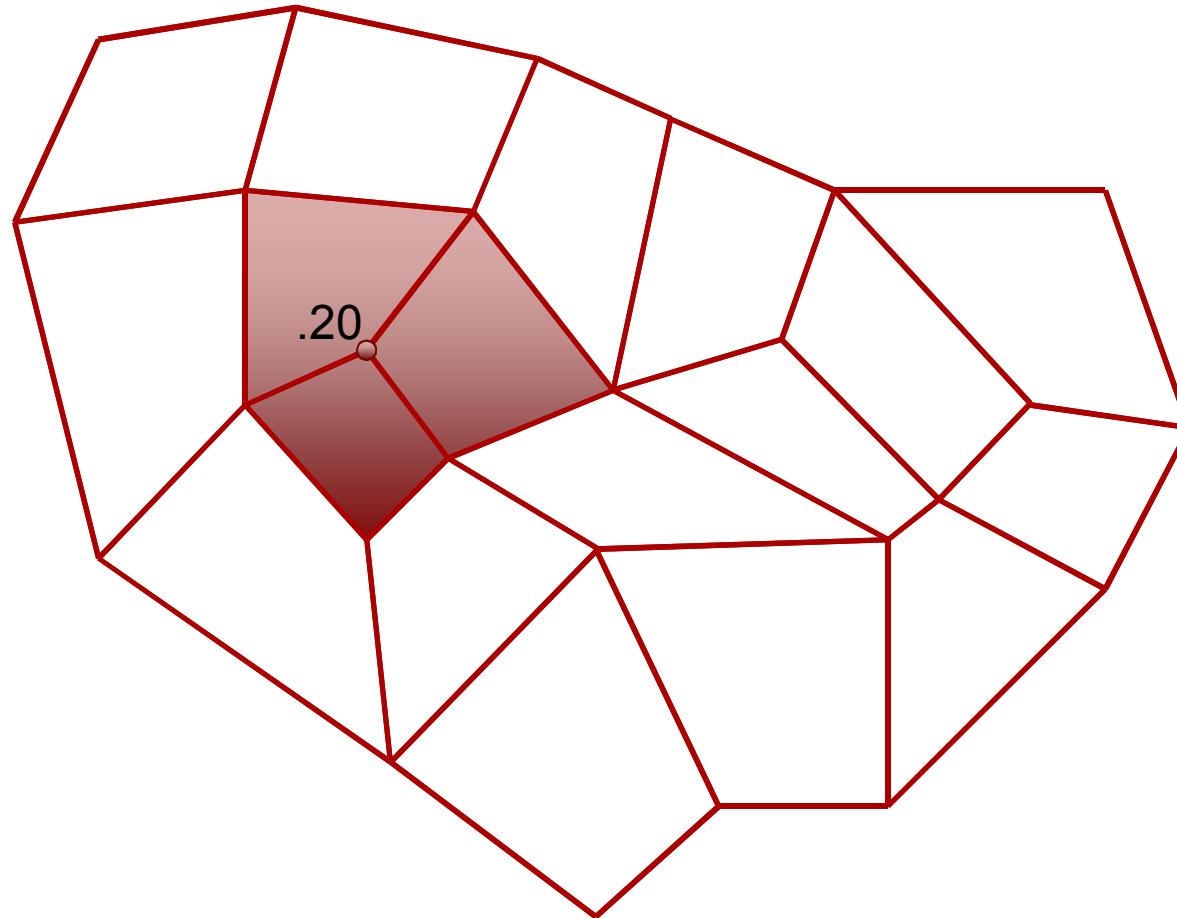


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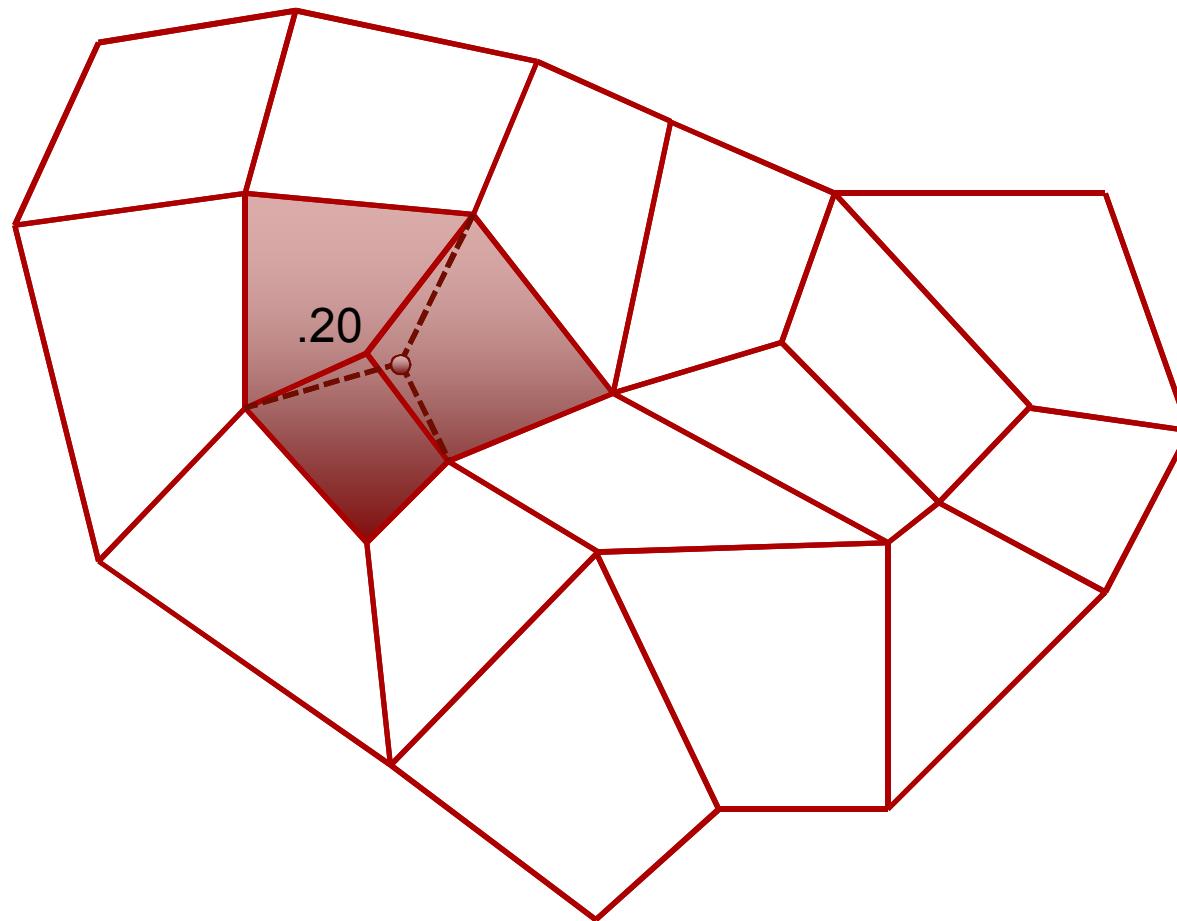
Parallel Coloring



