

A Hybrid ALE-Level Set Approach to Modeling Foaming in Punch Molds

Rekha Rao, David Noble, Lisa Mondy, Victor Brunini, Christine Roberts,
Scott Roberts

*Sandia National Laboratories
Albuquerque, NM*

James Tinsley
*Honeywell Kansas City Plant
Kansas City, MO*

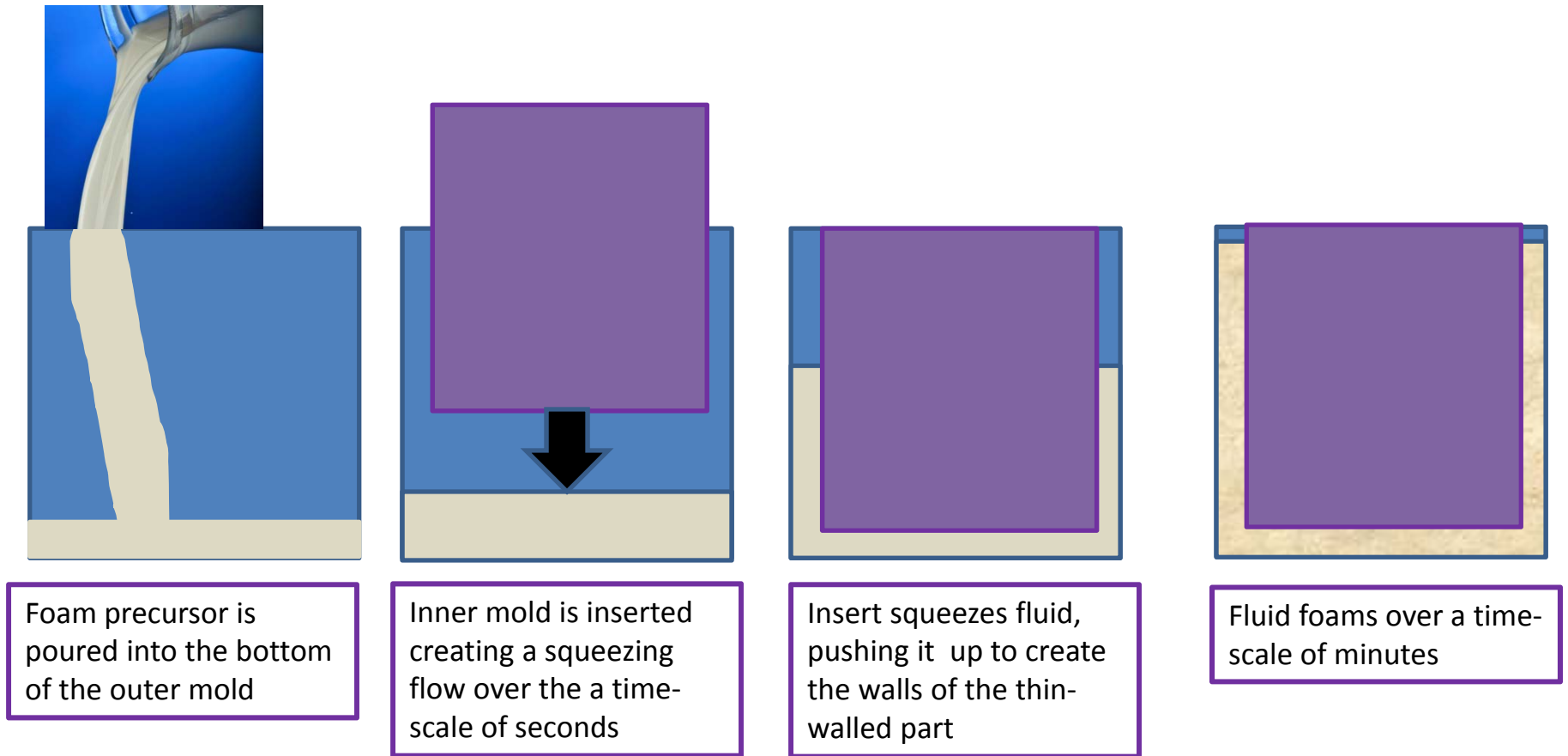
13th US National Congress on Computational Mechanics
San Diego, California
July 26 - 30, 2015

SAND2015-????C

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under contract DE-AC04-94AL85000.



Motivation: What Is a Punch Mold?



Approach:

- Decouple punching fluid mechanics from foam expansion
- Punch simulations use a Newtonian, incompressible fluid and give an initial conditions for foaming simulation
- For punch mold, couple a ALE moving mesh algorithm for the evolving geometry of the mesh insert to a level set for the fluid motion

Numerical Solution Methods for Interfacial Motion

Tracking motion of interface between two distinct phases appears often:

Phase changes

Film growth

Fluid filling

Interface tracking:

Explicit parameterization of location

Interface physics more accurate

Moving mesh

Limits to interface deformation

No topological changes

Examples:

Spine methods (*Scriven*)

ALE

Embedded Interface Capturing:

Interface reconstructed from
higher dimensional function

Fixed mesh

“Diffuse” interface physics

Interface deformation
theoretically unconstrained

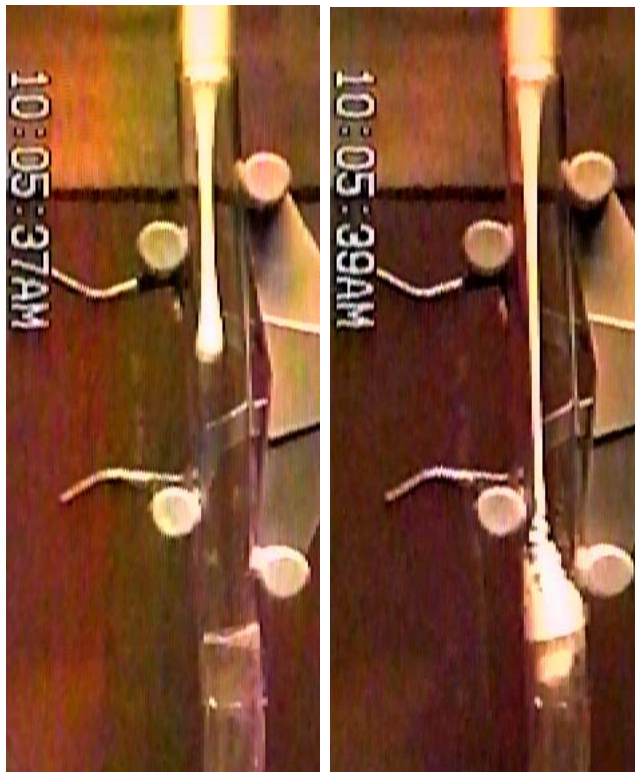
Examples:

Volume-of-Fluid (*Hirt*)

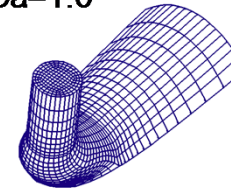
Level Sets (*Sethian*)

