

# A Hybrid ALE-Level Set Approach to Modeling Foaming in Punch Molds

Rekha Rao, David Noble, Lisa Mondy, Victor Brunini, Christine Roberts,  
Scott Roberts

*Sandia National Laboratories  
Albuquerque, NM*

James Tinsley  
*Honeywell Kansas City Plant  
Kansas City, MO*

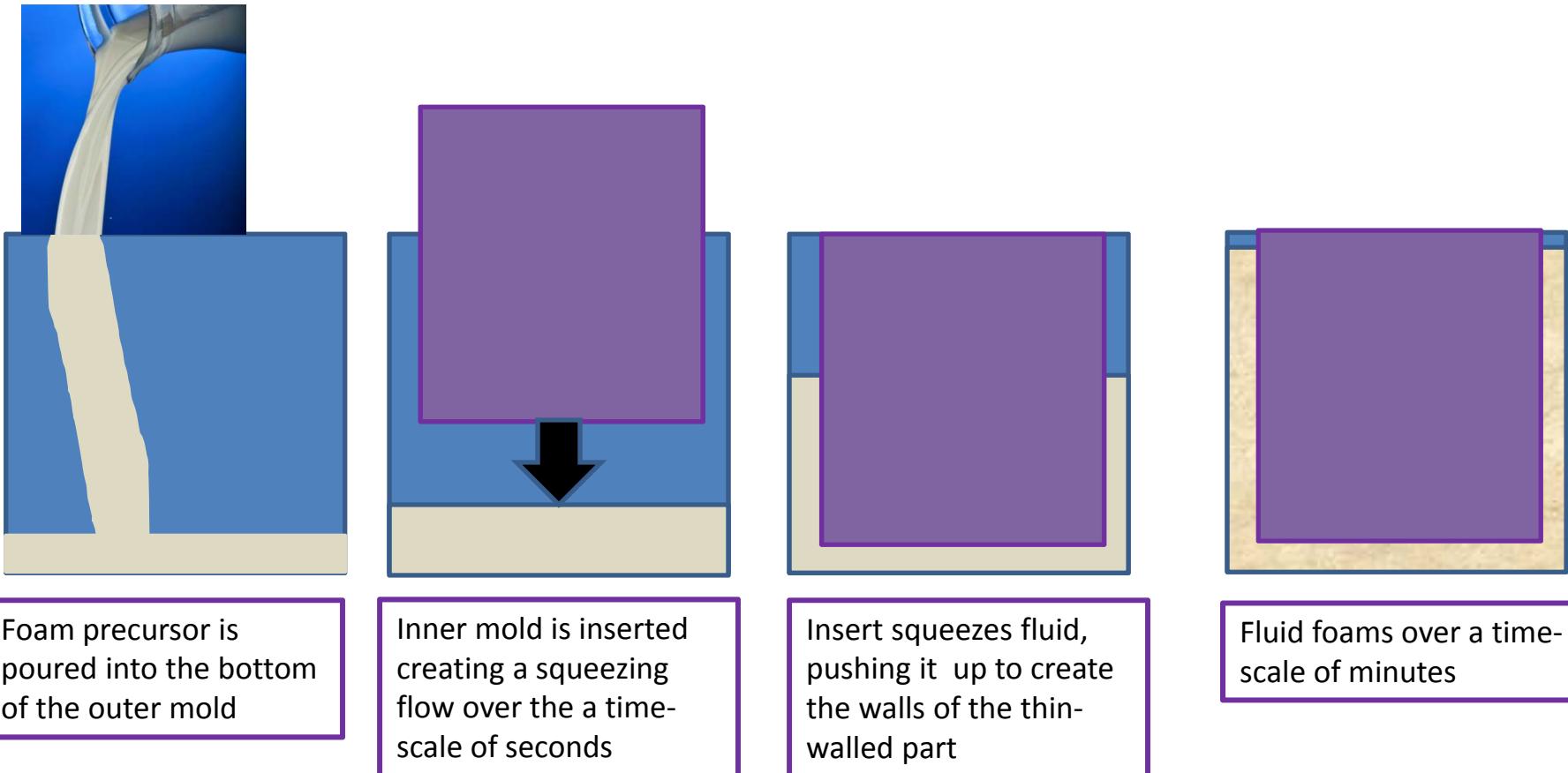
**13<sup>th</sup> US National Congress on Computational Mechanics**  
San Diego, California  
July 26 - 30, 2015

SAND2015-????C

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under contract DE-AC04-94AL85000.



# Motivation: What Is a Punch Mold?



## Approach:

- Decouple punching fluid mechanics from foam expansion
- Punch simulations use a Newtonian, incompressible fluid and give an initial conditions for foaming simulation
- For punch mold, couple a ALE moving mesh algorithm for the evolving geometry of the mesh insert to a level set for the fluid motion

# Numerical Solution Methods for Interfacial Motion

Tracking motion of interface between two distinct phases appears often:

Phase changes

Film growth

Fluid filling

## Interface tracking:

Explicit parameterization of location

Interface physics more accurate

Moving mesh

Limits to interface deformation

No topological changes

## Examples:

Spine methods ( *Scriven* )

ALE

## Embedded Interface Capturing:

Interface reconstructed from higher dimensional function

Fixed mesh

“Diffuse” interface physics

Interface deformation

theoretically unconstrained

## Examples:

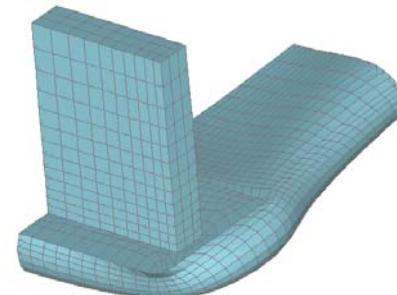
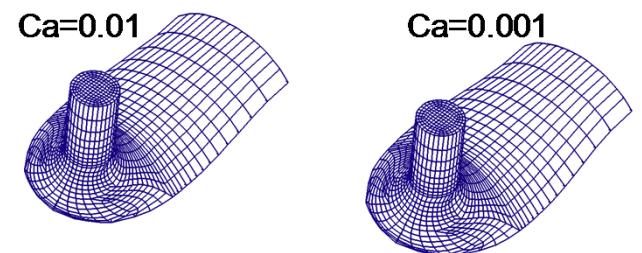
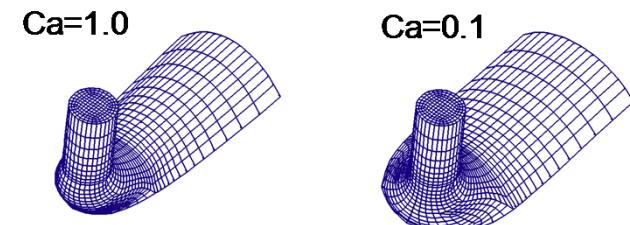
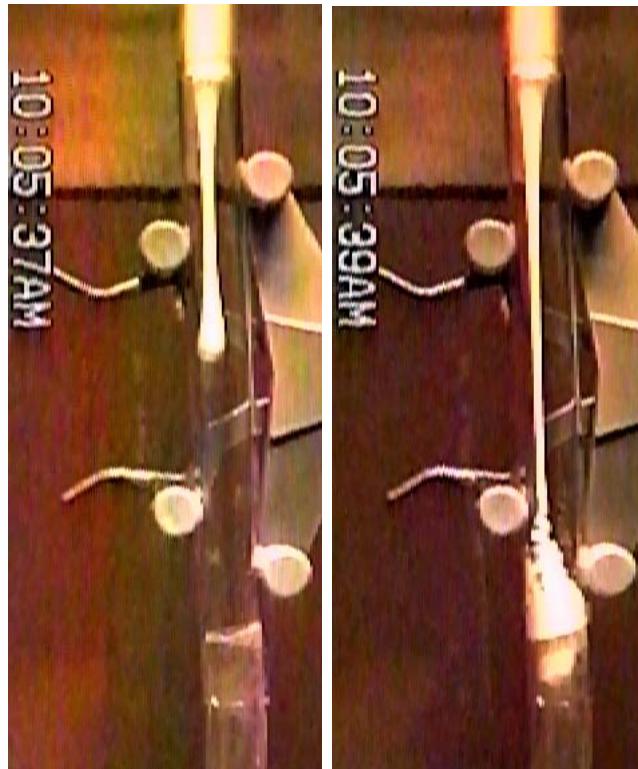
Volume-of-Fluid ( *Hirt* )