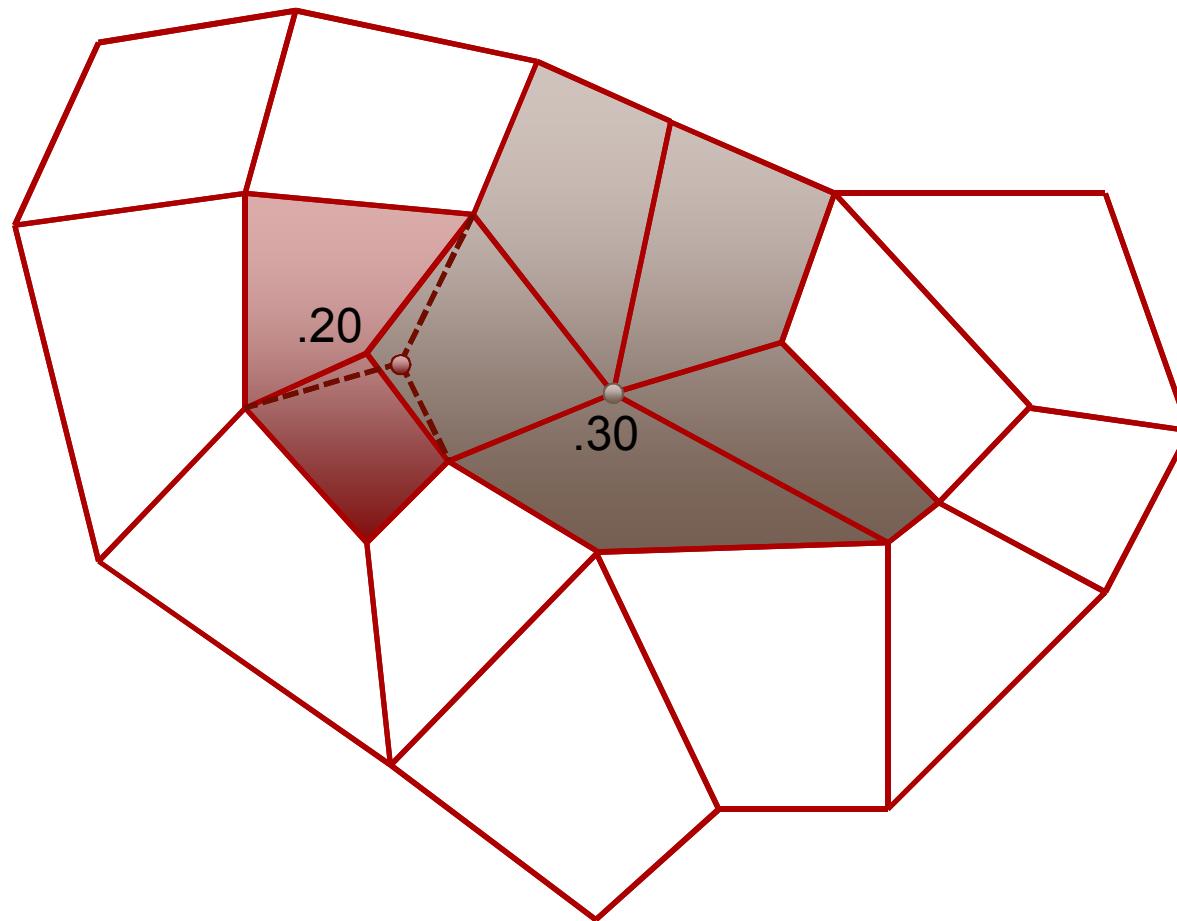
Parallel Coloring



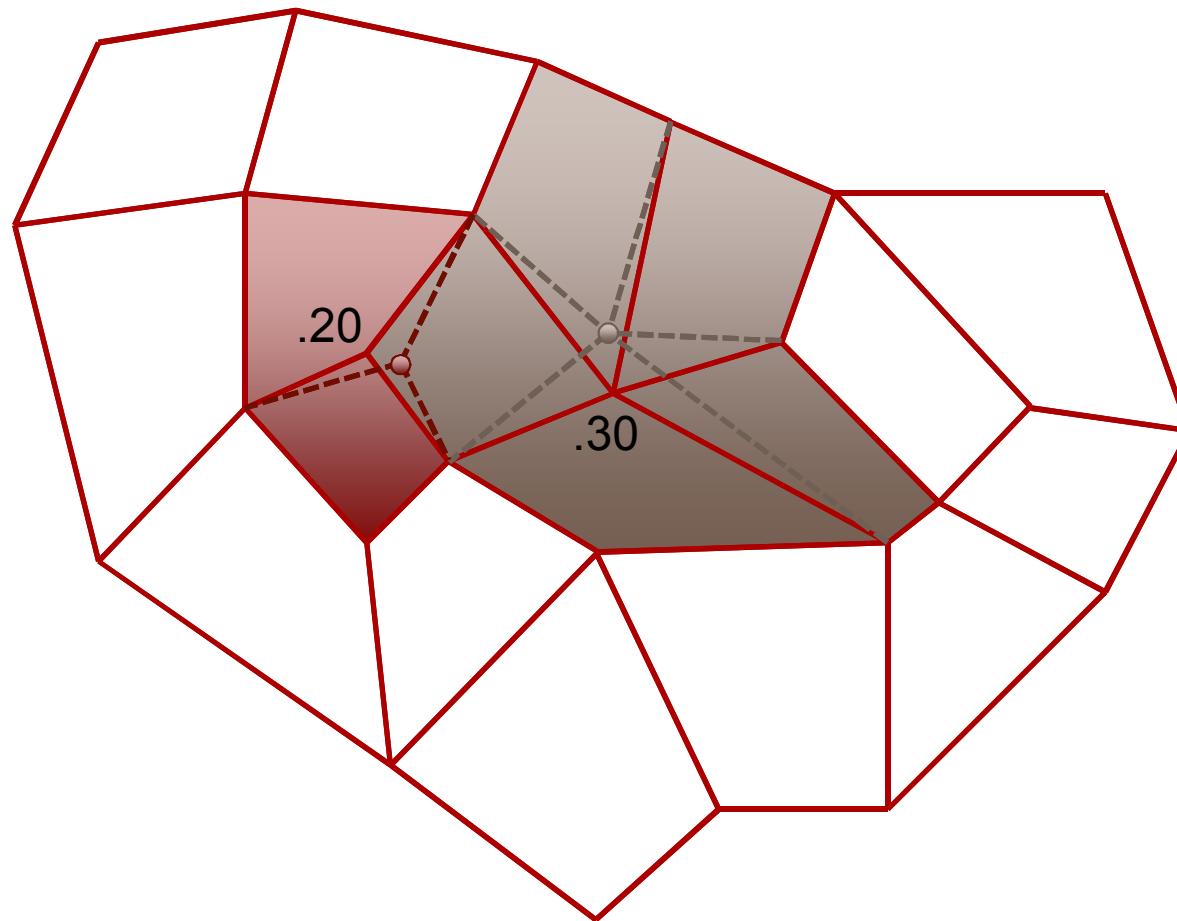
Parallel Coloring



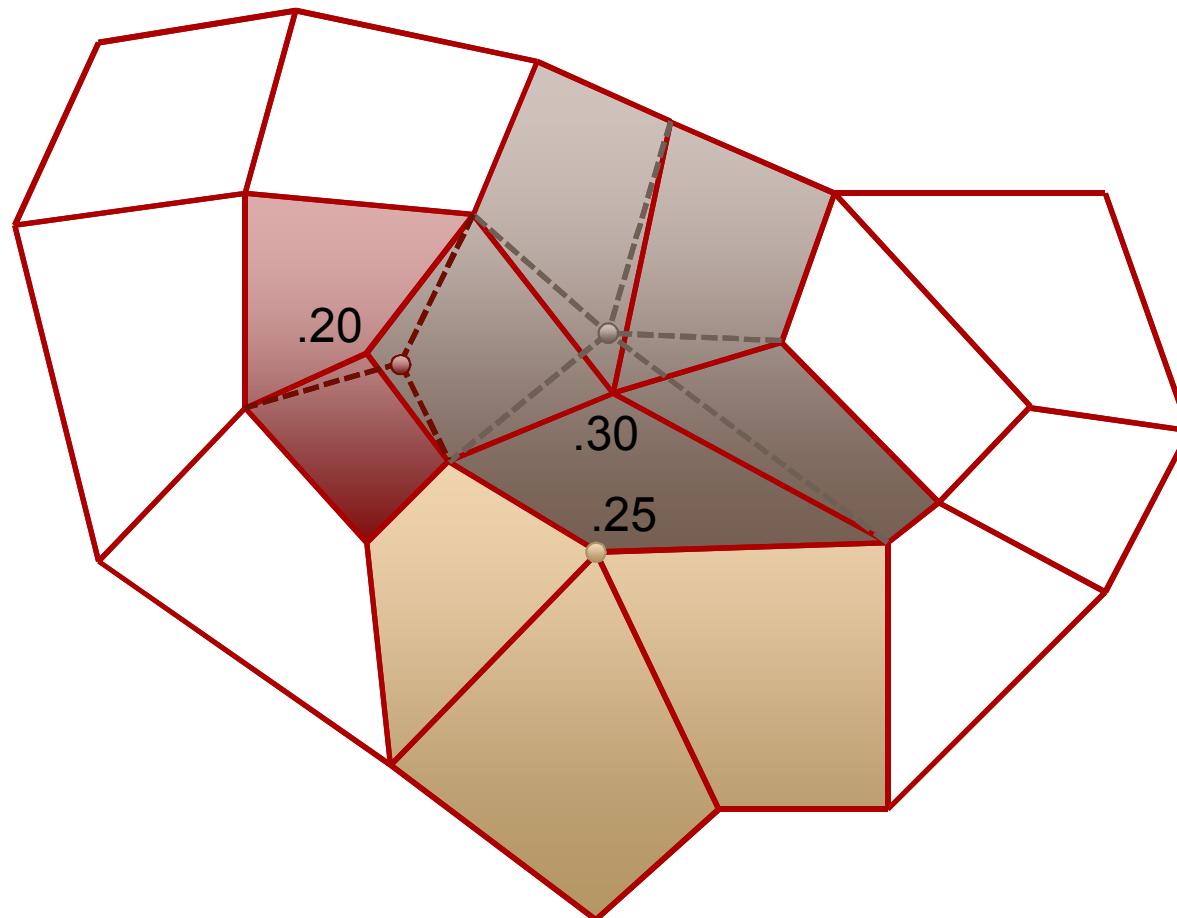