Embedded Interface Methods Can Capture Topological Changes

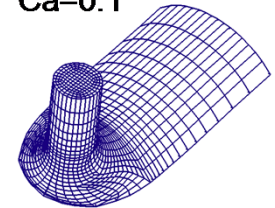
Level set method has possibility of modeling “Dairy Queen” effect



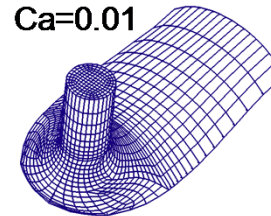
$Ca=1.0$



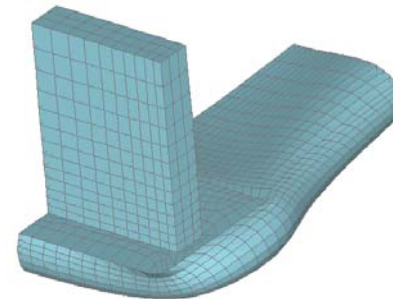
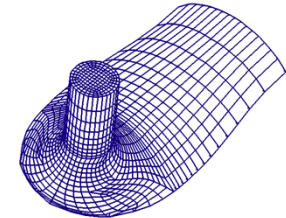
$Ca=0.1$



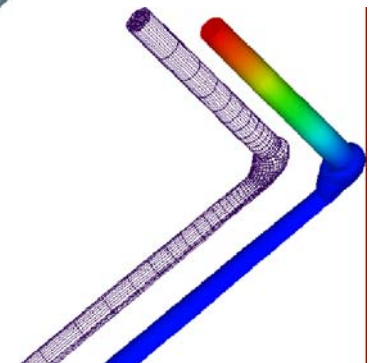
$Ca=0.01$



$Ca=0.001$



Tom Baer, P&G



Free Surface Flows: Coupling Fluid Flow to Pseudo-Solid Mesh Motion

- Technique for mapping mesh nodes in response to boundary deformation
- Displacement of nodes determined by solution of quasi-static problem: Neo-Hookean constitutive equation for pseudo-solid

$$\nabla \cdot \mathbf{T}_{mesh} = 0, \quad \mathbf{T}_{mesh} = f(\lambda_{ps}, \mu_{ps}; \nabla d_{mesh})$$

- Mesh node displacements are solved for simultaneously with other variables
- Deformation driven by boundary constraints:

Geometric

$$P(x, y, z) = 0$$

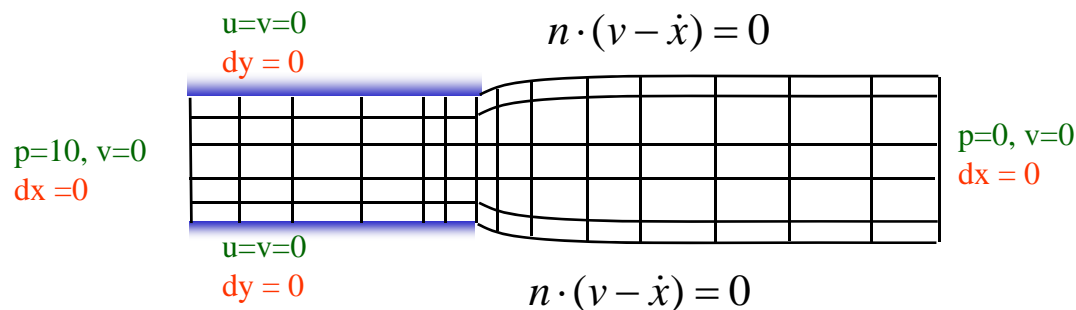
$$\vec{d} = \vec{D}_0$$

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = \cos(\theta)$$

Coupled

$$\mathbf{n} \cdot (\mathbf{v} - \dot{\mathbf{x}}) = 0$$

$$\mathbf{T} = \mathbf{T}_{melt}$$



Arbitrary Lagrangian Eulerian (ALE) mesh motion: The mesh moves with the material in the normal direction at boundaries and arbitrarily, as a nonlinear elastic solid, elsewhere.

Free Surface Flow: Level Set Method

Given fluid velocity field, $u(x,y,z)$, evolution on a fixed mesh is according to:

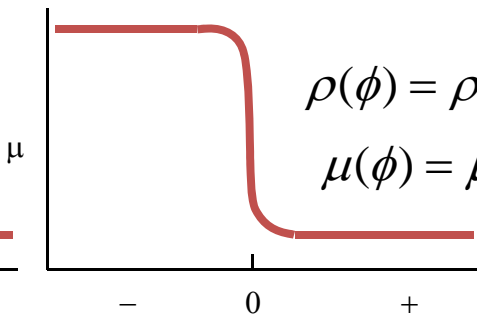
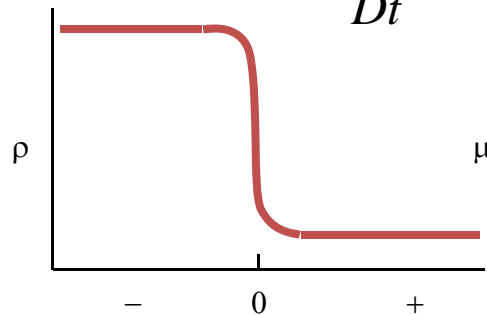
$$\frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = 0 \quad \vec{n} = \nabla \phi, \kappa = \nabla \cdot \nabla \phi$$

Purely hyperbolic equation ... fluid particles on $\phi(x,y,z) = 0$ should stay on this contour indefinitely

- Does not preserve $\phi(x,y,z)$ as a distance function
- Introduces renormalization step.

Fluid velocity evolves as one-phase fluid with properties that depend on ϕ

$$\rho(\phi) \frac{Du}{Dt} = -\nabla P + \nabla \cdot (\mu(\phi) \dot{\gamma}) + \rho(\phi) g + I.T., \quad \nabla \cdot u = 0$$



$$\rho(\phi) = \rho_- (1 - H_\alpha(\phi)) + \rho_+ H_\alpha(\phi)$$

$$\mu(\phi) = \mu_- (1 - H_\alpha(\phi)) + \mu_+ H_\alpha(\phi)$$

$$\underline{T}_\sigma = \sigma \delta_\alpha(F) (\underline{I} - \vec{n} \vec{n})$$

Coupling Level Set Method and ALE Method

- The motion of the fluid, $u(x,y,z)$, is now with respect to the mesh, and the mesh velocity enters the advection term
- Segregated solve at each time step in three different matrix systems
- First solve mesh equations, then level set, and then momentum and continuity
- Method implemented in Sierra Mechanics Aria

Mesh Motion

$$\nabla \cdot \mu_s \left(\nabla d + \nabla d^t \right) + \nabla \left(\lambda_s \nabla \cdot d \right) = 0$$

$$x_{new} = x_{old} + d, \quad \dot{x} = \dot{d}$$

Level Set

$$\frac{\partial \phi}{\partial t} + (u - \dot{x}) \cdot \nabla \phi = 0$$

Fluid Mechanics

$$\rho(\varphi) \left(\frac{\partial u}{\partial t} + (u - \dot{x}) \cdot \nabla u \right) = -\nabla P + \nabla \cdot (\mu(\varphi) \dot{\gamma}) + \rho(\varphi) g$$

$$\nabla \cdot u = 0$$

Finite Element Implementation

- Approximate variables with trial function, e.g.