Level Sets ( *Sethian* )

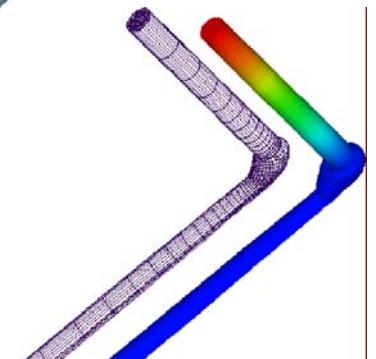
# Embedded Interface Methods Can Capture Topological Changes



Level set method has possibility of modeling “Dairy Queen” effect



Tom Baer, P&G



# Free Surface Flows: Coupling Fluid Flow to Pseudo-Solid Mesh Motion

- Technique for mapping mesh nodes in response to boundary deformation
- Displacement of nodes determined by solution of quasi-static problem: Neo-Hookean constitutive equation for pseudo-solid

$$\nabla \cdot \mathbf{T}_{mesh} = 0, \quad \mathbf{T}_{mesh} = f(\lambda_{ps}, \mu_{ps}; \nabla d_{mesh})$$

- Mesh node displacements are solved for simultaneously with other variables
- Deformation driven by boundary constraints:

Geometric

$$P(x, y, z) = 0$$

$$\vec{d} = \vec{D}_0$$

$$n_1 \cdot n_2 = \cos(\theta)$$

$$\begin{array}{l} \mathbf{u} = \mathbf{v} = \mathbf{0} \\ \mathbf{dy} = \mathbf{0} \end{array}$$

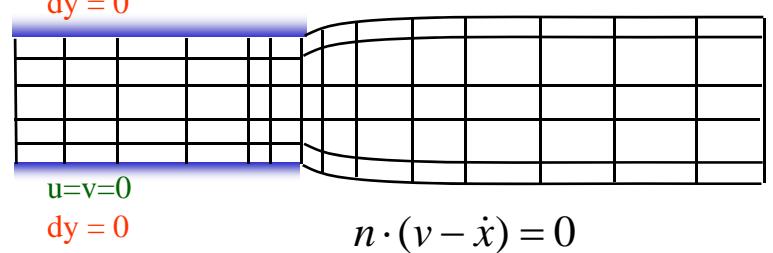
$$\begin{array}{l} p = 10, v = 0 \\ \mathbf{dx} = \mathbf{0} \end{array}$$

$$n \cdot (v - \dot{x}) = 0$$

$$n \cdot (v - \dot{x}) = 0$$

$$T = T_{melt}$$

$$\begin{array}{l} p = 0, v = 0 \\ \mathbf{dx} = \mathbf{0} \end{array}$$



Coupled

Arbitrary Lagrangian Eulerian (ALE) mesh motion: The mesh moves with the material in the normal direction at boundaries and arbitrarily, as a nonlinear elastic solid, elsewhere.

Sackinger, Schunk, and Rao, 1994; Cairncross et al, 2000; Baer et al, 2000; Notz et al, 2013

# Free Surface Flow: Level Set Method

Given fluid velocity field,  $u(x,y,z)$ , evolution on a fixed mesh is according to:

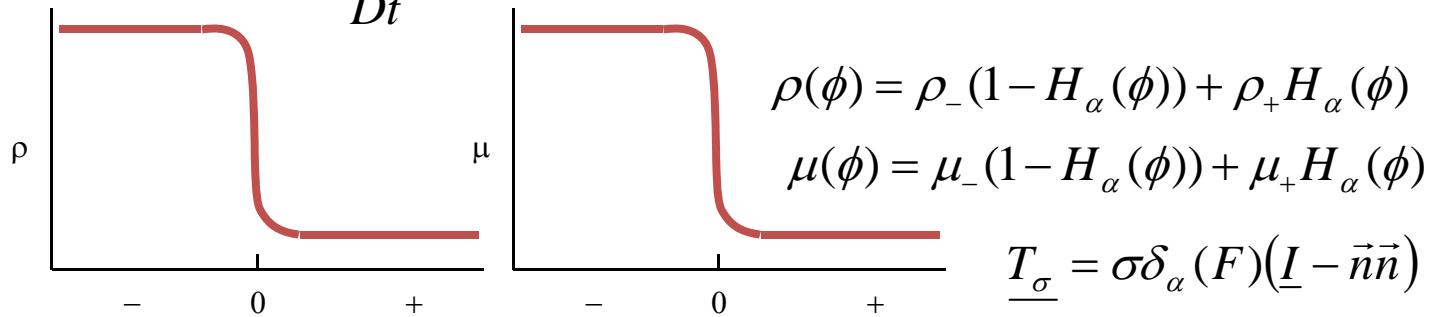
$$\frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = 0 \quad \vec{n} = \nabla \phi, \kappa = \nabla \cdot \nabla \phi$$

Purely hyperbolic equation ... fluid particles on  $\phi(x,y,z) = 0$  should stay on this contour indefinitely

- Does not preserve  $\phi(x,y,z)$  as a distance function
- Introduces renormalization step.

