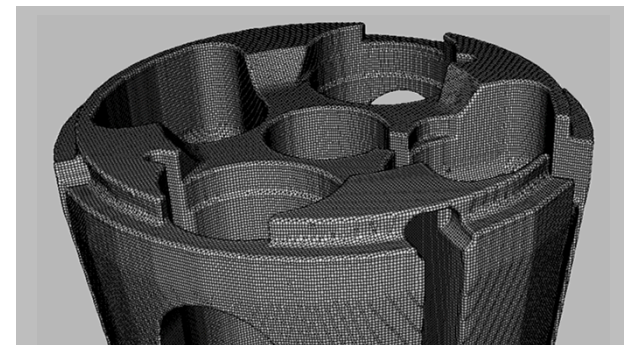
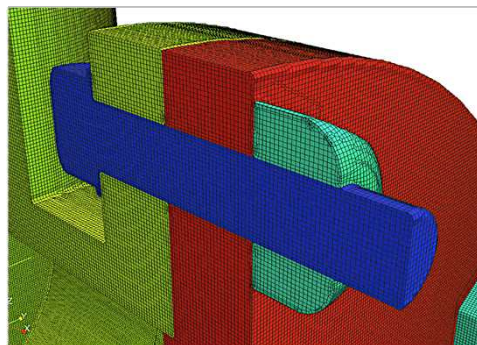
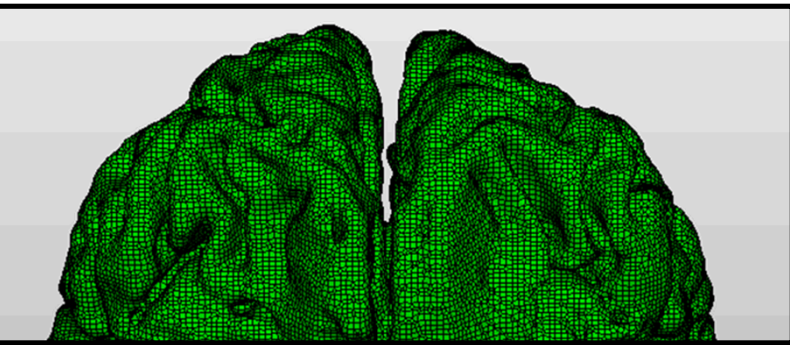


*Exceptional service in the national interest*



# Parallel Mesh Optimization for Grid-based Hex Meshes

US National Congress on  
Computational Mechanics

San Diego, CA

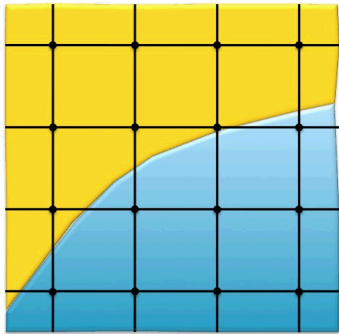
July, 26<sup>th</sup> 2015

Steven J. Owen

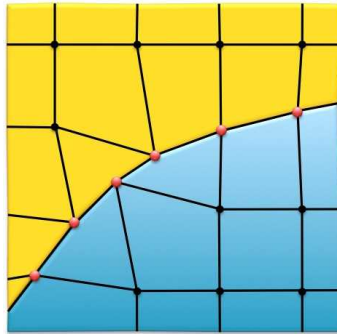
Sandia National Laboratories,

Albuquerque, NM

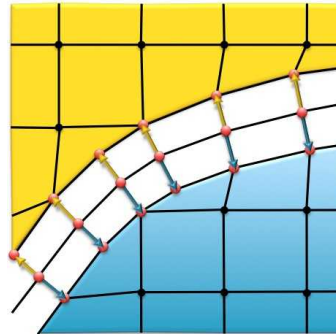
# Overlay Grid Hex Meshing Method



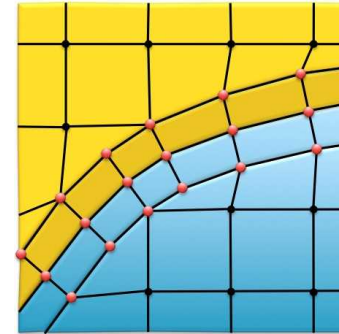
Overlay  
Cartesian  
Grid on  
Geometry



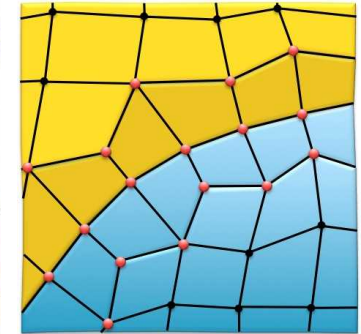
Nodes  
projected to  
interfaces



Nodes  
duplicated at  
interfaces and  
moved  
orthogonally



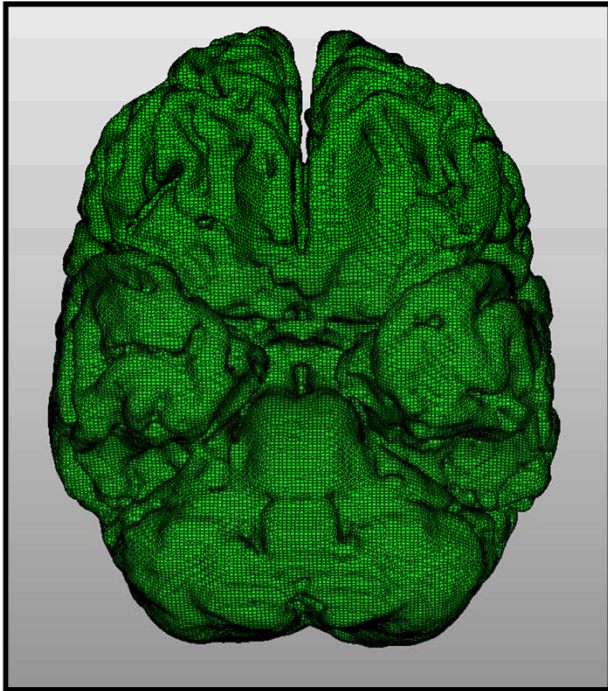
Layers of  
hexes created  
at interfaces



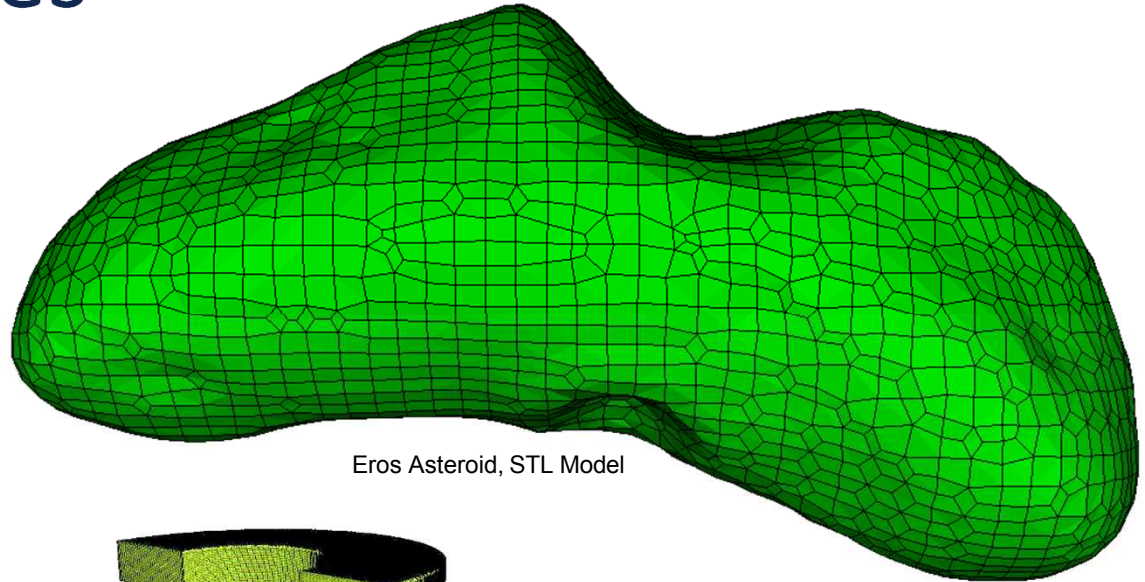
Smoothing  
performed



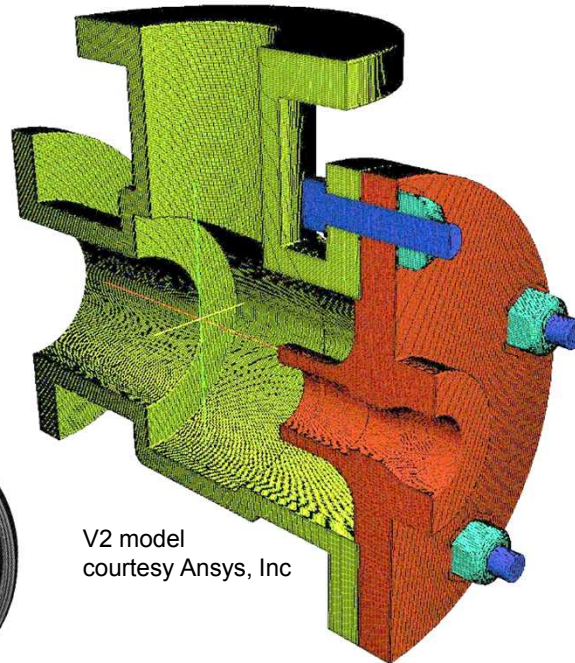
# Sculpt Examples



STL MRI Brain Model, Courtesy Bryce Owen  
Brigham Young University, Provo, UT



Eros Asteroid, STL Model

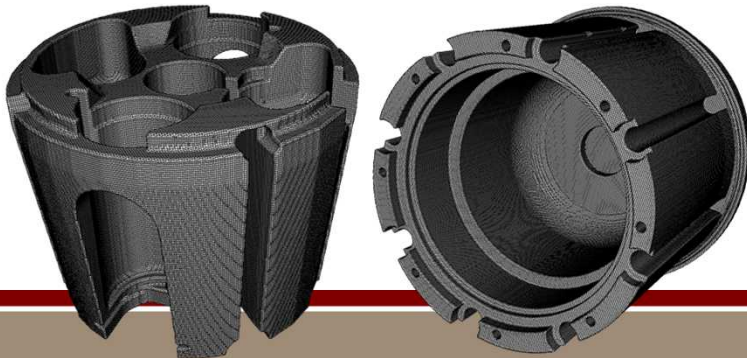


V2 model  
courtesy Ansys, Inc

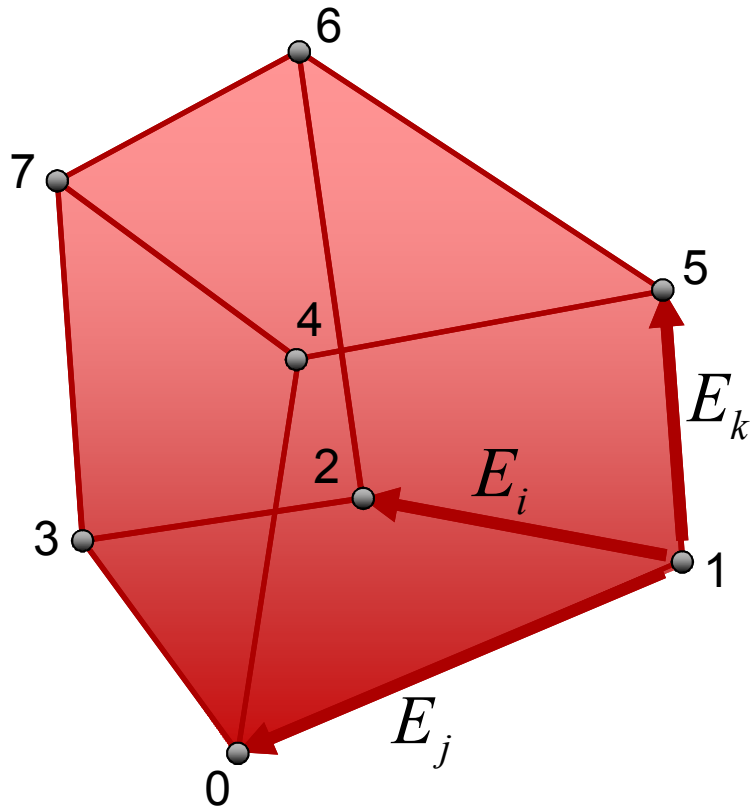
Weapon Component  
model, Sandia Labs



Weapon Component Models Courtesy Stephen Recchia,  
US Army, Picitinni. Used with Permission



# Scaled Jacobian



$$(J_s)_I = \det \left\{ \hat{E}_i \hat{E}_j \hat{E}_k \right\}^\top$$

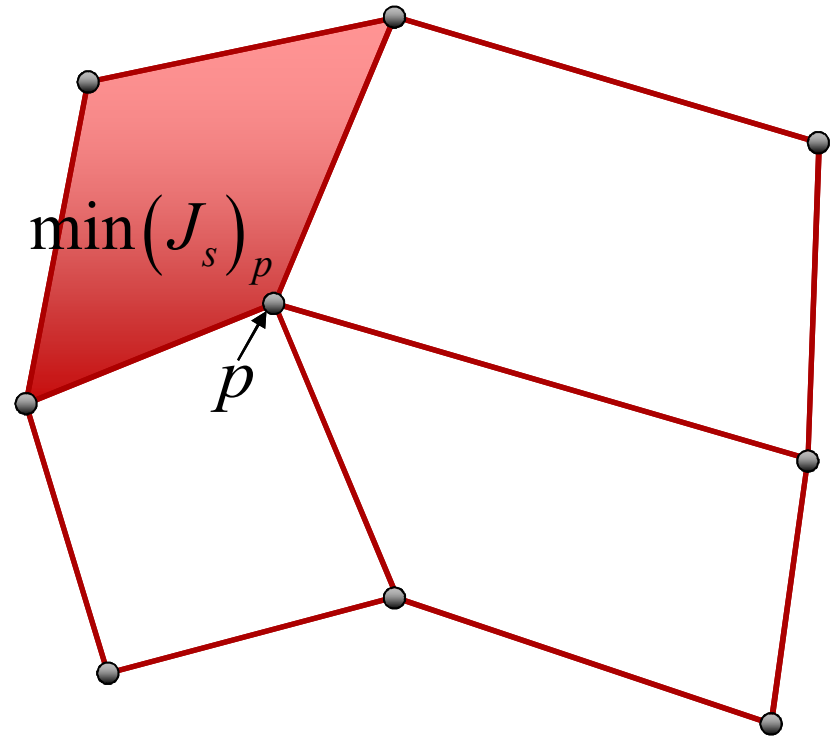
$$J_s = \min \left( (J_s)_I, I = 0, 1, \dots, 7 \right)$$

“Acceptable”

$$J_s \geq 0.2$$

# Optimization

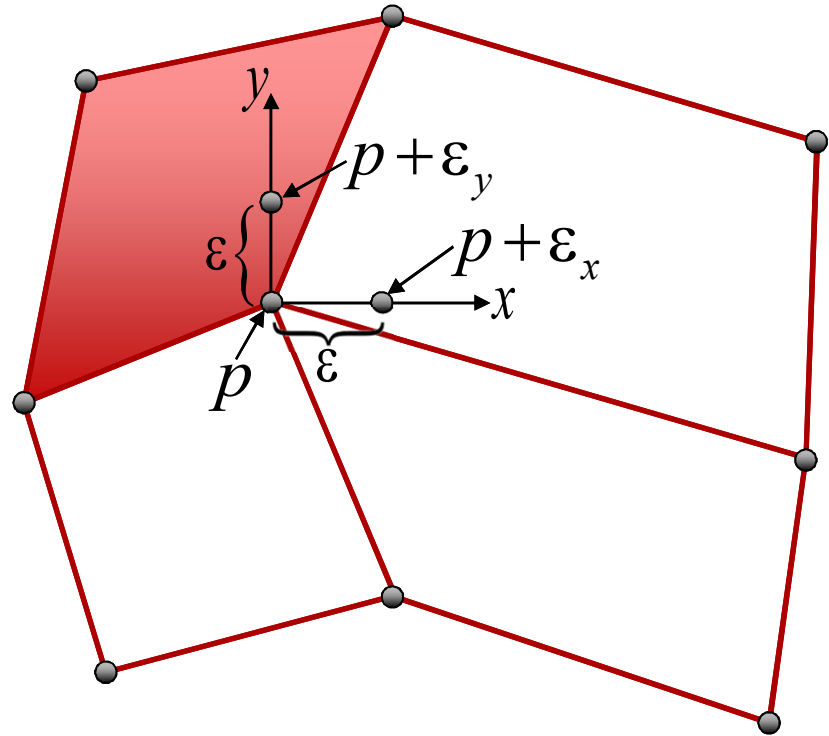
Compute minimum scaled  
Jacobian,  $(J_s)_p$ , of node  $p$  in all  
attached hexes



# Optimization

Compute minimum scaled  
Jacobian,  $(J_s)_p$ , of node  $p$  in all  
attached hexes

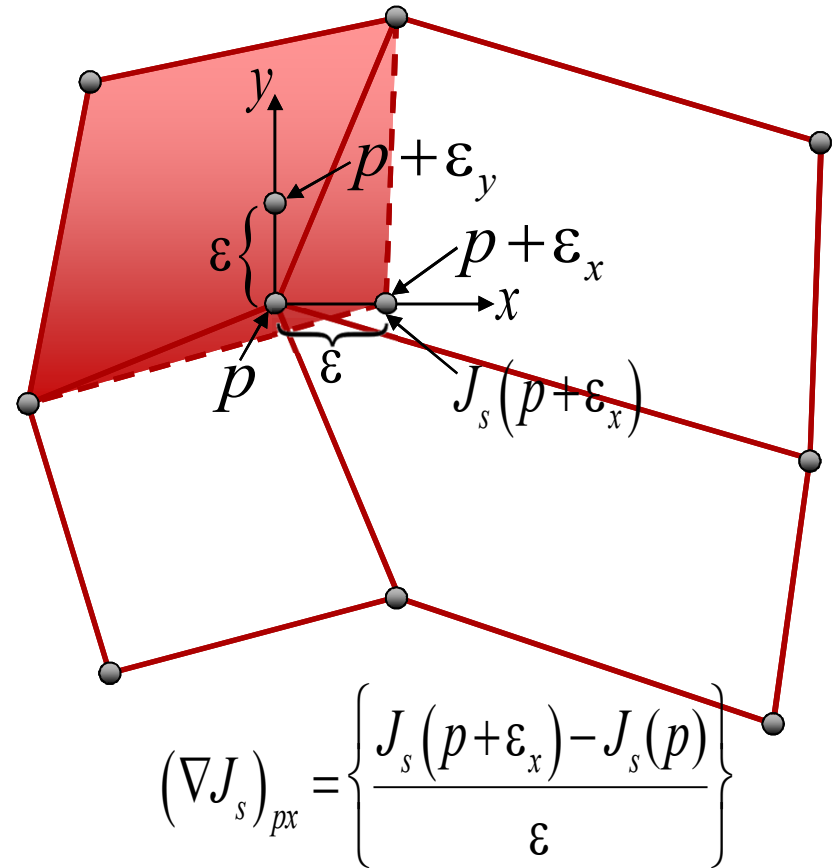
Compute numerical  
gradient  $(\nabla J_s)_p$



# Optimization

Compute minimum scaled  
Jacobian,  $(J_s)_p$ , of node  $p$  in all  
attached hexes

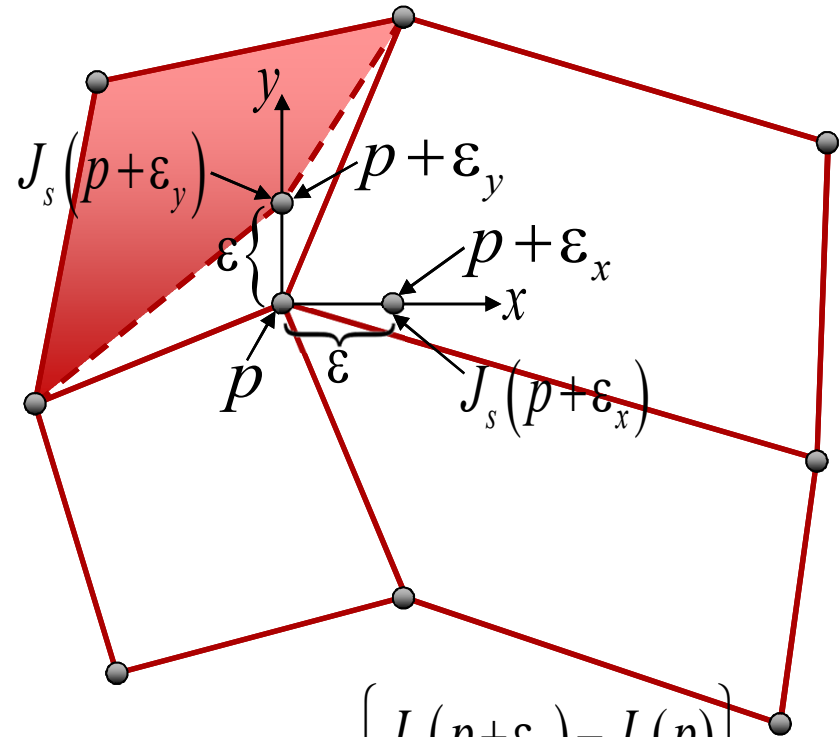
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# Optimization

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Jacobian,  $(J_s)_p$ , of node  $p$  in all  
attached hexes

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gradient  $(\nabla J_s)_p$



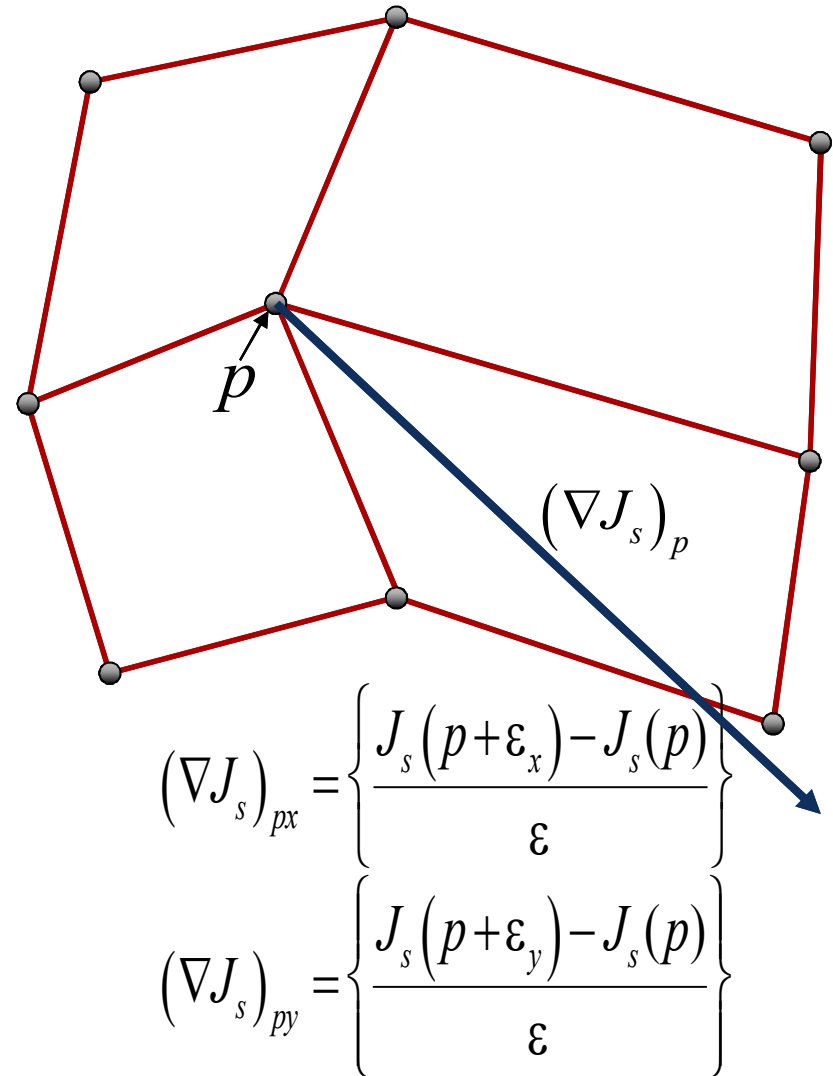
$$(\nabla J_s)_{px} = \left\{ \frac{J_s(p + \epsilon_x) - J_s(p)}{\epsilon} \right\}$$
$$(\nabla J_s)_{py} = \left\{ \frac{J_s(p + \epsilon_y) - J_s(p)}{\epsilon} \right\}$$



# Optimization

Compute minimum scaled  
Jacobian,  $(J_s)_p$ , of node  $p$  in all  
attached hexes