$$u \approx \sum_{i=1}^n u_i N_i \quad v \approx \sum_{i=1}^n v_i N_i \quad w \approx \sum_{i=1}^n w_i N_i \quad p \approx \sum_{i=1}^m p_i N_i'$$

- Substitute into equations of motion, weight residual with shape function for Galerkin implementation

$$\text{Weighted - Residual} = \int N_i R_i dV$$

- Gaussian quadrature
- Solve discretized system

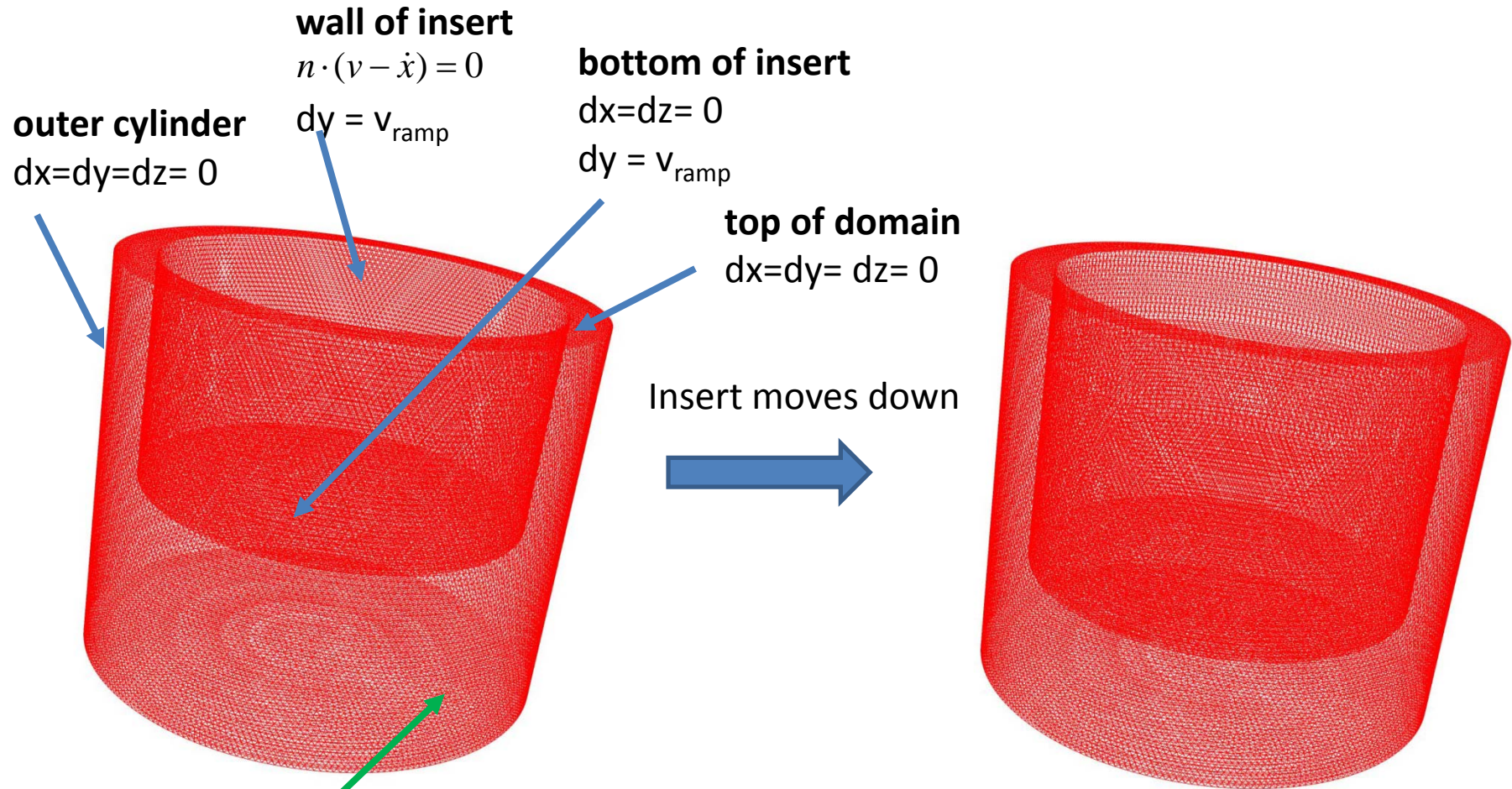
$$\underline{\underline{A}} \underline{\underline{x}} = \underline{\underline{b}}$$

- Issues: Linear system solved with Krylov-Based iterative solvers => require stabilization Dohrman-Bochev Stabilization (2004)

$$R_i^c = \int_D \phi^i [\nabla \cdot u] dV + \sum_{Elem} \tau_{pspp} (\phi^i - \pi \phi^i) (p - \pi p) dV$$

$$\pi p = \int_{V_e} p dV / \int_{V_e} dV$$

Fluid and Mesh Boundary Conditions



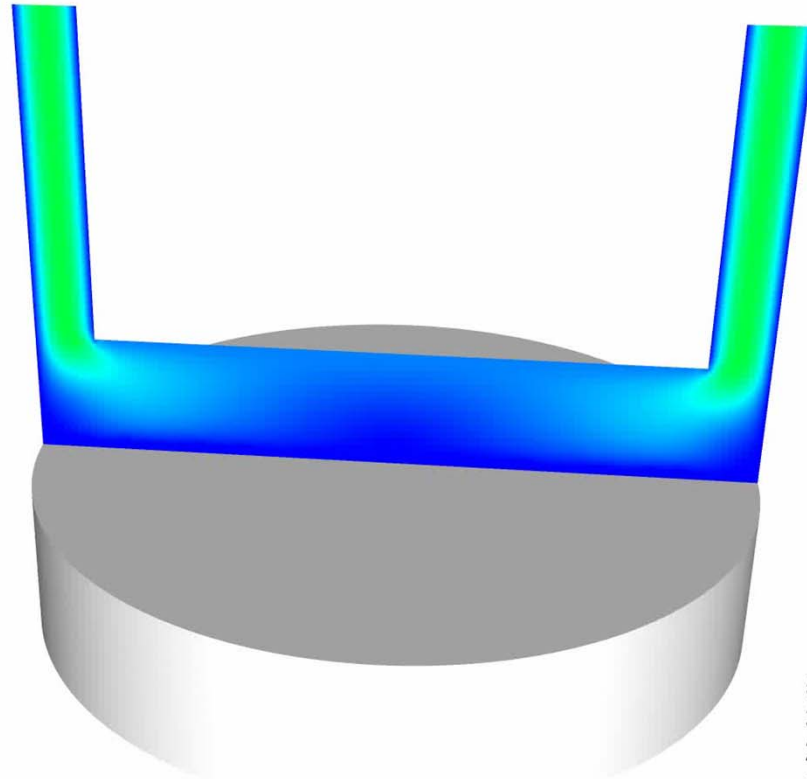
Solid surfaces: Rotated BCs

- Normal: No penetration
- Tangential: Navier slip condition

Goal is to have the mesh stretch and deform without tangling or inverting element