Parallel Coloring



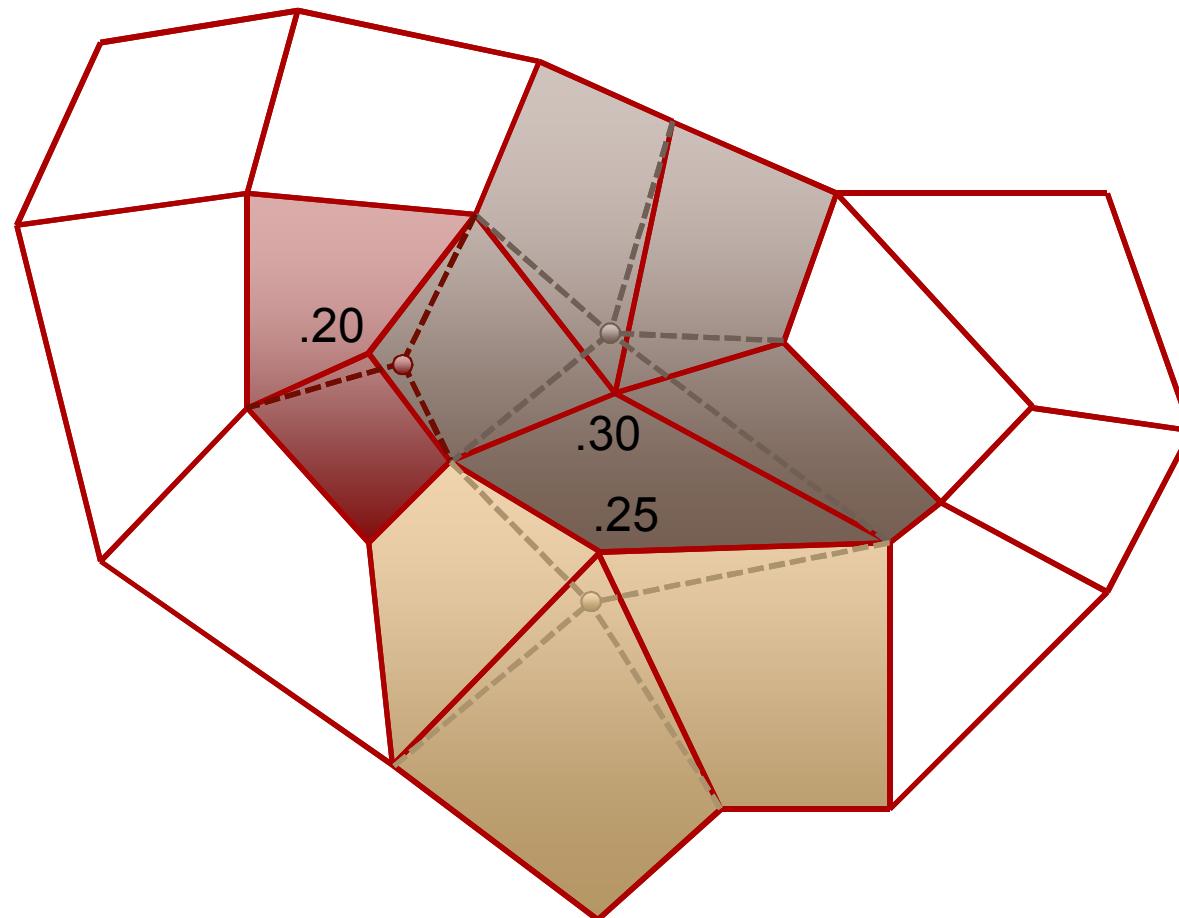
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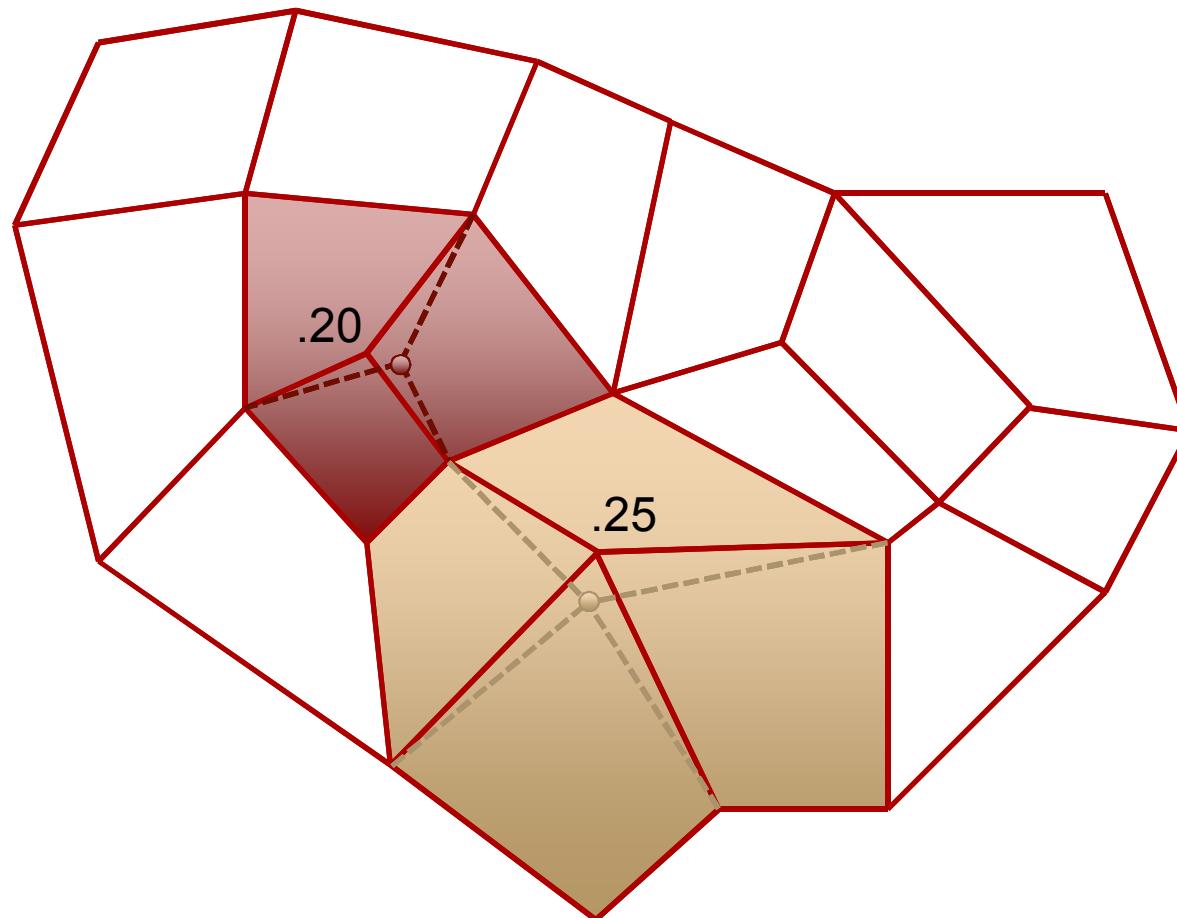
Parallel Coloring