Compute numerical  
gradient  $(\nabla J_s)_p$

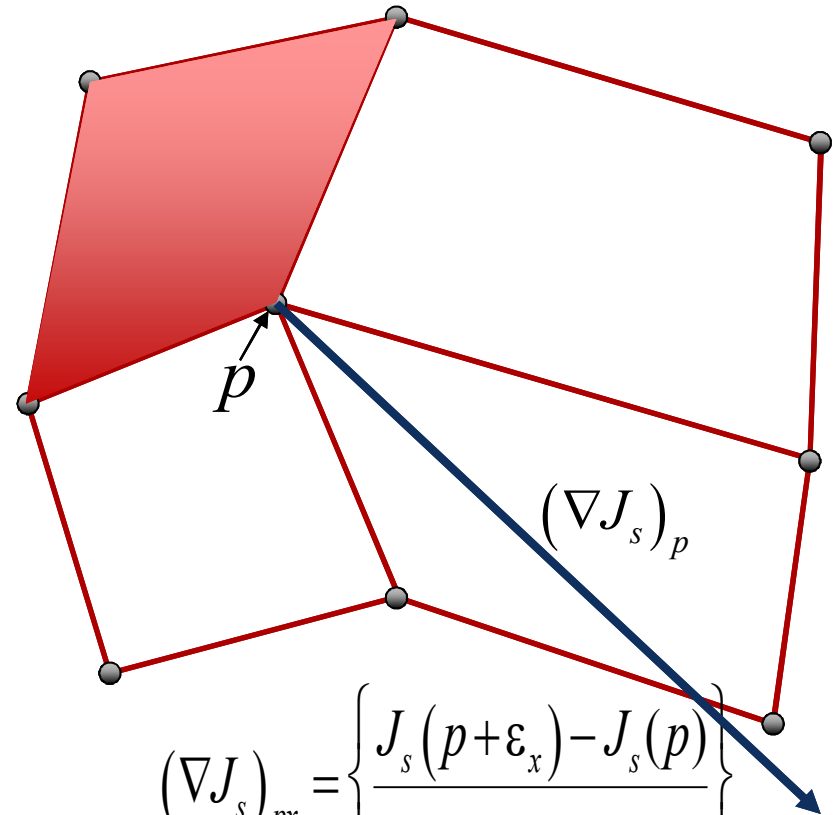


# Optimization

Compute minimum scaled  
Jacobian,  $(J_s)_p$ , of node  $p$  in all  
attached hexes

Compute numerical  
gradient  $(\nabla J_s)_p$

Find improved  $(J_s)_p$  by searching  
along vector  $(\nabla J_s)_p$



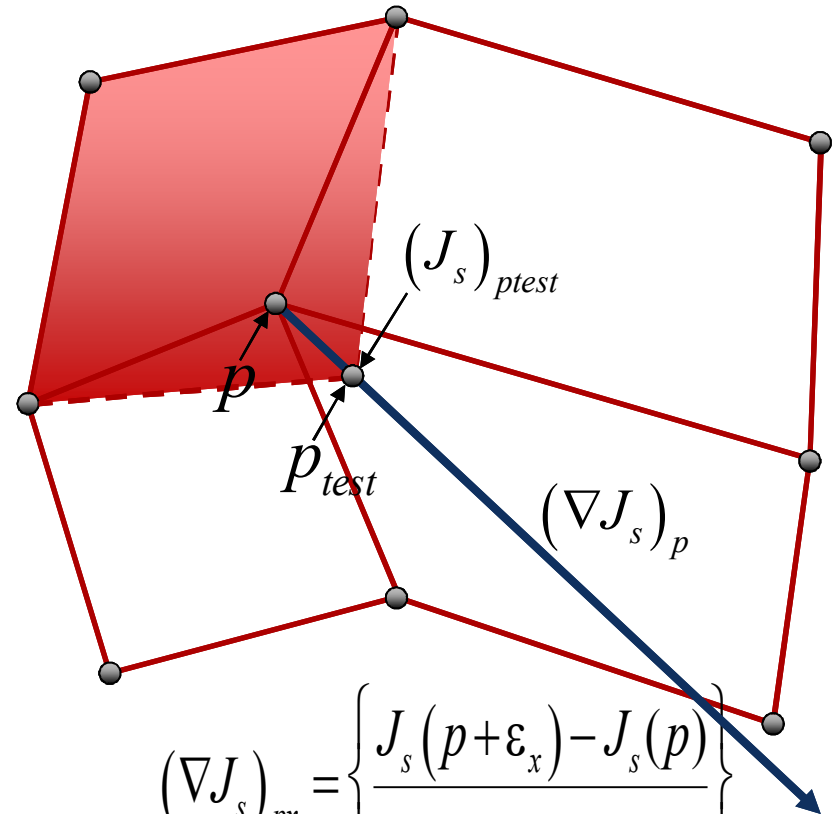
$$(\nabla J_s)_{px} = \left\{ \frac{J_s(p + \varepsilon_x) - J_s(p)}{\varepsilon} \right\}$$
$$(\nabla J_s)_{py} = \left\{ \frac{J_s(p + \varepsilon_y) - J_s(p)}{\varepsilon} \right\}$$

# Optimization

Compute minimum scaled Jacobian,  $(J_s)_p$ , of node  $p$  in all attached hexes

Compute numerical gradient  $(\nabla J_s)_p$

Find improved  $(J_s)_p$  by searching along vector  $(\nabla J_s)_p$



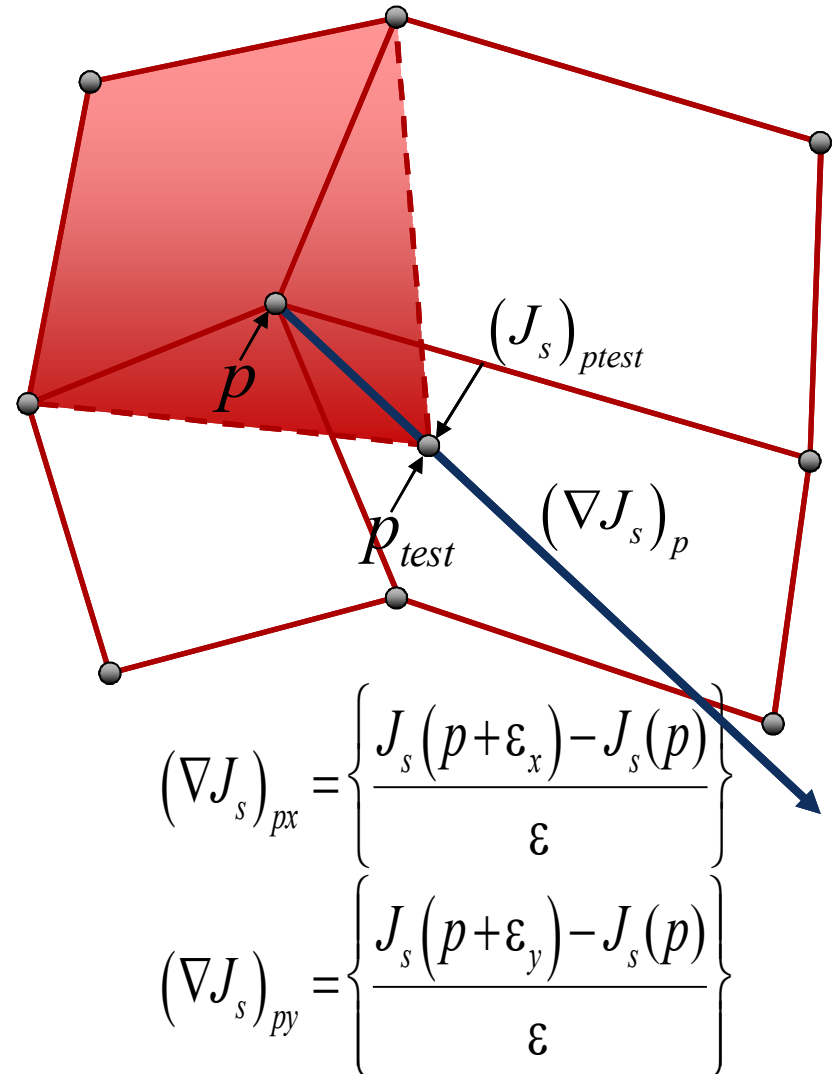
$$(\nabla J_s)_{px} = \left\{ \frac{J_s(p + \varepsilon_x) - J_s(p)}{\varepsilon} \right\}$$
$$(\nabla J_s)_{py} = \left\{ \frac{J_s(p + \varepsilon_y) - J_s(p)}{\varepsilon} \right\}$$

# Optimization

Compute minimum scaled Jacobian,  $(J_s)_p$ , of node  $p$  in all attached hexes

Compute numerical gradient  $(\nabla J_s)_p$

Find improved  $(J_s)_p$  by searching along vector  $(\nabla J_s)_p$

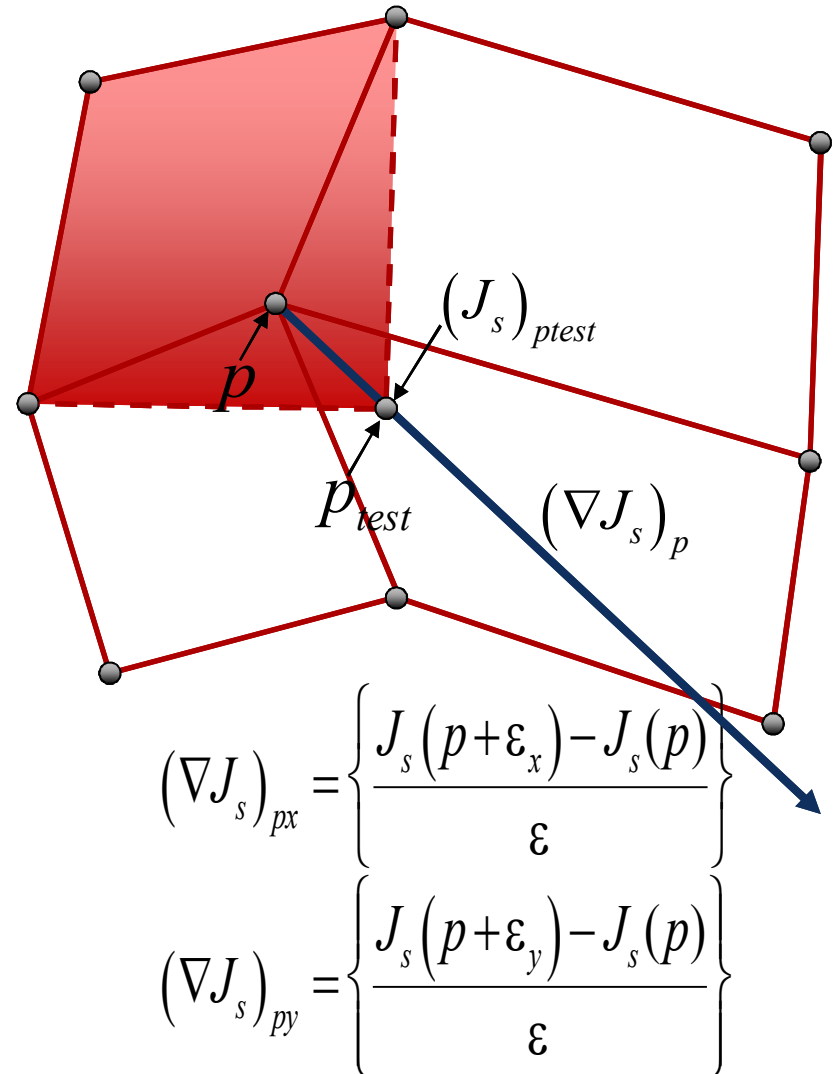


# Optimization

Compute minimum scaled Jacobian,  $(J_s)_p$ , of node  $p$  in all attached hexes

Compute numerical gradient  $(\nabla J_s)_p$

Find improved  $(J_s)_p$  by searching along vector  $(\nabla J_s)_p$

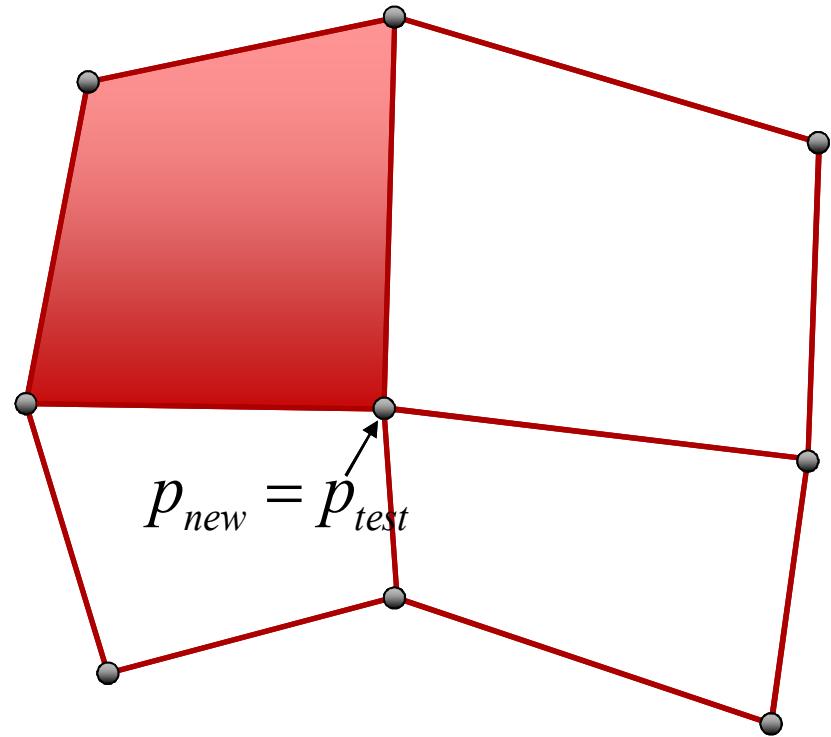


# Optimization

Compute minimum scaled  
Jacobian,  $(J_s)_p$ , of node  $p$  in all  
attached hexes

Compute numerical  
gradient  $(\nabla J_s)_p$

Find improved  $(J_s)_p$  by searching  
along vector  $(\nabla J_s)_p$





# Optimization

Compute minimum scaled Jacobian,  $(J_s)_p$ , of node  $p$  in all attached hexes

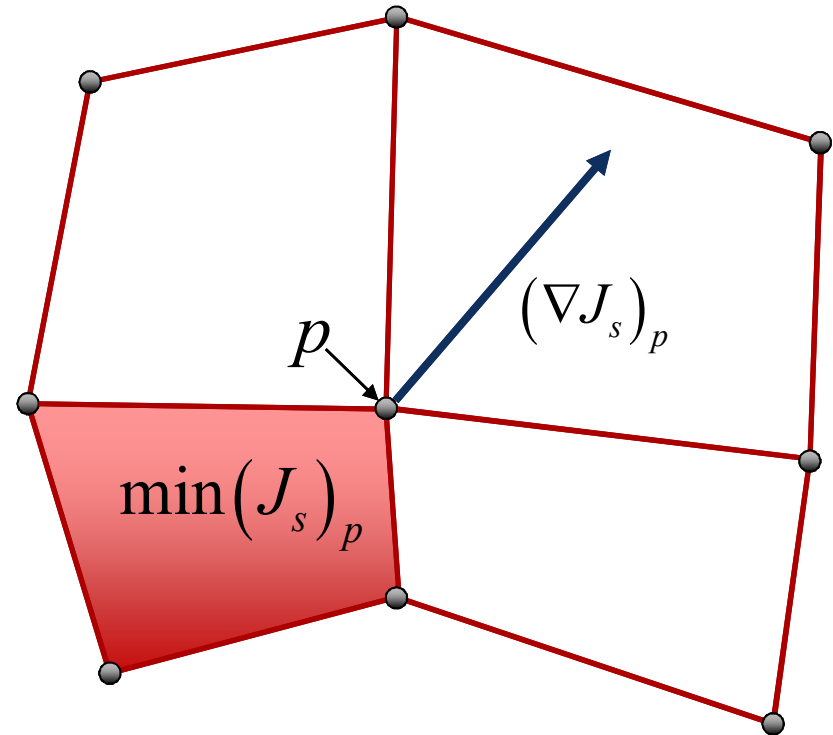
Compute numerical gradient  $(\nabla J_s)_p$

Find improved  $(J_s)_p$  by searching along vector  $(\nabla J_s)_p$

Stopping Criteria

$$(J_s)_p > 0.2$$

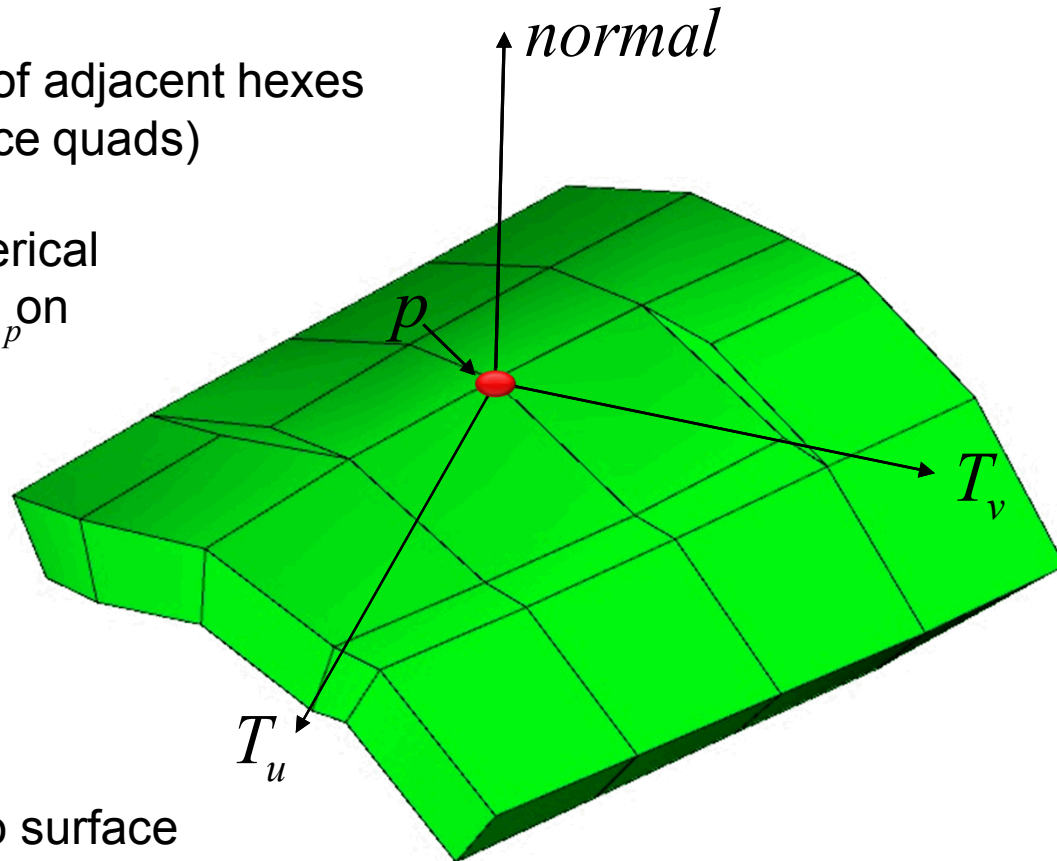
*Maximum 3 iterations*



# Optimization for Surface Nodes

Optimize  $(J_s)_p$  of adjacent hexes  
to  $p$  (not surface quads)

Compute numerical  
gradient  $(\nabla J_s)_p$  on  
tangent plane

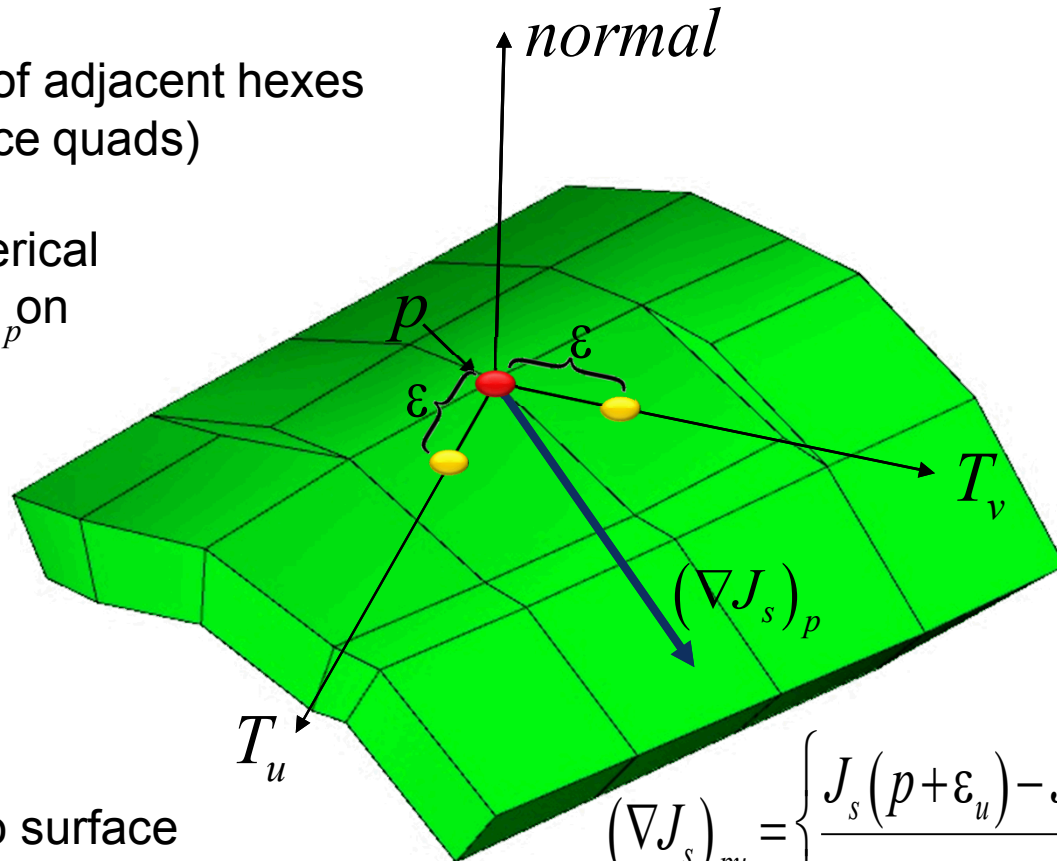


Project  $p_{new}$  to surface

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Optimize  $(J_s)_p$  of adjacent hexes  
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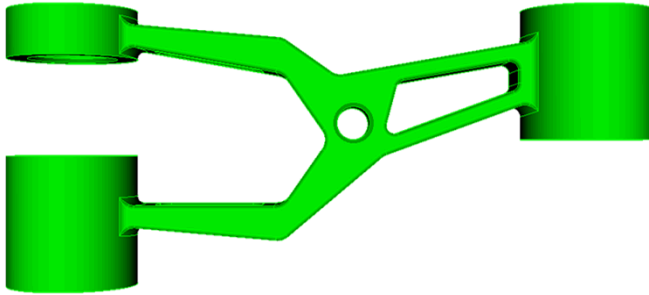


Project  $p_{new}$  to surface

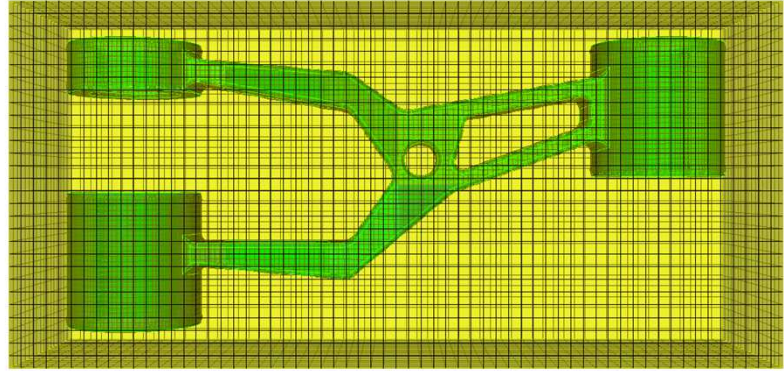
$$(\nabla J_s)_{pu} = \left\{ \frac{J_s(p + \epsilon_u) - J_s(p)}{\epsilon} \right\}$$

$$(\nabla J_s)_{pv} = \left\{ \frac{J_s(p + \epsilon_v) - J_s(p)}{\epsilon} \right\}$$