Fluid velocity evolves as one-phase fluid with properties that depend on  $\phi$

$$\rho(\phi) \frac{Du}{Dt} = -\nabla P + \nabla \cdot (\mu(\phi) \dot{\gamma}) + \rho(\phi) g + I.T., \quad \nabla \cdot u = 0$$



# Coupling Level Set Method and ALE Method

- The motion of the fluid,  $u(x,y,z)$ , is now with respect to the mesh, and the mesh velocity enters the advection term
- Segregated solve at each time step in three different matrix systems
- First solve mesh equations, then level set, and then momentum and continuity
- Method implemented in Sierra Mechanics Aria

## Mesh Motion

$$\nabla \cdot \mu_s (\nabla d + \nabla d^t) + \nabla (\lambda_s \nabla \cdot d) = 0$$

$$x_{new} = x_{old} + d, \quad \dot{x} = \dot{d}$$

## Level Set

$$\frac{\partial \phi}{\partial t} + (u - \dot{x}) \cdot \nabla \phi = 0$$

## Fluid Mechanics

$$\rho(\varphi) \left( \frac{\partial u}{\partial t} + (u - \dot{x}) \cdot \nabla u \right) = -\nabla P + \nabla \cdot (\mu(\varphi) \dot{\gamma}) + \rho(\varphi) g$$

$$\nabla \cdot u = 0$$

# Finite Element Implementation

- Approximate variables with trial function, e.g.

$$u \approx \sum_{i=1}^n u_i N_i \quad v \approx \sum_{i=1}^n v_i N_i \quad w \approx \sum_{i=1}^n w_i N_i \quad p \approx \sum_{i=1}^m p_i N_i'$$

- Substitute into equations of motion, weight residual with shape function for Galerkin implementation

$$\text{Weighted - Residual} = \int N_i R_i dV$$

- Gaussian quadrature
- Solve discretized system

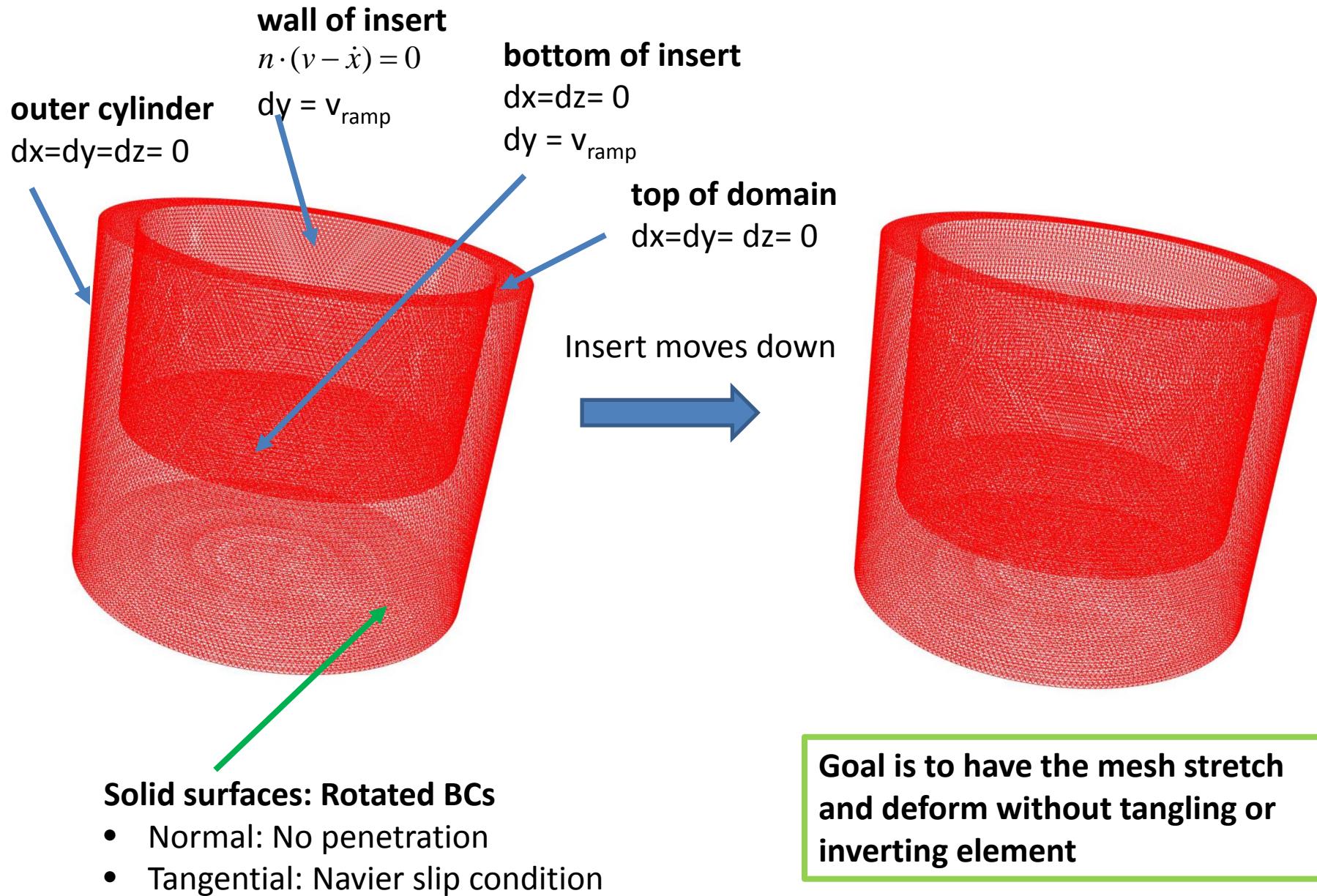
$$\underline{A}\underline{x} = \underline{b}$$

- Issues: Linear system solved with Krylov-Based iterative solvers => require stabilization Dohrman-Bochev Stabilization (2004)

$$R_i^c = \int_D \phi^i [\nabla \cdot u] dV + \sum_{Elem} \tau_{pspp} (\phi^i - \pi \phi^i)(p - \pi p) dV$$

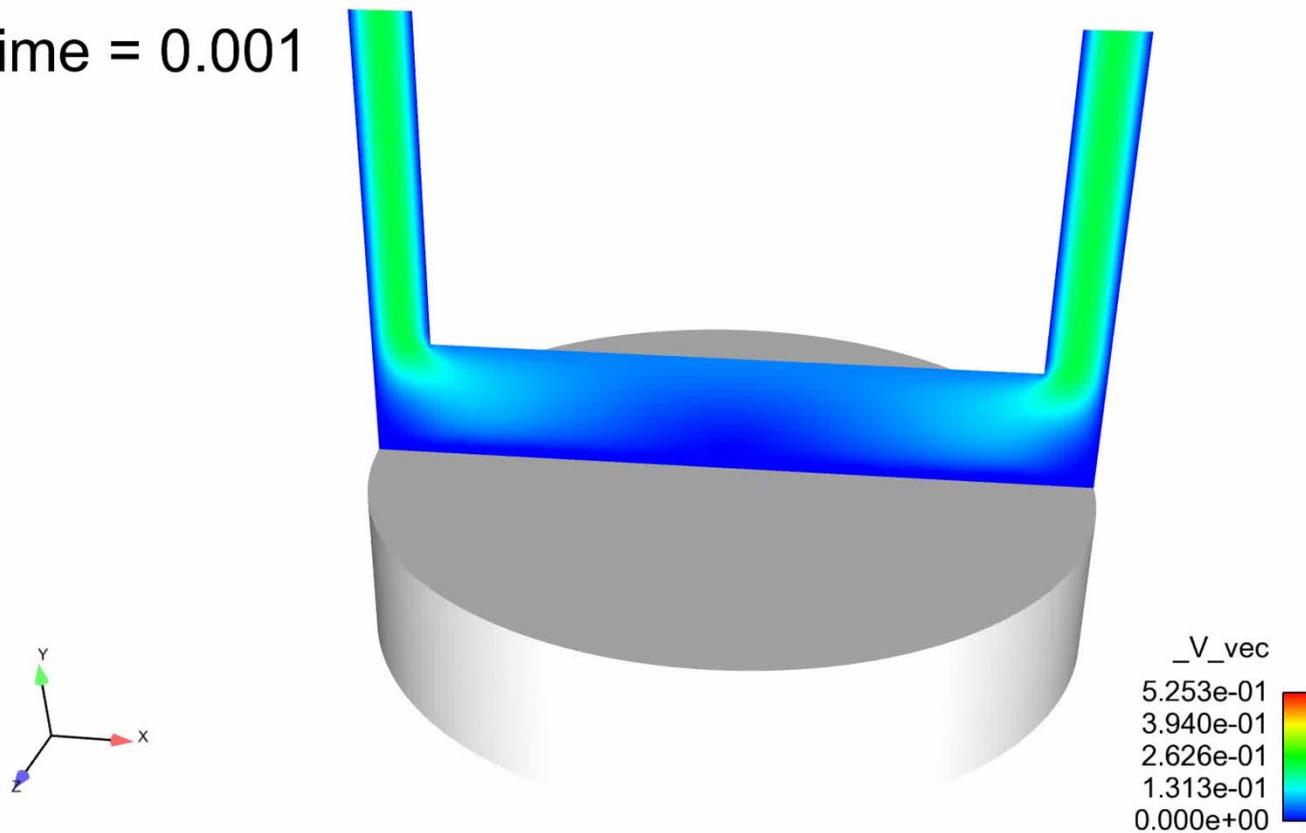
$$\pi p = \int_{V_e} p dV / \int_{V_e} dV$$