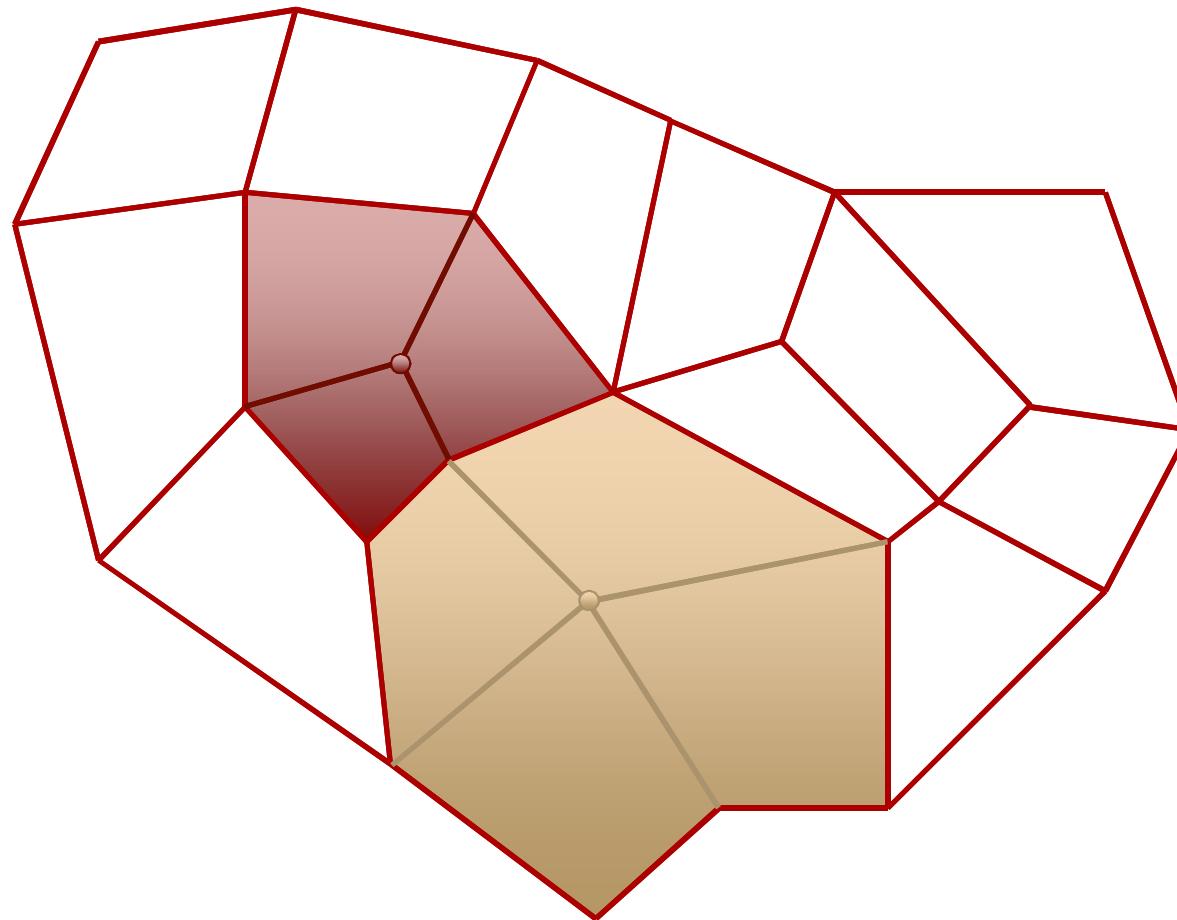
Parallel Coloring



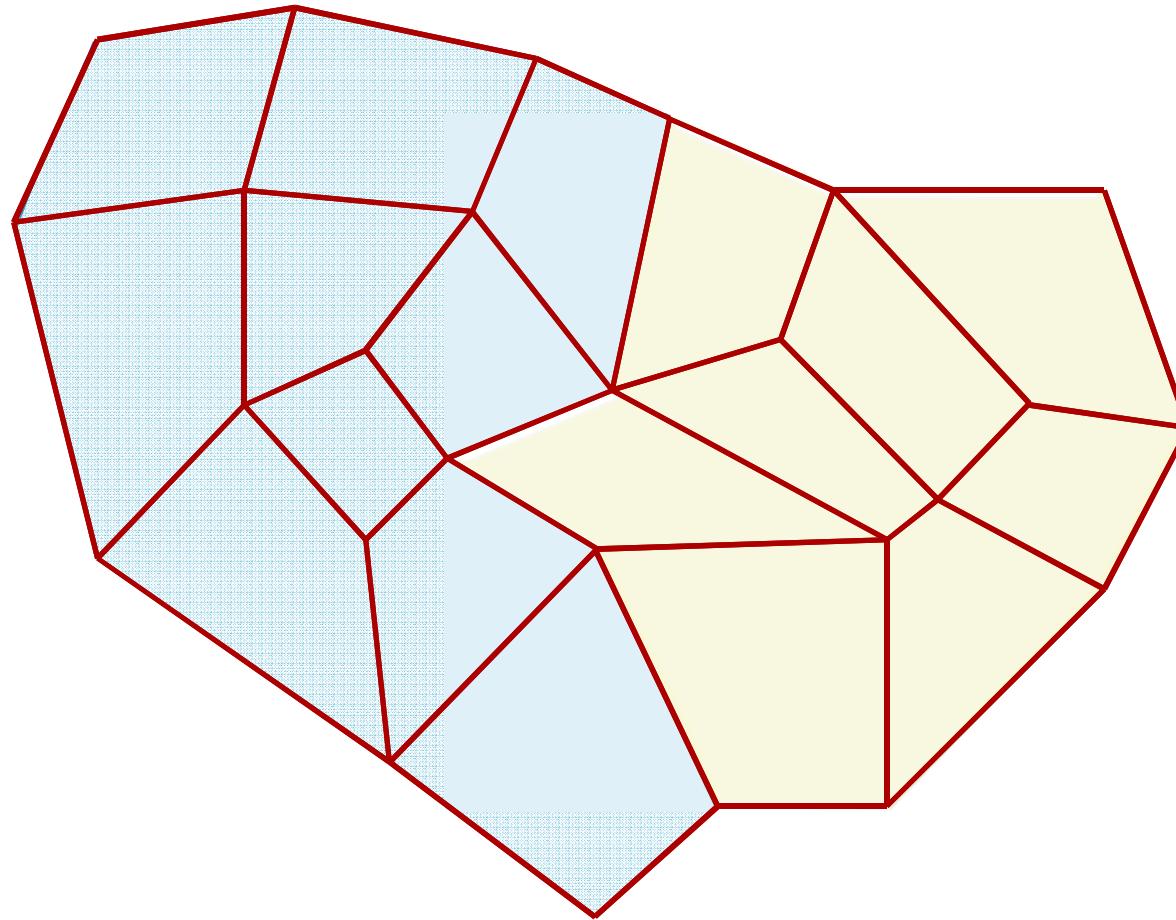
Parallel Coloring



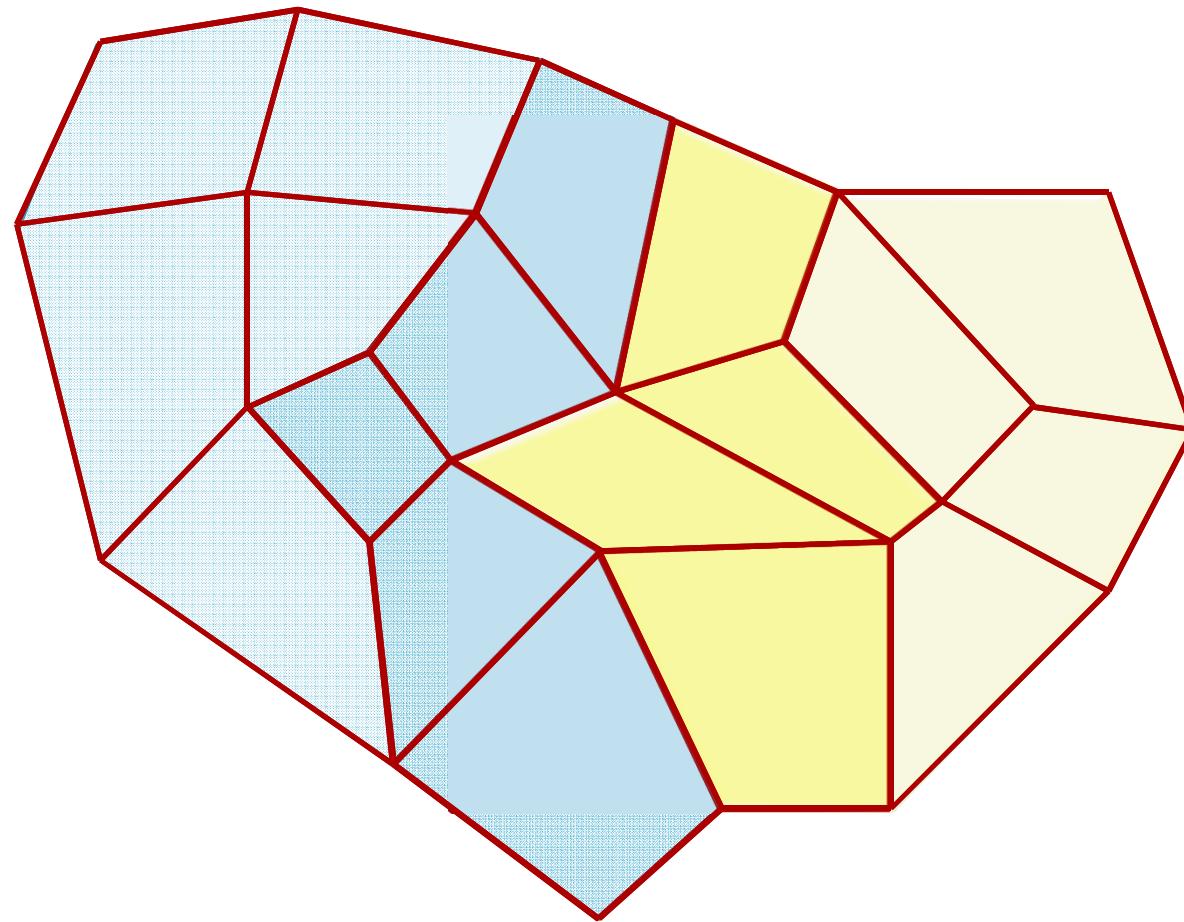
Parallel Coloring



Parallel Coloring



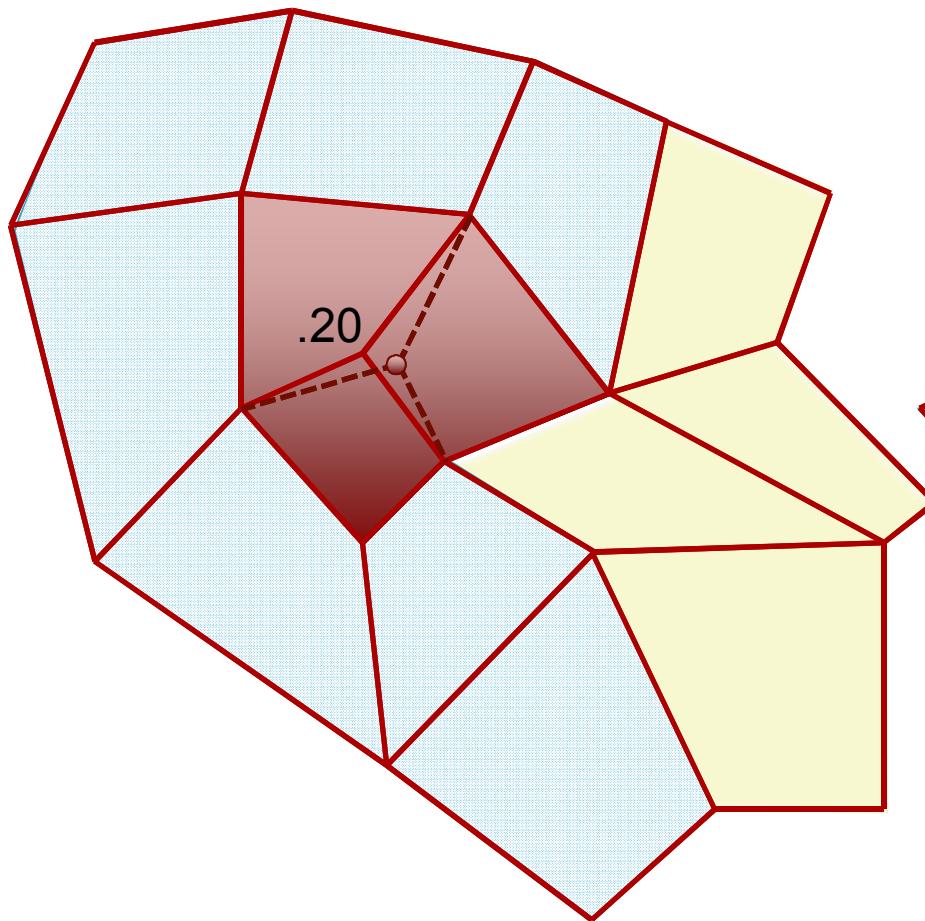
Parallel Coloring



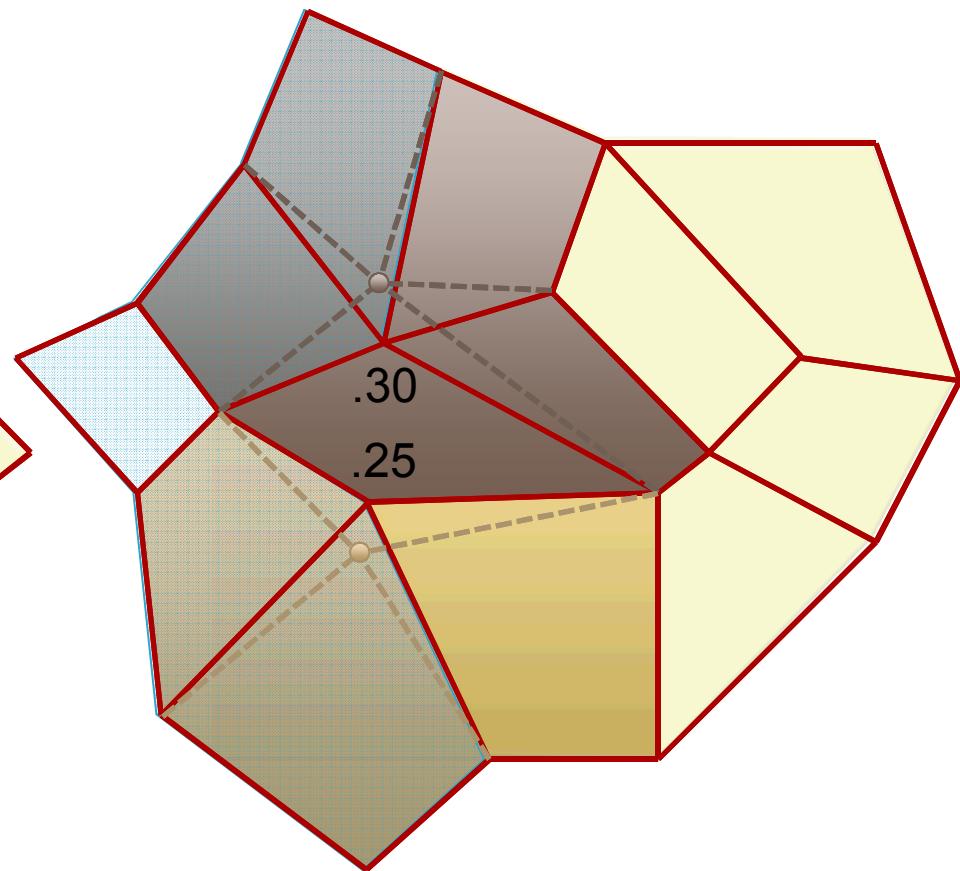
P0

P1

Parallel Coloring