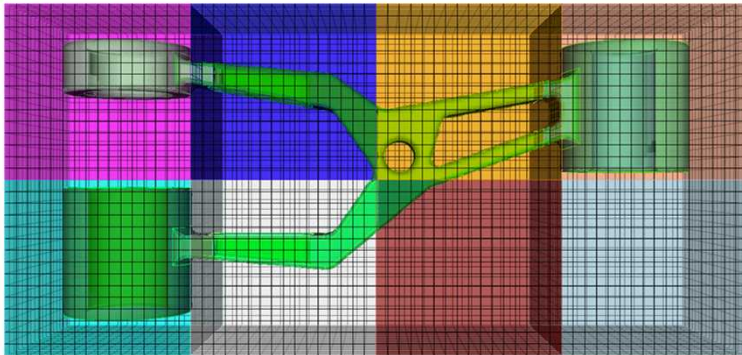
# Distributed Meshing



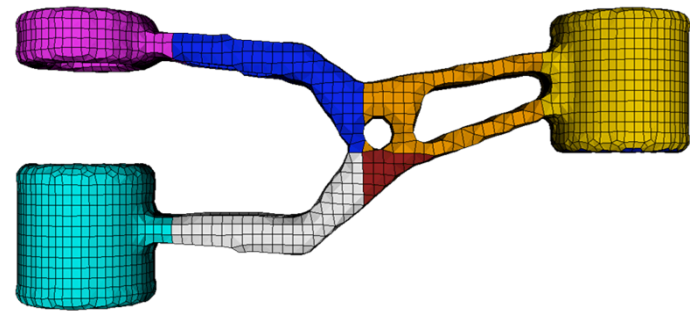
CAD geometry



Global overlay Cartesian grid

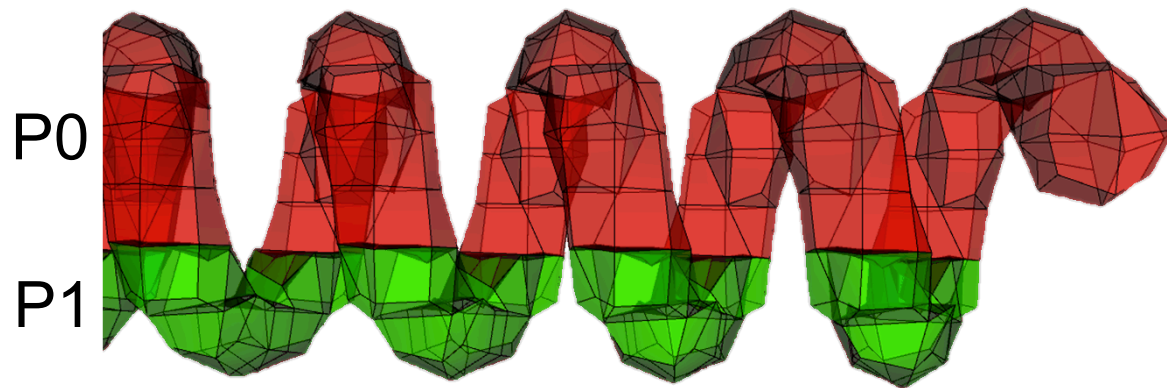


Cartesian grid decomposed and  
distributed amongst many  
processors

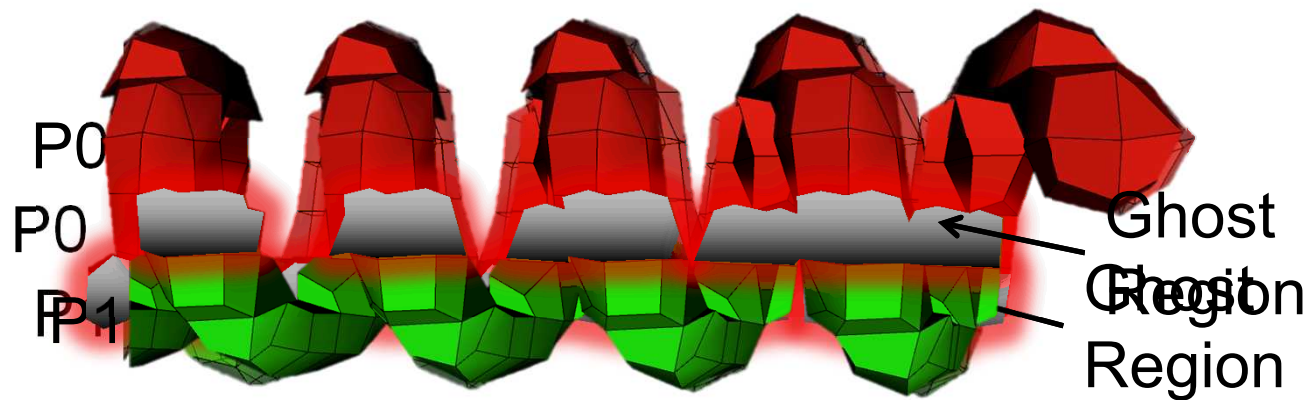


Each processor independently  
meshes its portion of Cartesian grid

# Communication for Smoothing

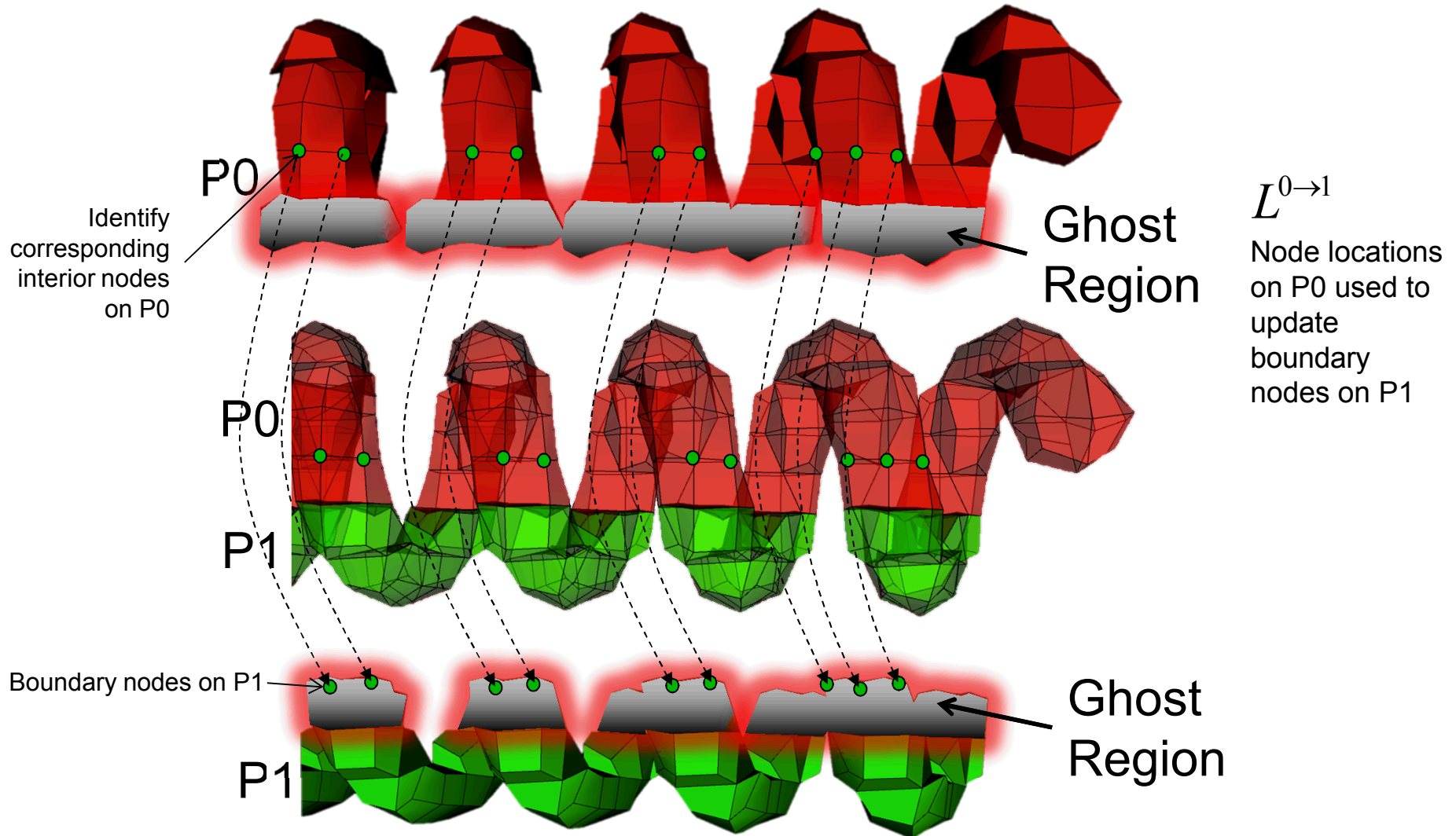


# Communication for Smoothing

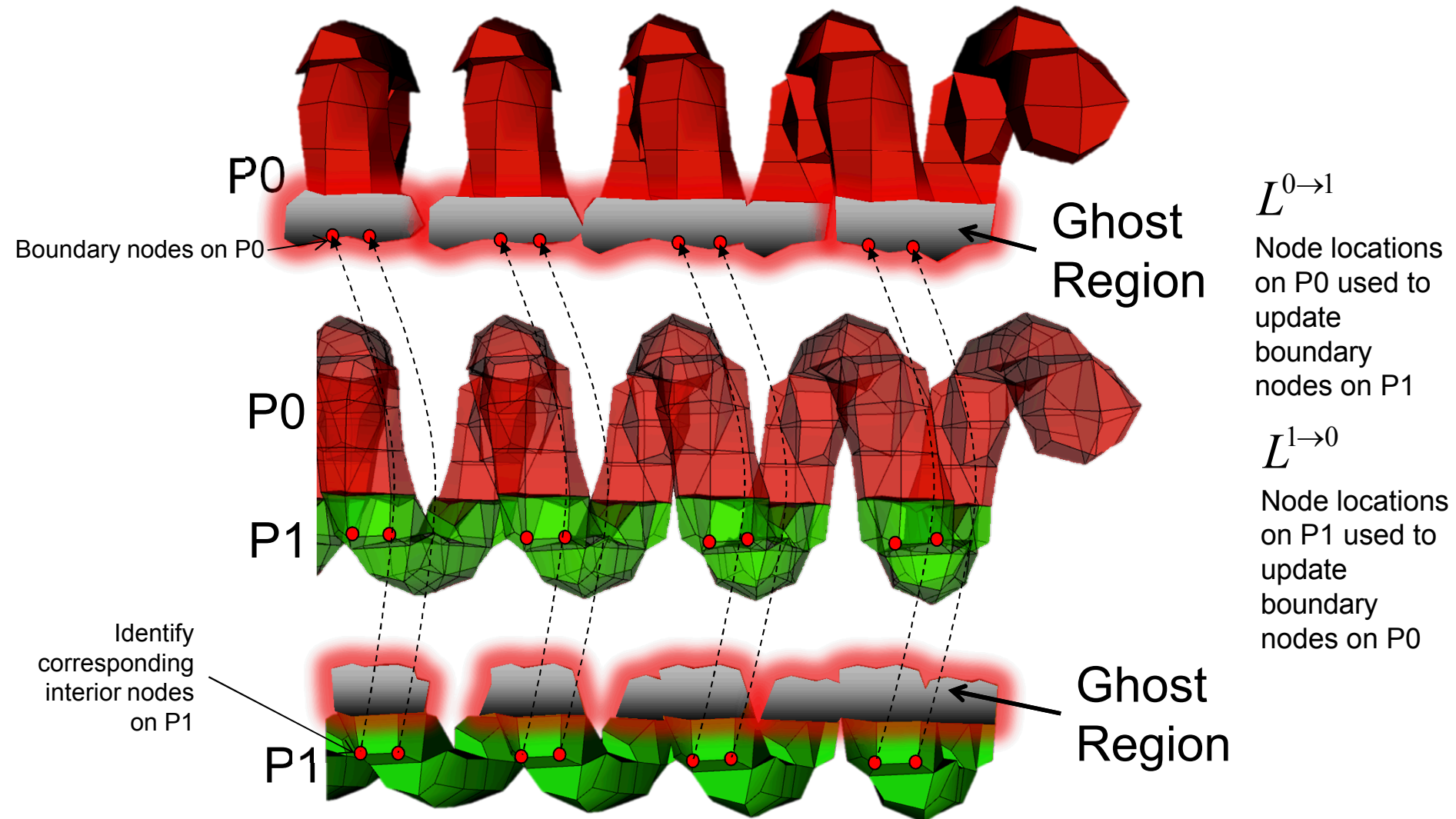




# Communication for Smoothing

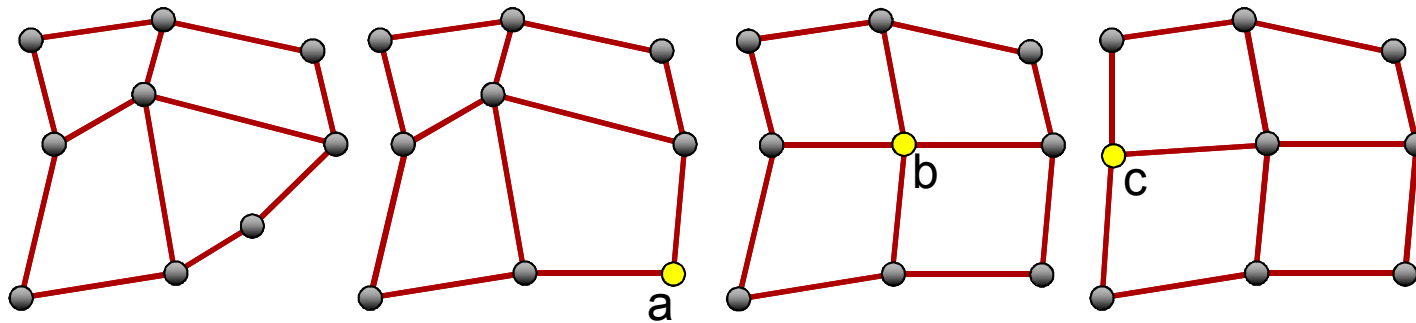


# Communication for Smoothing



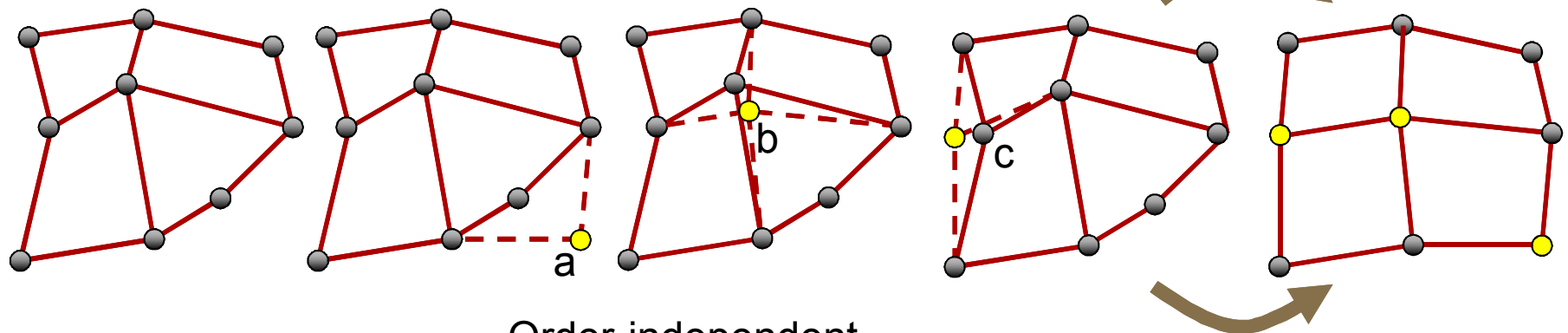
# Smoothing Strategies

## Gauss-Siedel



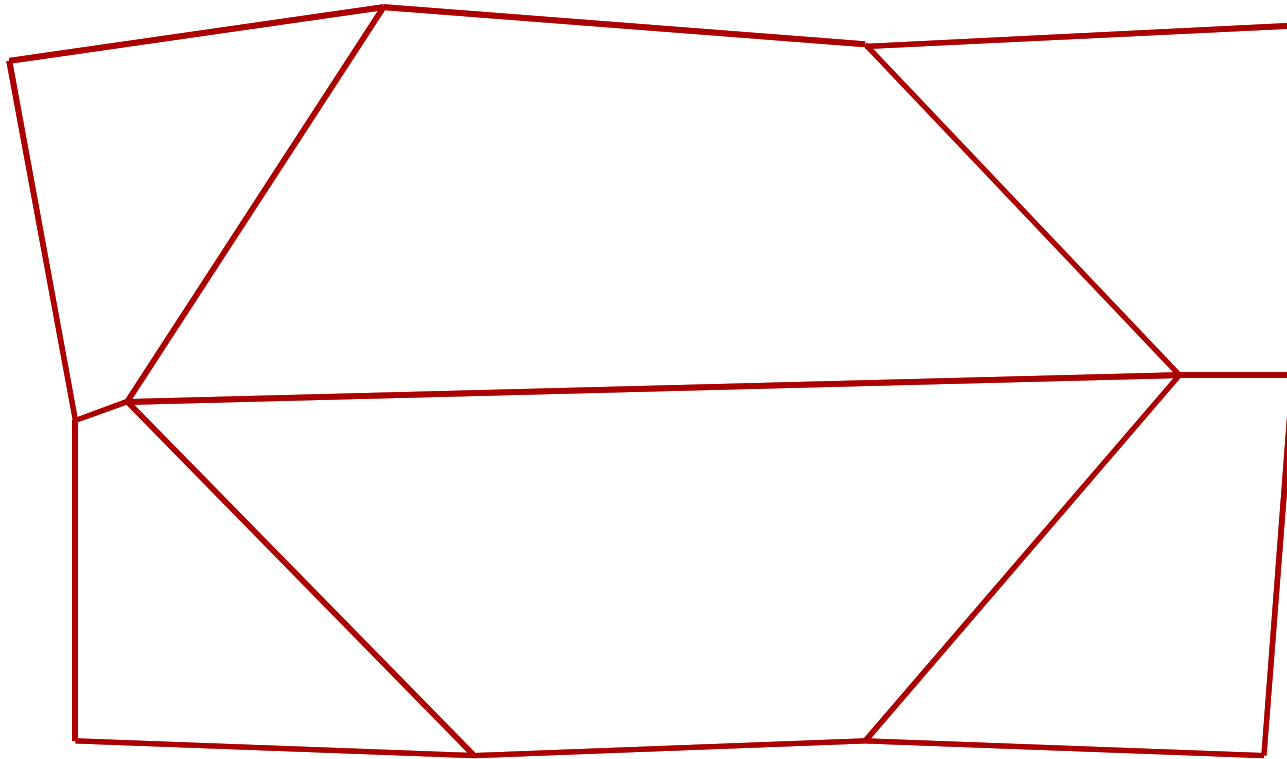
Order-dependent

## Jacobi

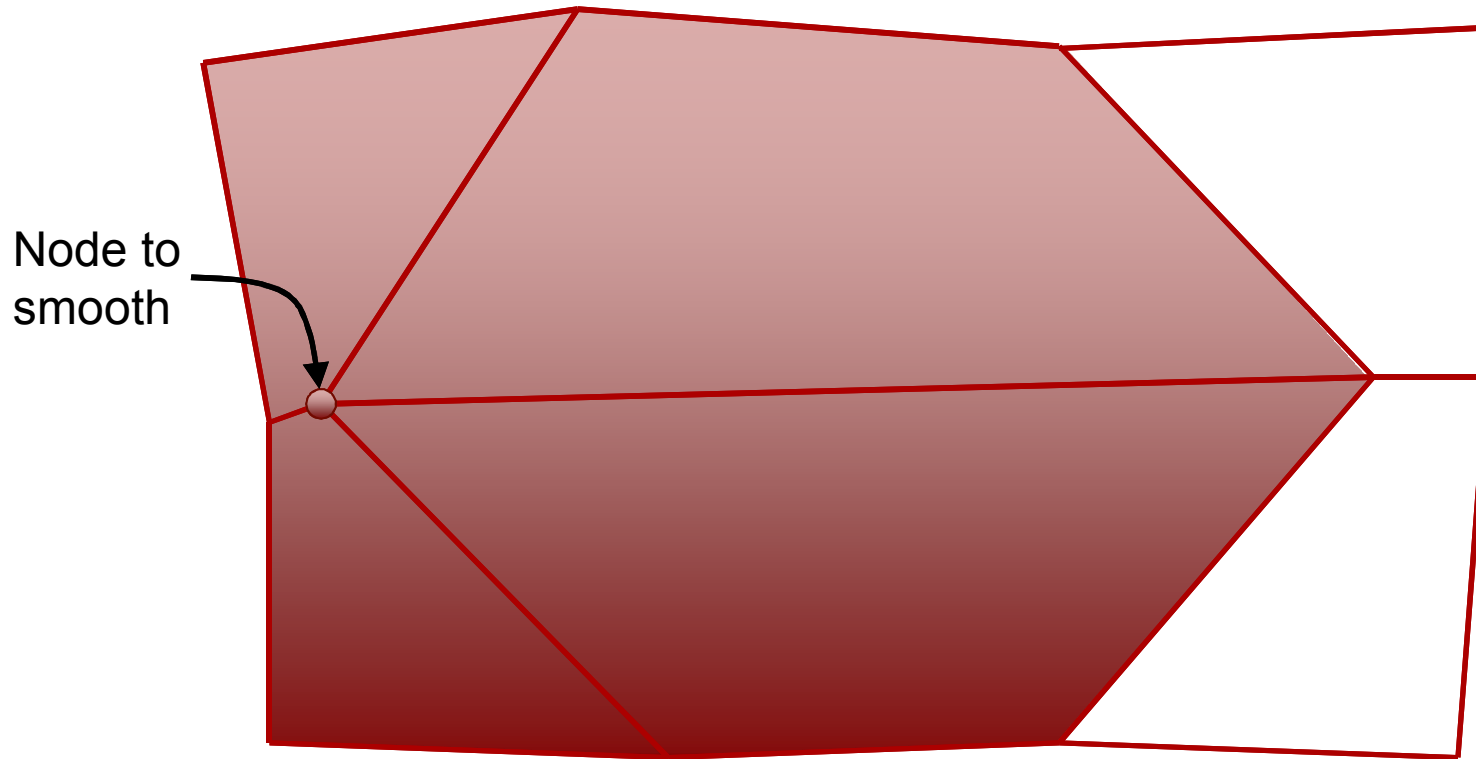