Results

Time = 0.001

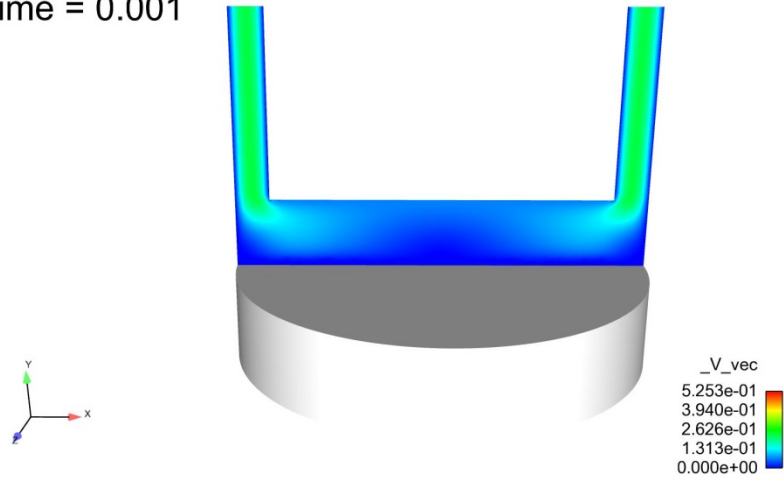


_V_vec
5.253e-01
3.940e-01
2.626e-01
1.313e-01
0.000e+00

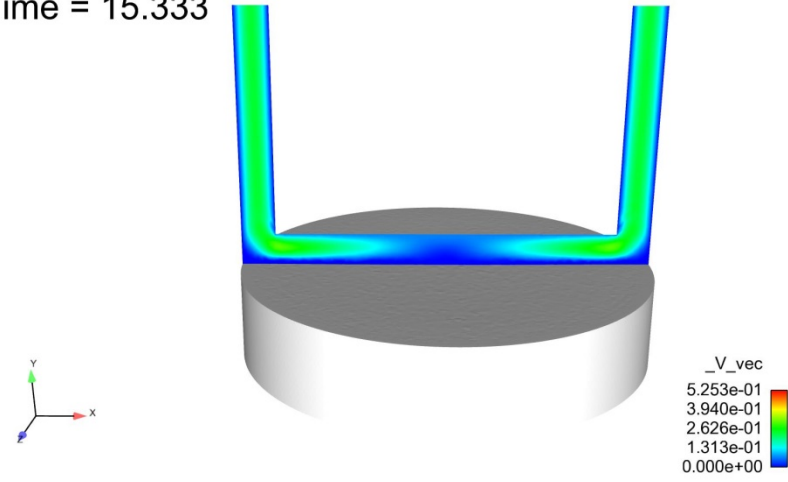


Results

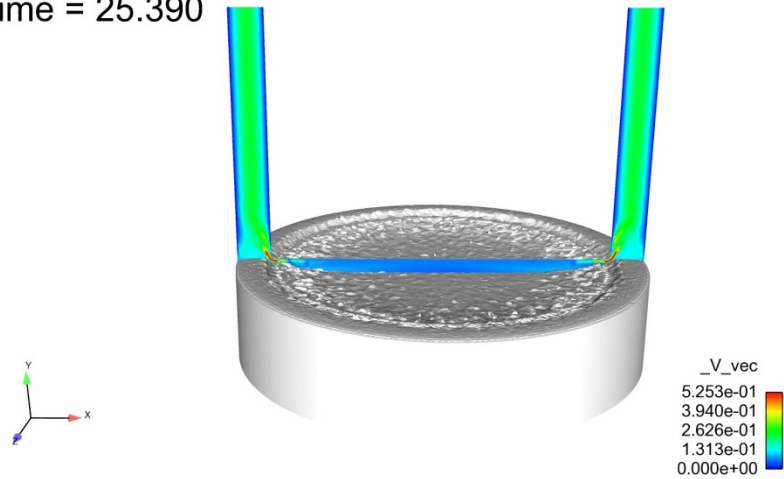
Time = 0.001



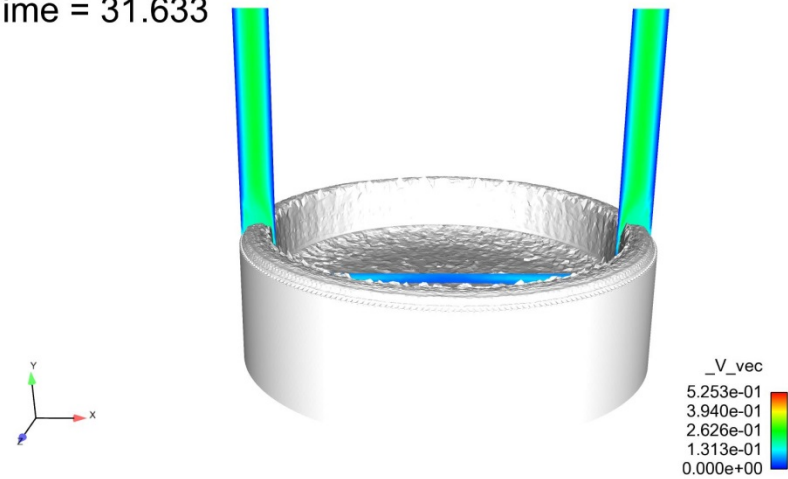
Time = 15.333



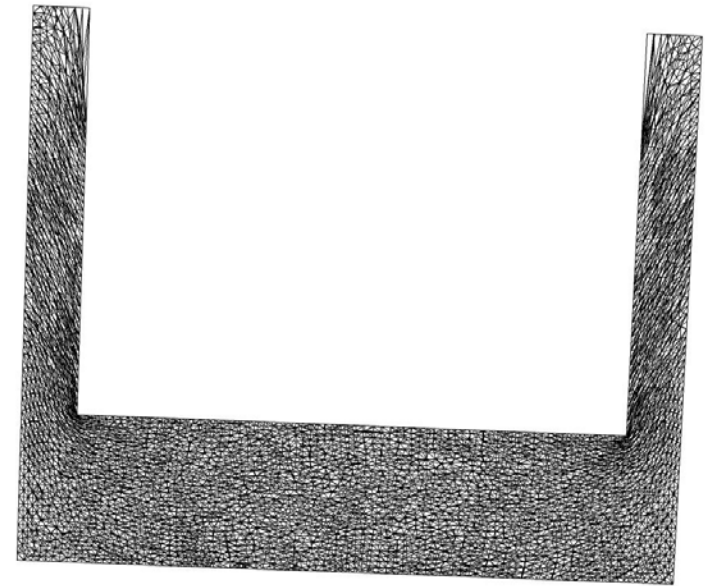
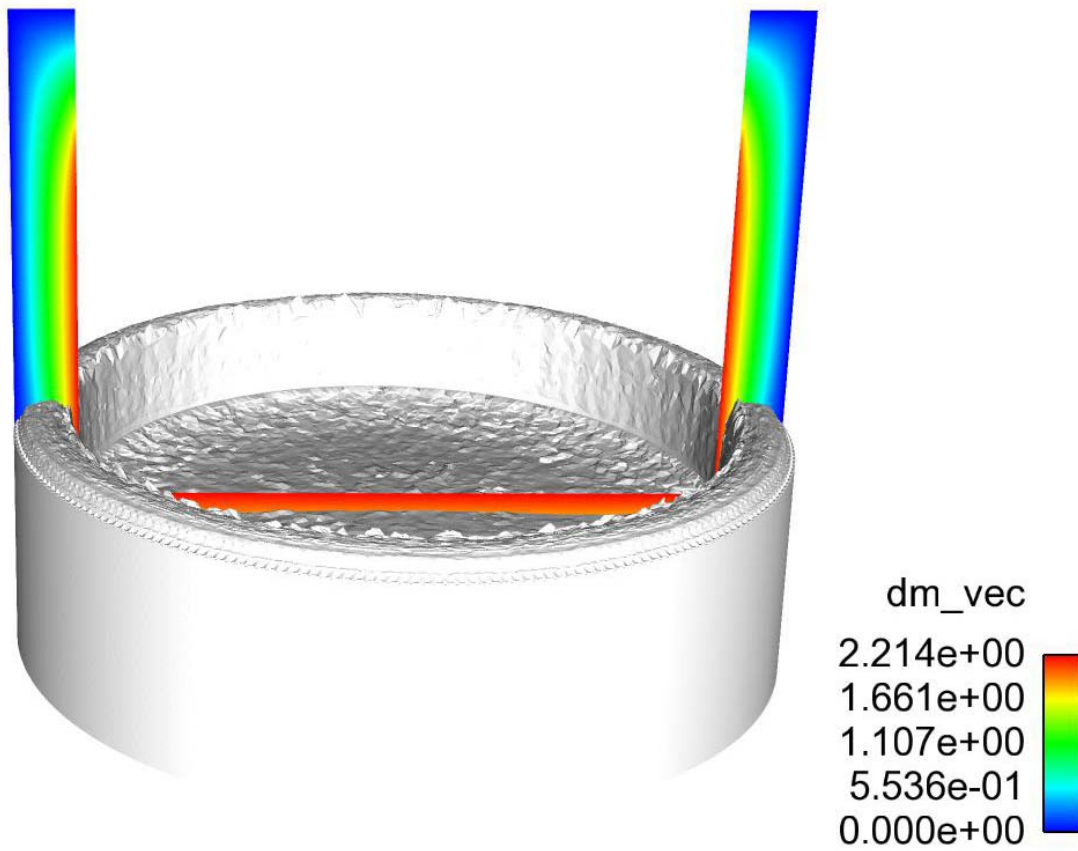
Time = 25.390