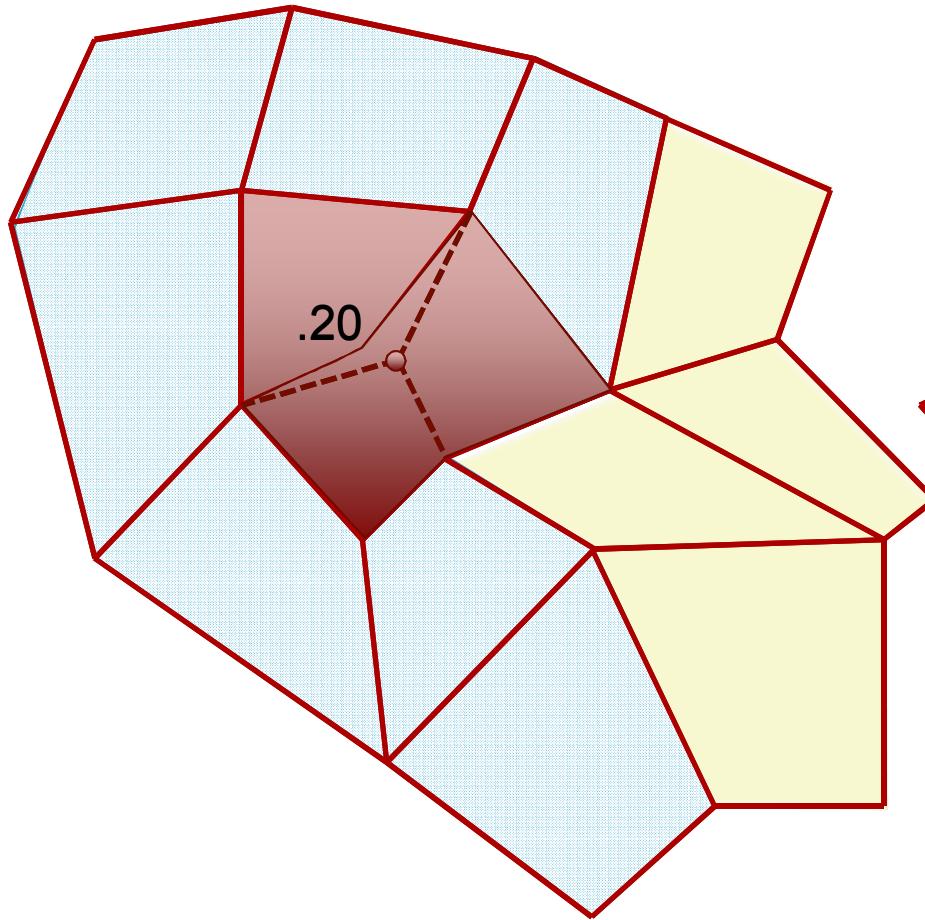


P0

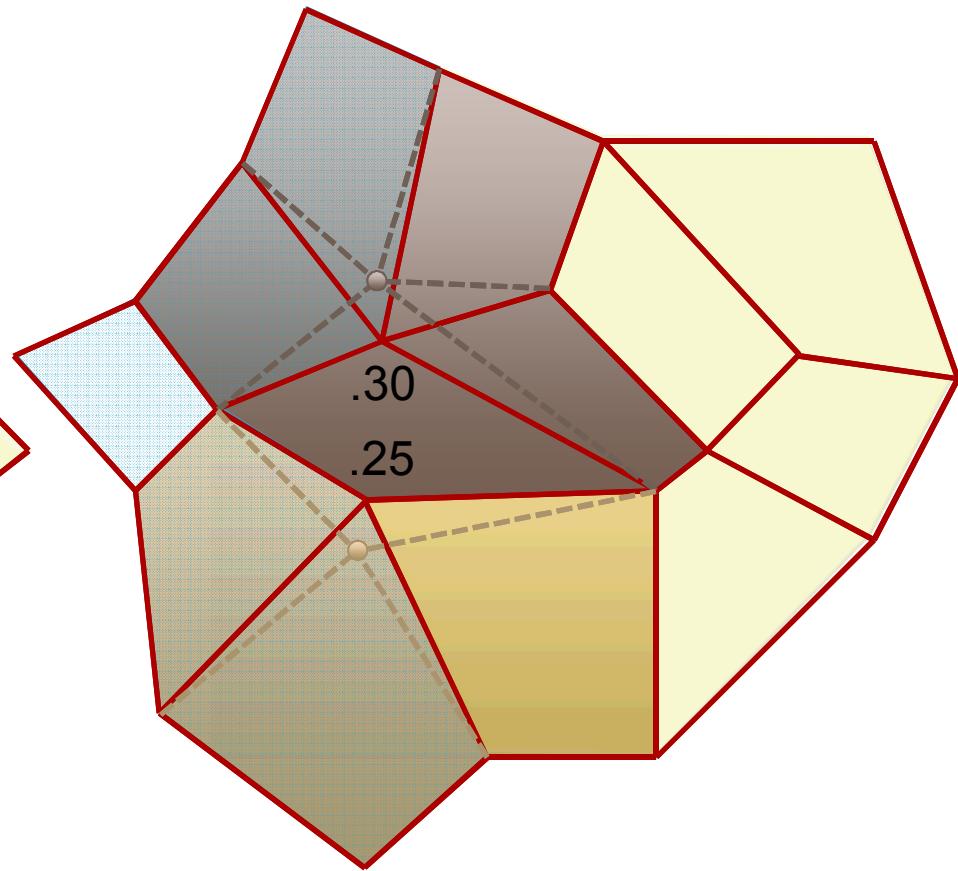


P1

Parallel Coloring

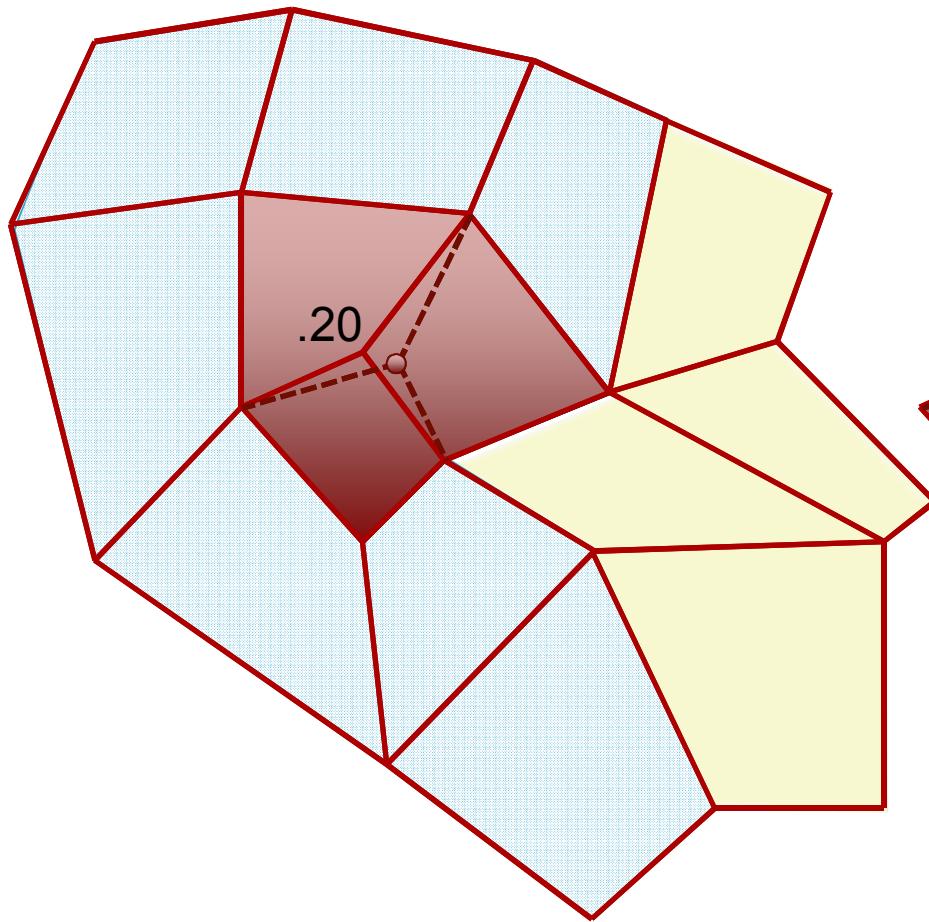


P0

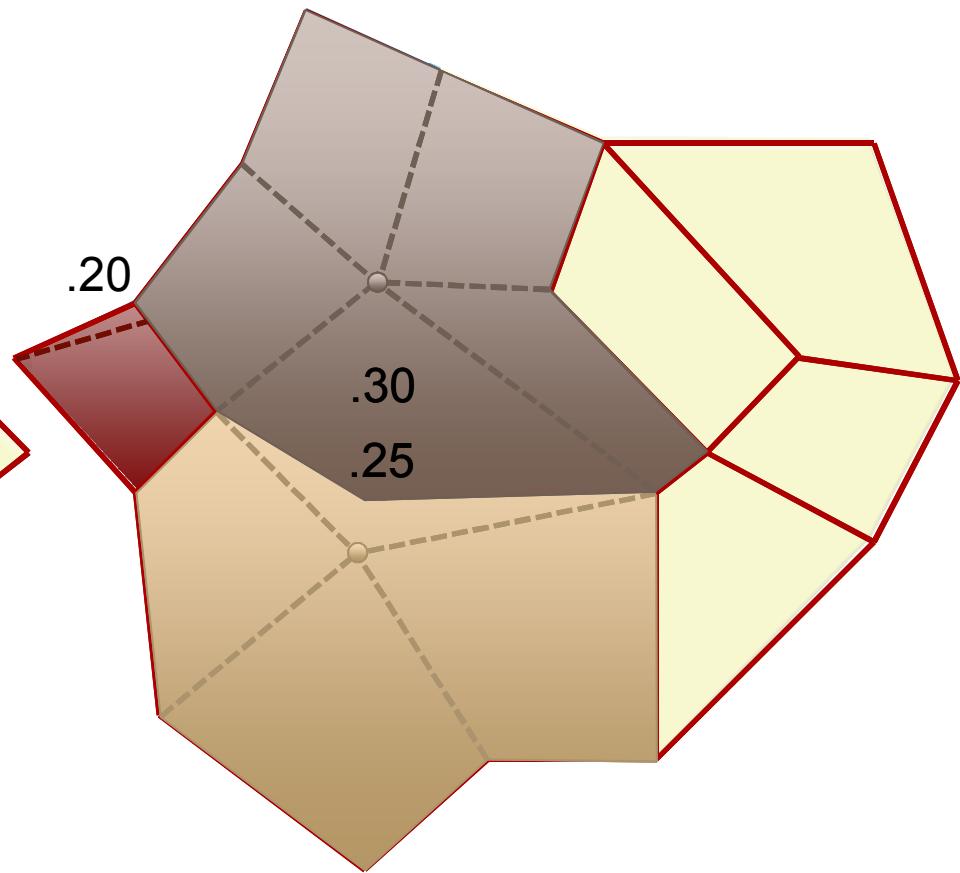


P1

Parallel Coloring

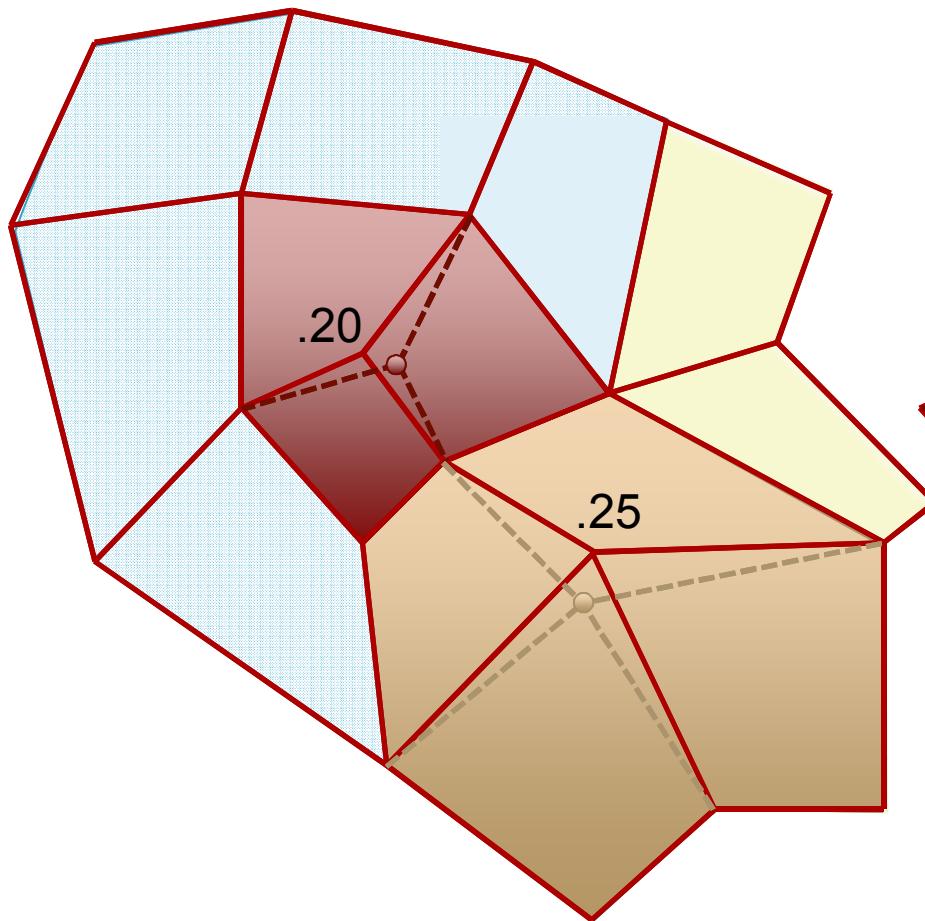


P0

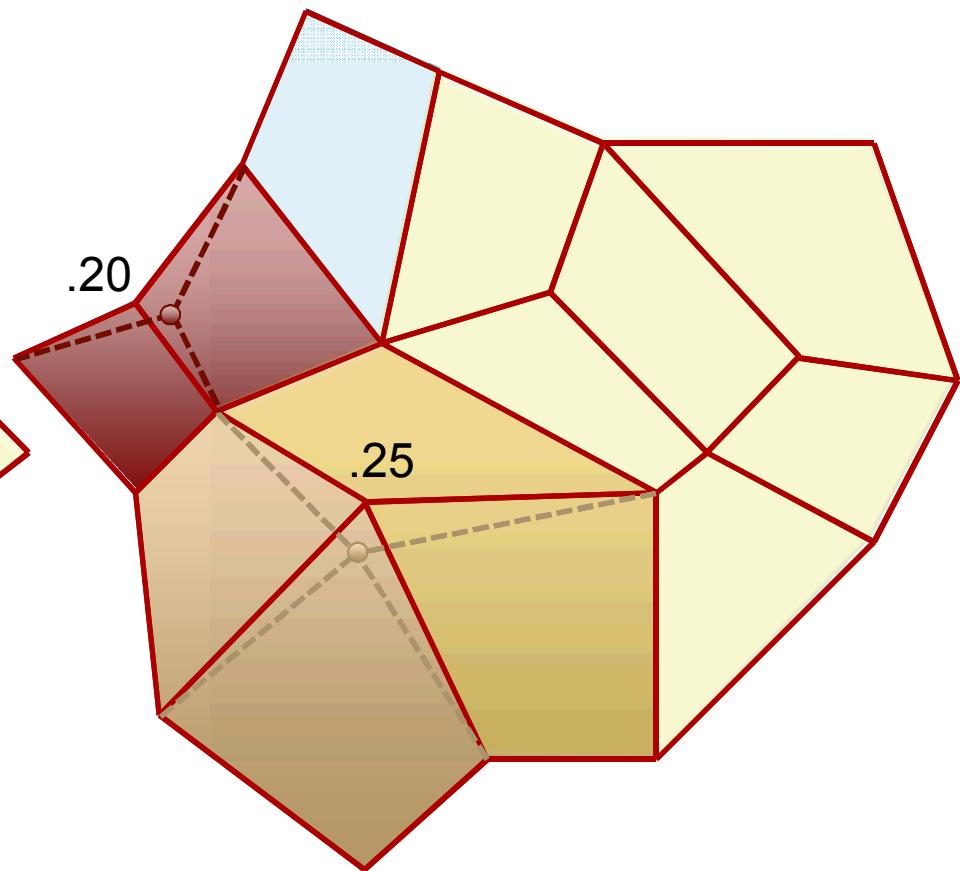


P1