Order-independent

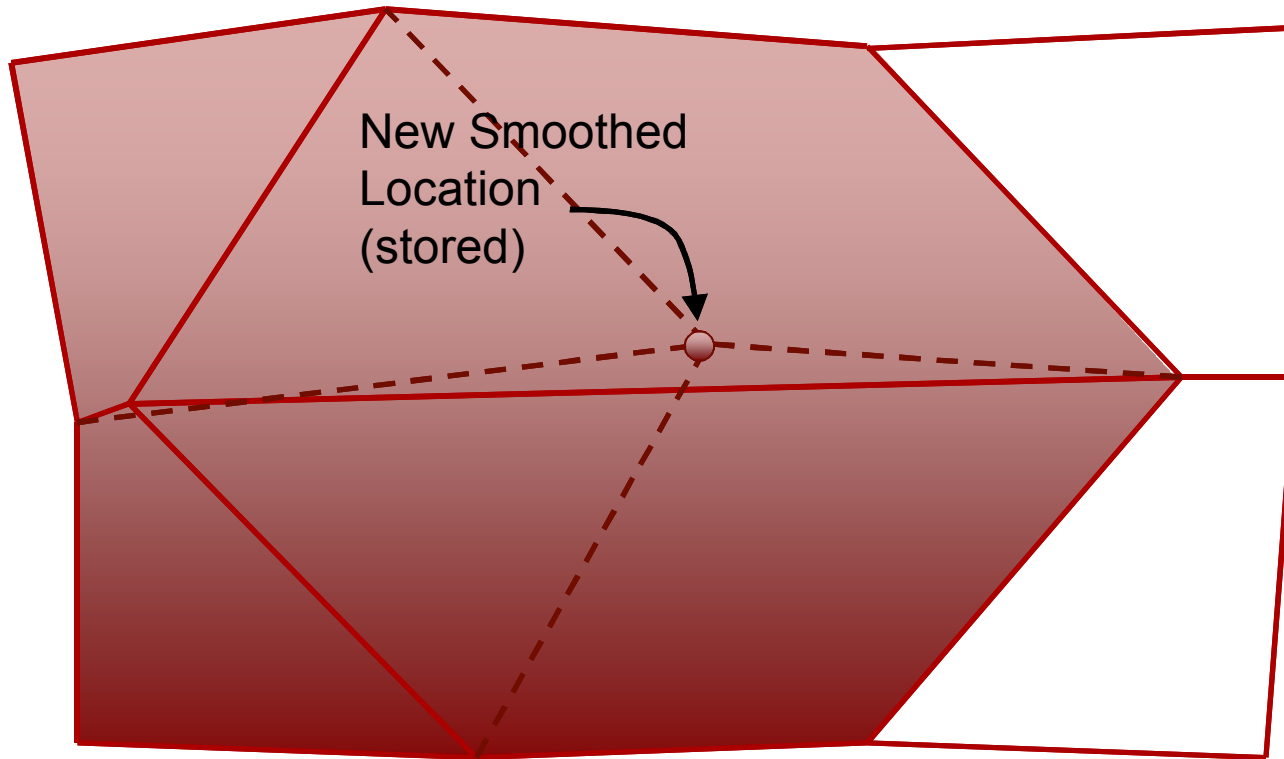
# Jacobi Smoothing



# Jacobi Smoothing

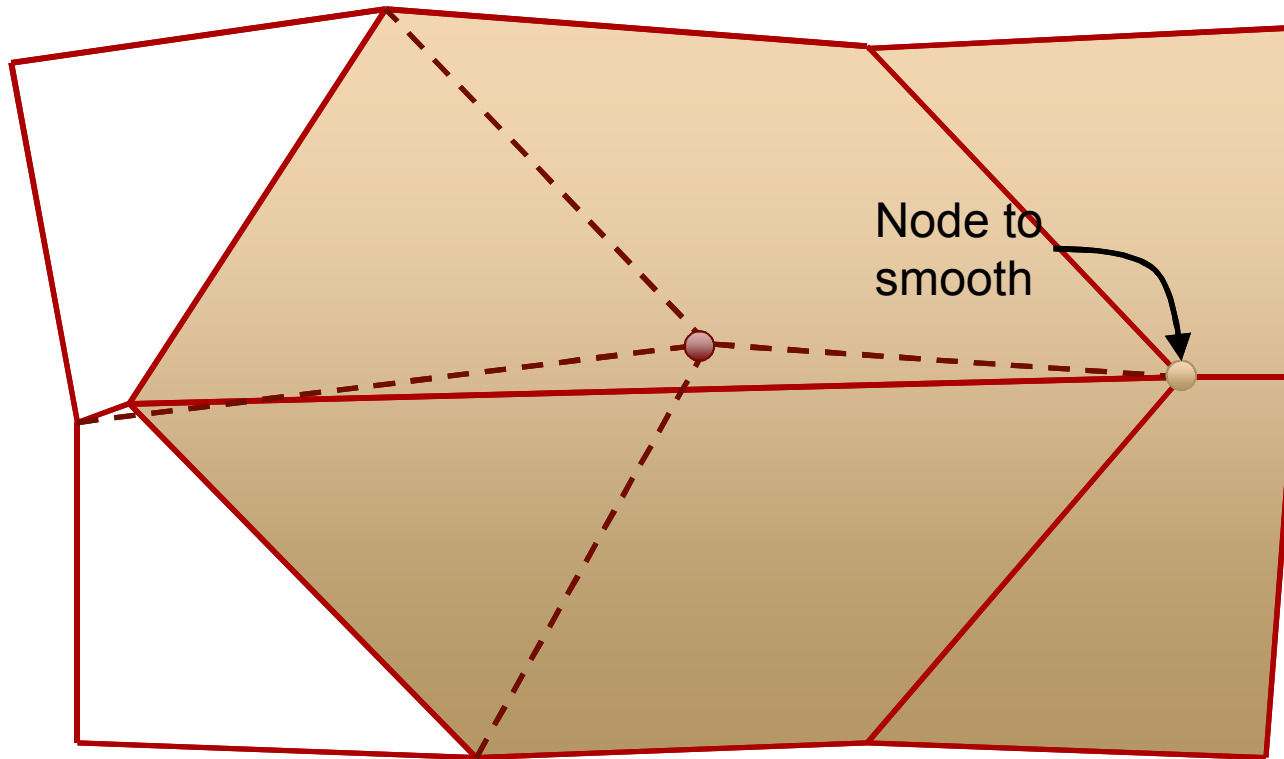


# Jacobi Smoothing

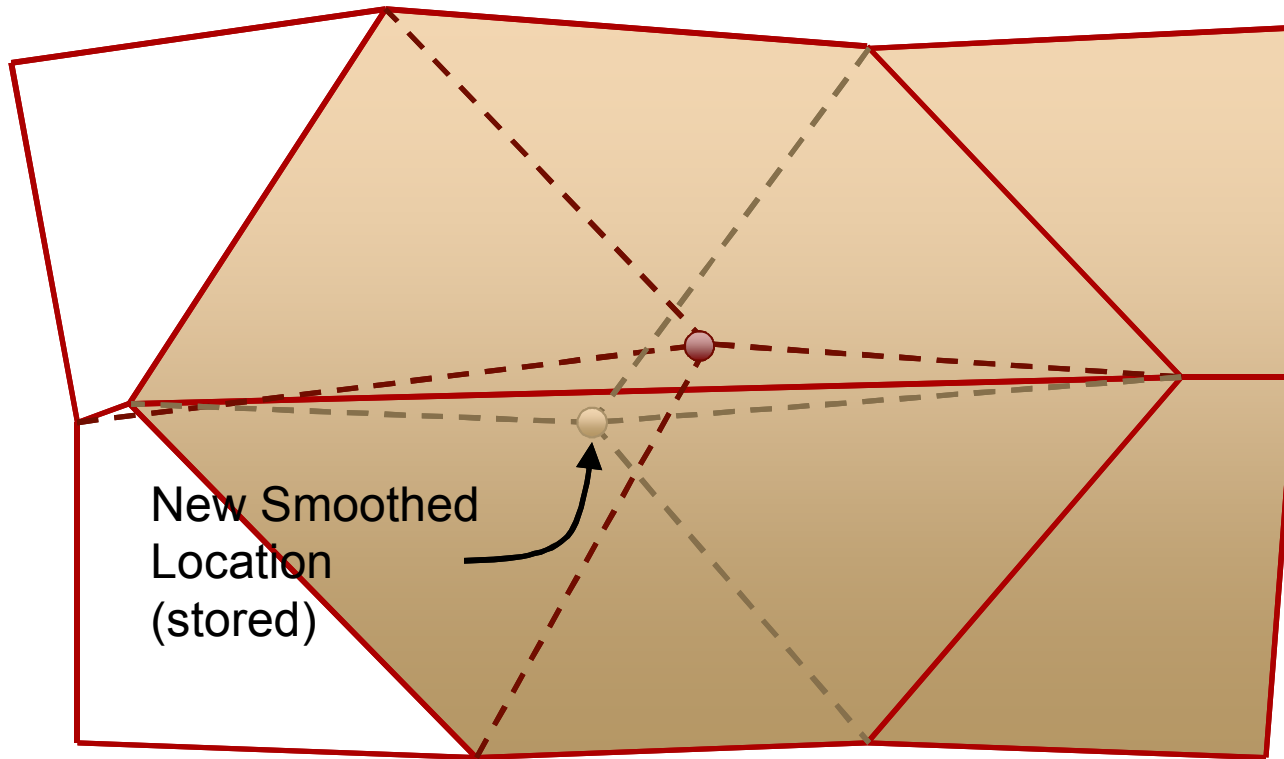




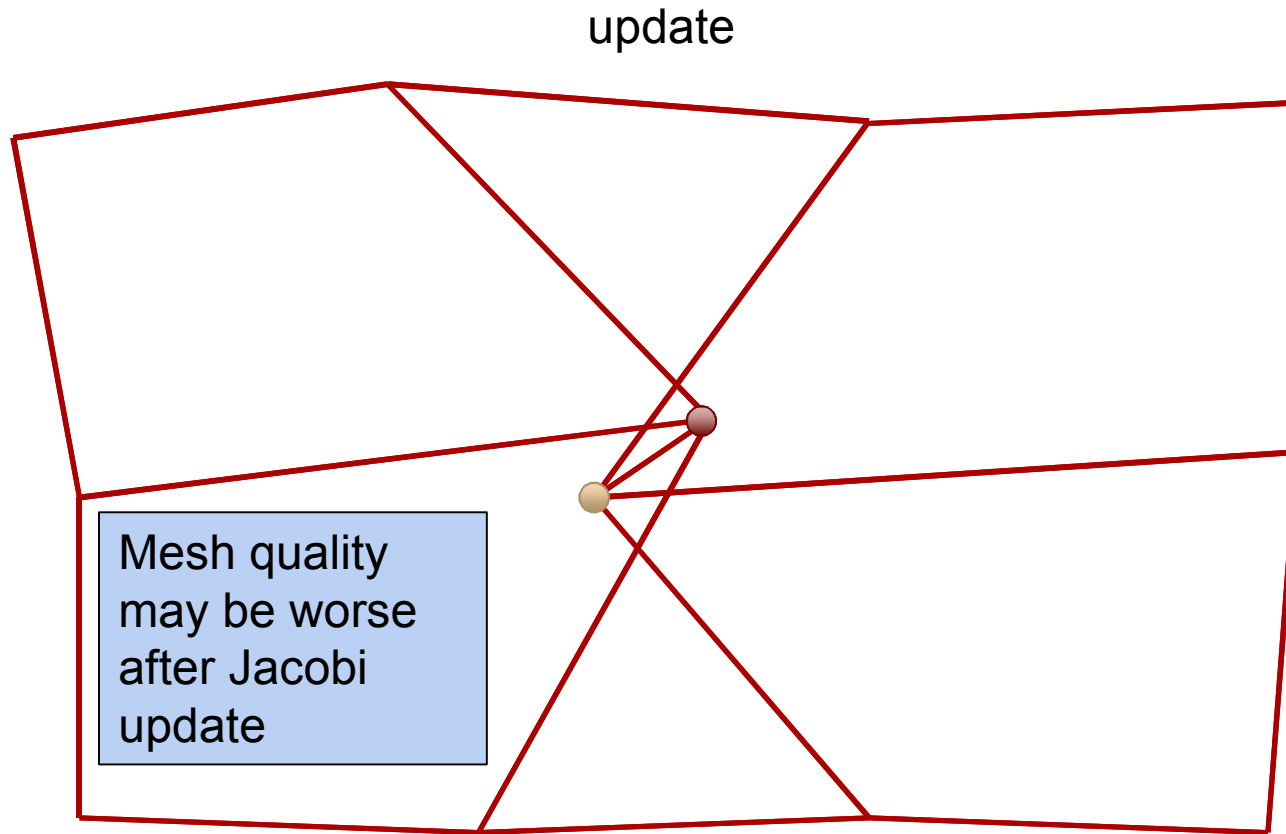
# Jacobi Smoothing



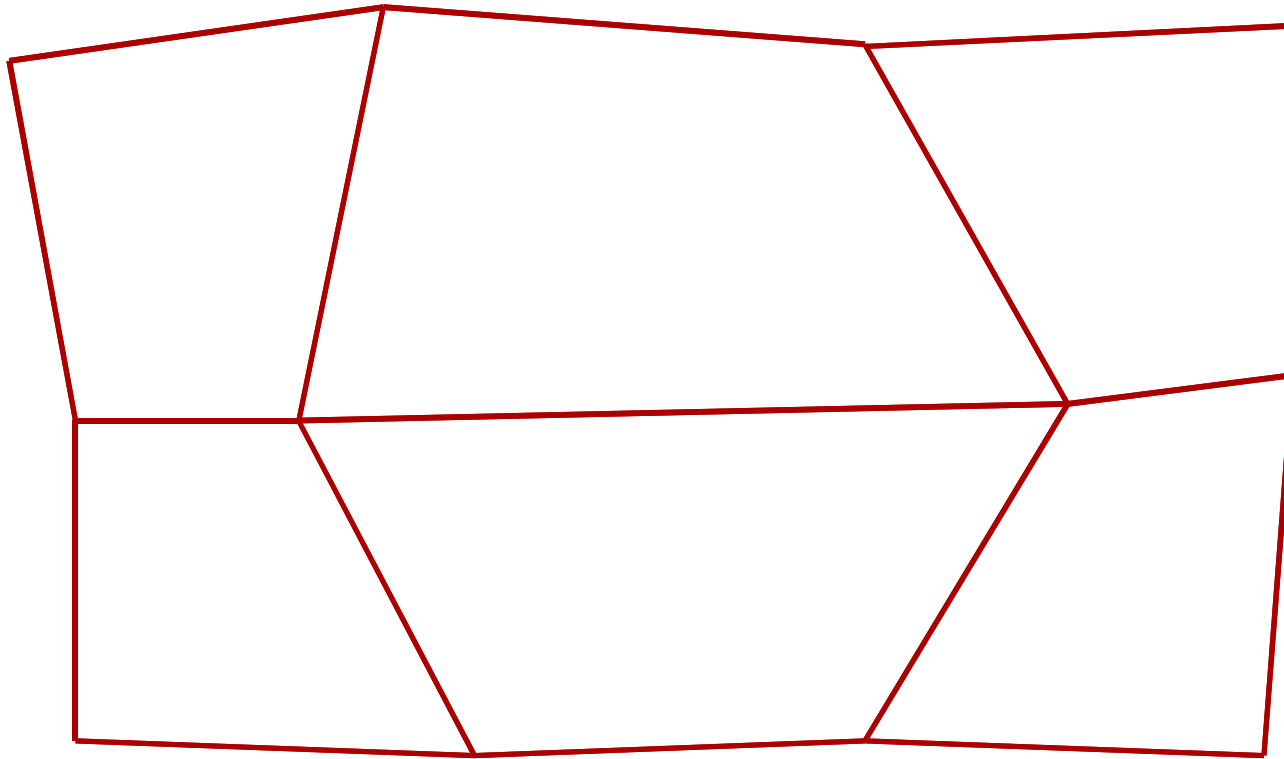
# Jacobi Smoothing



# Jacobi Smoothing

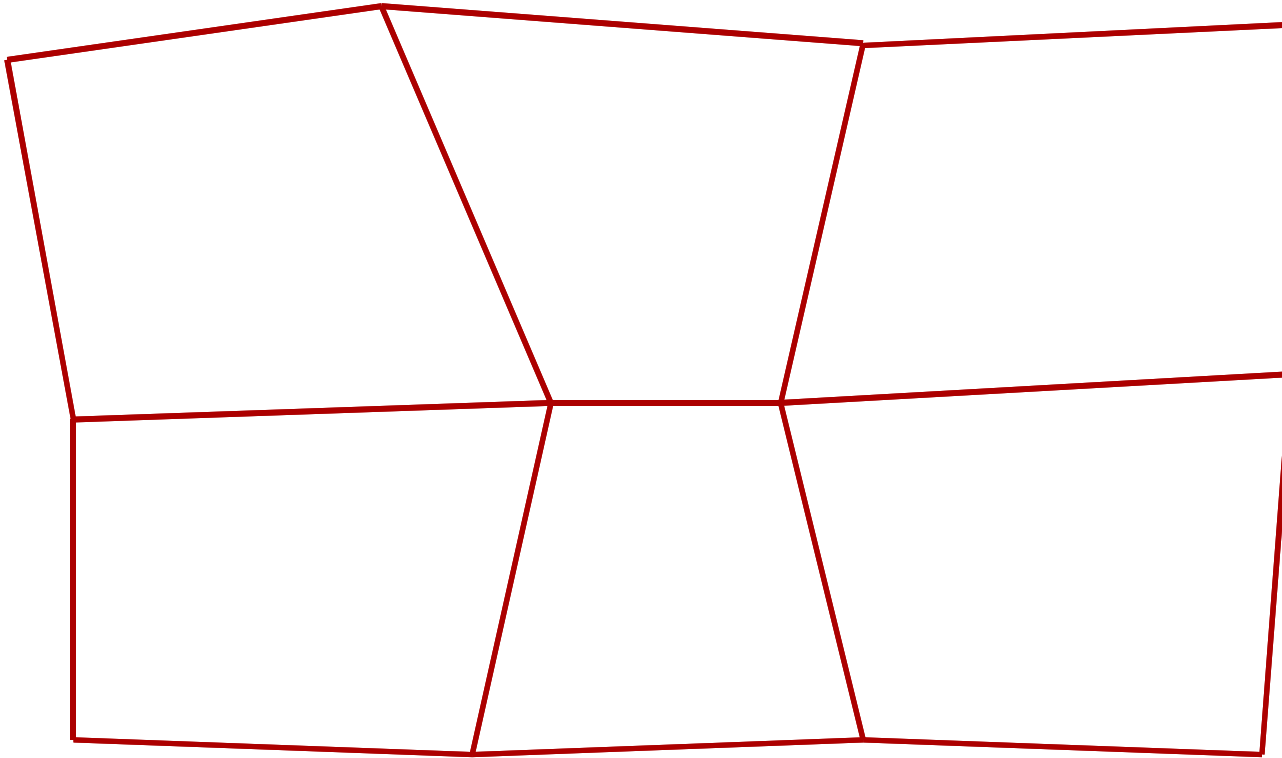


# Jacobi Smoothing



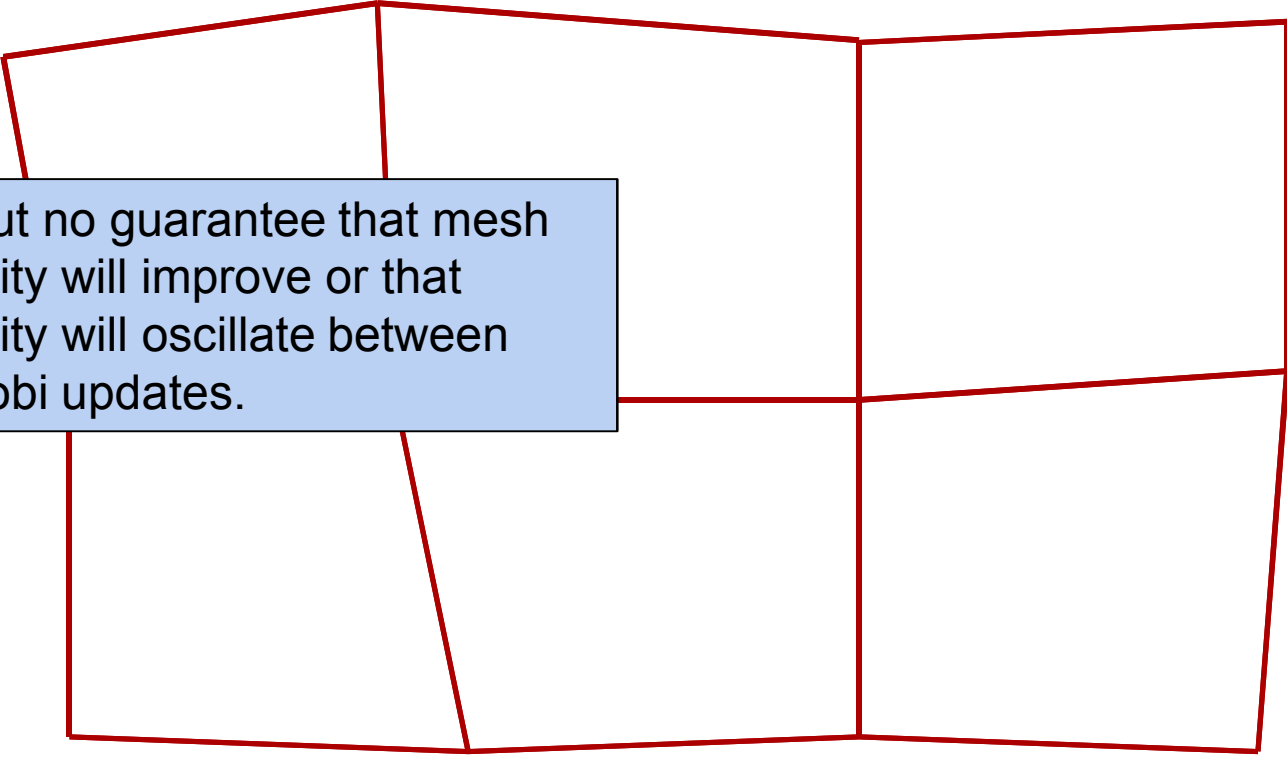
After several iterations, quality in *most* cases will improve

# Jacobi Smoothing



After several iterations, quality in *most* cases will improve

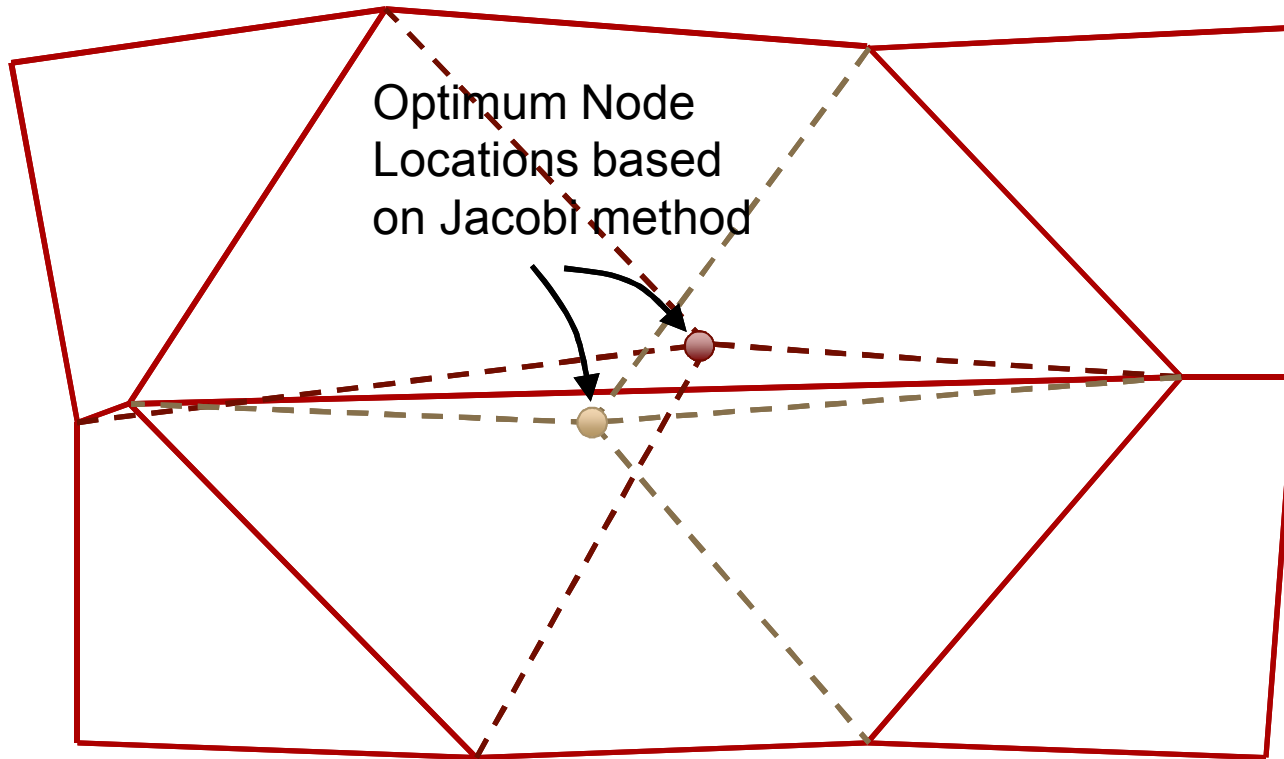
# Jacobi Smoothing



...But no guarantee that mesh quality will improve or that quality will oscillate between Jacobi updates.