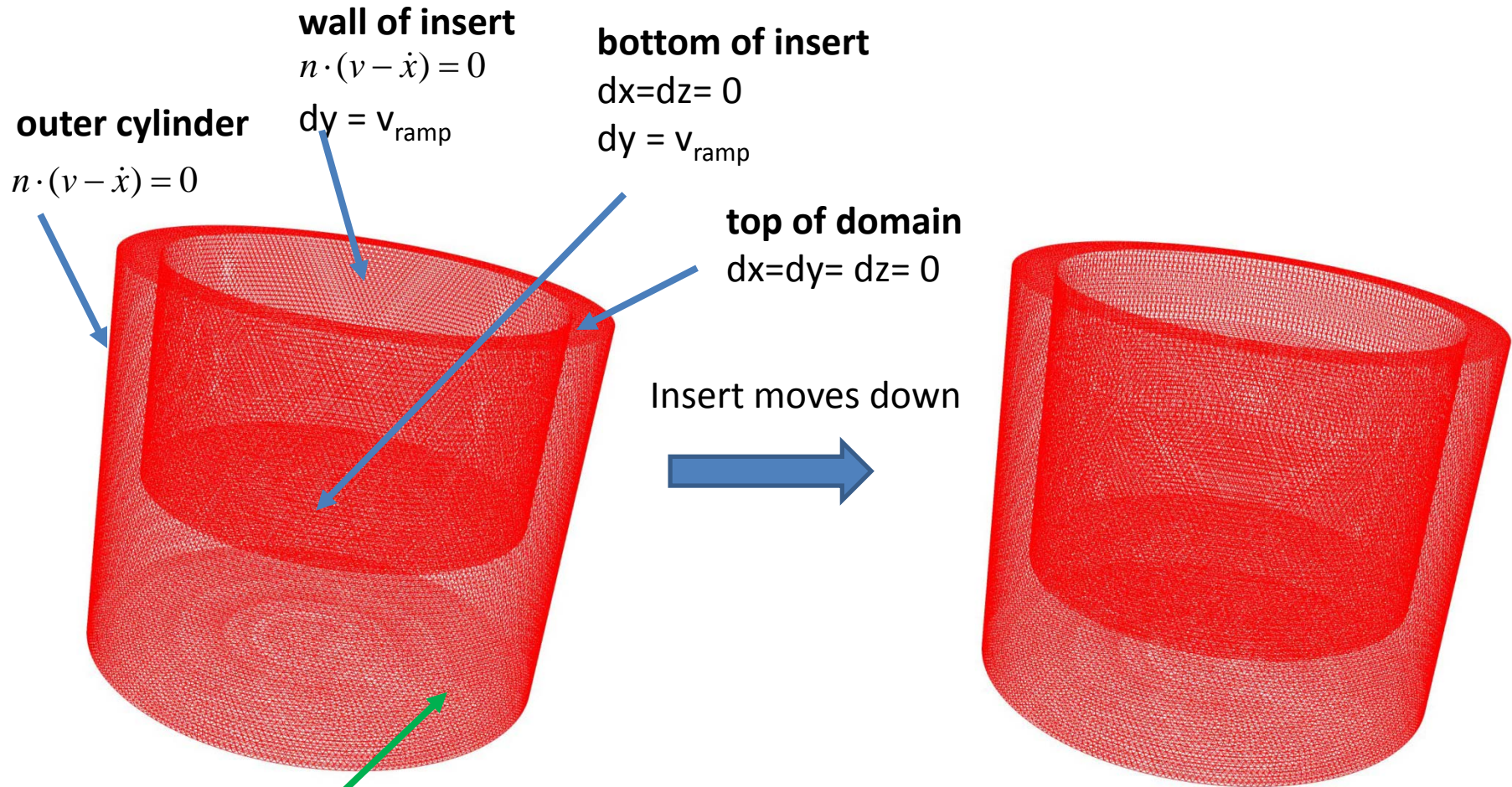
Time = 31.633



Mesh Shears Over Time



Improved Fluid and Mesh Boundary Conditions

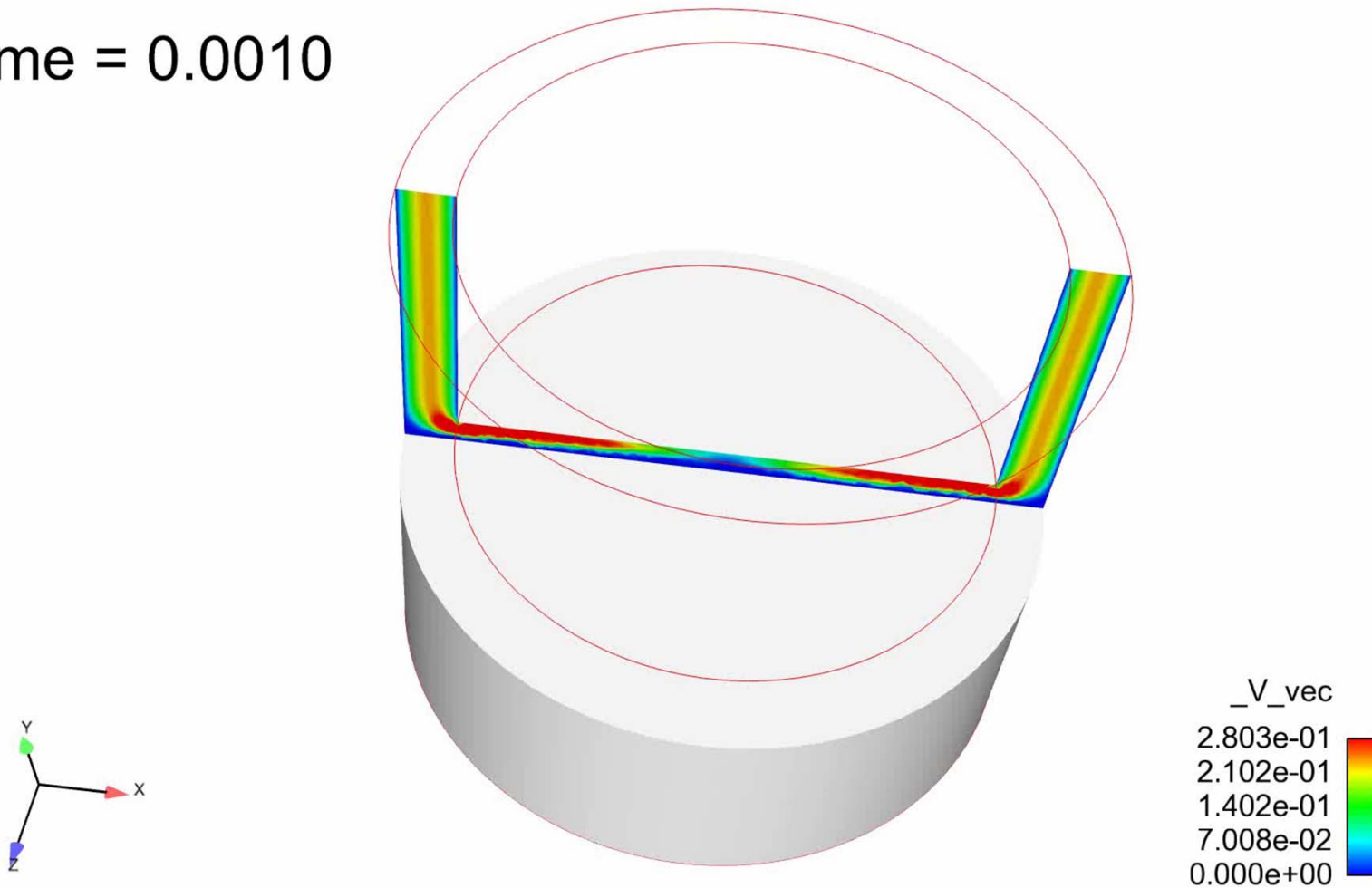


Solid surfaces: Rotated BCs

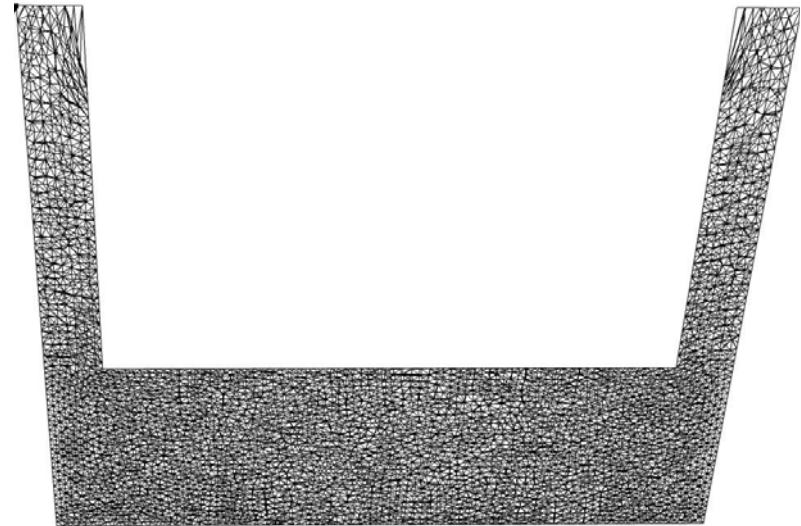