# Fluid and Mesh Boundary Conditions



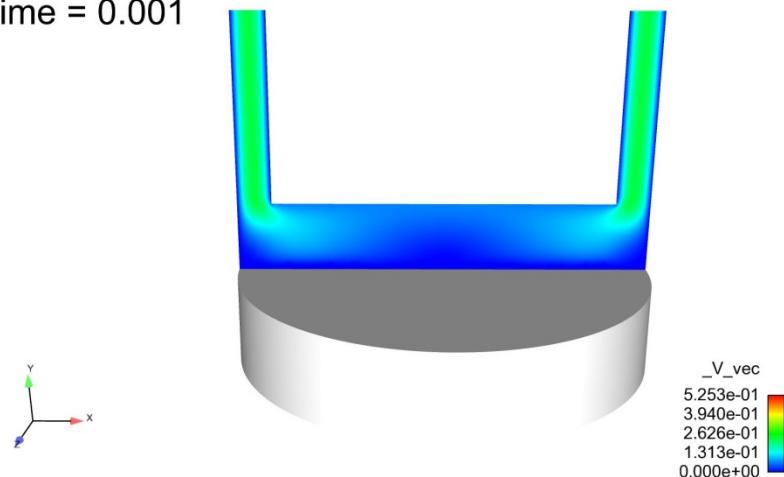
# Results

Time = 0.001

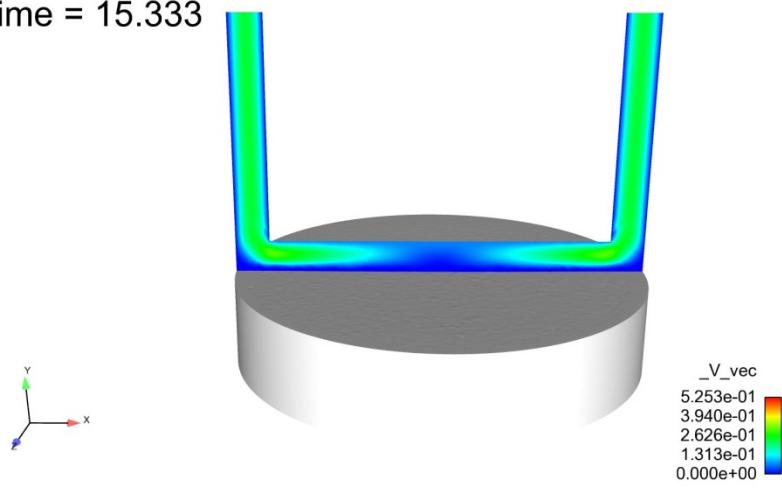


# Results

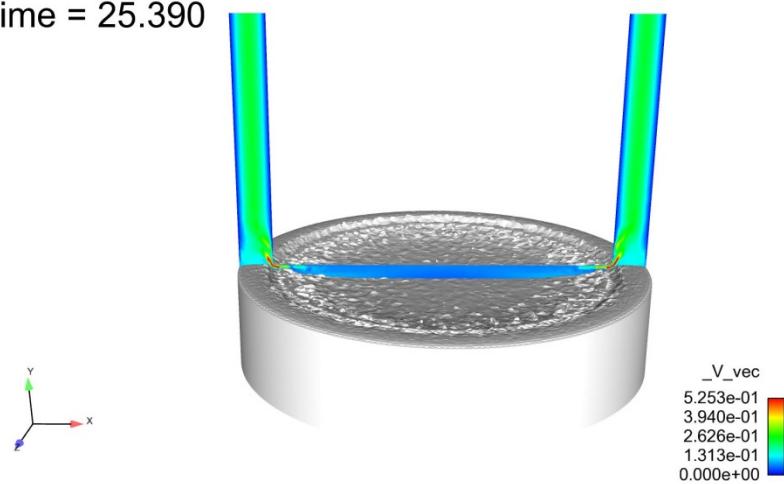
Time = 0.001



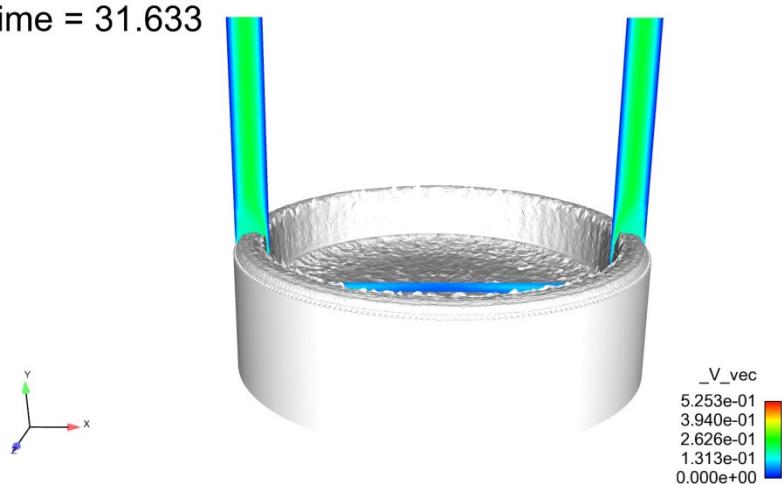
Time = 15.333



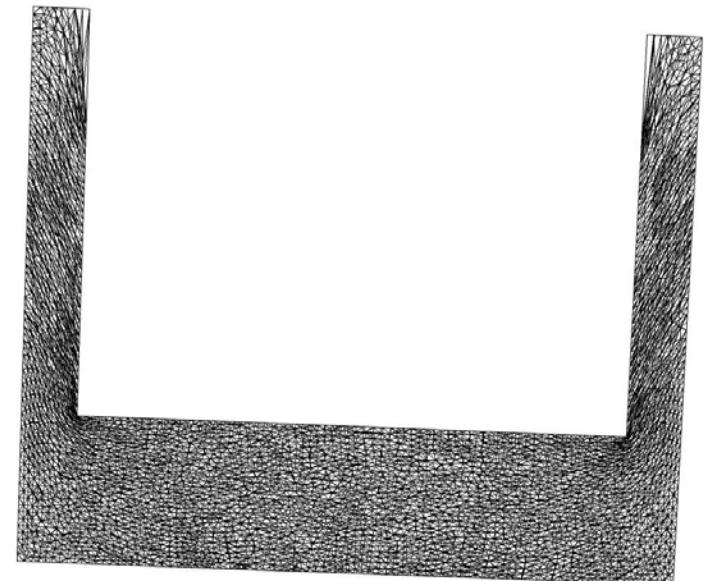
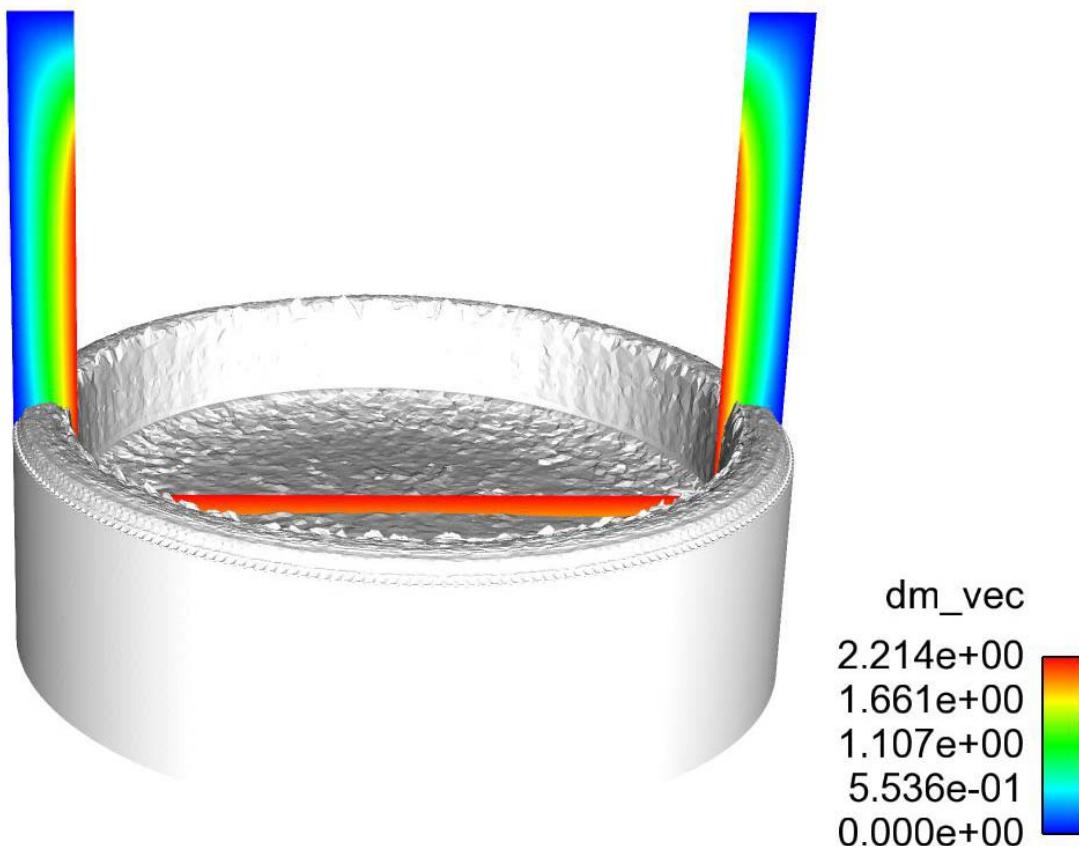
Time = 25.390