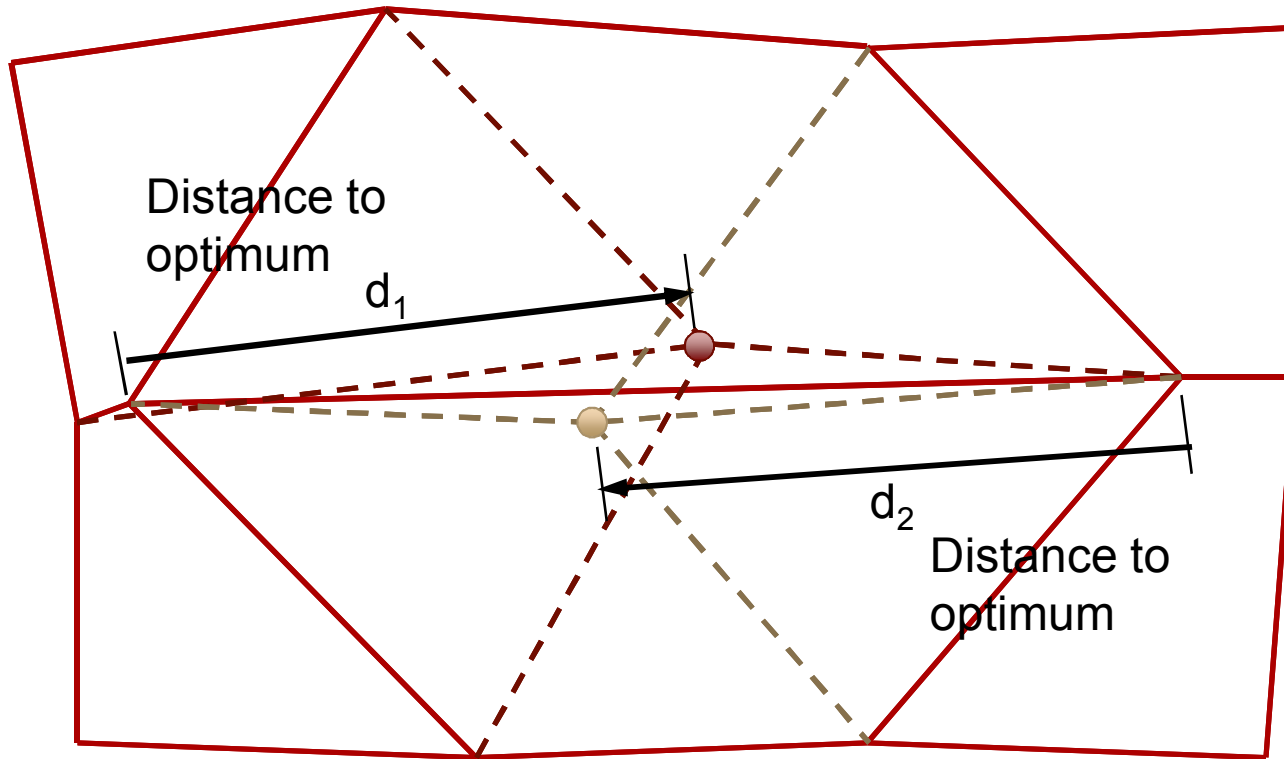
After several iterations, quality in *most* cases will improve

# Jacobi Smoothing



Solution: Apply *damping* to node movement

# Jacobi Smoothing

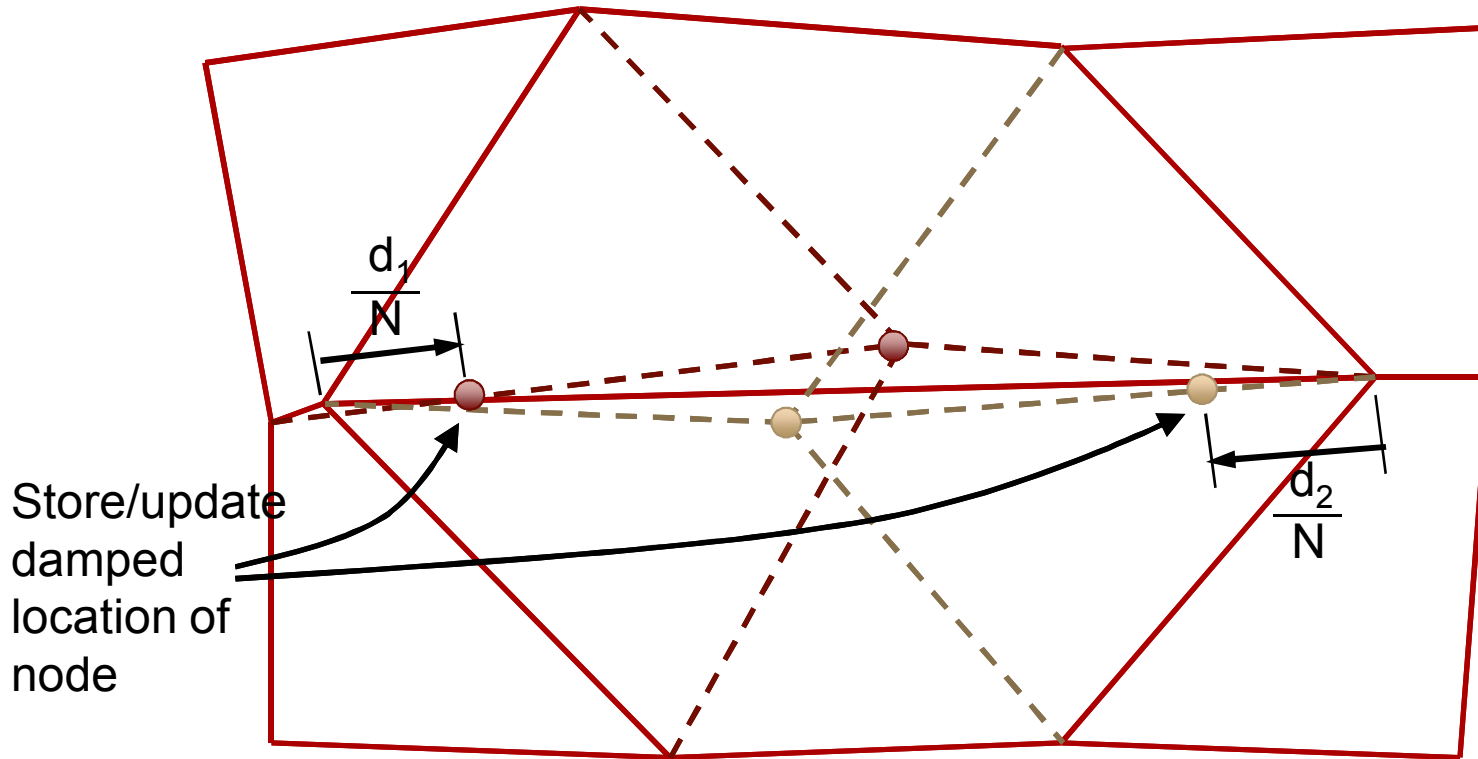


Solution: Apply *damping* to node movement



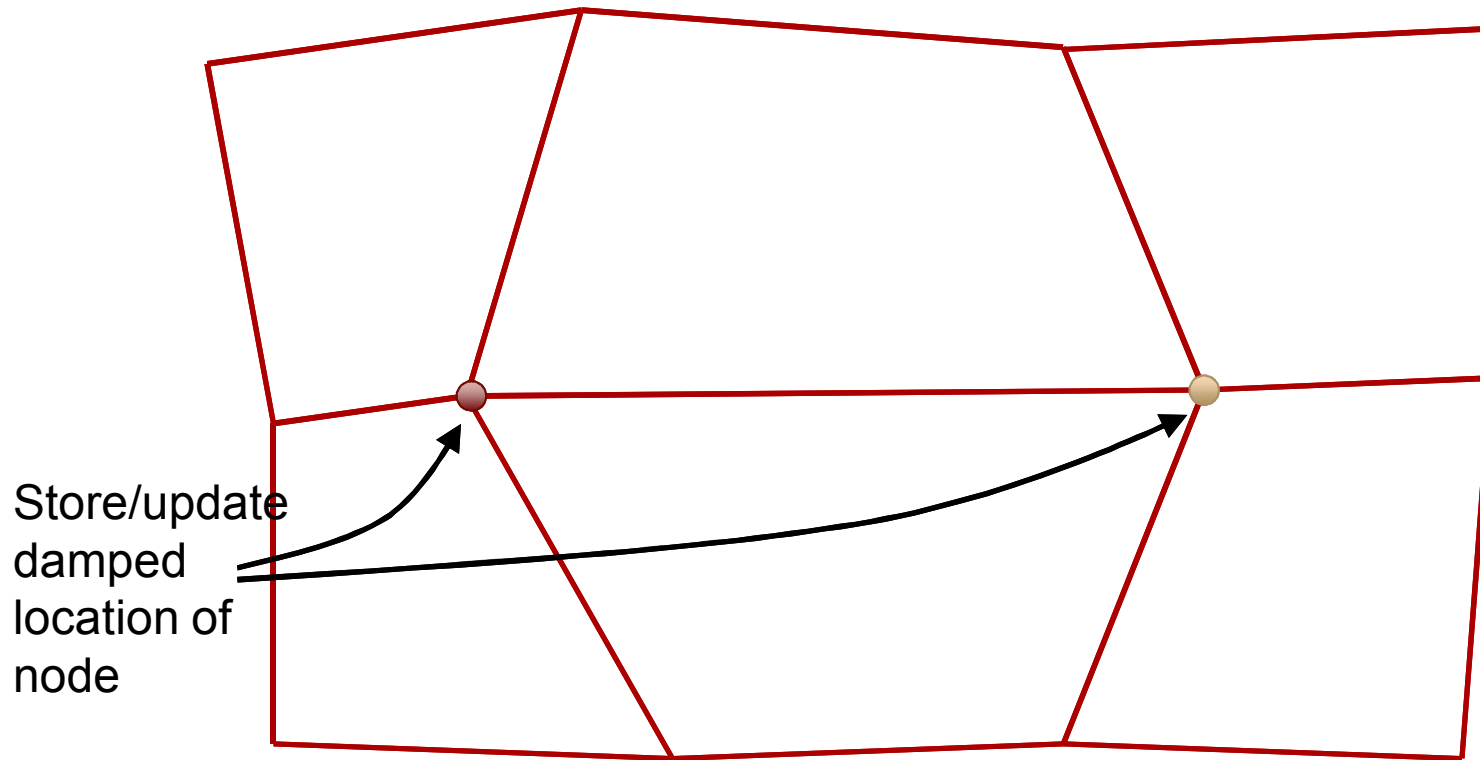
# Jacobi Smoothing

$N$  = number of remaining  
smoothing iterations



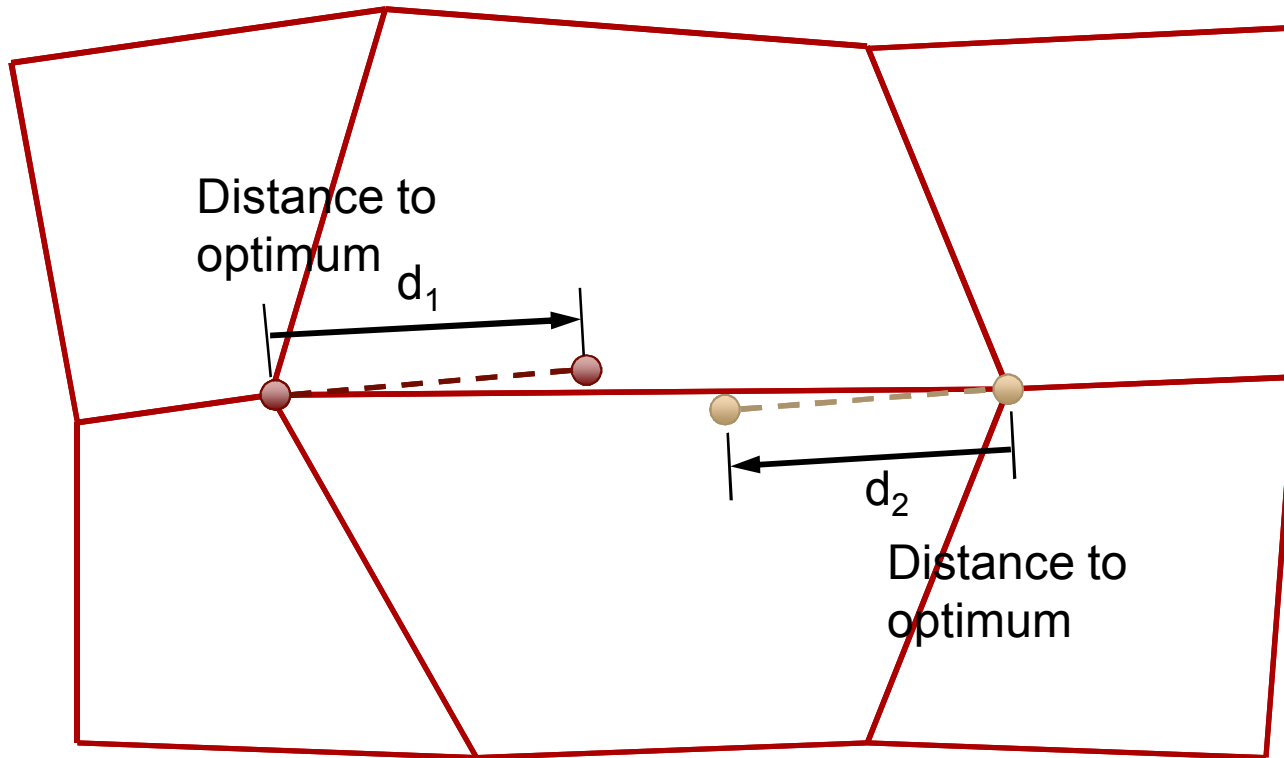
Solution: Apply *damping* to node movement

# Jacobi Smoothing



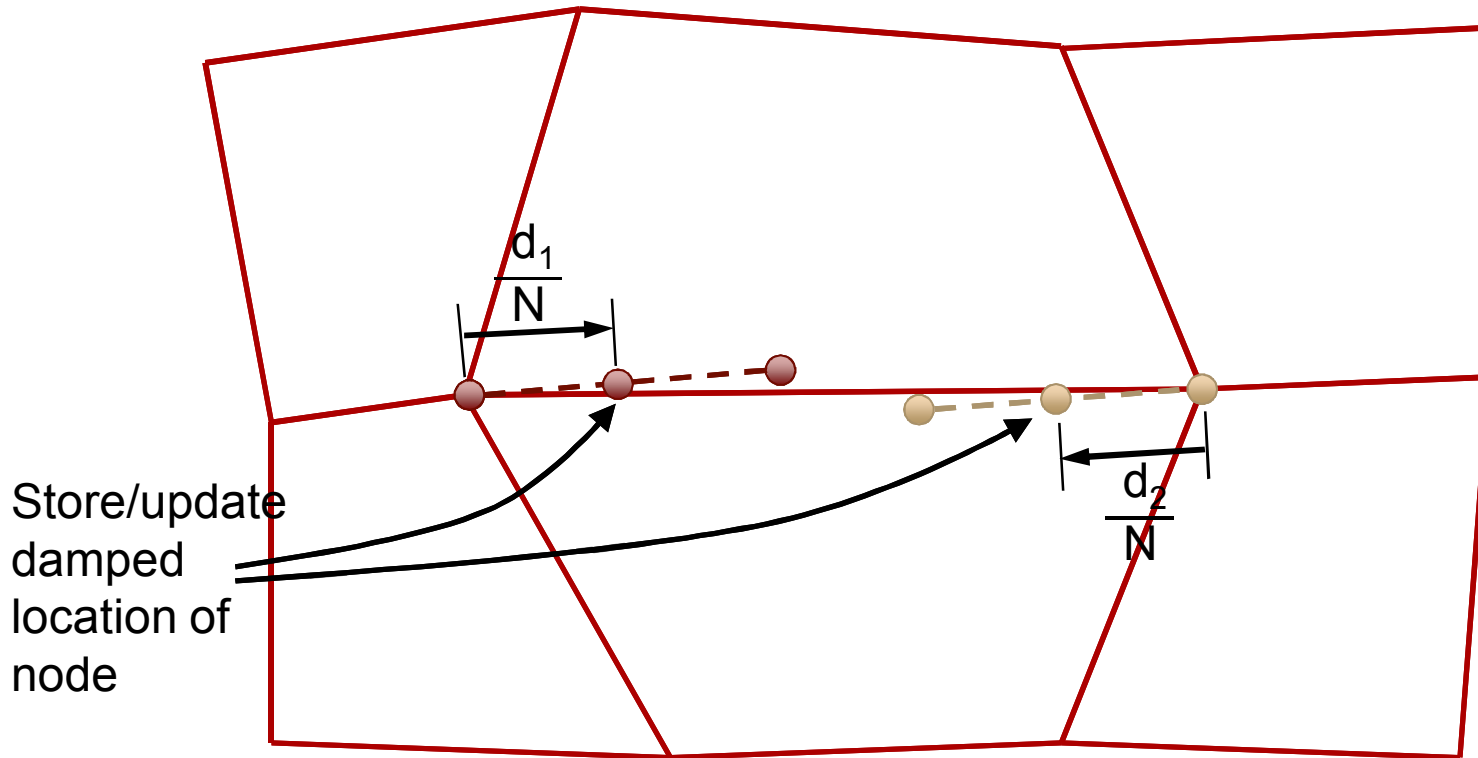
Solution: Apply *damping* to node movement

# Jacobi Smoothing



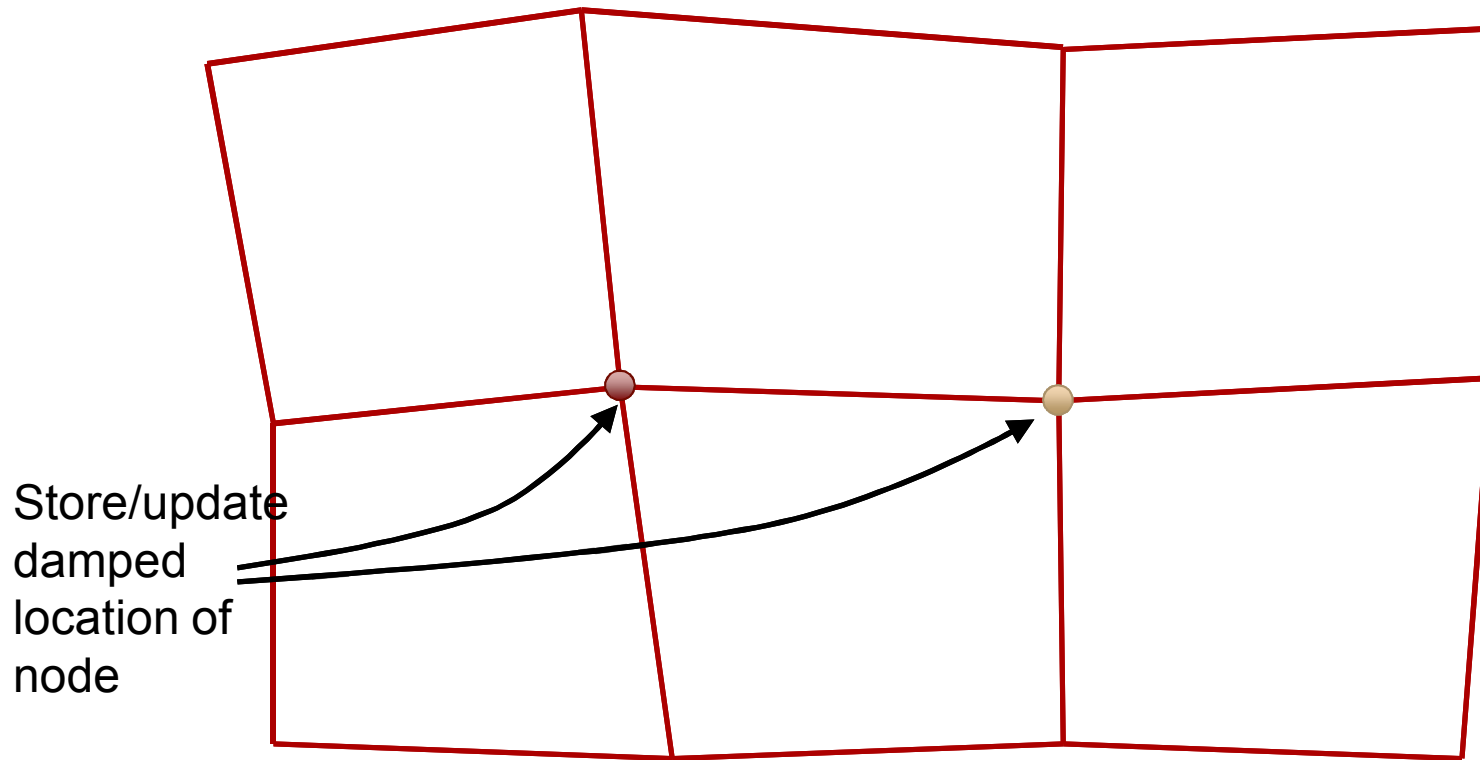
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$N$  = number of remaining  
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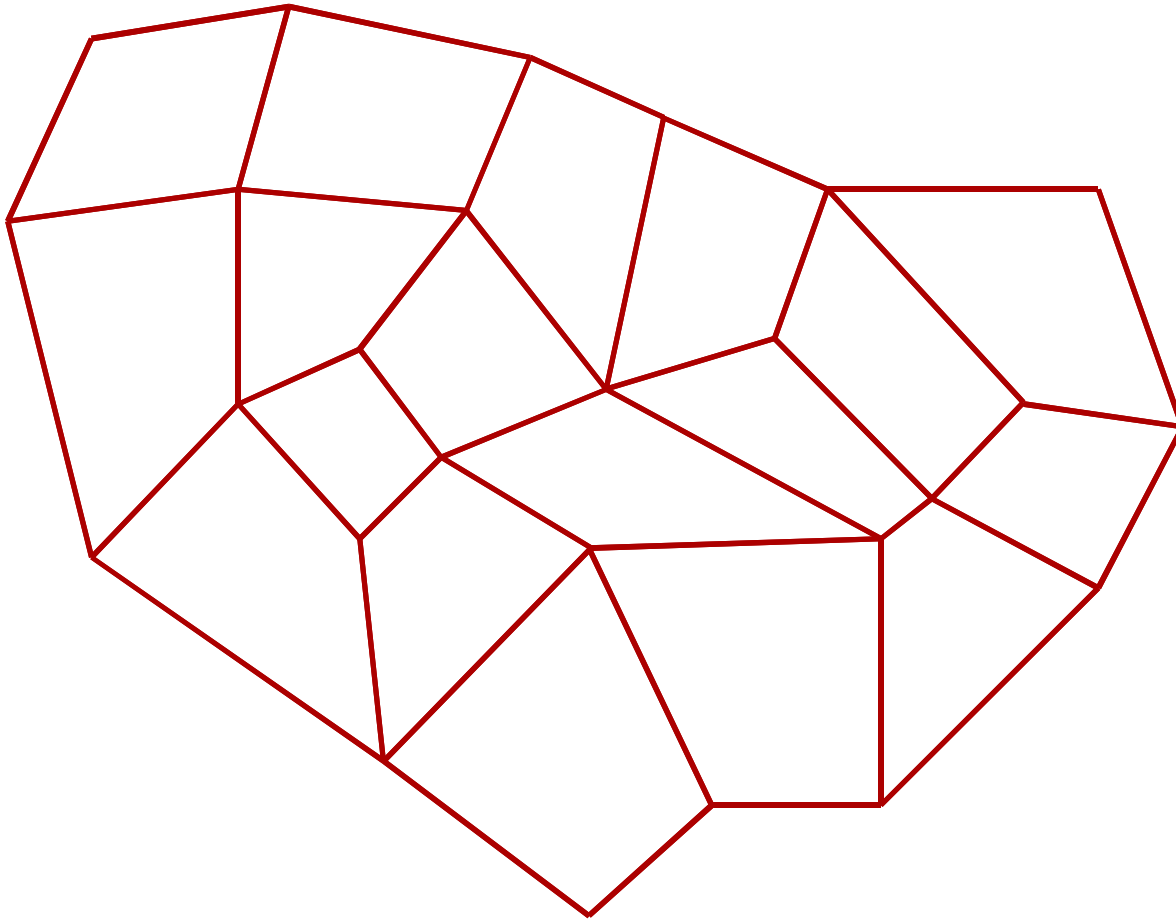
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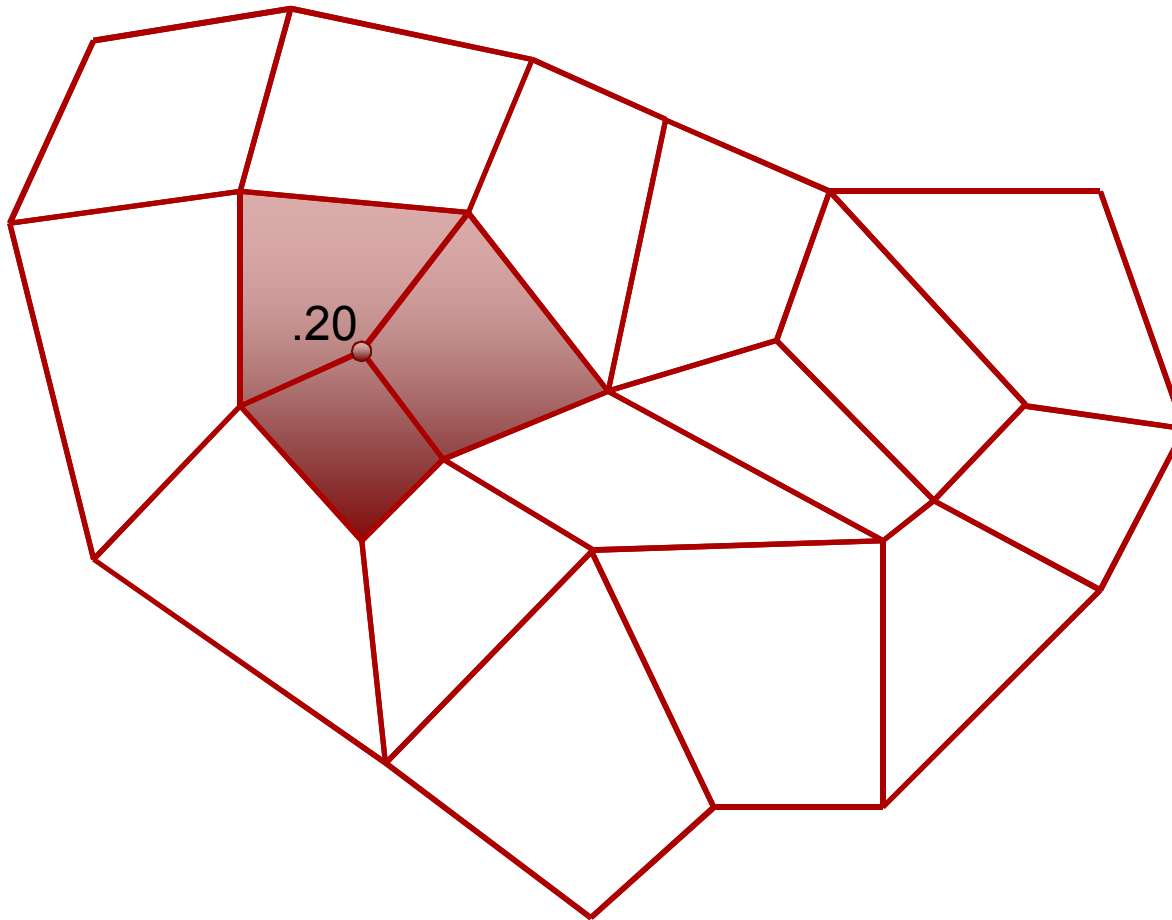