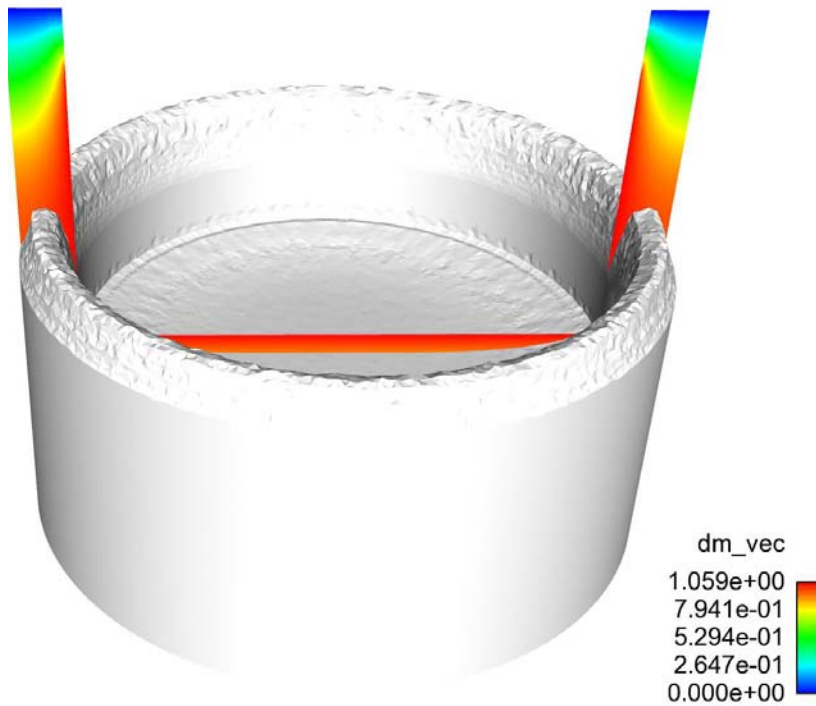
- Normal: No penetration
- Tangential: Navier slip for with phase dependent slip parameter: $\beta_{\text{gas}} = 200 \beta_{\text{fluid}}$

Can Boundary Conditions Improve Results?