Parallel Coloring

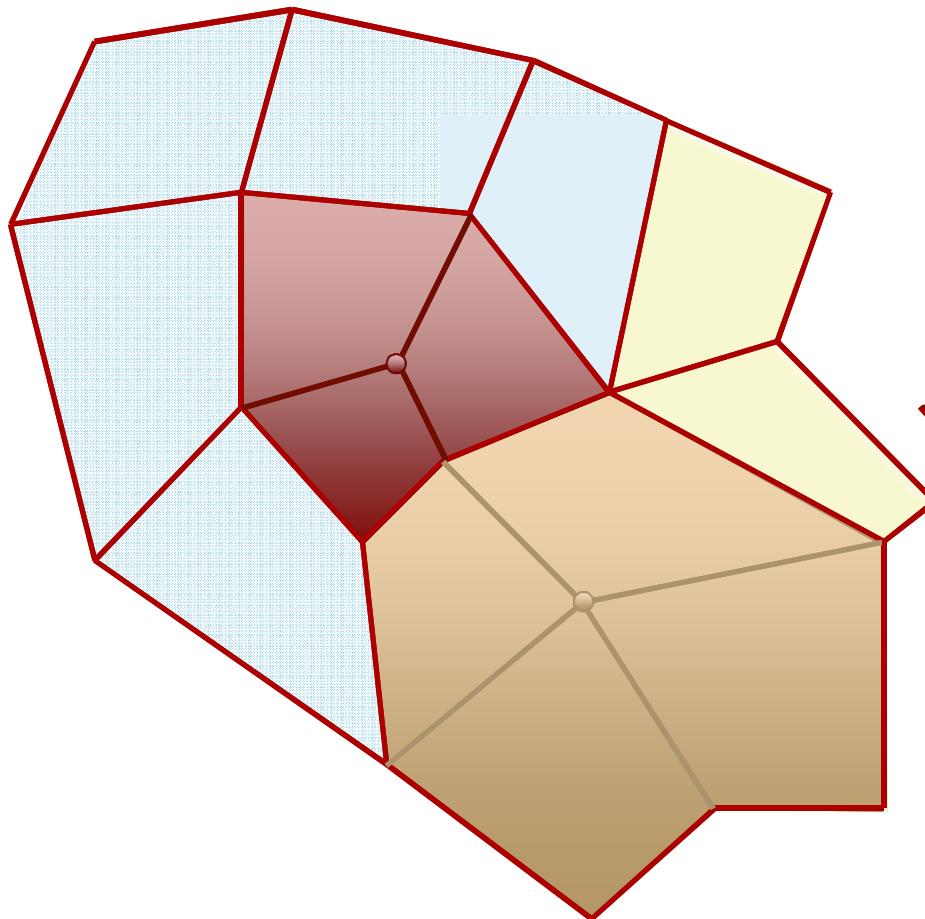


P0

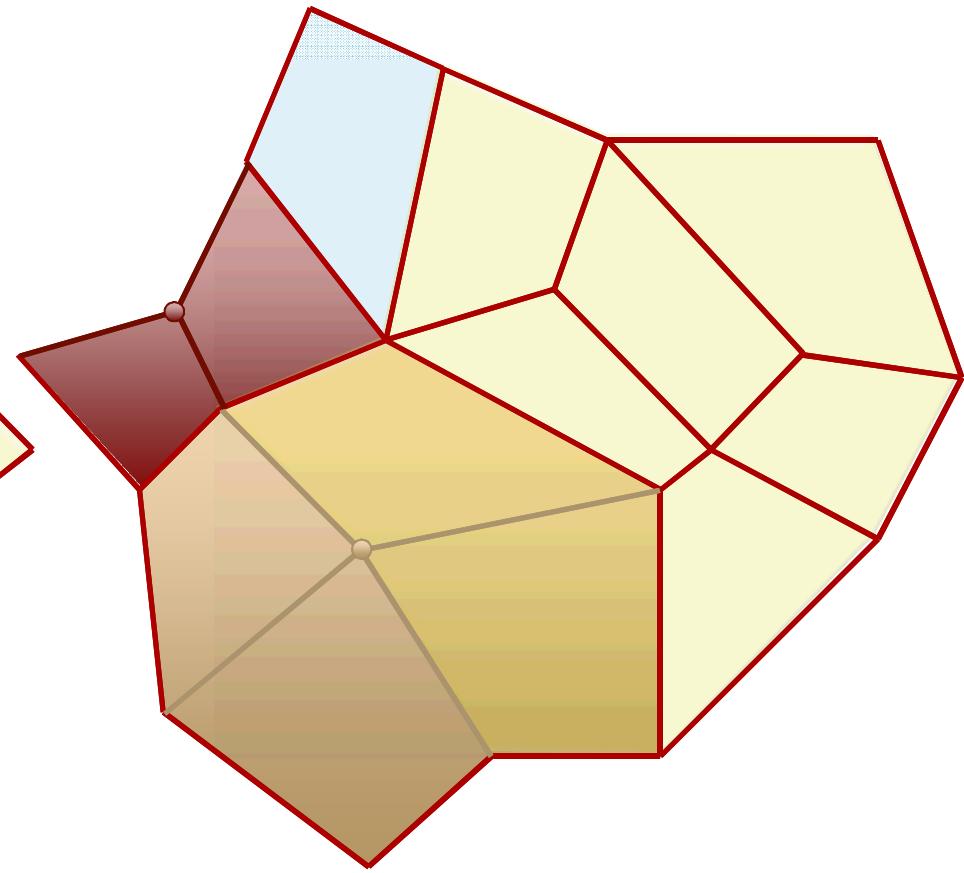


P1

Parallel Coloring



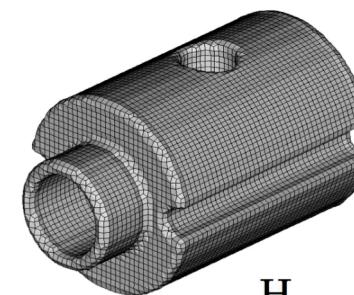
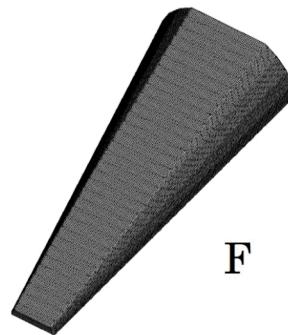
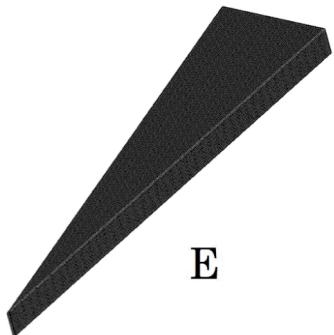
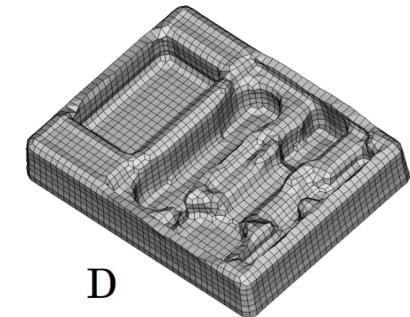
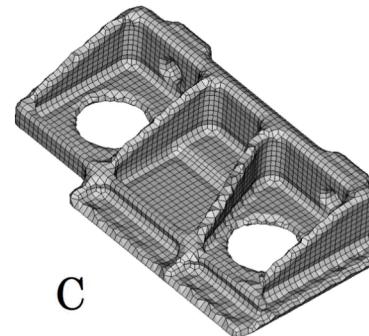