Solution: Apply *damping* to node movement

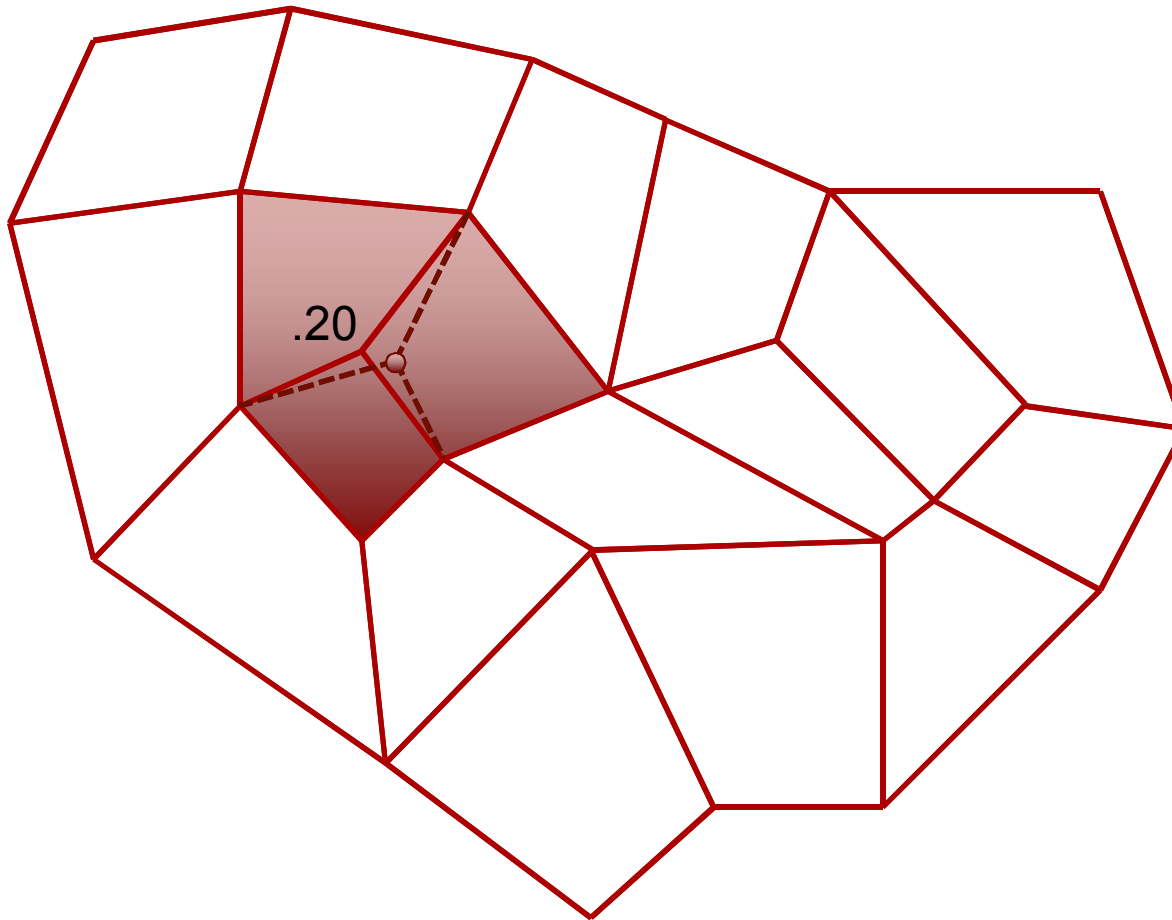
# Parallel Coloring



# Parallel Coloring

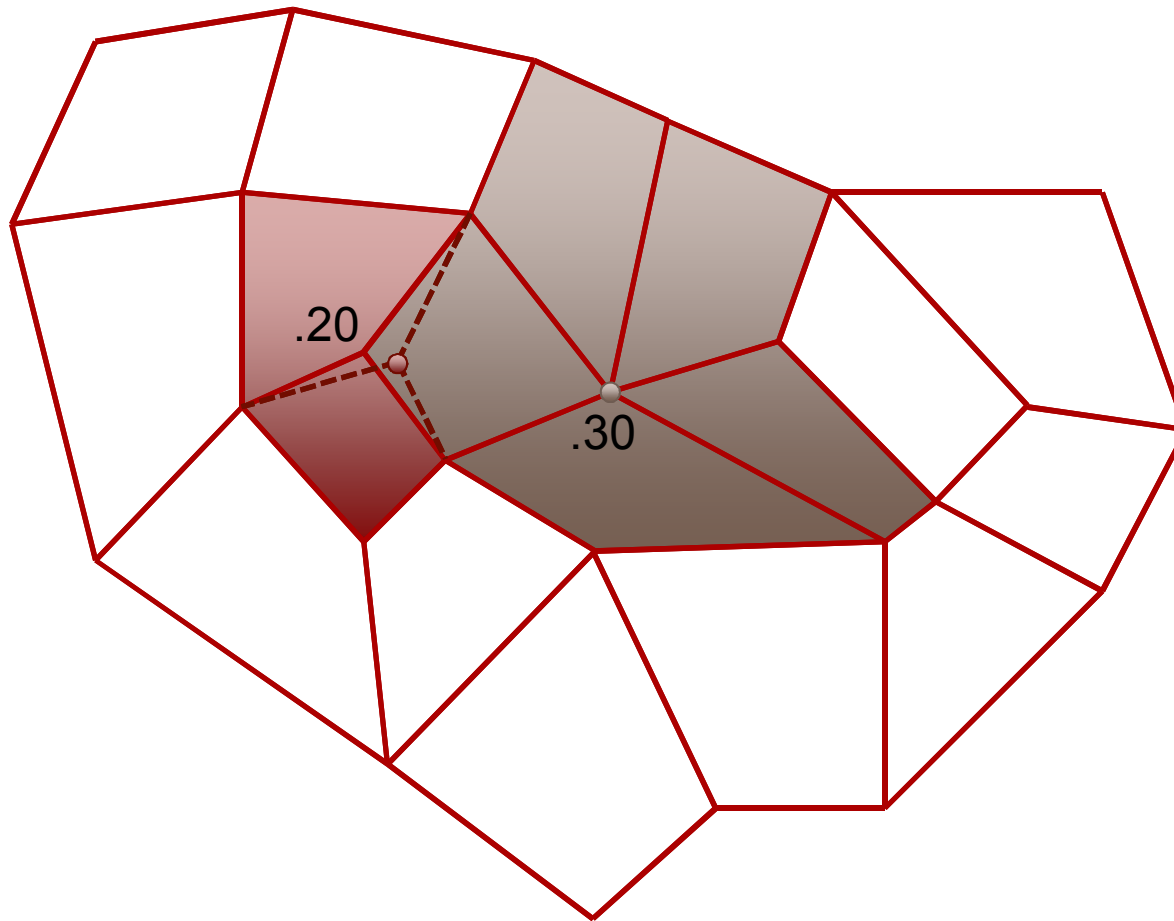


# Parallel Coloring

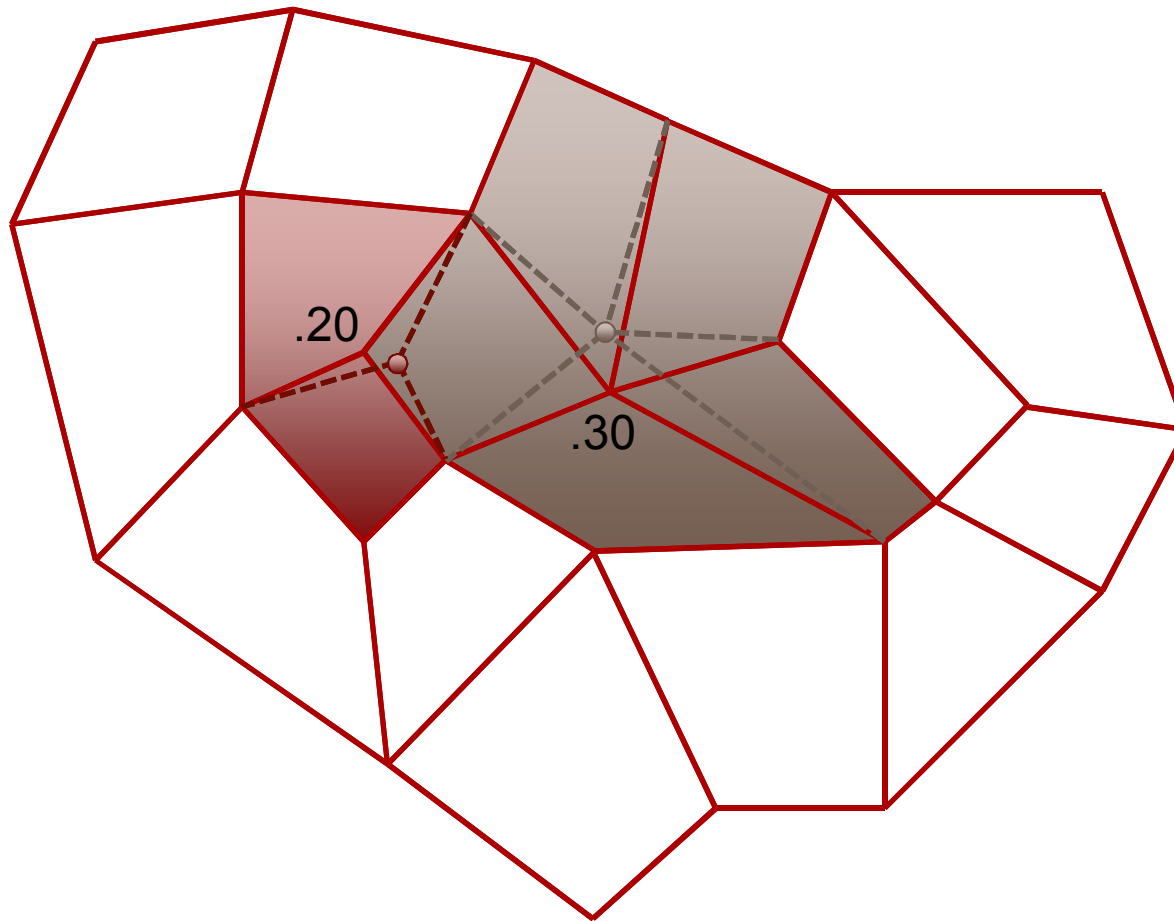




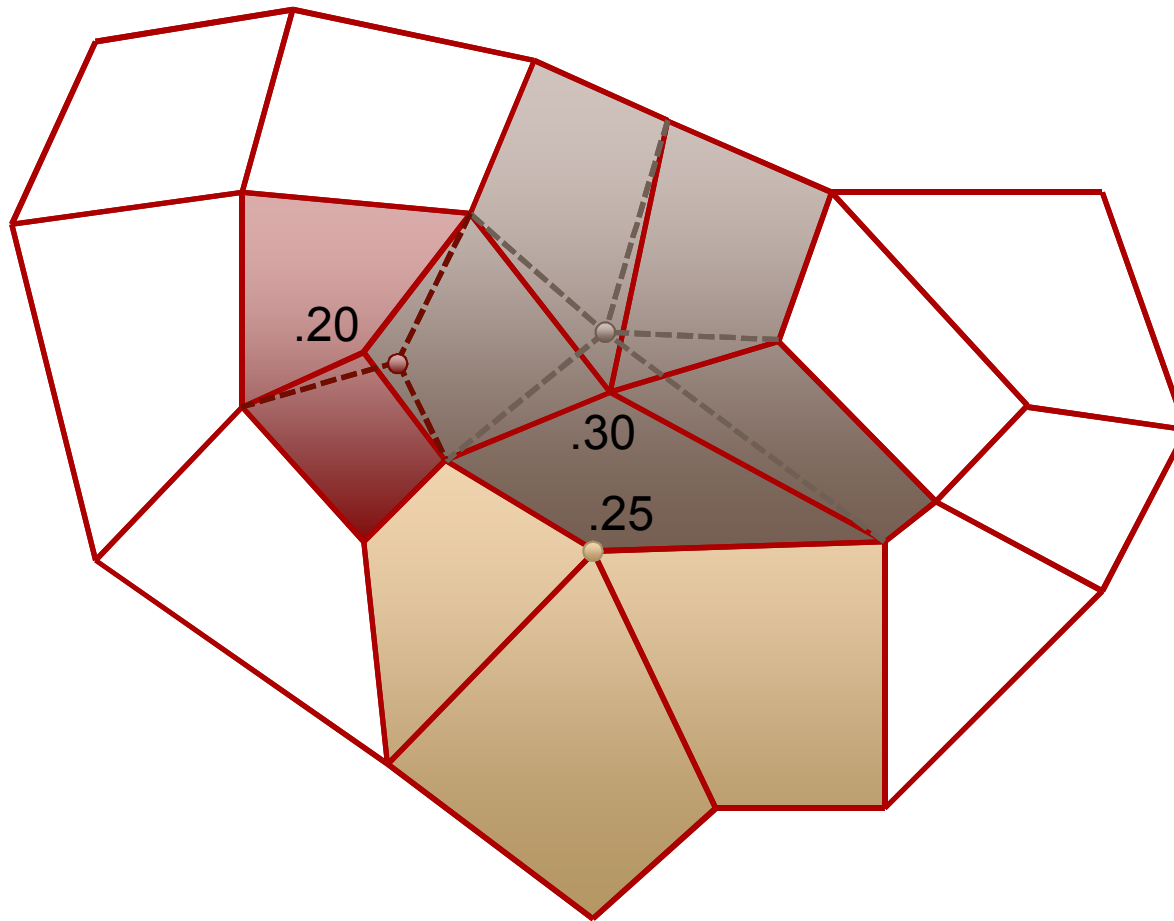
# Parallel Coloring



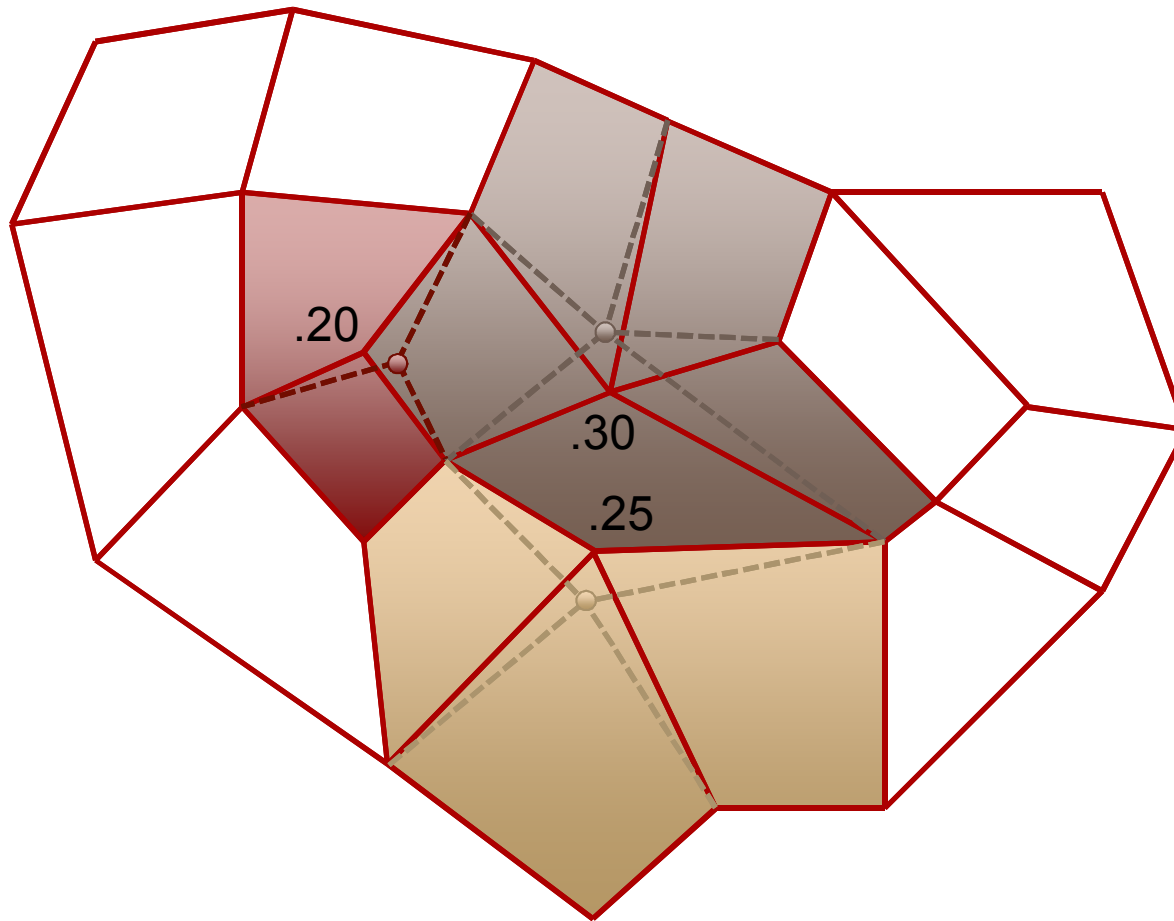
# Parallel Coloring



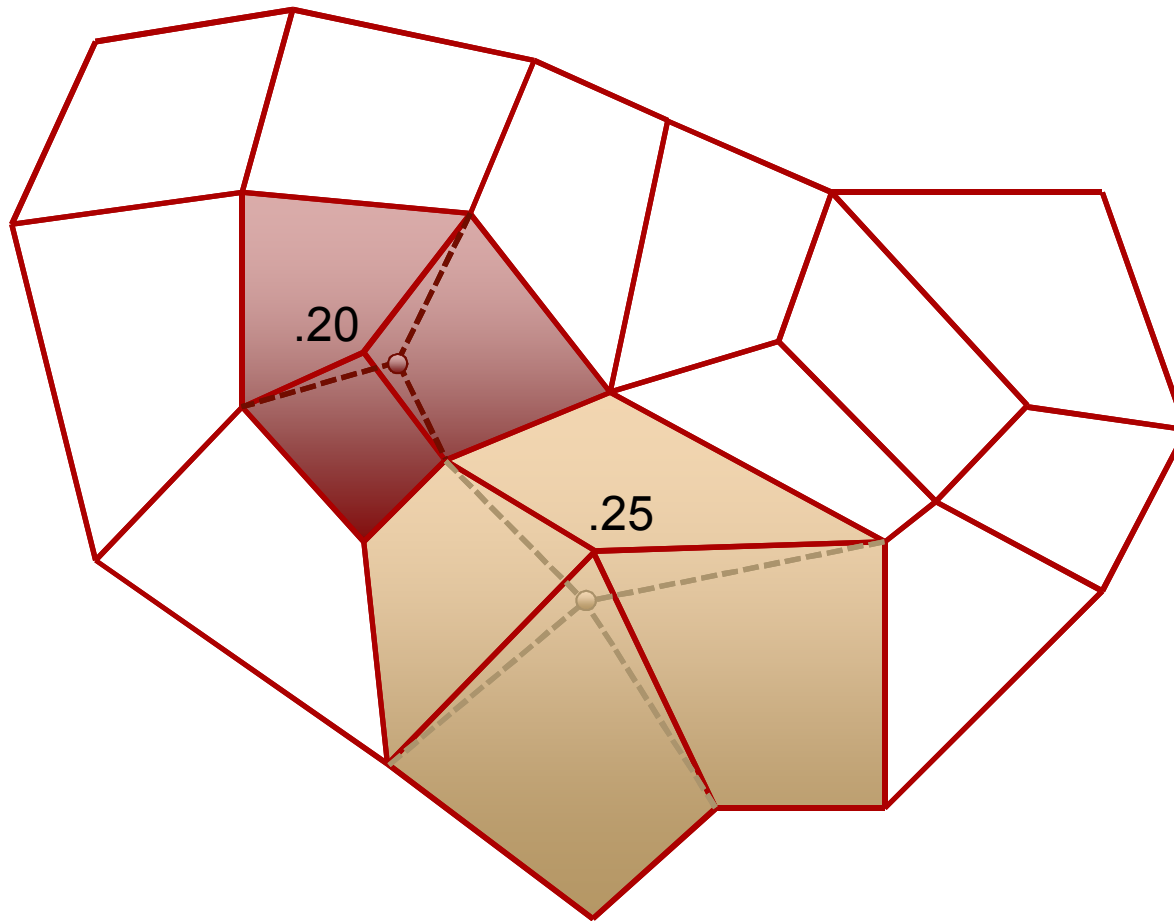
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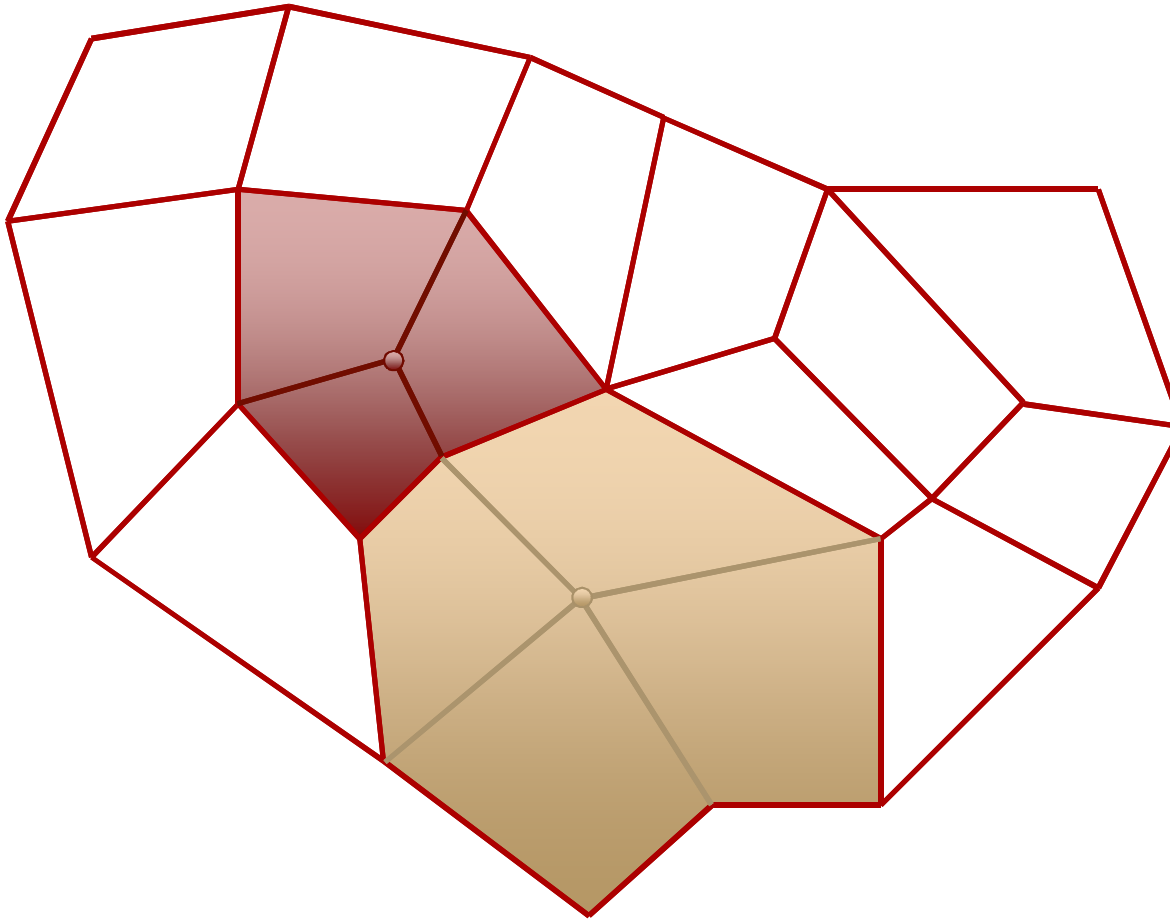
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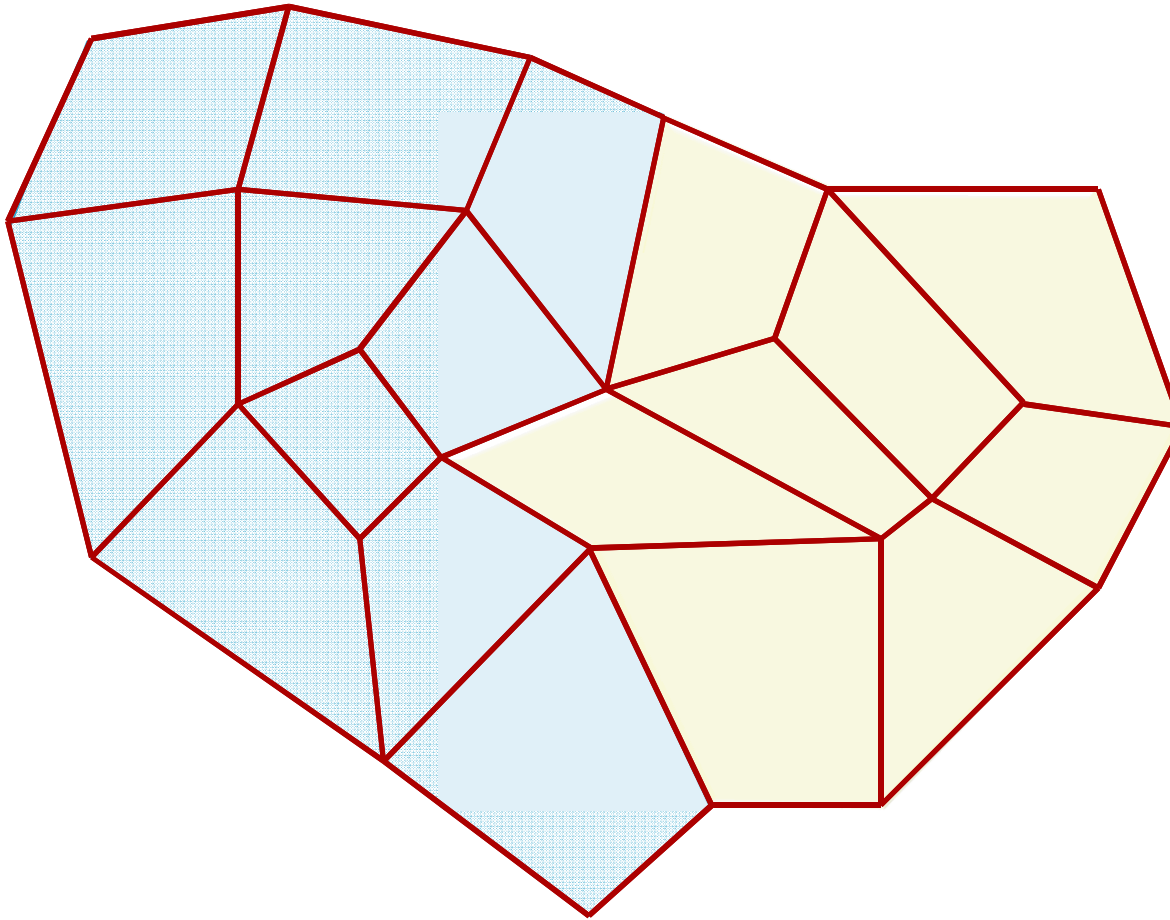
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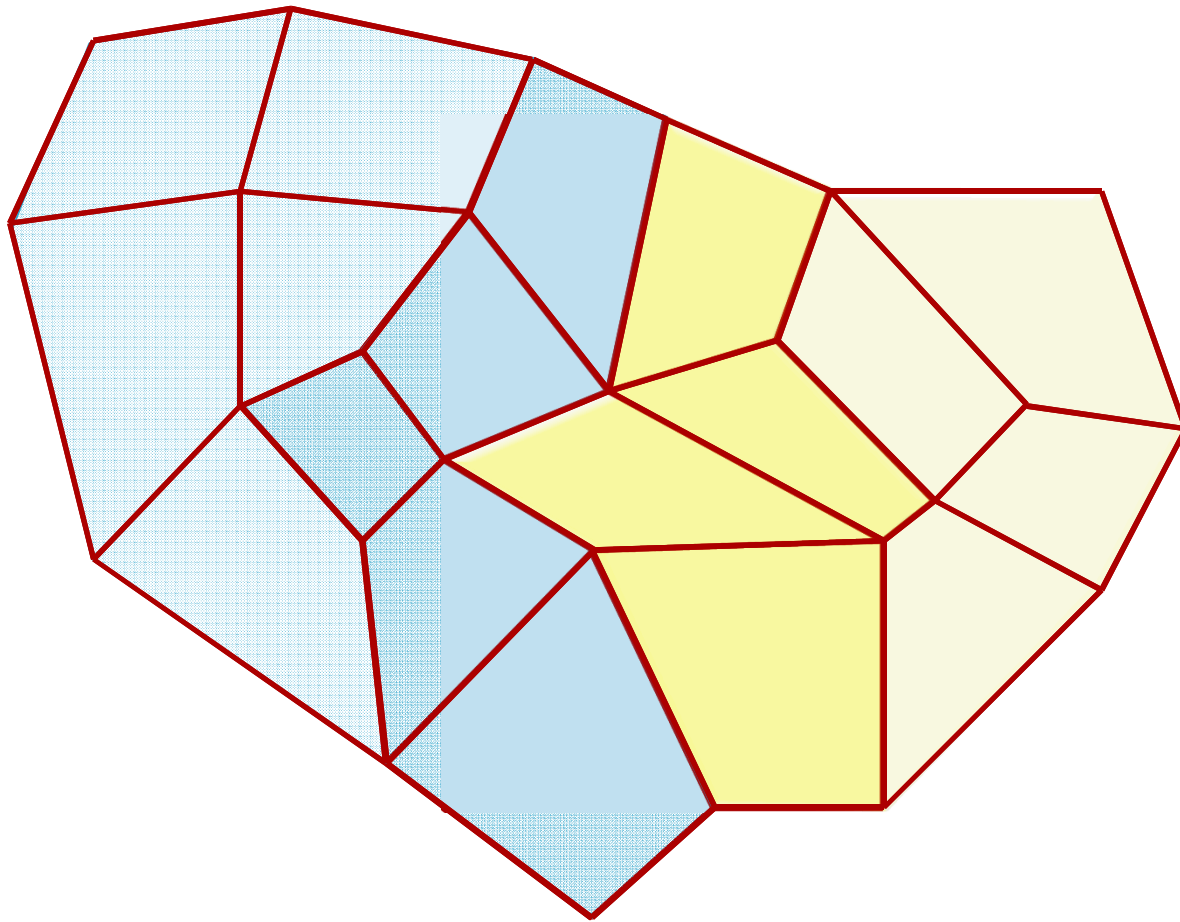
# Parallel Coloring



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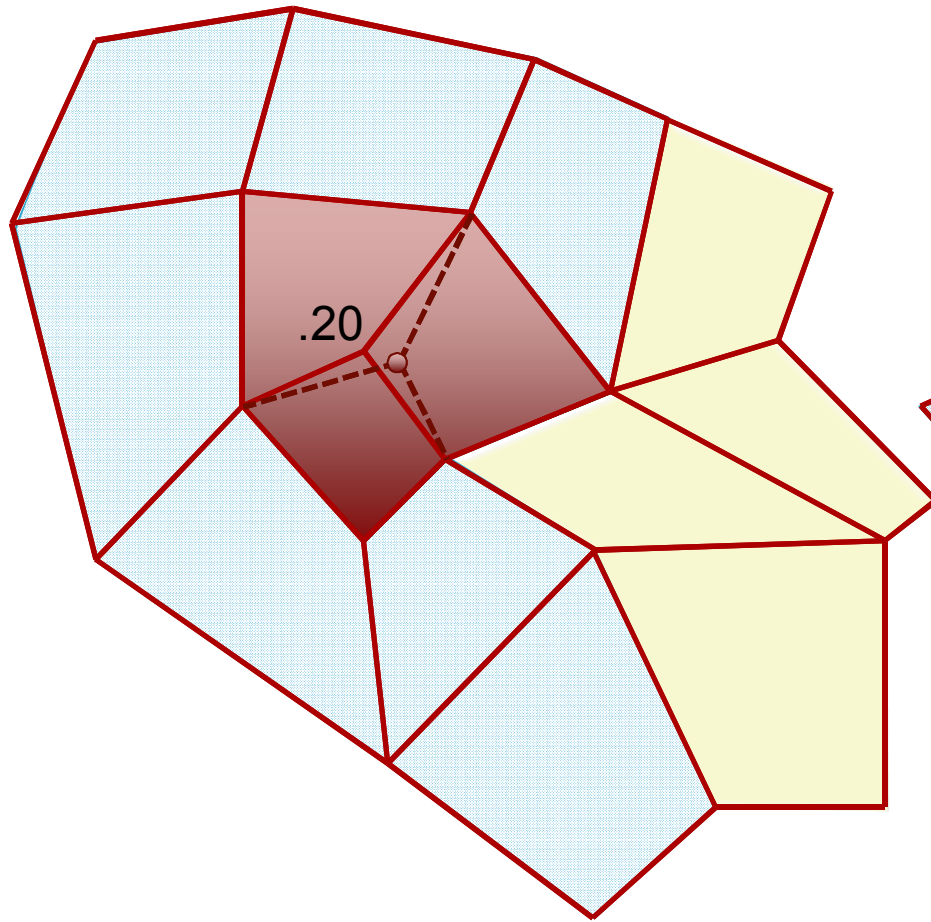