Time = 0.0010

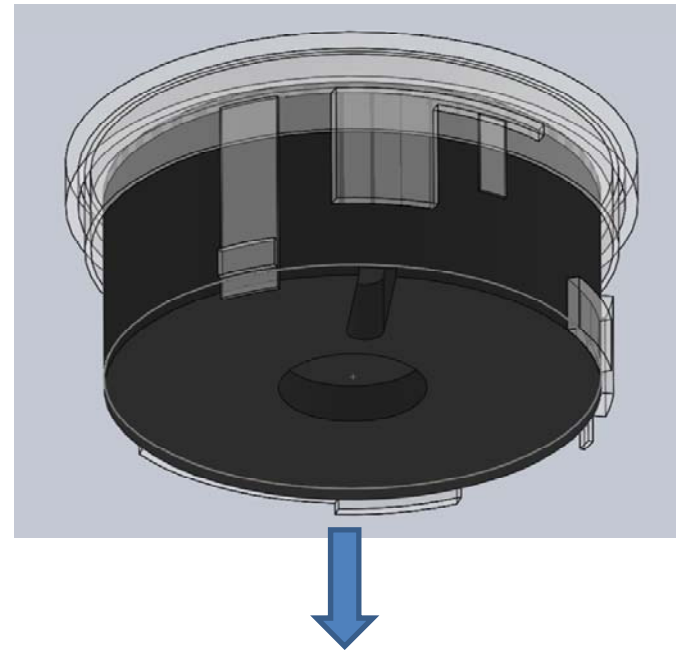
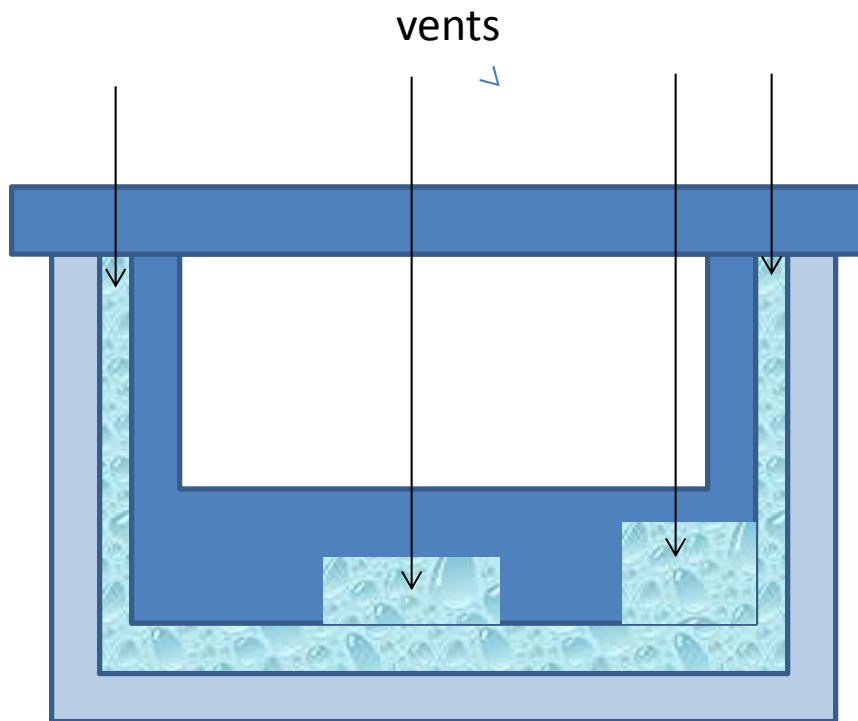


Can Boundary Conditions Improve Results?



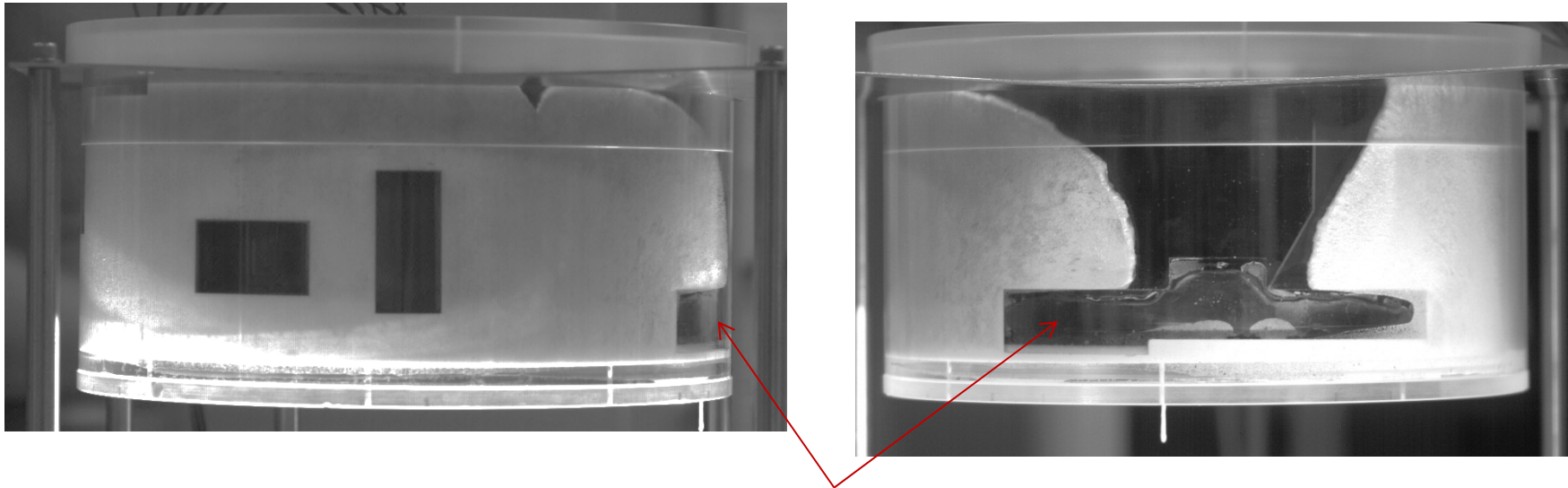
Simplified Structural Support Mold Test 3