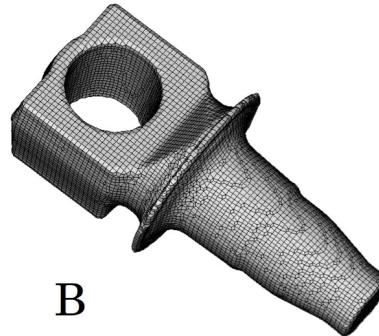
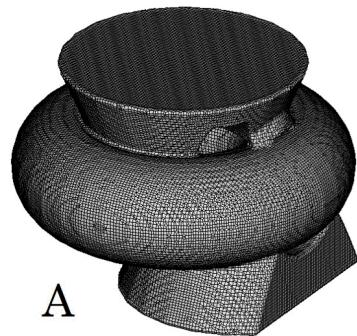
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P1

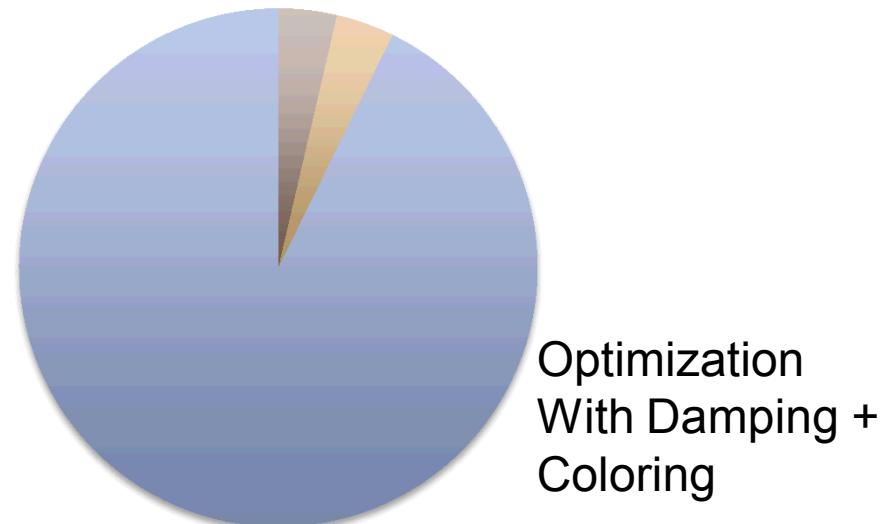
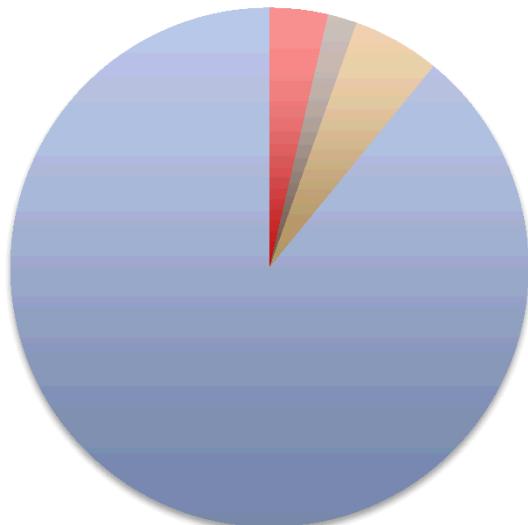
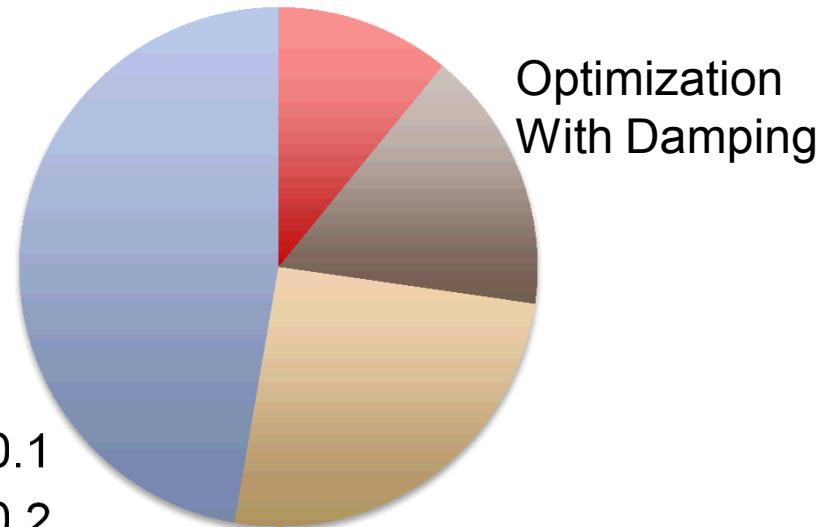
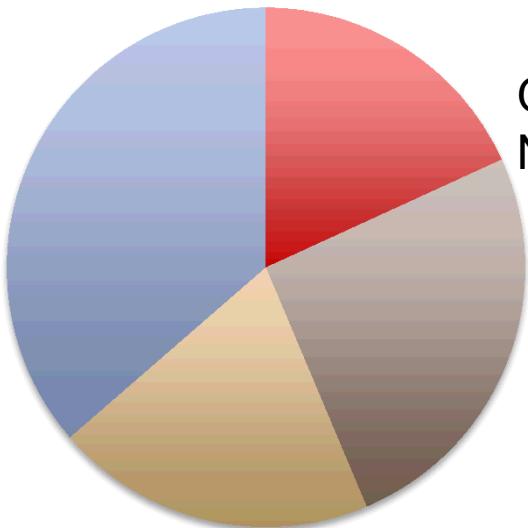
Sculpt Smoothing Comparison