P0

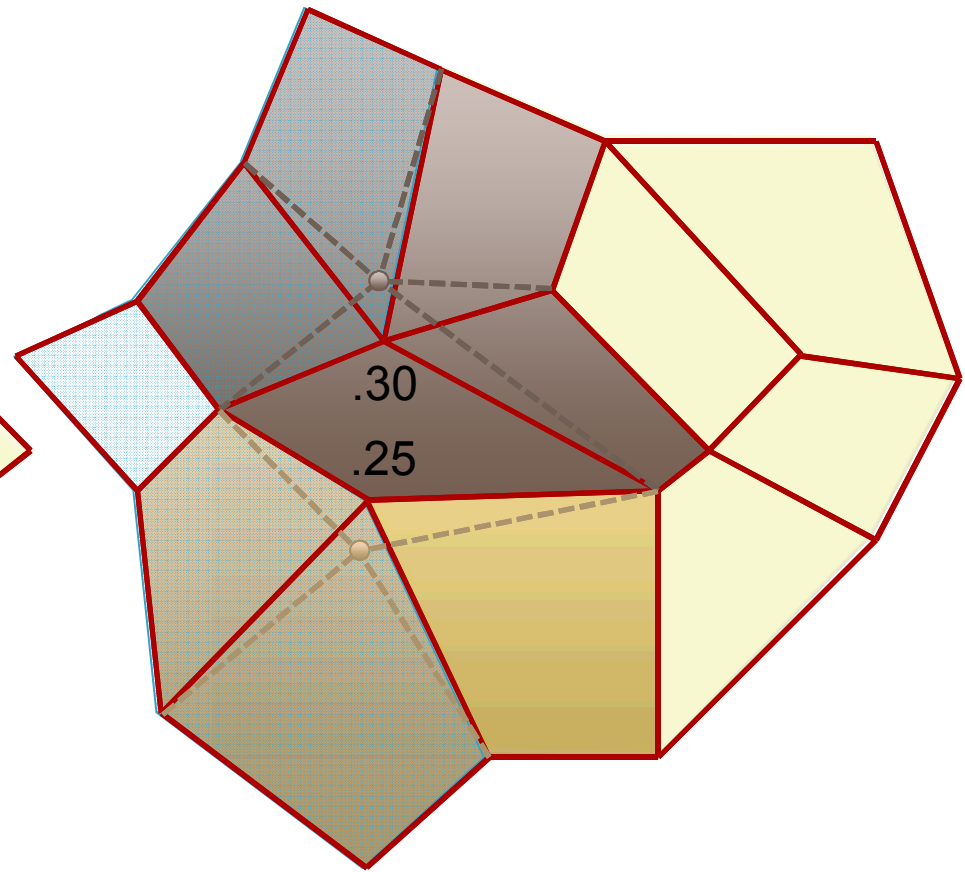
P1



# Parallel Coloring

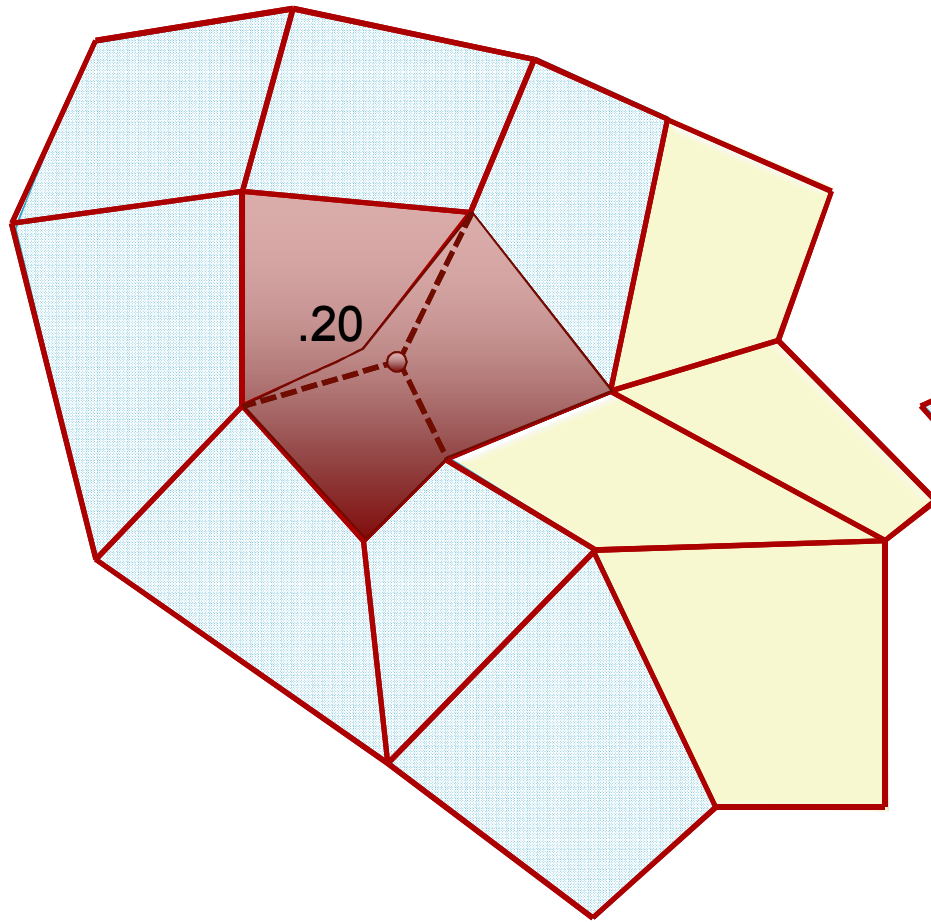


P0

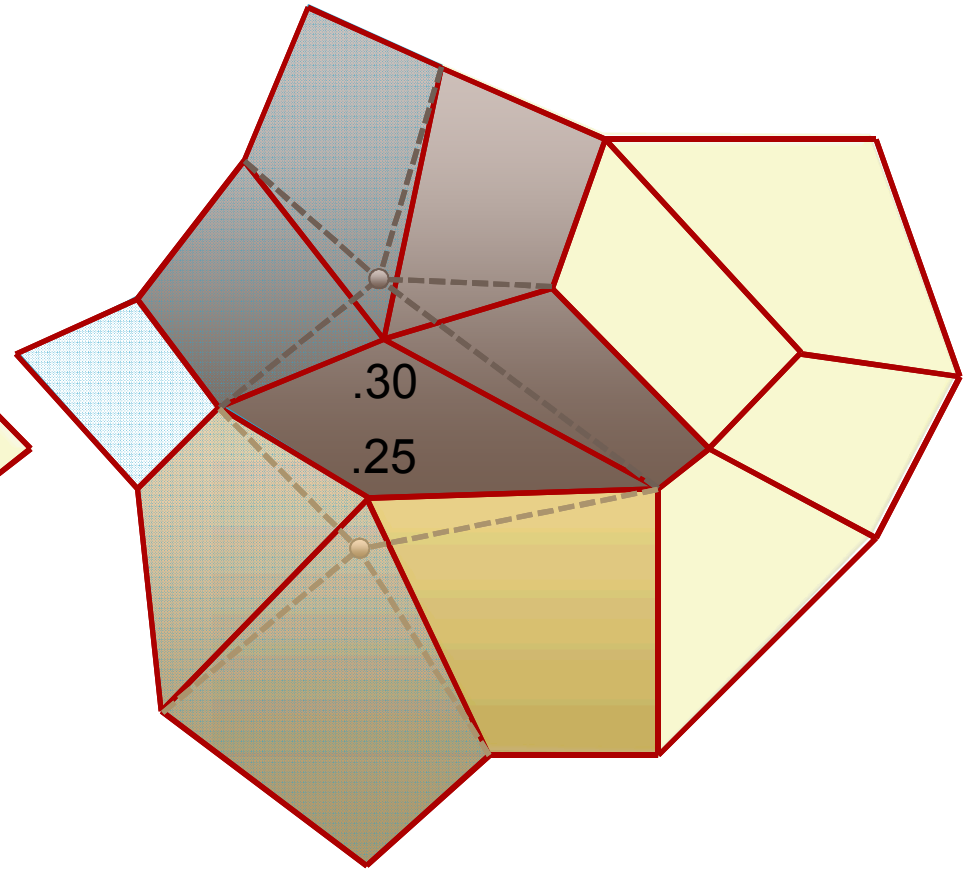


P1

# Parallel Coloring

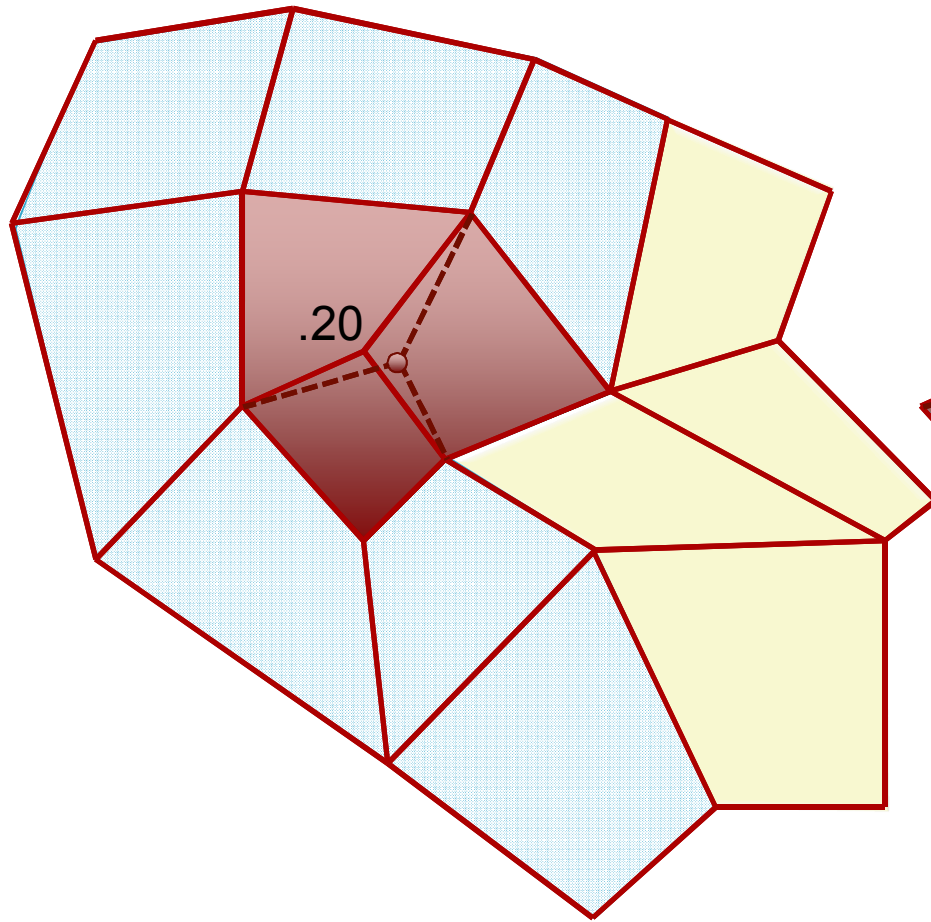


P0

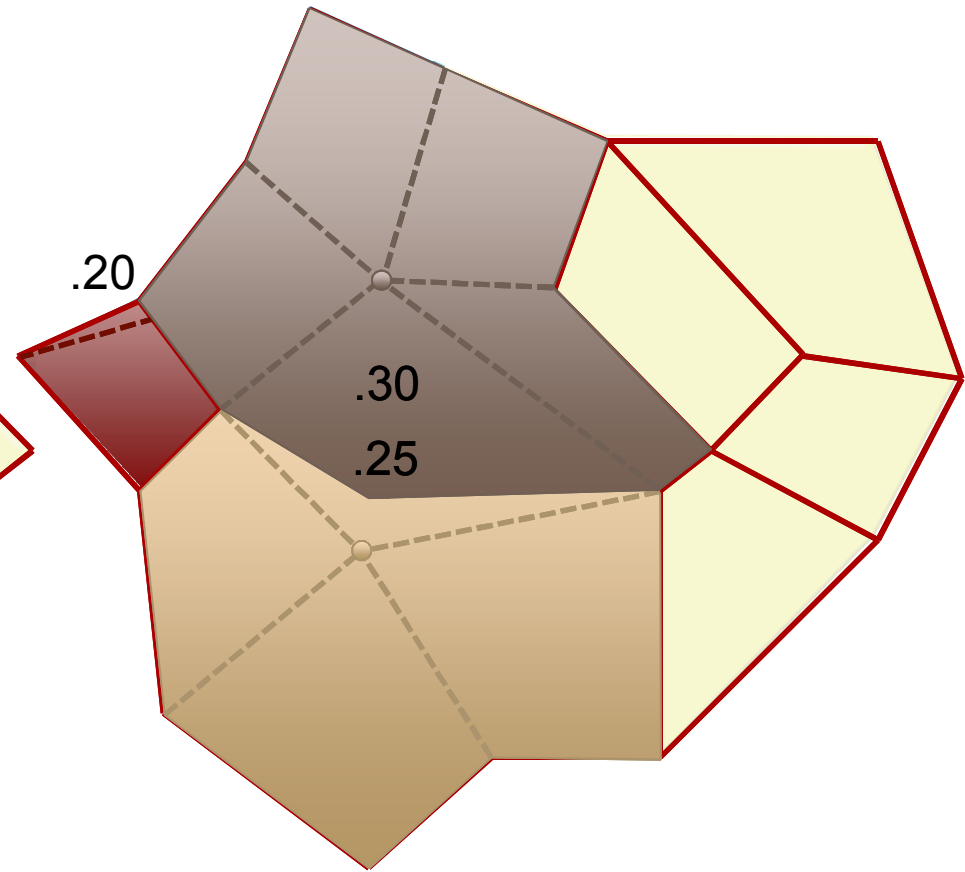


P1

# Parallel Coloring

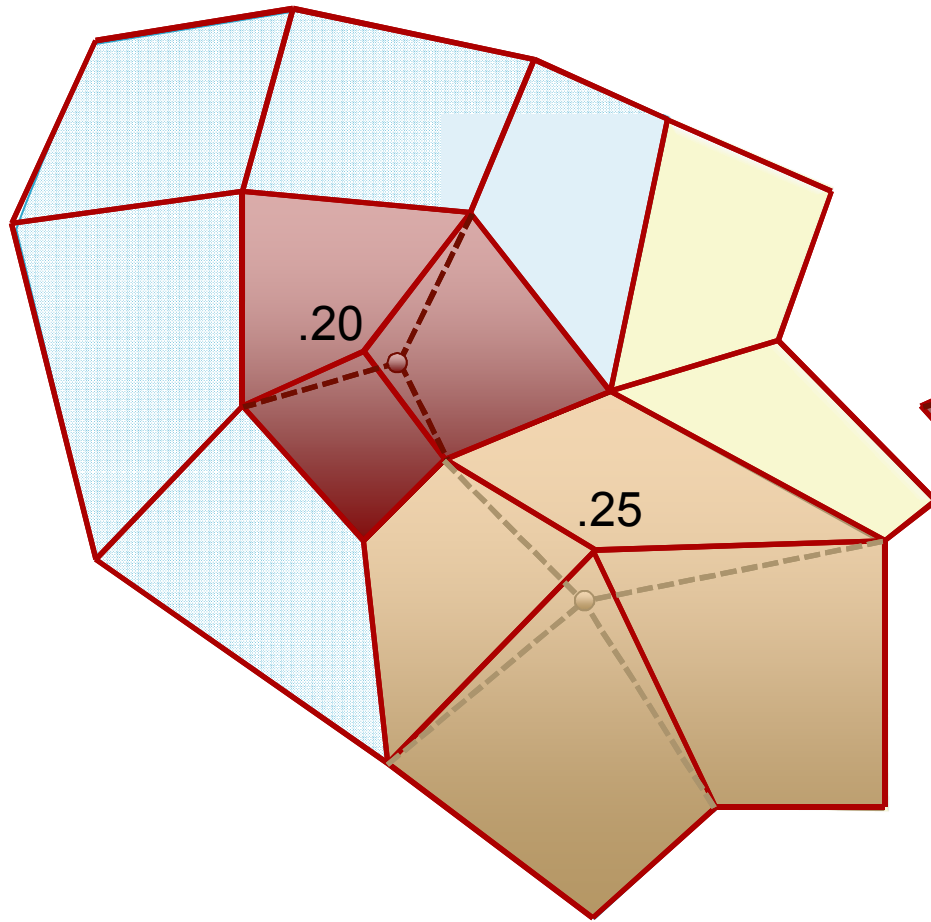


P0

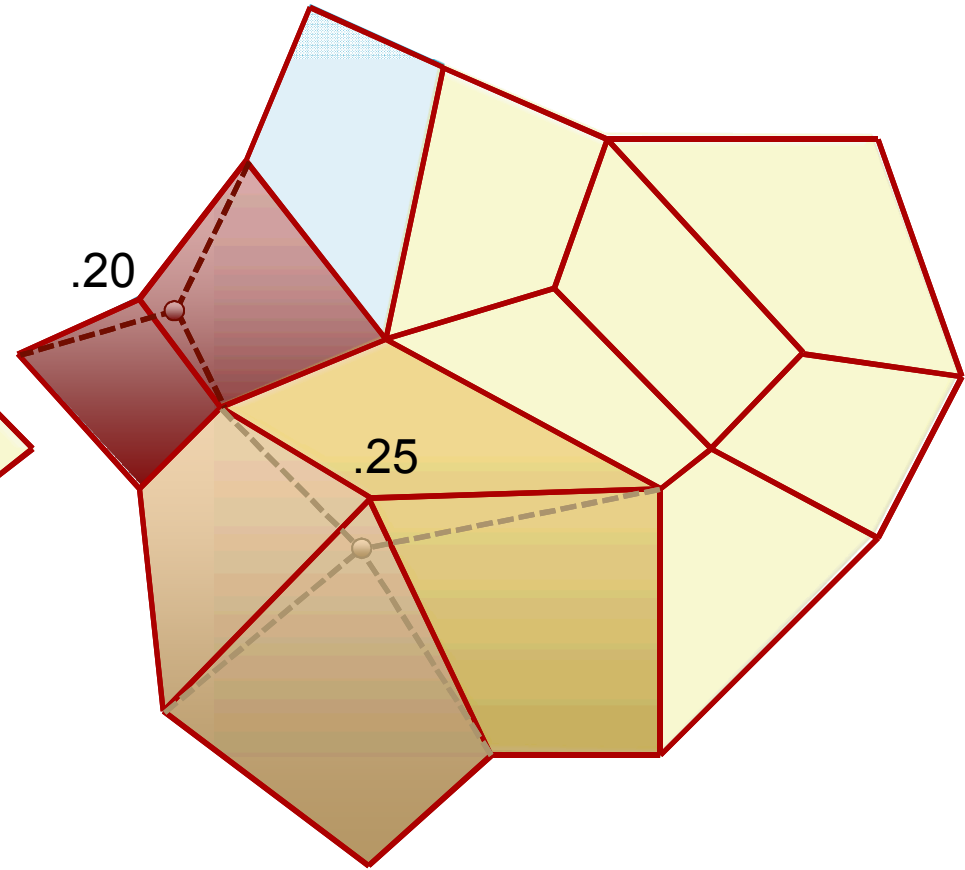


P1

# Parallel Coloring

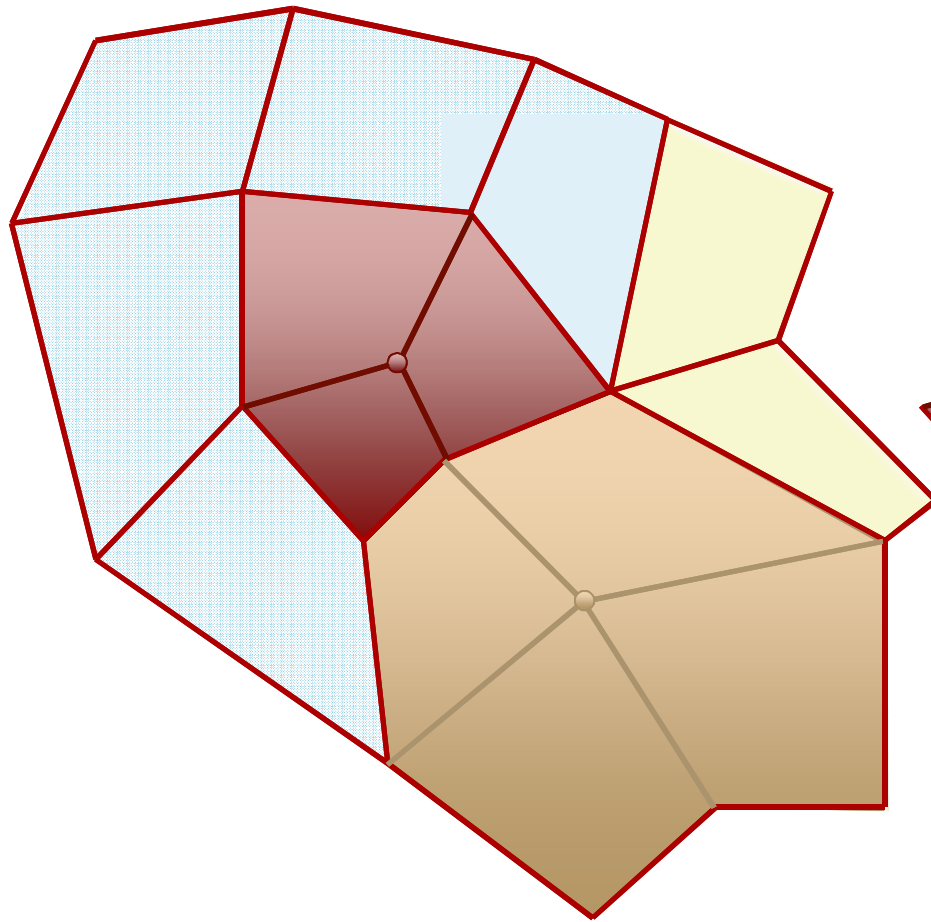


P0

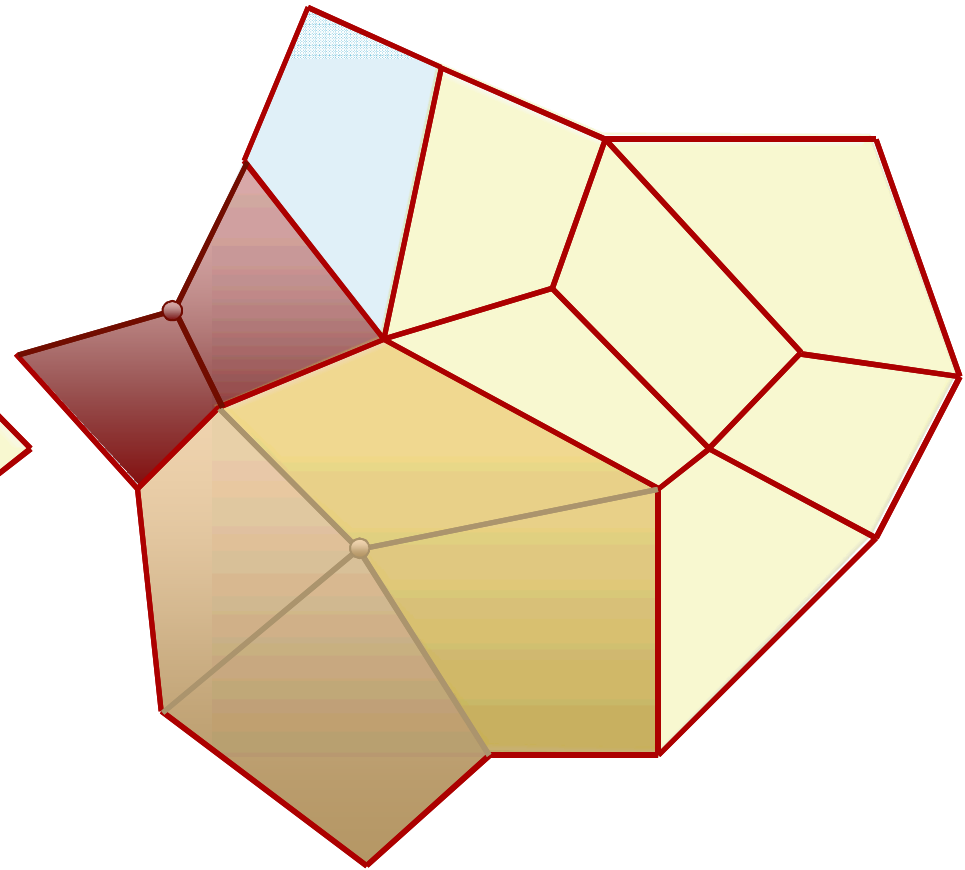


P1

# Parallel Coloring



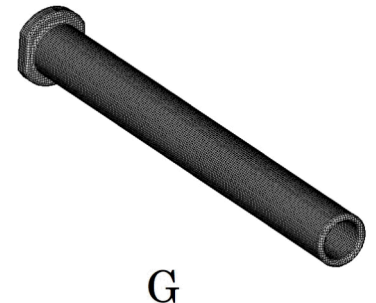
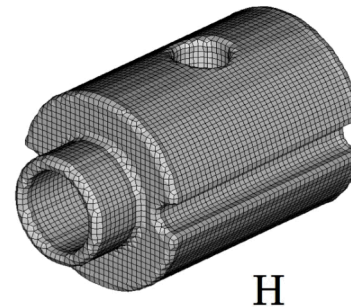
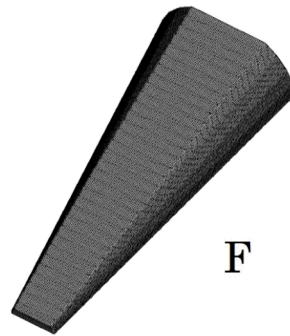