- Used 10 pcf free rise structural PMDI foam, filled to produce a 13 pcf part
- To speed up process and slow down foam reaction rates:
 - No preheats
 - Mixed 30 seconds instead of 1 minute
 - Pour all foam into one reservoir, the lid of the upside down part
- Temperature instrumented with four camera views



Push inside mold down into bowl that once was the lid

Last Place to Fill on Top of Largest Feature

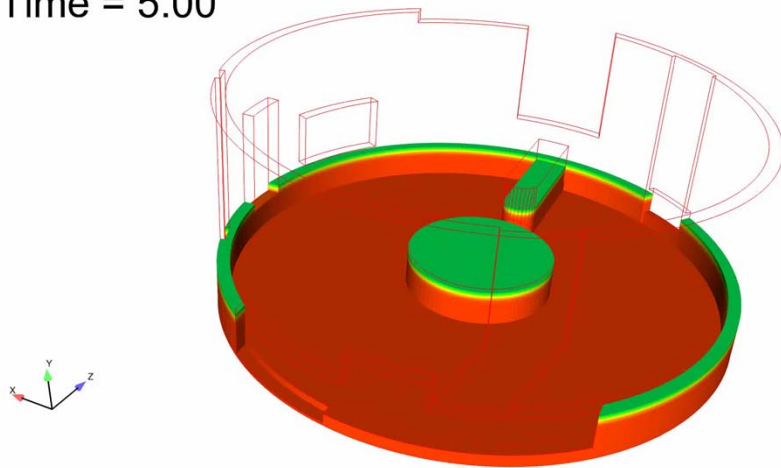


Largest feature

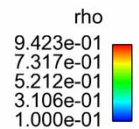
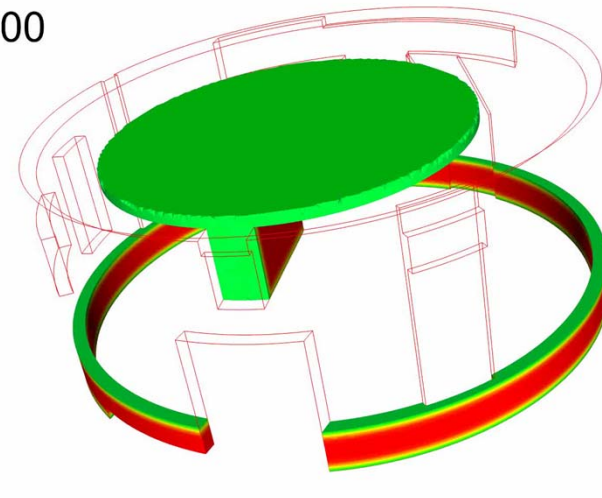
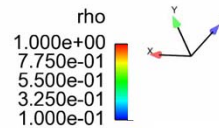
Short shot: less foam than encapsulation test 1, to see where last places to fill would occur. Reaction proceeded faster gelling foam before could finish rising.

If We Know the Initial Condition, Filling Models Can Predict Dynamics

Time = 5.00



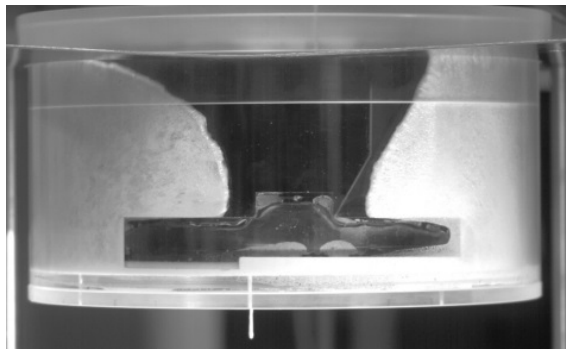
Time = 5.00



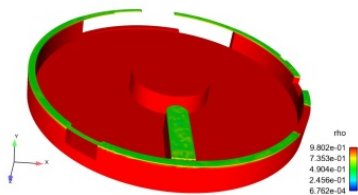
Model Give More Physics than Just the Filling Locations

Models developed for foam filling and curing
=> density/cure

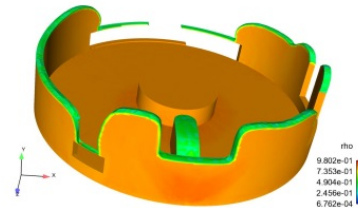
- The model allows us to look inside the mold
- New kinetics show water depletion and CO₂ variations
- Density variations are seen in the mold
- Foam exotherms significantly even and early times



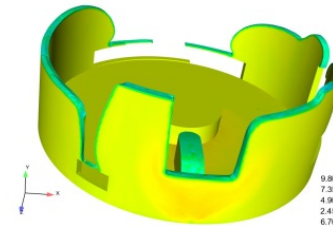
Time = 24.531



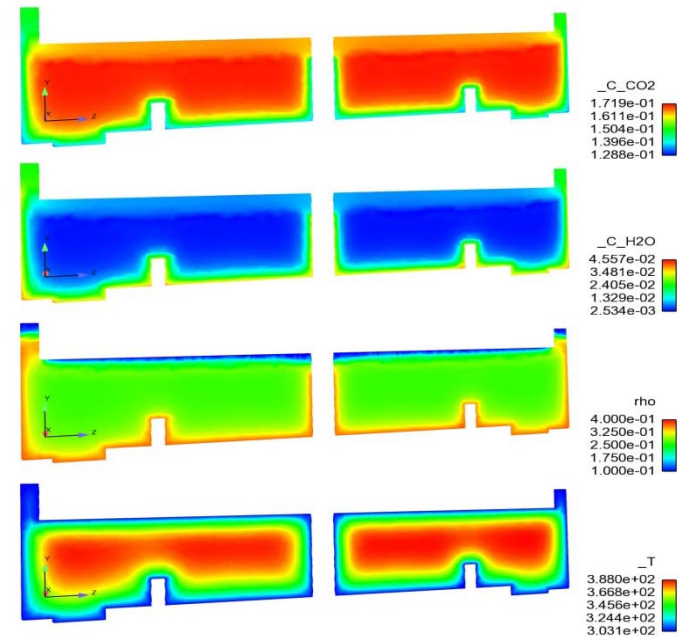
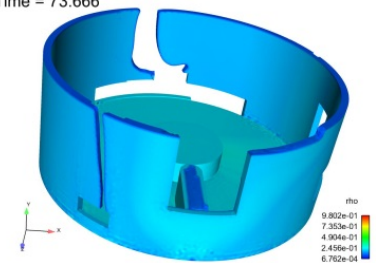
Time = 29.315



Time = 32.136



Time = 73.666



Conclusions and Future Work

Conclusions:

- Level set equations have been coupled to an ALE moving mesh algorithm to model fluid flow in a punch mold
- The dynamics of simple punch molds with idealized geometries have been investigated
- Compressible gas models are needed to be more predictive
- Coupled boundary conditions must be developed to improve performance of the punch and reduce mesh shearing
- To simulate more complex geometries, we may have to include solid-solid contact algorithms

Next Steps:

- Use CDFEM for fluid motion
- More realistic geometries
- Transfer initial conditions to foaming simulations