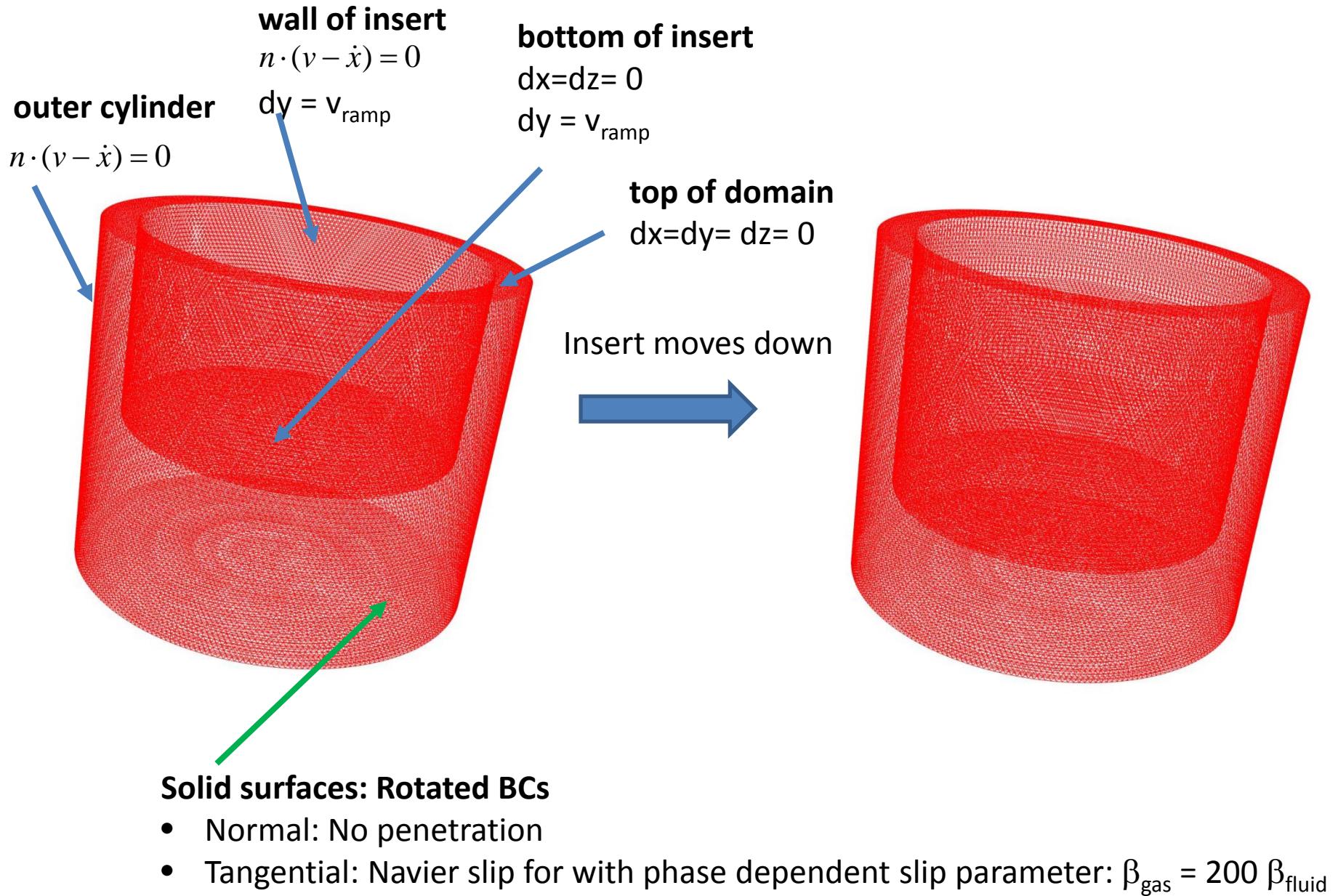
Time = 31.633



# Mesh Shears Over Time

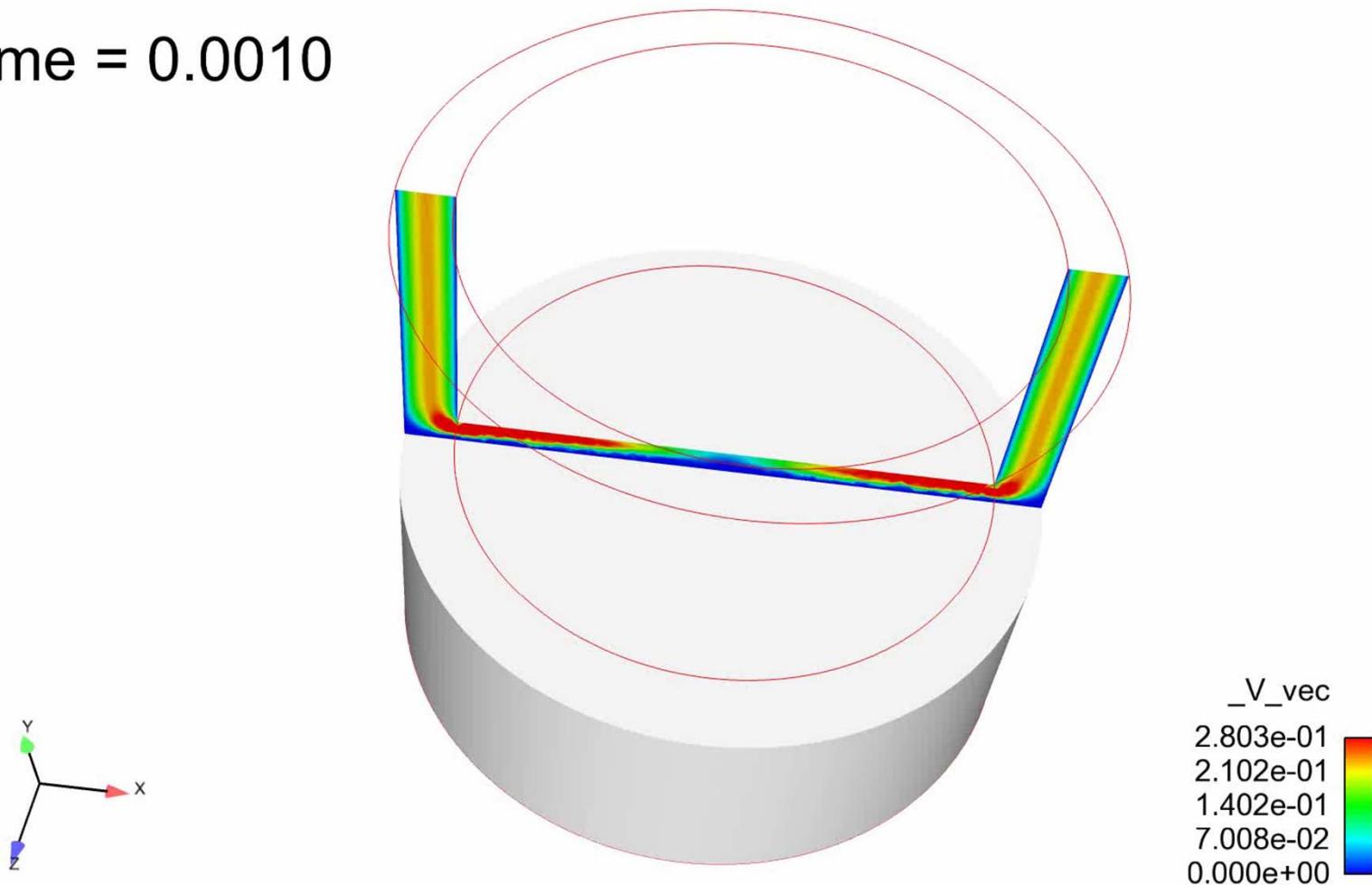