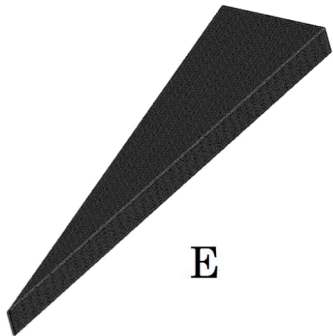
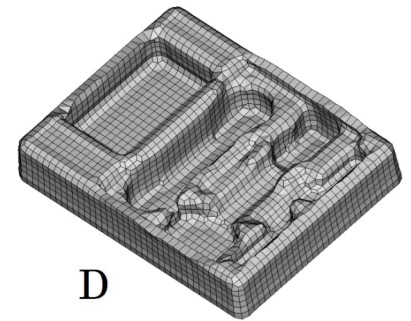
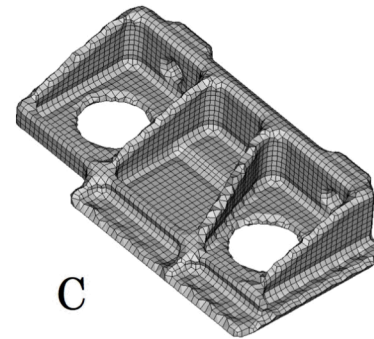
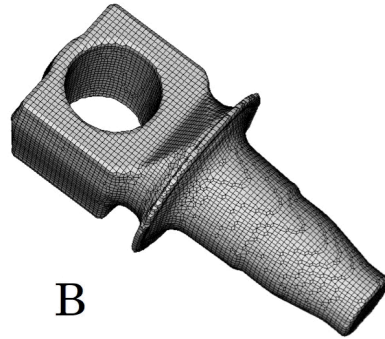
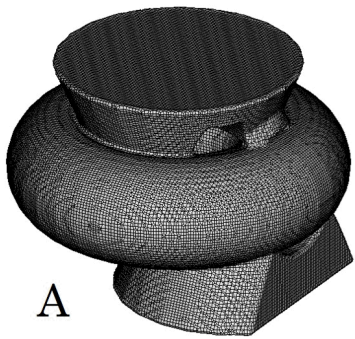
P0



P1

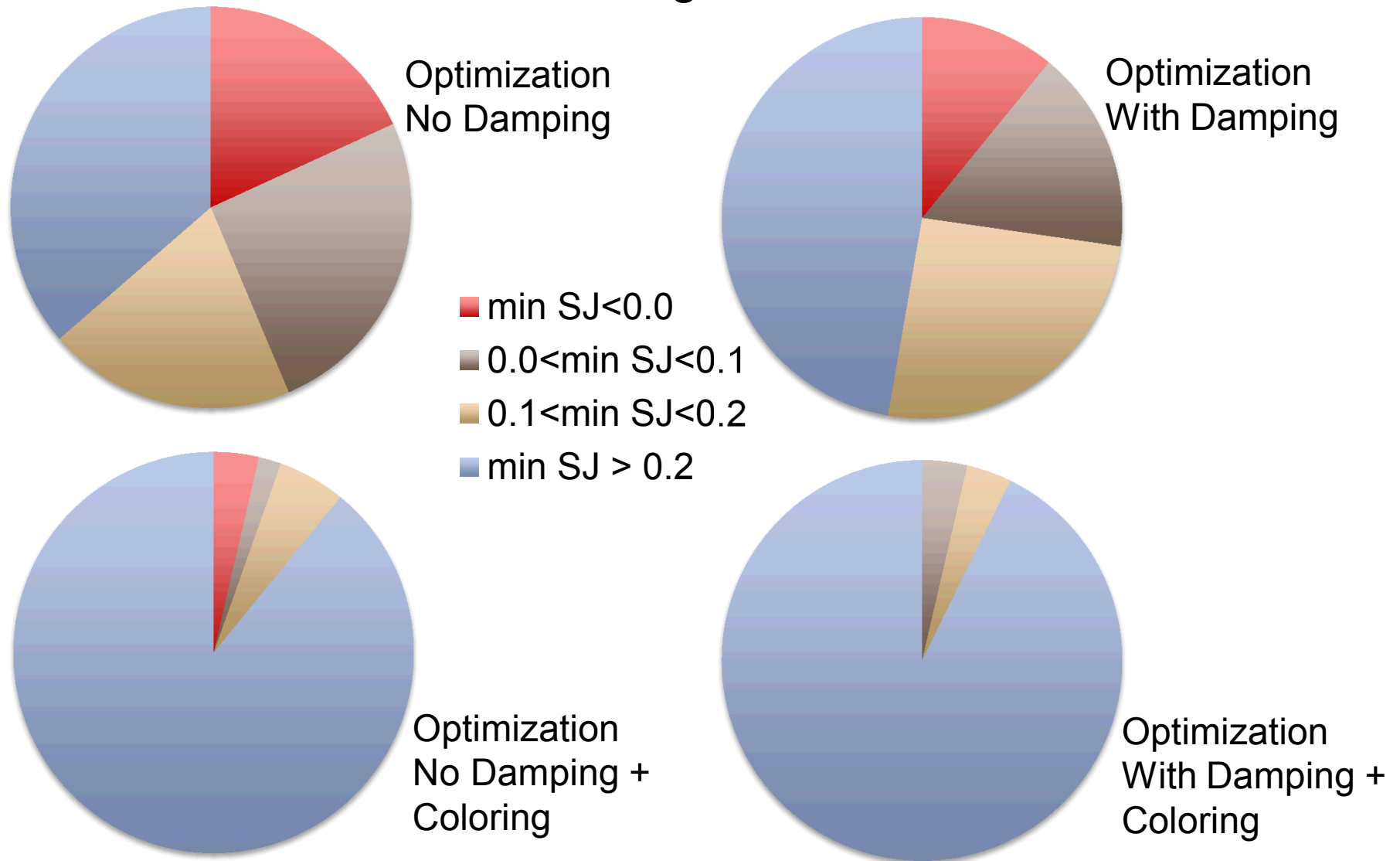
# Sculpt Smoothing Comparison

Test Suite: 52 Single Part CAD Models



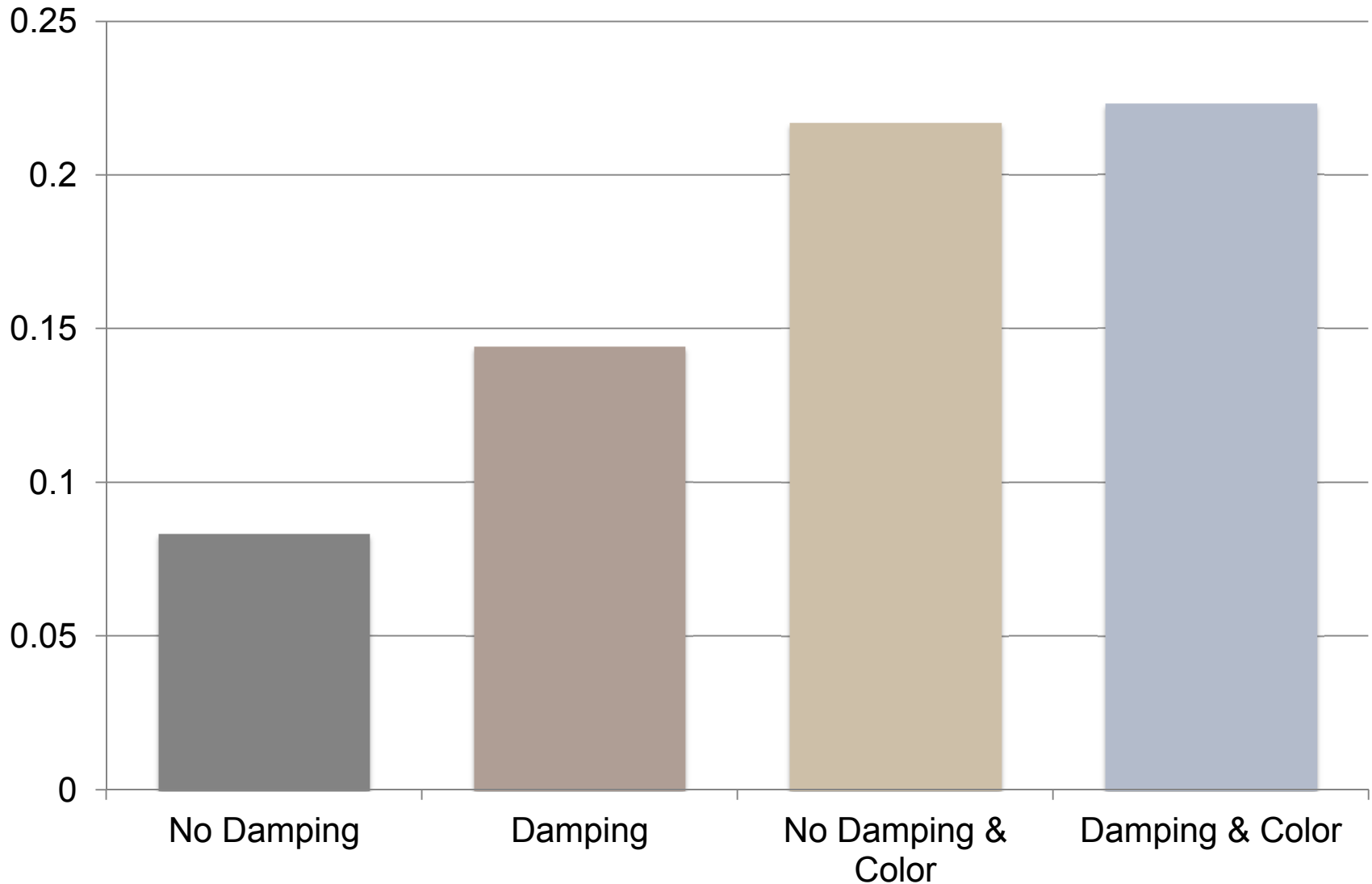
# Minimum Mesh Quality

## Test Suite: 52 Single Part CAD Models



# Average Minimum Mesh Quality

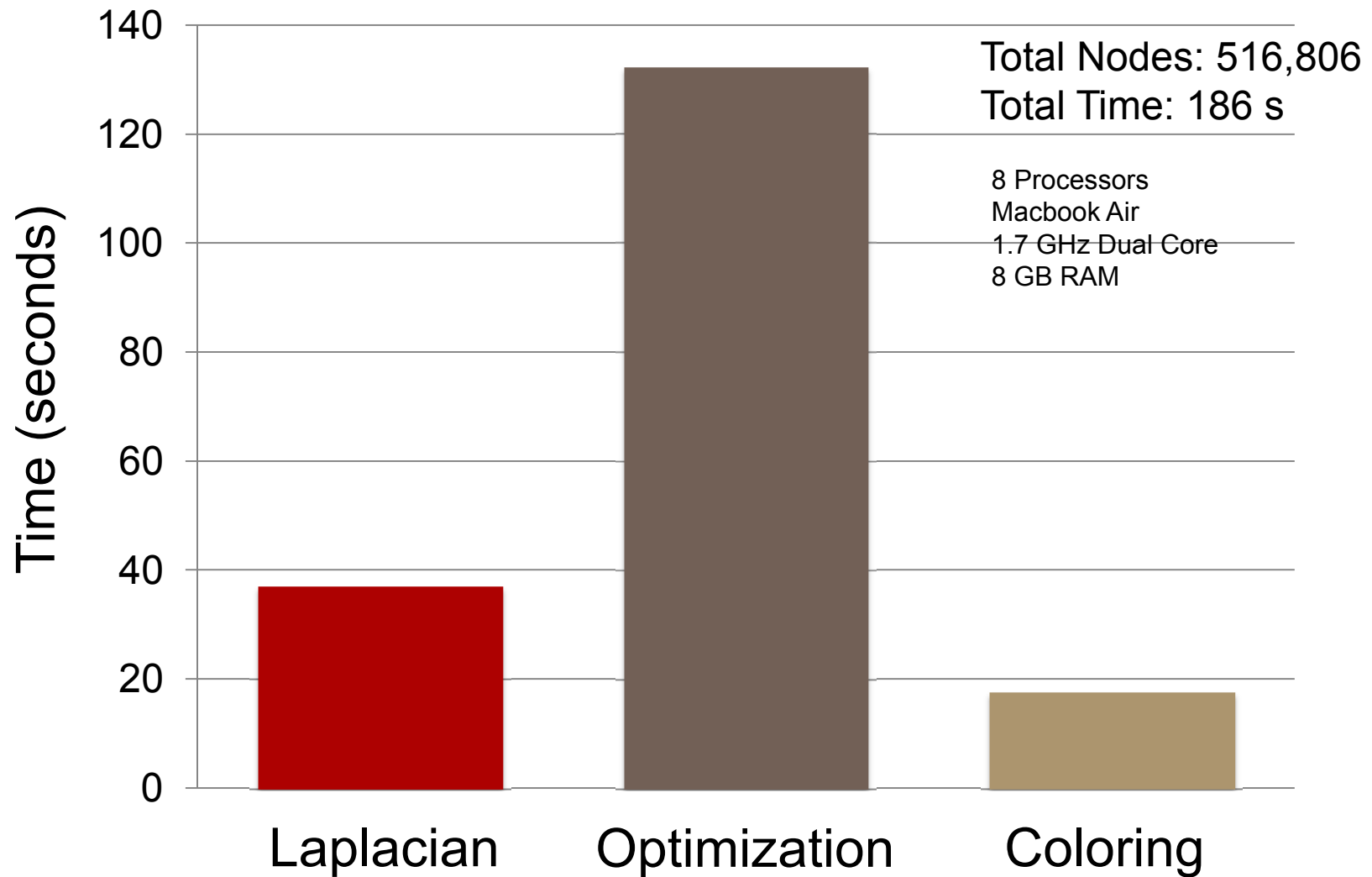
## Test Suite: 52 Single Part CAD Models





# Total Time in Smoothing

## Test Suite: 52 Single Part CAD Models



# Smoothing Procedure