Test Suite: 52 Single Part CAD Models



Minimum Mesh Quality

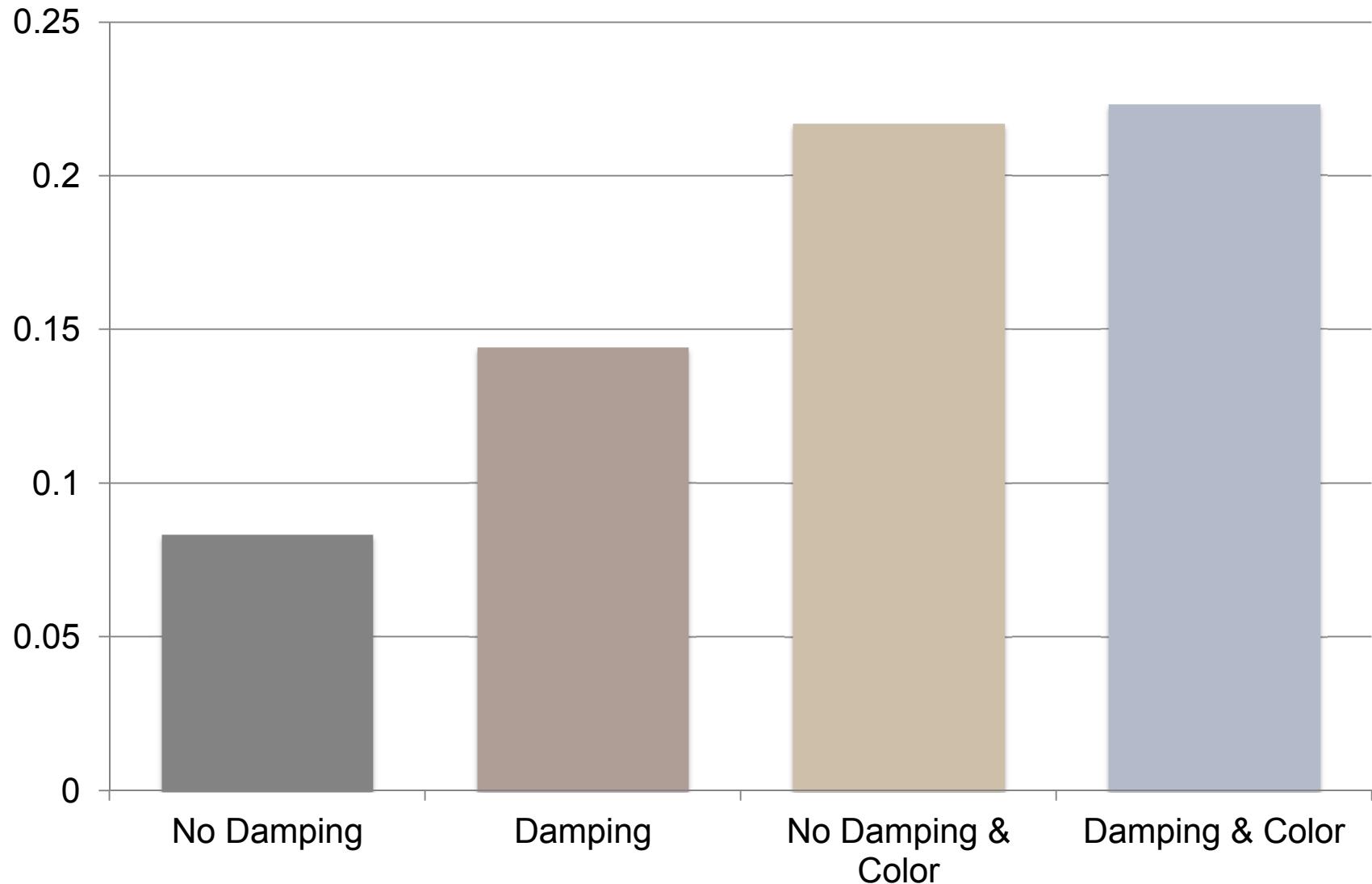
Test Suite: 52 Single Part CAD Models



- $\text{min SJ} < 0.0$
- $0.0 < \text{min SJ} < 0.1$
- $0.1 < \text{min SJ} < 0.2$
- $\text{min SJ} > 0.2$

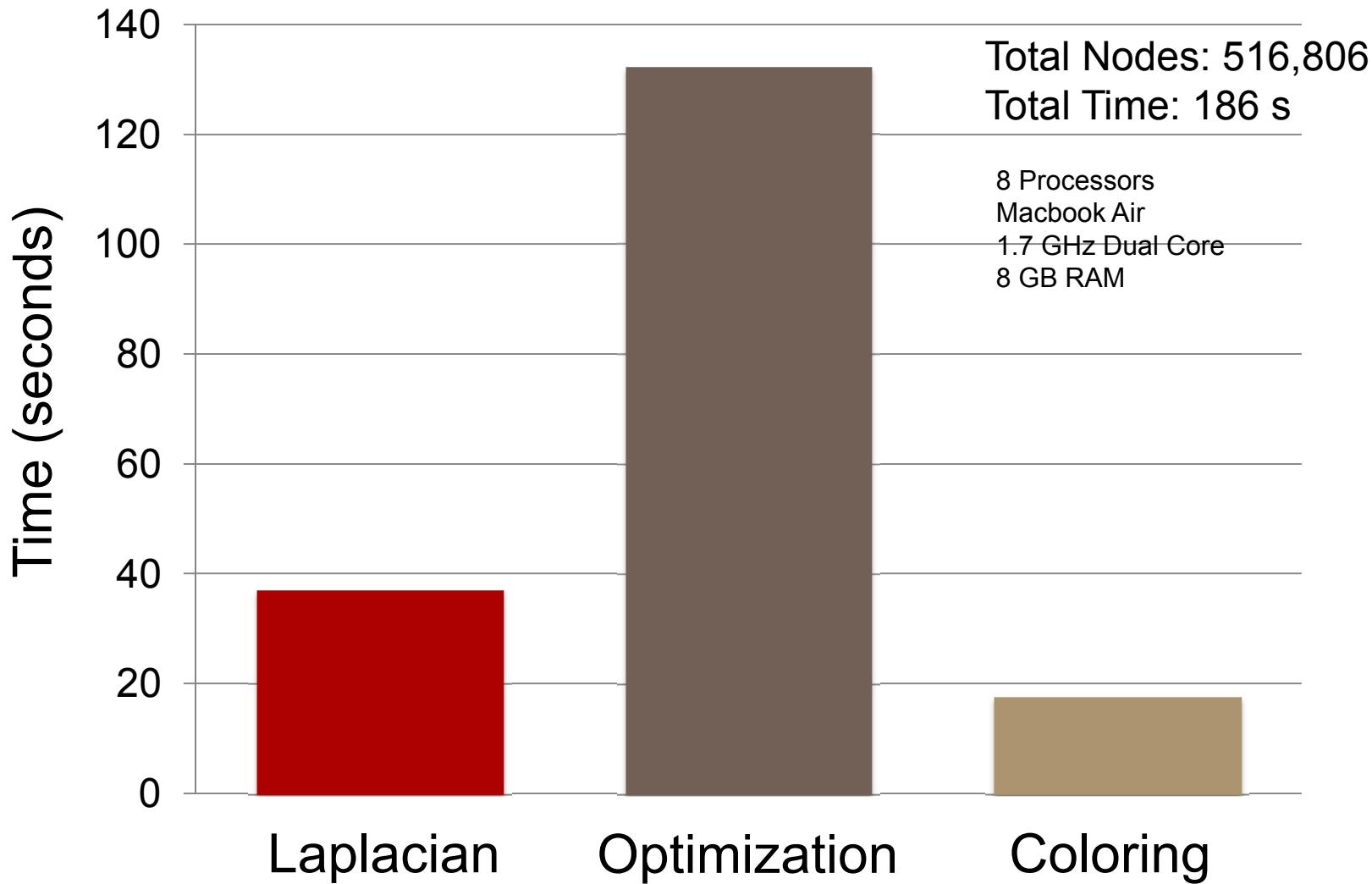
Average Minimum Mesh Quality

Test Suite: 52 Single Part CAD Models



Total Time in Smoothing

Test Suite: 52 Single Part CAD Models



Smoothing Procedure