# Improved Fluid and Mesh Boundary Conditions

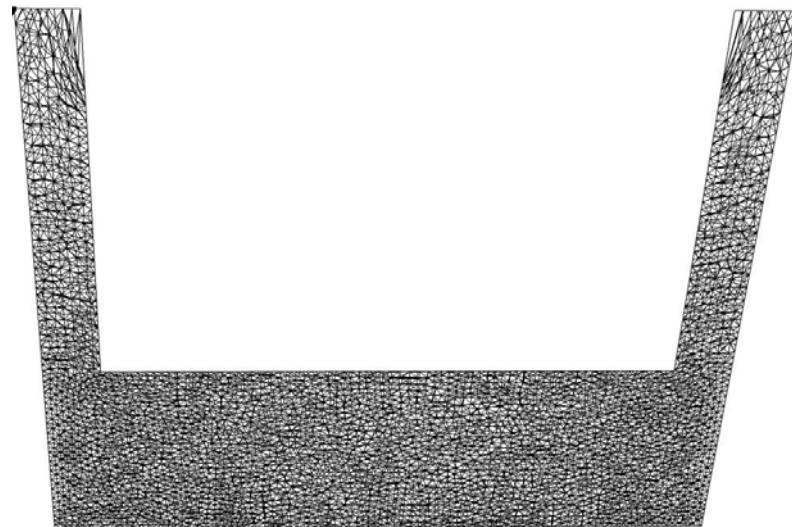
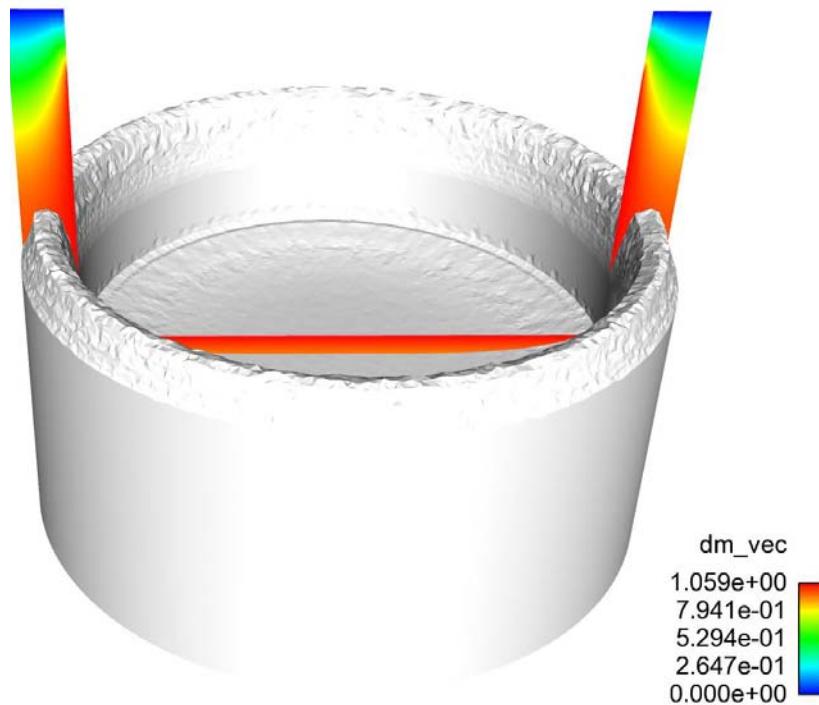


# Can Boundary Conditions Improve Results?

Time = 0.0010

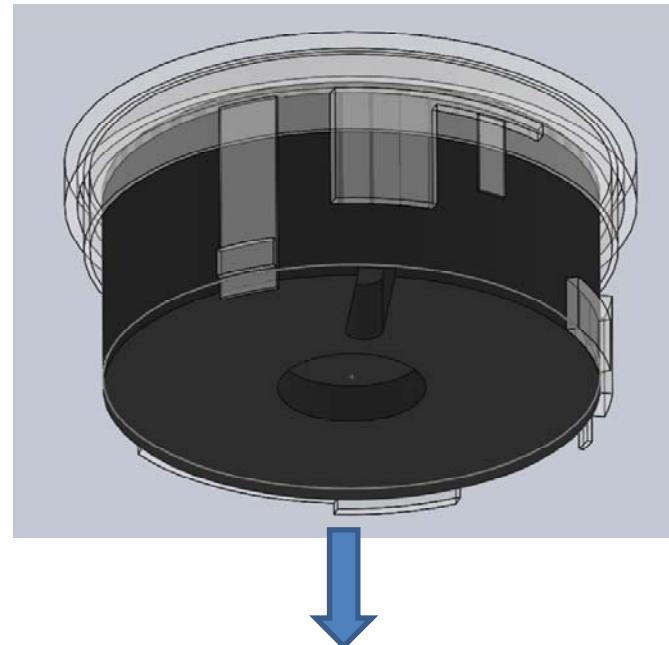
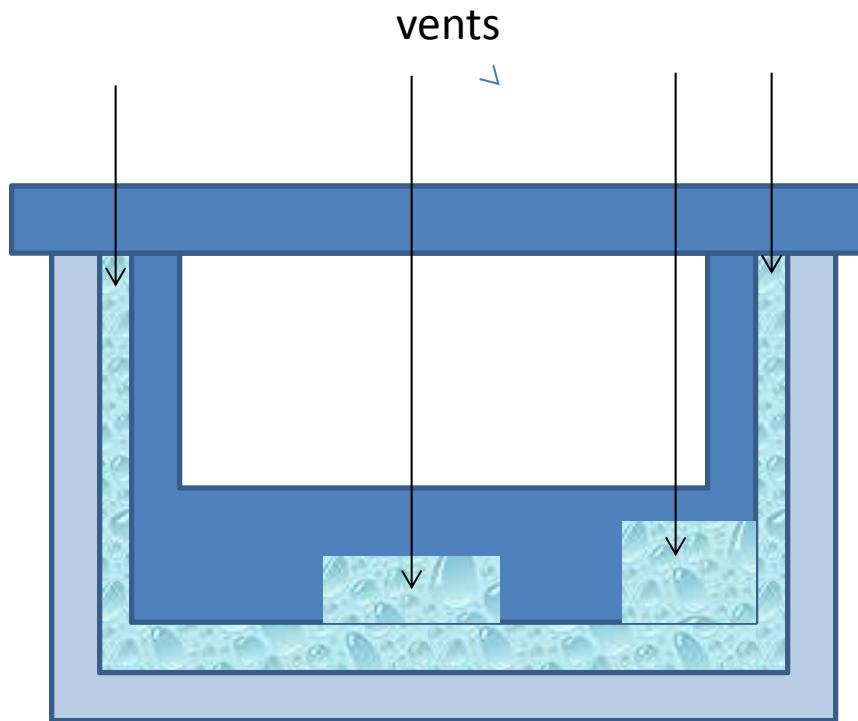


# Can Boundary Conditions Improve Results?



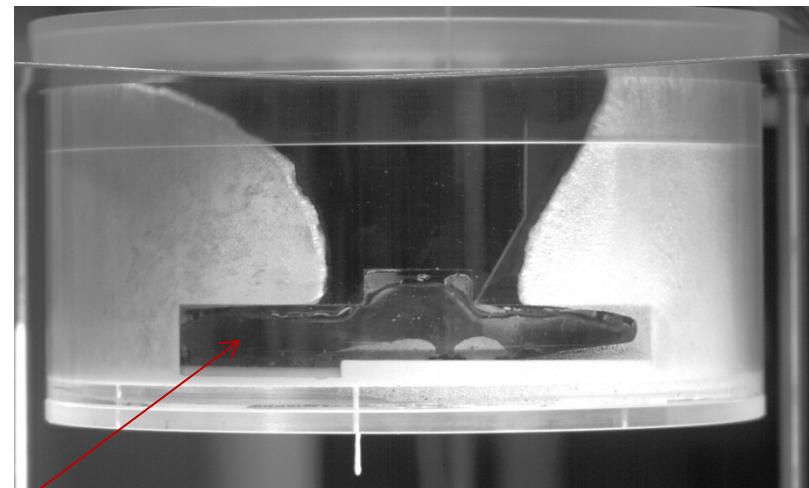
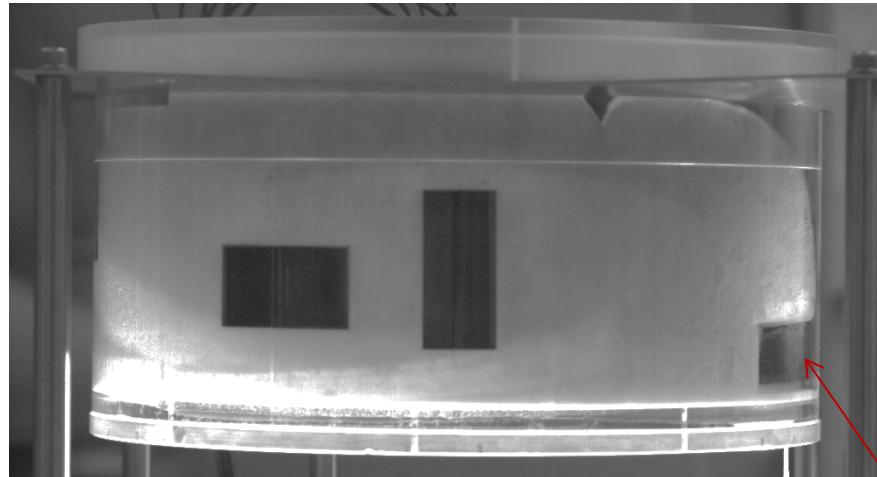
# Simplified Structural Support Mold Test 3

- Used 10pcf free rise structural PMDI foam, filled to produce a 13pcf part
- To speed up process and slow down foam reaction rates:
  - No preheats
  - Mixed 30 seconds instead of 1 minute
  - Pour all foam into one reservoir, the lid of the upside down part
- Temperature instrumented with four camera views



Push inside mold down into bowl  
that once was the lid

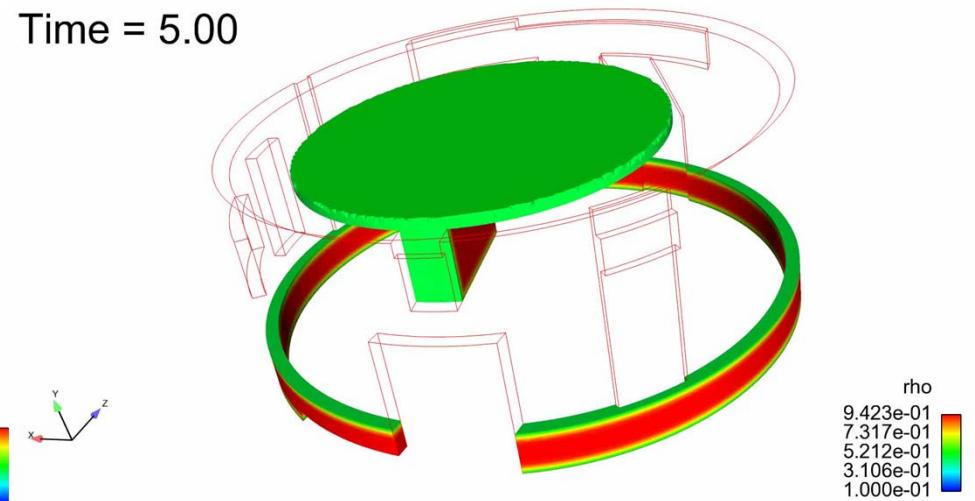
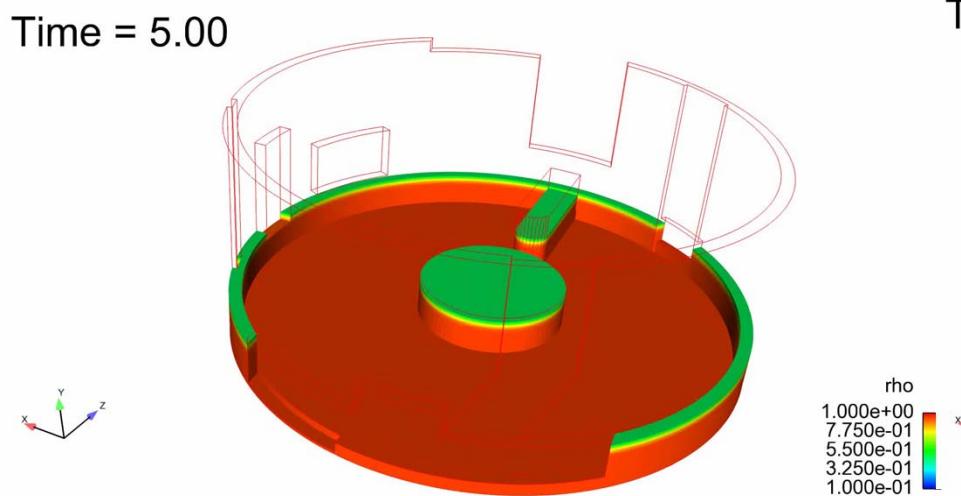
## Last Place to Fill on Top of Largest Feature



Largest feature

Short shot: less foam than encapsulation test 1, to see where last places to fill would occur. Reaction proceeded faster gelling foam before could finish rising.

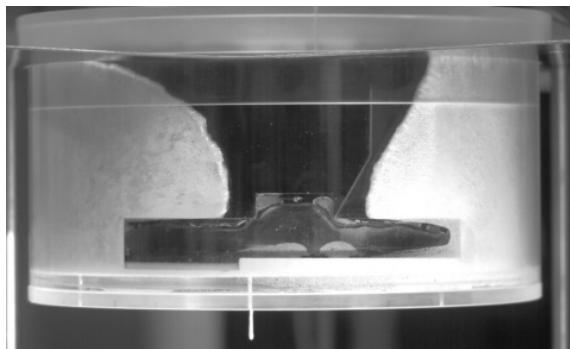
# If We Know the Initial Condition, Filling Models Can Predict Dynamics



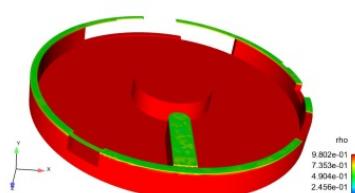
# Model Give More Physics than Just the Filling Locations

Models developed for foam filling and curing  
=> density/cure

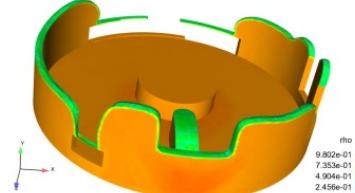
- The model allows us to look inside the mold
- New kinetics show water depletion and CO<sub>2</sub> variations
- Density variations are seen in the mold
- Foam exotherms significantly even and early times



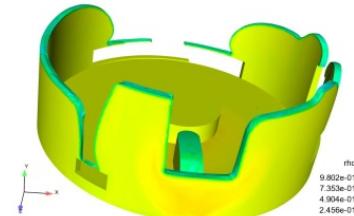
Time = 24.531



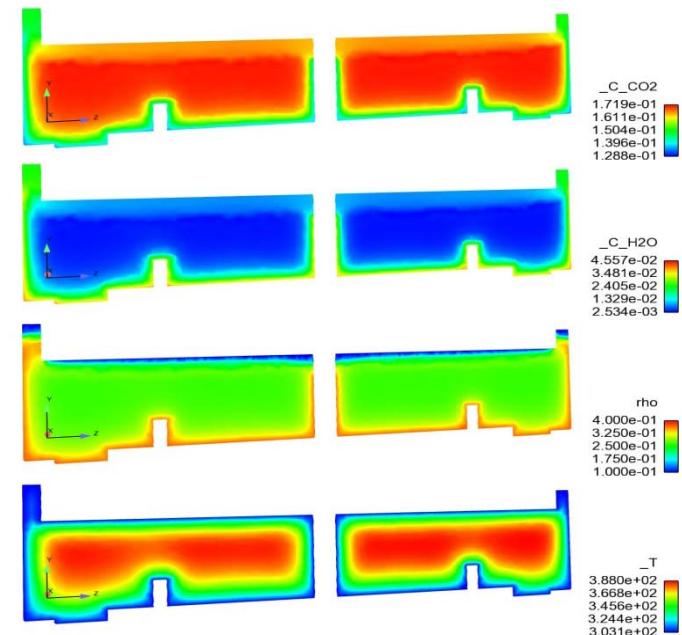
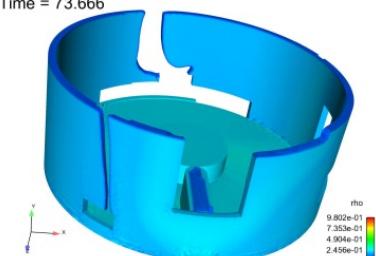
Time = 29.315



Time = 32.136



Time = 73.666



# Conclusions and Future Work

## Conclusions:

- Level set equations have been coupled to an ALE moving mesh algorithm to model fluid flow in a punch mold
- The dynamics of simple punch molds with idealized geometries have been investigated
- Compressible gas models are needed to be more predictive
- Coupled boundary conditions must be developed to improve performance of the punch and reduce mesh shearing
- To simulate more complex geometries, we may have to include solid-solid contact algorithms

## Next Steps:

- Use CDFEM for fluid motion
- More realistic geometries
- Transfer initial conditions to foaming simulations